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The Dual Space: Concept and Applications in Cultural Heritage

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***Abstract*—An important question in cultural heritage concerns the make process of an artifact. Understanding the make process provides insight related to the origin, techniques and craftsmanship used to make the artifact. Searching for tool marks or traces left by the artist's hand is one way of retrieving clues related to the make process.**

X-ray computed tomography (CT) is a non-destructive tool that produces volumetric images of structures inside an artifact. However, interactively searching in large volumetric images for tool marks is a difficult, tedious and time consuming task.

In this article, we introduce the concept of a dual space. The governing idea is that the dual space represents the air in the interior of an object. In the context of cultural heritage, the dual space represents those materials that first belonged to the object but have been removed during the make process. Our main goal of creating the dual space is to facilitate searching, inspection and interpretation of tool marks.

We provide two examples of how the dual space can be used to study the make process.

***Index Terms*—cultural heritage, make process, x-ray computed tomography.**

I. INTRODUCTION

Many artifacts in cultural heritage have a rich 3D internal structure. While the visible outer surface has been crafted with a focus on aesthetics, a broad range of properties of the artifact are hidden beneath the surface, invisible to the naked eye.

X-Ray computed tomography (CT) is a non-destructive tool that can produce a detailed volumetric image of structures inside a scanned object. Although X-ray CT was originally developed for medical purposes, it has also been used in cultural heritage for diagnostics of art objects, [13]. With CT it is possible to detect morphological features, such as cracks and voids, that reside in the interior of an artifact. These morphological features can provide invaluable information regarding the degradation, conservation or restoration of an artifact. A plethora of examples have been reported in the literature, ranging from detecting growth rings in wooden painting frames and applying dendrochronology for dating [5], locating foreign materials, such as epoxy, in metal statues in order to diagnose past restorations [14], and detecting fingerprints in interiors of terracotta statues in authenticity purposes [9].

An important question in cultural heritage concerns the *make process* of an artifact. Answers to questions such as the sequence of steps that the artisan carried out whilst making the artifact, which materials were used, and which tools were used, can provide insight related to the origins and craftsmanship used to make the artifact, [4], [18].

Modeling the make process is a complex task, as it entails mechanical characteristics (which materials and which tools were used), as well as

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the creative input from the artisan. Here, we can distinguish between works of art that have been crafted according to a *recipe*, prescribing in detail the sequence of steps, and works of art where the creative freedom of the artisan is guiding the full make process.

Searching for tool marks or traces left by the artist's hand is one way of retrieving clues related to the make process. A tool mark is any indentation made when contact occurs between a tool and the artifact. Examples of tool marks are incisions made by carving knives on a soft surface, cuts made by a saw along the edge of a wooden object, or fingerprints in a terracotta sculpture.

In addition to providing an art historian with insights in the make process, tool marks can also be used to perform *quantitative measurements* related to the tools that were used. For example, if tools of different sizes were used, quantification of distances between tool marks can recover the individual tool used at a specific location of the artifact. As such, identifying all major tool marks is a necessary step towards a data-driven workflow for identifying the make process.

3D CT images are potentially useful for visualizing and measuring tool marks. By using standard iso-surface extraction techniques, a surface polygonal model can be extracted from the CT image and standard interactive computer graphics rendering tools can be used to display the model from different camera positions. However, interactively searching in a polygonal model for tool marks is not straightforward. Since tool marks are almost always located in the interior of the artifact, the tool mark will reside on the inner surface of the polygonal model. Since it is unknown where a tool mark can reside, searching for tool marks is a difficult, tedious and time consuming task.

In this article, we introduce the concept of the dual space visualization of an cultural heritage artifact. Whilst the dual space visualization is based on a sequence of well-known image processing steps, its novelty resides in the application to crafted objects in cultural heritage, where its interpretation has a direct correspondence to the make process of the object. Intuitively, the dual space is defined as the regions of air within an object and around its surface. The governing idea is that the structure of the dual space represents those materials that first belonged to the object but have been removed by the artisan during the make process. Tool marks on the inner surface of the object will also appear on the outer surface of the dual space. The aim of creating the dual space is to facilitate the inspection and interpretation of the make process.

In various subfields of zoology, *endocasts* are 3D representations that are used to visualize and measure the space of an organ in animal skeletons or fossils. Endocasts have been used for classification of different species, [17]. In skeletons this space is typically filled with air, whereas in fossils this space is filled with sediment. Procedures have been developed to construct virtual endocasts from an X-ray CT volume, [15], [2]. Casting is also used in the forensic sciences as it allows a crime scene investigator to collect an identical copy of a mark or print from a scene, which can then be compared to a seized tool, shoe, or tire in order to establish a link between a suspect and a crime scene, [7], [11]. Castings for forensic applications rarely use X-ray CT volumes, but are made by shaping a material in a mold.

Although the governing ideas of casting are similar to the dual space, the interpretation and usage of the dual space differs greatly. Whereas both representations are used for measuring size of features in the data, the dual space is used as a visual and interactive representation that facilitates to reason about the make process.

II. Dual Space

Hand crafted cultural heritage objects will typically have two surfaces; an outer surface which encloses the entire object, and an inner surface which encloses the air within the object. The dual space of an cultural heritage object is defined as the volumetric regions of air within the object. Intuitively, air pockets were first materials that belonged to the object but have been removed by the artisan during the make process.

The structure of the inner surface of an object surface resembles the outer surface of the dual space. A concave texture on the inner surface of an object will be represented as a convex texture on surface of the dual surface. In particular, tool marks on the inner surface of the object will be represented

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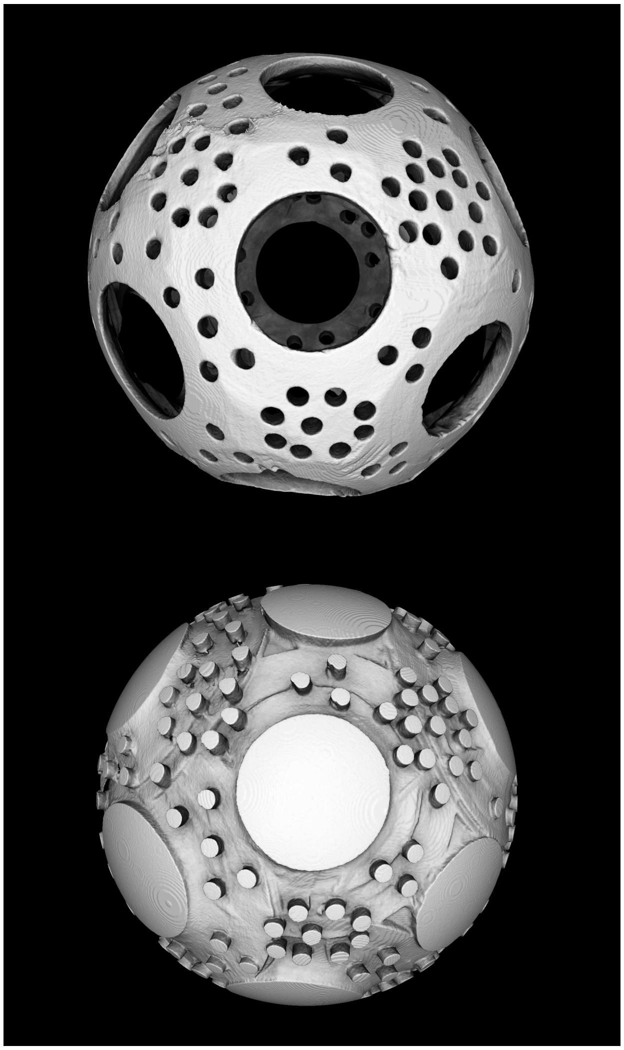

Fig. 1. An object and its dual space: a hollow sphere object with various large and small holes(top), and its dual space (bottom). The holes in the object are bulges in the dual space. Tool marks are clearly visible in the dual space.

as an 'inverted' tool mark on the surface of the dual space.

As an example, consider a hollow sphere object in the top panel of figure 1. The outer surface of the sphere contains many large and small holes. The, partially visible, inner surface shows the holes and also various tool marks shown as scratches on the inner surface. The dual space of the sphere is shown in the bottom panel. Here, the large and small holes are shown as bulges on the outer surface. The tool marks are clearly visible as concentric circles around each large hole.

In the appendix we discuss how the dual space can be computed.

A. From Tool Marks to Tool Morphology

The dual space is a representation that facilitates searching and measuring tool marks. Instead of visually searching for tool marks on a conventional visualization of the inner surface of the object, we search for tool marks on the surface of the dual space. It is important to note that in the sense of digital information, the dual space representation does not add new data to the original CT-dataset. Instead, it enables to introduce expert domain knowledge in an intuitive and effective way. Often a conservator can deduce properties of make process when one or more tool marks are clearly visualized. For example, depending on the shape of the tool mark, a conservator can deduce which tool was used and the when tool mark was made.

Once the tool marks are located, standard image processing techniques can be used to measure the shape and size of each mark and link these measurements to the morphology of the tools used to make the artifact.

In the bottom panel of figure 1, image processing techniques have been applied to measure the radius of the bulges, and the radius of the concentric circles around each large bulge on the surface of the dual space. These measurements are linked to the morphology of the drills and carving tools that were used to create the circles.

B. From Tools to Make Process

The make process can be defined as a sequence of steps which eventually result in the creation of the object. Each step is executed by the artisan with a specific tool.

The dual space is also used as a representation to gain insight into the ordering of the steps of the make process. When two distinct tool marks overlap, then it can often be deduced which tool mark was made first.

Consider the two concentric circles around each large bulge in the right panel of figure 1. The smaller circle is 'on top' of the larger circle. This means that the task which resulted in the smaller concentric circle was executed before the task which resulted in the larger concentric circle.

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III. Two Examples

In this section we illustrate through two case studies how the dual space enables effective discovery of tool marks, which in turn enables to derive quantitative features of the make process.

A. Openwork patterns on Chinese puzzle balls

Fig. 2. Nine layer puzzle ball.

Eighteenth century Chinese ivory puzzle balls are known for their beauty, finesse and their ability to captivate the curiosity of the viewer. A puzzle ball consists of several freely rotating concentric spheres, called *layers*. The outer layer is decorated with a floral scroll carving and each inner layer is decorated with a different geometric openwork pattern. Figure 2 is a photograph of a 18th century nine layer ivory puzzle ball from the Rijksmuseum (object AK-NM-7019). The radius of the outer layer is 4.3 cm.

Chinese puzzle balls were crafted by turning ivory, using only a lathe and a set of primitive drilling and carving tools. In a previous paper we have shown how the lathe and specialized carving tools were used to separated layers , [16].

After acquisition of the X-ray projections and 3D reconstruction into a 3D volume, the nine layers were segmented, and polygonal models were extracted from each layer. The top panel of figure 3 is a rendering of the polygonal model of the second layer. The pattern is visually complex and it is not obvious how it was carved. The surface of the dual space is rendered in the bottom panel of figure 3.

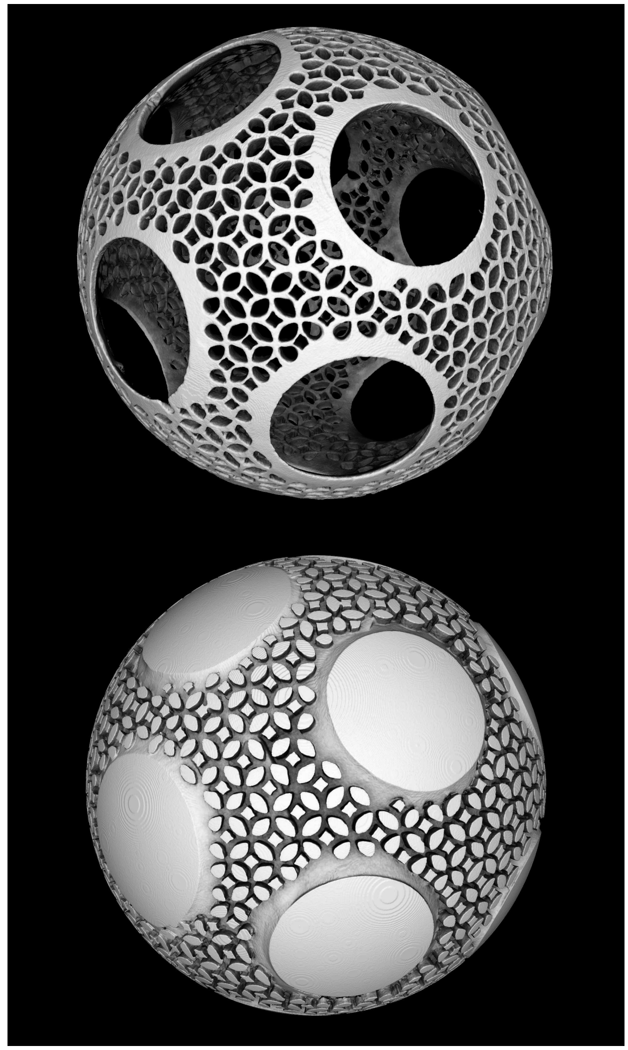

Fig. 3. Top: 3D rendering of the openwork pattern on the second layer. The carved pattern is visually complex. Bottom: 3D rendering of the dual space. The dual space shows that the pattern is carved as a collection of square and elliptical elements.

Inspection of this rendering reveals that the pattern is made by carving a large number of small square and ellipse shaped elements.

Due to the fact that carving is done by hand, there is variation in the sizes of shapes of each element. From the data, the average edge length of the square elements is 2 *mm*. The square can be made with 4 'punches' of a carving knife. Each punch is followed by rotating the knife 90 degrees to punch the next edge. In this layer the average length of the ellipse element's edge is 3 times as long as the average edge of the square element. An ellipse element is made with 6 punches. Thee punches for

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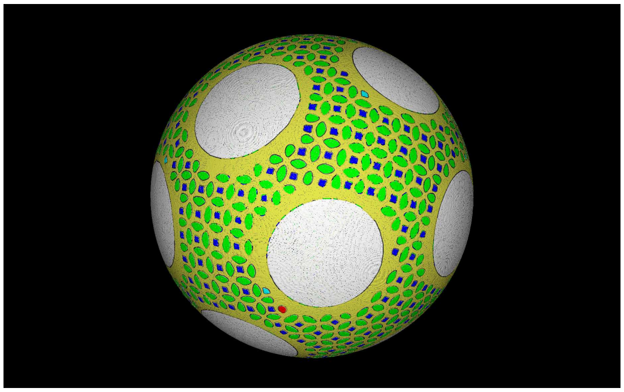

Fig. 4. Classification of the elements of the pattern. Square elements are drawn in blue and ellipse elements are green. This rendering shows how the pattern has been carved. First a set of equidistant squares on a line are carved and then four ellipses around each square are carved.

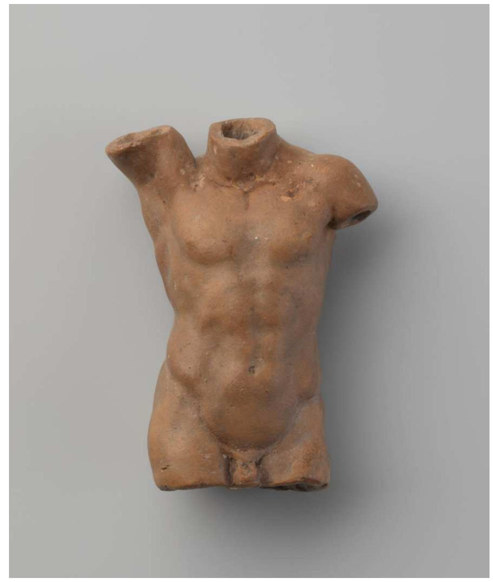

Fig. 5. van der Schardt torso.

one edge, rotate 180 degrees followed by 3 punches. From these measurements, we hypothesize that only one curve shaped carving tool was used to carve the element. The size of the tool's head is 2 *mm*.

Supervised machine learning techniques have been employed to classify every element in different classes. Figure 4 shows elements from four different classes; 383 square shaped elements (drawn in blue) , 816 elliptical shaped elements (green), 1 circle shaped (red) and 2 triangular (cyan) shaped elements.

By analyzing the position and neighborhood of each element, we can hypothesize on the carving ordering of the pattern. We believe that the artisan will fill a region by first carving square elements along a line (see blue elements). The distance between square elements is chosen such that an elliptical element can be placed between two square elements. After carving a number of lines, the artisan will fill the region with four ellipse elements around each square. When the artisan runs out of space, then a different element is sometimes chosen for filling. In this example, a circle shaped element (drawn in red) is carved instead of an ellipse, and two triangles (drawn in cyan) are carved instead of squares.

The geometric patterns on other layers are carved using different elements but following a similar procedure.

B. van der Schardt 16th century terracotta statue

Johan Gregor van der Schardt was 16th century Dutch sculptor who is well known for his works in painted terracotta busts, [9]. Terracotta objects are usually made by first shaping the exterior of the object then, using various sculpting tools, hollowing the object by removing some of the interior terracotta, and finally fire the object in a kiln. Figure 5 is a photograph of a van der Schardt terracotta torso (Rijksmuseum object BK-2016-44-5). The height of the object is 11.3 cm.

After acquisition, reconstruction and segmentation a polygonal model of the the torso is extracted from the data. Figure 6 shows the front of the torso (yellow) and its dual space (green).

The zoomed-in panel in figure 6 of the dual space shows many features that are difficult to see on the inner surface of the object. Very thin sharp ridges can be seen around the arm and shoulder regions, indicating incisions cuts made with a sharp knife. Cylinder shaped features in the arm regions indicate that a small spoon was used to scrape away terracotta. Fingerprints can be seen in the chest region, indicating the fingers were used to pat down and smooth the terracotta. Minutiae points can be extracted from the fingerprints, which can be used for authenticity purposes. The shape and size of the cylinder shaped features can be measured and the

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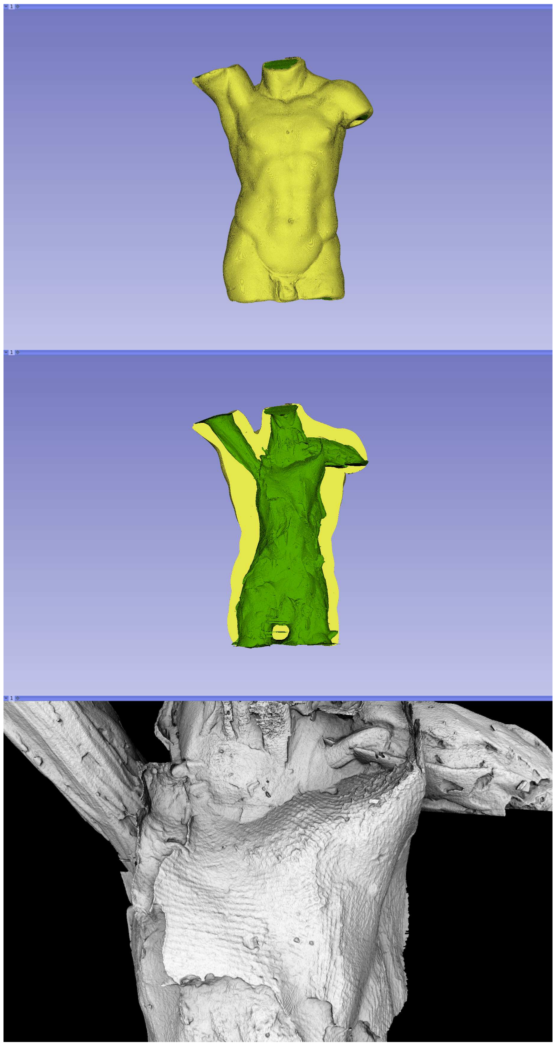

Fig. 6. Renderings of the van der Schardt torso). Top: the torso object drawn in yellow and the dual space (green) Middle: torso object front clipped away (yellow) revealing the dual space (green). Bottom: Zoomed in view of chest and arm region. The zoomed view shows small features such as fingerprints on the chest region. knife incisions shown as sharp ridges, spoon scrapings in the arm regions.

sizes can be linked to the size of the spoons that van der Schardt used.

By analyzing how features overlap it is possible to derive an ordering of operations used during the make process. For example, the knife incision operations in the arm pit region were made after the spoon scraping operations because the ridge shaped features intersect the cylinder shaped features. Also, the patting operations were applied after the spoon scraping operations because the fingerprint features cover the cylinder shaped features.

IV. Discussion

The dual space enables visualization-assisted discovery of tool marks and other imprints, that in turn can enable the deduction of key parameters of the make process (e.g. tool sizes, and applied operations). We envision that combining these observed data points with concrete models for the operations (as outlined in section II-B) will facilitate the algorithmic discovery of the make process, efficiently integrating expert knowledge of conservators into the data-driven process.

Any artifact, once completed by the artisan, is the result of a series of operations using one or more tools. In case of sculptures, the concept of "tool" should be extended to include impressions made by the artisan's hands. During the make process, each such operation takes the unfinished object one step further towards completion. At an abstract level, we can therefore view the make process as a concatenation of operators T_i that take the object from one state O_i to another state O_{i+1}:

$$O_N = T_N(T_{N-1}(...(T_1(O_0)))).$$

In a digital representation of the object, one can think of O_0 as a binary volume describing the raw material that was the starting point for creating the object. In the common case of tools that remove material from the object, each consecutive state in the sequence $\{O_1, O_2, \ldots, O_N\}$ then corresponds to a binary volume with an increasing number of voxels set to 0.

If the artisan works with a small set of tools, the options available at each step (i.e. the *parameters* of each operation) can be highly constrained. For instance, when applying stencil operations to an artwork, the only free parameters to choose at each step are the choice of the stencil and its location. In other cases, in particular when using human touch to modify the object, an enormous set of possible options can be chosen at each step.

A key feature common in artworks is that the order in which the operations are applied matters for the final result. In mathematical terms, the operations are not commutative. For example, when applying a knife to cut out material from the object, the first application of the knife will create a concavity that allows the subsequent operations to go

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deeper into the object. Clearly, reversing the order of operations would not work in this case. As a result, observing the tool marks in the finished object can potentially lead to reconstruction of the type and ordering of intermediate steps of the make process.

This leads to a wide range of general questions, including:

- Uniqueness: is the series of steps followed in the make process uniquely defined by the observed tool marks?
- Consistency: Could the object have been crafted by applying a particular set of operations (e.g. using a certain tool)?
- Reconstructability: Which parameters of the make process (e.g. tool sizes and types) can be deduced from the observed tool marks?

Such questions are common to the mathematical field of inverse problems, where they are studied based on a precise, mathematically defined, relationship between observed variables y and unknown model parameters x, following the relation y = F(x). As such, the problem of reconstructing the make process can be considered as an inverse problem, where the goal is to recover both the order and additional parameters (e.g. tool sizes) of the make process.

To the best knowledge of the authors, the problem of reconstructing the make process in cultural heritage has thus far not been considered as an integral computational problem, like we propose here, but rather has been focused either on recovering individual quantities, or on building hypothesis through a combination of observations and expert knowledge.

Whereas the real-valued parameters that describe the tools (e.g. tool size) and the positioning of the tool marks on the object (spatial coordinates) can in many cases be directly measured or inferred from the CT data using the dual space visualization, reconstructing the ordering of the make process will require more elaborate algorithmic approaches. A sensible model to capture the available information obtained from studying the tool marks is that of a *dependency graph*, [8], a Directed Acyclic Graph (DAG) where each operation is represented by a node, and a directed edge between node X and Y indicates that operation X must have preceded operation Y in the make process. The process of enumerating valid orderings can then be seen as an instance of the *Topological Sort* problem on the dependency graph, which can be solved in polynomial time. To deal with the complexities of real-world data, such basic models must then be generalized and made robust to incomplete data (tool marks may only be observable for certain operations) and noisy or inconclusive measurements.

Although the dual space does not provide other information than the information in the object, it does present the information in a different way. There are two reasons why the dual space facilitates searching for tool marks. First, tool marks on an inner surface of the object will be directly visible. More importantly, since tool marks are always (sometimes deep) concave features on a surface, tool mark features will be convex in the dual space. From vision research it is known that convex surfaces produce much greater perceived depth than concave surfaces with comparable relief [12].

In addition to tool mark discovery, the dual space visualization can also provide art historians and conservators with a range of other insights in the internal state of an object. As an example, consider the case of a scanned cornetto, a wind instrument. The top panel of figure 7 shows a photograph of a 16th century cornetto (Rijksmuseum object BK-AM-62-B). The dimensions is 58 x 4 x 4 cm. This cornetto is constructed out of two sections of different wood species; boxwood and cherry wood. Unfortunately, there is extreme structural damage due to wood-eating larva that infiltrated the instrument, [6]. The middle panel shows a 2D slice taken from the reconstructed tomogram. The dual space is drawn as a red line. The small dark regions indicate where larva have eaten the wood. The bottom panel shows the 3D representation of the dual space, clearly showing the larva tracks inside the instrument. Larva tracks can also be computed from the original X-Ray tomogram dataset using standard image processing vessel extraction techniques. However, these techniques often require prior information and are computationally expensive. Using the dual space to visualize the tracks does not require prior information of the track.

V. Conclusion

The dual space is a useful representation for obtaining insight to the make process of an artifact.

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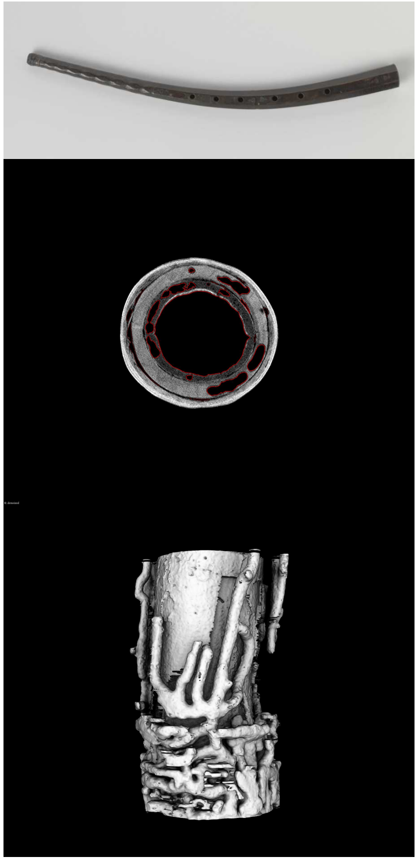

Fig. 7. Top: Cornetto Middle: An orthogonal slice of the cornetto tomogram. Holes made by wood-eating larva are visible in the slice as low intensity regions. A red line is drawn surrounding the dual space. Bottom: 3D rendering of the dual space. Worm tracks are clearly visible.

It is used as a representation to facilitate searching for tool marks, as well as reconstructing the steps performed by the artisan when making the artifact.

We have applied the dual space in two different cases. We have studied primitive carving operators and their ordering in openwork patterns on Chinese puzzle balls. Similarly, scraping operators and their ordering in a terracotta sculpture was investigated. Finally, we have discussed the prospects of using the dual space visualization as a tool towards formulating the make process discovery as an inverse problem.

VI. Acknowledgments

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APPENDIX

DUAL SPACE COMPUTATION

The voxel values in a 3D CT image correspond to X-ray attenuation, which reflects the proportion of X-rays absorbed as they pass through each voxel. X-ray attenuation is primarily a function of X-ray energy and the density of the object being imaged. Voxel values of air regions will have attenuation coefficients close to 0, whereas voxels belonging to an object will have larger values.

To compute the dual space, the 3D image is first thresholded into voxels representing the object and voxels representing air. Often this step will be proceeded with pre-processing filtering steps that mitigate noise and CT reconstruction artifacts, such as beam hardening and ring artifacts. Filtering is an essential step since tools marks may be obfuscated when not performed accurately. For example, valleys and ridges of a fingerprint in a 3D X-ray image may be difficult to detect without accurate noise mitigation filtering. The current implementation uses software beam hardening [12], a median filter [9] , and the Otsu method [14] to threshold the 3D image into regions of the object and air.

We denote the reconstructed image as the real valued matrix $\mathrm{I}[x, y, z]$ and the thresholded image as the matrix $\mathrm{B} = I' > \tau$, in which τ is the otsu computed threshold value and I' is the reconstructed image after applying beam hardening correction and a median filter.

After thresholding, a 3D convex hull [1] is used to define an outer boundary of the object. The convex hull surrounds the object, even in cases when the object has multiple holes. Next, the distance transform [10], of the object and its convex hull is computed. Voxels outside the convex hull and voxels belonging to the object are assigned the distance 0.

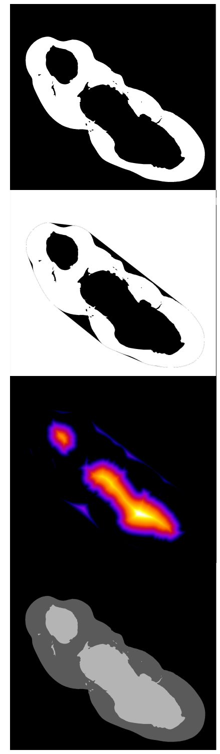

Fig. 8. Four steps in the dual space computation pipeline : after artifact mitigation and otsu, after convex hull, after distance transform, after region classification.

Let CHB[x,y,z] denote the image that combines the convex hull and object, and DT as the distance transform of CHB.

DT is a real valued image which consists of one or more regions of connected air voxels. Air regions that do not intersect the convex hull are regions that are fully enclosed by the object. These regions are classified as belonging to the dual space. Air regions which intersect the convex hull belong completely outside the object or enter the object through a hole in the surface. Air regions which intersect the convex hull are classified as belonging to the dual space if and only if the maximum distance of the region is larger than a user defined value.

Let $DS = \{r_i\}$ be the set of air regions which belong to the dual space. Region r_i belongs to the dual space if and only if the expression

$$\neg\ (\, r_i \cap \mathrm{C}HB) \vee (\, r_i \cap \mathrm{C}HB\ \wedge\ max(r_i) > \tau_u)$$

is true.

Figure 8 illustrates the procedure in a cross section of the 3D image. The top panel is the result

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of the binarization step. Voxel values are binary, indicating if the voxel belongs to the object or to the background. The second and third panel shows step of creating the the convex hull and the distance transform. The last panel show the dual space after region classification.