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Preference-based allocation of patients to nursing homes

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ABSTRACT

In many countries, the rapid aging of the population leads to an additional burden on already stretched long-term care systems. This often manifests itself in excessive waiting times for long-term care centers, and in abandonments (i.e., patients passing away while they are waiting). Interestingly, in practice, long waiting times are not caused by a lack of available total capacity in the system, but by systematic inefficiencies in the allocation of patients, each with their personal preferences and (in)flexibility, to geographically distributed care centers.

Motivated by this, we propose a new and easy-to-implement method for the optimal allocation of patients-in-need to nursing homes, balancing the trade-off between the waiting time performance and the individual patients' preferences and levels of flexibility. The optimal placement policy found by solving a Markov Decision Process demonstrates that for small instances, the mean optimality gap of the allocation model is equal to 1.3%. We validate a simulation model for a real-life use case of allocating somatic patients to nursing homes in the Amsterdam area. The results show that if more patient replacements are approved, the allocation model can reduce the abandonment fraction under the current policy from 32.2% to 7.4% and waiting times at the same time. Moreover, with the allocation model individual preferences can be served better, which thus provides a powerful means to face the increasing need for patient-centered and sustainable long-term care solutions.

1. Introduction

Between 2015 and 2050, the worldwide proportion of people aged 60+ will nearly double [1]. This imposes major challenges on long-term healthcare systems. Growing public expectations regarding the quality of long-term care, together with an increasing demand, lead to a growing need for housing facilities, skilled personnel, and medical equipment, which puts an additional strain on the system. At the same time, health expenditures need to be kept at an acceptable level. Therefore, serious concerns have been raised about the (financial) stability of the long-term care system [2]. Policy makers are saddled with the difficult task of designing the system in such a way that admission to appropriate care can be guaranteed.

Today, deficits in local nursing home capacity lead to overcrowded waiting lists [3,4], and the aging population causes longer waiting times in prospect. Lack of timely access to long-term care is a crucial issue due to the resulting confusion, distress, and anxiety of patients [5]. In addition, during the delay in admission, informal caregivers experience depressive symptoms and feelings of burden [6]. In addition, high costs are involved for the healthcare system because waiting patients occupy hospital beds [7], and patients may suffer injuries due to omission of proper care [8].

In addition to the costly solution of increasing facility capacity, waiting list management can be used to get patients to the right place more quickly. However, little research has been done on how long-term care waiting lists should be managed or organized [9,10]. Even from this small amount of research, almost all studies focus on improving available capacity (e.g., implementing the use of transitional care facilities [11]) and not on the organization of the waiting list itself. The Operational Research community is explicitly called upon to develop models for effective healthcare waiting lists [12]. Nevertheless, no mathematical models have yet been developed that can be used to study this highly socially relevant issue.

A core characteristic that distinguishes long-term care from other domains in terms of waiting list management is that patients have specific *preferences* for nursing homes. The possibility of including preferences in designing a waiting list policy is currently lacking in relevant research fields such as the queueing literature on routing and admission control. The contribution of this paper is two-fold. First, we propose a waiting list management method that incorporates routing preferences to parallel facilities. Second, we provide a solution to the currently existing waiting list problem in nursing home care. More specifically, our research objective is to design an allocation model

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for waiting list management policies that is (i) easy to implement and understandable for healthcare employees, (ii) scalable as the number of patients and nursing home beds can be large, and (iii) able to combine the goals of retaining preferences best and keeping waiting times at a small level.

2. Literature

Long-term care is provided for patients who have physical and/or mental deficits that prevent them from performing the usual daily tasks. The majority of long-term care is utilized by the elderly (65+), and therefore most long-term care facilities are designed for this subpopulation. Long-term care can be divided into three categories: (1) *informal care*, (2) *community care*, and (3) *institutional care* [13]. Informal care is provided by the patient's family, friends, and neighbors and consists mainly of basic activities such as cooking a meal or walking outside. It has been found that 92% of patients who live at home receive voluntary help [14]. Community care involves paid services targeted at patients who live at home and consists of a broad range of services from nursing care at home to day care at an external location. If home is not an appropriate living environment for the patient's condition, institutional care is required. Institutional facilities in which specialized care is provided by nurses are referred to as typical 'nursing homes'.

Proper access to nursing homes cannot always be guaranteed due to excessive waiting times. The European Commission reports high proportions of older people in need of institutional care who are currently on waiting lists in the EU [4]. For example, these fractions of older people on the waiting list are 16% in the Netherlands, 33% in Bulgaria, and 53% in Hungary. The problem exists in countries all over the world; see, for example, [15–17] for some reports in Australia, Canada, and South Africa. The lack of regional nursing home capacity can further exacerbate the problem. For example, in Copenhagen, the waiting time for a bed in a nursing home, which is not necessarily a preferred one, is over 3.5 years. Even in countries where waiting lists for nursing homes are not (yet) an issue, we observe regional and popularity issues with certain nursing homes. In the USA, only 29% of all residential care communities reported having a waiting list, but among those, the average waiting time for admission was around 5 months [18].

In the following, we address the literature regarding the management of waiting lists for nursing homes from different angles: waiting for nursing homes and waiting for healthcare in general. Thereafter, we elaborate on the relation of our paper with the literature on routing and conclude with our contribution.

2.1. Waiting-list management in long-term care

The long-term care waiting process starts with the selection of the right nursing home to apply for, based mainly on patient preferences, such as location [19] and cultural factors [20]. Specific preferences, such as cultural background, significantly influence waiting times [20, p. 1348]. If a patient's application to a nursing home is not immediately accepted, the waiting period begins. To better manage the waiting period, interventions such as transition care settings and adjustment of home care levels are explored. For example, Crotty et al. [11] found that off-site transitional care units can be utilized during the waiting period without adversely affecting patients' conditions. However, some patients may prefer not to use such units, suggesting a voluntary approach. Furthermore, research by Pedlar and Walker [21] highlights the preference for increased home care over nursing home admission by 90% of patients on waiting lists. This preference can significantly reduce nursing home demand and lead to substantial cost savings, given the higher expense of nursing home care compared to home care.

Limited research exists on the management of waiting lists themselves. In a meta-analysis by Chafe et al. [9], only two papers on this topic were identified. The first paper by Burkell et al. [22] examined the change from a first-come-first-served policy to one that prioritizes patients based on urgency. They observed significant variability in patient care needs, suggesting that a needs-based criterion would drastically alter priority. Similarly, in a study by Meiland et al. [23], priority was based on urgency, revealing that non-urgent patients experienced longer waits without deterioration of their condition. Moreover, their satisfaction levels remained consistent while waiting at home, indicating that prioritizing urgent patients had minimal negative consequences for nonurgent ones.

In [10], waiting-list management is discussed in a more general setting, including patients discharged from hospitals who need residential care. A Markov Decision Process was developed to determine when patients in hospitals should receive priority for nursing home placement over those waiting at home. However, lowering the threshold simply shifts the waiting time problem to patients at home. Furthermore, a simulation tool estimated that nursing home stays must decrease by 2–3 years to meet service level targets.

2.2. Waiting-list management in healthcare

The impact of priority settings is also studied in other health contexts. Bowers [24] aimed to simulate the waiting lists of the UK's National Health Service. By studying empirical waiting-time distributions, it was found that a FCFS policy did not correctly describe the behavior of the waiting lists. Motivated by this, a model was developed that includes priority parameters to differentiate between urgent and routine patients and to include specified target waits for those different groups of patients. Although the goodness-of-fit test revealed that the developed model could not adequately describe empirical data, the model can be used to predict waiting times when such alternative policies are implemented.

The priority setting can depend on more factors than just urgency. However, determining the right priority instruments is complex. In 1996, the International Society for Priorities in Health was founded to investigate health priorities for both theoretical and practical purposes [25]. Priority settings may even be more complicated when demand is heterogeneous, such as in the case of surgical procedures for which the expected outcome depends on the characteristics of the patient. There are even settings in which both demand and supply are heterogeneous, such as in the allocation of kidneys to patients where each kidney-patient combination determines the expected lifetime of the patient, as seen in Zenios et al. [26]. In those cases, a regular waiting list will not suffice, and an allocation model is needed.

2.3. Routing models

The issue of waiting-list management can also be examined from a queueing-theoretic perspective. There is now a substantial body of literature on routing customers to parallel queues. For our purposes, it is most intriguing to consider queues with both heterogeneous customers and heterogeneous server pools. Such systems have traditionally been studied in the call center domain [27,28], where the assignment of customers to server pools is known as skill-based routing (SBR). As indicated in [29], the analysis of SBR is typically intractable. Therefore, optimal routing decisions often rely on asymptotic approximations (see, e.g., [30–32]) and special cases, such as the V, inverted V, N, and X designs. For additional background and references, we refer the reader to the recent survey [29]. This survey also discusses the complexities of SBR models in healthcare applications, focusing on in-hospital patient flow from emergency departments to hospital wards. It should be noted that [29] also explores an MDP formulation for optimal routing. Due to the explosion of the state (and action) space, the authors only considered small-scale examples. For a recent study using approximate dynamic programming techniques for somewhat larger instances, see [33].

In computer server applications, the number of server pools N is generally considerably larger than in our nursing home setting. To avoid excessive communication overhead, power-of-d routing policies have been suggested. For such a policy, $1 \leq d(N) \leq N$ of the N server pools are randomly selected, and the customer is routed to the shortest of those queues. Even in the case of sampling two queues ($d(N) = 2$), waiting times are drastically reduced (see, e.g. [34]). In fact, [35], among others, demonstrate that for relatively small $d(N)$, asymptotically optimal behavior can be preserved. We note that these models fundamentally differ from our setting, as customers are routed to a single queue upon arrival, and there are no preferences (server pools are homogeneous). Nonetheless, the power-of-d results indicate the enormous potential to add some flexibility in the allocation of patients to nursing homes.

2.4. Our contribution

Long waiting lists for nursing homes are a significant societal problem, yet existing research often overlooks a crucial aspect: incorporating patient preferences into nursing home assignments. To fill this gap, we propose a method that integrates patient preferences into waiting-list management policies. Our aim is to develop a system that is easy for healthcare employees to implement and understand, that is scalable to accommodate varying numbers of patients and nursing homes, and that effectively balances patient preference retention with minimizing waiting times.

In addition to individual nursing home preferences, we also consider patients' preferences regarding willingness to wait. Drawing from the concept of utilities, as studied by Nomura et al. [36] in content delivery networks, we formalize this aspect. Using utilities as a measure of patient preferences, our model offers a practical and comprehensible approach for healthcare managers. This patient-centered allocation method not only aligns with the individuality of patients but also allows efficient bed allocation by leveraging explicit preferences.

3. Context and problem description

In this section, we define the scope of the problem, which involves the system that incorporates the placement of patients in nursing homes.

3.1. Background and terminology

For the sake of clarity, we elucidate the terms used throughout the paper.

Patients: We consider patients on a waiting list for nursing home placement, excluding high-emergency cases. All patients require the same type of care, allowing nursing home beds to be used interchangeably. Patients enter the system upon expressing readiness for placement, but remain at home while waiting.

Nursing home capacity: According to Ribbe [37], nursing homes are institutions that provide nursing, psycho-social, and personal care, as well as room and board. The capacity of a nursing home location is expressed in terms of the number of beds available at the location. The available number of beds may either be determined by the physical beds present at the nursing home or may depend on the availability of other resources such as personnel and services.

Preferences: In practice, certain nursing homes may be more appealing to patients due to factors such as proximity to their home, familiarity with the coordinating care organization, and specialization in specific subpopulations (e.g., LGBTQ+ patients or patients with the same ethnic background [20]). We assume that each patient has at least one preferred nursing home where they would ideally like to reside permanently. If a patient is placed in a nursing home other than one of their *preferred* choices, the placement may be temporary.

3.2. Problem outline

The conventional procedure involves patients enrolling in the waiting list(s) of their preferred nursing home(s) upon arrival. However, we propose an alternative that allocates patients to nursing homes to satisfy as many individual preferences as possible. Let $N = 0, 1, \ldots, n_{\max}$ denote the set of nursing homes, where we define ***nursing home 0*** as the location of the patient's home. We denote by b_n the number of beds available in the nursing home n, and let $b = \sum_{n \in N \setminus 0} b_n$ be the total number of beds. Moreover, let P be the set of patients who need a bed in a nursing home. Each patient $p \in P$ has a set of preferred nursing homes, $L_p \subseteq N \setminus 0$, in which they would like to reside permanently. If a patient is placed in a nursing home in the set $M_p := N \setminus L_p$, the placement is temporary and the patient still waits for a bed in a preferred nursing home.

Patients are assumed to arrive according to a Poisson process with rate λ. The service times in nursing homes are exponentially distributed with mean $1/\mu$. These assumptions are validated in Section 6 using data for our case study. Finally, the time until abandonment from the queue is exponentially distributed with mean $1/\theta$.

The patient placement process unfolds over time and space, as shown in Fig. 1. Initially, the patient joins the waiting list for their preferred nursing home while remaining at home. As time elapses and the patient's condition worsens, a preference for temporary placement in a nursing home may arise. If a bed is available in a temporary facility, the patient can be relocated there while still awaiting placement at their preferred home. Upon availability, the patient moves to the preferred nursing home, staying until they leave the system, often due to death or transfer. Each patient's journey is unique, with factors like disease affecting progression.

Objective and performance measures

The goal is to efficiently allocate patients to nursing homes while incorporating their preferences. This involves optimizing the aggregate utility of patients based on preferences for nursing homes and willingness-to-wait. We evaluated the trade-off between adhering to preferences and efficiency using the key metrics detailed in Table 1. Each patient p is assigned a preference, expressed in terms of g_{pn}, for each nursing home n, the preferred nursing homes being those with the highest values. Negative values ($g_{pn} < 0$) indicate a patient's unwillingness to be placed in a specific nursing home. We also account for patients' willingness to wait, formalizing this trade-off through utility functions; see Section 4.1 for a further discussion with an emphasis on the waiting aspect.

We allow each patient to have unique utility structures, offering customization appreciated by healthcare professionals. However, in practice, employing a limited set of patient profiles can streamline operations. These profiles capture common utility structures, simplifying the acquisition of patient preferences. Our model presents individual utility structures, while practical examples in Section 5 illustrate the use of patient profiles.

4. Allocation model and method

In this section, we first formalize the preference profiles and willingness-to-wait using utility functions in Section 4.1. These utility functions are used in our allocation model for assigning patients to nursing homes in Section 4.2. In Section 4.3, we evaluate the allocation model using simulation. We discuss some benchmark policies in Section 4.4.

4.1. Utility functions

We established the utility functions in collaboration with and with approval of experts in the elderly care domain. Specifically, the

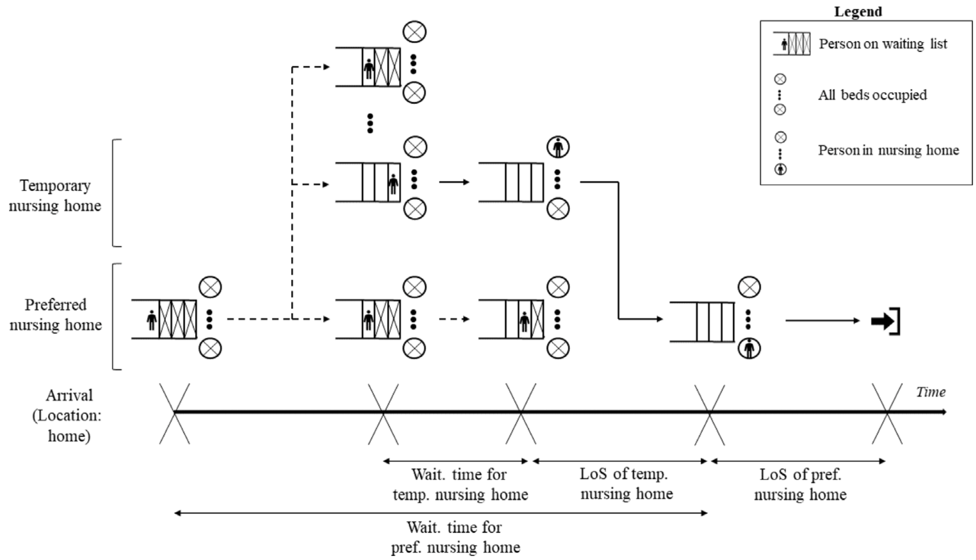


Fig. 1. Example patient journey.

Table 1
Performance measures.

Performance measure	Description
Waiting time	Time between arrival to waiting list and first and/or final placement
Abandonments	Fraction of patients that abandon the system
% preferred NH	Fraction of patients that end up in their preferred nursing home
Replacements	Average number of replacements per patient

application of utility functions is an intuitive and attractive way for health professionals to reflect the impact of preferences for nursing homes and waiting on the choices of individual patients. In this setting, the utility for each nursing home $n \in N$ represents the 'level of happiness' to reside there.

The utility depends on the initial preferences of the patient p, the current location l_p and the elapsed waiting time w_p, $\forall\ p \in P$. We denote the utility function for patient p toward nursing home n as $u_{pn}(l_p, w_p), \forall\ p \in P, n \in N$. In particular, it should comprise the following four elements (with corresponding notation):

(i) Each patient $p \in P$ can indicate initial preferences for each nursing home $n \in N$. These preferences are denoted by g_{pn}.

(ii) The patient's $p \in P$ willingness to be placed in a temporary nursing home $n \in M_p \setminus 0$ increases with the waiting time w_p. This is described by the function $v^U_{pn}(w_p)$, which is strictly increasing in w_p.

(iii) The patient's $p \in P$ willingness to be placed in preferred nursing home $n \in L_p$ increases with the waiting time w_p. This is described by the function $v^F_{pn}(w_p)$, which is strictly increasing in w_p.

(iv) Patient's $p \in P$ relocation between two temporary nursing homes results in a utility loss. This utility loss is modeled by the replacement penalty K.

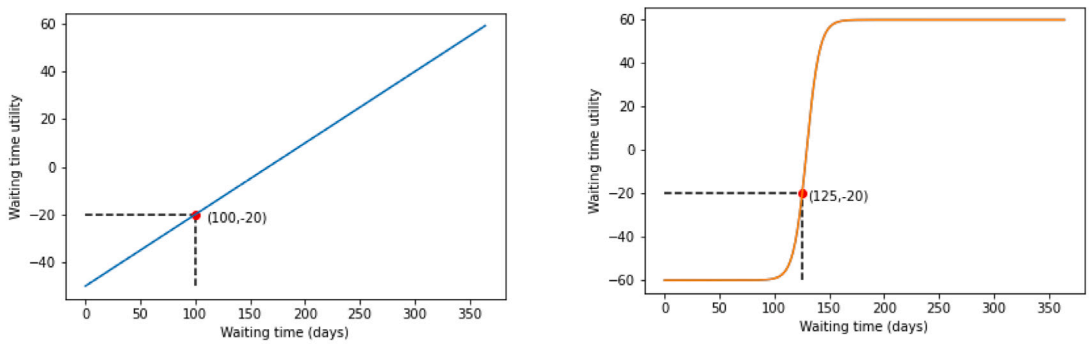
The preferences (i) remain constant over time, but if a patient's preferences change during the waiting period due to new information about nursing home features, they can update their preferences. The impact of waiting time is captured by elements (ii) and (iii). Specifically, $v^U_{pn}(w_p)$ reflects the flexibility of a patient in waiting time. Patients who prefer a quicker placement in a temporary nursing home will see their utility increase more rapidly with waiting, ensuring that they are placed sooner. However, this faster placement comes at the expense of their opportunity for rapid placement in a preferred nursing home, as reflected in a less steep increase in $v^F_{pn}(w_p)$ for preferred nursing homes. The waiting time utility function $v^U_{pn}(w_p)$ is added to the utility of temporary nursing homes if a patient resides at home. Thus, a patient may initially have a negative utility for a temporary nursing home, but as the waiting time increases, their utility may become positive, leading them to prefer the temporary nursing home over staying at home.

Regarding element (iv), patient relocations between temporary nursing homes are only considered if the increase in utility outweighs a replacement penalty K. Patients typically only relocate between temporary nursing homes if the utility gain exceeds this penalty. However, if a patient's preference is a preferred nursing home, the replacement penalty is not included.

Waiting-time utility

The waiting-time utility functions can be chosen so that the preferences of individual patients are best represented. In the following, we discuss two different forms of utility functions. First, a simple choice for waiting-time utility is a linear function of the elapsed waiting time w_p of the form $v^U_{pn}(w_p) = \alpha_p w_p - \beta_p$, with $\alpha_p > 0,\ \beta_p \geq 0,\ p \in P$. In this case, the increase in waiting-time utility always grows at the same speed. However, according to elderly care experts, the increase in waiting-time utility is not constant. They describe the willingness to be placed in a nursing home as follows: Initially, the waiting time utility increases slowly, whereas after some 'breaking point' (e.g. an abrupt loss in functionality) the utility increases drastically, after which the utility increases slowly again. For that reason, a second more natural choice for the waiting time utility function is to use a sigmoidal function, thus of the form $v^U_{pn}(w_p) = \frac{\alpha_p}{1+e^{-(\epsilon_p w_p - \gamma_p)}} - \beta_p\ \forall\ p \in P$.

The parameter values of the utility functions determine the speed with which a person is willing to be placed in a nursing home. Patient $p \in P$ wants to be placed in nursing home $n \in N$ if $u_{pn}(l_p, w_p) > 0$, thus in the linear case if $g_{pn} + \alpha_p w_p - \beta_p > 0$. If $\beta_p > 0$, only if w_p is large



(a) Linear: $v^U_{pn}(w_p) = 0.3w_p - 50$
(b) Sigmoidal: $v^U_{pn}(w_p) = \frac{120}{1+e^{-(0.17w_p-22)}} - 60$

Fig. 2. Example waiting time utility functions.

enough we have $g_{pn}+\alpha_p w_p-\beta_p > 0$. An example of this for both waiting time utility functions is provided in Fig. 2.[1]

Now, the utility function of patient $p \in P$ at location $l_p \in M_p$ towards location $n \in N$ with elapsed waiting time w_p is defined as follows:

$$u_{pn}(l_p, w_p) = \begin{cases} 0 & \text{if } n := l_p \\ g_{pn} + v^U_{pn}(w_p) & \text{if } l_p = 0, n \in M_p \setminus 0 \\ g_{pn} - g_{pl_p} + v^F_{pn}(w_p) & \text{if } n \in L_p \\ g_{pn} - g_{pl_p} - K & \text{if } l_p \neq 0, n \in M_p \setminus l_p \\ -\infty & \text{if } n = 0, l_p \in M_p \setminus 0. \end{cases} \tag{4.1}$$

In words, the utility is 0 if patient $p \in P$ stays at the same location. The utility is $g_{pn} + v^U_{pn}(w_p)$ if the patient lives at home and goes to a temporary nursing home. The utility is $g_{pn} - g_{pl_p} + v^F_{pn}(w_p)$ if the patient goes to a preferred nursing home. Furthermore, the utility is $g_{pn} - g_{pl_p} - K$ if a patient resides in a temporary nursing home and goes to another temporary nursing home. Finally, if the patient no longer lives at home, i.e., $l_p \in M_p \setminus 0$, the utility for home is set to $-\infty$, which prohibits a return placement. Adding the waiting-time utilities in this way ensures that if there are two patients $p', p'' \in P$ with the same utility functions and where $l_{p'} = l_{p''}$, but $w_{p'} > w_{p''}$, then patient p' is given priority over patient p'' for nursing home n. This fosters fairness in the system.

4.2. Allocation model

We propose a Binary Integer Programming (BIP) model to allocate patients to nursing homes. The allocation model is based on a static setting (*snap shot*) of the dynamic process. The model solves the optimal allocation for the static setting so that the utility sum of all patients is maximized. For our allocation model, we are not interested in the patients who are already residing in a preferred nursing home, since these patients do not have to be re-allocated to another nursing home. Thus, we have that for each patient $p \in P$, the current location is $l_p \in M_p$. Note that the set M_p also includes the home location 0.

Similarly, we define the capacity c_n of nursing home $n \in N$ as the number of beds not assigned to patients $p \in P$ for whom $n \in L_p$. This is based on the reasoning that these patients already have a bed allocated. The capacity c_n thus includes beds that could be occupied by patients who temporarily reside there. Utility functions ensure that temporary patients are primarily allocated to the nursing home where they already reside, allowing them to stay in the same bed.

[1] In Fig. 2, we assume that $g_{pn} = 20$ for patient $p \in P$ and nursing home $n \in N$. Then, only if $v^U_{pn}(w_p) > -20$ we have $u(l_p, w_p) > 0$, after which the patient prefers to be placed in nursing home $n \in N$ over staying at home. In the linear case this holds if $w_p > 100$ and in the sigmoidal case if $w_p > 125$.

For the allocation problem, we define the decision variable x_{pn}, which equals 1 if patient $p \in P$ is placed at location $n \in N$, and 0 else. An overview of the notation is provided in Table 2.

Now, the allocation problem can be formulated as follows:

$$\max \sum_{p\in P}\sum_{n\in N} u_{pn}(l_p, w_p)x_{pn} \tag{4.2}$$

$$\text{s.t.} \sum_{p\in P} x_{pn} \le c_n \qquad \forall n \in N \tag{4.3}$$

$$\sum_{n\in N} x_{pn} = 1 \qquad \forall p \in P \tag{4.4}$$

$$x_{pn} \in \{0,1\} \qquad \forall p \in P, n \in N. \tag{4.5}$$

The objective of the allocation problem (4.2) is to maximize the total utility of all patients $p \in P$. This must be done under the constraints that (4.3) no more patients can be allocated to a nursing home than the capacity allows and that (4.4) each patient can only be allocated to one location. Constraints (4.5) are binary constraints.

Since the constraint matrix is totally unimodular and the elements on the right-hand side of the constraints are integral, the solution of the linear problem is integral. Therefore, the allocation model can be solved using a linear solver, which makes it fast and scalable. Furthermore, the (binary) allocation problem is a special case of the Generalized Assignment Problem, in which the capacity needed for each job is equal to 1 [38].

Moreover, our waiting-time utilities structure not only caters to patient preferences but also promotes efficiency. If a patient faces prolonged waiting times for a specific nursing home, their utility for alternative options may increase, causing placement in another facility. This design ensures optimal bed utilization, balancing patient preferences with efficient resource allocation.

4.3. Simulation model

In this section, we describe how the allocation model performs in a dynamic setting and can be evaluated using simulation. First, in line with the current procedure, we assume that patients can only be admitted to a nursing home at fixed regular moments (e.g. every morning). We define Δ as the time between two consecutive allocation moments. Then, the (deterministic) sequence of allocation moments can be described as $t_n = t_{n-1}+\Delta$, for $n = 1, 2, \ldots$. For example, if patients can enter once a day, Δ equals one day. The set of all allocation moments is denoted by $T = \{t_n\}_{n=1}^{\infty}$.

The time interval between the decision moments, Δ, determines the speed with which entry takes place. If $\Delta \to 0$, then entering a nursing home can be done instantaneously, such that the nursing home n behaves as an M/M/b_n+M queue. If Δ is chosen to be rather large, a bed might remain empty for some time, resulting in inefficiency in capacity use. On the other hand, a larger Δ implies that during a time

Table 2
Notation allocation model.

Sets	
P	Patients
N	Nursing homes, with $n = 0$ the home location
$L_p \subseteq N \setminus 0$	Preferred nursing homes of patient $p \in P$
M_p	Non-preferred nursing homes, i.e. $N \setminus L_p$
Parameters	
$u_{pn}(l_p, w_p)$	The utility of patient $p \in P$ at location l_p towards location $n \in N$, after waiting time w_p
l_p	The current location of patient $p \in P$, where $l_p \in M_p$
g_{pn}	The fixed utility of patient $p \in P$ for location $n \in N$
$v^U_{pn}(w_p)$	The waiting time utility of patient $p \in P$ until placed in nursing home after waiting time w_p
$v^F_{pn}(w_p)$	The waiting time utility of patient $p \in P$ until placed in permanent nursing home after waiting time w_p
K	The replacement penalty
b_n	The capacity of nursing home $n \in N$, where $b_0 = \infty$
c_n	The remaining capacity of nursing home $n \in N$, i.e. the number of beds not occupied by patients $p \in P$ for which $n \in L_p$
Decision variables	
x_{pn}	Binary variable to indicate if patient $p \in P$ is placed at location $n \in N$

interval more arrivals and departures have taken place, which increases the allocation possibilities.

More specifically, the model above with entering possibilities can be related to a queueing model with discharges at inspection instances [39]. In case of a single nursing home n and infinite patience (i.e., the M/M/b_n variant), the stability condition is

$$b_n \left(1 - e^{-\mu\Delta}\right) > \lambda\Delta,$$

where the term on the left-hand side corresponds to the maximum number of departures per Δ time units, which follows a Binomial(b_n, $(1 - e^{-\mu\Delta})$) distribution. Intuitively, $1 - e^{-\mu\Delta}$ can be interpreted as the effective service rate. Observe that $1-e^{-\mu\Delta} \approx \mu\Delta$ for $\mu\Delta$ small, such that the capacity lost due to entering possibilities is small.

Now, in order to evaluate the allocation model in the dynamic setting, we implemented a simulation model, where the allocation model of Section 4.2 is executed based on a rolling horizon, i.e., at all instants t_n after updating arrivals, departures, and utilities. The procedure consists, after the initialization, of two steps: (1) incorporating the dynamic features of the simulation, and (2) running the allocation model. The two-step procedure is iteratively run until a stopping criterion is met. We refer to Appendix A for a more detailed description of the simulation procedure.

4.4. Benchmark policies

As mentioned, the aim of the allocation model is to have the ‘best of both worlds’, i.e., short waiting times, few abandonments, and placement in a preferred nursing home. Clearly, there are extreme policies that are best for one of such performance measures. Such policies provide insight into the best value that can be achieved, i.e., serve as a lower or upper bound. Ideally, the allocation model yields a performance close to those bounds.

Shared queue. This refers to the situation in which there is a single waiting list for all nursing homes (and preferences are completely neglected). The system is similar to the Erlang C or A model, with the capacity equal to the total number of beds of all nursing homes. This corresponds to the most efficient system design. Hence, the corresponding mean waiting times and fraction of abandonments provide lower bounds for the actual performance.

Separate queues. This corresponds to the other extreme, namely dedicated waiting lists for each individual nursing home. The queue of a single nursing home is similar to the Erlang C or A model, with capacity equal to the number of beds of the corresponding nursing home. Clearly, the number of preferred placements is now maximized (provides an upper bound), at the cost of waiting times and abandonments.

Optimal policy. An optimal policy can be formulated using a Markov Decision Process (MDP) that considers both direct and future costs in terms of preferred placements, abandonments, and waiting. For any reasonably sized instance, solving such an MDP is computationally prohibitive due to the curse of dimensionality. The goal of the optimal policy is to verify the optimality gap of the allocation model for very small instances. The precise MDP formulation and the size of the state space can be found in Appendix B.

5. Experiments for small setting

In this section, we present numerical results from applying the allocation model (AM) to a small-scale setting. The aim is to gain insights into the model’s performance and behavior under various parameter settings. Since our model facilitates in serving individual needs, we want to show the benefits of the allocation model for individual patients. For that purpose, selecting the same utility function for all patients will not suffice. Therefore, we created two characteristic groups of patients with similar utilities. The two utility groups are “Fast placement” (FP) and “Preferred placement” (PP). FP patients prefer quick placement in a (temporary) nursing home, while PP patients opt to wait for availability at their preferred nursing home. We denote the FP group as elements of the set $C^{FP} \subseteq C$ and the PP group as elements and consequently $\subseteq C$.

5.1. Instance specification

The small setting contains four nursing homes with 20 beds each. First, we elucidate the patients’ utilities and then the dynamic parameters.

For all patients $p \in P$ and nursing homes $n \in N \setminus 0$, we set the fixed utility to $g_{pn} = 30$. The FP and PP utility groups can be identified by their unique waiting-time utilities. For the FP group, the waiting time utilities equal $v^U_{pn}(w_p) = 0.1w_p + 100$ for $p \in P^{FP}$. This implies that in combination with $g_{pn} > 0$ for $p \in P, n \in N \setminus 0$, if a bed is available in a nursing home, a patient of the FP group will always be placed there. For the FP group, this results in a fast placement in a nursing home. For the PP group, the waiting time utilities are $v^U_{pn}(w_p) = 0.1w_p - 500$ for $p \in P^{PP}$. In this case, patients have to wait extremely long before the utility for temporary nursing homes becomes positive, which means that patients only receive a placement in their preferred nursing home. The probabilities of arrival of a patient from the FP and PP groups are indicated by p^{FP} and p^{PP}, respectively. Thus, the arrival rates for the groups are λp^{FP} and λp^{PP}.

The waiting time utilities for the preferred nursing homes are set to differ between the two groups. Namely, $v^F_{pn}(w_p) = 0.1w_p$ for $p \in P^{FP}$, and $v^F_{pn}(w_p) = 0.15w_p$ for $p \in P^{PP}$. This implies that the utilities for the preferred nursing homes of PP increase slightly faster than those of FP. In this way, at some moments, a PP group patient might surpass the waiting FP patients, although the waiting time is shorter. The utilities

Table 3
Parameter values small setting.

Parameter	Value	Parameter	Value
$\lvert N \rvert$	4	K	1000
$\lvert L_p \rvert$	$1\ \forall\ p \in P$	$v^U_{pn}(w_p)$	$0.1w_p + 100\ \forall\ p \in P^{FP}$
θ^{-1}	$\{\infty, 730\}$	$v^U_{pn}(w_p)$	$0.1w_p - 500\ \forall\ p \in P^{PP}$
μ^{-1}	1095	$v^F_{pn}(w_p)$	$0.1w_p\ \forall\ p \in P^{FP}$
M	1000	$v^F_{pn}(w_p)$	$\{0.25w_p, 0.15w_p\}\ \forall\ p \in P^{PP}$
c_n	Initial value: $20\ \forall\ n \in N$	p^{FP}	0.5
Δ	1 day	p^{PP}	0.5
g_{pn}	$30\ \ \forall\ p \in P, n \in N$		

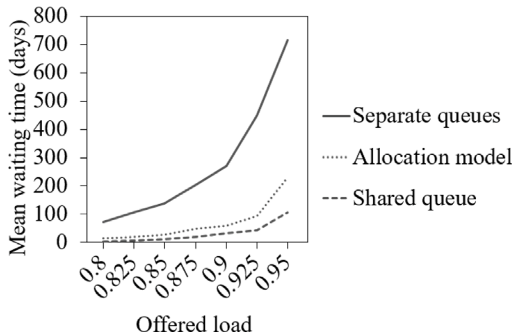


Fig. 3. Comparison allocation model to extremes ($\theta = 0$).

are chosen in this way to place PP patients faster in their preferred nursing home. Finally, the replacement penalty K is set to 1000, which prohibits replacements to be performed between temporary nursing homes.

Dynamic parameters are set with realistic values, where available. The mean length-of-stay in nursing homes is approximately two years, reflecting real-world data [40]. Estimating the abandonment rate at home, θ, is challenging due to data scarcity. The abandonment rate at home is assumed to be higher than the service rate in nursing homes, given care limitations and the potential for patients to seek alternatives outside the region. Thus, θ^{-1} is set to two years. In the absence of abandonments, $\theta = 0$. This adjustment prompts a modification in the waiting-time utility function, specifically for patients who prioritize placement in preferred nursing homes. Here, $v^F_{pn}(w_p) = 0.25w_p\ \forall\ p \in P^{PP}$, accommodating the altered dynamics in waiting times characteristic of abandonment-free scenarios.

We define the offered load per server as $\rho = \frac{\lambda}{c\mu}$. In case there are no abandonments ($\theta = 0$), we need at least that $\rho < 1$ to attain stability of the system. However, the assumption only applies to parts of our small-scale setting. For the allocation model, we allow $\rho \geq 1$ as long as $\theta > 0$. In our experiments, when studying the impact of the offered load, we will only vary the arrival rate λ and keep the service rate μ fixed. An overview of the parameter settings is given in Table 3.

5.2. Results for the small setting

First, we investigate the situation without abandonments, i.e., $\theta = 0$ and $v^F_{pn}(w_p) = 0.25w_p\ \forall\ p \in P^{PP}$. The performance of the allocation model can be compared with respect to the two extreme policies *separate* and *shared* queues (see Section 4.4), which is shown in Fig. 3. It can be seen that the mean waiting time of the allocation model is much shorter than the mean waiting time of the separate queues, whereas it is only slightly longer compared to the shared queue (i.e., the lower bound). This observation is in line with the principle that "a little flexibility goes a long way"; see, e.g. [28] for SBR examples in which some flexibility in resource pooling suffices to obtain most of the efficiency gain.

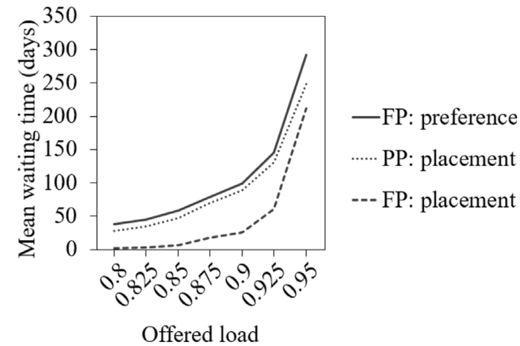


Fig. 4. Comparison patient groups within allocation model ($\theta = 0$).

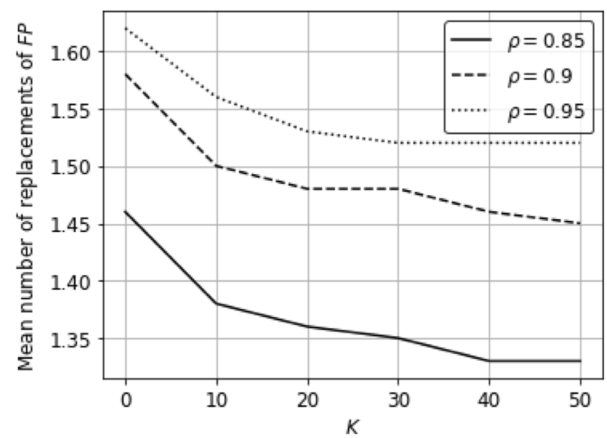


Fig. 5. Effect of increasing K on the number of replacements of FP.

To get more insight into the performance measures of the two groups of patients, we split the waiting times of the allocation model into groups, as shown in Fig. 4. We see that for all different loads, the mean waiting time until the placement of the FP group is the lowest. In addition, the time until a patient of this group is placed in their preferred nursing home is longer than for the PP group. In that respect, the needs of both groups are indeed served: FP is placed quickly, whereas PP is placed slightly faster in their preferred nursing home.

The next experiment is conducted on the effect of the replacement penalty K. Note that in our parameter setting, K only influences the replacement potential of the FP group, as the PP group is not temporarily placed. In addition, for this experiment, we diversify our g_{pn}-values into $g_{pn} \in \{10, 20, 30, 40\}$, where each value is selected with probability 0.25. The results of the different replacement penalties are shown in Fig. 5. Clearly, an increase in K-penalty is observed to lead to fewer replacements of the FP group. In addition, we see that the number of replacements between two temporary nursing homes increases as the system is more congested. This seems in line with the fact that congestion leads to longer waiting times and consequently to more potential benefits of temporary replacements.

The results of the simulation model, including abandonments, are detailed in Table 4.[2] Across various load levels, the allocation model exhibits slightly higher abandonment rates compared to the shared queue but significantly lower rates than separate queues, indicating efficient capacity utilization. The mean occupancy levels for AM are

[2] The values in Table 4 are the results of the simulation model that incorporates uncertainty. Since the model ran for a sufficient duration, the uncertainty around the results is negligible and therefore omitted.

Table 4
Results small setting.

Offered load	Policy	% abandonments	% died at temp. NH	% died at pref. NH	MWT till placement (d)	MWT till preferred (d)	Mean nr. of replacements	Mean occupancy
0.9	Shared queue	1.5%	0.0%	98.5%	10.3	10.3	0.98	0.89
	AM: tot	2.8%	2.5%	94.6%	18.4	44.2	1.14	0.87
	AM: FP	0.6%	5.1%	94.3%	4.3	55.5	1.35	–
	AM: PP	5.1%	0.0%	94.9%	33.1	33.1	0.95	–
	Separate queues	5.8%	0.0%	94.2%	38.4	38.4	0.94	0.85
1	Shared queue	5.2%	0.0%	94.8%	37.7	37.7	0.95	0.95
	AM: tot	6.3%	3.5%	90.2%	41.2	78.3	1.18	0.94
	AM: FP	2.5%	7.0%	90.5%	17.4	89.6	1.47	–
	AM: PP	10.0%	0.0%	90.0%	67.0	67.0	0.90	–
	Separate queues	9.7%	0.0%	90.3%	65.6	65.6	0.90	0.90
1.1	Shared queue	10.2%	0.0%	89.8%	75.8	75.8	0.90	0.98
	AM: tot	11.5%	3.3%	85.1%	76.3	115.4	1.14	0.97
	AM: FP	6.7%	6.7%	86.6%	46.4	121.0	1.44	–
	AM: PP	16.3%	0.0%	83.7%	109.7	109.7	0.84	–
	Separate queues	14.6%	0.0%	85.4%	101.6	101.6	0.85	0.94

Table 5
Parameter values for randomly generated instances.

Parameter		Value from distribution
μ		Unif(0.01, 0.03)
$\lambda = p_\lambda \cdot \mu \cdot \sum_{n \in N\setminus\{0\}} b_n$		With $p_\lambda \sim$Unif(0.5, 0.99)
g_{in}	$i \in \{FP_1, FP_2\}, n \in M_i$	Unif(20, 40)
g_{in}	$n \in L_i$	Unif(50, 70)
K_i	$i \in \{FP_1, FP_2\}, n \in N \setminus \{0\}$	Unif(1, 10)
w_i	$i \in \{FP_1, FP_2\}$	Unif(0.5, 1.5)
w_i	$i \in \{PP_1, PP_2\}$	Unif(1.5, 3)

only marginally lower than those for the shared queue. The efficiency is also apparent from the mean waiting times until placement, which are only slightly higher for AM compared to the shared queue. In addition, the waiting times for the preferred AM placement are only slightly longer than those of separate queues. AM is the only model using temporary nursing home placements. However, the fraction of patients who end up in their preferred nursing home is similar to both separate and shared queue strategies. For the shared queue policy, patients are treated as having no preferences (all nursing homes are preferred), providing a clear upper bound. Naturally, the shared queue policy does not facilitate patient preferences.

Based on these results, the allocation model is found to have 'the best of both worlds': both short waiting times are obtained next to preferences being retained well (both are relatively close to their lower and upper bound, respectively). However, these achieved gains are at the cost of the number of replacements, which increases by more than 20% for all offered loads. Hence, the allocation model yields promising results only if more patient replacements are allowed.

5.3. Comparison to optimal solution

To compare the allocation model with the optimal solution, we developed an MDP that gives us the optimal policy for small instances. More information on the MDP setup can be found in Appendix B. We run an experiment with 50 randomly generated small instances, where we have two nursing homes with two beds each. We choose the parameters as much as possible in line with previous experiments. For example, we choose μ such that the maximum total departure rate is equal to that of the small-scale example, i.e., we let $\mu = \frac{80}{4} \cdot \frac{1}{1095} = 0.018$, where the first ratio is the number of beds in the small-scale example divided by the number of beds in the current experiment. To create diversity in the numerical experiments, we add randomization and let $\mu \sim$ Unif(0.01, 0.03). All parameter values are realizations of the values provided in Table 5.

To align the allocation model with the optimal solution, we use the same classes, g_{in} values, and similar waiting utility, that is, $v^U_{in}(w) = v^F_{in}(w) = w_i w$, $\forall\ i \in I, n \in N$ for the generated w_i in the instances. This ensures that the waiting costs for the MDP correspond to the increase in waiting utility in the allocation model. The resulting long-term average rewards for the allocation model are determined through simulation, using the parameter values described in Table 5, to ensure consistency with the optimal solution found by the MDP. The optimality gap between the long-term average reward of the allocation model (denoted g^{AM}) and the optimal solution (denoted g^*) can now be defined as $\frac{g^{AM}-g^*}{g^*}$.

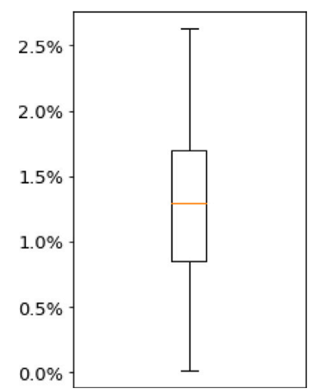


Fig. 6. Optimality gap in %.

For the 50 instances, we observe a mean optimality gap of 1.6%, which is rather small. Fig. 6 illustrates the distribution of this gap. Furthermore, Fig. 7 shows the percentages of patients who died in preferred nursing homes, while Fig. 8 presents the waiting time distributions. Although the allocation model slightly lags behind in "accurate placement" (with an average of 92.7% compared to 94.2% for the MDP solution), it compensates with shorter waiting times. The mean waiting time for the allocation model is marginally lower at 17.6 days, attributed to its immediate placement policy once a suitable bed is available.

We conducted 50 instances of a slightly larger MDP, maintaining the parameter values from Table 5 but with 3 beds per nursing home instead of 2. The mean optimality gap for this setting was 1.0%, suggesting a potential decrease in the gap as the problem scale increases (from 1.6% to 1.0%). These additional results are detailed in Appendix C.

Overall, the allocation model shows close to optimal performance with a mean optimality gap of 1.3%. Several factors contribute to its excellence. First, the ability to relocate patients post-placement allows for rectification of undesirable placements, which benefits static or

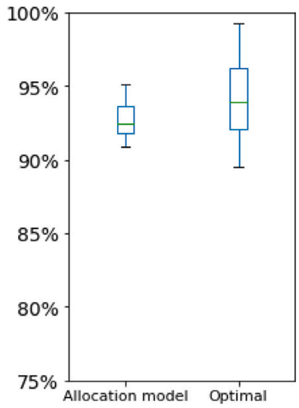


Fig. 7. % died in pref. nursing home.

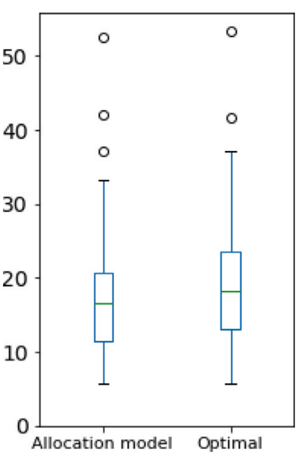


Fig. 8. Waiting time (d).

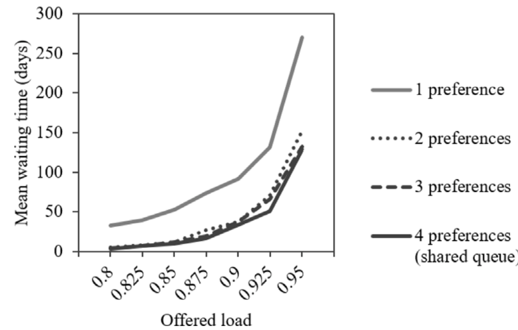


Fig. 9. Mean waiting time AM multiple preferences ($\theta = 0$).

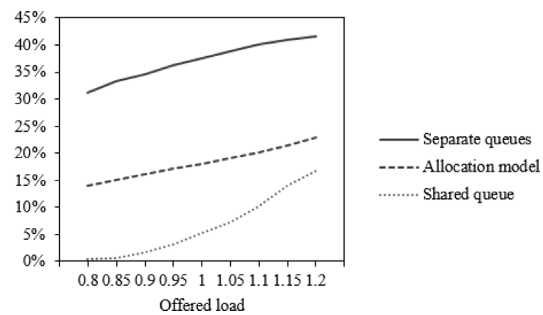


Fig. 10. Percentage abandonments scenario popular nursing homes.

greedy policies (cf. [41]). Second, the structure of waiting-time utilities exhibits similarities to threshold policies, known for their optimality in dynamic systems [42,43]. Third, uniformity in length of stay and abandonment rate between patient types creates a more homogeneous system. Lastly, it is worth noting that static policies have been proven optimal in various multi-class queue scenarios, such as the well-known $c\mu$ or $c\mu/\theta$ rule [44].

5.4. Effect of multiple preferred nursing homes

In the small setting, the number of nursing homes preferred by all patients is set at one. However, patients may be amenable to selecting multiple nursing homes as their preferred options. For that reason, we show the effect of choosing multiple nursing homes as their preferred ones. The results for the small setting without abandonments are presented in Fig. 9.

In Fig. 9 it becomes clear that if patients select two preference homes, the waiting times for various offered loads are nearly as short as the (minimal) waiting times of the shared queue. In this setting, increasing the number of preferred nursing homes to two is a very effective intervention, which is in line with the principle that adding a little flexibility provides most of the benefits of complete flexibility [28,45].

5.5. Effect of popular nursing homes

Although the preceding results are obtained for nursing homes with equal demand, in practice some nursing homes are more popular than others. Therefore, we define a scenario in which demand is not equally distributed. In this scenario, a nursing home $j^* \in N$ is selected with probability 0.5 as the preferred nursing home for each patient $p \in P$, hence $P(l_p := j^*) = 0.5$. The comparison between the number of abandonments under the allocation model and the extremes is shown in Fig. 10.

First, note that the policy for the shared queue does not change, since in this policy we do not take into account individual preferences. In contrast, we see in Fig. 10 that the percentage of abandonments for the separate queue policy and the allocation model has increased significantly. Compared to the results without popular nursing homes, the abandonment fraction $\rho = 1$ has increased for the separate queues from 9.7% to 37.7% and for the allocation model from 6.3% to 18.0%. We thus see that the allocation model is far better in retaining the waiting times, and therefore fraction of abandonments, at an acceptable level.

6. Case study: Amsterdam

To show the results of the model in a realistic parameter setting, we applied the model to the situation in Amsterdam, the capital of the Netherlands. First, we describe the current policy in Amsterdam, the obtained data and then we validate our model by validating the assumptions and using information on current waiting times. Thereafter, the results of our proposed alternative – the allocation model – are discussed.

6.1. Current policy

In Amsterdam, the current allocation policy allows patients to sign up for one preferred nursing home and wait at home until placement. However, if the waiting time becomes excessive, patients may opt for immediate placement due to impatience. Since bed availability is not centrally managed and information is limited, a regional office manager contacts nursing homes individually to inquire about temporary

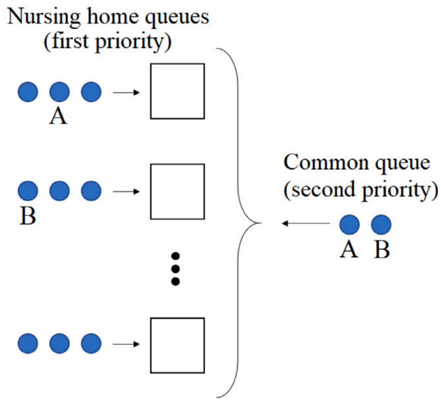


Fig. 11. Current policy model.

residency options, ceasing calls once a suitable bed is found. Under this policy, patients are initially placed on a waiting list for their preferred nursing home and later added to a secondary waiting list shared among all regional nursing homes, where beds are allocated if the primary list is empty. This policy framework is depicted in Fig. 11.[3]

We stress that the policy introduced here is the current *policy*, which may differ from the current *practice*. Namely, no centralized administration system exists that prohibits patients to register for multiple nursing homes, although this is against the rules. This phenomenon is recognized by experts as the ‘grey waiting list’.

6.2. Data

For our case study, we focus on Amsterdam and a specific patient group: those with severe somatic conditions, categorized as ZZP 6 and ZZP 8 in the Dutch healthcare system. These patients exhibit physical symptoms and require intensive care. In Amsterdam, there are 39 nursing homes with dedicated somatic departments, accommodating these patients (see Fig. 12). Utilizing non-public microdata from Statistics Netherlands, we derive essential parameters. The arrival rate (λ = 1.24/day) is determined from nursing home allowance requests, while the capacity (c = 775) reflects the maximum number of simultaneous residents. Considering a balanced distribution of capacity across nursing homes, each is assumed to have 20 beds, resulting in 780 beds. The average length-of-stay ($\frac{1}{\mu}$ = 666 days) is obtained from patient declaration data. With the offered load calculated at ρ = 1.07, we adjusted the arrival rate to 1.25 accordingly so that the offered load remained the same.

For the simulations, patient arrivals are generated using demographic data from data.amsterdam.nl. The total number of people in Amsterdam older than 65 is circa 110,000. Moreover, we identify the number of people older than 65 on neighborhood level. Then, for each neighborhood we calculate the fraction f_r of the total number of elderly people who live in the specific neighborhood $r \in R$. We generate patients for neighborhood $r \in R$ with rate λf_r. The fractions of each neighborhood are shown in Fig. 13.

The preference groups correspond to the Dutch ‘waiting classification’ of patients: the group FP corresponds to the ‘Active Placement’ group and PP corresponds to ‘Wait For Preference’ group [46]. From this report we know that in October 2022, 20.1% of the patients on a waiting list is of the FP category. However, as we do not know the waiting times of the different subgroups, we cannot calculate their arrival probabilities. Due to the lack of this information, we roughly set the arriving patients probabilities to $p^{PP} = 0.5$ and $p^{FP} = 0.5$.

[3] Note that all patients initially enter one queue. Then, after waiting >15 months, the patient additionally takes place in the common queue with second priority, which is displayed for patient A and B.

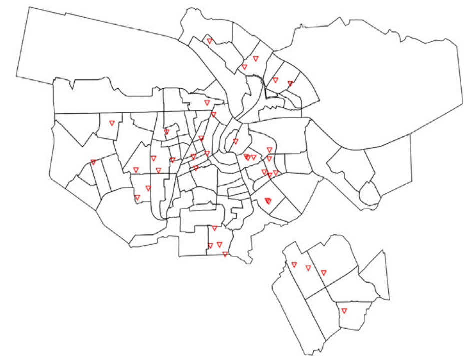


Fig. 12. Somatic nursing homes.

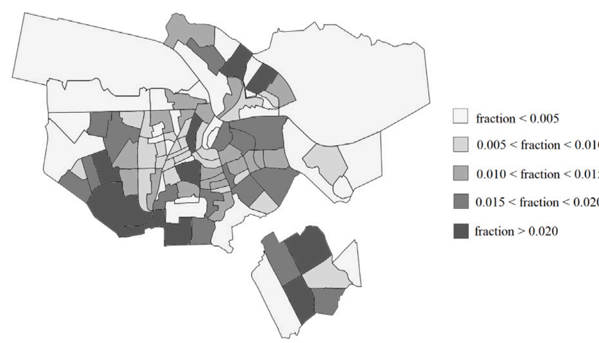


Fig. 13. Fractions for each neighborhood.

6.3. Model validation

We aim to validate our model in two ways. First, we use the real-life data to verify the assumptions that we provided in the dynamic model. Thereafter, we show that our model of the current policy is a decent representation of reality, by comparing the resulting waiting times of our model by existing data on waiting times.

Assumptions validation

We validate assumptions of our dynamic model using real-world data. Firstly, we assess the assumption that the length-of-stay in nursing homes follows an exponential distribution. Survival analysis is employed on data comprising lengths-of-stay, accounting for censored data where patients remain alive beyond the observation period. The resulting Kaplan–Meier curve, depicted in Fig. 14, reveals that after three years, approximately 15% of patients are still present. A two-sample Kolmogorov–Smirnov test compares this curve with an exponential distribution fitted to the lowest 85% of values. With a p-value exceeding .01, we cannot reject the null hypothesis, indicating similarity between the distributions. Thus, we conclude that the length-of-stay for this patient group conforms to an exponential distribution.

The second assumption that we aim to validate is that the arrival process is Poisson. As data source for the arrival process, we use acceptance data for ZZP 6 and 8 indications, i.e. after acceptance patients can apply for a nursing home. A histogram with the number of arrivals per week is displayed in Fig. 15. Moreover, a chi-square test indicated that the number of arrivals per week can be according to a Poisson distribution $\chi^2(df = 12, N = 52) = 24.8$, $p > .01$.

Current policy validation

To validate the results of the current policy model, data about waiting lines for long-term care are used. The best data source found contains information on the waiting time distribution of all patients to receive long-term care in the Netherlands [47], and thus not specified for our somatic patient population, although it is noted that “the

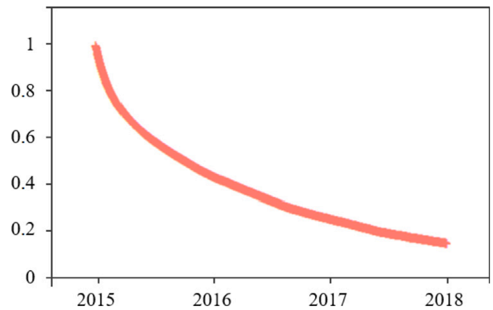


Fig. 14. Kaplan–Meier curve: survival probability.

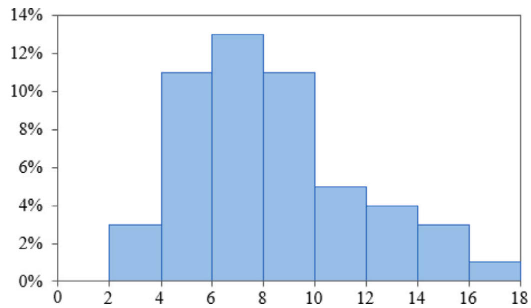


Fig. 15. Histogram arrival data per week 2016.

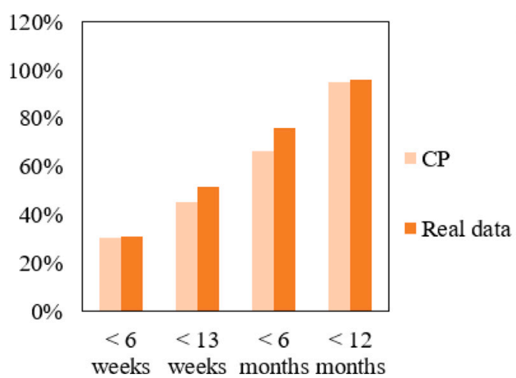


Fig. 16. Fraction of patients waiting for some amount of time.

individual indications show a similar distribution" [47, p. 1348]. The data used is from April 1, 2020 and thus pre-COVID-19. We use this information to compare to our current policy model, as displayed in Fig. 16.

From Fig. 16, we conclude that the waiting times resulting from the current policy model are similar to the real-life data. However, it was found that the occupancy level resulting from the current policy model is 0.76, which is unrealistically low. Therefore, we also run a scenario that represents the current *practice* better, which is the scenario in which (against the rules) 10% of the patients register for three nursing homes instead of one. In this case, the waiting times can also be approximated fairly well, as can be seen in Fig. 17. Next to that, the occupancy level is found to be the realistic value of 0.93. Hence, we validated the current policy by introducing a more realistic scenario for the current practice. In the following, we will use the current *policy* model as reference, since this model contains the least assumptions and is the official protocol.

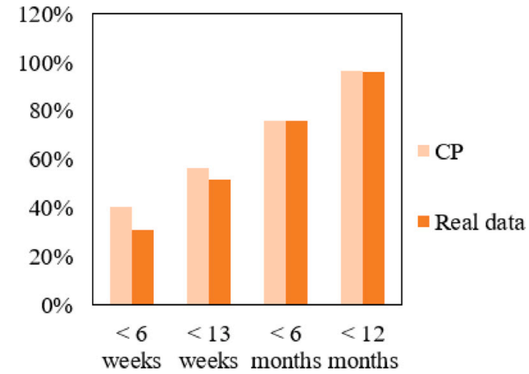


Fig. 17. Fraction of patients waiting for some amount of time, with P(3 pref. NHs) = 0.1.

6.4. Parameter setting for the allocation model

For the allocation model, we have defined the utilities for the patients in close collaboration with experts from the elderly care domain. To obtain the initial utilities g_{pn} for the patients to the nursing homes, we assume that preferences for certain nursing homes mainly depend on the travel distance between the home location of the patient and the nursing home, dist_{pn}, where the distance is defined as travel distance *by car*. This was motivated by research on nursing home selection where *location* was the "single most frequently cited factor" [19]. The home locations of the patients are chosen as the midpoints of the neighborhood from which they were generated. The midpoints of the neighborhoods are calculated in the following way:

$$\text{Midpoint latitude} = \frac{1}{2}(\text{maximum latitude} - \text{minimum latitude}),$$

$$\text{Midpoint longitude} = \frac{1}{2}(\text{maximum longitude} - \text{minimum longitude}).$$

After this, we used the following utility scheme for the region of Amsterdam for patient $p \in P$ towards nursing home $n \in N$:

$$g_{pn} = \begin{cases} 100 & \text{if } \text{dist}_{pn} \leq 5 \text{ min} \\ 50 & \text{if } 5 < \text{dist}_{pn} \leq 15 \text{ min} \\ 10 & \text{otherwise.} \end{cases}$$

If $\text{dist}_{pn} \leq 5$ min, the nursing home is in the same neighborhood, which is preferred by most patients. A drive between 5 and 15 min can still be seen as close by, whereas more than 15 min driving is rather far. We also developed corresponding waiting-time utilities based on the presumed preferences of patients in Amsterdam. We use the same groups FP and PP as for the small setting, i.e. the same interpretation but different parameter values. For the FP group, we have for both placement and preferred placement a linear waiting time utility, namely $v^U_{pn}(w_p) = 0.1 w_p \ \forall \ p \in P^{FP}$ and $v^F_{pn}(w_p) = 0.1 w_p \ \forall \ p \in P^{FP}$. For the PP group, we have that the waiting time utility to be placed in a preferred nursing home increases slightly faster, thus $v^F_{pn}(w_p) = 0.15 w_p \ \forall \ p \in P^{PP}$.

For the placement waiting time utility, we developed a sigmoidal function such that the preferences of this group are best described according to elderly care experts. This function equals $v^U_{pn}(w_p) = \frac{100}{1+e^{-(0.09 w_p - 13)}} - 101 \ \forall \ p \in P^{PP}$, such that in combination with $g_{pn} \ \forall \ p \in P, n \in N$ we have at $w_p = 0$, all resulting utilities are negative except those of the preferred nursing homes. Then, after approximately 3 months, the utility becomes positive for the nursing homes with $\text{dist}_{pn} \leq 5$ min, after approximately 4.5 months, the utility becomes positive for the nursing homes with $\text{dist}_{pn} \leq 15$ min as well, and after approximately 6 months, the utility becomes positive for all (resulting) nursing homes.

Preferred nursing homes for patients are selected based on the highest utilities, with priority given to those with the highest scores.

Table 6
Parameter values case study Amsterdam.

Parameter	Value	Parameter	Value
$\|N\|$	39	$v_{pn}^{U}(w_p)$	$0.1w_p \ \forall \ p \in P^{FP}$
λ^{-1}	1.25/day	$v_{pn}^{F}(w_p)$	$0.1w_p \ \forall \ p \in P^{FP}$
θ^{-1}	666 days (≈2 years)	$v_{pn}^{U}(w_p)$	$\frac{100}{1+e^{-(0.09w_p-13)}} - 101 \ \forall \ p \in P^{PP}$
μ^{-1}	666 days (≈2 years)	$v_{pn}^{F}(w_p)$	$0.15w_p \ \forall \ p \in P^{PP}$
M	1000	c_n	Initial value: 20 $\forall \ n \in N$
Δ	1 day		
g_{pn}	$\begin{cases} 100 & \text{if } \text{dist}_{pn} \leq 5 \text{ min} \\ 50 & \text{if } 5 < \text{dist}_{pn} \leq 15 \text{ min} \\ 10 & \text{else} \end{cases}$		

Table 7
Allocation model Amsterdam.

Policy	% abandonments	% died at temp. NH	% died at pref. NH	MWT till placement (d)	MWT till preferred (d)	Mean nr. of replacements	Mean queue length	Mean occupancy
Shared queue	6.0% (5.3%–6.7%)	0.0% (0.0%–0.0%)	91.3% (87.4%–95.2%)	40 (36–43)	40 (36–43)	0.91 (0.87–0.95)	91 (82–100)	1.0 (1.0–1.0)
AM: tot	7.4% (7.2%–7.7%)	25.1% (24.8%–25.3%)	67.5% (67.2%–67.8%)	47 (46–48)	191 (189–193)	1.34 (1.33–1.34)	85 (83–87)	0.99 (0.99–0.99)
AM: FP	0.4% (0.4%–0.5%)	33.1% (32.8%–33.4%)	66.4% (66.1%–66.7%)	3 (2–3)	196 (192–199)	1.58 (1.58–1.58)		
AM: PP	14.5% (14.0%–14.9%)	16.9% (16.6%–17.2%)	68.6% (68.2%–69.0%)	99 (97–101)	186 (184–188)	1.09 (1.09–1.1)		
CP	32.2% (32.0%–32.4%)	4.5% (4.3%–4.6%)	63.3% (63.1%–63.6%)	234 (232–235)	257 (255–259)	0.86 (0.86–0.87)	313 (311–315)	0.72 (0.72–0.73)
Separate queues	36.9% (36.6%–37.1%)	0.0% (0.0%–0.0%)	63.1% (62.9%–63.4%)	256 (254–258)	256 (254–258)	0.63 (0.63–0.63)	350 (348–353)	0.67 (0.67–0.68)

Note: The values in parentheses are the 95% confidence interval.

Table 8
Allocation model multiple preferences Amsterdam.

Policy	% abandonments	% died at temp. NH	% died at pref. NH	MWT till placement (d)	MWT till preferred (d)	Mean nr. of replacements	Mean queue length	Mean occupancy
AM(2): tot	6.6% (5.7%–7.5%)	8.5% (8.0%–9.1%)	84.8% (84.0%–85.6%)	40 (37–44)	98 (93–102)	1.35 (1.33–1.36)	75 (66–84)	0.99 (0.99–1.0)
AM(2): FP	1.1% (0.5%–1.6%)	14.6% (13.7%–15.4%)	84.3% (83.6%–85.1%)	6 (4–9)	102 (98–105)	1.69 (1.67–1.7)		
AM(2): PP	12.2% (10.9%–13.5%)	2.5% (2.1%–3.0%)	85.3% (84.1%–86.5%)	79 (73–85)	94 (88–100)	1.0 (0.98–1.03)		
AM(3): tot	6.3% (5.3%–7.3%)	2.5% (2.1%–2.9%)	91.2% (90.3%–92.1%)	43 (39–47)	61 (58–64)	1.27 (1.25–1.3)	86 (75–97)	1.0 (1.0–1.0)
AM(3): FP	4.2% (2.9%–5.4%)	5.0% (4.2%–5.9%)	90.8% (89.7%–91.9%)	26 (21–31)	61 (58–64)	1.63 (1.59–1.67)		
AM(3): PP	8.5% (7.6%–9.4%)	0.0% (0.0%–0.0%)	91.5% (90.6%–92.4%)	60 (57–64)	60 (57–64)	0.92 (0.91–0.93)		

Note: The values in parentheses are the 95% confidence interval.

Table 9
Current policy multiple preferences Amsterdam.

Policy	% abandonments	% died at temp. NH	% died at pref. NH	MWT till placement (d)	MWT till preferred (d)	Mean nr. of replacements	Mean queue length	Mean occupancy
CP(1)	32.2% (32.0%–32.4%)	4.5% (4.3%–4.6%)	63.3% (63.1%–63.6%)	234 (232–235)	257 (255–259)	0.86 (0.86–0.87)	313 (311–315)	0.72 (0.72–0.73)
CP(2)	14.7% (14.4%–15.0%)	0.0% (0.0%–0.0%)	85.3% (85.0%–85.6%)	100 (98–102)	100 (98–102)	0.85 (0.85–0.86)	164 (161–166)	0.91 (0.91–0.91)
CP(3)	7.5% (6.6%–8.4%)	0.0% (0.0%–0.0%)	92.5% (91.6%–93.4%)	51 (47–56)	51 (47–56)	0.92 (0.92–0.93)	100 (89–111)	0.98 (0.98–0.99)
CP(4)	6.8% (5.8%–7.7%)	0.0% (0.0%–0.0%)	93.2% (92.3%–94.2%)	46 (41–51)	46 (41–51)	0.93 (0.92–0.94)	94 (83–106)	0.99 (0.99–0.99)
CP(5)	6.7% (6.4%–7.0%)	0.0% (0.0%–0.0%)	93.3% (93.0%–93.6%)	45 (44–47)	45 (44–47)	0.93 (0.93–0.94)	97 (93–101)	0.99 (0.99–1.0)

In cases where multiple nursing homes have equal utility, a random selection is made. To ensure fairness in comparing the model to the current situation, we selected preferred nursing homes for the current policy using the same utility scheme. The parameter values for the case study can be found in Table 6.

6.5. Results

We analyzed various policies to assess their impact on performance measures. The results comparing the allocation model to the two extremes and the current policy are summarized in Table 7. Abandonment rates range from 6.0% for the shared queue to 36.9% for separate queues, indicating significant differences. Notably, under the separate queue policy, only 63.1% of individuals end up in their preferred nursing home, highlighting diverse bed demand across facilities in Amsterdam.

Table 7 illustrates the overall favorable performance of the allocation model. Compared to the current policy, abandonment rates decrease significantly from 32.2% to 7.4%, with the allocation model showing only slightly higher abandonment rates than the shared queue (which sets a lower bound). In addition, mean waiting times until placement and preferred placement are significantly reduced. However, the number of replacements increases in the allocation model compared to the current policy. Observe that the performance of the FP and PP groups is well in line with their goals in terms of time to placement and preference.

Furthermore, in a scenario with 'popular' nursing homes, where demand is considerably higher for some locations, the allocation model's superiority over the current policy becomes even more evident. In this scenario, the allocation model results in a less substantial increase in abandonments compared to the current policy. Details for this experiment can be found in Appendix D.

Scenario of multiple preferred nursing homes

The results of the allocation model, applied to Amsterdam, when patients have two or three preferences are presented in Table 8. As anticipated, allowing two preferences leads to notable improvements, such as a considerable reduction in mean waiting time until placement in a preferred nursing home (from 177 to 105 days). In addition, the percentage of patients who end up in their preferred nursing home increases from 71.2% to 82.5%, albeit with an increase in the mean number of replacements. When the number of preferred nursing homes is increased to three, overall performance improves further, although not as dramatically as observed between one and two preferences. Therefore, the decision to adopt multiple preferences depends on balancing these improvements against the loss of specific nursing home preferences, particularly given that bed efficiency approaches its maximum level with two preferences (mean occupancy of 0.99). Healthcare managers must carefully consider these factors to determine the optimal strategy.

The impact of multiple preferred nursing homes within the current policy setting is explored in Table 9. These results reveal significant reductions in abandonment rates and mean waiting times when the current policy allows for more preferred nursing homes. However, compared to the allocation model, the current policy's performance is inferior even when allowing the same number of preferred nursing homes. For example, with two nursing homes, the abandonment fraction is 18.5% for the current policy (CP(2)) and only 5.7% for the allocation model (AM(2)). Thus, the allocation model significantly outperforms the simple alternative of allowing patients to choose more than one preferred nursing home.

In summary, we find that if the system allows more patient replacements, the allocation model outperforms the current policy, significantly reducing waiting times in Amsterdam while also accommodating individual patient preferences.

7. Conclusion and discussion

The allocation model proposed in this article offers an alternative to conventional waiting lines for nursing home placements, boasting a patient-centered approach. By prioritizing individual preferences and allowing flexibility in waiting times, the model achieves a balance between minimizing waiting times, similar to shared queues, and enabling patients to choose their preferred nursing home, as if separate queues were employed. Moreover, the optimal placement policy found by solving a Markov Decision Process shows that, for small instances, the mean optimality gap of the allocation model equals 1.3%. Hence, the model is found to have 'the best of both worlds', resulting in a quicker placement of patients into the nursing home of their preference.

Applied to a population of somatic patients in Amsterdam, the allocation model significantly reduced the abandonment fraction from 32.2% to 7.4% and decreased the mean time until placement by five months. However, these improvements required a higher rate of patient replacements, increasing from 0.89 to 1.44 on average. Furthermore, allowing patients to choose two preferred nursing homes further improved the model's performance, reducing the abandonment fraction to 5.7%. These findings underscore the relevance of the model in addressing the pressing need for efficient and sustainable long-term care solutions.

Future research avenues include incorporating predictive information about bed availability to enhance allocation accuracy and expanding the model's scope to encompass broader elderly care contexts, such as addressing bed-blocking in hospitals or exploring the impact of transitional care units. Furthermore, the promising outcomes of the allocation model underscore the importance of centralized monitoring of nursing home waiting lists. In privatized care settings such as the Netherlands, where nursing homes operate independently, information sharing and collaborative strategies in the elderly care sector can significantly enhance efficiency for both patients and nursing homes. A potential direction towards practical implementation is to investigate the set-up of a regional care center that manages the available beds, waiting list, and placement of patients, using a scientifically-based allocation model. In addition, refinement of the utility functions warrants further investigation to ensure that they align with the diverse needs of patients. These steps seem to be important prerequisites for a successful practical implementation.

Although our research focuses primarily on the elderly care domain, the allocation policy framework is relevant for various other domains facing bed scarcity and trade-offs between preferences and waiting times. From long-term care for mentally disabled people to psychiatric patient care, applying logistic perspectives to address excessive waiting times is imperative. We hope that this paper inspires Operations Research experts to contribute their expertise to these domains, ultimately enhancing patient well-being regardless of future care challenges.

CRediT authorship contribution statement

R.J. Arntzen: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization. **R. Bekker:** Conceptualization, Methodology, Validation, Writing – original draft, Writing – review & editing, Validation, Formal analysis, Supervision. **R.D. van der Mei:** Conceptualization, Methodology, Validation, Formal analysis, Writing – original draft, Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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Appendix A. Simulation model

In this section, we discuss the simulation procedure of the allocation heuristic in more detail. First, denote I^t as the set of patients arrived and not yet left before time $t \in T$, where we see that the pth arrival is denoted as patient $p \in I^t$. We define the patient sets that reside at time $t \in T$ at a certain location:

$$F_n^t = \{p \mid p \in I^t, l_p \in L_p\} \qquad \forall t \in T, n \in N \tag{A.1}$$

$$G_n^t = \{p \mid p \in I^t, l_p \in M_p\} \qquad \forall t \in T, n \in N. \tag{A.2}$$

Thus, at time $t \in T$, F_n^t denotes patients who reside permanently in (final) nursing home $n \in N$. G_n^t is the set with temporary patients at nursing home $n \in N$. Note that G_0^t is the set of patients at home.

The simulation procedure can be described as follows; see Fig. 18 for a schematic representation. First, the initialization takes place, which includes a warming-up period for the model. The warming period begins with adding patients to nursing homes until an occupancy of 90%; this is to speed up the process of reaching steady state. The determination of when the steady state was reached was made visually using a graph with confidence intervals of the waiting times until placement. For the small instances, the warming period was ended after 1000 clients had left the system, and for the full-size instances, this was reached after 10,000 clients had left.

Then, in the dynamic setting, the time is set to the next time moment (A.3) and the system is updated according to all events that occurred during the interval of length Δ. The corresponding output values are updated. Then, for the current $t \in T$ the sets I^t, G_n^t and F_n^t $\forall n \in N$ are defined in (A.9). The set of patients P is updated in a way that includes new arrivals and excludes departures (A.10). The capacity of all nursing homes is updated so that it includes only empty beds and beds occupied by temporary patients (A.11). At the end of the dynamic setting, the utilities are updated according to the new waiting times (A.12).

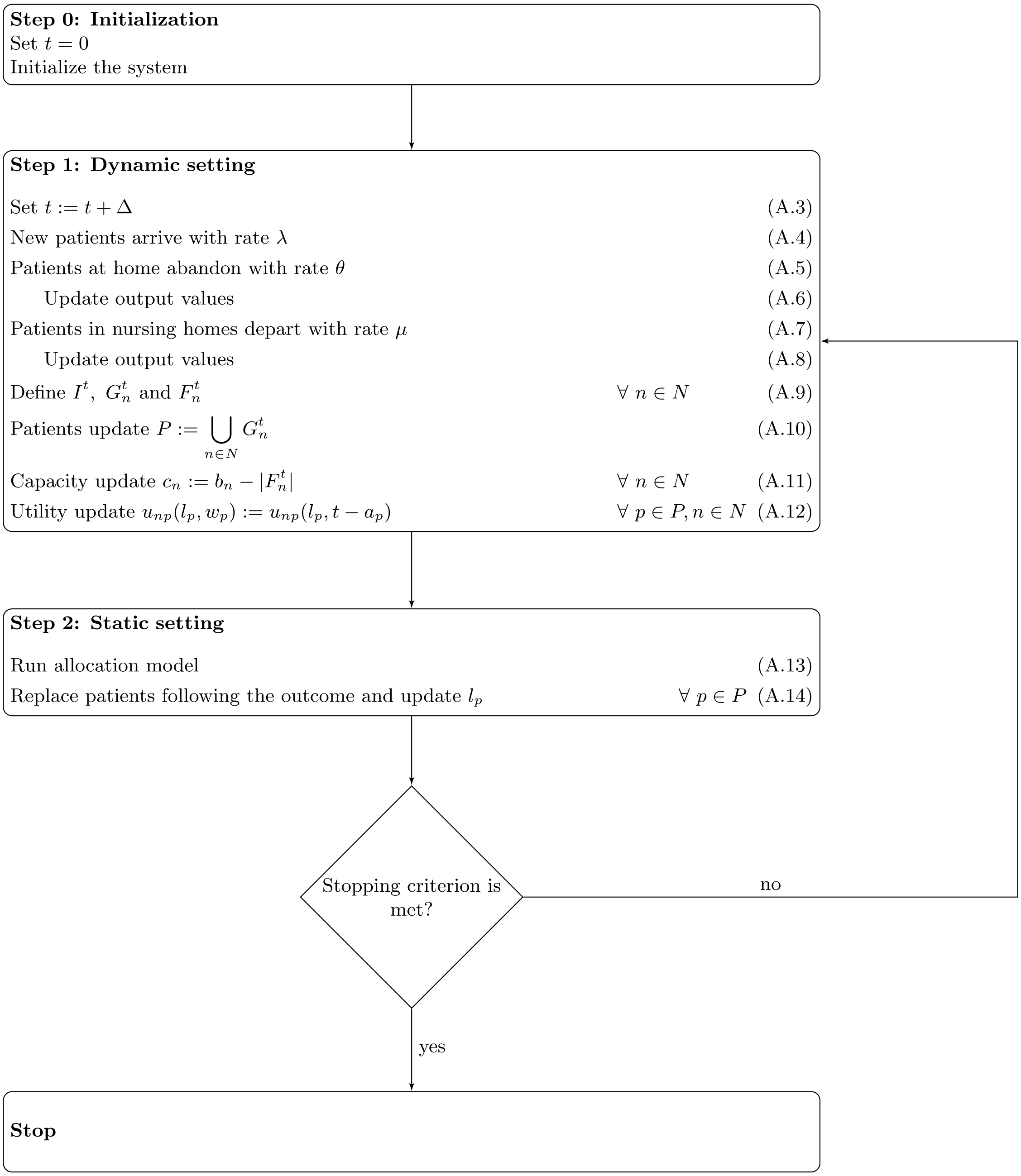

Fig. 18. Simulation procedure.

In the next step, the allocation model is run (A.13). Finally, the resulting replacements are carried out, including an update of the locations l_p of all patients $p \in P$ (A.14).

Finally, two stopping criteria were defined on the basis of the instance size. For the toy examples, the simulations were run until the width of the confidence interval for waiting times was less than 1% of the mean waiting time for placement. For realistic full-size instances, this was not achievable, and therefore we ran the simulations for at least 12 h and presented the results accompanied by their confidence intervals.

Appendix B. Markov decision process for optimal allocation

Let X_t denote the state of the system at times $t \geq 0$, defined in the following, such that $\{X_t\}_{t \in T}$ represents the stochastic process. Moreover, let Π denote the set of allocation policies. Then, policy $\pi \in \Pi$ has a long-term average reward

$$r_\pi = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} r_\pi(X_t).$$

The optimal policy is a policy π for which we have $max_\pi r_\pi$.

B.1. Problem formulation

Now we discuss all components of the MDP: the state space, action space, transition probabilities, and rewards. All notation used in this section can be found in Table 10.

State space

In the allocation model, we have that each patient is unique. In an MDP framework, this leads to an excessive state space. Therefore, we

Table 10
MDP notation.

Sets	
I	Patient classes
N	Nursing homes, with $n = 0$ the home location
$L_i \subseteq N$	Preferred nursing homes of class $i \in I$
$M_i \subseteq N$	Non-preferred nursing homes of class $i \in I$, i.e. $N \setminus L_i$
Parameters	
λ_i	Arrival rate of class $i \in I$
μ	Service rate
r_{inm}	Lumpsum reward of class $i \in I$ residing at location n moving to location m
X_{in}	The number of (temporary) patients of class $i \in I$ at location $n \in N$
y_n	The number of preferred persons at location $n \in N$
w_i	Waiting penalty of class $i \in I$
c_{rej}	Rejection costs
b_n	Number of beds available at location $n \in N$
A_{imn}	Number of patients of class $i \in I$ allocated from location m to location $n \in N$
g_{in}	(Initial) willingness of class $i \in I$ to be placed in nursing home $n \in N$
K_i	Replacement penalty of class $i \in I$

define a set of patient classes I for the MDP. The patients of class $i \in I$ arrive according to a Poisson process with rate λ_i, have sets of preferred nursing homes L_i and nonpreferred nursing homes M_i.

For the state space, we introduce the variable $X_{in} \in \mathbb{N}$ defined as the number of patients of class $i \in I$ residing in nursing home $n \in N$. Let $\mathbf{X} \in \mathbb{N}^{|I|\times|N|}$ be the matrix with the entries X_{in}. In addition, we only need to keep track of the classes of patients that are not yet in their preferred nursing home, since only those are candidates to be replaced. For the preferred placed patients (that is, patient classes i for which $n \in L_i$), only the number of patients $y_n \in \mathbb{N}$ satisfies. We store those elements in a vector $\mathbf{y} \in \mathbb{N}^{|N|}$, with y_n as the nth entry. Note that we truncate the total number of patients on the waiting list at b_0, such that the state space remains finite. This implies that patients arriving who find b_0 patients on the waiting list are rejected.

Hence, a state can be described as $\{\mathbf{X}, \mathbf{y}\}$, where the state space is given by

$$S = \left\{ \{\mathbf{X}, \mathbf{y}\} \mid \sum_{i\in I} X_{in} + y_n \le b_n \ \forall n \in N \right\}.$$

Action space

As actions we define the number of temporarily placed patients of a certain class $i \in I$ that are replaced from nursing home m to $n \in N$, denoted by A_{imn}. We define the action matrix $\mathbf{A} \in \mathbb{N}^{|I|\times|N|\times|N|}$ with elements A_{imn}. Note that we allocate all temporary patients, which can also be in their current location. Now, let $Y(\{\mathbf{X}, \mathbf{y}\})$ denote the set of all possible actions for state $\{\mathbf{X}, \mathbf{y}\}$. Then, we have

$$Y(\{\mathbf{X}, \mathbf{y}\}) = \left\{ \mathbf{A} \mid X_{in} = \sum_{m\in N} A_{inm}, \ \sum_{i\in I}\sum_{m\in N} A_{imn} + y_n \le b_n \ \forall n \in N \right\}.$$

Transition probabilities

The transition probabilities consist of two ‘consecutive’ parts. First, based on the actions to allocate patients to other locations, we move from state $s \to s^*$. Then, based on probabilities induced by the stochastic process, we arrive at our final state s'.

For the first part, we have the following state transitions from $\{\mathbf{X}, \mathbf{y}\}$ to $\{\mathbf{X}^*, \mathbf{y}^*\}$:

$$X^*_{in} = \sum_{m\in N} A_{imn}, \quad \text{for } n \in M_i,$$
$$y^*_n = y_n + \sum_{m\in N} A_{imn}, \quad \text{for } n \in L_i.$$

Now consider the second part, that is, the transition probabilities out of the state s^*. Without loss of generality, we rescale the time such that $\sum_{i\in I} \lambda_i + \sum_{n\in N\setminus\{0\}} b_n\mu = 1$, in which case the rates can be interpreted as transition probabilities. Moreover, for notation purposes, we define the single-entry matrix E_{ij} as a matrix with zeros and a 1-entry in the ith row and the jth column, and the unit vector e_j as a vector with zeros and a 1 as the jth element. The final transition probabilities are

$$P_{(\{\mathbf{X},\mathbf{y}\}),(\{\mathbf{X}',\mathbf{y}'\})}(\mathbf{A}) = \begin{cases} \sum_{i\in I} \lambda_i & \text{if } \mathbf{X}' = \mathbf{X}^*, \mathbf{y}' = \mathbf{y}^*, \sum_{i\in I} X^*_{i0} = b_0 \\ \lambda_j & \text{if } \mathbf{X}' = \mathbf{X}^* + E_{j0}, \mathbf{y}' = \mathbf{y}^*, \sum_{i\in I} X^*_{i0} < b_0, \ j \in I \\ \mu X^*_{jm} & \text{if } \mathbf{X}' = \mathbf{X}^* - E_{jm}, \mathbf{y}' = \mathbf{y}^*, \ j \in I, m \in N \setminus \{0\} \\ \mu y^*_m & \text{if } \mathbf{X}' = \mathbf{X}^*, \mathbf{y}' = \mathbf{y}^* - e_m, m \in N \setminus \{0\} \\ 1 - \mu \left(\sum_{i\in I} \sum_{n\in N\setminus\{0\}} X^*_{in} + y^*_n \right) & \\ \quad - \sum_{i\in I} \lambda_i & \text{if } \mathbf{X}' = \mathbf{X}^*, \mathbf{y}' = \mathbf{y}^*. \end{cases}$$

Rewards

The reward at each decision epoch depends on the action reward r_{imn} (defined below), waiting costs w_i and rejection costs c_{rej}. The formula for this is the following:

$$r(\{\mathbf{X}, \mathbf{y}\}, \mathbf{A}) = \underbrace{\sum_{i\in I}\sum_{m\in N}\sum_{n\in N} r_{imn} A_{imn}}_{\text{action reward}} - \underbrace{\sum_{i\in I} w_i X_{i0}}_{\text{waiting costs}} - \underbrace{c_{rej} \sum_{i\in I} \lambda_i \mathbb{1}_{\{\sum_{i\in I} X^*_{i0} = b_0\}}}_{\text{rejection costs}}.$$

The action rewards are based on the utilities for placement in nursing homes, as provided in (4.1), without the waiting times utilities. For notational convenience, we set $g_{pn} := g_{in}$ for the patients p that belong to class i. The precise relation between the utilities and the action rewards r_{imn} is given by

$$r_{imn} = \begin{cases} g_{in} & \text{if } m = 0, \ n \in N \setminus \{0\} \\ g_{in} - g_{im} - K_i & \text{if } m \in M_i, \ n \in L_i \\ 0 & \text{if } m = n \\ -\infty & \text{otherwise.} \end{cases} \tag{B.1}$$

B.2. Value iteration

In order to solve the MDP and find the optimal long-term average reward g^*, we use value iteration. Hence, we need to find the value function $V(s)$ for all states $s \in S$. This is done by iteratively computing the value function $V_n(s)$ until convergence occurs. For this purpose, we initialize $V_0(s) = 0 \ \forall s \in S$ and then iteratively solve the Bellman equations for $i \in S$,

$$V_{n+1}(i) = \max_{a\in A(i)} r(i, a) + \sum_{j\in S} P_{ij}(a) V_n(j).$$

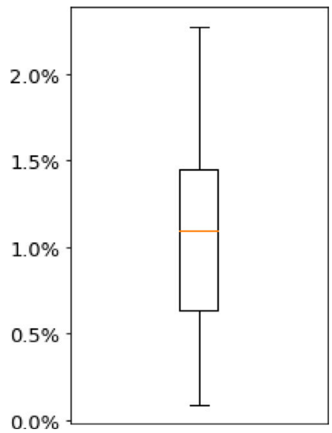

Fig. 19. Optimality gap in %.

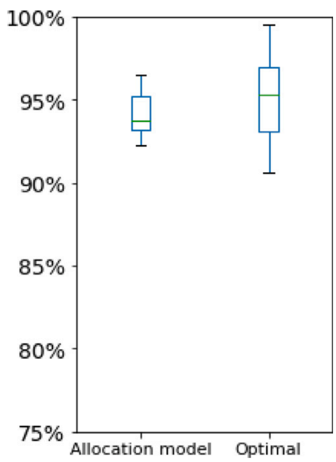

Fig. 20. % died in pref. nursing home.

After each iteration, we compute $h_n = \inf_i |V_n(i) - V_{n-1}(i)|$ and $H_n = \sup_i |V_n(i) - V_{n-1}(i)|$. The value functions are converged if $H_n - h_n < \epsilon h_n$, where ϵ is a prespecified accuracy. As the value function has converged, we have $\forall\ s \in S,\ V(s) = V_n(s)$ and $g^* = V_n(s) - V_{n-1}(s)$.

B.3. State space size

To illustrate the impact on the state space, we define the following instances to compare the allocation model with the optimal solution. We take two nursing homes, $N = \{0, 1, 2\}$ with 0 the home location, and four classes, $I = \{\mathrm{FP}_1, \mathrm{FP}_2, \mathrm{PP}_1, \mathrm{PP}_2\}$. The class subscripts correspond to the preferred nursing home. For this example, the number of possible states for $\mathbf{X}$ and $\mathbf{y}$ provides an upper bound for the size of the state space:

$$|\mathbf{X}| = \binom{b_0 + 4}{b_0}\binom{b_1 + 2}{b_1}\binom{b_2 + 2}{b_2}$$

$$|\mathbf{y}| = (b_1 + 1)(b_2 + 1)$$

As can be seen, the size of the state space grows quickly. Moreover, we see that the number of possible actions is also large, since the empty beds can be filled by all possible combinations of waiting patients. For these reasons, we are only able to solve small instances.

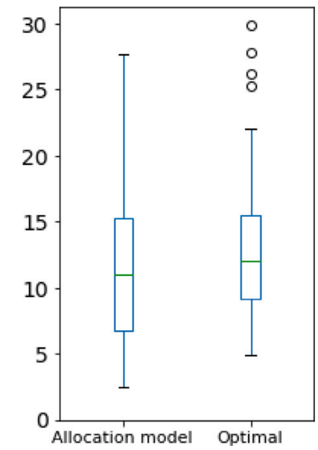

Fig. 21. Waiting time (d).

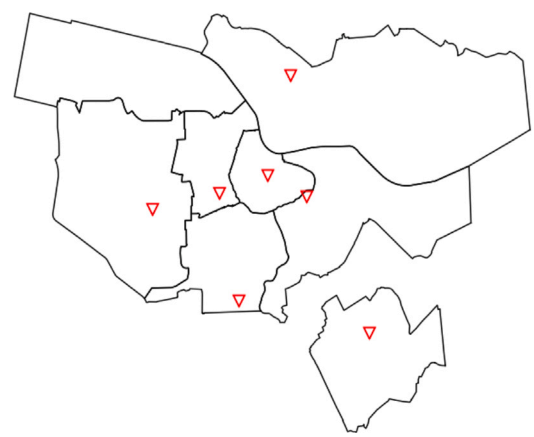

Fig. 22. Selected popular nursing homes in city parts. Note: In the city part in the north-west (Westpoort), no popular nursing home is selected, since no nursing home is located in this part.

Appendix C. Additional results MDP instances

In this appendix, we present the results for the instances defined in Section 5.3, with 3 beds per nursing home. As can be seen in Fig. 19, the optimality gap is lower for these instances than for the instances with 2 beds per nursing home. For the other output measures, the results for the instances with 3 beds show a similar behavior to the instances with 2 beds per nursing home; see Figs. 20 and 21.

Appendix D. Scenario with popular nursing homes

For the region of Amsterdam, we also consider a scenario with more popular nursing homes. More specifically, in each of the eight parts of the city (called ‘stadsdelen’), we randomly select a really popular nursing home. That is, with probability $p_{pop} = 0.25$, this nursing home is selected to be the preferred nursing home of a patient who belongs to that part of the city. The parts of the city and the popular nursing homes chosen are provided in Fig. 22.

In Table 11 we show the results for both the current policy and the allocation model. The table illustrates that in a scenario with popular nursing homes, the number of abandonments increases for both policies (recall that the abandonment fractions were 7.4% and 32.2% for the allocation model and the current policy, respectively). However, we see

Table 11
Results scenario popular Amsterdam.

Policy	% abandonments	% died at temp. NH	% died at pref. NH
AM: tot	8.6% (7.7%–9.4%)	72.5% (71.5%–73.5%)	18.7% (18.4%–19.2%)
AM: FP	0.7% (0.4%–0.9%)	79.2% (78.3%–80.1%)	20.1% (19.3%–21.1%)
AM: PP	16.3% (14.7%–17.7%)	66.3% (64.4%–68.0%)	17.5% (16.6%–18.5%)
CP	37.4% (36.7%–38.1%)	42.0% (41.0%–43.2%)	20.5% (19.4%–21.6%)

Note: The values in parentheses are the 95% confidence interval.

that compared to the situation without popular nursing homes, the percentage of abandonments under the allocation model only increases by 1.3%, while the percentage of abandonments under the current policy increases by 7.2%. This implies that the current policy is considerably more sensitive to a change in the popularity of nursing homes than the allocation model.

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