TENTH CONFERENCE ON THE MATHEMATICS OF OPERATIONS RESEARCH

January 9-11, 1984
CONFERENCE CENTRE "DE BLIJE WERELT"
LUNTEREN, THE NETHERLANDS

PROGRAM and SUMMARIES

Centre for Mathematics and Computer Science
Amsterdam, The Netherlands

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Conference Centre "De Blije Werelt", Westhofflaan 2, Lunteren, The Netherlands

Invited Speakers

J.B. Orlin (Cambridge (U.S.A.)/Rotterdam):
1. A survey of periodic scheduling problems;
2. Periodic graphs: algorithms, complexity, and applications.

M.J.D. Powell (Cambridge (U.K.)):
1. Variable metric algorithms for nonlinearly constrained optimization calculations;
2. Recent research on variable metric algorithms.

G.P. Prastacos (Athens):
1. Perishable inventory theory: a review;
2. Blood inventory management.

R.R. Weber (Cambridge (U.K.)):
1. Stochastic scheduling problems;
2. Stochastic scheduling on parallel machines.

Minicourse on Nonlinear Optimization

G. van der Hoek (Rotterdam):
A nonlinear decision support system.

G. van der Laan (Amsterdam) and A.J.J. Talman (Tilburg):
Simplicial algorithms for solving systems of nonlinear equations.

F.A. Lootsma (Delft):
Performance evaluation of nonlinear optimization methods.

J. Ponstein (Groningen):
Duality in optimization.

Summaries

See page 6 ff.

Program

Wednesday, January 9

11.25 - 11.30	Opening
11.30 - 12.30	Orlin (1)
14.15 - 15.00	Minicourse: Ponstein
15.30 - 16.15	Minicourse: Van der Hoek
16.15 - 17.00	Minicourse: Van der Laan/Talman

Thursday, January 10

9.00 - 10.00	Powell (1)
10.30 - 11.30	Weber (1)
11.30 - 12.30	Prastacos (1)
14.15 - 15.15	Powell (2)
15.45 - 16.30	Minicourse: Lootsma
16.30 - 17.00	Minicourse: discussion
17.00 - 18.00	Meeting Research Community

Friday, January 11

9.00 - 10.00	Orlin (2)
10.30 - 11.30	Prastacos (2)
11.30 - 12.30	Weber (2)

Organization

The conference is organized by the Centre for Mathematics and Computer Science under the auspices of the Dutch Research Community in the Mathematics of Operations Research and System Theory, and with financial support by the Dutch Mathematical Society and the Netherlands Society of Operations Research. Contact: E.A. van Doorn, Centre for Mathematics and Computer Science, P.O. Box 4079, 1009 AB Amsterdam, The Netherlands, tel. (020) 5924094.

Speakers

G. van der Hoek	Econometric Institute Erasmus University P.O. Box 1738, 3000 DR Rotterdam
G. van der Laan	Department of Actuarial Sciences and Econometrics Free University P.O. Box 7161, 1007 MC Amsterdam
F.A. Lootsma	Department of Mathematics and Informatics Delft University of Technology P.O. Box 356, 2600 AJ Delft
J.B. Orlin	Sloan School of Management Massachusetts Institute of Technology Cambridge, MA 02139, U.S.A.
G.P. Prastacos	The Athens School of Economics and Business Science 76, Patission Street Athens 104, Greece
J. Ponstein	Econometrics Institute University of Groningen P.O. Box 800, 9700 AV Groningen
M.J.D. Powell	Department of Applied Mathematics and Theoretical Physics University of Cambridge Silver Street, Cambridge CB3 9EW, U.K.
A.J.J. Talman	Department of Econometrics Tilburg University P.O. Box 90153, 5000 LE Tilburg
R.R. Weber	Engineering Department Cambridge University Mill Lane, Cambridge CB2 1RX, U.K.

Participants

J.M. Anthonisse	Centre for Mathematics and Computer Science, Amsterdam
A.J.M. Beulens	Erasmus University, Rotterdam
H.C. Boersma	University of Technology, Eindhoven
O.J. Boxma	University of Utrecht
A. Broere	C.S.U., Breda
J.W. Cohen	University of Utrecht
I.J. Curiel	Catholic University, Nijmegen
E.E.C. van Damme	University of Technology, Delft
H. Daniels	N.L.R., Marknesse
R. Dekker	University of Leyden
N.Ph. Dellaert	University of Technology, Eindhoven
J.J.M. Derks	Catholic University, Nijmegen
Y.M.I. Dirickx	University of Technology, Enschede
K. van Donselaar	University of Technology, Eindhoven
E.A. van Doorn	Centre for Mathematics and Computer Science, Amsterdam
J.B.M. van Doremalen	University of Technology, Eindhoven
B. Dorhout	University of Technology, Enschede
F.A. van der Duyn Schouten	Free University, Amsterdam
R. Heuts	Catholic University, Tilburg
G. van der Hoek	Erasmus University, Rotterdam
J.L. de Jong	University of Technology, Eindhoven
L.C.M. Kallenberg	University of Leyden
G.A.P. Kindervater	Centre for Mathematics and Computer Science, Amsterdam
S.J. de Klein	University of Utrecht
J. Koene	Rabobank, Utrecht
M. Kok	University of Technology, Delft
A.G. de Kok	Free University, Amsterdam
M.B.M. de Koster	University of Technology, Eindhoven
J. Kriens	Catholic University, Tilburg
L.G. Kroon	Erasmus University, Rotterdam
G. van der Laan	Free University, Amsterdam
B.J. Lageweg	Centre for Mathematics and Computer Science, Amsterdam
J.K. Lenstra	Centre for Mathematics and Computer Science, Amsterdam
J.Th. van Lieshout	Catholic University, Tilburg
F.A. Lootsma	University of Technology, Delft
H.W. van den Meerendonk	Erasmus University, Rotterdam

E. van der Meyden	Erasmus University, Rotterdam
H. Nauta	Groningen
J.K. van Ommeren	Free University, Amsterdam
J. Ponstein	University of Groningen
C. van Putten	Philips, Eindhoven
A. Ridder	University of Leyden
C. Roos	University of Technology, Delft
J.J. van Rotterdam	KIWA, Rijswijk
M.W.P. Savelsbergh	Centre for Mathematics and Computer Science, Amsterdam
J.A.M. Schreuder	University of Technology, Enschede
J.H. van Schuppen	Centre for Mathematics and Computer Science, Amsterdam
J.H.A. de Smit	University of Technology, Enschede
A.J.J. Talman	Catholic University, Tilburg
H.W.J.M. Trienekens	Erasmus University, Rotterdam
J. van der Wal	University of Technology, Eindhoven
J. Wessels	University of Technology, Eindhoven
J.R. de Wit	Erasmus University, Rotterdam
M. Zijlstra	Philips, Eindhoven
W.H.M. Zijm	Philips, Eindhoven

A NONLINEAR DECISION SUPPORT SYSTEM

G. van der Hoek

Model designers dealing with nonlinear optimization are focussed with two main problems: the application environment should recognise and formulate the nonlinearities in the problem and, after having formulated a nonlinear optimization model, the relatively difficult software for nonlinear programming should be understood and applied. Both these problems arose in the nonlinear decision support system to be discussed.

In the engineering environment of the Dredging Division of Royal Boskalis Westminster an economic model was developed describing the design and the operation of trailer suction dredgers in their working environment. The technical conditions on the operation of the vessel defined several nonlinear expressions in the decision variables. Management decided to buy Lasdon's GRG2 code (an implementation of a reduced gradient algorithm) to solve the nonlinear programming problem on the HP 3000 minicomputer. However, the CPU costs required to solve the original model appeared to be unacceptably high. This could be explained by the model characteristics (dimensions, nonlinear equations) and/or by the efficiency in using the code (starting point, precision parameters and options in the code). A more close examination of the results obtained by GRG2 suggested the reformulation or even the removal of several constraints. The discussions led to a feedback to the engineering departments (civil engineering, construction engineering, shipbuilding engineering) which supplied the modules of the model. As a result the project stimulated new research approaches which resulted in new insights and in a deeper understanding of the underlying physical processes.

A further reduction in the CPU costs was achieved by using so-called 'acceptable' inexact line search steps in the 1-dimensional searches of GRG2, in combination with the BFGS update formula for unconstrained optimization.

Both the DSS aspects and the nonlinear programming algorithms used will be discussed.

References:

W. van Donselaar and G. van der Hoek (1982) Decision support to the design and operation of trailer dredges, an application of Lasdon's GRG-code for NLP. Report 8229/0, Econometric Institute, Erasmus University Rotterdam.

G. van der Hoek and R.Th. Wymenga (1984) Acceptable steplength restrictions in constrained nonlinear programming. Paper presented at the NATO ASI on Computational Mathematical Programming, Bad Windsheim, Germany.

SIMPLICIAL ALGORITHMS FOR SOLVING SYSTEMS OF NONLINEAR EQUATIONS

G. van der Laan
A.J.J. Talman

In order to compute approximate solutions of a nonlinear system of n equations one can apply simplicial algorithms. Such an algorithm triangulates R^n in n-dimensional simplices and searches for a simplex which yields an approximate solution. The variable dimension restart (v.d.r.) algorithm starts with an arbitrary point, a zero-dimensional simplex, and generates a sequence of adjacent simplices of varying dimension of the triangulation. Since no simplex can be generated more than once the algorithm terminates within a finite number of steps if some convergence assumption holds such as Merrill's condition. If the accuracy of the approximation is not sufficient, some quasi-Newton steps can be taken after which, if necessary, the algorithm can be restarted in the last found approximation with a finer triangulation, in the hope to find within a few steps a better one. The several v.d.r. simplicial algorithms can be characterized by the number of rays along which the starting point can be left (with one-dimensional adjacent simplices). Thus far known are the 2-ray, the (n+1)-ray, the 2n-ray, the 2^n-ray and the (3^n-1)-ray algorithms. The efficiency of these algorithms will be illustrated by some computational results.

PERFORMANCE EVALUATION OF NONLINEAR OPTIMIZATION METHODS

F.A. Lootsma

We use pairwise-comparison methods to evaluate the performance of algorithms for nonlinear optimization. First, weights are calculated for the predominant performance criteria: generality, robustness, efficiency, capacity, conceptual simplicity, and shortness of code. Thereafter, we use the comparative studies in nonlinear optimization to assign weights to the methods under each of the performance criteria separately. Because we only have a fuzzy notion of the relative performance, we extend the above, deterministic evaluation: we carry out pairwise comparisons using fuzzy estimates (fuzzy numbers with triangular membership functions) to obtain fuzzy scores for the methods under consideration. To illustrate matters, we rank and rate five methods (geometric programming and four general methods) for solving polynomial geometric-programming problems. The procedure outlines how multi-criteria decision analysis can be used for an integrated assessment of algorithms, taking into account both the performance criteria of decision makers and the results of comparative studies.

A SURVEY OF PERIODIC SCHEDULING PROBLEMS

J.B. Orlin

We will consider a number of different deterministic periodic scheduling problems with applications in several different realms, including the following: vehicle routing, workforce scheduling, computer or machine scheduling, and inventory replenishment. In general, the most realistic models cannot be solved to optimality in practice, and one must rely on a variety of heuristic methodologies. Here, we will focus on the following five models.

(1) "The Tramp Steamer Problem". This model addresses the problem of finding an optimal tour for a single ship that may choose which ports it is to visit. Dantzig et al. (1967) introduced this problem in 1967 and gave the first efficient solution technique.

(2) "Minimum Fleet Size Problems". Airlines, buslines, and other travel-related service companies are often interested in minimizing their fleet size while maintaining sufficiently high service levels. Here we consider models proposed by Simpson (1968) with applications to airplane scheduling. We also consider non-periodic fleetsize problems solved by Dantzig and Fulkerson (1954) and Ford and Fulkerson (1958).

(3) "Minmum Workforce Problems". Here we consider problems of minimizing the workforce size while maintaining sufficiently high service levels. In particular, we will discuss the methodology of Bartholdi et al. (1980) and Bartholdi (1981) for cyclic staffing problems.

(4) "Machine Scheduling Problems". We will consider single and multiple processor scheduling with the criterion of satisfying all due dates.

(5) "Inventory Replenishment". We will discuss the 'economic order quantity' model and some of its variations.

References:

J.J. Bartholdi, J.B. Orlin and H.D. Ratliff (1980) Cyclic staffing via integer programs with circular ones. Oper. Res. 24, 1074-1085.

J.J. Bartholdi III (1981) A guaranteed accuracy round-off algorithm for cyclic staffing and set covering. Oper. Res. 29, 501-510.

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R.W. Simpson (1969) Scheduling and routing models for airlines systems. Technical Report 1268-3, Flight Transportation Laboratory, MIT.

PERIODIC GRAPHS: ALGORITHMS, COMPLEXITY, AND APPLICATIONS

J.B. Orlin

A periodic graph is a graph G = (V,E), where V = {1,2,3,...} and E has the following two properties: (1) each vertex is incident with a finite number of edges, and (2) the edges occur periodically with some fixed p; i.e., (i,j) ∈ E if and only if (i+p,j+p) ∈ E for $i,j \geq 1$. These graphs are appropriate for modeling a variety of periodic problems. In particular we will discuss applications to vehicle routing and cyclic staffing.

There is no guaranteed method for assessing the complexity of a problem on a periodic graph given only the complexity of the problem on finite graphs. Nevertheless, there are certain 'rules of thumb' that are generally accurate.

(1) If X is a graph problem solvable in polynomial time, then problem X as applied to periodic graphs may also be solved in polynomial time.
(2) If X is a graph problem that is NP-complete, then problem X as applied to periodic graphs is PSPACE-hard. (We will describe PSPACE-hardness in the talk. In general, these problems are even more intractible than NP-complete problems.)

We will present a general methodology for solving the 'easy periodic' problems in polynomial time. We will also present a general methodology for proving the intractability of the hard periodic problems.

References:

G.B. Dantzig, W. Blattner and M.R. Rao (1967) Finding a cycle in a graph with minimum cost to times ratio with application to a ship routing problem. In Theory of Graphs, P. Rosentiehl ed., Dunod, Paris, Gordon and Breach, New York, 77-84.

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S.C. Graves and J.B. Orlin (1980) The infinite-horizon dynamic lot-size problem with cyclic demands and costs. Working Paper, Operations Research Center, MIT.

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DUALITY IN OPTIMIZATION

J. Ponstein

Various forms of duality in optimization will be reviewed, such as the classical 'right-hand side' duality, Fenchel duality, duality involving modified Lagrangians, duality in integer programming, duality in monotropic programming.

Common elements will be emphasized, as well as implications towards algorithmic procedures and perhaps the part (to be) played by generalized gradients.

VARIABLE METRIC ALGORITHMS FOR NONLINEARLY CONSTRAINED OPTIMIZATION CALCULATIONS

M.J.D. Powell

Algorithms for minimizing a function $F(\underline{x})$ subject to constraints $\{c_i(\underline{x}) = 0;\ i = 1,2,\ldots,m\}$ are considered, where $\underline{x}$ is a vector of n real variables. Mostly they extend to inequality constraints. It is assumed that first derivatives can be calculated, and that, for each $\underline{x}$, the gradients $\{\underline{\nabla} c_i(\underline{x});\ i = 1,2,\ldots,m\}$ are linearly independent. Therefore, if $\underline{x}^*$ is optimal, there exist Lagrange multipliers $\{\lambda_i^*;\ i = 1,2,\ldots,m\}$ such that

$$\underline{\nabla} F(\underline{x}^*) = \Sigma \lambda_i^* \underline{\nabla} c_i(\underline{x}^*).$$

Penalty function algorithms can be highly useful. Here one usually adds to $F(\underline{x})$ a function of the constraints, such as $100\Sigma|c_i(\underline{x})|$, and one applies to the resultant function a procedure for unconstrained minimization. These techniques are reviewed briefly, including the augmented Lagrangian method. Often a sequence of unconstrained problems has to be solved.

Mainly, however, we consider iterative methods that make linear approximations to the constraints, and quadratic approximations to the objective function, in order to improve estimates of the required solution. All linear terms come from calculated first derivatives, but each second derivative matrix of a quadratic approximation has to be chosen automatically. Further, although this information can be used to estimate a change to $\underline{x}$, one sometimes reduces this change in order to force convergence. These reductions are made by "line search" or "trust region" techniques, which are mentioned briefly, including the role of "line search objective functions".

The second derivative matrices of the quadratic approximations to $F(\underline{x})$ should be chosen to give fast convergence. Suitable choices are derived from the "Kuhn-Tucker conditions", and we note the importance of taking account of constraint curvature. However, the matrices have to be derived from first derivative information, so this problem is discussed.

Finally, some numerical results are presented. Although good efficiency usually occurs, some examples are chosen to show slow convergence in order to motivate parts of the second lecture.

References:

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P.E. Gill, W. Murray and M.H. Wright (1981) Practical Optimization, Academic Press.

M.J.D. Powell (1983) Variable metric methods for constrained optimization. In Mathematical Programming: The State of the Art, A. Bachem, M. Grötschel and B. Korte eds., Springer-Verlag.

RECENT RESEARCH ON VARIABLE METRIC ALGORITHMS

M.J.D. Powell

The variable metric algorithms for constrained optimization, considered in the first lecture, have the property that there is not a close relation between the quadratic programming calculation that gives the trial change to $\underline{x}$ and the line search objective function. Several researchers, however, believe that greater efficiency can be obtained by bringing these factors together, and thus one can avoid some difficulties due to linearly dependent constraint gradients. We discuss these questions.

However, as shown in the previous lecture, slow convergence can be caused by line search functions that have derivative discontinuities. A published remedy, that we consider briefly, is to allow iterations to make a further change to the variables to allow for constraint violations. A similar idea, which we also study, is to divide the trial change in $\underline{x}$ into two parts, one parallel and one orthogonal to the constraints, which are calculated separately. This approach has the advantage of requiring only an $(n-m)\times(n-m)$ matrix of second derivative information. We also comment on a complication that has received much attention recently, namely that the $(n-m)\times(n-m)$ matrix depends on a choice of basis of the space of vectors that are parallel to the constraints. These considerations are important to the efficient solution of calculations in which $(n-m)$ is much smaller than n.

We also give some attention to Fletcher's differentiable exact penalty function, although each function value depends on first derivatives. Some new work is described that allows this function to be used as the line search function of a variable metric algorithm without the calculation of any second derivatives. Thus several disadvantages of the more usual non-differentiable line search function are avoided. Is this algorithm equivalent to a penalty function method? It will be claimed that it is different because of the attention that is given by variable metric methods to linear approximations to the constraints.

References:

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PERISHABLE INVENTORY THEORY: A REVIEW

G.P. Prastacos

Most inventory models assume that stock items can be stored indefinitely to meet future demands. However, certain types of inventories undergo change in storage so that in time they may become partially or entirely unfit for consumption. In this lecture we review the relevant results on the problem of determining suitable ordering policies for these perishable inventories.

BLOOD INVENTORY MANAGEMENT

G.P. Prastacos

Blood Inventory Management has attracted significant interest from the Operations Research profession during the last 15 years. A number of methodological contributions have been made in the areas of inventory theory and combinatoric optimization that can be of use to other products or systems. These contributions include the development of exact and approximate ordering and issuing policies for an inventory system, the analysis of LIFO or multi-product systems, and various forms of distribution scheduling. In addition, many of these results have been implemented, either as decision rules for efficient blood management in a hospital, or as decision support systems for hierarchical planning in a Regional Blood Center.

In this talk we attempt a review of the recent Operations Research contributions to blood inventory management theory and practice. Whereas many problems have been solved, others remain open and new ones keep being created with advances in medical technology and practices. Our approach is not to present an exhaustive review of all the results in the field, but rather to address several important issues from a unified perspective of theory and practice, and point out new areas for further research.

STOCHASTIC SCHEDULING PROBLEMS

R.R. Weber

Scheduling problems [1] are concerned with the optimal ordering of the processing of a number of jobs on a set of machines in order to optimize certain performance criteria. There are many variations of scheduling problems. Among the many factors to be considered are: precedence constraints amongst the jobs, due dates and conflicting objectives. Given the combinatorial intricacy of the relationships among such factors it is not surprising that many scheduling problems turn out to be quite intractible.

In this talk we consider scheduling problems in which the processing times of the jobs are not known, but described only by probability distributions. For example, the processing time of each job may assumed to be a realization of a random variable having an exponential distribution with a known mean. There are now many models of such problems which have been studied by researchers [2]. The talk will review some of the main successes in the analysis of stochastic scheduling models. Interestingly, some stochastic scheduling problems can be solved by efficient algorithms, whereas their deterministic counterparts are NP-complete.

The principal technique for the study of stochastic scheduling models is stochastic dynamic programming [3]. Other useful mathematical techniques will be illustrated, including: the use of interchange arguments, reduction to a deterministic problem, application of the Gittin's Index, analysis by continuous deformation of the model, and reduction to an optimal control or calculus of variations problem.

References:

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[2] M.A.H. Demster, J.K. Lenstra and A.H.G. Rinnooy Kan (1981) Deterministic and Stochastic Scheduling, Reidel.

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STOCHASTIC SCHEDULING ON PARALLEL MACHINES

R.R. Weber

The talk will principally concern the discussion of a new result in stochastic scheduling on parallel machines.

A collection of n jobs is to be processed by m ($m < n$) machines which operate in parallel. The processing times of the jobs are unknown, but it is known that they are stochastically ordered, with job i having a processing time which is distributed stochastically less than the processing time of job j for all $i < j$. It is desired to minimize the expected flowtime (the sum of completion times). Processing is to be nonpreemptive: once the processing of a job has been started its processing may not be interrupted. It is shown [3] that the scheduling strategy which minimizes the expected flowtime is SEPT, the strategy of processing the jobs according to the shortest expected processing time first order: 1,2,...,n. This result generalizes a well-known result in deterministic scheduling, and similar results in deterministic scheduling under more restrictive conditions than stochastic ordering, [1] and [2].

The method of proof is by the technique of "continuous deformation of the model". This will be explained. Other results and open problems in stochastic scheduling on parallel processors will be discussed.

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[2] R.R. Weber (1982) Scheduling jobs with stochastic processing requirements on parallel machines to minimize makespan or flowtime. J. Appl. Prob. 19, 167-182.

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