NINETEENTH CONFERENCE
ON THE
MATHEMATICS OF OPERATIONS RESEARCH

&

FIFTH INTERNATIONAL WORKSHOP
LANDELIJK NETWERK
MATHEMATISCHE BESLISKUNDE

CONFERENCE CENTER 'DE BLIJE WERELT'
LUNTEREN, THE NETHERLANDS

JANUARY 11-14, 1994

Organized by
CWI (Centrum voor Wiskunde en Informatica)
LNMB (Landelijk Netwerk Mathematische Besliskunde)

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Nineteenth Conference on the Mathematics of Operations Research & Fifth International Workshop Landelijk Netwerk Mathematatische Besliskunde

January 11–14, 1994, Lunteren, The Netherlands

Aim and scope

The aim of the meeting is to promote research activities and cooperation between senior and junior researchers in the mathematics of operations research in the Netherlands. The program offers high quality research and applications and should appeal to both academic researchers and to management consultants in trade and industry. For the first time a special mini-symposium on operations research in practice is organized, for which special invitations have been send out to management consultants.

The program should give ample opportunity for informal discussions. The conference center is located in the scenic surroundings of Lunteren, in the center of the Netherlands.

Conference plan

- An important part of the conference consists of three minicourses, each consisting of four lectures, in which prominent experts from abroad (R.E. Bixby, E.G. Coffman Jr., and P.W. Diaconis) will present introductions to their field of research.
- A second component of the conference involves eight lectures by international specialists (A. Frank, R. Fourer, G. Latouche and B. Peleg) about new developments in their specialism.
- On Wednesday afternoon there will be the mini-symposium: *New developments in the operational planning of organisations: modelling and optimisation.* The mini-symposium consists of three lectures by leading experts: R.E. Bixby, R. Fourer, and K. Jőrnsten.
- On Tuesday and Thursday there will be presentations by Ph.D. students (aio's and oio's).
- Wednesday evening, after the mini-symposium, there will be an informal drink and the conference dinner. During dinner there will be a talk by J.A.P. Hoogervorst (KLM).

Organization

The conference is organized by CWI (Centrum voor Wiskunde en Informatica) in Amsterdam and the Landelijk Netwerk Mathematische Besliskunde under the auspices of the Dutch Research Community in the Mathematics of Operations Research and System Theory, with financial support by the Dutch Mathematical Society. The Organizing Committee consists of A.M.H. Gerards and L.C.M. Kallenberg.

About the Speakers

Robert E. Bixby (Rice University, Houston, TX, USA and CPLEX Optimization, Inc., Incline Village, NV, USA) is the author of *CPLEX Linear Optimizer*, the recent, highly competitive, linear programming package. His current research areas are: solution of large-scale linear programming problems; computational integer programming, particularly methods for general 0/1 mixed integer programming, both sequential and parallel. Professor Bixby is John von Neumann Professor at the Institute for Operations Research and Econometrics of the University of Bonn and received a Humboldt Senior Scientist Award last year. He has been Area Editor of *Mathematics of Operations Research* and is Editor-in-Chief of *Mathematical Programming.*

Edward G. Coffman Jr. is a Distinguished Member of Technical Staff of AT&T Bell Laboratories (Murray Hill, NJ, USA). His research has concentrated on the mathematical modeling and analysis of algorithms and system performance. Specific interests in operations research include combinatorial scheduling and packing theory, probabilistic analysis of algorithms, stochastic scheduling, and the application of queueing theory to the analysis of computer/communication systems.

Persi W. Diaconis (Harvard University, Cambridge, MA, USA) is a prominent researcher in applied probability and Bayesian methods. His work on rates of convergence of Markov chains has led to sub-specialities in group theory and non-commutative Fourier analysis. He has never been able to live down a previous carreer as professional magician. This shows up in his current work on the mathematics of shuffling cards.

Robert Fourer's (Northwestern University, Evanston, IL, USA) research concerns computational issues in large-scale optimization. He has a long-standing interest in the design of languages and computing environments to support optimization, including the *AMPL* modeling language and system, and database interfaces for mathematical programming. For their work on *AMPL*, Robert Fourer and his co-workers have been awarded the 1993 prize of the Computer Science Technical Section of ORSA. He has also studied algorithms for specially structured linear programs, including trust-region methods for nonlinear equations, heuristics for finding and exploiting embedded networks, and robust approaches to solving symmetric indefinite systems for interior-point methods.

András Frank (Eőtvős Loránd University, Budapest, Hungary) is a well-known expert in combinatorial optimization and graph theory. His research interests include: polyhedral combinatorics, submodular functions, disjoint paths, connectivity and reliability. He has published over 60, frequently cited, papers in these areas. He held many visiting positions all over the world, including the Institute for Operations Research and Econometrics of the University of Bonn (from 1984 to 1986 and from 1989 to 1993).

Kurt Jőrnsten is Research Manager at the Chr. Michelsen Institute for Science and Intellectual Freedom in Fantoft, Norway. He has a wide experience in many areas of operations research, including various aspects of transportation issues and applications from the oil industry. The Petroleum Economics Group of the Chr. Michelsen Institute has had a number of research and development projects for the Norwegian Oil Industry, mainly for the Norwegian oil companies and the Norwegian and Danish government energy agencies.

Guy Latouche works at the Département d'Informatique of the Université Libre de Bruxelles in Belgium. His research activity is mainly concerned with matrix-analytic methods for Markov processes, a major branch of Computational Probability. He has occasionally conducted forays in the analysis of nearly-completely decomposable Markov chains by perturbation methods. In recent years, he collaborated with researchers at Bellcore, on the design of models for packet traffic in telecommunication networks.

Bezalel Peleg (Department of Mathematics, The Hebrew University of Jerusalem; temporarily CentER, Tilburg University) is one of the leading game theoretists in the world with more than 80 papers on his name and the book: *Game Theoretical Analysis of Voting in Committees* (Cambridge University Press). Professor Peleg was one of the initiators of NTU-game theory and of effectivity functions. He applied game theory in economic situations and also in biology. He is a member of the editorial boards of *Social Choice and Welfare*, *International Journal of Game Theory*, *Games and Economic Behavior* and *Economic Theory.*

Program

Tuesday January 11, 1994

10.15	*Registration*
10.45	Opening
11.00–11.45	Edward Coffman Jr.: Stochastic matching theory: its guises and tools, with applications I
12.00–12.45	Edward Coffman Jr.: Stochastic matching theory: its guises and tools, with applications II
12.45	*Lunch*
14.30–15.10	Parallel sessions aio's/oio's
15.20–16.00	Parallel sessions aio's/oio's
16.15–17.00	András Frank (1): Designing survivable networks
17.15–18.00	András Frank (2): Minimal edge-coverings of pairs of sets
18.30	*Dinner*
20.00	*Ledenvergadering* Landelijk Netwerk Mathematische Besliskunde

Wednesday January 12, 1994

9.00– 9.45	Persi Diaconis: Geometry and Markov chains, aimed at applications I
10.00–10.45	Persi Diaconis: Geometry and Markov chains, aimed at applications II
11.00–11.45	Bezalel Peleg (1): Ring formation in auctions: the general model
12.00–12.45	Guy Latouche (1): Markovian models for arrival processes in queueing theory
12.45	*Lunch*
15.00–18.00	**Mini-symposium: Modelling and Optimization**
15.15–16.00	Robert Bixby: Linear programming: the computational state of the art I
16.15–17.00	Robert Fourer (1): Trends in the design of modeling languages and systems of mathematical programming
17.15–18.00	Kurt Jőrnsten: An exposé of applications of OR models in the Norwegian oil industry
18.00	*Informal Drink*
19.00	*Conference Dinner*

Thursday January 13, 1994

9.00– 9.45	Robert Bixby: Linear programming: the computational state of the art II
10.00–10.45	Robert Bixby: Linear programming: the computational state of the art III
11.00–11.45	Bezalel Peleg (2): Ring formation in auctions: a detailed analysis for the grand coalition
12.00–12.45	Guy Latouche (2): Divide-and-conquer methods in certain classes of Markov chains
12.45	*Lunch*
14.30–15.10	Parallel sessions aio's/oio's
15.20–16.00	Parallel sessions aio's/oio's
16.15–17.00	Edward Coffman Jr.: Stochastic matching theory: its guises and tools, with applications III
17.15–18.00	Edward Coffman Jr.: Stochastic matching theory: its guises and tools, with applications IV
18.30	*Dinner*
20.00	*Meeting* aio's and oio's

Friday January 14, 1994

9.00– 9.45	Persi Diaconis: Geometry and Markov chains, aimed at applications III
10.00–10.45	Persi Diaconis: Geometry and Markov chains, aimed at applications IV
11.00–11.45	Robert Fourer (2): Recent developments in algorithms and interfaces for stochastic programming and robust optimization
12.00–12.45	Robert Bixby: Linear programming: the computational state of the art IV
12.45	Closing
13.00	*Lunch*

Parallel Sessions AIO's/OIO's

Tuesday January 11, 1994

	Room A:	Room B:
14.30	WILBERT VAN DEN HOUT (KUB) *The power-series algorithm for a wide class of Markov processes*	ANDRÉ VAN VLIET (EUR) *Algorithms and lower bounds for on-line scheduling*
14.50	ANNEKE LOEVE (RUL) *Algorithm for computing locally optimal periodic policies in Markov decision chains with partial information*	EDO VAN DER POORT (RUG) *Sensitivety analysis and the k-best TSP*
15.10	Coffee break	
15.20	GERT-JAN OTTEN (KUB) *An alternative view on the uniform rule*	RALPH WILDEMAN (EUR) *Joint replacement in an operational planning phase*
15.40	HANS REIJNIERSE (KUN) *on finding an envy free pareto optimal division*	DOUWE-FRITS BROENS (RUG) *Investment planning robustness using the commercial scope*

Thursday January 13, 1994

	Room A:	Room B:
14.30	ROB VAN DER MEI (KUB) *Multiple-server polling systems*	JOS STURM (EUR) *An $O(\sqrt{n}L)$-iteration primal-dual cone affine scaling method*
14.50	SEM BORST (CWI) *Polling systems with multiple coupled servers*	BENJAMIN JANSEN (TUD) *Primal-dual target following algorithms for linear programming — Dikin steps, analytic centers and weighted centers*
15.10	Coffee break	
15.20	FRANK COMPAGNER (UvA) *Two relaxations for the edge-disjoint Steiner problem in graphs*	DRIES VERMEULEN (KUN) *Consistency & equilibrium selection*
15.40	KOOS HEERINK (UT) *Integration of the modeling and solution phase for combinatorial optimization problems*	JACQUELINE BLOEMHOF-RUWAARD (LUW) *Generalized bilinear programming: an application in farm management*

Extended Abstracts of the Minicourses

Linear Programming: the Computational State of the Art

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In a recent survey of computational progress in linear programming [1], the speaker documented the remarkable progress that has occurred over the last five years. This talk will focus on several of the issues examined in that paper, with particular attention given to the most recent improvements. The topics to be covered will include the following:

1. Problem reductions prior to solution.

2. The ratio test, and dealing with degeneracy and numerical stability.

3. Pricing in both the primal and the dual, particularly steepest-edge pricing.

4. Full-featured implementations of the dual simplex method.

5. Primal-dual log barrier methods with predictor corrector.

6. Crossover methods from optimal barrier to optimal basic solutions.

Particular emphasis will be placed on developments in simplex pricing, the new implementations of the dual simplex method, and basis crossover.

Preprocessing

Preprocessing methods are not new [4], but aggressive implementations were not available in many codes until more recently. One reason was that smaller model instances, such as those represented by the NETLIB set, did not seem to benefit markedly from preprocessing. The emergence of barrier methods has also motivated further developments. They are very sensitive to extraneous model components that may, for example, make the interior of the feasible region empty. They are also sensitive to practices such as splitting free variables, which necessarily make the set of optimal solutions unbounded (if nonempty).

Some of the important preprocessing ideas, each simple in its own right, will be quickly reviewed. The most significant reductions result from: (1) Bound analysis within constraints, resulting in the fixing of variables and removal of redundant constraints, and (2) variable aggregation where the nonnegativity of the "aggregated" variable is implied by the nonnegativity of the remaining variables.

Ratio Test

In [8] Paula Harris suggested a modified ratio test that uses feasibility tolerances to expand the set of possible leaving variables in an iteration of the simplex method. This modification, which has long been used, in some form, in production simplex codes, improves numerical stability and helps reduce the effects of degeneracy. However, it also may introduce small infeasibilities during the solution process, and it is sometimes unavoidable that these infesibilites exceed tolerances. The effect is that the step sizes at some simplex iterations may become negative. Using an idea suggested by [6], these infeasibilities can be dealt with by explicitly shifting bounds, and increasing the size of the feasible region. This approach is, in effect, a kind of "bound perturbation", and acts as a further measure to improve numerical stability and prevent degenerate pivots.

Pricing

The most important improvements in primal simplex implementations over the last several years have come in the pricing step. Consider a linear programming problem of the form

$$\text{(LP)} \qquad \begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax = b \\ & x \geq 0 \end{array}$$

where A is an $m \times n$ matrix. A *feasible basis* B of (LP) is an $m \times m$ nonsingular submatrix of A such that the basic variables have nonnegative values: $x_B = \mathbf{B}^{-1} b \geq 0$. The ordered set B denotes the index-set of the variables in the basis, and its bold-face counterpart $\mathbf{B}$ denotes the corresponding submatrix of A. The *nonbasic variables* x_N are set to zero.

Let π be the solution of

$$\mathbf{B}^T \pi = c_B,$$

and set

$$d = c - A^T \pi.$$

The elements of d are called the *reduced costs*, and are the central quantities in *pricing*: If $d \geq 0$, the current basis is optimal; otherwise, we select some $j \in N$ such that $d_j < 0$, and designate x_j as the *entering variable*. The two fundamental questions in connection with pricing are to determine (1) how many of the components of d are actually computed and each iteration, and (2) which criterion is used to select the entering variable from among the x_j with $d_j < 0$.

For many years *full pricing*, the computation of all of d_N at each iteration, was considered computationally too expensive; however, essentially all of the recent advances in pricing use full pricing in some form.

We will begin by discussing the efficient computation of d. The two essential ideas are, first, to *update* d rather than computing it anew at each iteration, and, second, to re-store the nonbasic part of A, A_N, by row so that the inner products $\pi^T A$, or their equivalent, can be more efficiently computed. These ideas are essential not only to primal simplex computations, but to the dual simplex method, and to basis crossover.

Once d is known, it is not proposed to implement the classical *most-negative rule*: select as the entering variable x_j the one that minimizes d_j. The performance of that rule, both in time and, perhaps surprisingly, in the number of iterations is inferior to sensible implementations of partial pricing.

While direct application of the most-negative rule is not a good idea, various kinds of *normalized pricing*, in which the reduced costs are scaled before selecting the entering variable, are effective. The normalizations we have in mind replace d_j by

$$\text{(SE)} \qquad \frac{d_j}{(\bar{A}_j^T \bar{A}_j + 1)^{\frac{1}{2}}}$$

where $\bar{A}_j = \mathbf{B}^{-1} A_j$.

Using the exact form (SE) is called ***steepest-edge pricing.*** The fundamental paper on steepest-edge algorithms is [7]. We will also consider a particular alternative to (SE) called devex [8], which can be viewed as an approximation to (SE). Finally, we will consider a hybrid pricing approach using both "multiple-partial pricing" and devex. Hybrid approaches have been adopted as the default in several optimizers.

Dual Simplex Method

The dual simplex method was introduced in 1954 [9]. The idea is to solve the dual problem using the primal representation. The actual algorithm may be viewed as a *working basis* method in which all the linear algebra is carried out on a naturally associated primal basis embedded within the dual basis.

If the primal problem is as given in (LP), then the corresponding dual problem is

$$\text{(DLP)} \qquad \begin{array}{ll} \text{maximize} & b^T y \\ \text{subject to} & A^T y + s = c \\ & y \text{ free}, s \geq 0. \end{array}$$

Assuming that the rows of A are linearly independent, it is easy to prove that all y variables may be assumed basic in any optimal solution. Thus, it may be assumed that any basis for (DLP) has the form

$$\begin{pmatrix} \mathbf{B}^T & 0 \\ A_N^T & I \end{pmatrix}$$

where I is an $(n-m) \times (n-m)$ identity matrix. B is then the associated "primal" basis. The details of the dual simplex method may be worked out by assuming a basis has the above form and decomposing the solutions of all required linear systems to ones using $\mathbf{B}$ alone.

That the dual simplex method is not new is apparent. However, in the past, its use was restricted largely to integer programming. What is new is that now the dual method is considered as one of the basic alternatives for solving any linear program. That development has led to full-featured implementations of the dual simplex method. We will discuss the important features of such implementations, with emphasis on pricing. The main result to be discussed is a very efficient steepest-edge alternative to "biggest infeasibility" is presented in [5]. This pricing paradigm is proposed as the correct default pricing in implementations of the dual simplex method.

We note that dual steepest-edge algorithms have been particularly useful in theoretical work on integer programming. Most of the leading work that is now being carried out on branch-and-cut approaches to combinatorial optimization problems would likely not be possible without them.

Barrier and Basis Crossover

We will discuss primal-dual log barrier methods with predictor corrector, following the description given in [10]. The emphasis will be on the principal computational aspects of these methods, and their effect on the solution of real-world linear programs. We will then consider some new results on the *basis crossover* problem: Given feasible, complementary primal and dual solutions x and (y, s) for (LP) and (DLP), respectively, find a basis B that is both primal and dual feasible. Thus, B must satisfy $x_B = \mathbf{B}^{-1}b \geq 0$ and $d = c - A^T\pi \geq 0$, where $\mathbf{B}^T\pi = c_B$. Clearly these conditions imply that B is optimal for (LP). Of course, in practice one can only assume that x and (y, s) are "nearly feasible," and "nearly complementary," in the same way that the simplex method must allow for small nonnegativity violations in x_B and d.

The crossover to be discussed is an implementation of the proof of the main theorem in [11]. This implementation is joint work of the speaker and Irv Lustig [3]. It uses both the primal and the dual solutions in a fundamental way. Its efficient implementation depends in a fundamental way on the same techniques used in the efficient computation of the vector of reduced costs d in the primal simplex method.

Conclusion

We will conclude with a computational comparison of primal and dual simplex methods and barrier methods for a sampling of large real-world linear programming problems.

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Stochastic Matching Theory its Guises and Tools, with Applications

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Since the pioneering work on multi-dimensional packing by Karp, Luby, and Marchetti-Spaccamela [KLM] in 1984, stochastic matching problems in two and three dimensions have surfaced in the analysis of a surprising number of algorithms and systems, with applications in operations research, electrical engineering, and computer science. Thus, the related theory has provided an important analysis tool, one that, unfortunately, has yet to be applied by more than a handful of researchers. Of the several variants of stochastic matching that we will discuss, the following two have played fundamental roles. Let n plus points and n minus points (i.e., points labeled with +'s and −'s) be chosen independently and uniformly at random in the unit square. Let M_n denote a matching of the plus points to minus points and let (P^-, P^+) denote a pair of matched points (or an edge) in M_n.

Euclidean Matching: Let $d(P^-, P^+)$ denote the Euclidean distance between P^-, P^+, and let M_n be a matching that minimizes the total distance between matched points, $D_n = \sum_{(P^-,P^+)\in M_n} d(P^-, P^+)$. Find $E[D_n]$.

Up-Right (or Ordered) Matching: Consider only up-right matchings M_n such that $(P^-, P^+) \in M_n$ only if P^+ is up and to the right of P^-, i.e., if $P^- = (x_1, y_1)$, $P^+ = (x_2, y_2)$, then $x_1 \le x_2$, $y_1 \le y_2$. Let U_n be the number of unmatched points in an up-right matching of maximum cardinality. Find $E[U_n]$.

As stated, these problems have proven to be quite difficult, a property that is inherited in particular cases from the problems in which stochastic matching finds application. However, tight asymptotic bounds have been derived. Specifically, for Euclidean matching, Ajtai, Komlos, and Tusnády [AKT] proved that $E[D_n]$ grows like $\sqrt{n \log n}$ for large n; to be more precise, we can express this in the standard $\Theta(\cdot)$ notation,

$$E[D_n] = \Theta(\sqrt{n \log n}) \ .$$

For up-right matching, Shor [Sho], Leighton and Shor [LS] and Rhee and Talagrand [RT] proved that

$$(*) \qquad E[U_n] = \Theta(\sqrt{n} \log^{3/4} n) \ .$$

Subsequent, simpler proofs were given by Coffman and Shor [CS] and Talagrand [Tal]; see Chapter 3 in [CLu] for a general treatment. Asymptotic bounds on tail probabilities are also available; indeed, it is often the case that such bounds provide the desired estimates of expected values. The result in (*) has also been generalized to the ordered matching problem in $d \geq 3$ dimensions. (If (P^-, P^+) is in a d-dimensional ordered matching, then in every dimension the coordinate of P^- must be at most that of P^+.) As shown in [KLM], one obtains $E[U_n] = \Theta(n^{(d-1)/d})$, $d \geq 3$.

As is common with asymptotic results of this sort, the absence of information about multiplicative constants is compensated by a greater generality of the results. Obvious examples of robustness are rescalings: n can be replaced by αn for any given $\alpha > 0$ and, in the planar case, the unit square can be replaced by an $a \times b$ rectangle for any given $a, b > 0$; such changes modify only the hidden multiplicative constant. For less obvious examples, we note that the following changes to problem instances also leave (*) unchanged:

(i) the numbers of plus and minus points are independently Poisson distributed with mean n; or the total number of points is Poisson distributed with mean n, and each point is equally likely to be a plus or minus independently of the others;

(ii) the points are restricted to lie on an $n \times n$ lattice, or the problem is similarly discretized in just one of the dimensions; or

(iii) n plus points are fixed at the vertices of a $\sqrt{n} \times \sqrt{n}$ lattice and n minus points are chosen at random as before.

These lectures will take results of the above type as background theory (proofs of such results would require at least four lectures by themselves). Instead, we concentrate on techniques that exploit these results in analyzing a variety of mathematical models of engineering problems. The techniques require only a modest background in applied probability, e.g., certain basic probability inequalities, elementary results from the theory of random walks, central limit theorems, and Chernoff bounds.

As illustrated in the problems below (e.g., see Problem 1), stochastic matching often arises quite naturally in the analysis; in such cases, some form of matching can usually be recognized in the sample functions of the underlying stochastic process. In other cases, however, as in Problem 2 below, the matching problem may be well disguised; it can be seen only after a substantial problem reduction has been made. Even after a stochastic matching problem has been identified, a nontrivial analysis may still be required to account for new variations in problem instances. For example, in Problem 3 below, an instance of up-right matching arises, but the horizontal components of the minus points do not have a uniform distribution, except in a certain asymptotic sense.

The lectures will expand at length on the above matters, as they emerge in the analysis of the problems listed below. If time permits, we shall also discuss a matching problem in processor-ring communications.

Problem 1. A Selection-Replacement Process on the Circle [CGS].

Given n points on a circle, a selection-replacement operation removes one of the points and replaces it by another. To select the removed point, an extra point P, uniformly distributed, arrives at random and starts moving counterclockwise around the circle; P removes the first point it encounters. A new random point, uniformly distributed, then replaces the removed point. The quantity of interest is $d = d(n)$, the distance that the searching point P must travel to select a point. In particular, consider the mean of d in the stationary version of the selection-replacement process. Sample functions of the process can be represented by $+$ and $-$ points on a cylinder representing the product space of the circle and a time axis; $+$'s denote the selected points on the circle and $-$'s indicate the points P that remove selected points. Up-right matchings of $-$'s to $+$'s pair off points P with the points they remove. The expected horizontal component of the distance between matched points gives $E[d] = \Theta\left(\frac{\log^{3/2} n}{n}\right)$, as shown in [CGS].

In a computer application, the circle represents a track on a disk memory, P is a read-write head, the n points mark the beginnings of n files and d determines the time wasted as the head moves from the end of the last file processed to the beginning of the next. The number n is a parameter of the service rule (the next service goes to one of the n customers waiting the longest).

Problem 2. First Fit Bin Packing with Discrete Item Sizes [CJSW].

A list L of n items is to be packed into a sequence of unit capacity bins. The first-fit (FF) rule packs each successive item into the first bin of the sequence that has room for it. We discuss an average-case analysis of FF in the discrete uniform model: the item sizes are drawn independently and uniformly at random from the set $\{1/k, \ldots, (k-1)/k\}$, for some $k > 1$. Let $FF(L)$ denote the wasted space in the FF packing of L, i.e., the total space still available in the occupied bins. It is proved in [CJSW] that $E[FF(L)] = O(\sqrt{nk})$, i.e., there exists a constant $c > 0$ such that $E[FF(L)] \leq c\sqrt{nk}$ for all n, k sufficiently large. In the proof of this result, items in a problem instance are represented by points in 2 dimensions, one denoting the item's index in the list L and the other denoting its "folded" size, i.e., an item of size s is plotted as a $-$ with coordinate s if $s \leq 1/2$ and as a $+$ with coordinate $1-s$ if $s > 1/2$. (Note that a $+$ item can fit in a bin with a $-$ item only if the size coordinate of the $+$ is larger than that of the $-$.) A certain class of up-right matchings of problem instances corresponds to the matchings produced by a modified FF rule that packs at most two items in a bin and never uses fewer bins than FF. The bound on $E[FF(L)]$ is then obtained from a bound on the expected number of unmatched points in such up-right matchings.

Problem 3. Dynamic Storage Allocation [CLe].

A computer storage device is represented by a sequence of adjacent cells with sizes s_i nondecreasing in $i = 1, 2, \ldots$. Files arrive and depart by Poisson processes, with each item placed at its time of arrival into a smallest empty cell large enough for the item. The known, average number of files in storage in the stationary regime is denoted by n, and assumed to be an integer. One assumes a given distribution of file sizes such that s_i, $1 \leq i \leq n$, can be chosen so that a file size is equally likely to fall in any of the intervals $[0, s_1]$, $[s_1, s_2], \ldots, [s_{n-1}, s_n]$; for all $i > n$, one chooses $s_i = s_n$, a largest file size. Trajectories of the stationary storage process are represented in the two dimensions of time t and cell index i; a $-$ point at (i, t)

denotes a new arrival at time t with a size in $(s_{i-1}, s_i]$, $s_0 = 0$. At plus point at (i,t) denotes a departure from cell i at time t. Up-right matchings pair arriving files (−'s) to the departures (+'s) creating the empty cells in which the files are placed. Of interest is the expected number of interior unused cells, i.e., the number W_n of empty cells among the first m, where cell m is the highest indexed occupied cell. An analysis of the up-right matchings gives $E[W_n] = \Theta(\sqrt{n}\log^{3/4} n)$, as shown in [CLe].

Problem 4. Probabilistic Analysis of a Vehicle Routing Problem [BCSS].

Consider n points distributed uniformly at random in some rectangular region. The points represent customers with demands for some commodity supplied by a depot, which is represented by an additional point with a given location in the region. An unlimited number of equal capacity vehicles are available at the depot for delivery of customer demands. Vehicles are to be routed to the n customers, with each customer being visited by at most one vehicle, and all vehicles making a round-trip tour, so that all demands are satisfied and the total route length L_n of all vehicles is minimized. The demands are i.i.d. uniform draws from $[0, 1]$, each giving the required fraction of a vehicle's capacity.

In a third (vertical) dimension plot customer demands directly above the corresponding customer locations in the (horizontal) plane, using the folding convention of Problem 2 to decide whether a point is labeled with a + or −. Now construct a (three-dimensional) *upward matching* of +'s and −'s that minimizes the number of unmatched points; the only requirement that must be met by matched + and − points is that the + must be above the −. This yields an obvious heuristic routing algorithm. Vehicles serve at most two customers in a round-trip tour; matched customers are served by the same vehicle, and unmatched customers are served by vehicles that serve no other customer. It is proved in [BCSS] that $E[L_n] = nE[d] + \Theta(n^{2/3})$, where $E[d]$ is the average distance between the customers and the depot. Moreover, it is shown that this is a best possible asymptotic result in the sense that the expected total route length under an optimal routing algorithm is also equal to $nE[d] + \Theta(n^{2/3})$.

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Geometry and Markov Chains, Aimed at Applications

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I. Reversible Markov chains and honest rates of convergence

In recent years there has been an explosion of applied interest in Markov chains. Much of this is for reversible chains such as random walk on graphs. I will explain the basics: metropolis algorithms, Gibbs samplers, and set out the basic machinery of eigen analysis. Part of the interest is mathematical: I will explain rapid mixing and the cut of phenomena through examples.

II. Eigenvalues and geometry

This lecture explains tools like Poincaré and Cheeger inequalities for bounding eigenvalues. Borrowed from differential geometry and differential equations, these techniques have a distinctly geometric flavor. They require an understanding of diameter and covering numbers of the underlying graph. This talk reports joint work with Dan Stroock and Laurent Saloff-Coste.

III. Constructing Markov chains

Here the problems are: given a natural combinatorial object (for example the set of all 3 way arrays with given line sums and non-negative integer entries) construct a Markov chain which allows you to pick an object at random. For a large class of problems, the construction can be done using techniques of computational algebraic geometry (Gröbner Bases). This reports joint work with Bernd Sturmfels.

IV. Nash inequalities

This talk reports on advanced techniques for bounding eigenvalues and rates of convergence using geometric estimates of moderate growth of the underlying graph. The techniques give sharp rates of convergence for many problems. This is joint work with Laurent Saloff-Coste.

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Abstracts of the Invited Lectures

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TRENDS IN THE DESIGN OF MODELING LANGUAGES AND SYSTEMS FOR MATHEMATICAL PROGRAMMING — WEDNESDAY 16.15

Modeling languages have become a familiar tool in the application of mathematical programming. Numerous established commercial software systems accept linear and nonlinear optimization models in more-or-less standard algebraic notation, translate models and data to forms required by efficient solvers, and translate the results back to tables and graphs for convenient analysis. As experience with these systems has grown, moreover, many features of their modeling languages have proved to have valuable uses beyond the description of algebraic objectives and constraints.

Continuing advances in computing and software technology are encouraging further extensions to modeling languages, and to the systems that incorporate these languages. This presentation will survey recent and continuing trends, with examples drawn in part from the AMPL language developed by the speaker, David Gay and Brian Kernighan. Among the language design issues to be covered are features for network and combinatorial optimization, stochastic programming extensions, and facilities for iterative solution of models within grand iterative schemes. Broader system issues of interest include graphical interfaces, integration with spreadsheet and database packages, links to analytic software (for such purposes as parametric analysis and infeasibility diagnosis), and support of case management.

RECENT DEVELOPMENTS IN ALGORITHMS AND INTERFACES FOR STOCHASTIC PROGRAMMING AND ROBUST OPTIMIZATION — FRIDAY 11.00

Although stochastic programming (and related robust optimization) techniques are appealing in principle, their application has been discouraged both by the complexity of formulating the associated optimization models, and by the expense of solving the very large problem instances that result. These difficulties are being addressed by two different research projects to be described in this talk.

Linear programs that arise in two-stage stochastic programming might seem to be an ideal application for interior-point methods, whose cost tends to grow slowly with problem size. But in fact, stochastic LPs offer a particularly difficult test of the robustness of interior-point implementations, because they incorporate "dense columns"—corresponding to the first-stage variables—that rule out the standard approach of solving the so-called normal equations. Research in collaboration with Joseph Czyzyk and Sanjay Mehrotra has shown that an alternative "augmented system" approach consistently requires an amount of work that is very nearly linear in the number of scenarios, whereas a "column-splitting" approach requires a much more variable amount of work that grows worse than quadratically in some cases. The differences between the two approaches can be viewed as a consequence of the way in which myopic elimination strategies deal with the block-structured matrices involved. Our analysis also leads to a useful comparison of the augmented system approach with a "Schur complement" approach considered in other recent research.

Stochastic programming techniques might also be more widely applied if they were directly supported by easy-to-use modeling languages for mathematical programming. Several extensions to the AMPL language have been proposed for this purpose, in joint work with David Gay and Brian Kernighan. We consider a variety of alternatives for specifying the stage and scenario structure of the model, for adding variance terms and soft constraints, for representing scenario trees in multi-stage models. Because stochastic programs arise in different ways and are used for different purposes, each alternative is likely to be attractive to some group of users.

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DESIGNING SURVIVABLE NETWORKS TUESDAY 16.15

A network may be considered reliable it it satisfies specified connectivity requirements, In this talk we survey the theoretical and algorithmical aspects of the problem.

MINIMAL EDGE-COVERINGS OF PAIRS OF SETS TUESDAY 17.15

A new min-max theorem concerning bi-supermodular functions is proved. As special cases, we derive an extension of Gyori's theorem on intervals, Mader's theorem on splitting edges in digraphs, Edmonds' theorem on matroid partitions and a min-max formula for the minimum number of new edges whose addition makes a digraph k-edge-connected. As a new corollary, we solve the corresponding node-connectivity augmentation problem for digraphs (joint work with T.Jordan).

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AN EXPOSÉ OF APPLICATIONS OF OR MODELS IN THE NORWEGIAN OIL INDUSTRY

WEDNESDAY 17.15

The presentation will focus on some operations research applications from the Norwegian Oil Industry which I have come across in my work as research manager at Chr. Michelsen Institute and as professor at the Norwegian School of Economics and Business Administration in the period from 1985 to present. Instead of discussing traditional applications such as applications of linear programming in refineries and exploration models I will concentrate my presentation of other more nontraditional applications.

The main problem I will focus on is a problem concerning optimal relinquishment according to the Norwegian Petroleum Law. This is of course not the most central problem for a petroleum economist but an interesting application of mathematical programming generated from the law text. Also the applications give rise to many interesting pure graph theoretical problems.

A short description of the problem is as follows. A Norwegian petroleum licence is a rectangular area of 15 times 20 distance minutes divided into 300 squares of size 1 by 1 minute. According to the Norwegian Petroleum Law the licensee must after 6 years have passed relinquish at least 50% of the area. The relinquished area and the area kept have to fulfill a number of requirements of which one is that the relinquished area has to be connected.

The licensee of course try to keep the most valuable area without violating the requirements given in the law text.

I will present some possible formulations for this problem and corresponding theoretical problems and algorithms for their solution. Some results on "practical" problems will also be given.

The second application is also an application which starts of from a legal document and concerns a mathematical description af a gas sales contract. Here I am not allowed to be very specific but I will try to present the basic ideas and results generated from this study.

If time permits I will continue by discussing some applications of mathematical programming in petroleum field optimization, oil tanker scheduling to a refinery, project sequencing and oil field unitization.

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MARKOVIAN MODELS FOR ARRIVAL PROCESSES IN QUEUEING THEORY

WEDNESDAY 12.00

The Poisson process is widely used in queueing theory as a model of the arrival process of customers to a queue. The main reason for its popularity is that it is a process without memory and is therefore well-suited for the building of Markovian models. There are numerous circumstances, however, where the Poisson process is thoroughly inadequate; of particular interest nowadays are telecommunication networks, for which the Poisson process may not serve as a faithful model of the traffic.

A number of processes have been introduced in time, such as the interrupted or Markov modulated Poisson processes, which keep to some extent the Markovian property of the Poisson process and yet are more versatile. These efforts have culminated in recent years with the introduction of the so-called 'Markovian Arrival Process' or MAP.

The analysis of MAP queueing systems proceeds along lines which are analoguous to those of queues with Poisson arrivals. This is very clearly seen when one adopts a matrix formalism, as I shall try to demonstrate.

DIVIDE-AND-CONQUER METHODS IN CERTAIN CLASSES OF MARKOV CHAINS

THURSDAY 12.00

One particular class of Markov processes which lend themselves well to a matrix formalism are called Quasi-Birth-and-Death processes, or QBDs. They have been used in various areas of queueing theory, such as computer performance evaluation and teletraffic modeling.

For QBDs, the state space is typically bi-dimensional, infinite in one dimension, finite in the other, say $\{(n, j); n = 0, 1, 2, \ldots; j = 1, 2, \ldots K\}$. The transitions are such that the process acts as an homogeneous birth-and-death process in the "n" dimension. This homogeneity property allows one to use divide-and-conquer methods to determine various quantities of interest: the stationary distribution, expected time to blocking (in the finite buffer case), the distribution of the maximum queue length during a busy period, etc.

In this talk, I shall principally discuss the case of the stationary distribution.

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RING FORMATIONS IN AUCTIONS: THE GENERAL MODEL

WEDNESDAY 11.00

We assume that a unique and indivisible commodity is to be sold by an auction with bidders $i = 1, \dots n$, where $n \geq 2$. To have something specific in mind think of a unique piece of art or a building at a certain location. A ring is a non-empty subset C of $N = \{1, \dots n\}$. Bidders in the complement $\overline{C} = \{i \in N | i \notin C\}$ are called outsiders. Our analysis is based on the following assumption about the market decision process: First all ring members $i \in C$ choose bids $b_i \geq 0$. (We assume that 0 is the reservation value of the seller.) On the basis of those bids a member $r \in C$ — the representative of the ring — is chosen. This step is called the preauction. In the second step — the main auction — each outsider $j \in \overline{C}$ bids $b_i \geq 0; r$ repeats his bid (in the preauction) b_r; all other ring members $i \in C$, $i \neq r$, bid 0.

Observe that each bidder $k \in N$ submits just one (non-trivial) bid b_k. Let $b = (b_1, \dots, b_n)$. To uniquely determine the results of the auction we must specify for every bid vector b the following results:

(a) the winner $w(b)$ in the main auction;

(b) the price $p(b)$ that the winner must pay to the seller;

(c) the representative $r(b)$ of the ring C in the main auction;

(d) the transfer payments $t_i^{r(b)}(b)$ of the ring's representative to the other ring members i with $i \in C$ and $i \neq r(b)$.

Thus an auction mechanism with ring information is described by

$$\mu = (N; C; w(\cdot), p(\cdot), r(\cdot), (t_i^{r(b)}(\cdot) | i \in C, \ i \neq r(b))).$$

We use two axioms, "envy-freenes with respect to bids" and "independent weights" to define the class of mechanisms that we investigate. Also, we consider auctions as games with incomplete information in the sense of HARSANYI (1967/8). Moreover, we assume that the true values of the players are independent and identically distributed. The solution for two bidders, under foregoing assumptions, will be presented in this part of the lecture.

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RING FORMATIONS IN AUCTIONS: A DETAILED ANALYSIS FOR THE GRAND COALITION — THURSDAY 11.00

Again let N be the set of bidders and let $C \subset N$ be the ring. The beliefs within the ring satisfy the IID assumption, that is, v_i, i's true value for $i \in C$, satisfies $0 \le v_i \le 1$ and all $j \in C \setminus \{i\}$ consider v_i to be a random variable with distribution $F_i(v_i)$. Furthermore, $(v_i)_{i \in C}$ are independent and $F_i(.) = F_j(.) = F(.)$ for all $i, j \in C$. The behavior of $N \setminus C$ is described by a distribution $G(x), 0 \le x \le 1$, where

$$x = max\{b_j \mid j \notin C\}$$

The foregoing model includes all IID bidding games on N where the ring members play the "preauction game' and the outsiders play an equilibrium of the original auction. We seek pure, strictly increasing, symmetric, and differentiable equilibria of ring games. The following procedure is used to find equilibria with the foregoing properties.

1. Find the differential equation for the ring's bidding strategy.

2. Solve the equation (if possible).

3. Prove that the solution of (2) is a bidding strategy for the ring.
4. Prove that the foregoing strategy is more profitable than competitive bidding.
5. Prove that the ring's strategy is immune against formation of subrings.

In this lecture we shall wee the results of the application of the foregoing procedure for the case

$$C = N \text{ and } F(v) = v, 0 \leq v \leq 1$$

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Abstracts of the AIO/OIO Presentations

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Thursday 15.40
Room B

GENERALIZED BILINEAR PROGRAMMING: AN APPLICATION IN FARM MANAGEMENT

A popular method in farm management and land use planning is linear programming (LP). The traditional linear farm management model determines the number of hectares with a certain farm set-up to maximize the profitability given some restrictions on e.g. total land, available labour, etc.

In this study, the farm management modelling is extended with a manure and fertilizer aspect. The nutrients in the manure can be used to fertilize the meadow by applying the manure on the land. However, the government gives bounds for the allowed utilization of nutrients on the meadow, due to environmental reasons.

The resulting problem is an example of a generalized bilinear programming model: a model formulation of which the objective function is bilinear and the constraint set is non-convex, caused by bilinear inequality constraints. These models often have several local optima.

Some solution approaches for the introduced problem are given. Two heuristics to obtain local solutions are considered, and a branch and bound procedure is outlined to obtain the global optimum. An example is used to illustrate the structure of the problem and the quality of the solution methods.

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Thursday 14.50
Room A

POLLING SYSTEMS WITH MULTIPLE COUPLED SERVERS

A multiple-server polling system is a multiple-queue system attended to by multiple servers in a cyclic manner. Like the single-server versions, multiple-server polling systems find applications in computer systems, communication networks, and manufacturing environments. So far there are hardly any exact results known for these systems, apart from some mean value results for global performance measures like cycle times. In the talk we investigate the class of systems that allow an exact analysis. For these systems we present distributional results for the joint queue length at polling epochs, the marginal queue length, and the waiting time. The class in question includes two-queue two-server systems with exhaustive service and exponential service times, two-queue systems with 1-limited or semi-exhaustive service and general service times, as well as infinite-server systems with exhaustive or gated service and deterministic service times.

Douwe-Frits Broens Tuesday 15.40
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INVESTMENT PLANNING ROBUSTNESS USING THE COMMERCIAL SCOPE

The yearly production planning problem of a natural gas trading and transporting company is modelled as a system of linear (in)equalities in terms of the production decision vector y, the commercial scenario vector s and the investment decision vector x. Eliminating y from these restrictions, we get the *commercial scope of* x: the (polyhedral) set of commercial future scenarios s for which a feasible production plan exists, given x. Measures of the commercial robustness or risk of an investment plan x are given in terms of this commercial scope. Examples are based either on a central forecast scenario or on a probability distribution of s. As a by–product of the calculation of these measures, one easily derives inequalities in s and x, inducing facets of the scope. These so–called *induced constraints* give useful insight in the effectivity of investment alternatives.

Frank Compagner Thursday 15.20
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TWO RELAXATIONS FOR THE EDGE-DISJOINT STEINER PROBLEM IN GRAPHS

In a weighted graph G, the Steiner Problem is the problem of constructing a tree of minimum total weight spanning a certain subset S of the vertices of G. In order to increase the reliability of the solution, we require that the solution consists of two or more edge-disjoint trees spanning S. Deciding whether a feasible solution to this problem exists is an NP-complete problem. We consider two relaxations of the problem, the first one based on the (normal) Steiner Problem in Graphs and the second one based on the disjoint minimum spanning tree problem. By applying subgradient optimization to these relaxations we compute a lower bound to the problem. The results from both these optimization programs are also used to calculate an upper bound. Computational results are given for graphs with up to 100 vertices and 4950 edges.

Koos Heerink Thursday 15.40
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INTEGRATION OF THE MODELING AND SOLUTION PHASE FOR COMBINATORIAL OPTIMIZATION PROBLEMS

We propose an extension to an algebraic modeling language that allows the modeler (1) to control the underlying enumeration algorithm and, (2) to model combinatorial problems using pre-defined combinatorial structures (e.g. (ordered) subset, partition, etc.). With the help of such an extended modeling language it is possible to model problem dependent solution processes.

At first we will present a framework for a general Branch & Bound method which contains several control mechanisms through yet undefined parameters, functions and procedures. We distinguish 14 such mechanisms and show through some examples how they can be incorporated into a modeling language.

We will formulate the enumeration process for 8 different types of combinatorial structures using the earlier mentioned control mechanisms. Embedding these formulations into an enumeration code makes it possible to model in terms of these combinatorial structures. The model will not only be more concise and transparent but the model can also be solved more efficiently. Knowledge of the specific combinatorial structures will be used to decrease the size of the enumeration tree.

Wilbert van den Hout Tuesday 14.30
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THE POWER-SERIES ALGORITHM FOR A WIDE CLASS OF MARKOV PROCESSES

The power-series algorithm is a method to find the steady-state distribution of multi-dimensional continuous-time Markov processes. The large or infinite set of balance equations is transformed with a parameter γ, resulting in steady-state probabilities as functions of γ. If the transformation is done in an appropriate way, these steady-state probabilities are analytic functions of γ, and the coefficients of the power-series expansions can be calculated recursively.

The method has been applied to a variety of queueing models. The generalization of the method to more general Markov processes will be discussed, and some comments will be made on the theoretical justification of the method.

Benjamin Jansen Thursday 14.50
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PRIMAL-DUAL TARGET FOLLOWING ALGORITHMS FOR LINEAR PROGRAMMING — DIKIN STEPS, ANALYTIC CENTERS AND WEIGHTED CENTERS

Primal-dual algorithms for linear programming (LP) are among the most widely used interior point methods, since they appear to have excellent theoretical and practical behaviour. Most of these methods follow the central path of the LP problem more or less closely, others use weighted paths. The two main goals in interior point methods are to obtain an optimal solution (optimality) and to ensure that this solution is strictly complementary (centrality). In this talk we propose a method for LP with the property that, starting from an initial non-central point, it generates iterates that simultaneously get closer to optimality and closer to centrality. The iterates follow paths that in the limit are tangential to the central path. These paths are motivated by the new primal-dual affine scaling algorithm of Jansen, Roos and Terlaky and hence the algorithm is called to be an algorithm using Dikin (affine) steps. Along with the convergence analysis we provide a general framework which enables us to analyze various primal-dual algorithms in the literature in a short and uniform way. Among these methods are central and weighted path-following methods; algorithms of Den Hertog and Mizuno for computing a point on the central path given a non-central point; and algorithms of Mizuno and Atkinson/Vaidya for computing a weighted center given a point on the central path.

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Tuesday 14.50
Room A

ALGORITHM FOR COMPUTING LOCALLY OPTIMAL PERIODIC POLICIES IN MARKOV DECISION CHAINS WITH PARTIAL INFORMATION

The optimal policy in a Markov decision chain can be found by successive approximation. However, if it is not possible to observe all states of the system, the policy found in this way is not useful in practice. This is one of the reasons for looking at Markov decision chains with partial state information.

For example, this is the case in decentralized queueing networks. In such a queueing network there are several nodes or centres and each centre has its own decision maker who controls that centre. Each decision maker can observe only one centre, so the decisions are based on the information about that particular centre.

We can model this kind of situation by making a partition of the state space. Now the information on the state of the system available on each decision moment is the set of the partition in which the state is contained. So the exact state is not known and the decisions in all states in a set of the partition have to be the same.

A new algorithm to obtain good periodic policies in Markov decision chains with partial information is designed. It will be applied to a queueing network where arriving customers have to be assigned to one of a number of parallel queues and to a queueing network with server control.

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Thursday 14.30
Room A

MULTIPLE-SERVER POLLING SYSTEMS

A polling system basically consists of a number of queues, attended by a single server. Polling systems find many applications in computer systems, communication networks and manufacturing environments. In this paper we consider polling systems with multiple servers. Such systems are generally very hard to analyze by means of mathematical techniques, because multiple-server polling systems combine the complexity of single-server polling systems and multiple-server systems. Hence, there is a need for numerical algorithms to analyze the behaviour of these systems. We have implemented a class of multiple-server polling models into the so-called power-series algorithm, a tool for the numerical evaluation and optimization of a broad class of multiple-queue models. Numerical experiments give rise to interesting observations which provide an insight into the behaviour of multiple-server polling systems.

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Tuesday 15.20
Room A

AN ALTERNATIVE VIEW ON THE UNIFORM RULE

The uniform rule is considered to be the most important rule for the problem of allocating a positive amount of an infinitely divisible good between agents who have single-peaked preferences. The uniform rule was studied extensively in the literature and several characterizations were provided. The aim of this note is to provide a different view on the uniform rule. Our main result shows that the uniform rule maximizes the product of the amounts allocated to the different agents over the set of Pareto optimal allocations. Instead of giving a direct proof of this result using standard optimization techniques, we provide a proof using several properties of the uniform rule and the Nash bargaining solution.

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Tuesday 14.50
Room B

SENSITIVITY ANALYSIS AND THE k-BEST TSP

In this paper we reveal the connections between two problems related to the Traveling Salesman Problem (TSP), namely the tolerance problem and the problem of finding the k smallest Hamiltonian tours (k-best TSP).

Given is a minimal Hamiltonian tour $H_{(1)}$ in an undirected weighted complete graph K_n with weight function d. In posterior sensitivity analysis applied to the TSP it is studied how much each edge weight can be changed individually without affecting the optimality of $H_{(1)}$. The maximum increment (decrement) of an edge weight that preserves the optimality of $H_{(1)}$ is called the tolerance.

The k-best TSP is the problem of finding k pairwise different Hamiltonian tours $H_{(k)}$ in K_n given weight function d such that $l(H_{(1)}) \leq l(H_{(2)}) \leq \ldots \leq l(H_{(k)}) \leq l(H)$ for all H in

K_n and $H \neq H_{(1)}, \ldots, H_{(k)}$.

In this paper we present a new method for solving the k-best TSP based on tolerances. The new method solves the 2-best tour $H_{(2)}$ using $H_{(1)}$ and the tolerances in polynomial time. However, solving the 3-best tour $H_{(3)}$, given $H_{(1)}$, $H_{(2)}$, and the tolerances is $\mathcal{NP}$-complete.

The previously mentioned method for solving the 2-best tour can be used to improve existing algorithms for solving the k-best TSP. Moreover, many of the results can be applied to other Combinatorial Optimization problems.

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Tuesday 15.40
Room A

ON FINDING AN ENVY FREE PARETO OPTIMAL DIVISION

We will describe an algorithm to find an envy free pareto optimal division in the case of a finite number of homogeneous divisible goods and linear utility functions. The general cake division problem can be roughly phrased as follows: Suppose we have a cake C and n people who may value different parts of the cake differently. Can one find a way to divide C among the n people so that each is satisfied with his share?

As Brahms and Taylor (1992) remark: "A consideration of the cake division problem requires addressing the following issues: (i) definition of "satisfactory" and (ii) a choice of the mathematical framework in which to formalize the problem." We have chosen for a strong definition of "satisfactory". Therefore we need rather strong conditions on the framework.

Our aims are: to divide the cake so that every agent likes his share best (envy freeness) and so that there is no other division that is strictly better for somebody and not worse for anybody (pareto optimality).

We consider the cake as the interval $C = [0, 1]$. The involved people are called agents. Each agent i has a non-negative Lipschitz-continuous density function f_i. These give us utility functions μ_i on the measurable subsets of the cake, i.e. agent i's utility of $D \subset C$ equals:

$$\int_0^1 f_i(x) 1_D(x) dx := \mu_i(D), \tag{1}$$

where 1_D is the indicator-function of D.

We divide C into a finite number of small intervals and approximate each density with a step-function, constant at each of those intervals. We give an algorithm that provides us a pareto optimal envy free allocation, if the densities were these step-functions. Because of the Lipschitz-continuity, this allocation is as close as desired to a real envy free pareto optimal one.

We have no upper bound for the amount of iterations the algorithm needs. In tests it terminated fast.

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Thursday 14.30
Room B

AN $O(\sqrt{n}L)$-ITERATION PRIMAL-DUAL CONE AFFINE SCALING METHOD

A primal-dual affine scaling method is introduced. The method uses a search-direction that is obtained by minimizing the duality gap in the primal-dual scaled space over the region in which the Jan-Fang cone constraint [1] is fulfilled. This direction neither coincides with known primal-dual affine scaling direction [4, 2], nor is it a combination of the affine scaling direction and the center direction in the generic primal-dual method of Kojima, Mizuno and Yoshise [3].

I explain the ideas behind this new efficient algorithm and I show the complexity result.

[1] G-M Jan and S-G Fang, A new variant of the primal affine scaling algorithm for linear programs, *Optimization* **22** (1991) 681–715.

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[3] M. Kojima, S. Mizuno and A. Yoshise, A primal-dual interior point algorithm for linear programming, in Megiddo ed., *Progress in Mathematical Programming: interior point and related methods* Springer Verlag, New York, 1989, pp. 29–48.

[4] R.D.C. Monteiro, I. Adler, M.G.C. Resende, A polynomial-time primal-dual affine scaling algorithm for linear and convex quadratic programming and its power series extension, *Mathematics of Operations Research* **15** (1990) 191-214.

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Thursday 15.20
Room B

CONSISTENCY & EQUILIBRIUM SELECTION

We consider two classes of games, namely the class of mixed extensions of finite games in strategic form and the class of games with compact and convex strategy spaces and continuous payoff functions.

A well-known solution concept for both types of games is the set of Nash equilibria. However, many of these Nash equilibria are not "reasonable" according to several authors: Nash equilibria may use dominated strategies and/or tend to be unstable against certain types of perturbations of the game involved.

In order to eliminate such "unreasonable" Nash equilibria several authors introduced selection procedures. One can think of the theory of equilibrium selection (cf. Harsanyi and Selten (1982)) or stable set theory (cf. Kohlberg and Mertens (1986)) in the case of mixed extensions of finite games, and perfection in the other case.

Yet we prove that the Nash solution concept is in both cases characterized by non-emptiness (NEM), utility maximization (UM) and consistency (CONS). This implies that all selection procedures are bound to lead to inconsistent solutions.

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ALGORITHMS AND LOWER BOUNDS FOR ON-LINE SCHEDULING

In solving a problem of optimization, the amount of available information of the problem instances can directly affect the quality of the solutions. On-line algorithms are in principle strategies for getting approximate solutions under the provision of little information. In this paper we investigate the problem of on-line scheduling a set of independent jobs on a number of parallel and identical machines with the objective of minimizing the overall finishing time. We derive the inevitable deviations from the optimum in the worst case by any on-line algorithm. Furthermore, we provide a fast on-line algorithm that has its worst-case performance ratios quite close to the lower bounds.

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JOINT REPLACEMENT IN AN OPERATIONAL PLANNING PHASE

We consider the problem of combining replacements of multiple components in an operational planning phase. Within a given infinite or finite time horizon, decisions concerning replacement of components are made at discrete time epochs. Optimal solving of this problem is computationally very difficult and is limited to a small number of components. We present a heuristic rolling horizon approach that decomposes the problem; at each decision epoch an initial plan is made that addresses components separately, and subsequently a deviation from this plan is allowed to enable joint replacement. This approach provides insight in the actions taken and the time needed to determine an action at a certain epoch is only quadratic in the number of components. After elimination of certain negative effects, our approach yields average costs less than 1% above the minimum value.

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