P.J. van der Houwen

# CONFERENTIE
# VAN NUMERIEK WISKUNDIGEN

24 - 26 september 1997

CONFERENTIECENTRUM WOUDSCHOTEN
ZEIST

Werkgemeenschap Numerieke Wiskunde

## TWEEENTWINTIGSTE CONFERENTIE NUMERIEKE WISKUNDE

### Doel van de conferentie

De Conferentie van Numeriek Wiskundigen wordt eenmaal per jaar gehouden onder auspiciën van de Werkgemeenschap Numerieke Wiskunde. Het doel van de conferentie is kennis te nemen van recente ontwikkelingen binnen de numerieke wiskunde. Hiertoe worden jaarlijks meestal twee, voor dit jaar drie, thema's vastgesteld. Internationaal bekende deskundigen worden uitgenodigd over deze thema's lezingen te houden.

### Thema's

1. nietlineaire randwaardeproblemen (met aandacht voor aspecten als continueringsmethoden en bifurcatie)
2. gegeneraliseerde-eigenwaardeproblemen
3. numerieke behandeling van financiële modellen

### Organisatie

De organisatie is in handen van de voorbereidingscommissie bestaande uit
P.J. van der Houwen (CWI/UvA) (voorzitter), D. Roose (KU Leuven), P. Wesseling (TUD), en J. Kok (CWI) (secretaris). Organisatorische medewerking is verleend door het Centrum voor Wiskunde en Informatica. Financiële ondersteuning is gegeven door de Stichting Wiskunde Onderzoek Nederland (SWON).

### Adres organisatie


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**Conferentie-URL**: `http://www.cwi.nl/~jankok/woudschoten.html`

Twenty-second Dutch Conference on Numerical Analysis

**Themes and Speakers**

| | |
|---|---|
| Theme 1. | *nonlinear boundary-value problems (with special attention to continuation methods and bifurcation)* |
| | Eusebius Doedel (Concordia University, Montreal)<br>Herbert B. Keller (CalTech, Pasadena)<br>Hans Mittelmann (Arizona State University, Tempe) |
| Theme 2. | *generalized eigenvalue problems and singular-value decomposition* |
| | Alan Edelman (M.I.T.)<br>Danny Sorensen (Rice University, Houston) |
| Theme 3. | *numerical treatment of financial models* |
| | Michel Hoevenaars (ING-bank, Amsterdam) |

Half-hour, contributed presentations by:

Johan Cambré, KU Leuven (Theme 2)
Laurence Cherfils, Université J. Fourier, Grenoble (Theme 1)
Koen Engelborghs, KU Leuven (Theme 1)
Hilda van der Veen, Delft University of Technology (Theme 1)

Twenty-second Dutch Conference on Numerical Analysis

## Programme and titles of lectures

*Wednesday, September 24, 1997*

| | | |
|---|---|---|
| 10.00 - 11.05 | **arrival, coffee** | |
| 11.10 | **opening** v.d. Houwen. | |
| 11.15 | **H.B. Keller** | |
| | 11.15 - 12.05: | Path following and bifurcations in scientific computing, I |
| | 12.05 - 12.15: | discussion |
| 12.20 | **lunch** Wesseling v.d. Vorst 3 | |
| 13.45 | **A. Edelman** | |
| | 13.45 - 14.35: | Algorithms for nearest canonical forms |
| | 14.35 - 14.45: | discussion |
| 14.50 | **D.C. Sorensen** | |
| | 14.50 - 15.40: | New approaches to large scale eigenanalysis, I |
| | 15.40 - 15.50: | discussion |
| 15.50 | **tea** Roose 3 | |
| 16.15 | **H.D. Mittelmann** | |
| | 16.15 - 17.05: | A software tool for nonlinear parameter-dependent PDE's |
| | 17.05 - 17.15: | discussion |
| 17.20 | **L. Cherfils** | |
| | 17.20 - 17.45: | A parallel and adaptive continuation method for semilinear bifurcation problems |
| | 17.45 - 17.50: | discussion |
| 18.15 | **dinner** | |
| 20.00 | **WNW Committee meeting,** followed by **Woudschoten Committee meeting** | |

*Thursday, September 25, 1997*

| | | |
|---|---|---|
| 08.00 | **breakfast** | Axelsson 9 |
| 09.00 | **H.B. Keller** | |
| | 09.00 - 09.50: | Path following and bifurcations in scientific computing, II |
| | 09.50 - 10.00: | discussion |
| 10.05 | **K. Engelborghs** | |
| | 10.05 - 10.30: | Continuation and bifurcation analysis of periodic solutions of delay differential equations |
| | 10.30 - 10.35: | discussion |
| 10.35 | **coffee** | v.d. Vorst Walle 9 |
| 11.10 | **A. Edelman** | |
| | 11.10 - 12.00: | The geometry of eigenvalue algorithms |
| | 12.00 - 12.10: | discussion |
| 12.15 | **J. Cambré** | |
| | 12.15 - 12.40: | Algorithms for the structured total least squares problem based on the Riemannian SVD |
| | 12.40 - 12.45: | discussion |
| 12.50 | **lunch** | te Riele 9 |
| 14.45 | **M. Hoevenaars** | |
| | 14.45 - 15.35: | Financial modeling and numerical techniques, I |
| | 15.35 - 15.45: | discussion |
| 15.45 | **tea** | Matthij Spijker 9 |
| 16.15 | **E.J. Doedel** | |
| | 16.15 - 17.05: | Collocation methods for ODE bifurcation problems |
| | 17.05 - 17.15: | discussion |
| 17.20 | **General Assembly of the Dutch** "Werkgemeenschap Numerieke Wiskunde" | |
| 18.00 | **dinner** | |

*Friday, September 26, 1997*

| | | |
|---|---|---|
| 08.00 | **breakfast** ~~Spijker~~ ~~Matthey~~ Schilders | |
| 09.00 | **H.D. Mittelmann** | |
| | 09.00 - 09.50: | Solution of continuation and bifurcation problems for PDE's |
| | 09.50 - 10.00: | discussion |
| 10.05 | **H. van der Veen** | |
| | 10.05 - 10.30: | Post-bifurcation analysis of soil plasticity with eigenvector perturbations |
| | 10.30 - 10.35: | discussion |
| 10.35 | **coffee** | |
| 11.10 | **M. Hoevenaars** Veldman | |
| | 11.10 - 12.00: | Financial modeling and numerical techniques, II |
| | 12.00 - 12.10: | discussion |
| 12.15 | **lunch** | |
| 13.30 | Wesseling **D.C. Sorensen** | |
| | 13.30 - 14.20: | New approaches to large scale eigenanalysis, II |
| | 14.20 - 14.30: | discussion |
| 14.35 | **E.J. Doedel** | |
| | 14.35 - 15.25: | New collocation methods for PDEs |
| | 15.25 - 15.35: | discussion |
| 15.35 | **closure, tea, departure** | |

## Page numbers of abstracts

# Algorithms for the structured total least squares problem based on the Riemannian SVD

Cambré Johan, Bart De Moor

The Structured Total Least Squares problem (STLS) can be formulated as follows:

> Given A matrix $A$ with a given affine structure (e.g. Hankel, Toeplitz, symmetric,...) find a **rank deficient** approximation $B$ of $A$ such that $\|A - B\|_F$ is **minimal.**

The natural way to tackle constrained optimization problems is through Lagrange multipliers. Careful treatment of these equations leads to a set to the so called **Riemannian SVD** equations, which can be seen as a nonlinear generalization of the ordinary SVD equations.

Let $B_1, \ldots, B_n$ be the basis of the linear structure of our matrix such that $A$ can be written as $A = a_1 B_1 + a_2 B_2 + \ldots + a_n B_n$, with rank($A$)=$q$ and we are looking for a matrix $B = b_1 B_1 + b_2 B_2 + \ldots + b_n B_n$ with the desired properties stated above. Then the Riemannian SVD equations look as follows:

$$\left\{ \begin{array}{lcl} Av & = & D_v u \tau \\ A^t u & = & D_u v \tau \end{array} \right. \quad \text{under the conditions} \quad \left\{ \begin{array}{lcl} u^t D_v u & = & 1 \\ v^t v & = & 1 \end{array} \right.$$

where $D_v = \sum_{k=1}^{n} \frac{1}{W(i)} B_k v (B_k v)^t$ and $D_v = \sum_{k=1}^{n} \frac{1}{W(i)} B_k^t u (B_k^t u)^t$ are square positive definite matrices that are quadratic in $v$ and $u$ respectively, and $W(i)$ is the Frobenius norm of the $i$-th basis matrix $B_i$.
These equations are identical to the ordinary SVD equations except for the square matrices $D_u$ and $D_v$.

As solving the SVD equations is equivalent to finding the minimal eigenvalue of a symmetric matrix, solving the Riemannian problem.
In our talk we will present a series of algorithms that converge to a suboptimal solution along with some examples.

# A parallel and adaptive continuation method for semilinear bifurcation problems

Laurence Cherfils

*LMC-IMAG, Université J. Fourier, Grenoble, France*

In the structural mechanics area for instance, predictor-corrector continuation methods are frequently used for the following of solution branches of parametrized nonlinear elliptic systems. In spite of their usefulness, these methods have a serious disadvantage: the exploration, without any *a priori* information, of one or more branches, requires the generation of many solution points, and therefore yields to a large computation time.

We present a continuation method for the resolution of parametrized two-dimensional nonlinear PDE. It has been elaborated with a double aim: to allow the exploration, at a low computation time, of an entire branch of accurate solutions, and to be especially suited to problems whose solutions contain boundary layers or singularities. To this end, we introduce two different tools to improve the usual Allgower-Georg's continuation method (see [1]). Firstly we use an adaptive finite element strategy, as in the software PLTMG [2], in order to handle *minimal meshes*, and thereby to limit the size of the nonlinear systems without a loss of accuracy for the solutions. Such approach can also be found in the software AUTO [3]. Moreover, the time required for the computation of an accurate solution branch can be noticeably reduced with an implementation on a computer network. One computer is devoted to the continuation on a coarse mesh, the others share out the correction of every roughly generated solution on a sequence of *a posteriori* refined meshes.

A numerical example illustrating the performance of the method is given.

## References

[1] ALLGOWER E.L., GEORG K.,*Numerical continuation methods, an introduction*, Springer Verlag, Berlin, 1990.

[2] BANK R.E., *PLTMG: A Software Package for Solving Elliptic Partial Differential Equations, User's guide 6.0*, SIAM, 1990.

[3] DOEDEL E., *AUTO: Software for continuation and bifurcation problems in ordinary differential equations*, User's guide, 1986.

# Collocation Methods for ODE Bifurcation Problems

Eusebius Doedel
Concordia University, Montreal

Woudschoten Conference, 24 – 26 September 1997

## Abstract

The method of orthogonal collocation with piecewise polynomials is recalled. A particular implementation is described, with emphasis on the numerical linear algebra. Application to a singularly perturbed boundary value problem and to the computation of periodic solutions in Plant's model of bursting nerve cells are described in some detail.
We also formulate an efficient algorithm for computing loci of period-doubling bifurcations.
Time permitting, the numerical computation of connecting orbits will also be discussed.

Key references will be given in the lecture. Part of the material covered can be found in the following tutorial articles:

E. J. Doedel, H. B. Keller, J. P. Kernevez,
**Numerical Analysis and Control Of Bifurcation Problems**,
Part I : Int. J. Bifurcation and Chaos, Vol. 1, No. 3. 1991, 493–520.
Part II : Int. J. Bifurcation and Chaos, Vol. 1, No. 4. 1991, 745–772.

# New Collocation Methods for PDEs

Eusebius Doedel
Concordia University, Montreal

Woudschoten Conference, 24 – 26 September 1997

## Abstract

Algorithmic aspects of a class of finite element collocation methods for the approximate numerical solution of elliptic partial differential equations are described. Locally, for each finite element, the approximate solution is a polynomial. Polynomials corresponding to adjacent finite elements need not match continuously, but their values and normal derivatives match at a discrete set of points on the common boundary. High order accuracy can be attained by increasing the number of matching points and the number of collocation points for each finite element. For linear equations the collocation methods can be equivalently defined as generalized finite difference methods. The linear (or linearized) equations that arise from the discretization lend themselves well to solution by the method of nested dissection. An implementation is described and some numerical results are given.

*References:*
W.-J. Beyn, E. J. Doedel, **Stability and multiplicity of solutions to discretizations of nonlinear ordinary differential equations**, SIAM J. Sci. Stat. Comput. 2, No. 1, 1981, 107–120.

E. J. Doedel, **On the construction of discretizations of elliptic partial differential equations**, J. Difference Equations and Applications, 1997, (to appear).

# Algorithms for Nearest Canonical Forms

Prof. Alan Edelman
Massachusetts Institute of Technology

Woudschoten Conference, 24 – 26 September 1997

## Abstract

Nearness of a matrix to a multiple eigenvalue is indicated by a small cosine between left and right eigenvectors, as for example is computed by MATLAB's condeig routine or the RCONDE variable in LAPACK. What can we supply the user who wishes to *exactly* obtain the nearest matrix with a multiple eigenvalue or more generally the nearest matrix (or pencil) with a particular canonical form? Such problems have been motivated from problems in systems and control, and have been studied by many experts in numerical linear algebra. We are working towards solving these problems so that the information could be available easily to users of library software.

In this talk, we will indicate how new geometrical approaches are being successfully applied towards the solution of this problem. Our proposed solution combines the power of a new understanding of the perturbation theory of so-called "staircase algorithms" with manifold techniques for optimization on the Grassmann and Stiefel manifolds.

# The Geometry of Eigenvalue Algorithms

Prof. Alan Edelman
Massachusetts Institute of Technology

Woudschoten Conference, 24 – 26 September 1997

## Abstract

We show how the geometrical framework, specifically the knowledge of geodesics, covariant derivatives, and the geometry of the Grassmann manifold gives penetrating new insights into algorithms allowing us to create, understand, and compare algorithms not as isolated entities, but as objects with a coherent mathematical structure. This understanding may never replace the value of numerical experiments, but does provide a top level view of so many algorithms that can circumvent the need for some experiments. It is our hope that developers of new algorithms and perturbation theories will benefit from the theory, methods, and examples that we propose.

In particular we will show how knowledge of the geometry allows us to clearly understand in new ways such algorithms as Rayleigh Quotient Iteration (and also Davidson's method), any number of conjugate gradient algorithms for the linear and nonlinear eigenvalue problem, sequential quadratic programming and eigenvalue optimization.

[These lectures survey joint work with Demmel, Elrmoth, Kagstrom, Ma, and Ripper.]

# Continuation and bifurcation analysis of periodic solutions of delay differential equations

Koen Engelborghs
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Computer science department
Celestijnenlaan 200A
B-3001 Leuven

August 18, 1997

**Abstract**

In this talk we consider a numerical method to compute periodic solutions of delay differential equations and their stability. A periodic solution can be computed by a 'shooting' approach in a functional space. In each Newton step a linear operator equation must be solved which contains the linearized Poincare operator. When discretizing the state on the delay interval a high-dimensional system is obtained. However zero is the only cluster point of the spectrum of the linearized Poincare operator and thus only a few eigenmodes of this operator are important. Our numerical method exploits this fact by combining shooting with straightforward time integration. Especially in a continuation procedure the method is very efficient. The method also computes all dominant, stability determining Floquet multipliers and thus allows robust detection and calculation of bifurcations.

# Financial Modeling and Numerical Techniques

Michel Hoevenaars
ING Nederland, ITResearch
Amsterdam, July 1997

Contribution to the Dutch Conference of Numerical Mathematicians, 24 - 26 September 1997

**Abstract:** Different mathematical models have been developed for example to forecast the behaviour of stocks and bonds but also the value at risk in option portfolios, to show dependencies between different stocks and clusters of stocks or to find dependencies between market and social/economic factors and the value of stocks. In an early stage, implementation of these models made clear that high performance computers would be required to process the amount of data encountered in 'real life'. This paper concentrates on the mathematical aspects and on the parallel implementation of forecasting models, not on the economic validity.

## A Stock Market Forecasting Model

A stock market forecasting model aims to preview the rates or the trend of stocks in the future. Different techniques are available today to do this, for example 'self-building' neural networks, 'Feed-forward' networks or combinations of neural networks, statistical techniques and algorithms. Most of these solutions do have drawbacks which mainly have to do with long learning cycles and availability of the right data. In the worst-case scenario this requires a trained network for each stock, for each marketplace and for each particular question. The models under discussion here, are built under the assumption that price building of stocks is not a random walk and that stock markets do have the best information concerning stock price building. So a forecasting model cannot produce a better stock price than the stock exchange can, for example the Amsterdam or New York Stock Exchange.

Two characteristics are of general interest; forecasting trends or prices and tracing dependencies or so-called 'leader-follower' relation. We will discuss here the forecasting model. 'Leader-follower' and other models will be discussed during the presentations. For an outline of the model see figure 5.1, 'The enhanced forecasting model'. Having established a clear model, its inherent parallelism was exploited in order to implement most of the steps in such a way that no inter-node communication was needed. Out of the four algorithms that did fit a parallel implementation, the computation of the eigen-values an eigen-vectors, needed for the principal components analysis turned out to be the system's critical stage. Taking into account the hyper-cube architecture, the parallel block Jacobi algorithm was selected and implemented. As it turned out, each of the resulting sub-problems constituted a significant implementation problem in its own. In particular the parallel ordering to be used for a hyper-cube architecture was hard to grasp and to implement. All of these problems were solved resulting in a performance, that is comparable to highly optimized implementations of the sequential Jacobi algorithm. At a glance the performance of the parallel block Jacobi algorithm seems to offer just a small improvement over the sequentially superior combination of Householder plus QL algorithms. However, as the forecasting system will be executed on a daily basis and most of the data used in the computation will change only slightly from day to day, 'yesterday's' rotation matrix can be used to boost the performance of the Jacobi algorithm to that of the Householder plus QL algorithms. Parallel execution then results in a significant improvement of the performance.

**The Model**

Basically, each of the stocks is split into a number of orthogonal signals, each representing a specific frequency band. Then, all signals of a frequency band are taken together and the principal components of these signals are calculated. As these components are ortogonal as well, the stocks are now represented by a set of signals that are all independent of each other. Subsequently, for each of these signals a forecast is constructed. Finally, the stocks are reconstructed again, using inverse transforms. The different algorithms that have to be implemented for a practical implementation of the model are:

1. Regression analysis; to eliminate the linear component,
2. Discrete Wavelet Transform; the first algorithm computes the filtered series of signals, the second algorithm computes the trends and fluctuations given time series and wavelet filters,

3., 4. Normalization and Covariance,

5. Eigen-values and Eigen-vectors or principal components analysis, the purpose of this stage is to find the smallest set of (statistically independent) linear combinations of stocks, that explain a particular part of the variance present in all stocks. The linear combinations that make up this smallest set are called the principal components. Determining the n-th principal component is equivalent to solving the eigen-value problem. Because of the characteristics of the forecasting problem, all eigen-values found are of interest for further research.
6. Determining the number of principal components,
7. Inverse discrete Wavelet Transform,
8. The prediction phase, the type and length of the forecast depends on the type of indicators that have to be predicted and the length of the history that was taken into account. Different types of extension are possible, a very common one is a sine-wave type of extension.
9. Reconstruction of the filtered values,
10. De-normalization,

11., 12. Reconstruction of the unfiltered stocks and adding the linear component.

**Results**

Concerning the results there are two important issues:

1. Are the forecasts produced by the system in line with the model,
2. What resources does the system need in order to produce these forecasts.

Both items will be discussed during the presentations.

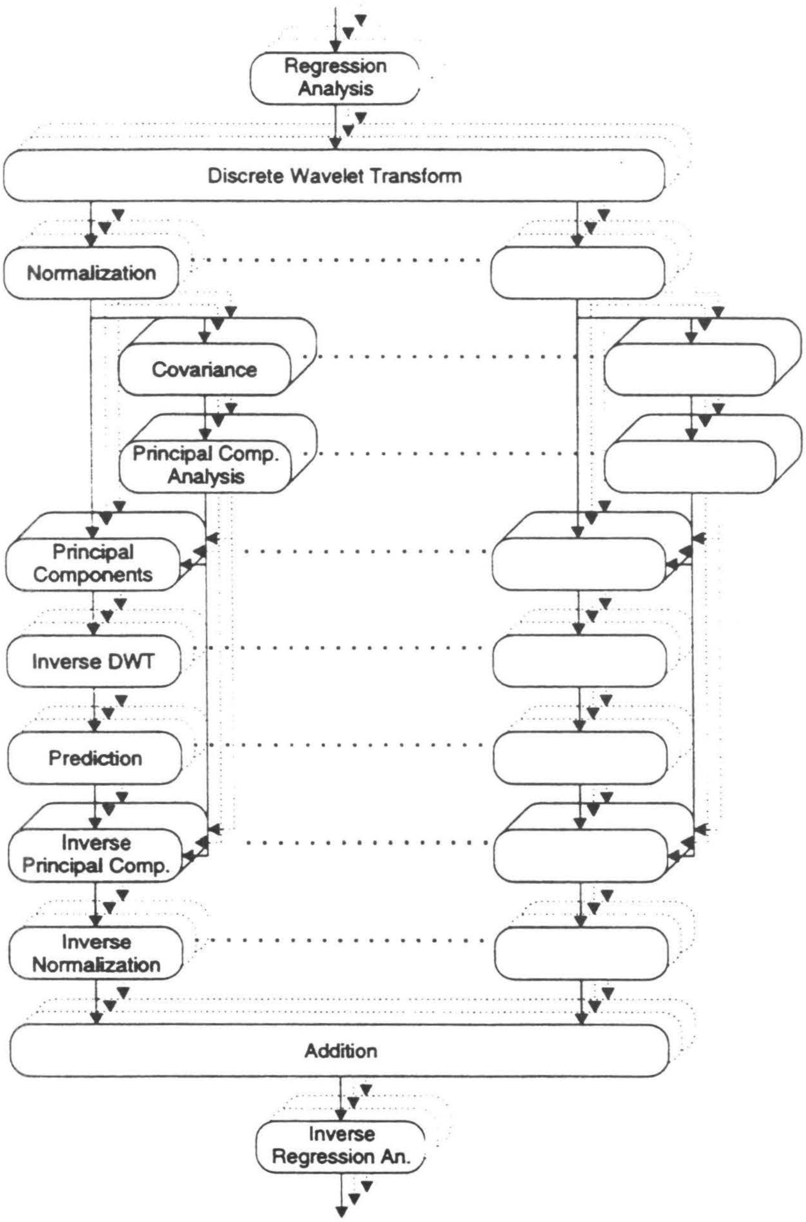


FIGURE 5.1. The enhanced forecasting model

# Path Following and Bifurcations in Scientific Computing, I and II

Herbert B. Keller
CalTech, Pasadena

Woudschoten Conference, 24 – 26 September 1997

## Abstract

I. General techniques for following regular paths will be introduced. Several different methods for introducing continuation parameters are shown. Global results and the sign-change lemma are covered. Degree theory and the homotopy invariance of degree follow with applications such as a constructive proof of the Brouwer fixed point theorem and the computation of periodic orbits.

II. The study of steady state bifurcations and several ways to switch branches at bifurcations are covered. The bordering algorithm and its applications in path following are introduced. Fold following and related matters in higher co-dimension bifurcations will be discussed.

# A Software Tool for Nonlinear Parameter Dependent PDE's

*Hans D. Mittelmann*
*Arizona State University*
*Department of Mathematics*
*P.O. Box 871804*
*Tempe, AZ 85287-1804, USA*

September 8, 1997

### Abstract

In this talk we introduce a package for the finite element solution of single, nonlinear elliptic partial differential equations. We have contributed to its feature of treating equations that in a general way may depend on several real parameters. Since the program is rather complex, it includes multilevel techniques, adaptivity, continuation, a graphical user interface etc, this description will go somewhat into detail which seems necessary even for a general audience.

## References

[1] R. E. Bank, PLTMG: A Software Package for Solving Elliptic Partial Differential Equations, User's Guide 7.0, SIAM, Philadelphia, 1994. (The version considered will be 7.8)

# Solution of Continuation and Bifurcation Problems for PDE's

*Hans D. Mittelmann*
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*Department of Mathematics*
*P.O. Box 871804*
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September 8, 1997

### Abstract

In continuation of the first talk, we will look in detail at the continuation features of the package. Some of both the basic underlying mathematical principles as well as implementation issues will be addressed. While the type of bifurcation problems that can be treated is not as general as those solvable by some ODE packages, the crucial point is that through the integration of the treatment of parameter–dependence with the other features of the program and through its interactive use via a convenient graphical user interface, this program overall represents a powerful tool to solve parameter–dependent PDE problems well. This will be demonstrated with a number of examples.

## References

[1] H. D. Mittelmann, A pseudo–arclength continuation method for nonlinear eigenvalue problems, *SIAM J. Numer. Anal.* 23(1986), 1007–1016.

[2] R. E. Bank and H. D. Mittelmann, Stepsize selection in continuation procedures and damped Newton's method, *J. Comp. Appl. Math.* 26(1989), 67–77.

## New Approaches to Large Scale Eigenanalysis - I
*Introduction to Krylov Projection Methods*

The past few years have seen significant advances in our abilities to compute partial eigendecompositions of large matrices. These new approaches have led to critical advances in several application areas including computational chemistry, semi-conductor laser design, linear stability analysis, and reduced basis techniques for large state space control systems.

This introductory talk will develop the background for Krylov subspace projection methods and then survey some of these new techniques. In particular, we shall discuss the implicitly restarted Arnoldi method which is the basis for the softare package ARPACK. This package has been used extensively in many application areas that require the solution of large scale symmetric and nonsymmetric (generalized) eigenvalue problems.

Introductory topics will include: Subspace Projection, The Lanczos/Arnoldi Process, Implicit Restarting, Spectral Transformation, Polynomial Preconditioning.

Prof. Danny C. Sorensen
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## New Approaches to Large Scale Eigenanalysis - II
*Truncated QZ-methods*

Recent research in large scale eigenanalysis has focused on the generalized eigenvalue problem

$$\mathbf{Ax} = \mathbf{Bx}\lambda.$$

The emphasis has been on intoducing pre-conditioning (or spectral enhancement) ideas that accelerate convergence without accurate solution to shift-invert equations. This talk will briefly survey some of these new approaches and then develp two algorithms for the generalized problem. These new methods are developed within a subspace projection framework as a truncation and modification of the $QZ$-algorithm for dense problems that is suitable for computing partial generalized Schur decompositions of the pair $(\mathbf{A}, \mathbf{B})$. A generalized partial reduction to condensed form is developed by analogy with the Arnoldi process. Then truncated forward and backward $QZ$ iterations are introduced to derive generalizations of the Implicitly Restarted Arnoldi Method and the Truncated $RQ$ method for the large scale generalized problem. Preliminary computational experience is presented along with a discussion of possibilities for inexact solutions of shift-invert equations.

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# Post-Bifurcation Analysis of Soil Plasticity with Eigenvector Perturbations.

Hilda van der Veen[1], Kees Vuik[2] and René de Borst[1].

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[2]Faculty of Technical Mathematics and Informatics, PO. Box 356, 2600 AJ Delft,
Delft University of Technology, the Netherlands.

Numerical modeling of soil deformation is a difficult task. However, it is technically important because of numerous applications, for example in the tunneling and oil drilling industry. In this contribution, attention is focused on the computation of post-failure behavior of soil bodies.

For the numerical treatment of material nonlinearities in soil, it is common practice to use an incremental formulation. Typically, the material behavior is described by incremental constitutive laws. These laws together with kinematic and equilibrium equations and the appropriate boundary conditions form the boundary value problem [1].

The deformation of soil is described by the evolution of stresses and strains. The relation between them, in the case of elasto-plasticity, is described by the elasto-plastic stiffness tensor $\mathbf{D}_{ep}$. In the considered case of non-associative plasticity, this tensor is non-symmetric. As a consequence, the global tangential stiffness matrix, that relates the nodal displacements to the external forces, will also be non-symmetric.

As an example, we modeled a biaxial compression test on soil, Figure 1. The structural and material data are summarized in Table 1. Up to failure the eigenvalues of the tangent stiffness matrix $\mathbf{K}$ are real and positive. At the precise moment of failure, due to loss of uniqueness of the solution, one or more eigenvalues of $\mathbf{K}$ are zero. This is called a bifurcation point. In numerical computations this point is never exactly found, but once a negative eigenvalue appears, we assume that a bifurcation point has been passed. The eigenvectors associated with negative eigenvalues are now computed,

Figure 2, and will be used to find a new solution related to a stable path. Such a path is characterized again by a tangent stiffness matrix with only real and positive eigenvalues. Several perturbation techniques have been examined, that make use of eigenvectors [2, 5]. Deformation after bifurcation is no longer homogeneous, but will have localized into a shear band of which the size and direction depends on the perturbation method used, Figure 3.

For the computation of the eigenvalues and eigenvectors we use the bi-Lanczos method with partial orthogonalization [3, 4]. This allows for efficient computation of the left and right eigenvectors that are both used in the computation of post-bifurcation behavior.

# References


[1] BATHE, K.-J. *Finite element procedures in engineering analysis.* Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1982.

[2] BORST, R. D. Computation of post-bifurcation and post-failure behavior of strain- softening solids. *Computers and Structures 25* (1987), 211–224.

[3] DAY, D. M. *Semi-Duality in the two-sided Lanczos algorithm.* PhD thesis, University of California at Berkeley, 1993. Third decendent.

[4] VEEN, H. V. D., AND VUIK, K. Bi-Lanczos with partial orthogonalization. *Computers & structures 56*, 4 (1995), 605–613.

[5] VEEN, H. V. D., VUIK, K., AND BORST, R. D. Computation of post-bifurcation behavior in soil plasticity with eigenvector perturbation. In *Finite elements in engineering and science; Proceedings of the second international DIANA conference on computational mechanics* (1997), pp. 529–536.

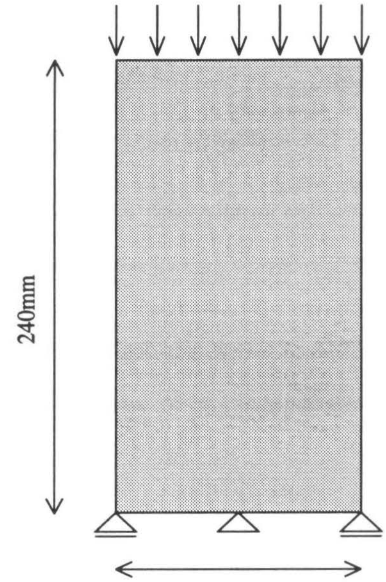

Figure 1: Biaxial compression.

| | |
|---|---|
| Size | 120mmx240mm |
| Elements | 12x24 8-noded plane strain |
| Degrees of freedom | 1847(1823) |
| Gauss integration | 3x3 |
| Cohesion | 0.01 MPa |
| Young's modulus E | 1.8625 MPa |
| Poisson's ratio $\nu$ | 0.3 |
| Friction angle | 17.46 degrees |
| Dilatancy angle | 11.54 degrees |

Table 1: Structural & Material data.

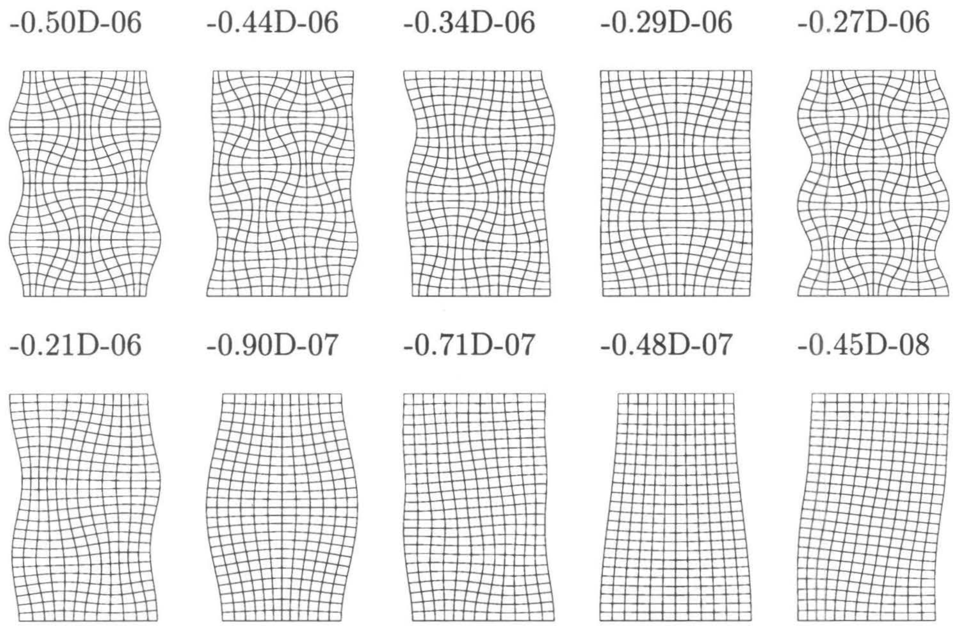


Figure 2: Eigenvalues and related eigenmodes.

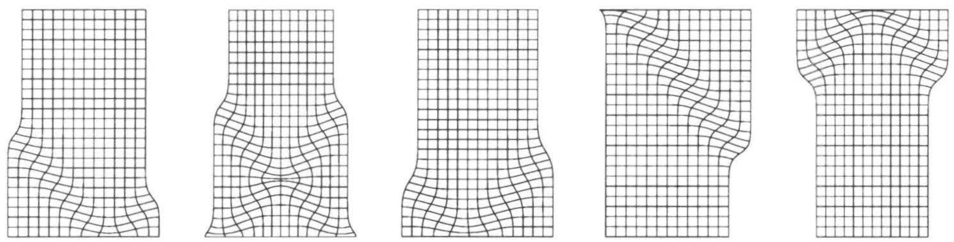

Figure 3: Deformation modes for different perturbation techniques.

## List of participants Woudschoten Conference 24-26 September 1997

| | | | | |
|---|---|---|---|---|
| 1. | Werner | Aernouts | KU Leuven | Leuven |
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| 31. | Rik | Kaasschieter | TU Eindhoven | Eindhoven |
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| 43. | Wim | Michiels | KU Leuven | Leuven |
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| 52. | W.H.A. | Schilders | Philips Research | Eindhoven |
| 53. | Jo | Simoens | KU Leuven | Leuven |
| 54. | Gerard | Sleijpen | Univ Utrecht | Utrecht |
| 55. | Ben | Sommeijer | CWI | Amsterdam |
| 56. | Danny | Sorensen | Rice University | Houston |
| 57. | Stefan | Spekreijse | NLR | Amsterdam |
| 58. | Marc | Spijker | RU Leiden | Leiden |
| 59. | Jacques | de Swart | CWI | Amsterdam |
| 60. | Stefan | Vandewalle | KU Leuven | Leuven |
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| 62. | Arthur E.P. | Veldman | RU Groningen | Groningen |
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