Quantum Majority Vote

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Abstract

Majority vote is a basic method for amplifying correct outcomes that is widely used in computer science and beyond. While it can amplify the correctness of a quantum device with classical output, the analogous procedure for quantum output is not known. We introduce *quantum majority vote* as the following task: given a product state $|\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle$ where each qubit is in one of two orthogonal states $|\psi\rangle$ or $|\psi^\perp\rangle$, output the majority state. We show that an optimal algorithm for this problem achieves worst-case fidelity of $1/2 + \Theta(1/\sqrt{n})$. Under the promise that at least $2/3$ of the input qubits are in the majority state, the fidelity increases to $1 - \Theta(1/n)$ and approaches 1 as n increases.

We also consider the more general problem of computing any symmetric and equivariant Boolean function $f : \{0,1\}^n \to \{0,1\}$ in an unknown quantum basis, and show that a generalization of our quantum majority vote algorithm is optimal for this task. The optimal parameters for the generalized algorithm and its worst-case fidelity can be determined by a simple linear program of size $O(n)$. The time complexity of the algorithm is $O(n^4 \log n)$ where n is the number of input qubits.

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References

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