# TWELFTH CONFERENCE ON THE MATHEMATICS OF OPERATIONS RESEARCH



CONFERENCE CENTRE 'DE BLIJE WERELT' LUNTEREN, THE NETHERLANDS

JANUARY 14-16, 1987

Organized by the Centre for Mathematics and Computer Science, Amsterdam, The Netherlands

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#### AIM AND SCOPE

The aim of the conference is to promote the research activities and the cooperation between researchers in the mathematics of operations research.

The emphasis of the 1987 conference is on three fields which are promising for further OR research: parallel computation, routing and reliability/maintenance.

Five non-Dutch specialists will give two lectures each on recent developments in their field of interest. They have been asked to present a tutorial survey of their area in the first talk and discuss their own recent work in the second lecture.

Two of the invited speakers will talk about parallel and distributed computations. In addition to these invited lectures the program includes a minicourse about this subject. Two of the other invited speakers will speak on routing and scheduling. The fifth invited speaker will address the subject of reliability and maintenance.

The program should give ample opportunity for informal discussions.

#### **ORGANIZATION**

**Organizers** 

The conference is organized by the Centre for Mathematics and Computer Science in Amsterdam, in particular by H.C. Tijms and B.J. Lageweg.

## Sponsors

The conference is organized under the auspices of the Dutch Research Community in the Mathematics of Operations Research and System Theory, with financial support of the Dutch Mathematical Society and the Netherlands Society of Operations Research.

#### **PROGRAM**

Invited speakers

- A. Bendell (Nottingham):
  - 1. Reliability modelling: a historical perspective
  - 2. Multi-state and continuous-state reliability models
- M. Desrochers (Montreal/Amsterdam):
  - 1. Solving logistic problems using column generation
  - 2. A column generation approach to the transit crew scheduling problem
- K. Mehlhorn (Saarbrücken):
  - 1. On local routing of two-terminal nets
  - 2. Deterministic simulation of idealized parallel computers on more realistic ones
- W.R. Pulleyblank (Waterloo/Bonn):
  - 1. Solving travelling salesman problems on a microcomputer
  - 2. International salesmen, weighted girth and three edge cutsets
- P. Toint (Namur):
  - 1. Parallel unconstrained nonlinear optimization and partitioned quasi-Newton updating algorithms
  - 2. A view of nonlinear optimization in a large number of variables

Minicourse: Parallel computing

J. van Leeuwen (Utrecht):

Parallel computers and computations

J.K. Lenstra (Amsterdam/Tilburg):

Parallel computing in combinatorial optimization

A.H.G. Rinnooy Kan (Rotterdam):

Parallel algorithms for partitioning methods

#### TIME SCHEDULE

10:20

11:10

12:00

12:30

#### Wednesday January 14, 1987 11:30 Opening Pulleyblank (1) 11:40 12:30 Lunch Minicourse (1): Van Leeuwen 14:30 15:20 Tea 15:50 Minicourse (2): Lenstra 16:40 Mehlhorn (1) Thursday January 15, 1987 9:00 Bendell (1) 9:50 Coffee Desrochers (1) 10:20 11:10 Pulleyblank (2) 12:00 Meeting Research Community 12:30 Lunch 15:00 Minicourse (3): Rinnooy Kan 15:50 Tea 16:20 Toint (1) 17:10 Mehlhorn (2) Friday January 16, 1987 9:00 Bendell (2) 9:50 Coffee

Desrochers (2)

Lunch

Toint (2) Closing

Bendell 1

## RELIABILITY MODELLING; A HISTORICAL PERSPECTIVE

Tony Bendell

Department of Mathematics, Statistics and Operational Research Trent Polytechnic Nottingham NG1 4BU UK

The paper reviews the historical development of reliability and replacement modelling, paying particular attention to generalization of the basic dichotomic reliablity model. Differences of approach in the engineering, statistical and operational research litarature are highlighted, and the literature is contrasted to current reliability engineering practice. Some unsubstantial practices and gaps in the literature are discussed.

Attention is also given to the diversity and complexity of the point processes underlying reliability data, and to the practical problem of identification of appropriate model structure. Reliability data rarely reflects the modeller's hypotheses. In this respect, the results of applying simple exploratory data analysis (EDA) techniques in reliability problems are described, together with the nature of more sophisticated methodologies. Perhaps the most important of these is the emergence of *Proportional Hazards Modelling* in the reliability setting with the ability to incorporate numerous explanatory variables.

## MULTI-STATE AND CONTINUOUS-STATE RELIABLITY MODELS

Tony Bendell

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A number of directions for generalising the familiar binary reliability model to incorporate partial degradation have now been explored by various authors. The literature is, however, largely unstructured with considerable duplication.

This paper highlights some missing aspects in the development of multistate and continuous-state models to date, and introduces the general jumpprocess component model of Bendell and Humble.

The model provides a unified conceptual and algebraic framework for both multi-state and continuous-state scenarios, and raises new aspects of previous models in the literature. Properties of the model are discussed, and component and system characteristics derived. The renewal theory for the model is also developed. The initial tuning of a partially degrading unit to an appropriate level of performance is formulated as an optimisation problem, and solved graphically for various submodels. A similar approach is also taken for the returning problem following a period of use.

#### REFERENCE

A. Bendell, S. Humble (1985). A reliability model with states of partial operation. Nav. Res. Log. Quart., 509-535.

#### SOLVING LOGISTIC PROBLEMS USING COLUMN GENERATION

Martin Desrochers

Centre for Mathematics and Computer Science Kruislaan 413 1098 SJ Amsterdam

Problems as different as satellite traffic assignment, one and two dimensional cutting stock, school bus routing, ship routing, dial-a-ride, airline crew scheduling and transit crew scheduling have been solved using column generation. We will survey logistic problem resolution by column generation by first giving an intuitive idea of the method, by reviewing the mathematical programming theory used by column generation and by presenting several examples of problem modeling and resolution using this approach.

These logistic problems divide work or demand between production plans, routes or work schedules subject to a set of feasibility conditions. All those problems are very large integer programming problems that can not be dealt with directly by a linear programming approach because either the size of the resulting problems is too big or of the poor quality of the lower bounds produced by linear programming. The column generation approach decompose a problem in two parts: the master problem divides the work or demand between known feasible production plans, routes or work schedules and the subproblem proposes new feasible production plans, routes or work schedules to improve the actual solution of the master problem.

The main issues in the development of a column generation approach are:

- 1) the definition of the master problem;
- 2) the definition of the subproblem;
- 3) how to obtain an integer solution.

In all the above problems, the master problem is a set covering or a multiple set covering problem and the subproblems are integer programming problems that can be solved efficiently using dynamic programming or another well known algorithm. We will present the special characteristics of Branch and Bound methods to obtain an integer solution when solving problems using column generation. One of the interesting empirical conclusions was that it was usually easy to obtain an integer solution. The case of the school bus routing problem will be used to present all aspects of the column generation method. We will also present how the master problem and the subproblem of the other problems mentioned are modeled.

# A COLUMN GENERATION APPROACH TO THE TRANSIT CREW SCHEDULING PROBLEM

Martin Desrochers

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The transit crew scheduling problem arises in mass transit corporations who have to create minimal cost bus driver schedules respecting both the collective agreement and the bus schedules. This problem is often modeled as a set covering problem where each column represents a driver's workday and each row represents a task i.e. a given bus travel. In previous solution methods, a subset of the feasible workdays was created and a heuristic solution using only these workdays was found by solving the resulting set covering problem.

We propose a column generation approach to solve the transit crew scheduling problem. The column generation approach decomposes the problem in two parts. The master problem is a set covering problem and chooses a schedule from already known feasible workdays. The subproblem is a shortest path problem with resource constraints and proposes new feasible workdays to improve the actual solution of the master problem.

We will present how the creation of new workdays is modeled using resource constraints. We will also outline the primal/dual method used to solve these special shortest path problems. The approach was tested successfully on real life problems.

Lenstra

## PARALLEL COMPUTING IN COMBINATORIAL OPTIMIZATION

J.K. Lenstra

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This lecture deals with theoretical models for parallel computations and with combinatorial optimization problems that are solvable in polynomial sequential time. The use of more realistic parallel architectures for broader classes of optimization problems is discussed by A.H.G. Rinnooy Kan in the third lecture of the minicourse.

Within the class of *tractable* or *easy* problems referred to above, a distinction will be made between the *very easy* problems, which are solvable in polylogarithmic space (and hence in polylogarithmic parallel time), and the *not so easy* problems, which are P-hard under log-space transformations.

These concepts are illustrated on a number of elementary problems. Shortest paths and minimum spanning trees can be found in polylog parallel time, but linear programming and even finding a maximum flow are P-hard. One can find optimal preemptive schedules and traveling salesman tours no worse than twice the optimum in polylog time on a polynomial number of processors, but simple heuristics like list scheduling and the nearest neighbor rule are likely to require at least linear time, irrespective of the number of processors. The matching problem is the most prominent open problem in this context, although it is solvable in random polylog time.

#### REFERENCE

G.A.P. Kindervater, J.K. Lenstra (1986). Parallel computing in combinatorial optimization. Report OS-R8614, CWI, Amsterdam.

#### ON LOCAL ROUTING OF TWO-TERMINAL NETS

K. Mehlhorn

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The planar rectangular grid consists of vertices  $\{(x,y); x,y \in Z\}$  and edges  $\{((x,y),(x',y')); |x-x'| + |y-y'| = 1\}$ . A routing region R is a finite subgraph of the planar rectangular grid.

In the sequel R always denotes a routing region. We call a bounded face F of R trivial if it is has exactly four vertices on its boundary and nontrivial otherwise. We use M to denote the nontrivial bounded faces and  $\overline{M}$  to denote M together with the unbounded face. Let B be the set of vertices of R with degree at most three. Note that a vertex  $v \in B$  lies on the boundary of a face  $F \in \overline{M}$ .

A local routing is a path in the routing region R connecting two vertices of B. The endpoints of the path are called its terminals. Two local routings p and q are elementary equivalent if there are paths  $p_1, p_2, q_2, p_3$  such that  $p = p_1 p_2 p_3$ ,  $q = p_1 q_2 p_3$ , and such that  $p_2 q_2^{-1}$  ( $q_2^{-1}$  is the reverse of path  $q_2$ ) is a boundary cycle of a trivial face. Two elementary equivalent paths are hence the same except that they take two different routes around a single trivial face. Two local routings p and q are equivalent if there is a sequence  $p_0, \ldots, p_k, k \ge 0$ , of paths such that  $p = p_0, q = p_k$  and  $p_i$  and  $p_{i+1}$  are elementarily equivalent for  $0 \le i < k$ . Note that if p and q are equivalent then p and q have the same terminals.

We use [p] to denote the equivalence class of local routing p, i.e., to denote the homotopy class of path p. A global routing or net is an equivalence class [p]; the terminals of the path p are also called the terminals of the net.

We are now ready to state the Local Routing Problem (LRP).

**Input:** A routing region R and a multi-set N of nets.

We assume that each net  $N \in \mathbb{N}$  is given by one of its representatives.

We use n to denote the number of vertices of R plus the total length of the representatives and call n the size of the problem.

**Output:** A local routing lr(N) for each net  $N \in \mathbb{N}$  such that

- 1)  $lr(N) \in N$  for all  $N \in N$
- 2)  $lr(N_1)$  and  $lr(N_2)$  are edge-disjoint for  $N_1$ ,  $N_2 \in \mathbb{N}$ ,

 $N_1 \neq N_2$  or an indication that there is no such set of local routings.

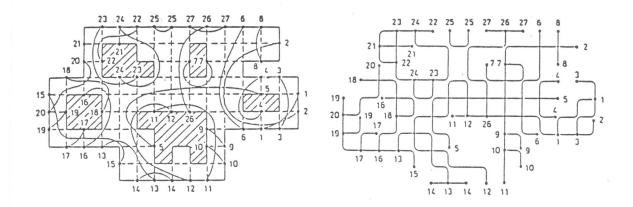


Fig 1. A Local Routing Problem and its solution. The global routings are shown as curves for added clarity.

Figure 1 gives an example.

We will prove the following theorem.

**Theorem 1:** Let P = (R, N) be an even bounded LRP of size n.

- a) P is solvable if and only if the free capacity of every cut is nonnegative.
- b) In time  $O(n^2)$  one can decide whether P has a solution and also construct a solution if it does.

It remains to define the terms 'even LRP', 'cut', 'free capacity of a cut' and 'bounded'.

## REFERENCE

Michael Kaufmann, Kurt Mehlhorn (1986), On local routing of two-terminal nets. Report 03/1986, FB 10 - Informatik, Universität des Saarlandes.

# DETERMINISTIC SIMULATION OF IDEALIZED PARALLEL COMPUTERS ON MORE REALISTIC ONES

K. Mehlhorn

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We describe a deterministic simulation of PRAM's on module parallel computers (MPC's) and on processor networks of bounded degree. The simulated machines have the same number n of processors as the simulated PRAM, and if the size of the PRAM's shared memory is polynomial in n, each PRAM step is simulated by  $O(\log n)$  MPC steps or by  $O((\log n)^2)$  steps of the bounded degree network. This improves upon a previous result by Upfal and Wigderson. We also prove an  $O((\log n)^2/\log \log n)$  lower bound on the number of steps needed to simulate one PRAM step on a bounded degree network under the assumption that the communication in the network is point to point.

As an important part of the simulation of PRAM's on MPC's, we use a new technique for dynamically averaging out a given work load among a set of processors operating in parallel.

## REFERENCE

H. Alt, T. Hagerup, K. Mehlhorn, F.P. Preparata (1985), Deterministic simulation of idealized parallel computers on more realistic ones. Report 36/1985, FB 10 - Informatik, Universität des Saarlandes.

# SOLVING TRAVELLING SALESMAN PROBLEMS ON A MICROCOMPUTER

W.R. Pulleyblank

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The Euclidean travelling salesman problem involves finding the shortest cycle passing through n points (cities) in the Euclidean plane. In a practical sense, it is usually sufficient to provide a solution which is close to the optimum, provided that some bound on this gap from optimality can be established.

We exhibit a microcomputer based system which both constructs *good* tours and provides *good* bounds. Usually on problems of 60-120 cities, solutions and bounds are provided for which the difference is less than 2%.

This project is joint work with Sylvia Boyd and Gerard Cornuéjols.

# INTERNATIONAL SALESMEN, WEIGHTED GIRTH AND THREE EDGE CUTSETS

W.R. Pulleyblank

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Suppose we are given a graph with arbitrary real edge costs in which the nodes are grouped into *countries*. The International Salesman Problem is to find a minimum cost cycle which enters and leaves each country exactly once, but is not restricted as to which nodes it must visit. If all cities belong to a single country then the problem becomes finding a cycle for which the sum of the costs is minimum, or the weighted girth problem.

We show how decomposition methods can be applied to solve these problems optimally, for certain classes of graphs. We also show how this enables us to solve the classical travelling salesman problem on a larger class of graphs.

This is joint work with David Hartvigsen.

Rinnooy Kan

#### PARALLEL ALGORITHMS FOR PARTITIONING PROBLEMS

A.H.G. Rinnooy Kan

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A natural application of parallel computation arises when an optimization problem lends itself for solution through partitioning of the set of feasible solutions.

Two such applications will be discussed: the solution of combinatorial optimization problems by branch-and-bound methods and the solution of continuous global optimization problems through stochastic methods involving sampling and local search.

Some limited computational results are available, but research in this area is generally in its infancy and many open problems remain.

Toint 1

# PARALLEL UNCONSTRAINED NONLINEAR OPTIMIZATION AND PARTITIONED QUASI-NEWTON UPDATING ALGORITHMS

Philippe Toint

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Partitioned updating quasi-Newton methods for unconstrained optimization allow the solution of large scale nonlinear problems by exploiting the partially separable structure of the objective function, that is its decomposition in element functions depending only on a small subset of the variables. The fact that these algorithms consider small dimensional subdomains could inhibit, at first sight, the vectorization of such algorithms.

A new implementation, mainly based on loops of dimension equal to the number of elements, shows that adapting these methods to vector and parallel processors may be quite efficient.

Tests on a FPS164, a simulated parallel processor on IBM4381 and a two-processor IBM3090-200 are reported and discussed.

Toint 2

## A VIEW OF NONLINEAR OPTIMIZATION IN A LARGE NUMBER OF VARIABLES

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The paper is intended as a personal survey of the recent developments in a rapidly growing field: optimization problems involving a large number of non-linear variables.

The broad classes of algorithms suitable for these problems are first considered, with emphasis on the various costs associated with their use, such as cpu-time, storage, etc.

Recent advances in unconstrained minimization are then examined, as well as the ongoing research in algorithms and software for constrained problems.

Finally, some open problems and areas where the author finds that valuable progress could be made are outlined and commented.

Van Leeuwen

## PARALLEL COMPUTERS AND COMPUTATIONS

J. van Leeuwen

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In the realm of scientific computing there is a continuing demand for faster computers. New developments in computer technology have led to a variety of supercomputers, which are available for scientific computing at many places, also in the Netherlands. More recent types of computer architectures are based on networks of cooperating processing units. From this new breed, the next generation of very fast production computers will emerge.

The new computer architectures all aim at a high degree of parallelism in data processing. This is achieved through the distribution of all activities, on the instruction level as well as on the program level. Vector computers, array processors, and processor networks are all types of parallel computers, each with its own approach to the (parallel) processing of instructions and distribution of data. The scientific programmer is thereby faced with the problem of designing application software which fully exploits the parallel processing power.

This introductory lecture will give a model oriented view of the possibilities of a few types of parallel computers, including vector computers and processor networks. Some examples will be given to illustrate the design of parallel algorithms, both on the system level and on the problem level. New theoretical developments will be indicated with respect to the parallelization of specific problems (such as sorting), parallel data storage, and network structures for fast data processing.

#### **REFERENCE**

J. van Leeuwen (1985). Parallel computers and algorithms. J. van Leeuwen, J.K. Lenstra (eds.). Parallel Computers and Computations, CWI Syllabus 9, Centre for Mathematics and Computer Science, Amsterdam, 1-32.