CONFERENCE ON THE MATHEMATICS OF OPERATIONS RESEARCH & SYSTEMS THEORY

January 14 - 16, 1981

Congress Centre 'De Blije Werelt' Lunteren

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Congress Centre "De Blije Werelt", Westhofflaan 2, Lunteren, The Netherlands

Speakers

E. Balas	(Pittsburgh/Cologne)	: Combinatorial Optimization
P. Brémaud	(Paris)	: Optimal Control of Point Processes
L.S. Lasdon	(Austin)	: Nonlinear Programming
I. Meilijson	(Tel Aviv/Amsterdam)	: Stochastic Optimization
S.K. Mitter	(Cambridge, MA/Florence)	: Stochastic Systems Theory
M.J. Todd	(Ithaca/Cambridge, UK)	: Piecewise Linear Homotopy Theory

Program

Wednesday		Thursday		Friday	
		09.00-10.00	Brémaud	09.00-10.00	Brémaud
11.15-11.30	opening	10.30-11.30	Balas	10.30-11.30	Balas
11.30-12.30	Meilijson	11.30-12.30	Todd	11.30-12.30	Todd
13.00	lunch	13.00	lunch	13.00	lunch
14.30-15.30	Mitter				
16.00-17.00	Lasdon	16.00-17.00	Mitter		
17.00-18.00	Meilijson	17.00-18.00	Lasdon		
18.30	dinner	18.30	dinner		

The program gives ample scope for informal discussions with the speakers and with other participants.

Registration Fee

The registration fee is Dfl. 215.-- and includes board and lodging from Wednesday, January 14, before lunch until Friday, January 16, after lunch.

Organization

Dutch Research Community in the Mathematics of Operations Research and Systems Theory. Secretary: J.K. Lenstra, Mathematisch Centrum, Kruislaan 413, 1098 SJ Amsterdam, The Netherlands, tel. (020)5924089.

Congress Centre "De Blije Werelt", Westhofflaan 2, Lunteren, The Netherlands

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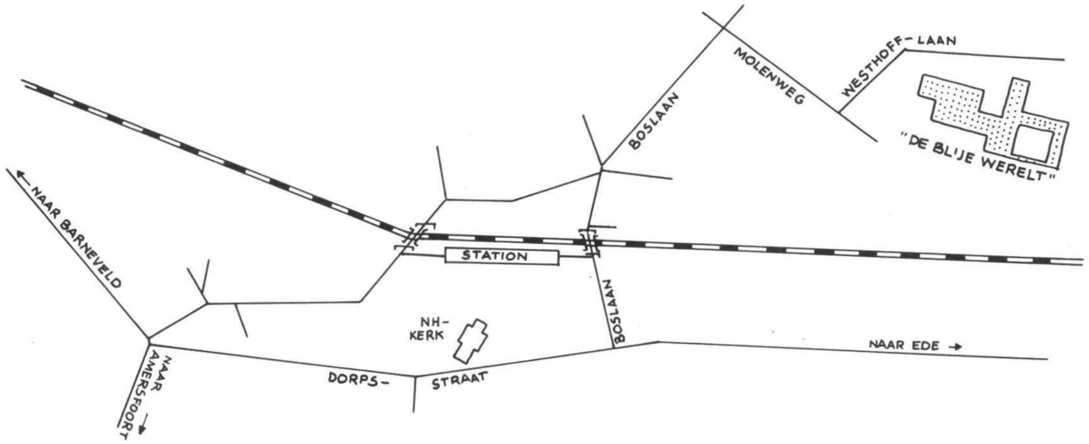
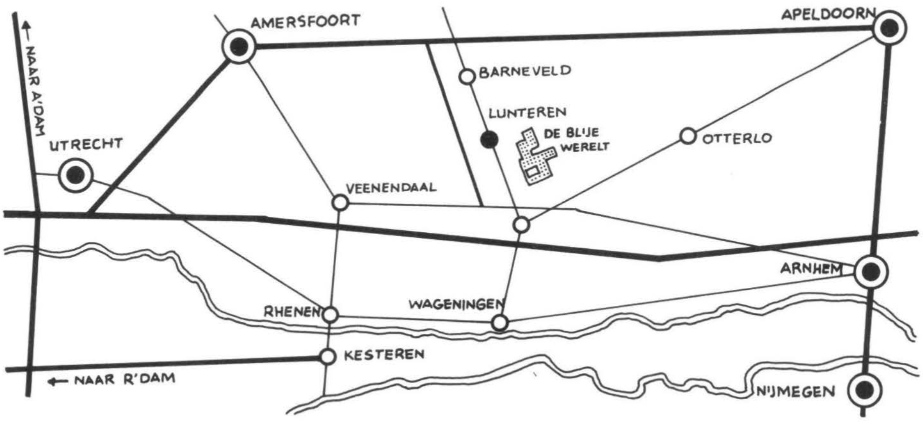


Lunteren can be reached by train from Amersfoort and from Ede-Wageningen.

The walk from the station to "De Blije Werelt" takes about 15 minutes.

Opposite to the station is a taxi company; tel. (08388)2319.



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CUTTING PLANES IN DISCRETE OPTIMIZATION: RECENT DEVELOPMENTS

E. Balas

Integer and combinatorial programming problems are difficult because their feasible sets are nonconvex. Cutting planes attempt to convexify the feasible set by approximating its convex hull. The first decade in the use of cutting planes was marked by lack of computational success, and disappointment. The last few years in turn have brought important theoretical advances and some undeniable computational successes.

Today there are three main theoretical approaches, with corresponding computational methods, for generating cutting planes: algebraic (subadditive), geometric (disjunctive), and graph-theoretic (combined with lifting). Each of these theories gives a description of the convex hull of feasible points and means of generating its facets. The difficulty lies in the fact that in general the number of facets, as well as the effort required to generate each one of them, grows exponentially with the number of variables. Some recent research efforts have focused on finding polynomially bounded procedures for generating facets or lower dimensional faces of the convex hull, that cut off a given point, and using them in combination with linear programming methods less affected by the number of constraints than the simplex method.

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SOME HEURISTICS AND EXACT ALGORITHMS FOR PACKING AND COVERING PROBLEMS

E. Balas

Breakthroughs in the theoretical analysis of the performance of algorithms and heuristics were accompanied in recent years by the discovery of empirically efficient heuristics and algorithms for certain combinatorial problems. However the procedures that are empirically found to be efficient are often different from the ones whose worst case and expected performance can be successfully analyzed. We will discuss this paradox on the example of the 0-1 knapsack and the set covering problems.

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OPTIMAL CONTROL OF POINT PROCESSES

P. Brémaud

I. POINT PROCESS AND QUEUEING SYSTEMS

Dynamical description of point process systems. Stochastic intensity in terms of conditional increments, or in terms of martingales. Stochastic point process integrals. Likelihood ratio for point processes. Description of queueing networks by means of stochastic integrals. Filtering.

II. CONTROLLED POINT PROCESSES

The 3 basic types of control, intensity control, optimal stopping, impulsive control. Controls reducible to intensity controls : impulse at the jumps, and attraction controls. Examples : optimal file allocation, routing in networks, non-preemptive priority disciplines.

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A SURVEY OF NONLINEAR PROGRAMMING ALGORITHMS AND SOFTWARE

L.S. Lasdon

The field of nonlinear programming (NLP) has been characterized by a wealth of theory and algorithms, but little knowledge regarding comparative algorithmic performance, and few good software packages. Consequently, applications have lagged behind our modeling and solution capabilities. In the last several years this situation has changed dramatically. Good software, both general and special purpose, is now available. Recent computational comparisons have clarified the relative efficiency of various kinds of algorithms, as well as identifying some good codes. Predictably, applications are increasing, but there is still a long way to go.

Part of this applications gap is due to a lack of information on what can be done and what benefits can be achieved. We attempt here to partially fill this gap by summarizing recent developments. The talk begins by briefly describing the four most promising classes of NLP algorithms: generalized reduced gradient (GRG), successive linear programming (SLP), Augmented Lagrangian (AL), and Succesive Quadratic Programming (SQP). Then we turn to NLP software, listing some desirable software features and describing a few codes in terms of these features. A summary of results of comparative studies of NLP codes is given, some of which are quite recent.

Progress has also been made in solving large nonlinear programs. Each of the four classes of algorithms listed above has been extended to large systems. However, work in this area is still in its early stages.

This talk draws on some material from two earlier survey papers, one on NLP software (Waren and Lasdon, 1979), the other on NLP applications (Lasdon and Waren, 1980).

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RECENT DEVELOPMENTS IN NONLINEAR PROGRAMMING SOFWARE AND APPLICATIONS

L.S. Lasdon

Recently, some experimental codes for solving large nonlinear programs have been developed. This lecture briefly describes experience with three such codes. The first, using a variant of the Successive Linear Programming Algorithm first proposed by (Griffith and Stewart, 1961), has proven surprisingly effective in experiments (Palacios-Gomez, 1980). The second, LSGRG (Lasdon and Waren, 1978) has been used to solve some large nonlinear macroeconomic planning models for Mexico, and some large problems in water resource planning. The third, OPTCON (Mantell and Lasdon, 1978), has been used to solve some large optimal control problems involving econometric models at the U.S. Federal Reserve Board.

A way to increase applications of any operations research tool is to imbed it within a user-oriented system which solves a particular class of problems. A new system of this type, called IFPOS (Interactive Financial Planning Optimization System), couples the financial planning language IFPS with a number of optimizers. (Roy and Lasdon, 1980).

In Chemical Engineering, some researchers are combining nonlinear programming routines with chemical process simulators (Westerberg, 1980).

Spiraling energy costs have spawned many new nonlinear programming applications. These include product blending, optimal design of drilling rigs and petrochemical process units, and optimal operation of process units. Problems of optimally heating and cooling a building have been formulated and solved as NLP's, as have some problems involving optimal management of oil and gas reservoirs (Lohrenz and Monash, 1979). The lecture will include brief descriptions of these problems.

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PROBABILISTIC PERT

I. Meilijson

For a PERT network with delay time X_i at node i, $(1 \le i \le n)$, and paths I_j $(1 \le j \le k)$, the completion time of the network is

(1) $$M = \max_{1 \le j \le k} \sum_{i \in I_j} X_i .$$

Suppose that $(X_1, X_2, \ldots, X_n)$ are random variables with an unknown joint distribution possessing marginal distributions $(F_1, F_2, \ldots, F_n)$ which are assumed known.

It would be desirable to obtain a smallest possible function ϕ, obtained by means of $(F_1, F_2, \ldots, F_n)$, such that for every joint distribution with those marginals and every positive x,

(2) $$P(M > x) \le \phi(x).$$

We will present a weaker bound, to wit, a ψ for which

(3) $$\int_x^\infty P(M > t)dt \le \psi(x).$$

A bound ϕ as in (2) is a stochastic bound.

A bound ψ as in (3) is a convex bound.

The concept will be explained, and its computation for the problem in question will be presented. It amounts to solving a separable convex minimization problem subject to linear constraints. Such a problem may be approximated by a Linear Programming one. There is a nice interpretation of the solution of the dual problem as the bottleneck probabilities of the paths of the network.

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GREED IS GOOD

I. Meilijson

A problem of optimal placement of data in a disk will be presented, in which the greedy algorithm is optimal.

Let 1,2,...,N be the names of the data points equally spaced along a memory disk. Let $p_1, p_2, \ldots, p_N$ be the frequencies with which they are called for.
Let there be n independent requests for data, and let D be the range of their names (maximal name minus minimal name). The so-called "organ pipe" permutation will be shown to be the order in which the data points should be placed along the disk so as to minimize the expected value of D, for every $n \geq 2$. A stronger result, for $n = 2$ only, appears in Bergmans (1972). This result will be seen to be a special case of the following:
Let ϕ be a symmetric concave function and see it as the roof of a house with a one dimensional floor extending from a to b, not necessarily satisfying $\phi(a) = \phi(b)$. We tell the builder the sizes of the rooms we want, but he may permute them at will. A "greedy" builder will try to minimize at every step the height of the next wall he builds, i.e., he will start by placing the shortest room at the lowest end of the house, and proceed inductively. In doing so, he actually minimizes the sum of the heights of the walls!
A counterexample will be presented in which ϕ is concave but not symmetric.

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THE CONCEPT OF STATE IN DETERMINISTIC AND STOCHASTIC SYSTEM THEORY

S.K. Mitter

In this lecture we present a state space theory for deterministic and stochastic systems. Intuitively, the concept of state may be defined as the minimal amount of information necessary to predict the future external behavior of a system. We give an abstract framework to study this and illustrate it with examples.

Reference

J.C. Willems: "System Theoretic Models for the Analysis of Physical Systems" to appear in Richerche di Automatica and the bibliography cited there.

RECENT RESULTS IN NON-LINEAR FILTERING

S.K. Mitter

Considerable progress has recently been made in the problem of estimating the state of a stochastic dynamical system from noisy observations. We give an account of these ideas, especially those developments which use functional integration and group representation methods.

Reference

S.K. Mitter: "On the Analogy Between Mathematical Problems of Non-Linear Filtering and Quantum Physics". To appear in Ricerche di Automatica, Special Issue on System Theory and Physics.

PIECEWISE-LINEAR HOMOTOPY ALGORITHMS: A SURVEY WITH APPLICATIONS TO OPTIMIZATION PROBLEMS

M.J. Todd

This talk will discuss the piecewise-linear homotopy algorithms (also called fixed-point or simplicial algorithms) that have been developed in the last fifteen years for the solution of zero-finding and fixed-point problems. For general references on these methods, see Scarf and Hansen [9], Todd [10], Eaves [2] and Allgower and Georg [1].

We will first describe how optimization and equilibrium problems can be formulated as zero-finding problems. Then, after a brief discussion of the early "fixed-grid" algorithms, the more recent methods of Eaves and Saigal [3], Merrill [5] and van der Laan and Talman [4] will be presented. If a zero of a continuous function $f:R^n \to R^n$ is sought, these methods choose a simple function $f^o:R^n \to R^n$ with a known zero x^o and introduce a homotopy function $h:R^n \times [0,1] \to R^n$ with $h(.,0) = f^o$ and $h(.,1) = f$. They then approximate a path of solutions to $h(x,t) = 0$ starting from $(x^o,0)$. For this purpose h is replaced by a piecewise-linear approximation ℓ using a triangulation of $R^n x[0,1]$.

Because these methods were first proposed for general fixed-point problems, they have very attractive global convergence properties for certain classes of problem. In addition, recent work of Saigal [6] and Saigal and Todd [8] has shown that they can be accelerated on smooth problems to achieve fast local convergence. However, on such problems they tend to be rather slower than more traditional methods in numerical analysis, and are certainly more complicated to implement, though computer codes are now available (Saigal [7], Todd [12]).

Throughout the talk, we will focus on applications to optimization problems. In such cases, piecewise-linear homotopy algorithms have two special drawbacks: they fail to use function values (only gradients being relevant) or the symmetry of Hessian matrices. On the other hand, the path followed by the algorithms does give insight into the original optimization problem [11].

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EXPLOITING SPECIAL STRUCTURE IN PIECEWISE-LINEAR HOMOTOPY ALGORITHMS

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Piecewise-linear homotopy algorithms [1,3,5,6] possess some attractive features for solving zero-finding and fixed-point problems. However, they tend to become increasingly inefficient as the dimension becomes large, especially in the point-to-set case (i.e. when solving $0 \in F(x)$, where F takes points of R^n into compact convex subsets of R^n). This talk will describe some recent enhancements to these algorithms to exploit special structure. Scarf (see [5]) developed the first specialized algorithms, but these were only applicable to certain economic equilibrium problems. Subsequently Kojima [4] considered the separable case ($f(x) = \sum_i f^i(x_i)$, where each f^i maps R into R^n), and Todd [7,8,9] provided a general framework embracing partial separability and sparsity. The main idea is that if ℓ^T is the piecewise-linear approximation with respect to the triangulation T of the homotopy $h(x,t) = tf(x) + (1-t)f^o(x)$, then the pieces of linearity of ℓ^T which must be traversed by the algorithm can be much larger than simplices of T. Of course, these large pieces depend on the special structure of f and the particular triangulation T employed. We illustrate this approach by considering the example of a separable function f.

One of the advantages of piecewise-linear homotopy algorithms is that they are capable of treating the zero-finding problem for a point-to-set mapping F, as arises for example in constrained optimization problems. However, as noted above, the efficiency of the algorithms is substantially degraded in this case. We conclude by discussing accelerated algorithms when F is a convex union of smooth functions, as it is in most applications. In this case, fast local convergence can be achieved (Awoniyi [2]); preliminary computational experience is encouraging.

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