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Accumulation Bias in meta-analysis

How to handle it ALL-IN

(and maybe subjective Bayesian)

Judith ter Schure

CWI, Machine Learning group

What's to come

Accumulation Bias

- The assumption of random sampling in a typical meta-analysis
- The assumption of fixed-series-size in a typical meta-analysis

Handling it ALL-IN

- Taking a random approach to series size
- Martingales

Empirical evidence for Accumulation Bias

What's to come

Accumulation Bias

- The assumption of random sampling in a typical meta-analysis
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Handling it ALL-IN

- Taking a random approach to series size
- Martingales

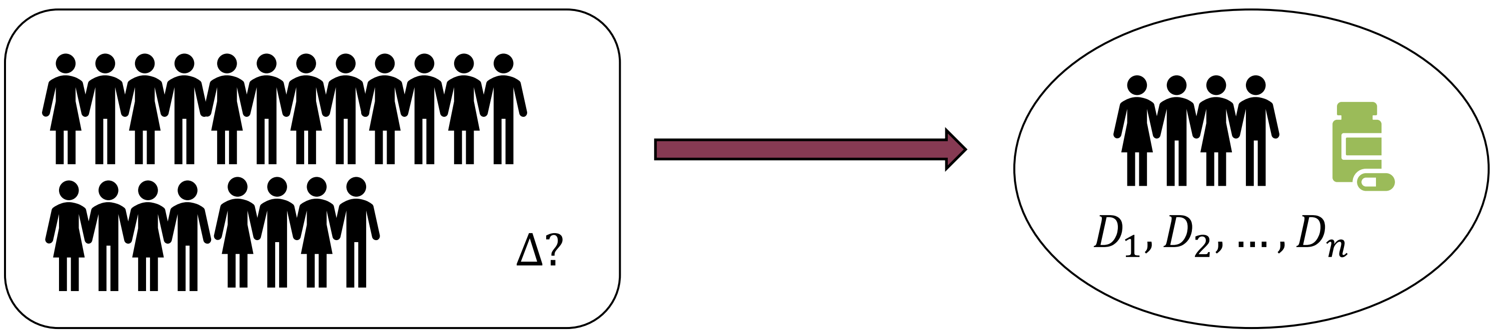
Empirical evidence for Accumulation Bias

(maybe) ***Handling it subjective Bayesian***

- Mixing a fixed-series-size approach with a prior

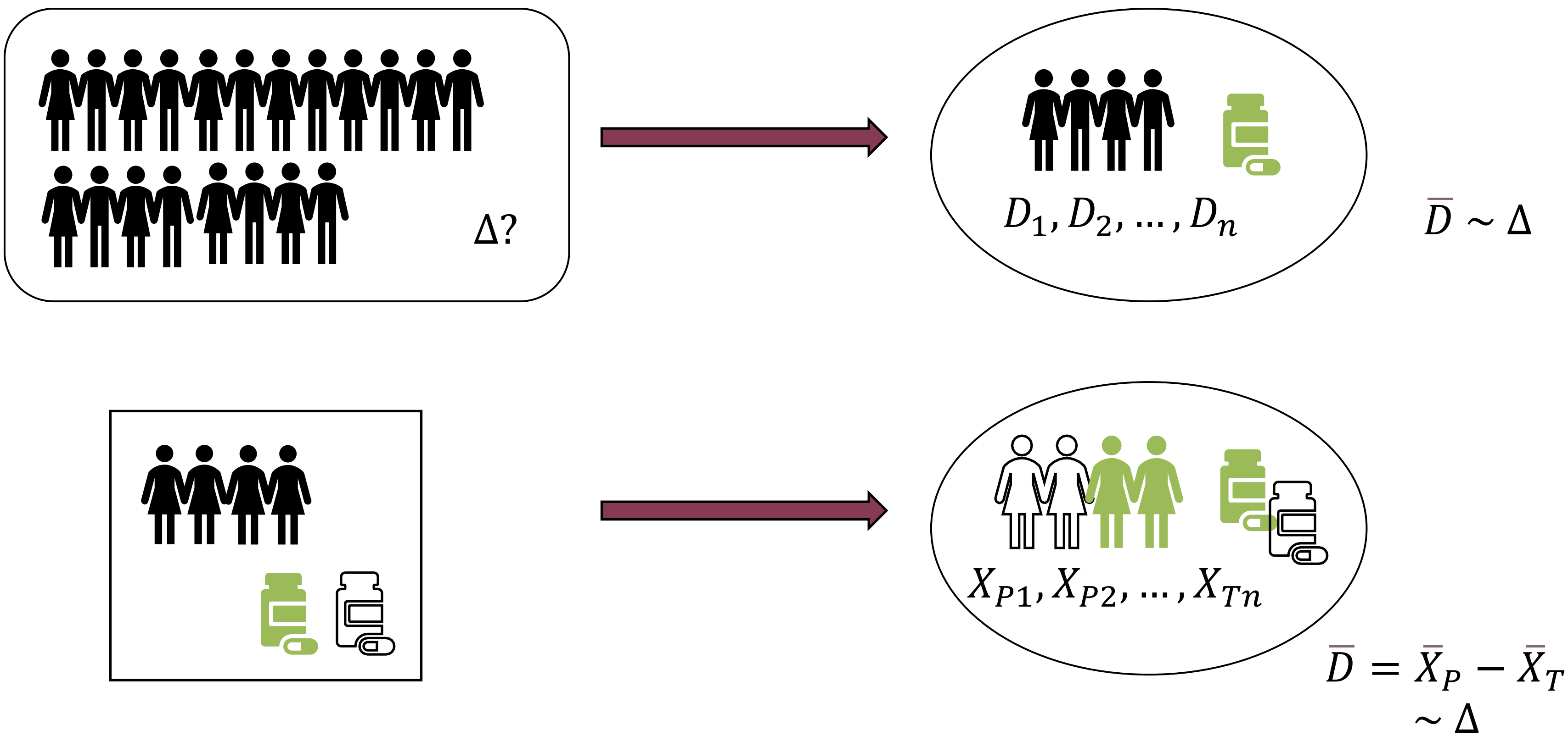
Random sampling in science

Random sampling of participants

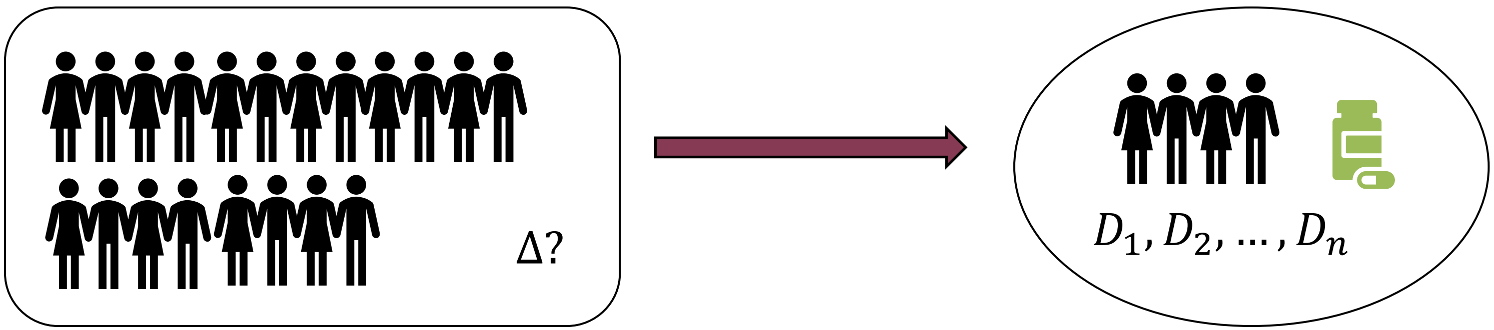


$\overline{D} \sim \Delta$

Random sampling of participants

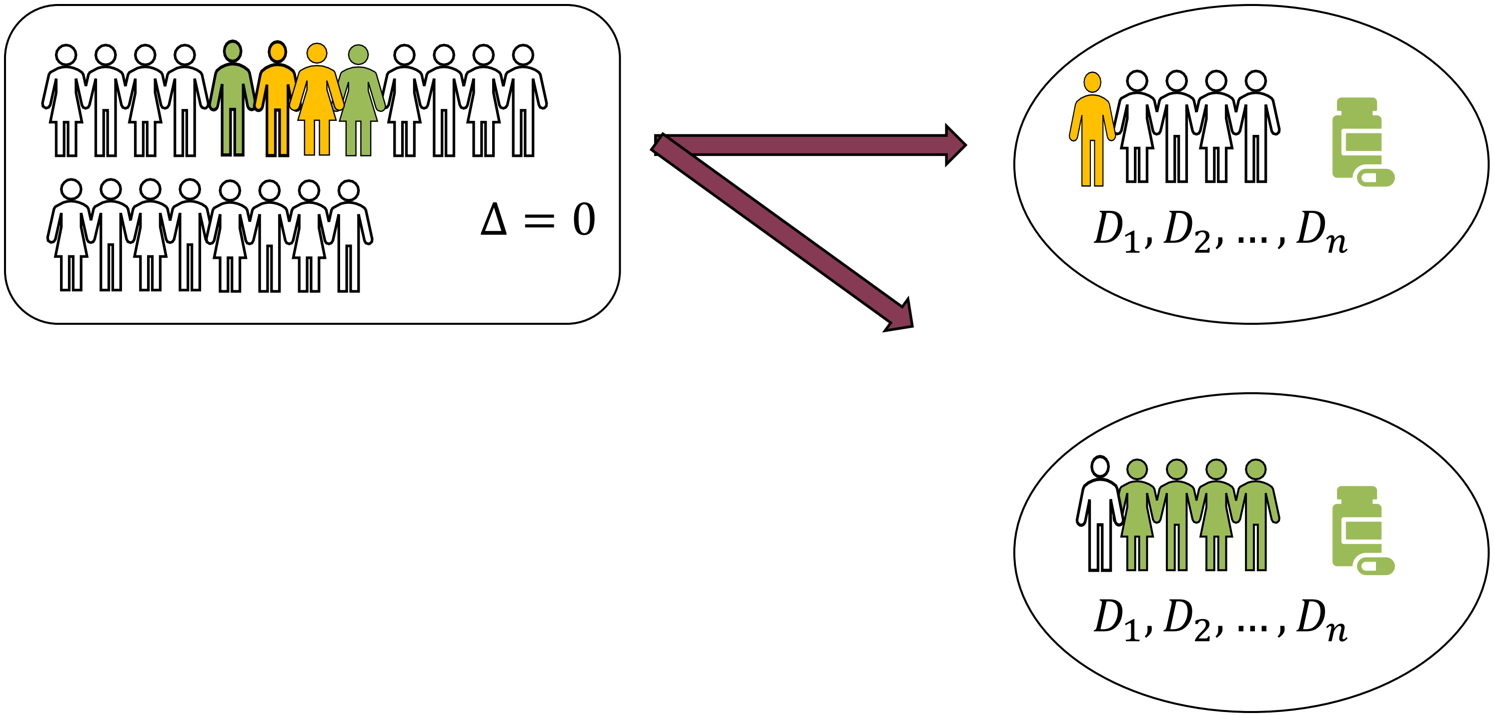


Random sampling of participants

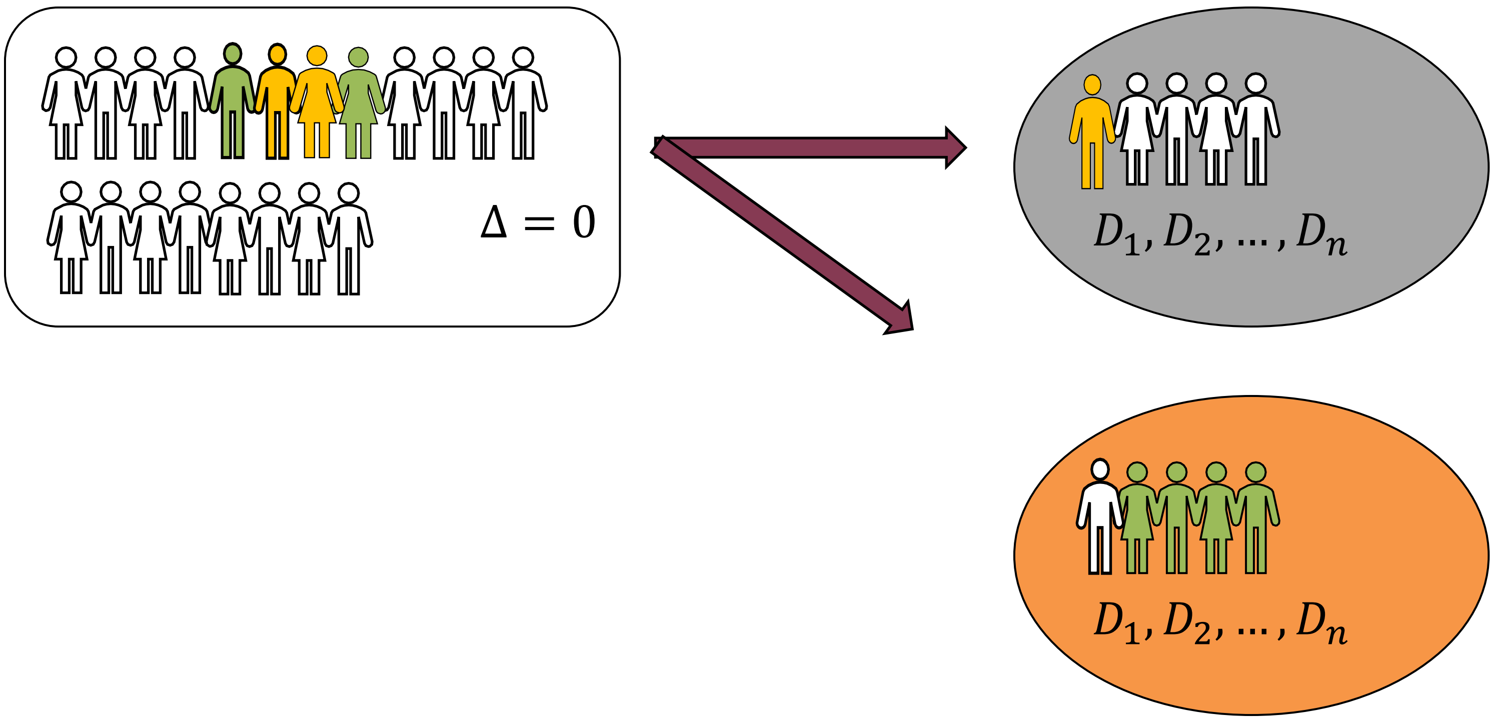


$\bar{D} \sim \Delta$

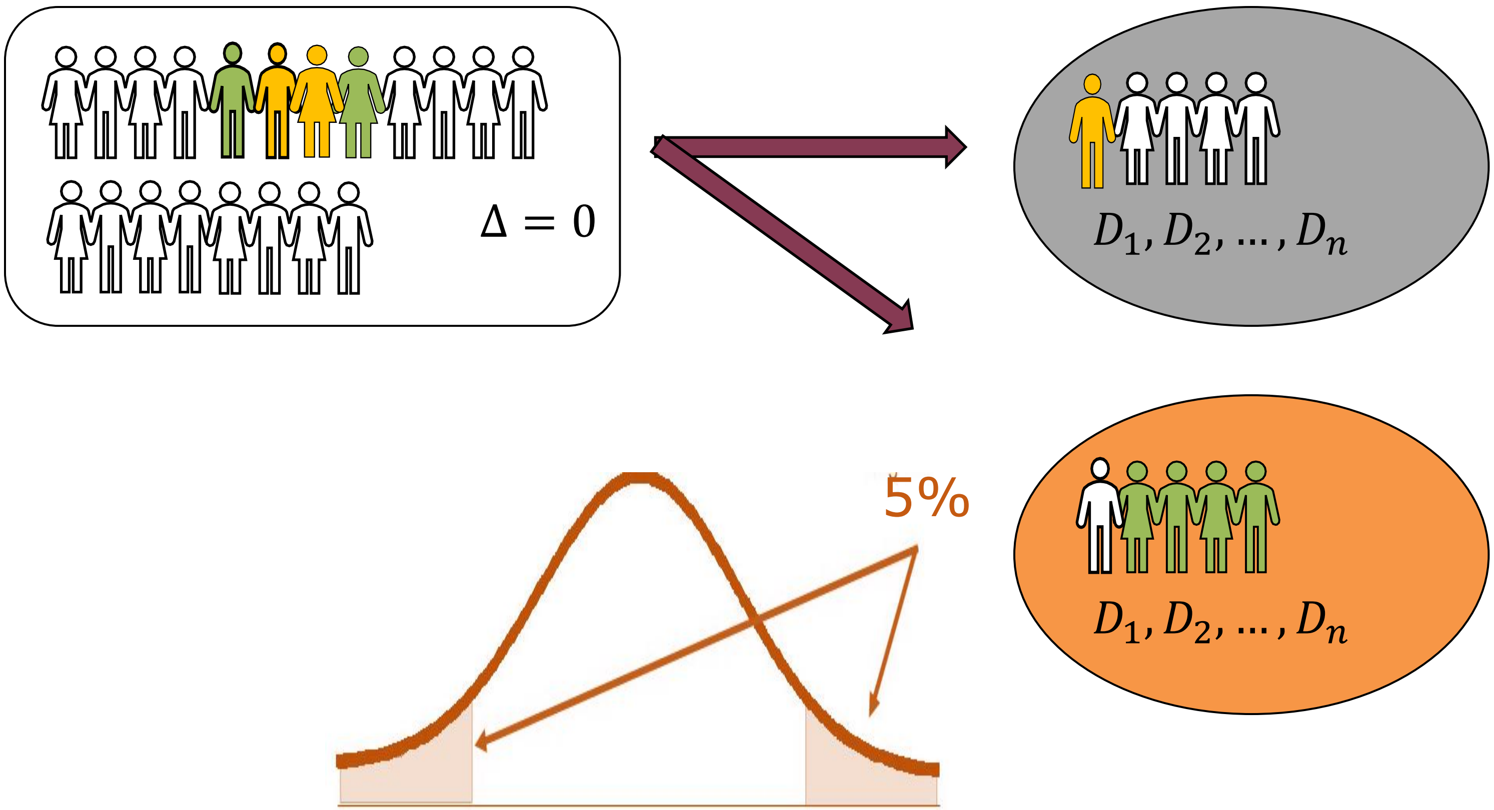
Random sampling of participants



Random sampling of participants

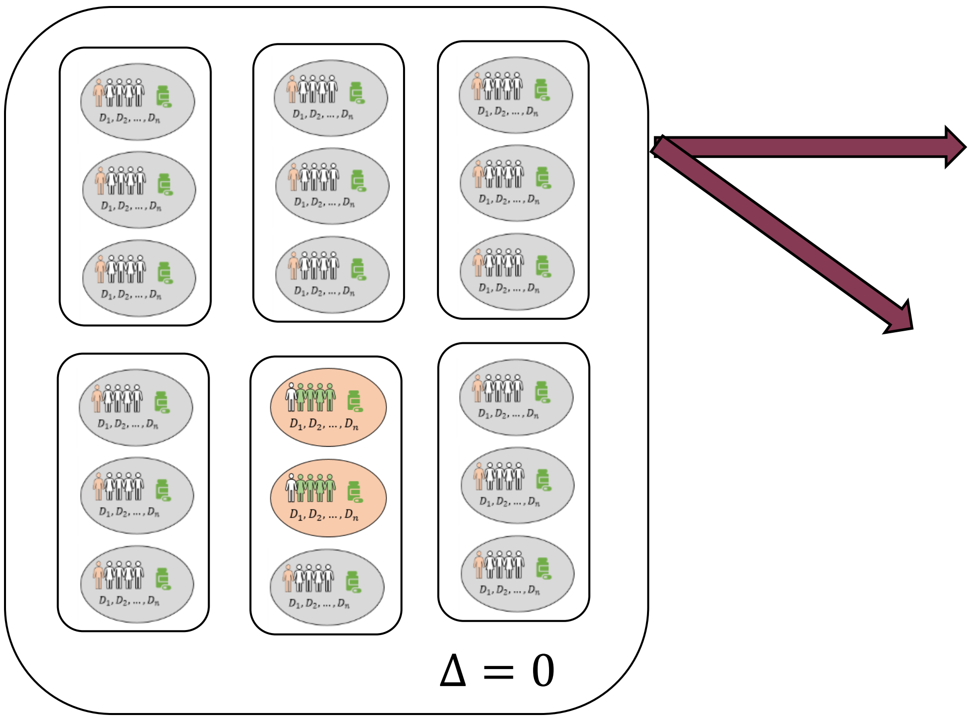


Random sampling of participants

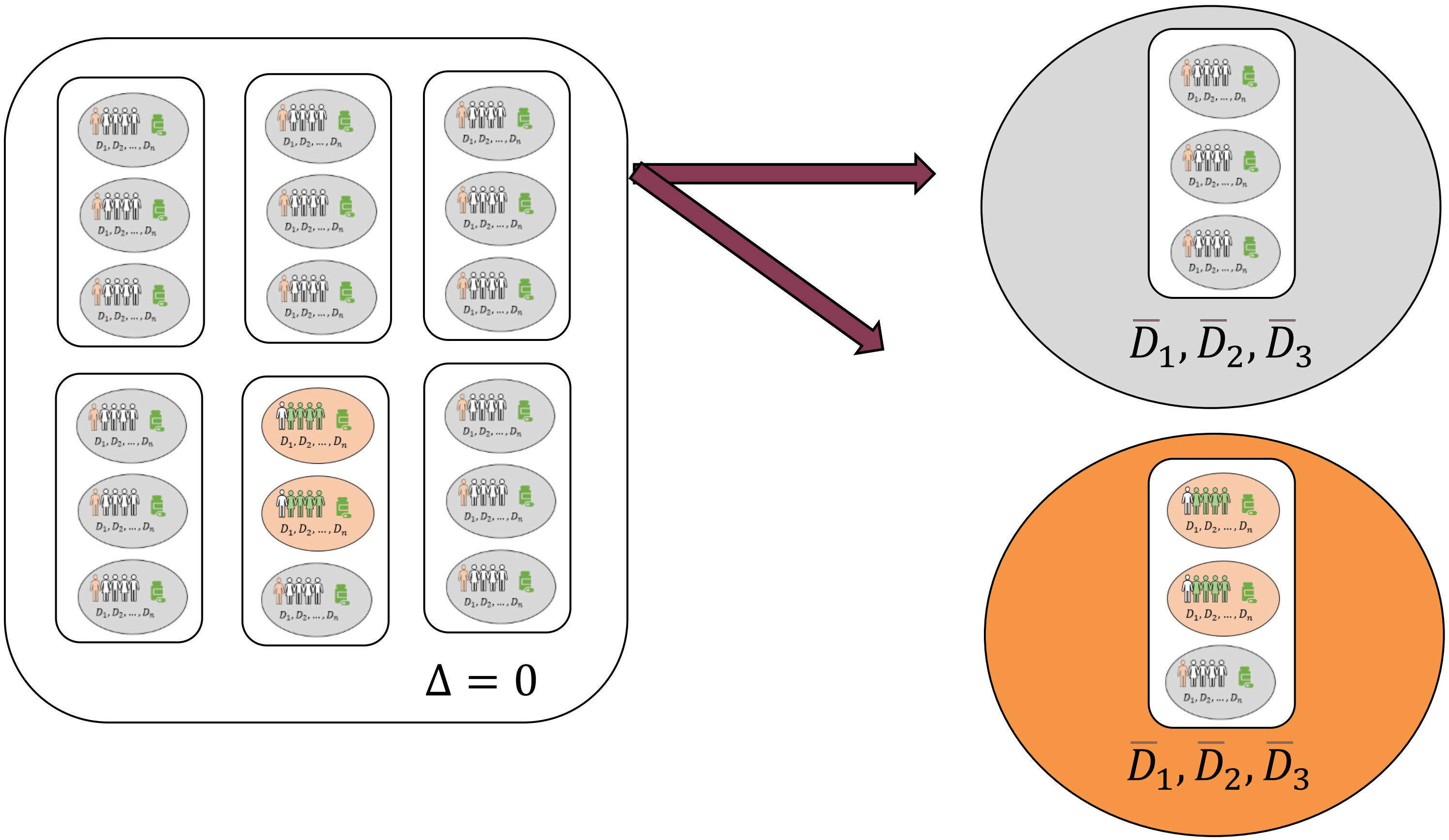


Random sampling of study series

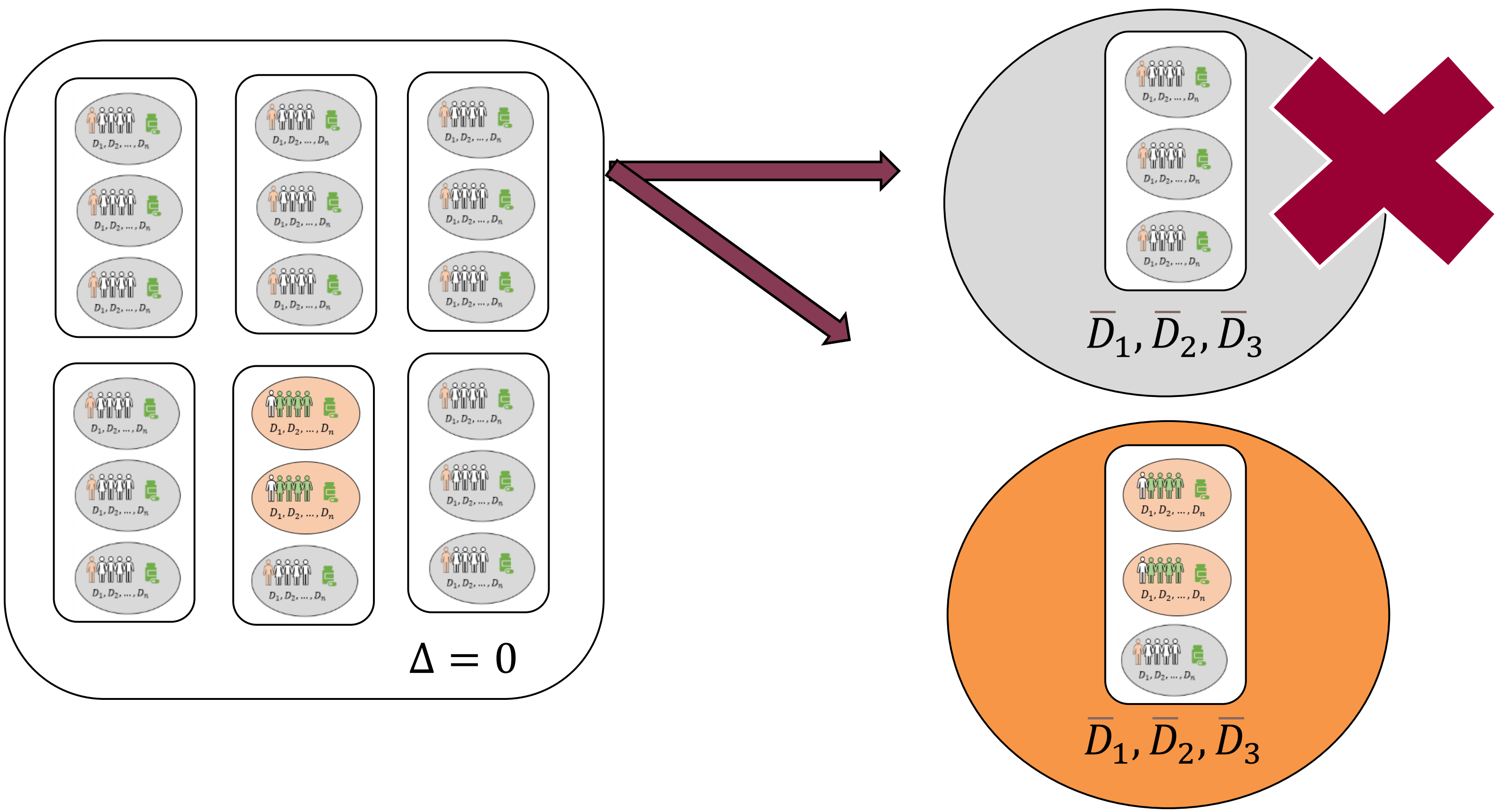
Random sampling of study series



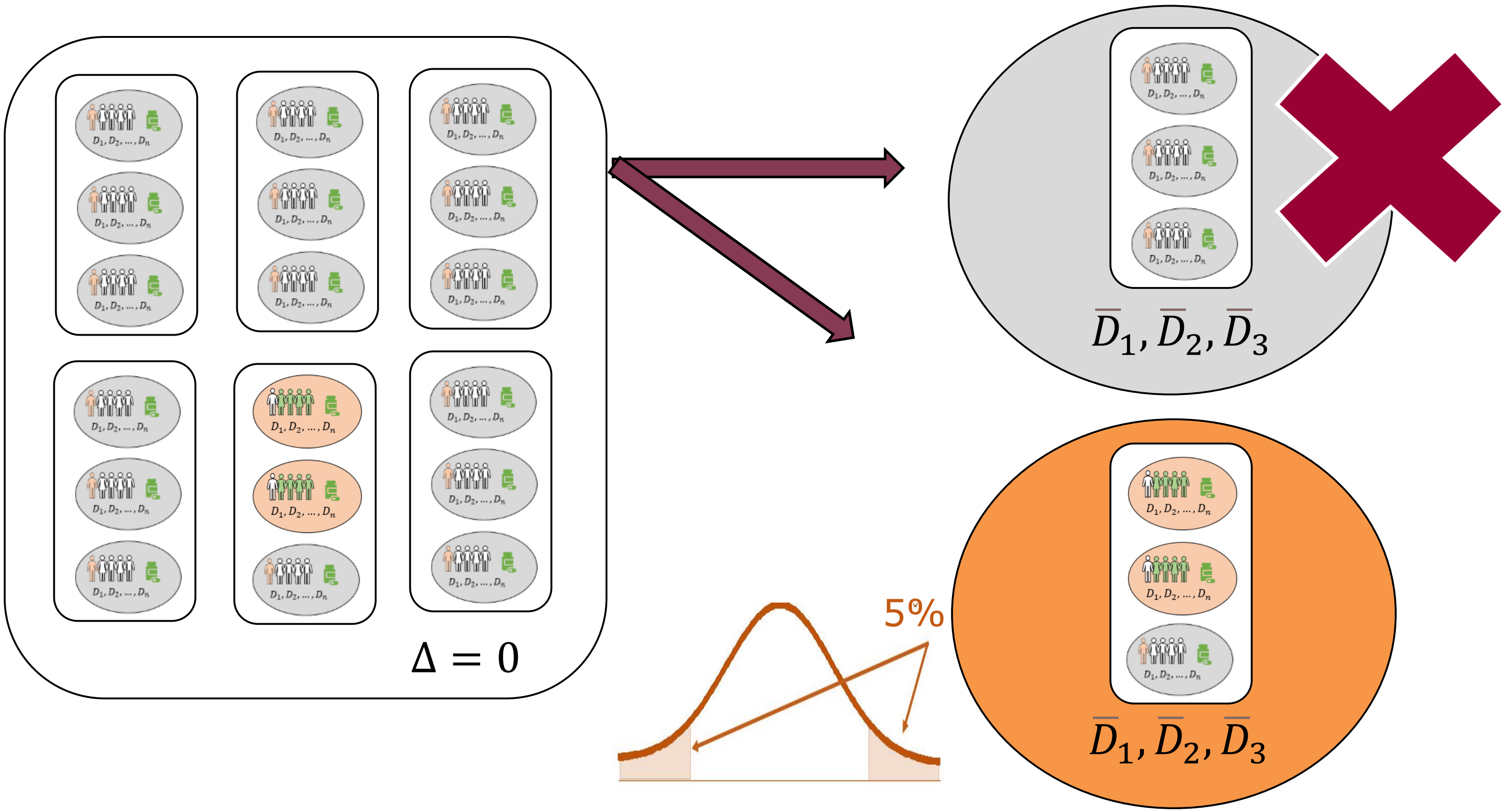
Random sampling of study series



Random sampling of study series



Random sampling of study series



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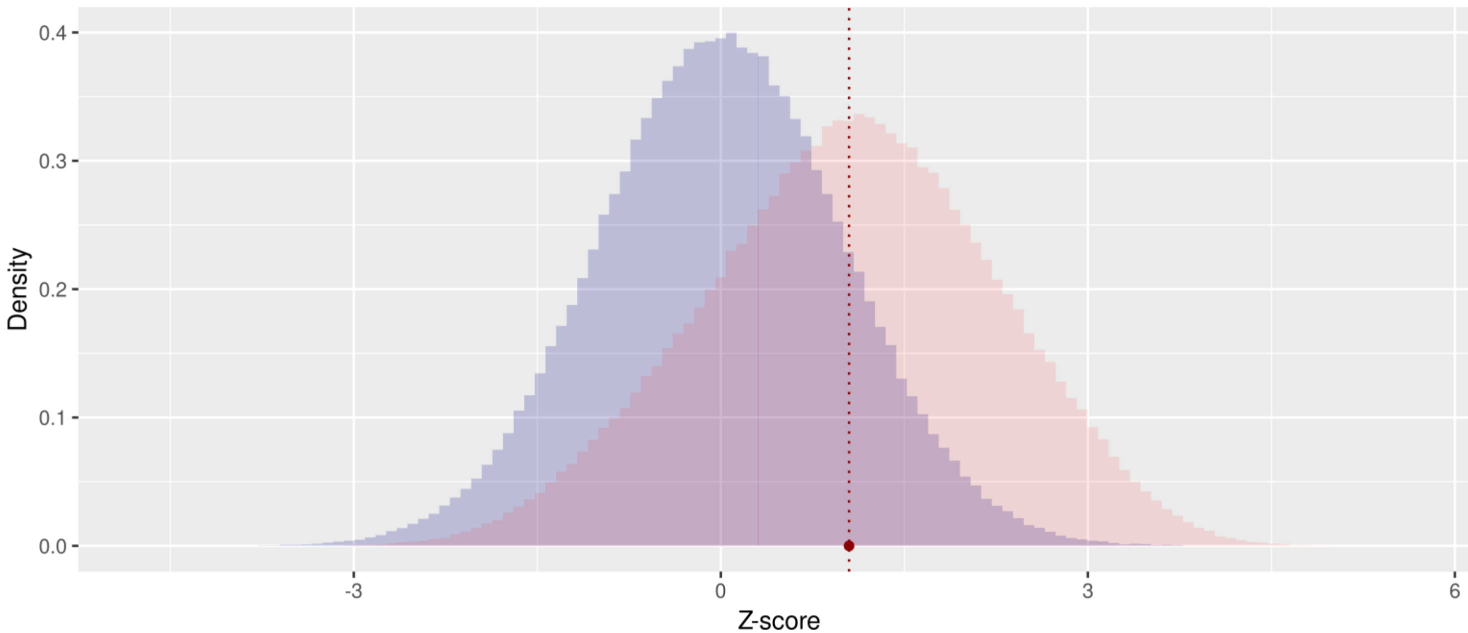
Accumulation Bias arises if

whether or not a series of studies accumulates

is dependent on the results within

Accumulation Bias



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Questions?

Questions?

1. Why do you think statisticians argue against meta-analyzing **replication studies** with their **original studies**?
2. How does **accumulation bias** relate to **publication bias**?

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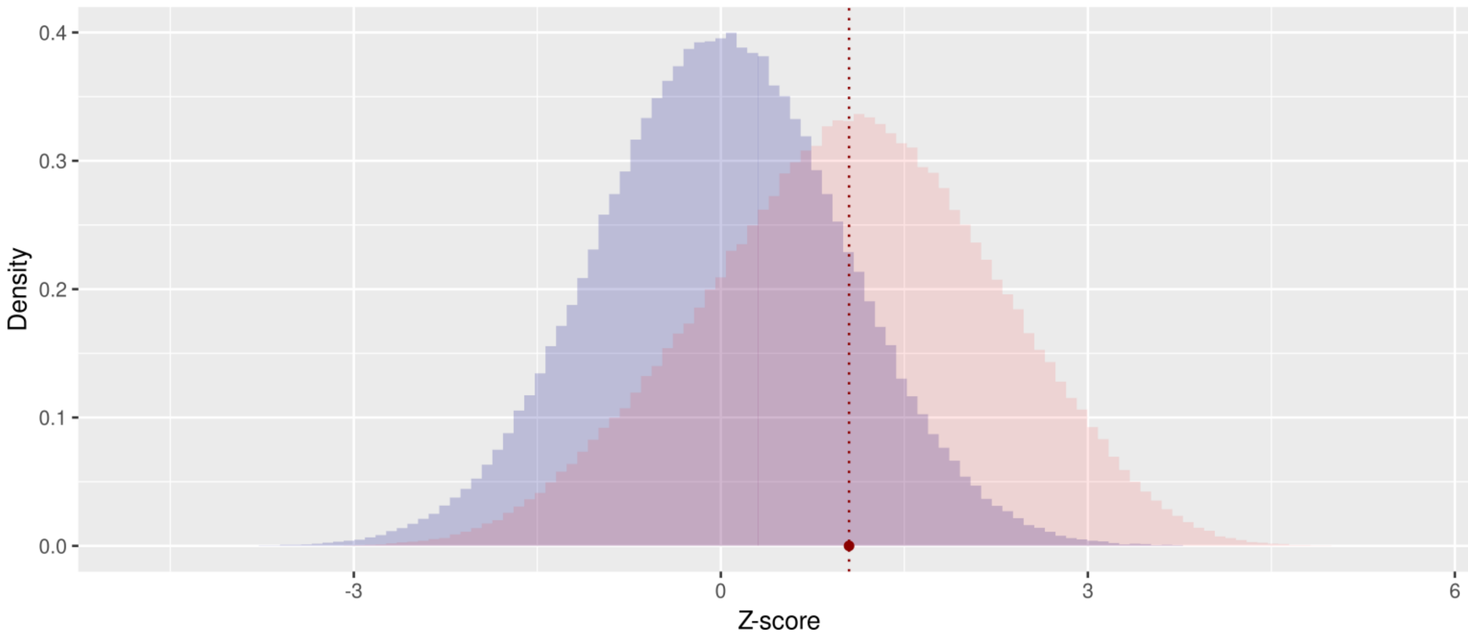
Accumulation Bias arises if

whether or not a series of studies accumulates

is dependent on the results within

Accumulation Bias



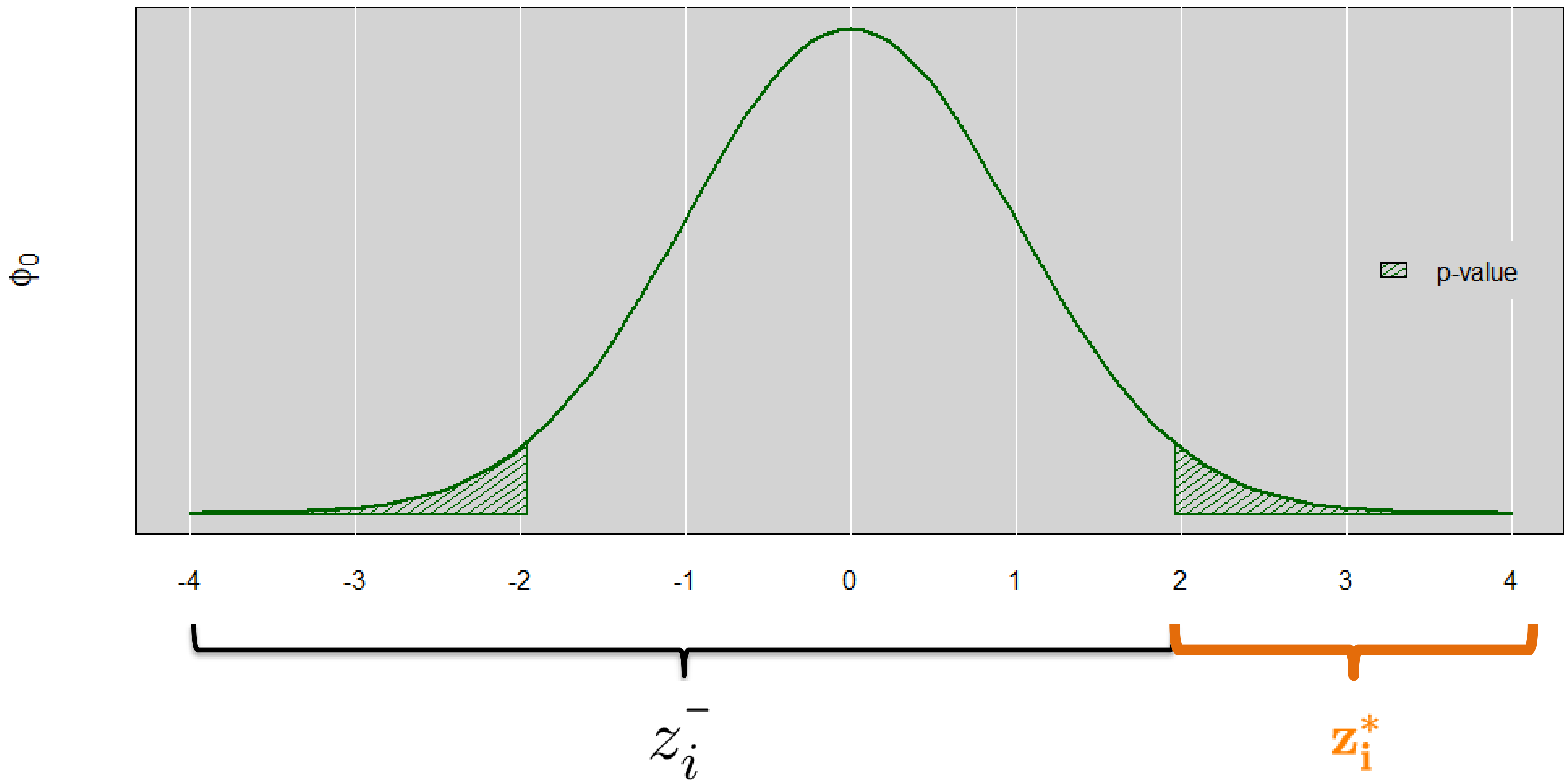
Accumulation Bias framework

$$A(t)$$

Gold Rush example

$$z_1, z_2, z_3, \ldots, z_t$$

Gold Rush example



Gold Rush example

$$z_1, z_2, z_3, \ldots, z_t$$

Gold Rush example

$$z_1, z_2, z_3, \ldots, z_t$$

$$A\,(t \,|\, z_1, \ldots, z_{t-1}) = \begin{cases} 1, & \text{if } t = 1 \quad \mathit{or} \quad \text{for all } i < t: \quad z_i = \mathbf{z_i^*}. \\ 0, & \text{otherwise.} \end{cases}$$

Gold Rush example

$$z_1^-$$

$$\mathbf{z}_1^*$$

$$\mathbf{z}_1^*, z_2^-$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^-$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*$$

Gold Rush example

$$
\begin{aligned}
z_1^- &\rightarrow z^{(1)} \\
\mathbf{z}_1^* &\rightarrow z^{(1)} \\
\mathbf{z}_1^*, z_2^- &\rightarrow z^{(2)} \\
\mathbf{z}_1^*, \mathbf{z}_2^* &\rightarrow z^{(2)} \\
\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^- &\rightarrow z^{(3)} \\
\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^* &\rightarrow z^{(3)}
\end{aligned}
$$

Gold Rush example

$$z_1^-$$

$$\mathbf{z}_1^*$$

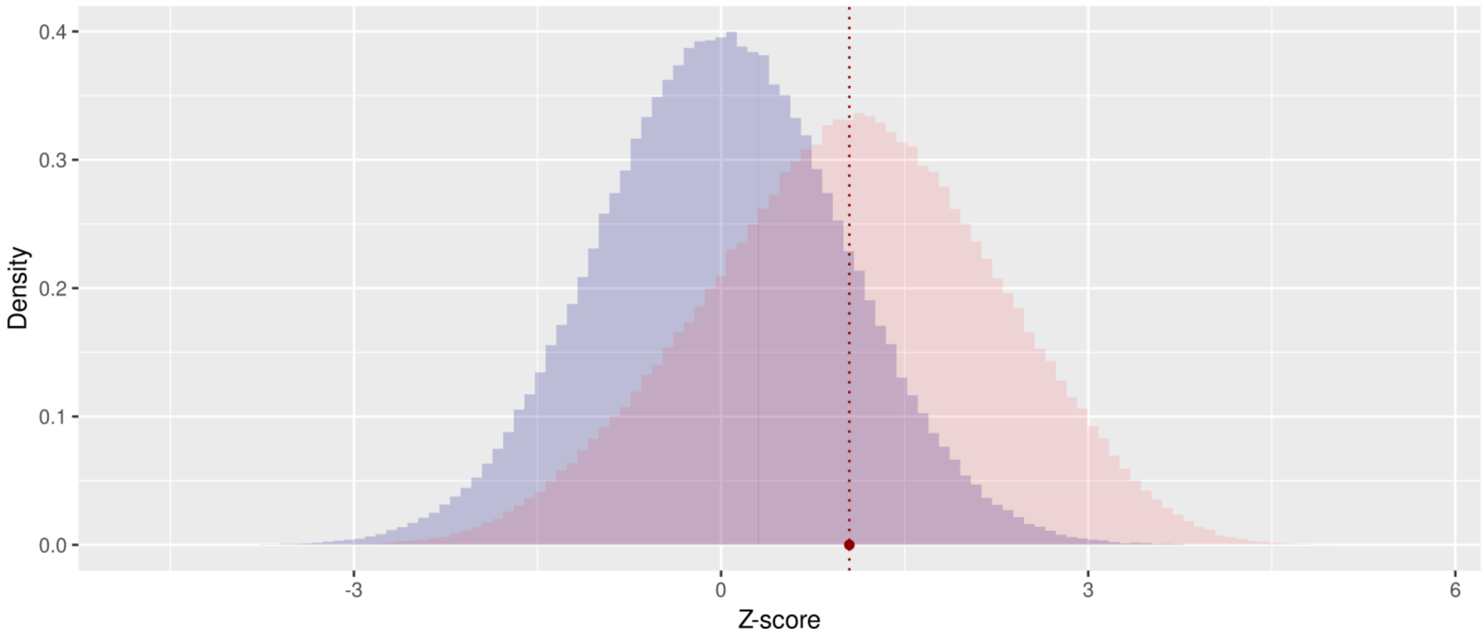
$$\mathbf{z}_1^*, z_2^-$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*$$

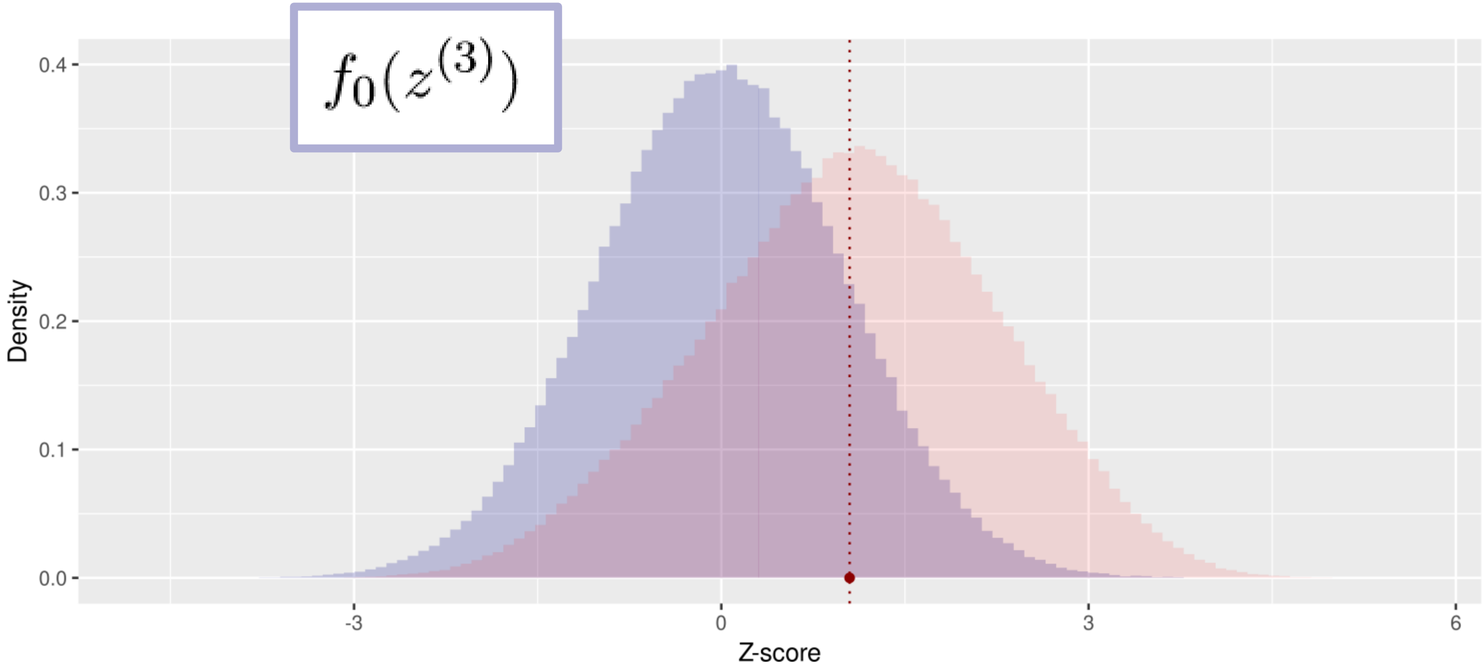
$$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^- \quad \to \quad z^{(3)} \qquad\qquad T \geq 3$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^* \quad \to \quad z^{(3)}$$

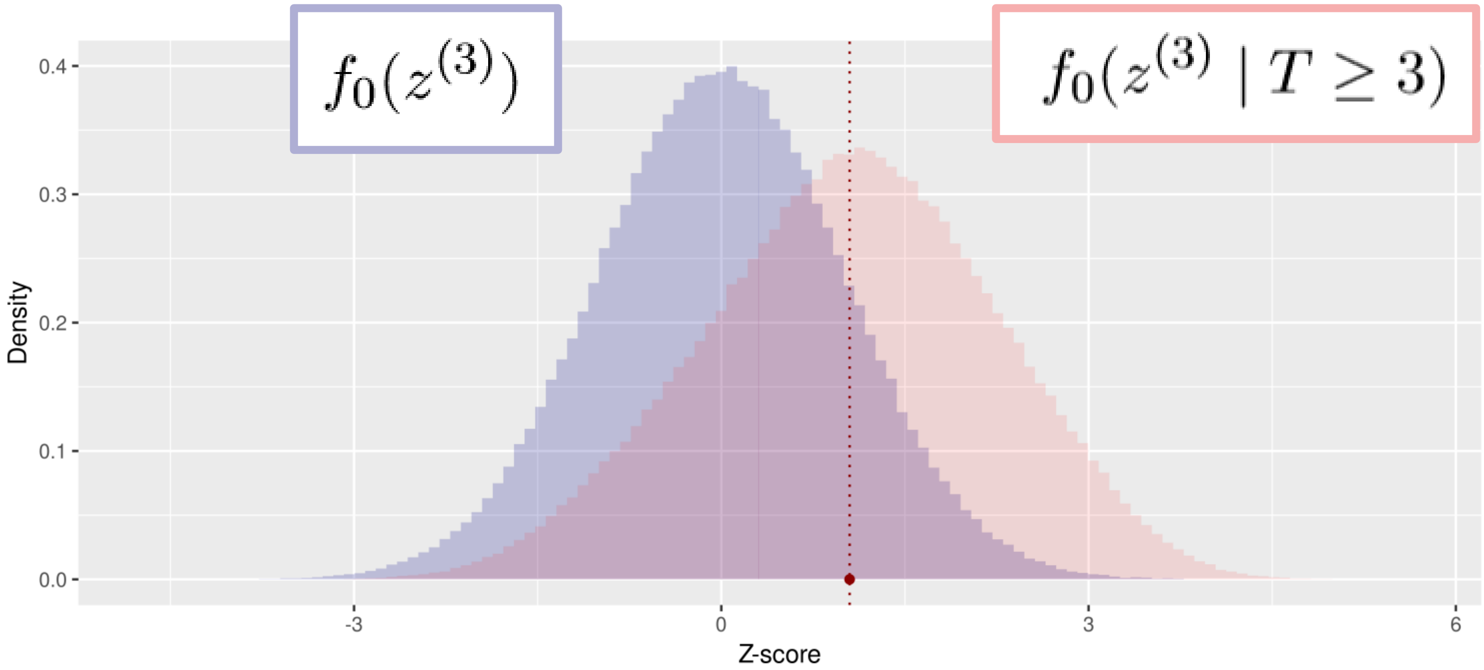
Accumulation Bias



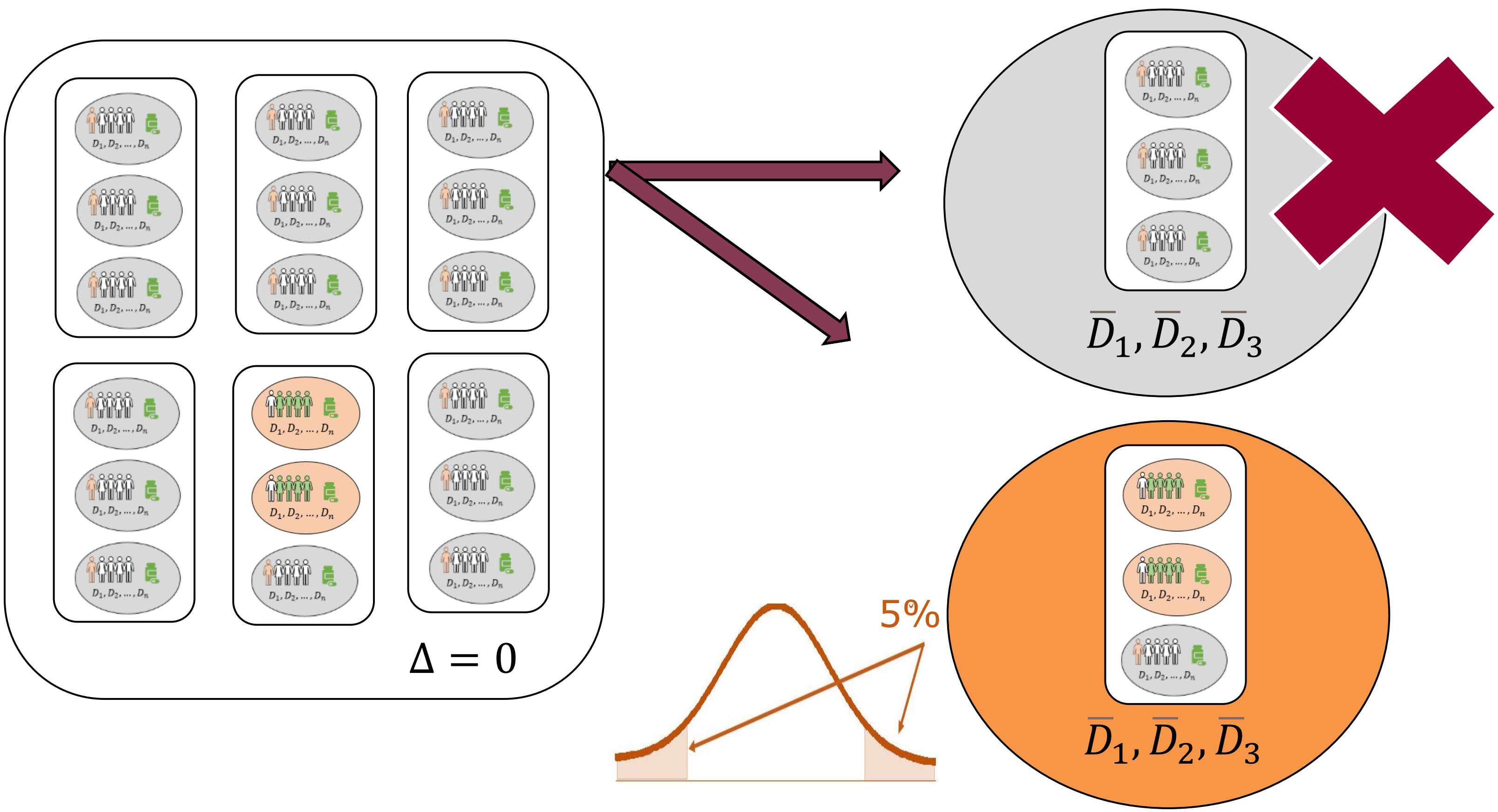
Accumulation Bias



Accumulation Bias



Random sampling of study series *of fixed size*



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A typical meta-analysis is a

conditional analysis:

given the available number of studies

Accumulation Bias invalidates

the assumed conditional sampling distribution

inflating type-I errors

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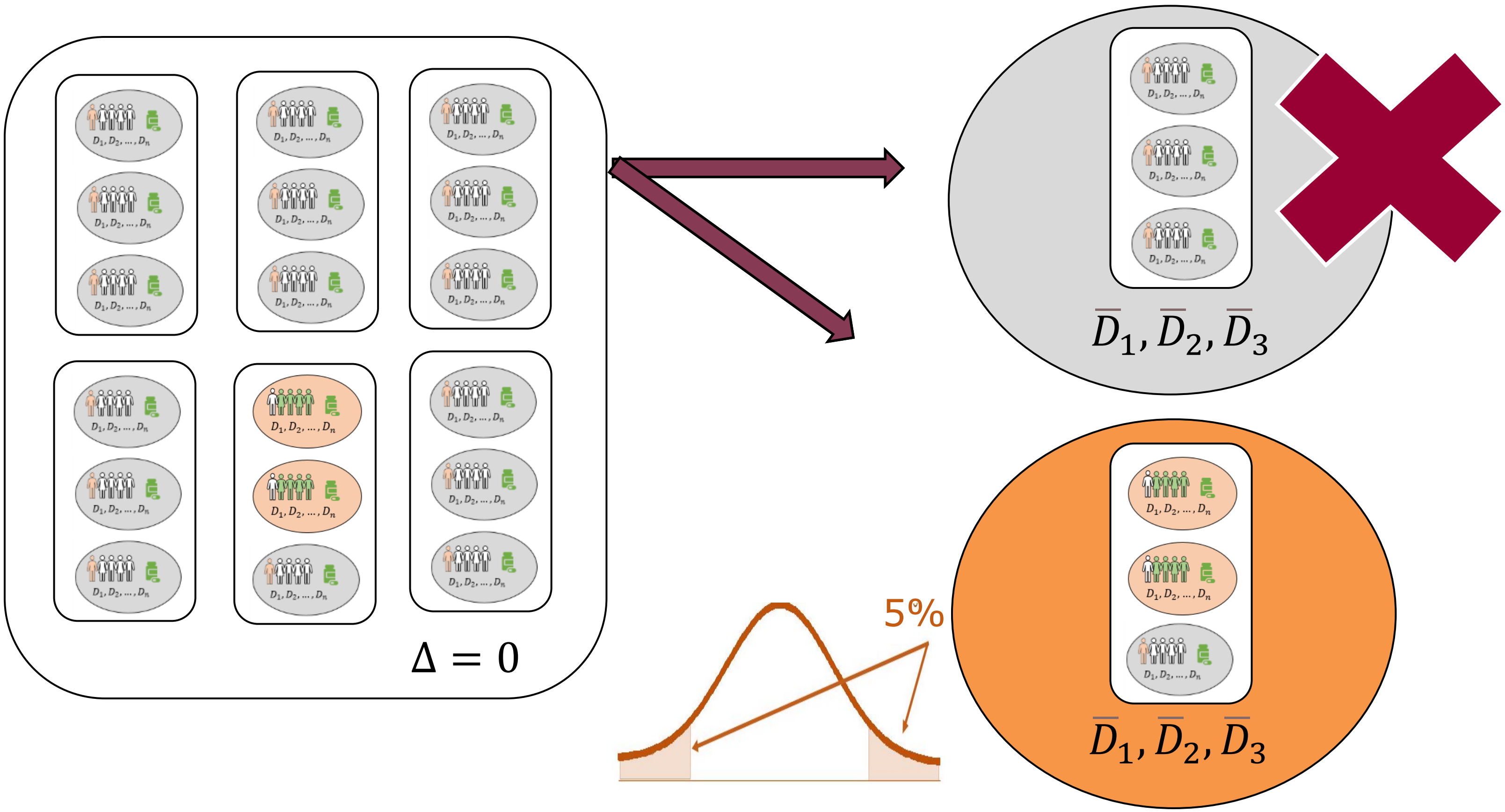
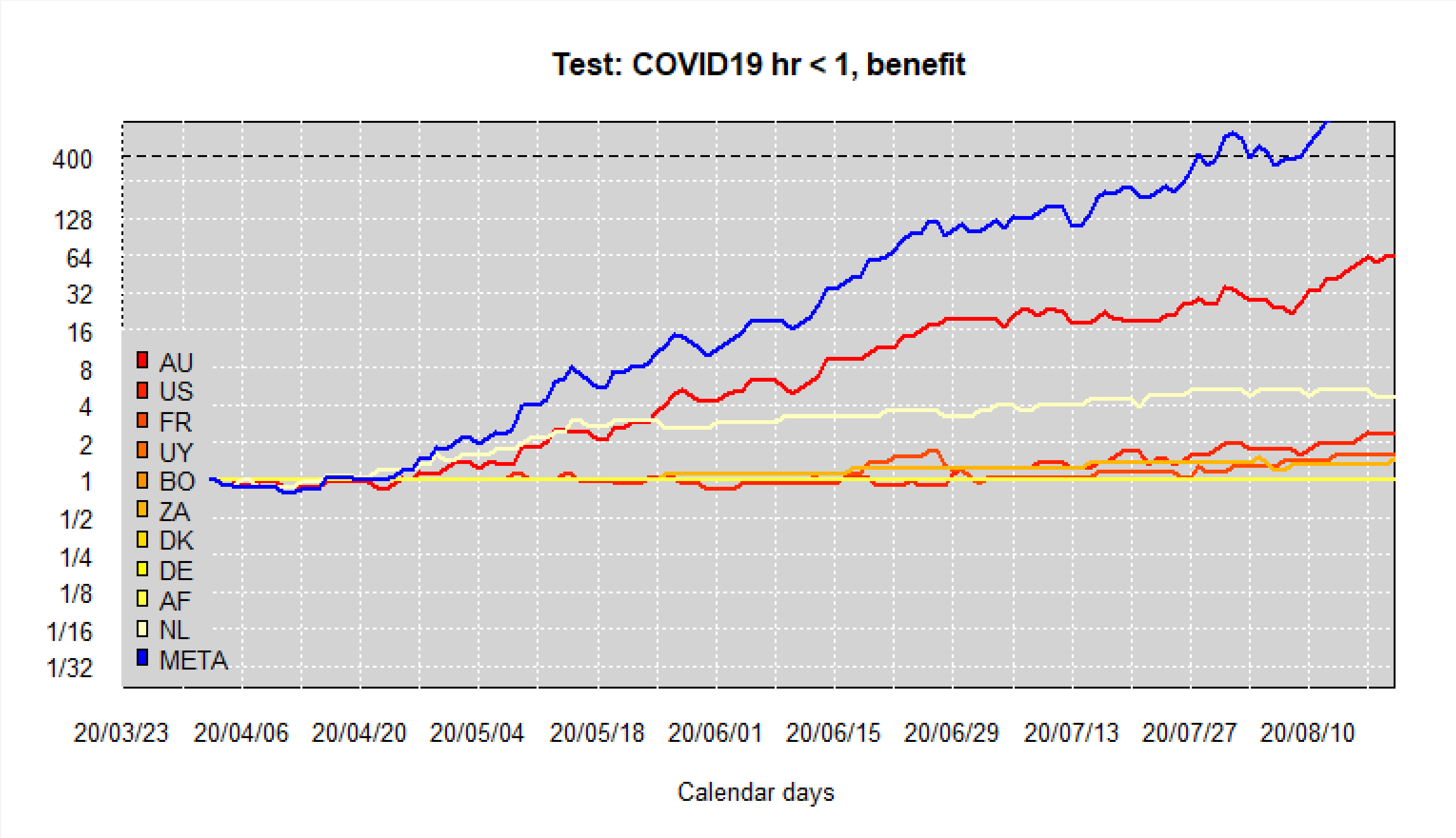
Questions?

Random sampling of study series *of fixed size*



ALL-IN meta-analysis



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ALL-IN meta-analysis

Any*time*
Live and
Leading
INterim meta-analysis

Judith ter Schure – UC Davis Quant Brown Bag Series Oct 22th 2020

ALL-IN meta-analysis is an

unconditional analysis:

assuming random number of studies

Gold Rush example

$$z_1^-$$

$$\mathbf{z}_1^*$$

$$\mathbf{z}_1^*, z_2^-$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^- \quad \rightarrow \quad z^{(3)} \qquad\qquad T \geq 3$$

$$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^* \quad \rightarrow \quad z^{(3)}$$

Gold Rush example

$$
\begin{array}{rcl}
z_1^- & \rightarrow & z^{(1)} \\
\mathbf{z}_1^* & \rightarrow & z^{(1)} \\
\mathbf{z}_1^*, z_2^- & \rightarrow & z^{(2)} \\
\mathbf{z}_1^*, \mathbf{z}_2^* & \rightarrow & z^{(2)} \\
\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^- & \rightarrow & z^{(3)} \\
\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^* & \rightarrow & z^{(3)}
\end{array}
$$

Gold Rush example

$$
\begin{array}{rcl}
z_1^- & \rightarrow & z^{(1)} \\
\cancel{\mathbf{z}_1^*} & \rightarrow & z^{(1)} \\
\mathbf{z}_1^*, z_2^- & \rightarrow & z^{(2)} \\
\cancel{\mathbf{z}_1^*, \mathbf{z}_2^*} & \rightarrow & z^{(2)} \\
\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^- & \rightarrow & z^{(3)} \\
\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^* & \rightarrow & z^{(3)}
\end{array}
$$

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Gold Rush example *ALL-IN* solution

t		*
1	z_1^-	0
2	$\mathbf{z_1^*}, z_2^-$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

Gold Rush example *ALL-IN* solution

t		$*$
1	z_1^-	0
2	$\mathbf{z_1^*}, z_2^-$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

$\mathbf{P}_0$

Gold Rush example — *ALL-IN* solution

t		*
1	z_1^-	0
2	$\mathbf{z}_1^*, z_2^-$	1
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^-$	2
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*$	3
$\sum$		

P_0
$1-\alpha$

Gold Rush example *ALL-IN* solution

t		*
1	z_1^-	0
2	$\mathbf{z_1^*}, z_2^-$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

P_0
$1-\alpha$
$\alpha(1-\alpha)$

Gold Rush example *ALL-IN* solution

t		$*$	P_0
1	z_1^-	0	$1-\alpha$
2	$\mathbf{z_1^*}, z_2^-$	1	$\alpha(1-\alpha)$
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	2	$\alpha^2(1-\alpha)$
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3	
$\sum$			

Gold Rush example *ALL-IN* solution

t		*
1	z_1^-	0
2	$\mathbf{z_1^*}, z_2^-$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

$\mathbf{P}_0$
$1-\alpha$
$\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$
α^3
1

Gold Rush example *ALL-IN* solution

t		$*$
1	z_1^-	0
2	$\mathbf{z}_1^*, z_2^-$	1
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^-$	2
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*$	3
$\sum$		

$\mathbf{P}_0$	$* \cdot \mathbf{P}_0$
$1-\alpha$	0
$\alpha(1-\alpha)$	$\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$	$2\alpha^2(1-\alpha)$
α^3	$3\alpha^3$
1	$\alpha+\alpha^2+\alpha^3$

Gold Rush example *ALL-IN* solution

t		$*$
1	z_1^-	0
2	$\mathbf{z_1^*}, z_2^-$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

$\mathbf{P}_0$	$* \cdot \mathbf{P}_0$	$\mathbf{t} \cdot \mathbf{P}_0$
$1-\alpha$	0	$1-\alpha$
$\alpha(1-\alpha)$	$\alpha(1-\alpha)$	$2\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$	$2\alpha^2(1-\alpha)$	$3\alpha^2(1-\alpha)$
α^3	$3\alpha^3$	$3\alpha^3$
1	$\alpha+\alpha^2+\alpha^3$	$1+\alpha+\alpha^2$

Gold Rush example *ALL-IN* solution

t		$*$
1	z_1^-	0
2	$\mathbf{z_1^*}, z_2^-$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

$\mathbf{P}_0$	$* \cdot \mathbf{P}_0$	$\mathbf{t} \cdot \mathbf{P}_0$
$1-\alpha$	0	$1-\alpha$
$\alpha(1-\alpha)$	$\alpha(1-\alpha)$	$2\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$	$2\alpha^2(1-\alpha)$	$3\alpha^2(1-\alpha)$
α^3	$3\alpha^3$	$3\alpha^3$
1	$\alpha+\alpha^2+\alpha^3$	$1+\alpha+\alpha^2$

$$\frac{\mathbf{E}_0[*]}{\mathbf{E}_0[t]} =$$

Gold Rush example *ALL-IN* solution

t		$*$
1	z_1^-	0
2	$\mathbf{z}_1^*, z_2^-$	1
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^-$	2
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*$	3
$\sum$		

$\mathbf{P}_0$	$* \cdot \mathbf{P}_0$	$\mathbf{t} \cdot \mathbf{P}_0$
$1-\alpha$	0	$1-\alpha$
$\alpha(1-\alpha)$	$\alpha(1-\alpha)$	$2\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$	$2\alpha^2(1-\alpha)$	$3\alpha^2(1-\alpha)$
α^3	$3\alpha^3$	$3\alpha^3$
1	$\alpha+\alpha^2+\alpha^3$	$1+\alpha+\alpha^2$

$$\frac{\mathbf{E}_0[*]}{\mathbf{E}_0[t]} = \frac{\alpha(1+\alpha+\alpha^2)}{1+\alpha+\alpha^2}$$

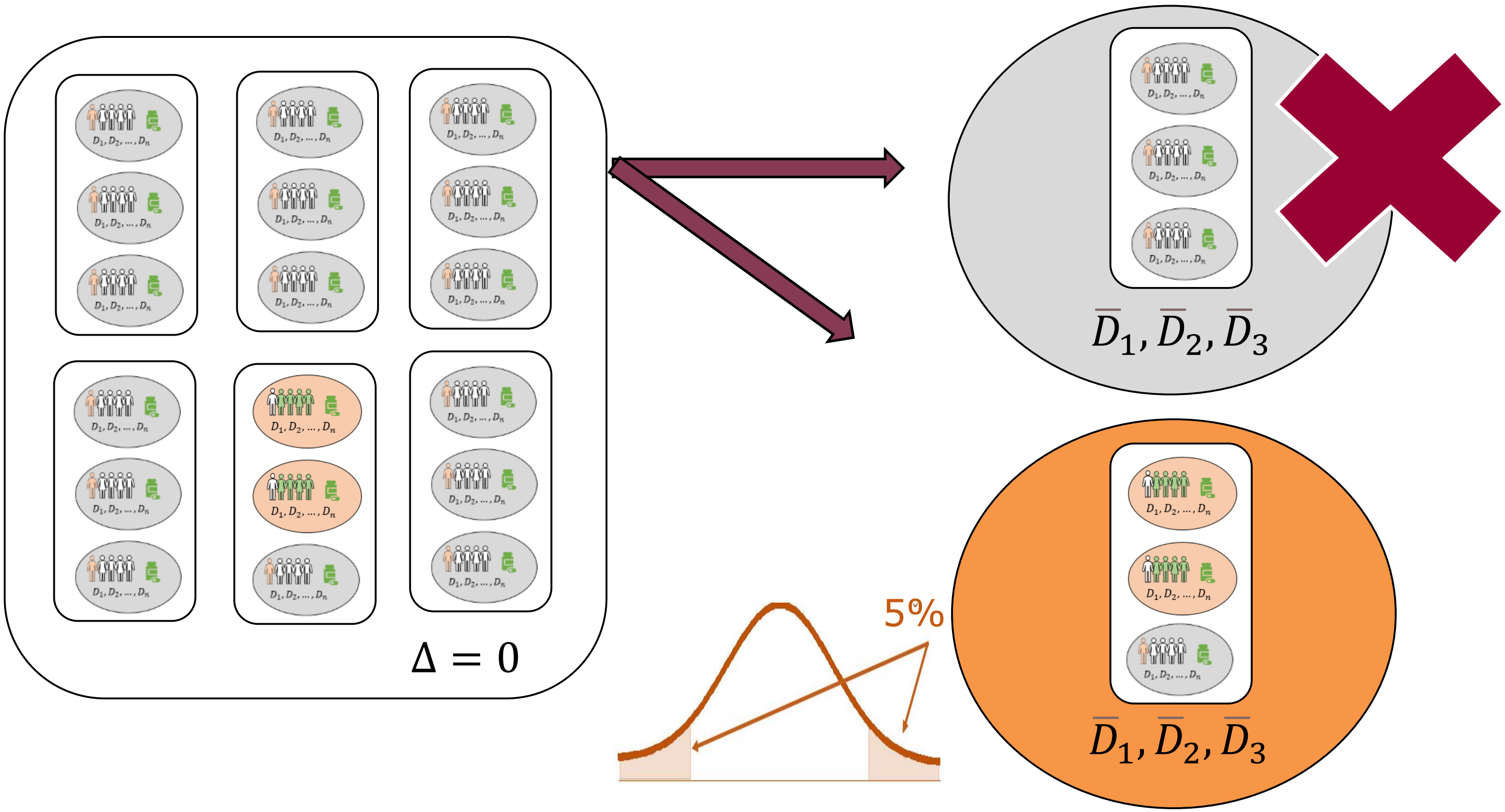
Gold Rush example *ALL-IN* solution

t		$*$
1	z_1^-	0
2	$\mathbf{z}_1^*, z_2^-$	1
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^-$	2
3	$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*$	3
$\sum$		

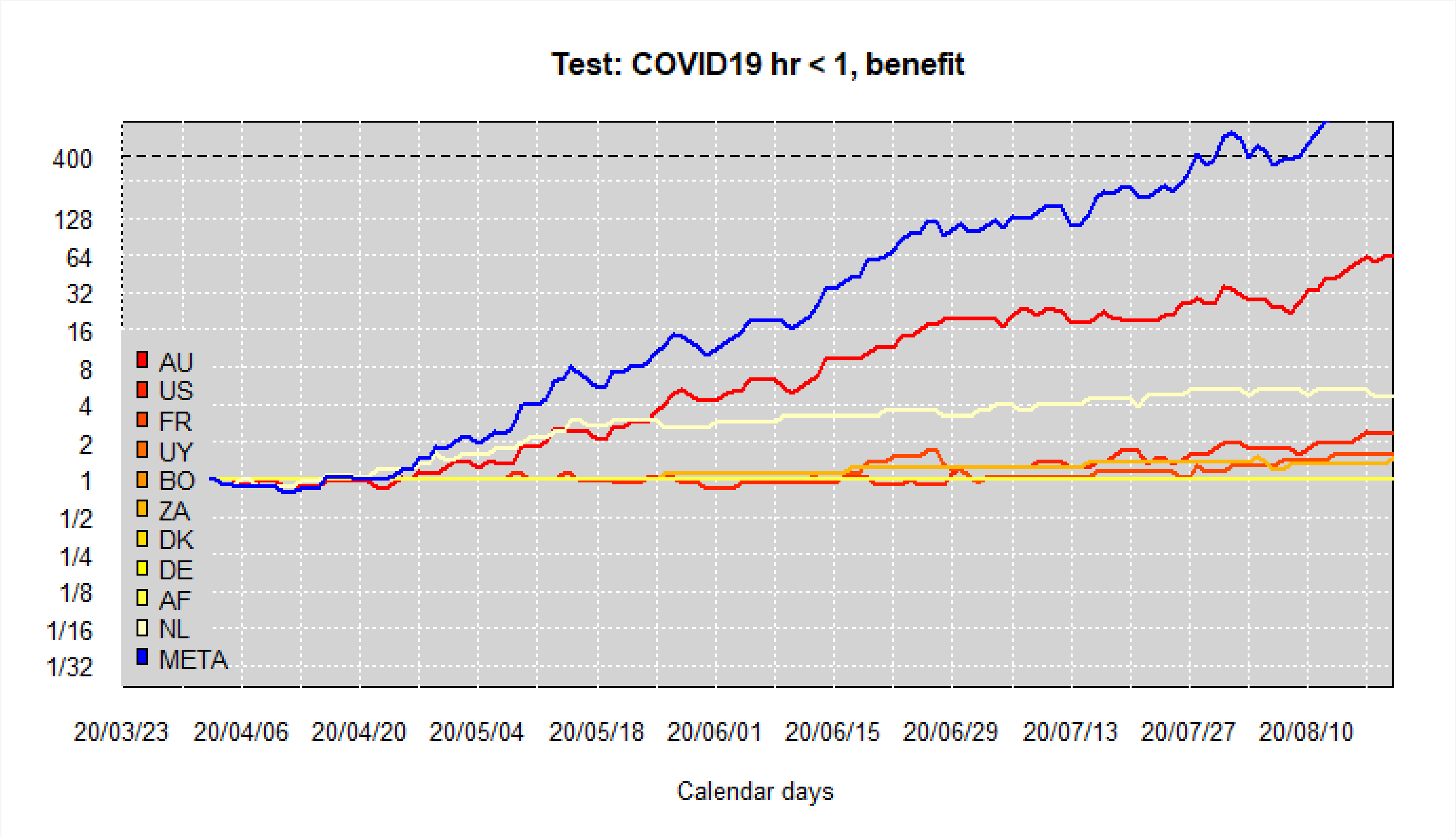
$\mathbf{P}_0$	$* \cdot \mathbf{P}_0$	$\mathbf{t} \cdot \mathbf{P}_0$
$1-\alpha$	0	$1-\alpha$
$\alpha(1-\alpha)$	$\alpha(1-\alpha)$	$2\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$	$2\alpha^2(1-\alpha)$	$3\alpha^2(1-\alpha)$
α^3	$3\alpha^3$	$3\alpha^3$
1	$\alpha+\alpha^2+\alpha^3$	$1+\alpha+\alpha^2$

$$\frac{\mathbf{E}_0[*]}{\mathbf{E}_0[t]} = \frac{\alpha(1+\alpha+\alpha^2)}{1+\alpha+\alpha^2} = \alpha$$

Random sampling of study series *of fixed size*



ALL-IN meta-analysis



ALL-IN meta-analysis uses

a stochastic process

instead of a probability density function

to control type-I errors

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Questions?

A test martingale

$$Z_1, \quad Z_2, \quad Z_3, \quad \ldots, \quad Z_{t-1}, \quad Z_t$$

The likelihood ratio as a test martingale

$\boxed{Z_1}$ $Z_2,$ $Z_3,$ $\ldots,$ $Z_{t-1},$ Z_t

$\mathbf{LR}_{10}{}^{(1)},$

$$\frac{f_1(z_1)}{f_0(z_1)}$$

The likelihood ratio as a test martingale

$$\boxed{\boxed{Z_1} \quad Z_2} \quad Z_3, \quad \ldots, \quad Z_{t-1}, \quad Z_t$$

$$\mathbf{LR}_{10}{}^{(1)}, \qquad \mathbf{LR}_{10}{}^{(2)},$$

$$\frac{f_1(z_1)}{f_0(z_1)} \qquad \frac{f_1(z_1, z_2)}{f_0(z_1, z_2)}$$

The likelihood ratio as a test martingale

$$Z_1 \qquad Z_2 \qquad Z_3 \qquad \ldots, \qquad Z_{t-1}, \qquad Z_t$$

$$\mathbf{LR}_{10}{}^{(1)}, \qquad \mathbf{LR}_{10}{}^{(2)}, \qquad \mathbf{LR}_{10}{}^{(3)},$$

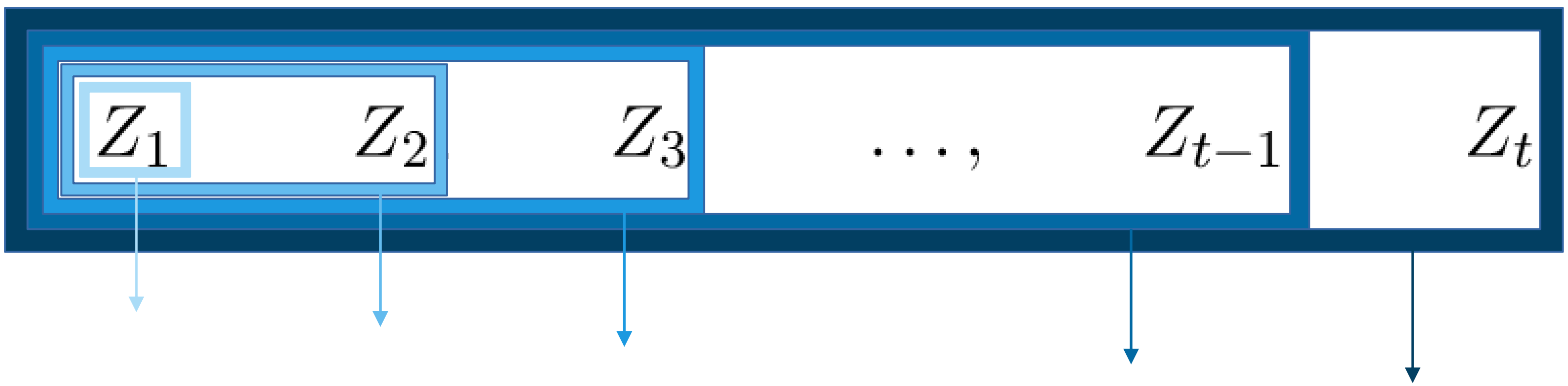
$$\frac{f_1(z_1)}{f_0(z_1)} \qquad \frac{f_1(z_1, z_2)}{f_0(z_1, z_2)} \qquad \frac{f_1(z_1, z_2, z_3)}{f_0(z_1, z_2, z_3)}$$

The likelihood ratio as a test martingale

$$Z_1 \quad Z_2 \quad Z_3 \quad \ldots, \quad Z_{t-1} \quad Z_t$$

$$\mathbf{LR}_{10}{}^{(1)}, \quad \mathbf{LR}_{10}{}^{(2)}, \quad \mathbf{LR}_{10}{}^{(3)}, \quad \ldots, \quad \mathbf{LR}_{10}{}^{(t-1)},$$

The likelihood ratio as a test martingale



$$\mathbf{LR}_{10}{}^{(1)}, \quad \mathbf{LR}_{10}{}^{(2)}, \quad \mathbf{LR}_{10}{}^{(3)}, \quad \ldots, \quad \mathbf{LR}_{10}{}^{(t-1)}, \quad \mathbf{LR}_{10}{}^{(t)}$$

Test martingales: control type-I error

$$\textit{reject } \mathscr{H}_0 \qquad \text{if } \mathbf{LR}_{10}{}^{(t)} > 20 \qquad \text{for } \alpha = 0.05 \text{ error control}$$

Universal bound over time (Ville's inequality):

$$\mathbf{P}_0\left[\mathbf{LR}_{10}{}^{(t)} \geq \frac{1}{\alpha} \quad \text{for some } t\right] \leq \alpha$$

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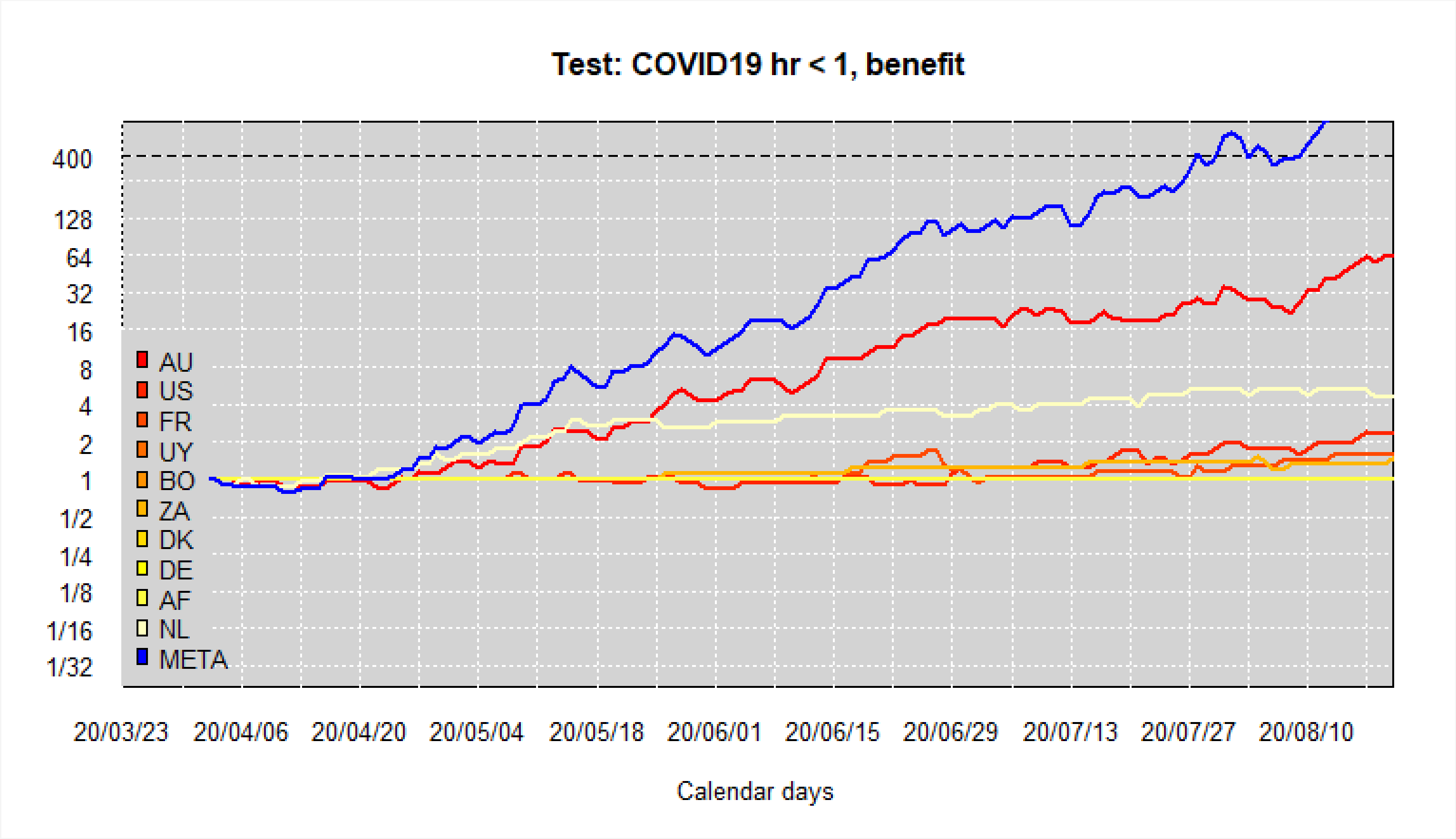
ALL-IN meta-analysis



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Questions?

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Questions?

1. Do you think accumulation bias actually affects real-life meta-analysis?

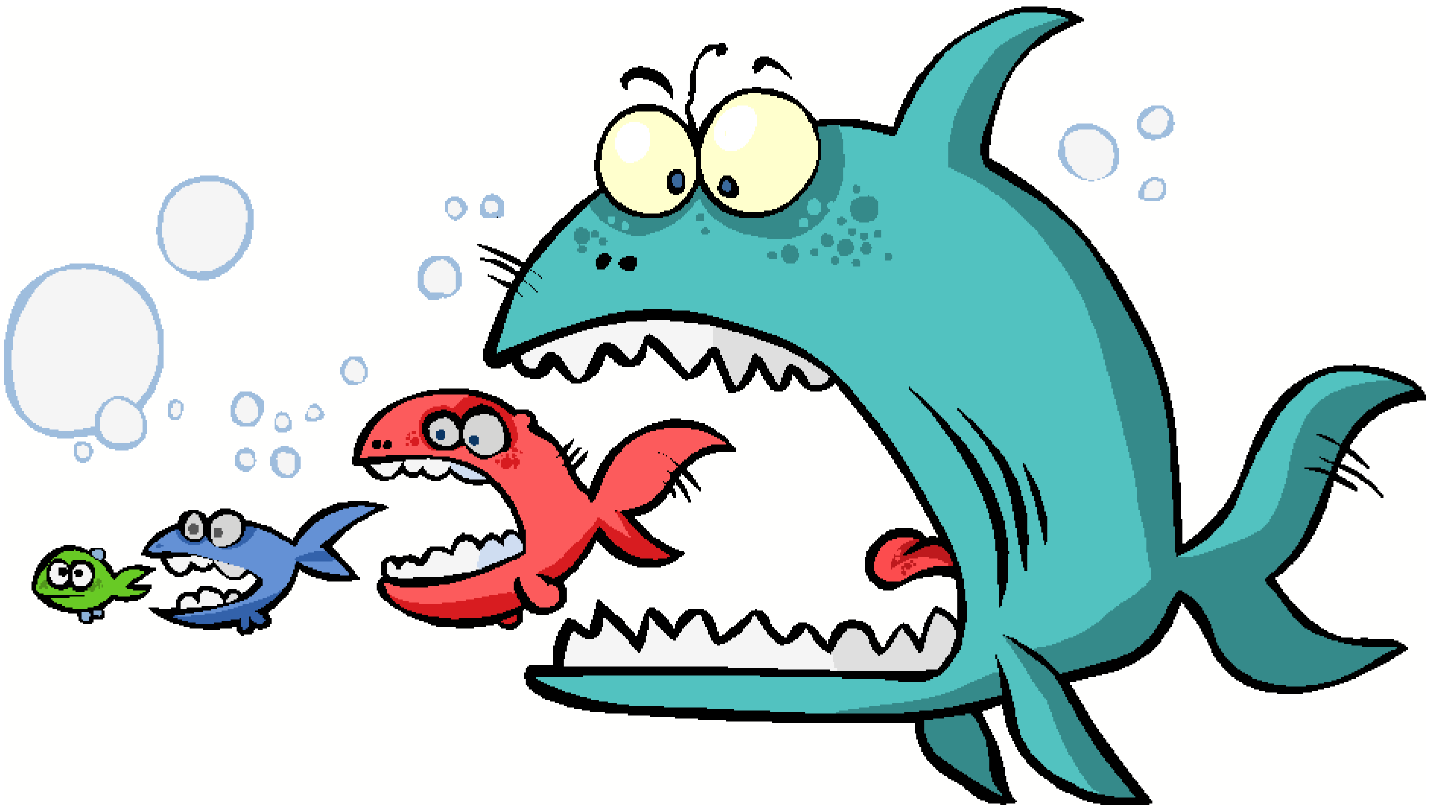
Viewpoint

Avoidable waste in the production and reporting of research evidence

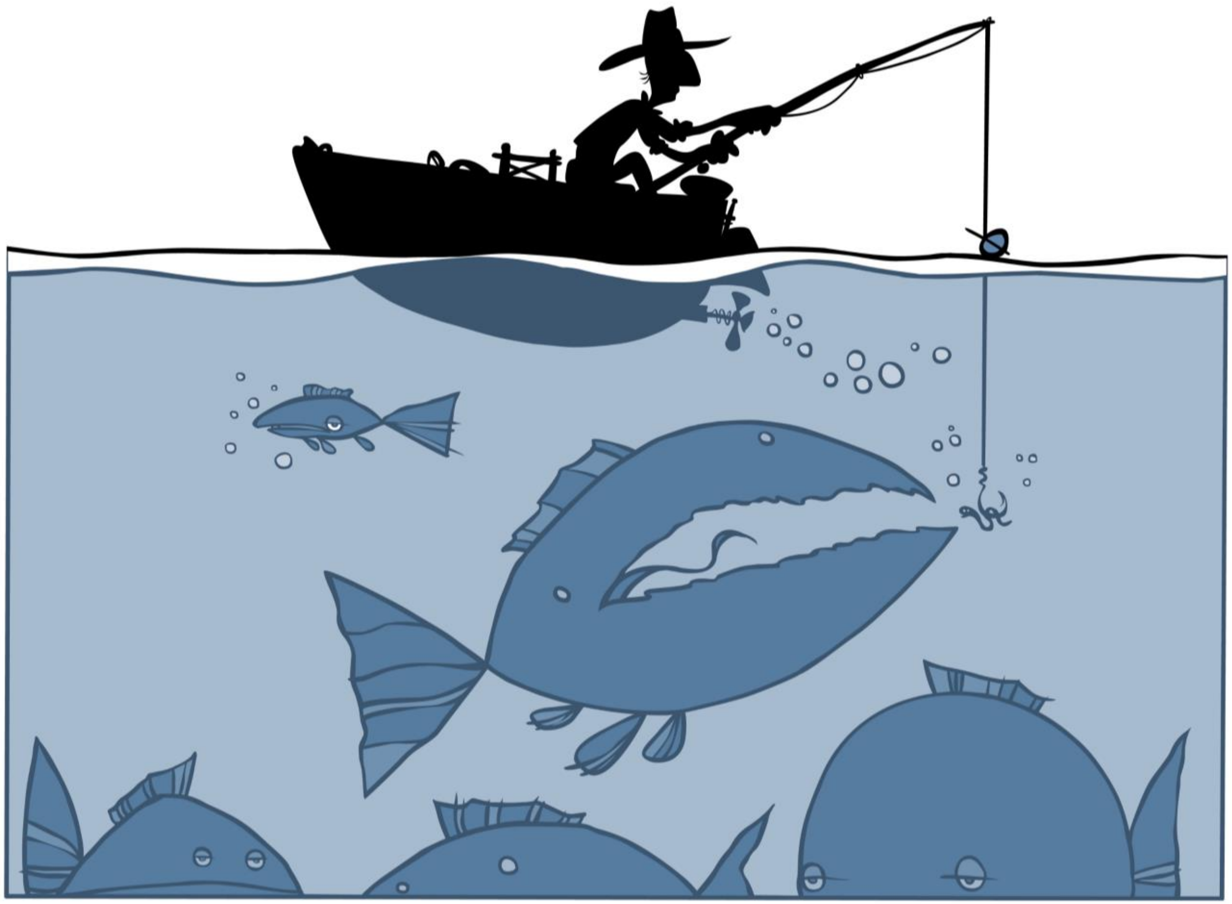
Iain Chalmers, Paul Glasziou

Lancet 2009; 374: 86–89

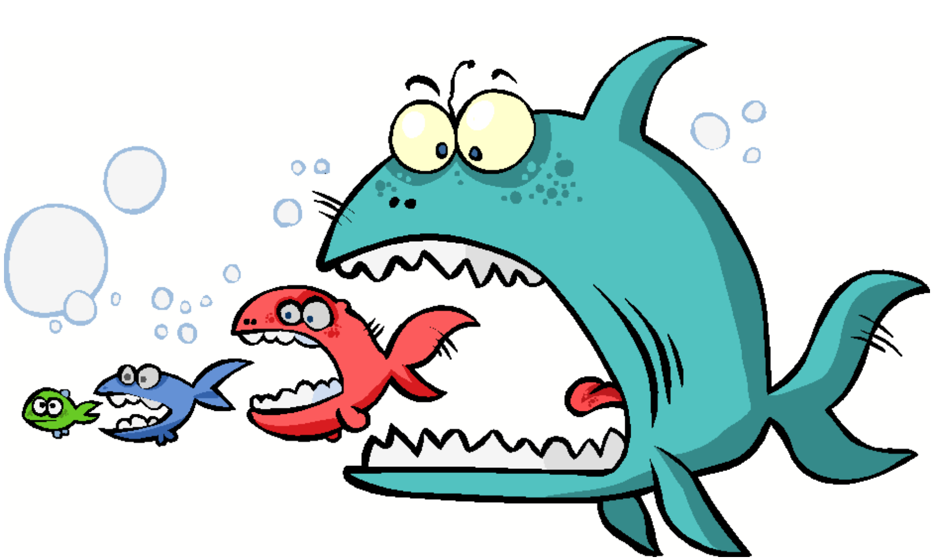
Accumulation Bias: introduced by the study series

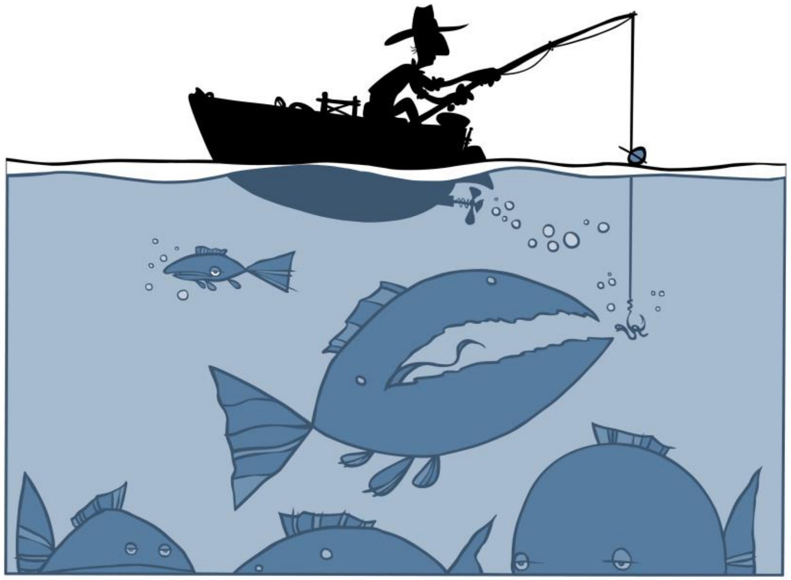


Accumulation Bias: introduced by the meta-analysis



Accumulation Bias





dependency on series size

dependency on meta-analysis timing

Accumulation Bias framework

$$A(t)$$

BMC Medical Research Methodology

BioMed Central

Research article

Open Access

The fading of reported effectiveness. A meta-analysis of randomised controlled trials

Bernhard T Gehr[1], Christel Weiss[2] and Franz Porzsolt*[3]

fading

Correspondence

fading

How systematic reviews cause research waste

Systematic reviews of small trials increase waste by ad[...]o the scientific community inflated, often significant treatment effects that become smaller or absent when large, high-quality trials are done. Effect estimates from systematic reviews often inform sample size calculations.[4] However, because most reviews provide exaggerated estimates of treatment effects due to inclusion

We declare no competing interests.

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fading

inflated,

exaggerated estimates

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Journal of Cardiac Failure Vol. 9 No. 5 2003

Perspective

Why do Phase III Trials of Promising Heart Failure Drugs Often Fail? The Contribution of 'Regression to the Truth'

HENRY KRUM, MBBS, PhD, FRACP, ANDREW TONKIN, MD, FRACP

Victoria, Australia

fading

inflated,

exaggerated estimates

'Regression to the Truth'

fading

inflated,

exaggerated estimates

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ELSEVIER

Journal of Clinical Epidemiology 58 (2005) 543–549

Journal of Clinical Epidemiology

REVIEW ARTICLES

Early extreme contradictory estimates may appear in published research: The Proteus phenomenon in molecular genetics research and randomized trials

John P.A. Ioannidis[a,b,c,*], Thomas A. Trikalinos[a,b]

'Regression to the Truth'

fading

inflated,

exaggerated estimates

Correspondence

How systematic reviews cause research waste

Systematic reviews of small trials increase waste by advertising to the scientific community inflated, often significant treatment effects that become smaller or absent when large, high-quality trials are done. Effect estimates from systematic reviews often inform sample size calculations.[4] However, because most reviews provide exaggerated estimates of treatment effects due to inclusion

We declare no competing interests.

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Systematic reviews and research waste

We share Ian Roberts' and Katharine Ker's frustration with the poor quality of much research (Oct 17,

their suggestion scientifically flawed, it is also unrealistic. Funders and regulators cannot be expected to support and endorse large studies without some reassurance from the results of smaller existing studies that the substantial investment needed is justified. We hope that, instead of promoting their

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Early extreme contradictory estimates

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'Regression to the Truth'

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$A(t)$

Early extreme contradictory estimates

The Proteus phenomenon

$A(t)$

'Regression to the Truth'

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inflated,

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exaggerated estimates

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$$A(t)$$

Systematic reviews and research waste

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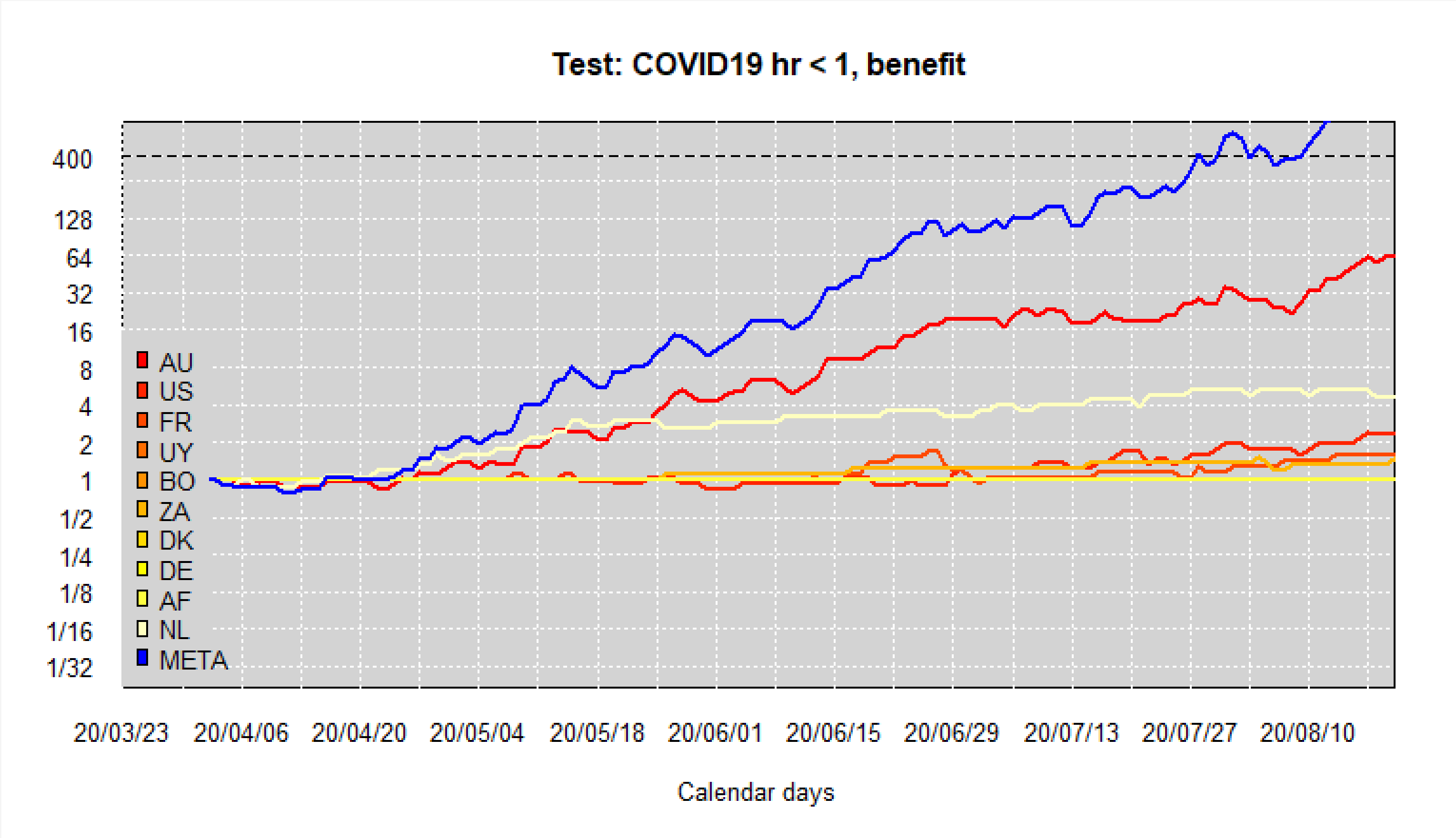
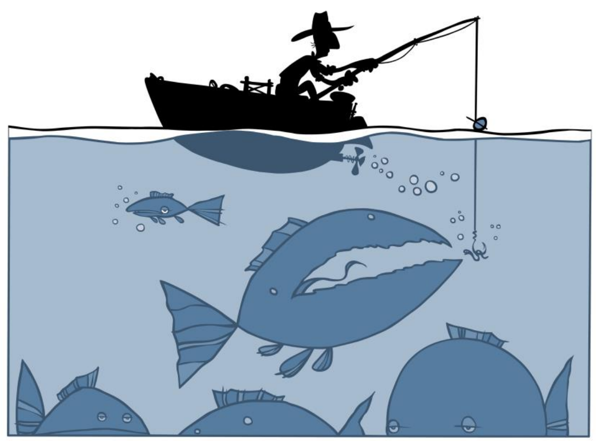
ALL-IN meta-analysis



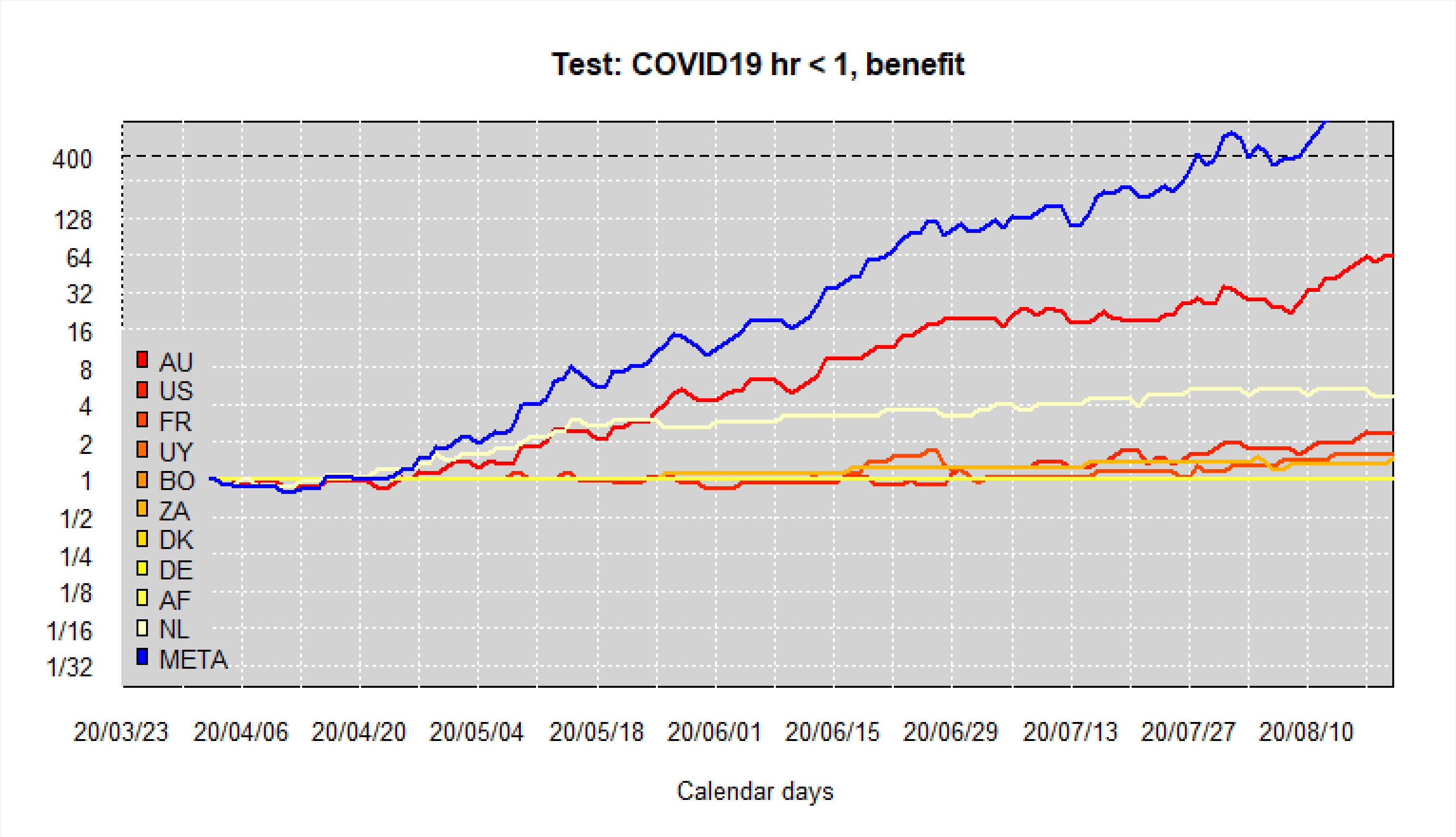
- ter Schure J and Grünwald P. (2019) Accumulation Bias in meta-analysis: the need to consider *time* in error control *F1000Research*, **8**:962.

Thank you!

Contact me on: schure@cwi.nl

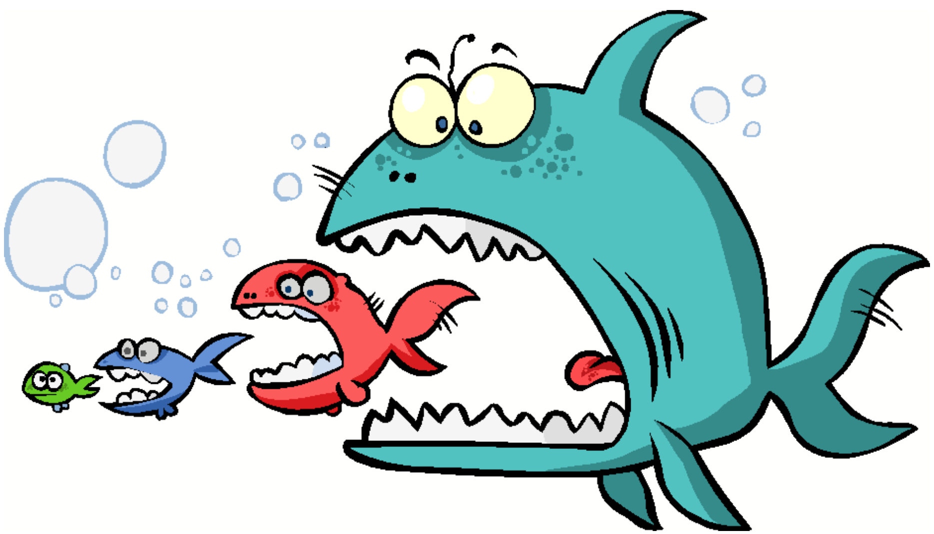


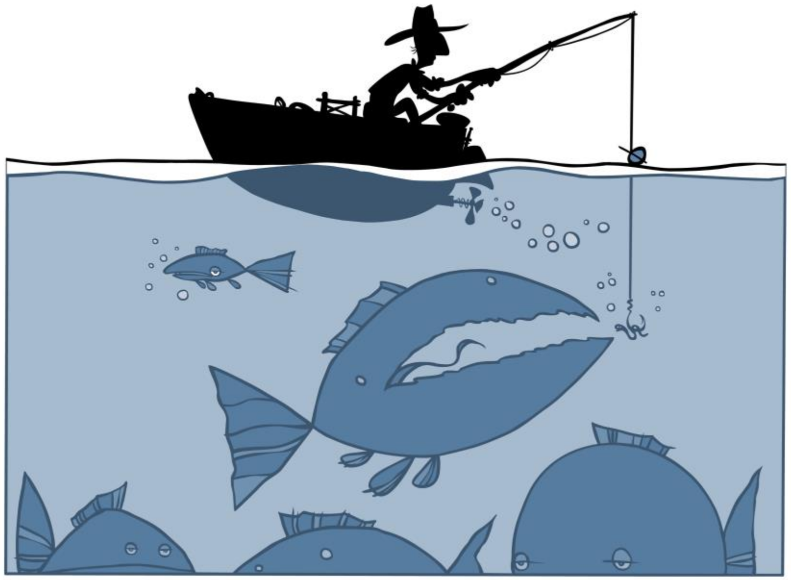
ALL-IN meta-analysis



$$\mathbf{LR}_{10}{}^{(t)}\left(z_1, \ldots, z_t, \mathscr{A}^{(t)}, T \geq t\right)$$

Accumulation Bias





dependency on series size

$$T \geq t$$

dependency on meta-analysis timing

$$\mathscr{A}^{(t)}$$

$$\mathbf{LR}_{10}{}^{(t)}\left(z_1,\ldots,z_t,\mathscr{A}^{(t)},T\geq t\right)$$
$$:=\frac{f_1\left(z_1,\ldots,z_t\right)\cdot\mathbf{P}_1(\mathscr{A}^{(t)},T\geq t\mid z_1,\ldots,z_t)}{f_0\left(z_1,\ldots,z_t\right)\cdot\mathbf{P}_0(\mathscr{A}^{(t)},T\geq t\mid z_1,\ldots,z_t)}$$

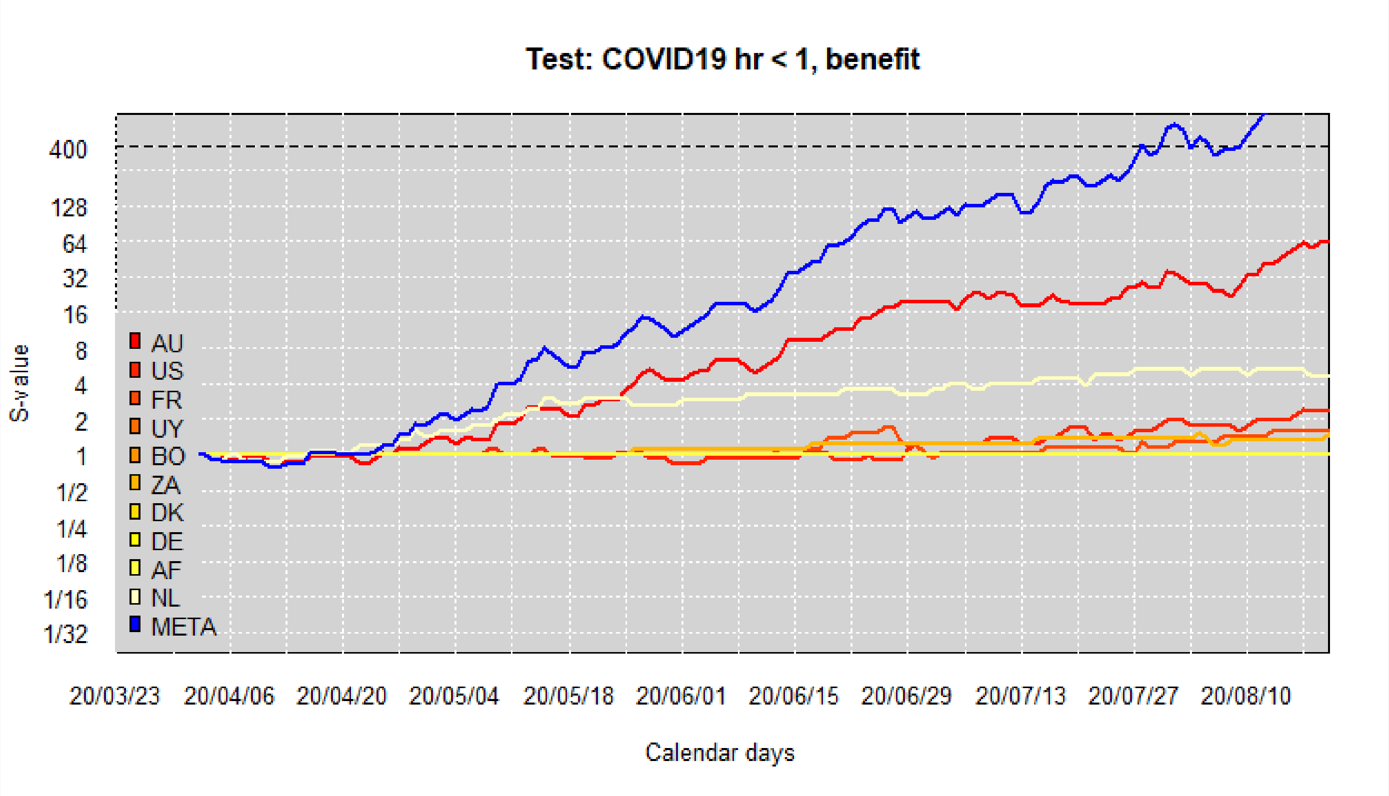
$$\mathbf{LR}_{10}{}^{(t)}\left(z_1,\ldots,z_t,\mathscr{A}^{(t)},T\geq t\right)$$

$$:=\frac{f_1(z_1,\ldots,z_t)\cdot\mathbf{P}_1(\mathscr{A}^{(t)},T\geq t\mid z_1,\ldots,z_t)}{f_0(z_1,\ldots,z_t)\cdot\mathbf{P}_0(\mathscr{A}^{(t)},T\geq t\mid z_1,\ldots,z_t)}$$

$$=\frac{f_1(z_1,\ldots,z_t)\cdot A(t\mid z_1,\ldots,z_t)}{f_0(z_1,\ldots,z_t)\cdot A(t\mid z_1,\ldots,z_t)}$$

$$
\begin{aligned}
&\mathbf{LR}_{10}{}^{(t)}\left(z_1,\ldots,z_t,\mathscr{A}^{(t)},T\geq t\right)\\
&:=\frac{f_1\left(z_1,\ldots,z_t\right)\cdot\mathbf{P}_1(\mathscr{A}^{(t)},T\geq t\mid z_1,\ldots,z_t)}{f_0\left(z_1,\ldots,z_t\right)\cdot\mathbf{P}_0(\mathscr{A}^{(t)},T\geq t\mid z_1,\ldots,z_t)}\\
&=\frac{f_1\left(z_1,\ldots,z_t\right)\cdot A(t\,|\,z_1,\ldots,z_t)}{f_0\left(z_1,\ldots,z_t\right)\cdot A(t\,|\,z_1,\ldots,z_t)}\\
&=\frac{f_1\left(z_1,\ldots,z_t\right)}{f_0\left(z_1,\ldots,z_t\right)}\\
&=\mathbf{LR}_{10}\left(z_1,\ldots,z_t\right).
\end{aligned}
$$

ALL-IN meta-analysis



Subjective Bayesian analysis uses

the same likelihood to update the prior

with or without Accumulation Bias

$$\frac{\mathbf{P}\left[H_1 \,\middle|\, z_1, \ldots, z_t, \mathscr{A}^{(t)}, T \geq t\right]}{\mathbf{P}\left[H_0 \,|\, z_1, \ldots, z_t, \mathscr{A}^{(t)}, T \geq t\right]}$$

$$\frac{\mathbf{P}\left[H_1 \,\middle|\, z_1,\ldots,z_t,\mathscr{A}^{(t)},T \geq t\right]}{\mathbf{P}\left[H_0 \,|\, z_1,\ldots,z_t,\mathscr{A}^{(t)},T \geq t\right]}$$

$$= \frac{f_1\left(z_1,\ldots,z_t,\mathscr{A}^{(t)},T \geq t\right)\cdot \pi}{f_0\left(z_1,\ldots,z_t,\mathscr{A}^{(t)},T \geq t\right)\cdot (1-\pi)}$$

$$= \mathbf{LR}_{10}{}^{(t)}\left(z_1,\ldots,z_t,\mathscr{A}^{(t)},T \geq t\right)\cdot \frac{\pi_1}{\pi_0}$$

$$= \mathbf{LR}_{10}\left(z_1,\ldots,z_t\right)\cdot \frac{\pi_1}{\pi_0}$$

Gold Rush example *ALL-IN* solution

t		*
1	z_1	0
2	$\mathbf{z_1^*}, z_2$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

$\mathbf{P}_0$	$* \cdot \mathbf{P}_0$	$\mathbf{t} \cdot \mathbf{P}_0$
$1-\alpha$	0	$1-\alpha$
$\alpha(1-\alpha)$	$\alpha(1-\alpha)$	$2\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$	$2\alpha^2(1-\alpha)$	$3\alpha^2(1-\alpha)$
α^3	$3\alpha^3$	$3\alpha^3$
1	$\alpha+\alpha^2+\alpha^3$	$1+\alpha+\alpha^2$

Gold Rush example *ALL-IN* solution

t		$*$
1	z_1	0
2	$\mathbf{z_1^*}, z_2$	1
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3$	2
3	$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	3
$\sum$		

$$\pi_0 = 1$$

$\mathbf{P}_0$	$* \cdot \mathbf{P}_0$	$\mathbf{t} \cdot \mathbf{P}_0$
$1-\alpha$	0	$1-\alpha$
$\alpha(1-\alpha)$	$\alpha(1-\alpha)$	$2\alpha(1-\alpha)$
$\alpha^2(1-\alpha)$	$2\alpha^2(1-\alpha)$	$3\alpha^2(1-\alpha)$
α^3	$3\alpha^3$	$3\alpha^3$
1	$\alpha+\alpha^2+\alpha^3$	$1+\alpha+\alpha^2$

Gold Rush example Bayesian solution

	π_1
z_1^- $\mathbf{z}_1^*$ $\mathbf{z}_1^*, z_2^-$ $\mathbf{z}_1^*, \mathbf{z}_2^*$ $\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^-$ $\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*$	

Gold Rush example Bayesian solution

	π_1				
z_1^-					
$\mathbf{z_1^*}$	$\overline{A}_1(1)$	$=$	1	$=$	1
$\mathbf{z_1^*}, z_2^-$					
$\mathbf{z_1^*}, \mathbf{z_2^*}$	$\overline{A}_1(2)$	$=$	0.8	$=$	0.8
$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$					
$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	$\overline{A}_1(3)$	$=$	$0.8 \cdot 0.8$	$=$	0.65

Gold Rush example Bayesian solution

	π_0				
z_1^-					
$\mathbf{z_1^*}$	$\overline{A}_0(1)$	$=$	1	$=$	1
$\mathbf{z_1^*}, z_2^-$					
$\mathbf{z_1^*}, \mathbf{z_2^*}$	$\overline{A}_0(2)$	$=$	0.05	$=$	0.05
$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$					
$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	$\overline{A}_0(3)$	$=$	$0.05 \cdot 0.05$	$=$	0.0025

Gold Rush example Bayesian solution

	π_1	π_0
z_1^- $\mathbf{z_1^*}$ $\mathbf{z_1^*}, z_2^-$ $\mathbf{z_1^*}, \mathbf{z_2^*}$ $\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$ $\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$	$\overline{A}_1(1) = 1$ $\overline{A}_1(2) = 0.8$ $\overline{A}_1(3) = 0.65$	$\overline{A}_0(1) = 1$ $\overline{A}_0(2) = 0.05$ $\overline{A}_0(3) = 0.0025$

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Subjective Bayesian analysis relies

on the prior so much that the same would hold

with or without *publication* bias

Publication Bias Bayesian solution

	π_1	π_0
z_1^-		
$\mathbf{z}_1^*$	$\overline{A}_1(1) = 1$	$\overline{A}_0(1) = 1$
$\mathbf{z}_1^*, z_2^-$		
$\mathbf{z}_1^*, \mathbf{z}_2^*$	$\overline{A}_1(2) = 0.8$	$\overline{A}_0(2) = 0.05$
$\mathbf{z}_1^*, \mathbf{z}_2^*, z_3^-$		
$\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*$	$\overline{A}_1(3) = 0.65$	$\overline{A}_0(3) = 0.0025$

Publication Bias **Bayesian solution**

	π_1	π_0
$\cancel{z_1^-}$		
$\mathbf{z_1^*}$		
$\mathbf{z_1^*}, z_2^-$		
$\mathbf{z_1^*}, \mathbf{z_2^*}$	$\overline{A}_1(2) = 0.8$	$\overline{A}_0(2) = 0.05$
$\mathbf{z_1^*}, \mathbf{z_2^*}, z_3^-$	$\overline{A}_1(3) = 0.65$	$\overline{A}_0(3) = 0.0025$
$\mathbf{z_1^*}, \mathbf{z_2^*}, \mathbf{z_3^*}$		