# Math in the city: designing a math trail for high school students

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*Chapter 4*

# Math in the City: Designing a Math Trail for High School Students

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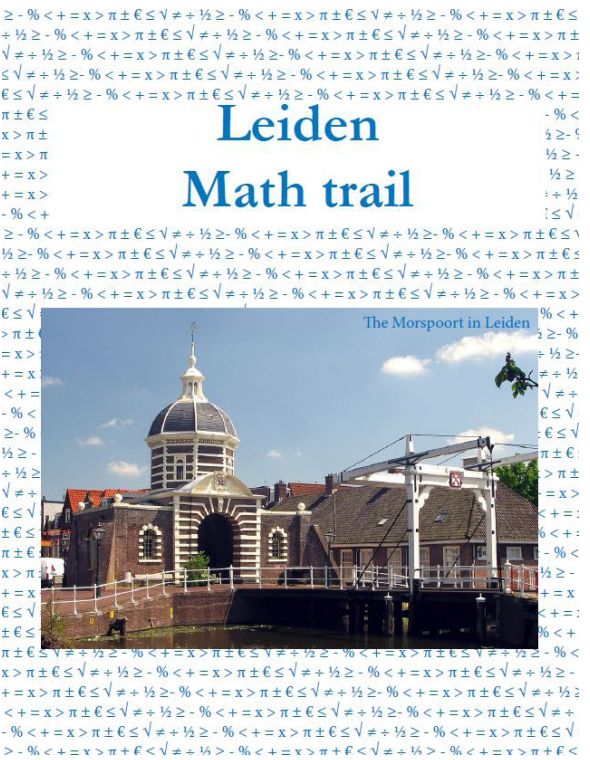
This chapter describes the development of a math trail for high school students. In 2016, we developed this trail through Leiden (The Netherlands) during a student project for the Science Communication and Society specialization, a track for master students at the Faculty of Science at Leiden University. Our aim was to provide a guided trail through the city that links everyday sights to mathematical concepts within the curriculum of high school students between 13 and 15 years old. The entire project was carried out in 3 weeks. We did background research, consisting of literature reviews, target audience surveys with school children, and focus groups with teachers. Based on the conclusions from this background research, we developed questions that suited both the goal to make the math trail a fun experience that makes math less abstract and the goal to include questions from across the curriculum. In this chapter, we would like to share our insights from the background research and our experiences in developing a math trail. We moreover aim to provide those who are interested in designing a math trail in their city with a practical step-by-step plan and checklist.

## Introduction

There are multiple ways to describe a mathematics trail. For example, Shoaf, Pollak, and Schneider (2004) write: "A mathematics trail is a walk to discover mathematics. (…) The math trail map or guide points to places where walkers formulate, discuss, and solve interesting mathematical problems" [1]. Zender writes: "A mathematics trail is like a sightseeing tour, but the objects are of mathematical interest. One can discover or practice math at the hand of these objects" [2].




Regardless of slight differences in definition and design, the goal of math trails is similar: to link mathematics to practical situations, and to promote learning and engaging with mathematics in a playful and motivating setting. Math trails are ultimately suitable to engage students in problem-solving, to make connections between concepts and applications, and to apply the skills in a meaningful context [3]. Cross describes the benefits that arise because the activity takes place outside the classroom: the problems are real and relevant, pupils gain confidence by applying their knowledge in practical situations, the discussion that will be stimulated helps them to communicate their knowledge effectively, they need to accurately record their data to be used later in the classroom and they will meet the same ideas and concepts in different contexts [4]. Specifically for a city walk, the assignments should not be solvable without the surroundings and the topics should have been discussed in class before [5]. Mathematics comes

to life, because students have to engage both physically and cognitively [6]. Math trails for example exist in The Netherlands[a] and in other countries.[b]

Previous literature on math trails details the development of trails for primary school students [3,6,7], high school students [5], teachers [8,9], a general audience [1], and the development itself as an exercise for the students [6,10]. Zender (2020) describes the development of math trails and research on their effectiveness in the last 40 years [2].

During a 3-week student project in December 2016, the Leiden Math Trail for high school students was developed.[c] The project was part of the Science Communication and Society specialization, a Master Track of the Faculty of Science at Leiden University, The Netherlands.[d] The first two authors and another student worked full-time during these 3 weeks to perform literature reviews and target audience research, supervised by the third author of this chapter. The goal was to develop a route through the city and design questions linked to sights with the appropriate underlying concepts and to combine all this into a booklet to be used by high school children to explore mathematics in the city.

The uniqueness of the student project described in this chapter lies in the integral scientific approach, from background and target audience research to evaluation in a short time period. Zender describes the lack of systematic research on math trails in [2]. The goal of the student project was to have a usable and tangible product based on scientific research and methods and to include as much background research and evaluation as the short time limit of 3 weeks allowed. The background research led to insights that changed the approach of the math trail and was a valuable part

[a] For example, in Utrecht: https://hanswisbrun.nl/winkel/wiskundewandeling-utrecht/, and Groningen: https://www.rug.nl/sciencelinx/wiskundeopstraat/ [Accessed on May 4, 2022].

[b] For example, in Cambridge: https://www.cambslearntogether.co.uk/cambridgeshire-school-improvement/cambridgeshire-english-and-maths/cambridgeshire-maths-trails/resources and https://mathcitymap.eu/en/ [Accessed on May 4, 2022].

[c] The math trail can be downloaded from the Leiden University website here: https://www.universiteitleiden.nl/en/news/2017/01/math-trail-leiden [Accessed on May 4, 2022].

[d] For more information on the specialization programme see: https://www.universiteitleiden.nl/en/science/science-communication-and-society [Accessed on May 4, 2022].

of the development of the project. An evaluation was moreover performed in 2020 for the purpose of making the research presented in this chapter as comprehensive as possible.

In this chapter, we will detail the research steps that were taken to ensure a match between the target audience and the product, elaborate on the designing process, and show the road that led to a successful final product. We will also include a short evaluation with teachers who have requested the teacher's guide in the time since the publication of the trail. The goal of this chapter is to show that in a relatively short period, a math trail can be developed that suits the needs of teachers and high school students, based on scientific ideas and background research. We hope to inspire future designers of math trails with the experiences and insights from this project and to show the added value of research. We will also provide them with a step-by-step plan, checklist and time schedule to make sure their contribution is of value to teachers, students, and of course any other interested people. We believe that our results can be an inspiration for other science communication projects, since it shows how to easily incorporate the target audience in the design of an activity.



## Goals

The math trail was developed with the aim to create more enthusiasm for mathematics amongst high school students by engaging them actively in a "treasure hunt"-style walk-through the city. The goal was to actively let them interact with math applications in daily life and show how math is present all around us. One of the underlying goals was to create a fun addition to high school lessons, fitting the curriculum of high school students, and that could be used by teachers as part of their program. The math trail aims to broaden pupils views on mathematics, with the hope to improve the general image of mathematics and the relation pupils have with this subject.

### Audience target audience description

The main target audience for the math trail is high school students between the ages of 13 and 15 of the Dutch middle (HAVO) and higher (VWO) level and by extension their math teachers. Although the trail is developed

for this main audience, it is freely available and there is therefore a large secondary audience, consisting of anyone who is interested in mathematics and city walks.

### Target audience research

To investigate the needs and interests of our main target audience, the teachers and the pupils, audience research consisted of two parts: focus groups with mathematics teachers and surveys with high school children.

#### *Focus groups with high school teachers*

We held two focus group discussions of 5–10 minutes in an informal setting with four or five high school teachers. The structure of the focus groups and possible questions can be found in Table 1. Most of the teachers taught the lowest 3 years (12–15) of the middle and higher level of high school.

The teachers in the focus groups agreed unanimously that their pupils would be eager to do a math trail during school hours and that a math trail is well suited to motivate pupils by showing them the applications of mathematics in the outside world. Especially in the context of an active assignment in small groups. One of the teachers remarked: "The

Table 1: Focus group structure.

1. Briefing
   - What is the goal of the focus group?
   - Ask permission for transcribing and possible use of quotes.
2. Open questions and discussion
   - What do you think of when we say "math trail"?
   - What would you like to be included in a math trail?
   - What are the conditions and requirements to do a math trail with your students?
   - Do you think the students would be interested in a math trail (during school hours)?
3. Survey: Small sheet of paper to get personal information
   - Name and email (optional for sending results and trail).
   - School and city.
   - Level of high school and grades of the taught classes.
   - Optional if a publication is foreseen: Do you wish your name to appear in acknowledgments of a possible publication?
4. Debrief
   - Thank them for their contribution and if they wrote down an email address they will receive the trail when finished.
   - Hand participants a small note with contact details.

goal would be that the students would get more context to the mathematics, which they otherwise only see in their textbook. I would like them to see it in real life, to make them experience it more."

The teachers agreed that the trail questions needed to have enough depth. The "puzzle" part of the trail needed to be challenging. One of the teachers said: "It is not a problem if they have to spend 10 minutes on a question". The questions should have more content than just calculations, but a one-to-one overlap with the curriculum was not a must. The teachers viewed the trail as an addition and extension of the already discussed theory. There should be a lot of variety in the questions and it would be an asset if the most interesting sights of the city were featured in the trail.

Most of the teachers would link the math trail to an assignment, add a competition element or give a mark for it. They expected that the students would not be motivated enough if there was no obligation attached. They estimated that the students would be able to concentrate on a trail of 1.5–2 hours maximum. The questions should have sufficient space in between them to allow groups of students to depart one after the other without all accumulating at the same question. All the teachers would be interested in the trail but saw different opportunities. One would like it to fit in one lesson (or a double lesson), for example, to complete in the last lessons before the holiday. Another thought of walking the trail in a project week, so that the students would have enough time for an extensive trail. One other teacher had the idea to make designing a trail for the 13–14-year-olds a project for the older students (15–16).

### *Surveys with high school students*

We conducted surveys with high school children in the second and third grade (13–15-year-olds) of the higher-level high schools. The survey questions can be found in Table 2. 97 students filled out the survey in total, with 47 in the second grade and 50 in the third grade.

The results of the survey showed clearly that most of the students would not do a math trail in their free time (92%) but would be interested in a math trail during school hours (95%). Most of the students had a neutral opinion about mathematics (55/97), 31 out of 97 said they liked

Table 2: Questions for the target audience survey.

1. Short introduction: Why this survey?
2. Math and math trails
   - What is your opinion on mathematics? Likert Scale, five choices from "I don't like it at all" to "I like it a lot". Give a neutral option.
   - What do you think of when we say "math trail"?
   - What would you like to do in a math trial through the city?
   - Would you like to do a math trail in your spare time? Yes/No.
   - Would you like to do a math trial instead of a regular lesson? Yes/No.
3. Demographic questions
   - What is your gender? Girl/Boy.
   - What is your age?
   - What school are you attending?
   - What is the level of the school?
   - What grade are you in?
4. Example questions (note, don't answer them, just mark the one you like best!).
   - We gave two options of questions and asked the students to pick the question that appealed to them most. We did this for two topics: geometry and combinatorics.



mathematics and only 10 did not like mathematics at all. Most of the students had quite an accurate idea of what a math trail would be; they expect it to be a walk during which exercises have to be done.

To find out their preference for a certain type of question, we included two example questions: one about calculating angles and one about combinatorics. For the question about angles, there were two options: a passive question involving a clock, which was only solvable by thinking, and an active question involving a lantern, which was only solvable by walking around. A slight majority of the students preferred the passive question (56%). The goal of the combinatorics question was to assess whether the pupils would prefer a question that was directly linked to a site over a question that could be answered anywhere. The first question was one about the possible combinations of the numbers in a pincode, the second a question about the possible ways to color the coat of arms of the city Leiden. The underlying idea was that the first could be done in any city, whereas the second was specific to Leiden. The pupils were in favor of the pincode question (75%). This disconfirmed our hypothesis that a link with the city would be preferred. We suspect that this can be partly attributed to the fact that the question on the coat of arms was much

Table 3: Checklist for the target audience survey.

- Ask for demographic details.
- Ask for their likes and dislikes.
- Be explicit in what you want them to do.
- Make sure example questions are similar, do they test what you want to test?
- If possible, do a test run with a few pupils.
- Try to get a varied audience, to avoid bias toward certain students or levels.



lengthier due to the historical background being included. We therefore think that this question may have tested whether students like longer or shorter questions, which was not the goal of this question. To assist future math trail developers, our experiences are summarized in a checklist for target audience research in Table 3.

Many pupils gave non-math-related answers to the question "What would you like to do in a math trial through the city?", such as "not being in school", "freedom", "eating during the walk". For the survey that took place after the first, we therefore added that we would like to hear any ideas they had on questions that we could use. Some noteworthy answers were: calculate the height of a church, binary puzzles, calculate how much can you buy in a shop when you've got 10 Euros, calculate how much water there is in the canals, logical thinking.

### *Conclusions from the target audience research*

Based on the target audience research, we concluded that the most important goal of a math trail to us is to create enthusiasm amongst the students and to bring them in contact with mathematics in real life, which can make math topics less abstract. An engaging activity in the open air gives the students a positive attitude toward mathematics, which is then hopefully transferred to the classroom. To achieve this, it can be useful if they are actively working on mathematics in small groups, independently from the teacher, to promote self-guided learning. A competition element or marked assignment attached to the trail could encourage the pupils to take the trail seriously. The surveys clearly showed that the students would not do the trail in their spare time but were very interested in it during school hours. Teachers are enthusiastic about the concept of a math trail, and they would like to see varying and challenging questions. It is

moreover important to link the questions to sights in the city and have enough space between questions. Lastly, the trail should take 1.5–2 hours.

## Format and development

### *Target audience*

The mathematician who suggested the idea for a math trail as a science communication student project, originally wanted to create a math trail for 15–16-year-olds, to do with their families in their pastime. We suspected that the students would not be overly enthusiastic to do extra math exercises in their pastime and our surveys confirmed this. The teachers of the lower grades expected their students to be motivated to do a math trail, during school hours and in groups. We therefore adjusted the target audience to 13–15-year-olds during school hours.

As the trail targets different grades and levels of mathematics, we tried to avoid the need for too much known theory. The questions should be answerable with a basic knowledge of mathematics and logical thinking. Where any extra theory was necessary, we added this theory to the description of the question, without giving the answer away. An example was providing the formula for the volume of a sphere. This also enables a much wider group outside the target audience to do the trail: families, students, and math aficionados.

### *Design process*

The teachers in our focus groups gave a time limit of 1.5–2 hours for a regular lesson, but others indicated that the trail could be longer for a project week or excursion. To accommodate both, we decided to design a trail of about 2 hours with an optional extension, which takes another 45 minutes.

The design process of the walk consisted of a number of steps (see also [3]):

1. Exploration and draft
2. Write route directions
3. Formulate questions
4. Design and check

### *Exploration and draft*

To decide on the route, we started by collecting sights in the city, which the trail should pass. The criteria for including sights was, for example, the historical value of the sight, an aesthetic value such as a beautiful house, the diversity of the trail (streets and a park), and inspiration for questions. Examples of this inspiration are a church, the coat of arms of Leiden (Figure 1 left), the Weigh House, and a beautiful house with orange stones (Figure 1 right).

We were moreover in contact with the local science museum Rijksmuseum Boerhaave, who had at that time a mathematics exposition and were interested in hosting the booklets of the trail, and providing those who walked the trail with the answer sheet at the end. This provided us with a good starting and end point for our trail, as the museum is centrally located in the city and within walking distance from the train station. It enhanced the possibility for an excursion, as the trail could be combined with a visit to the museum and also a good possibility for our secondary target audience to get hold of the trail and only receive the answers at the end. It was moreover possible to find a circular route starting at Rijksmuseum Boerhaave, with a shortcut after 2 hours to design the route plus extension route. During the exploration walks, we took many pictures to record possible question locations and noted any locations that should be included because of the historical value or inspiration for questions. We kept in mind that the route should be quiet enough, but not too secluded, and safe for the students to walk. Streets without cars had our preference

Figure 1: Inspiring sights in the city. Left: The coat of arms of Leiden on the side of the Hooglandse church, right: "The orange house".

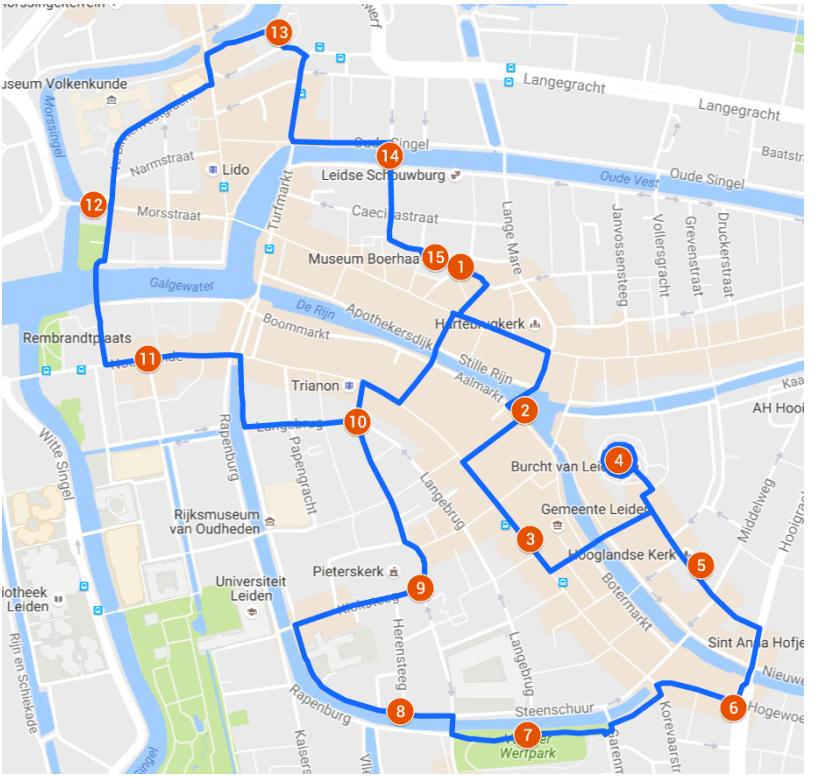



Figure 2: The route of the Math Trail in Leiden.

over streets with cars. Besides the math element, a trail is more of an outing if there are nice shops to peek into. Based on these criteria and our notes, we used a map to draw a route (see Figure 2). We moreover wrote down our draft questions and noted where any other questions should be inserted to make the time between stops approximately the same.

### *Write route directions*

The next step in the design process was to write clear directions for the chosen route. Two of the three students involved in the design process wrote the directions while walking the trail again. The third then used these directions to walk the trail and check whether the directions were clear. Next, we asked a few friends to walk the route alone or with one of us to identify any mistakes or unclear directions. In total, the route was

proof-walked four times. This is an important step in the design process, as the students should be able to follow the route by themselves without a teacher. When you design your own trail, keep in mind that over time some of the landmarks on your route description may change and you have to walk the route now and then to keep it up to date.

### *Formulate questions*

An important goal of the math trail is to make students enthusiastic for math and to show them the applications of math in daily life. Our aim was to keep the questions original and surprising, while also keeping a tight link with the city. Therefore, we have developed most of the questions ourselves. Some questions were inspired by an example from another math trail or an exercise in a textbook. The mathematics in the questions does not always correspond one-to-one to the curriculum, so as to disconnect from the textbooks and use any inspiration from city highlights. We decided, however, to design some questions based on the curriculum, to facilitate the discussion of certain theories by the teachers prior to or after walking the trail. While we extended the questions, we simultaneously wrote the answer sheet. We sometimes included short explanations to make them understandable for the interested people who would do this trail on their own, to meet the needs of the secondary audience.

Diversity in the questions is of importance; this was confirmed by both the background research and the literature. We have taken this into account in several ways. First of all, the questions are diverse in the underlying mathematical concepts. We have noticed in other trails that it was a pitfall to use mainly geometry questions in a trail. Geometry is the most visible in the surroundings, but it is important to have variety in the question topics [9]. One can think of, for example, statistics, probability, modeling, functions, derivatives, variables, and equations. Inspiration can be found in other math trails or in literature.

The second type of diversity can be found in the type of questions: open questions, multiple choice questions, calculations. Also, the underlying goal of a question can be different: application, analysis, and making connections between concepts.

Thirdly, there is a variety in subjects outside mathematics. There is an overlap in mathematics-related and physics-related questions, because many applications of mathematics are in physics. This overlap helps to put mathematics in perspective and combats its image of a subject that stands on its own. For example, we used an existing "wall formula"[e] to discuss Snell's law. In addition, history can be included in a math trail through a city, which may enhance interest in the subject.

As a last consideration, we have based our questions on the learning goals as published by the committee Toekomst Wiskunde Onderwijs (Future of Mathematics Education) [11]: problem-solving and analytical thinking, modeling and algebra, ordering and structuring, manipulation of formulas, abstraction, logical reasoning and proofs. We have ensured that all these concepts are present in at least one of the questions in the trail. In Table 4, we point out which concepts are present in each question of the trail.

Table 4: Learning goals per question.

| | **Problem-solving and analytical thinking** | **Modeling and algebra** | **Ordering and structuring** | **Manipulation of formulas** | **Abstraction** | **Logical reasoning and proofs** |
|---|---|---|---|---|---|---|
| 1. The Lottery | X | X | | | | X |
| 2. The Weigh House | X | X | | | | X |
| 3. The Rijnland foot | | | | | X | X |
| 4. The fortress | | X | | X | X | |
| 5. Leiden's coat of arms | X | X | | | | X |
| 6. Snell's law | | | | X | | |
| 7. The gunpowder disaster | | | | X | | X |
| 8. The orange house | X | | X | | | |

(*Continued*)

[e] https://muurformules.nl/?ln=en [Accessed on May 4, 2022].

Table 4: (*Continued*)

| | Problem-solving and analytical thinking | Modeling and algebra | Ordering and structuring | Manipulation of formulas | Abstraction | Logical reasoning and proofs |
|---|---|---|---|---|---|---|
| 9. The Pieterskerk | X | | | | | X |
| 10. The lantern | | X | X | | X | |
| 11. The stepped gable | | X | X | | | X |
| 12. The cannon | | | | X | X | |
| 13. The windmill | | | | | X | X |
| 14. Gables | | | X | | | |
| Bonus: The bridge problem | X | X | X | | X | X |



### *Design and check*

For the final design, we decided to choose the simple format of a booklet, answer sheet, and teachers' guide. One reason for this was that it made it possible to meet our time limit of 3 weeks. We moreover reasoned that this was the easiest way to make the trail available to a wider public, as it could be made available both through a free PDF on the website of the university and in printed form together with the answer sheet at Rijksmuseum Boerhaave. The final version of the math trail was designed using InDesign.

The booklet consists of a map with the route and on each page a question followed by the direction to the next stop. There is room for the answers on each page, there are some empty pages at the end to use as scrap paper, and there is a printed set square tool on the last page, so that the users of the trail only need to bring a pen.

As we decided to include historical sights in our route, we wanted to include some information about them for those who are interested. As

this information is not related to the mathematics in the trail, we put the information into frames on the side, to make it clear that it was additional information and could be easily ignored by those who were not interested in this extra context.

The main audience was high school students who would do this trail with the school, therefore we extended the answer sheet to a teachers' guide. In this guide, extra practical information was included, such as the presence of a small café in the park in the middle of the route, where the teacher could take post to meet the pupils halfway. Another tip was that the students could bring their lunch and eat that in the park for a break along the way. The teachers' guide also expands the answer sheet with suggestions for continuing the discussion about the questions in the lessons, using the trail as an original method to start a topic from the theory to be taught. The teachers' guide is available via email at the university upon request of the teacher.

The last step in the development of the math trail is to thoroughly check all the material and to ask for feedback from people who were not involved in the development process to proofread all the material, including if possible, high school teachers. Our trail was developed in Dutch and due to its success, it was afterwards translated to English by the Academic Language Center of Leiden University.[f]

## Evaluation

When the trail was just published, the local media showed interest. Three radio programs featured an interview with one of the developers,[g,h,i]

[f] https://www.universiteitleiden.nl/en/language-centre [Accessed on May 4, 2022].

[g] https://www.nporadio1.nl/fragmenten/nieuws-en-co/8beb7fab-e27c-45c9-bcdc-106f1ab2e1c1/2017-02-15-wiskunde-in-de-leidse-buitenlucht [Accessed on May 4, 2022].

[h] https://sleutelstad.nl/2017/01/25/nationalisme-pijnstillende-cannabis-en-knuffelhormoon/ [Accessed on May 4, 2022].

[i] Omroep West (local Dutch radio station), program 'Mogge Michiel', January 20, 2017.

it was featured on news and math websites,[j,k] and the local newspaper Leids Dagblad wrote a short article on the trail.[l] It was also published on the university website[m] and announced on the website of Rijksmuseum Boerhaave.[n] Directly after the publication of the trail, there was no evaluation research, as this was not part of the student project and, as it so often goes, we all started working on other projects. However, we have seen people walk the trail in the city and the booklets at Rijksmuseum Boerhaave had to be reprinted half a year later (the first edition consisted of 350 booklets), which shows that there is a considerable secondary audience.

Since the trail was published in 2016, over 40 teachers have requested the teachers' guide. In 2020, when this chapter was written, we sent out a survey to them to evaluate the goals of the math trail. In total 19 out of the 47 addressed persons filled in the survey. Of these, 11 did the math trail with their students and indicated that they would like to do so again in later years. The reactions were predominantly positive. Many teachers did the trail as part of an excursion to Leiden in a project week, combined with a visit to one of the local museums for a different course than mathematics. The number of students for each teacher ranged from 20 to 125, mostly 50 or more per group. There was no grade attached to the trail, but the teachers thought of fun ways of encouraging their students, such as including the unraveling of a code through the exercises with which they could open a pot with candy at a stop with teachers in the middle of the walk. Another teacher included the option of buying "hints" for a reduction in points, to enhance the feeling of competition. The opinion on the level of the questions differed, many mentioned the level to be fine, some too easy, some too hard. Of course, this also depended on the age and grade of

[j] https://www.omroepwest.nl/nieuws/3336446/Leiden-heeft-eigen-wiskundewandeling [Accessed on May 4, 2022].

[k] https://pyth.eu/wandelen-in-leiden [Accessed on May 4, 2022] and https://pyth.eu/de-pythagoras-wandeling [Accessed on May 4, 2022].

[l] https://www.leidschdagblad.nl/cnt/dmf20180926_61900602/leiden-is-weer-een-wandelroute-rijker?utm_source=google&utm_medium=organic [Accessed on May 4, 2022].

[m] https://www.universiteitleiden.nl/nieuws/2017/01/voor-de-liefhebbers-van-wiskunde-de-leidse-wiskundewandeling [Accessed on May 4, 2022]; in English: https://www.universiteitleiden.nl/en/news/2017/01/math-trail-leiden [Accessed on May 4, 2022].

[n] https://rijksmuseumboerhaave.nl/te-zien-te-doen/spelen-met-wiskunde-abc/ [Accessed on May 4, 2022].

the students, as these varied from the target group to older students. On average the teachers thought the students enjoyed the activity.

The evaluation clearly showed that many teachers were inspired by the trail. Some modified it, used certain questions for a shorter version of the route, or added extra assignments. One teacher extended the excursion with an assignment afterwards, for which the students had to design some math trail questions themselves and present them to each other and parents for a grade. All teachers thought the trail had definitely added value, because it was "different from other school activities, nice to be outside", "provided the insight that math is present in the world around us" and "showed the practical application of math". One question was mentioned often as too hard because of the practical issue that the students had to stand on exactly the right spot in the street to solve it. There were a few other useful remarks and tips. It was mentioned that at least one accompanying teacher should be a math teacher. To combine with an excursion to a museum, it might be useful to break the walk into even smaller trails than the two provided. This would also solve the issue that for lower grade students a shorter trail was preferred and that it is quite a challenge to guide the number of students (40 or more) along the same trail. One teacher mentioned it was useful for them to walk the trail themselves first, to assess the level of the questions and estimate how much time students would need.

## Discussion and Conclusion

In this chapter, we have described the development process of a successful mathematics trail in a 3-week student project. We have shown the added value of the target audience and the background research, and hope that this will inspire others to develop a much-used math trail. For the target audience research, it would have been good to have a wider spread amongst the different levels of education in the target audience. As the surveys in our research have only been filled by students in the higher educational level of high school (VWO), the results may be biased. It is moreover good to do a test run, to make sure one is testing what one wants to test and not, for example, length of the question instead of the topic.

To further the development of new math trails using the experience of this project, time schedules and checklists can be found in Tables 5 and 6.

It would be of great value to do a few test walks with students and teachers of the target group to assess the level of the questions and to find out which type of questions create the most enthusiasm. Moreover, evaluation is recommended to further increase the usefulness of the trail to teachers and students. Besides contacting teachers, for example, diagnostic interviews with students might be of interest if time allows [5]. In order to make sure the trail is of value over the years, it is advisable to walk it yourself every now and then to check the route and directions. The evaluation research has moreover shown that teachers gladly use the trail as input and in this

Table 5: Time schedule and step-by-step plan.

| **Week 1** |
|---|
| Preparation, exploration, and draft<br>• Design target audience research (i.e., surveys and focus groups).<br>• Contact the target audience to hold surveys, etc.<br>• Literature search.<br>• Perform target audience research and start analysis of results.<br>• Route draft: walk around the city and take many pictures, keeping track of any inspiration that could be developed into a question and other useful information. |
| **Week 2** |
| Research analysis, route directions, and questions<br>• Work out literature research and target audience research.<br>• Write down the route description and draw a map with the stops.<br>• Ask for and incorporate feedback on the route description.<br>• Draft questions and keep track of the choices you make.<br>• Make the visual additions to questions (pictures and graphs).<br>• Ask for and incorporate feedback on the questions (can be combined with the route description feedback).<br>• Make a first draft of the trail booklet design. |
| **Week 3** |
| Design and check<br>• Proof-walk the route and incorporate any improvements and extra inspiration.<br>• Gather any other information that should be included, such as practical additions (set square and scratch paper) and historical information on the city highlights.<br>• Finish the questions and visual additions.<br>• Write the answers to make an answer sheet and optional teacher's guide.<br>• Layout the booklet (for example in InDesign or another design programme).<br>• Send the end product to the people who were involved in the process (teachers in the focus group, proof-walkers, feedback-givers).<br>• If required or desired, write research and development report. |

Table 6: Checklist math trail.

| |
|---|
| Route<br>• Do you have a clear map, on which the stops are indicated?<br>• Is it circular and about 2 hours (or with an extension)?<br>• Is your route foolproof? Can anyone follow the route description?<br>• Does it include varied scenery: historical sights, a park, a church? |
| Questions<br>• Do the questions target the different math areas: problem-solving and analytical thinking, modeling and algebra, ordering and structuring, manipulation of formulas, abstraction, and logical reasoning and proofs?<br>• Are the questions linked to visual representations?<br>• Are the questions fun and linked to things that can be seen during the trail? |
| Design<br>• Does the design contain: scratch space, a set square, references if needed, information about contact, and how to get the answer sheet?<br>• Do you have a clear distinction between questions and route description?<br>• Did you include contact details for requesting the answers and for letting you know when there was a problem following the route as described? |

format can easily adjust it to their needs and insights. The development of the math trail was an interesting and successful student project and is, 4 years later, still put to good use.

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