A Model for Pricing Collateralized Derivatives

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ABSTRACT

This paper presents a model for pricing derivatives subject to collateral posting. We find empirical evidence that market fluctuation has a larger impact on value at risk when credit risk is taken into account. Our research highlights the linkage between market and credit risk. The empirical results shed light on the economic and statistical significance of collateralization. The increase in the explanatory power of swap premium spreads bears an interesting finding: It seems that credit risk alone has a modest explanatory power on premium spreads. Only the combination of credit risk and collateralization can sufficiently explain them.

Key words: collateralization, asset pricing, plumbing of financial system, swap premium spread, CVA, VaR, interaction between market and credit risk

Collateral arrangements are regulated by the Credit Support Annex (CSA) and are always counterparty-based as different counterparties may have different CSA agreements. Thus, financial institutions normally group derivatives into counterparty portfolios first and then process them separately. The difference between counterparties is determined by counterparty credit qualities whereas the difference in collateralization is distinguished by the terms and conditions of CSA agreements.

Due to the complexity of collateralization, the literature seems to turn away from direct and detailed modeling. For example, Johannes and Sundaresan [2007], and Fuijii and Takahahsi [2012] model collateralization via a cost-of-collateral instantaneous rate. Piterbarg [2010] regards collateral as a regular asset and uses the replication approach to price collateralized derivatives.

This article makes theoretical and empirical contributions to the study of collateralization by addressing several essential questions. First, how does collateralization affect swap rate?

Unlike generic mid-market swap rates, swap premia are determined in a competitive market according to the basic principles of supply and demand. A swap client first contacts a number of swap dealers for a quotation and then chooses the most competitive one. If a premium is too low, the dealer may lose money. If a premium is too high, the dealer may lose the competitive advantage.

To circumvent this difficulty, this article uses an indirect empirical approach. We define a *swap premium spread* as the premium difference between two swap contracts that have exactly the same terms and conditions but are traded with different CSA counterparties. We reasonably believe that if two contracts are identical except counterparties, the swap premium spread should reflect counterparty credit risk only, as all other risks/costs are identical.

We then assess the joint effect. Because implied or model-generated spreads take into account both counterparty risk and collateralization, we assign the model-implied spreads as the explanatory variable and the market spreads as the response variable. The new adjusted R^2 is 0.9906, suggesting that counterparty risk and collateralization together have high explanatory power on premium spreads. The finding leads to practical implications, such as collateralization modeling allows forecasting credit spread.

Finally, how do collateralization and credit risk, either alone or in combination, impact market risk? How do they interact with each other? Value at risk (VaR) is the regulatory measurement for market risk We compute VaR in three different cases – VaR without taking credit risk into account, VaR with credit risk, and VaR with both credit risk and collateralization. We find that there is a positive correlation between market risk and credit risk as VaR increases after considering counterparty credit risk. We also find that collateralization and market risk have a negative correlation, i.e., collateral posting can actually reduce VaR. This finding contradicts the prevailing belief in the market that collateralization would increase market risk (see Collateral Management – Wikipedia).

Pricing Collateralized Derivatives

A CSA is a legal document that regulates collateral posting. It specifies a variety of terms including threshold, independent amount, and minimum transfer amount (MTA). A threshold is the unsecured credit exposure that a party is willing to bear. A MTA is used to avoid the workload associated with a frequent transfer of insignificant collateral amounts. An independent amount plays the same role as an initial margin or haircut. We define the effective collateral threshold as the threshold plus the MTA. Collateral is called as soon as the mark-to-market (MTM) value rises above the effective threshold.

Since the only reason for taking collateral is to reduce/eliminate credit risk, collateral analysis should be closely related to credit risk modeling. There are two primary types of models that attempt to describe default processes in the literature: structural models and reduced-form models. Many practitioners in the market have tended to gravitate toward the reduced-from models given their mathematical tractability and market consistency.

It is well-known that the survival probability from time t to s in this framework is defined by

$$p(t,s) := P(\tau > s \mid \tau > t) = \exp\left(-\int_t^s h(u)du\right) \tag{2a}$$

The default probability for the period (t, s) is given by

$$q(t,s) := P(\tau \le s \mid \tau > t) = 1 - p(t,s) = 1 - \exp\left(-\int_t^s h(u)du\right) \tag{2b}$$

Let valuation date be t. Consider a financial contract that promises to pay a $X_T > 0$ at maturity date $T > t$, and nothing before the time.

Suppose that there is a CSA agreement between a bank and a counterparty in which the counterparty is required to deliver collateral when the mark-to-market (MTM) value arises over the effective threshold H.

It is worth noting that the default payment in equation (4) is always greater than the original recovery, i.e., $P^D(T) > \varphi(T)X_T$ because $\varphi(T)$ is always less than 1. Said differently, the default payoff of a collateralized contract is always greater than the default payoff of the same contract without a CSA. That is why the major benefit of collateralization should be viewed as an improved recovery in the event of a default.

For a discrete one-payment period (t, T) economy, at time T, if the contract survives, the survival value is the promised payoff X_T and the collateral taker returns the collateral to the provider. If the contract defaults, the default payment is defined in (4) where the future value of the collateral is $C(T) = C(t)/D(t,T)$. Since the most predominant form of collateral is cash according to ISDA [2013], it is reasonable to consider the time value of money only for collateral assets. The large use of cash means that collateral is both liquid and not subject to large fluctuations in value. The above collateral rule tells us that collateral does not have any bearing on survival payoffs; instead, it takes effect on default payments only. The value of the CSA contract is the discounted expectation of all the payoffs and is given by

$$V^C(t) = E\left\{D(t,T)\left[q(t,T)\left(C(T) + \varphi(T)(X_T - C(T))\right) + p(t,T)X_T\right] \mid \mathcal{F}_t\right\} \tag{6}$$

After some simple mathematics, we have the following proposition

***Proposition 1**: The value of a collateralized single-payment contract is given by*

$$V^C(t) = E\left[F(t,T)X_T \middle| \mathcal{F}_t\right] - G(t,T) \tag{7a}$$

where

$$F(t,T) = \left(1_{V^N(t) \le H(t)} + 1_{V^N(t) > H(t)} / \bar{I}(t,T)\right) I(t,T) D(t,T) \tag{7b}$$

$$G(t,T) = 1_{V^N(t) > H(t)} H(t) \bar{q}(t,T) \left(1 - \varphi(T)\right) / \bar{I}(t,T) \tag{7c}$$

where $\bar{I}(t,T) = E\left(I(t,T) \middle| \mathcal{F}_t\right)$. $I(t,T)$ and $V^N(t)$ are defined in (3).

Proof: See the Appendix.

We may think of $F(t,T)$ as the CSA-adjusted discount factor and $G(t,T)$ as the cost of bearing unsecured credit risk. Proposition 1 tells us that the value of a collateralized contract is equal to the present value of the payoff discounted by the CSA-adjusted discount factor minus the cost of taking unsecured counterparty credit risk. This proposition theoretically demonstrates that collateral posting changes valuation.

We discuss a special case where $H(t) = 0$ corresponding to full-collateralization. Suppose that default probabilities are uncorrelated with interest rates and payoffs[1]. From Proposition 1, we can easily obtain $V^C(t) = V^F(t)$ where $V^F(t) = E\left[D(t,T)X_T \middle| \mathcal{F}_t\right]$ is the risk-free value. That is to say: the value of a fully-collateralized contract is equal to the risk-free value. This conclusion is in line with the results of Johannes and Sundaresan [2007], Fuijii and Takahahsi [2012], and Piterbarg [2010].

***Proposition 2:** The value of a collateralized multiple-payment contract is given by*

$$V^C(t) = \sum_{i=1}^{m} E\left[\prod_{j=0}^{i-1}\left(F(T_j, T_{j+1})\right) X_i \middle| \mathcal{F}_t\right] - \sum_{i=0}^{m-1} E\left[\prod_{j=0}^{i-1}\left(F(T_j, T_{j+1})\right) G(T_i, T_{i+1}) \middle| \mathcal{F}_t\right] \tag{8a}$$

where

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[1] Moody's Investor's Service [2000] presents statistics that suggest that the correlations between interest rates, default probabilities and recovery rates are very small and provides a reasonable comfort level for the uncorrelated assumption.

$$F(T_j,T_{j+1}) = \left(1_{J(T_j,T_{j+1})\le H(T_j)} + 1_{J(T_j,T_{j+1})>H(T_j)} / \bar{I}(T_j,T_{j+1})\right) I(T_j,T_{j+1}) D(T_j,T_{j+1}) \tag{8b}$$

$$G(T_j,T_{j+1}) = 1_{J(T_j,T_{j+1})>H(T_j)} H(T_j) \bar{q}(T_j,T_{j+1}) \left(1-\varphi(T_{j+1})\right) / \bar{I}(T_j,T_{j+1}) \tag{8c}$$

$$J(T_j,T_{j+1}) = E\left[D(T_j,T_{j+1}) I(T_j,T_{j+1}) \left(V^C(T_{j+1}) + X_{j+1}\right) \middle| \mathcal{F}_{T_j} \right] \tag{8d}$$

where $\prod_{j=0}^{i-1} F(T_j,T_{j+1}) = 1$ is an empty product when $i = 0$. Empty product allows for a much shorter mathematical presentation of many subjects.

The valuation in Proposition 2 has a backward nature. The intermediate values are vital to determine the final price. For a payment period, the current price has a dependence on the future price. Only on the final payment date T_m, the value of the contract and the maximum amount of information needed to determine $J(T_{m-1},T_m)$, $F(T_{m-1},T_m)$ and $G(T_{m-1},T_m)$ are revealed. This type of problem can be best solved by working backward in time, with the later value feeding into the earlier ones, so that the process builds on itself in a recursive fashion, which is referred to as *backward induction*. The most popular backward induction algorithms are lattice/tree and regression-based Monte Carlo.

Empirical Results

In this subsection, we choose interest rate swaps for our empirical study. Ultimately, it is the objective of this subsection to test if counterparty credit risk and collateralization are sufficient to explain market swap premium spreads. We choose a statistical measurement R^2 to determine how much market spreads can be interpreted by model-implied spreads that take counterparty risk and collateralization into account.

Swap rate is the fixed rate that sets the market value of a swap at initiation to zero. ISDAFIX provides average mid-market swap rates based on a mid-day polling from a panel of dealers. In practice, the mid-market swap rates are generally not the actual swap rates transacted with counterparties, but are instead the benchmarks against which the actual swap rates are set. A swap dealer that arranges a contract and provides liquidity to the market involves costs. Therefore, it is necessary to adjust the mid-market swap rate to cover various transacting expenses and also to provide a profit margin to the dealer. As a result, the actual price agreed for a transaction is not zero but a positive amount to the dealer.

Prior research has primarily focused on the generic mid-market swap rates and results appear puzzling. Sorensen and Bollier [1994] believe that swap spreads are partially determined by counterparty default risk. Whereas Duffie and Huang [1996], Minton [1997] and Grinblatt [2001] find weak or no evidence of the impact of counterparty credit risk on swap spreads. Collin-Dufresne and Solnik [2001] and He [2001] further argue that many credit enhancement devices, e.g., collateralization, have essentially rendered swap contracts risk-free. Meanwhile, Duffie and Singleton [1999], and Liu, Longstaff and Mandell [2006] conclude that both credit and liquidity risks have an impact on swap spreads. Moreover, Feldhutter and Lando [2008] find that the liquidity factor is the largest component of swap spreads. It seems that there is no clear-cut answer yet regarding the relative contribution of various factors.

A swap premium is supposed to cover the expected profit and all the expenses, including the cost of bearing unsecured credit risk. Unfortunately, however, we do not know what percentage of the market swap premium is allocated to the unsecured credit risk, which makes a direct verification impossible.

The dataset contains derivative contract data, counterparty data (including collateral agreements, recovery rates, etc), and market data. The trading dates are from May 6, 2005 to May 11, 2012. We find a total of 1002 swap pairs in the dataset, where the two contracts in each pair have the same terms and conditions but are traded with different CSA counterparties. We arbitrarily select one pair shown in Exhibit 1.

Exhibit 1: A pair of 20-year swap contracts

This exhibit displays the terms and conditions of two swap contracts that have different counterparties but are otherwise the same. We hide the counterparty names according to the security policy of the investment bank while everything else is authentic.

	Swap 1		Swap 2	
	Fixed leg	**Floating leg**	**Fixed leg**	**Floating leg**
Counterparty	*X*		*Y*	
Effective date	15/09/2005	15/09/2005	15/09/2005	15/09/2005
Maturity date	15/09/2025	15/09/2025	15/09/2025	15/09/2025
Day count	30/360	ACT/360	30/360	ACT/360
Payment frequency	Semi-annually	Quarterly	Semi-annually	Quarterly
Swap rate	**4.9042%**	-	**4.9053%**	-
Roll over	Mod_follow	Mod_follow	Mod_follow	Mod_follow
Principal	25,000,000	25,000,000	25,000,000	25,000,000
Currency	USD	USD	USD	USD
Pay/receive	Bank receives	Party *X* receives	Bank receives	Party *Y* receives
Floating index	-	3 month LIBOR	-	3 month LIBOR
Floating spread	-	0	-	0
Floating reset	-	Quarterly	-	Quarterly

An interest rate curve is the term structure of interest rates, derived from observed market instruments that represent the most liquid and dominant interest rate products for certain time horizons. Normally the curve is divided into three parts. The short end of the term structure is determined using LIBOR rates. The middle part of the curve is constructed using Eurodollar futures. The far end is derived using mid swap rates. The LIBOR-future-swap curve is presented in Exhibit 2. After bootstrapping the curve, we get the continuously compounded zero rates.

Exhibit 2: USD LIBOR-future-swap curve

This exhibit displays the closing mid prices as of September 15, 2005

Instrument Name	**Price**

September 21 2005 LIBOR	3.6067%
September 2005 Eurodollar 3 month	96.1050
December 2005 Eurodollar 3 month	95.9100
March 2006 Eurodollar 3 month	95.8100
June 2006 Eurodollar 3 month	95.7500
September 2006 Eurodollar 3 month	95.7150
December 2006 Eurodollar 3 month	95.6800
2 year swap rate	4.2778%
3 year swap rate	4.3327%
4 year swap rate	4.3770%
5 year swap rate	4.4213%
6 year swap rate	4.4679%
7 year swap rate	4.5120%
8 year swap rate	4.5561%
9 year swap rate	4.5952%
10 year swap rate	4.6368%
12 year swap rate	4.7089%
15 year swap rate	4.7957%
20 year swap rate	**4.8771%**
25 year swap rate	4.9135%

As the payoffs of an interest rate swap are determined by interest rates, we need to model the evolution of floating rates. Interest rate models are based on evolving either short rates, instantaneous forward rates, or market forward rates. Since both short rates and instantaneous forward rates are not directly observable in the market, the models based on these rates have difficulties in expressing market views and quotes, and lack agreement with market valuation formulas for basic derivatives. On the other hand, the object modeled under the Libor Market Model (LMM) is market-observable. It is also consistent with the market standard approach for pricing caps/floors using Black's formula. They are generally considered to have more desirable theoretical calibration properties than short rate or instantaneous forward rate models. Therefore, we choose the LMM lattice proposed by Xiao [2011] for pricing collateralized swaps.

Exhibit 3: CDS premia and recovery rates

This exhibit displays the closing CDS premia as of September 15, 2005 and recovery rates

Counterparty name	**Bank**	**Company *X***	**Company *Y***
6 month CDS spread	0.00031	0.000489	0.000808
1 year CDS spread	0.000333	0.00056	0.001017
2 year CDS spread	0.000516	0.000866	0.00154
3 year CDS spread	0.000664	0.001147	0.002114
4 year CDS spread	0.000848	0.00147	0.002768
5 year CDS spread	0.001012	0.001783	0.003439
7 year CDS spread	0.001334	0.002289	0.004283
10 year CDS spread	0.001727	0.002952	0.005281
15 year CDS spread	0.001907	0.003283	0.005814
20 year CDS spread	0.002023	0.003266	0.006064
30 year CDS spread	0.002021	0.00336	0.006461
Recovery rate	0.39213	0.35847	0.33872

Exhibit 4: CSA agreement

This exhibit provides the collateral thresholds and MTAs under the CSA agreements.

CSA agreement	**1**		**2**	
Counterparty name	Bank	Company *X*	Bank	Company *Y*
Threshold	0	0	0	0

MTA	500000	500000	500000	500000

Given the above information, we are able to compute the collateralized swap rates. We first use the LMM to evolve the interest rates and then determine the associated CSA-adjusted discount factors as well as the cost of bearing unsecured credit risk according to Proposition 2. Finally, we calculate the collateralized swap rates via a backward induction method. The results are given in Exhibit 5.

Exhibit 5: Swap rate results

This exhibit presents the model-implied swap rates and premia as well as the dealer-quoted market swap rates and premia, where Swap premium (in bps) = Swap rate – Generic swap rate, and Premium spread = Premium of swap 2 – Premium of swap 1.

	Swap 1		**Swap 2**		**Premium spread**	**Generic swap rate**
	Swap rate	**Premium**	**Swap rate**	**Premium**		
Model-implied	0.048780	0.09 bps	0.048790	0.19 bps	0.10 bps	0.048771
Dealer quoted	0.049042	2.71 bps	0.049053	2.82 bps	0.11 bps	

The 20-year generic mid-market swap rate is 0.048771 shown in Exhibit 2. The swap rates of contracts 1 and 2 are given in Exhibit 1 as 0.049042 and 0.049053. Accordingly, the market swap premia are 2.71 (0.049042 - 0.048771) basis points (bps) and 2.82 (0.049053 - 0.048771) bps. These premia are charged for many expenses. Although we do not know what percentage of the premia are allocated to cover the unsecured credit risks, we reasonably believe that the market premium spread, 0.11 (= 2.82 – 2.71) bps in Exhibit 5, should solely reflect the difference between the two counterparties' unsecured credit risks, as other factors are identical.

Exhibit 5 shows that the model-implied spread is quite close to the dealer-quoted spread, suggesting that the model is fairly accurate in pricing collateralized financial instruments.

Exhibit 6: Summary statistics of model-implied swap premium spreads, market swap premium spreads and model-market swap premium spread differentials

All values are displayed in bps. Model-market swap premium spread differential = Model-implied swap premium spread – Market-quoted swap premium spread.

	Max	Min	Mean	Median	Std
Market quoted swap premium spreads	3.08	-5.25	-0.46	-0.15	1.80
Model-implied swap premium spreads	2.10	-5.33	-0.44	0.03	1.73
Model-market premium spread differentials	0.99	-1.18	-0.03	0.05	0.46

Next, we examine the effects of credit risk and collateralization, alone or combined, on swap premium spreads. First, we study the marginal effect of credit risk. For each swap pair, we obtain the counterparty CDS premia. Presumably, the differences in CDS premia should mainly represent the differences in counterparty risk. To determine the strength of the statistical relationship between market premium spreads and counterparty risk, we present the estimate of the following regression model.

$$Y = a + bX + \varepsilon \tag{9}$$

where Y is the market swap premium spread, X is the difference between the counterparty CDS premia, a is the intercept, b is the slope, and ε is the regression residual.

Exhibit 7: Marginal credit risk regression results

This exhibit presents the regression results for the following regression model:

MarketSwapPremiumSpreads = a + b * DifferencesBetweenCounterpartyCDSs + ε

where the market swap premium spreads are used as the dependent variable and the differences between the counterparty CDS premia as the explanatory variable.

Slope	**Intercept**	**Adjusted R^2**	**Significance F**	**T value**	**P value**

0.0127	-2.7E-04	0.7472	1.1E-05	18.6	1.51E-66

According to ISDA Margin Survey (ISDA [2013], 73.7% of OTC derivatives are subject to collateral agreements. For large firms, the figure is 80.7%. Accounting for collateralization has become increasingly important in pricing OTC derivatives. Since the implied spreads generated by our model take into account both credit risk and collateralization, the statistical relationship between the market spreads and the model-implied spreads should refer to the joint effect of counterparty credit risk and collateralization on market spreads. Thus, we present another regression model where the market spreads are regressed on the implied spreads. The regression results are shown in Exhibit 8.

Exhibit 8: Credit risk and collateralization combined regression results

This exhibit presents the regression results for the following regression model:

$$MarketSwapPremiumSpreads = a + b * ImpliedSwapPremiumSpreads + \varepsilon$$

where the market swap premium spreads are used as the dependent variable and the model-implied swap premium spreads as the explanatory variable.

Slope	**Intercept**	**Adjusted R^2**	**Significance F**	**T value**	**P value**
0.9857	4.48E-05	0.9906	3.21E-08	25.4	3.16E-110

Exhibit 8 shows that the adjusted R^2 value is 0.9906, implying that approximately 99% of the market spreads can be explained by the implied spreads. Also the small p-value suggests that the changes in implied spreads are strongly related to the changes in market spreads.

In this subsection, we study how collateralization affects credit risk by measuring CVA changes due to collateral posting. CVA is the market price of counterparty credit risk that has become a central part of counterparty credit risk management.

Second, we assume that there is counterparty credit risk but no collateral agreement. Based on the pricing model proposed by Xiao [2015], we compute the risky value of the portfolio as $V^N = 2{,}688{,}014$ after considering counterparty credit risk. By definition, the CVA without collateralization is equal to $CVA_N = V^F - V^N = 49{,}688$.

Exhibit 9. Impact of Collateralization on CVA

This exhibit shows that CVA increases with collateral threshold. The infinite collateral threshold is equivalent to no collateral agreement and the zero-value collateral threshold corresponds to full collateralization. An increase in collateral threshold leads to a decrease in collateralization.

Effective Threshold	0	2.1 Million	4.1 Million	6.1 Million	8.1 Mil	Infinite (∞)
CVA	0	12,608	23,685	33,504	42,254	49,688

Exhibit 9 tells us that collateral posting can reduce CVA. Full collateralization makes a portfolio appear to be risk-free. An increase in collateral threshold leads to a rise in unsecured credit exposure, and thereby an increase in CVA. In particular, CVA reaches the maximum when the threshold is infinite representing no collateral arrangement.

We study how collateralization impacts market risk by gauging VaR changes due to collateral arrangements. VaR is the regulatory measurement for assessing market risk. It is defined as the maximum loss likely to be suffered on a portfolio for a given probability defined as a confidence level over a given period time. In its most general form, VaR measures 10-day 99th percentile of potential loss that can be incurred.

For monitoring market risk, many organizations segment portfolios in some manner. They may do so by traders and trading desks. Typically, a market risk portfolio contains derivatives across multiple counterparties, while a counterparty portfolio comprises transactions among different traders and trading desks. We fortuitously select a trading portfolio and then partition it into single-counterparty sub-portfolios.

One sub-portfolio contains 172 interest rate swaps, 68 caps and floors, 25 European swaptions and 17 cancelable swaps.

For many reasons, both historical and practical, market and credit risk have often been treated as if they are unrelated source of risk: the risk types have been measured separately, managed separately, and economic capital against each risk type has been assessed separately.

Therefore, we further calculate VaR by taking credit risk into account. Let us first assume that there is no CSA agreement. Each trade is valued by the risky model developed by Xiao [2015]. The VaR with credit risk is computed as -418,948. It can be seen that VaR has increased from 386,570 to 418,948 after accounting for credit risk. The above empirical results show that market fluctuation has a larger impact on VaR when credit risk is taken into account. Our research highlights the linkage between market and credit risk.

Exhibit 10. Impact of Collateralization on VaR

This exhibit shows that VaR increases with collateral threshold. The infinite collateral threshold is equivalent to no collateral agreement and the zero-value collateral threshold corresponds to full collateralization.

Effective Threshold	0	2.1 Million	4.1 Million	6.1 Mil	Infinite (∞)
VaR	-386,570	-398,235	-404,401	-410,756	-418,950

Exhibit 10 tells us that collateral posting actually can reduce VaR. The results show that market risk exposure rises as collateral threshold increases. In particular, VaR reaches the maximum when the threshold is infinite representing no collateral arrangement. We come to the same conclusion after repeating this test for other portfolios.

Conclusion

This article addresses an important topic of the impact of collateralization on valuation and risk. We present a new model for pricing collateralized financial contracts based on the fundamental principal and legal structure of CSA. The model can back out market prices. This is very useful for pricing outstanding collateralized derivatives.

We find strong evidence that counterparty credit risk alone plays a significant but not overwhelming role in determining credit-related spreads. Only the joint effect of collateralization and credit risk has high explanatory power on unsecured credit costs. This finding suggests that failure to properly account for collateralization may result in significant mispricing of derivatives.

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