PHILOSOPHICAL TRANSACTIONS A

royalsocietypublishing.org/journal/rsta

Introduction: the mathematics of energy systems

Pierluigi Mancarella[1], John Moriarty[2], Andy Philpott[3], Almut Veraart[4], Stan Zachary[5] and Bert Zwart[6,7]

[1]School of Electrical and Electronic Engineering, University of Melbourne, Parkville VIC 3010 Australia
[2]School of Mathematical Sciences, Queen Mary University of London, London E1 4NS, UK
[3]Engineering Science, University of Auckland, Auckland 1142, New Zealand
[4]Mathematics, Imperial College, London SW7 2AZ UK
[5]Heriot-Watt University, Edinburgh EH14 4AS, UK
[6]CWI, Amsterdam, The Netherlands
[7]TU/e, Eindhoven, The Netherlands

JM, 0000-0002-4520-4133; SZ, 0000-0003-1221-9352

Introduction

Check for updates

Cite this article: Moriarty J, Philpott A, Veraart A, Zachary S, Zwart B. 2021 Introduction: the mathematics of energy systems. *Phil. Trans. R. Soc. A* **379**: 20190425. https://doi.org/10.1098/rsta.2019.0425

Accepted: 30 March 2021

One contribution of 11 to a theme issue 'The mathematics of energy systems'.

Subject Areas:
power and energy systems, applied mathematics

Keywords:
energy transition, energy markets, decarbonization

Author for correspondence:
John Moriarty
e-mail: j.moriarty@qmul.ac.uk

The urgent need to decarbonize energy systems gives rise to many challenging areas of interdisciplinary research, bringing together mathematicians, physicists, engineers and economists. Renewable generation, especially wind and solar, is inherently highly variable and difficult to predict. The need to keep power and energy systems balanced on a second-by-second basis gives rise to problems of control and optimization, together with those of the management of liberalized energy markets. On the longer time scales of planning and investment, there are problems of physical and economic design. The papers in the present issue are written by some of the participants in a programme on the mathematics of energy systems which took place at the Isaac Newton Institute for Mathematical Sciences in Cambridge from January to May 2019—see http://www.newton.ac.uk/event/mes.

This article is part of the theme issue 'The mathematics of energy systems'.

The first two articles are opinion pieces concerned with problems in the design of day-to-day markets. Karangelos & Panciatici [1] advocate for a progressive rethinking of the day-ahead/intra-day market arrangements to deal with power system security across

THE ROYAL SOCIETY PUBLISHING

multiple areas (e.g. different countries in the European interconnected system) while the grid becomes increasingly low-carbon. In fact, with more and more renewables in the system, the associated variability and forecast uncertainty create more challenges in terms of how power flows across different areas under the jurisdiction of different transmission system operators (TSOs). These flows need to be managed to preserve system security. The paper argues that, in order to fully exploit the value of existing grid flexibility to economically support security management across several interconnected areas, multi-TSO coordination could be achieved by adopting a new approach inspired by the principles of cooperative game theory. More specifically, the authors' proposed approach aims to evaluate how and to what extent each control area impacts in a positive or negative way on the common security performance of the multi-area system, so that clear and transparent economic incentives could be provided to achieve optimal coordination. Notably, while driven by economic principles, the approach facilitates the inclusion of more detailed physical modelling (e.g. non-convexities of power system operation) in the inter-TSO settlement of the security management cost of the interconnected system. This further ensures that the resulting economic incentives are also efficient to actually preserve secure physical operation. The approach is successfully demonstrated on a proof-of-concept test system and opens a number of discussions, including computational complexity and requirements, and the research and development pathways that are needed for real-scale applications.

The paper by Luna *et al.* [2] addresses a long-standing issue with pricing electricity supply that is subject to indivisibilities in production, primarily arising from start-up costs of thermal plant. System marginal prices are the Lagrange multipliers of supply–demand constraints of optimal economic dispatch models when these are convex optimization problems. In models with indivisibilities such as start-up costs, convexity is lost, and Lagrange multipliers might fail to exist at optimality, so marginal prices need to be defined so as to account for this. Over the past 25 years, electricity market designers have adopted different approaches to dealing with this issue. The renewed attention in the Luna *et al*. paper arises from the very recent development of economic dispatch and pricing algorithms for the Brazilian day-ahead electricity market. The authors propose a methodology for economic dispatch and subsequent approximation of marginal prices that meets two objectives (the opposite sides of the coin): first the dispatch should satisfy all system constraints and be at least cost and marginal prices should be close to a target determined from this solution; second each generator should receive remuneration (possibly with extra uplift payments) to meet their operating costs over a given dispatch period. The authors claim that this approach yields more stable prices over time than competing approaches, including the current approach adopted by the Brazilian system operator. This is illustrated in the paper using some numerical simulations.

The mathematics of energy systems (MES) programme's first 'research track', titled 'Look-ahead operational planning under uncertainty', had significant focus on the incorporation of weather forecast information within algorithms for operational planning. Part of the complexity here stems from a large state space: as operational time progresses, not only are new observations received, but forecasts of future states are periodically updated, and operational plans may also be modified. De Chalendar & Glynn's paper [3] makes a valuable contribution in this direction in the linear case. More precisely, the authors formulate an augmented state space with linear dynamics, within which dynamically updated forecast information can be incorporated alongside the underlying state variable. Clearly, the dynamics of the forecasts as they update should be consistent with the forecasted state variables themselves, and this is ensured by the use of the martingale model of forecast evolution. The ability to handle dynamically updating forecasts is of increasing importance as the generation side grows more variable and demand side participation becomes more active. The formulation provides a tractable setting to incorporate forecast information within a Markov decision process (MDP) framework and the authors discuss different possible approaches, each leading to a different, but computationally tractable, MDP. Notably, the authors' work also opens the door to the study of corresponding frameworks with nonlinear dynamics.

The role of consumers in the energy transition is the focus for Radoszynski and Pinson's contribution [4]. A main challenge here is that this role is changing and potentially becoming more active, but in a way that is not yet well understood. The idea of demand-side management is to incentivize customers to support the grid by being flexible, often by means of interaction between a central coordination mechanism and consumers with controllable loads. This can be viewed as a problem of decentralized control: that is, agents make decisions using their local information combined with partial information about the whole system. This perspective recognizes customer autonomy while offering the potential of both scalability and appropriate privacy preservation. The authors apply the framework of mean-field games, in which the decisions of each agent depend on the average state of the population (which is taken to be infinite) rather than one-to-one interactions. However, rather than assuming that agents are rational decision-makers, as is commonly done, the authors model deviations from optimal responses by assuming only bounded rationality. The specific problem under study is coordinating a large population of heat pumps through dynamic pricing, taking into account state and dynamic constraints. Through a suitable case study, bounded rationality is found to have a marked effect on the equilibria, and consequently on electricity prices, comfort and total costs, in this important problem of decentralized control.

Ensembles and scenarios are widely used in the context of dynamic and stochastic programming problems in a wide range of applications, including trading, storage and maintenance in energy and power systems. In order to make scenario-based approaches computationally tractable, typically their dimension needs to be reduced. Naturally, this raises the question of how such a reduction can be performed in an *optimal* way. The question of optimality is linked to the choice of a suitable distance measure. While the Wasserstein distance has been widely used in this context, the article by Ziel [5] proposes the use of the so-called *energy distance*, a special case of the maximum mean discrepancy, for ensemble and scenario reduction. This approach is supported by the finding that the energy distance can be used for testing for equality of arbitrary multivariate distributions or for their independence. Moreover, the statistical properties of the scenarios obtained when using the energy distance rather than the Wasserstein distance appear to be favourable. The methodological discussion is supported by three applications: a generic Bernoulli random walk and two applications in energy systems. For the latter, scenarios of electricity load profiles (based on quarter-hourly German electricity demand data) and of electricity prices (hourly German day-ahead electricity prices) are considered. The applications give indications of the advantages and disadvantages of the energy distance compared with the Wasserstein distance depending on which parts of the distribution are of interest in an application. This article presents an interesting alternative to the widely used Wasserstein metric for scenario reduction. Natural extensions of this work would include a comparison of the scenario-reduction techniques by means of a subsequent optimization problem.

The reliability of modern power systems was an important theme of the MES programme, and has been inspirational to several articles in this special issue. One of the most fundamental characteristics of a power system is its frequency: this needs to be kept close to its nominal value in order for a power system to function properly. Traditionally, this is done using the rotational inertia of synchronously-connected generators, but as power systems decarbonize, such generators decrease in prevalence. At the same time, fluctuations of solar and wind power result in fluctuations of the system frequency. The article by Goodridge *et al.* [6] studies the use of carefully reformulated random sampling techniques to study rare events in a power system where grid-scale batteries address these challenges by providing both regulation and emergency frequency control ancillary services. Using a model of random power disturbances at each bus, the authors employ the recently developed skipping sampler (a Markov Chain Monte Carlo algorithm for rare-event sampling) to construct conditional distributions of the power disturbances leading to two kinds of instability: frequency excursions outside the normal operating band, and load shedding. Notably, the authors' approach enables time-domain studies of power system robustness and resilience under uncertainty, taking into account detail such as feedback from emergency protection schemes, without any need for strong statistical assumptions.

A further fundamental theme of the MES programme was that of how to move energy through time: while electrical grids move energy through space, the scheduling of grid components such as generation, storage and demand-side response shifts energy along the time axis. The article by Zachary *et al.* [7] focuses on the potentially much greater role of storage in the balancing of future electricity systems. The authors show how heterogeneous stores, differing in capacity and rate constraints, may be optimally, or nearly optimally, scheduled to assist in such balancing, with the aim of minimizing the total imbalance (unserved energy) over any given period of time. The authors show that in many cases the optimal policies are such that the optimal decision at each point in time is independent of the future evolution of the supply–demand balance in the system, so that these policies remain optimal in a stochastic environment. Both rigorous results as well as numerical experiments are developed to illustrate their findings in this important area for future power and energy systems.

The paper by Wogrin *et al.* [8] addresses the problem of the stochastic optimization of transmission capacity expansion to maximize the expected total social welfare of consumers and producers. A very interesting feature of the model is that it allows generators to be modelled with behaviour between perfectly competitive and Cournot players, with the potential to also study the implications of such behaviour if it evolves randomly over time, for example in response to evolving regulatory mechanisms. The mathematical formulation is a large-scale mixed integer program that is tested on a 5-bus system considering different sizes of scenario trees. The problem is modelled in the JuDGE platform and the computational performance of the tool is thoroughly compared against the deterministic equivalent solved with state-of-the-art packages. Unique features, limitations and future developments of the modelling tool are also discussed in detail. Overall, it is emphasized how the deterministic equivalent problem is simply too large to be solved using the solvers currently available, while decomposition techniques such as the ones performed by JuDGE allow identification of optimal transmission plans for such large-scale problems while considering sources of uncertainty, which is of great relevance in the context of the energy transition.

Electricity trading in an environment characterized by high variability and uncertainty was the subject of much attention during the MES programme. The final two papers of the present issue study important aspects of this problem. High-frequency trading of electricity in an intra-day market has become a major component of electricity trading in recent years. Limited access to high-quality high-frequency trading data, however, has so far somewhat hindered research on new models for intra-day trading, and few models have so far been proposed in the literature. Kremer *et al.* [9] consider a novel and extensive dataset which contains high-frequency electricity price transaction data for 15 min contracts from the German market and linked fundamental supply and demand data. What is more, it includes the same intra-daily updated forecasts of wind and solar power generation available to traders on the intraday market. Their work contains a detailed econometric analysis of this dataset and proposes a new model for high-frequency electricity price changes. The new model is a regression model, including various regressors not previously considered, such as the slope of the merit order curve, the 15 min intra-day auction price and also price changes of neighbouring 15-min contracts. They propose calibrating the model using threshold regression and present important insights on the behaviour of intra-daily price changes and the impact of fundamentals, such as wind and solar power generation forecasts, on price changes. The model can be viewed as a first step towards developing trading strategies for intra-day electricity markets.

The rapidly increasing penetration of highly variable renewable generation brings many further mathematical and statistical challenges. While more attention is often paid to problems of balancing generation and demand, high variability also implies congestion in many networks. This is particularly so in large North American networks, where the problem is often managed by the introduction of locational marginal prices (LMPs). However, the high variability of the underlying driving processes means that these prices may fluctuate rapidly and unpredictably according to both location and time of day, causing considerable problems in energy markets. Price spikes in such networks are of concern and the problem of their prediction is extremely difficult. The paper by Nesti *et al.* [10] combines the use of multiparametric programming techniques to combine seamlessly the analysis of network flows—the *optimal power flow* problem—with large deviations theory to study the problem of predicting extreme price fluctuations in LMP-based energy markets. It is assumed that the modeller has a complete stochastic description of the generation and demand processes and of the physical characteristics of the network. Large deviations techniques have been successfully used in many fields—in particular, in applications of stochastic networks—for predicting the frequencies of extreme events of interest, from the perspective of both design and management. By adding a study of energy markets alongside recent work on line failures and blackouts, this work extends a growing list of exciting applications of this powerful mathematical technique in the context of electrical power systems.

Data accessibility. This article has no additional data.

Authors' contributions. All authors contributed equally.

Competing interests. We declare we have no competing interests.

Funding. The guest editors thank the Isaac Newton Institute and its staff for their excellent hosting and support of the MES programme, and acknowledge also the support of the Engineering and Physical Sciences Research Council grant no. EP/I017054/1.

References

1. Karangelos E, Panciatici P. 2021 'Cooperative game' inspired approach for multi-area power system security management taking advantage of grid flexibilities. *Phil. Trans. R. Soc. A* **379**, 20190426. (doi:10.1098/rsta.2019.0426)
2. Luna JP, Sagastizábal C, Silva PJS. 2021 A discussion on electricity prices, or the two sides of the coin. *Phil. Trans. R. Soc. A* **379**, 20190428. (doi:10.1098/rsta.2019.0428)
3. de Chalendar JA, Glynn PW. 2021 On incorporating forecasts into linear state space model Markov decision processes. *Phil. Trans. R. Soc. A* **379**, 20190430. (doi:10.1098/rsta.2019.0430)
4. Radoszynski AM, Pinson P. 2021 Electric demand response and bounded rationality: mean-field control for large populations of heterogeneous bounded-rational agents. *Phil. Trans. R. Soc. A* **379**, 20190429. (doi:10.1098/rsta.2019.0429)
5. Ziel F. 2021 The energy distance for ensemble and scenario reduction. *Phil. Trans. R. Soc. A* **379**, 20190431. (doi:10.1098/rsta.2019.0431)
6. Goodridge MP, Moriarty J, Pizzoferrato A. 2021 A rare-event study of frequency regulation and contingency services from grid-scale batteries. *Phil. Trans. R. Soc. A* **379**, 20190433. (doi:10.1098/rsta.2019.0433)
7. Zachary S, Tindemans SH, Evans MP, Cruise JR, Angeli D. 2021 Scheduling of energy storage. *Phil. Trans. R. Soc. A* **379**, 20190435. (doi:10.1098/rsta.2019.0435)
8. Wogrin S, Tejada-Arango D, Downward A, Philpott AB. 2021 Welfare-maximizing transmission capacity expansion under uncertainty. *Phil. Trans. R. Soc. A* **379**, 20190436. (doi:10.1098/rsta.2019.0436)
9. Kremer M, Kiesel R, Paraschiv F. 2021 An econometric model for intraday electricity trading. *Phil. Trans. R. Soc. A* **379**, 20190624. (doi:10.1098/rsta.2019.0624)
10. Nesti T, Moriarty J, Zocca A, Zwart B. 2021 Large fluctuations in locational marginal prices. *Phil. Trans. R. Soc. A* **379**, 20190438. (doi:10.1098/rsta.2019.0438)