FOURTEENTH CONFERENCE ON THE MATHEMATICS OF OPERATIONS RESEARCH

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CONFERENCE CENTRE 'DE BLIJE WERELT'
LUNTEREN, THE NETHERLANDS

FEBRUARY 13-15, 1989

Organized by the
Centre for Mathematics and Computer Science,
Amsterdam, The Netherlands

FOURTEENTH CONFERENCE ON THE MATHEMATICS OF OPERATIONS RESEARCH

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AIM AND SCOPE

The aim of the conference is to promote the research activities and the cooperation between researchers in the mathematics of operations research.

Eight non-Dutch specialists have been invited to give two lectures on recent developments in their field of interest. They have been asked to present a tutorial survey of their area in the first talk and discuss their own recent work in the second lecture.

Of the invited speakers H. Kaspi (Technion), M. Rubinovitch (Technion), M. Yadin (Technion) and U. Yechiali (Technion) will discuss various subjects in the area of stochastic optimization: Markov processes, geometric probability, stopping time problems. Futhermore two of the invited speakers will give lectures in the field of mathematical programming: A. Ben-Tal (Technion) about duality and non-smooth optimization, U. Passy about projective convexity. M. Maschler (Hebrew University, Jerusalem) will speak about game theory subjects, M. Hofri (Technion) about queueing theory.

The program should give ample opportunity for informal discussions.

ORGANIZATION

Organizers

The conference is organized by the Centre for Mathematics and Computer Science in Amsterdam, in particular by J.W. Cohen, H.C. Tijms and B.J. Lageweg.

Sponsors

The conference is organized under the auspices of the Dutch Research Community in the Mathematics of Operations Research and System Theory, with financial support of the Dutch Mathematical Society and the Netherlands Society of Operations Research.

PROGRAM

A. Ben-Tal (Technion, Haifa):
1. The role of duality in optimization
2. Non-smooth optimization: analysis, methods and applications

M. Hofri (Technion, Haifa):
1. Self-organizing lists and independent references: a statistical synergy
2. Analyzing a packet switch as a non-conservative exhaustive-service polling system

H. Kaspi (Technion, Haifa):
1. Stationary regenerative systems on the real line
2. Applications of regenerative systems to Markovian storage systems

M. Maschler (Hebrew University, Jerusalem):
1. Game theory: bargaining sets and related topics
2. Game theory and bankruptcy problems

U. Passy (Technion, Haifa):
1. Symmetric, reflexive duality for projectively-convex sets and production theory
2. Matrix criteria for pseudo P-convex quadratic forms

M. Rubinovitch (Technion, Haifa):
1. A model for mixing based on Poisson random measures

M. Yadin (Technion, Haifa):
1. The hunter problem and other applications of the theory of coverage (1)
2. The hunter problem and other applications of the theory of coverage (2)

U. Yechiali (Technion, Haifa):
1. Stopping-time problems with application to live-organ transplants
2. Stochastic sequential stopping-time problems with application to live-organ transplants

TIME SCHEDULE

Monday February 13, 1989

11.30	Opening
11.40	Passy (1)
12.30	Lunch break
14.10	Kaspi (1)
15.00	Maschler (1)
15.50	Tea break
16.20	Ben-Tal (1)
17.10	Yadin(1)
18.30	Dinner

Tuesday February 14, 1989

9.00	Rubinovitch (1)
9.50	Coffee break
10.20	Hofri (1)
11.10	Yechiali (1)
12.30	Lunch break
14.10	Yadin (2)
15.00	Kaspi (2)
15.50	Tea break
16.20	Ben-Tal (2)
17.10	Maschler (2)
18.30	Dinner

Wednesday February 15, 1989

9.00	Hofri (2)
9.50	Coffee break
10.20	Passy (2)
11.10	Yechiali (2)
12.00	Closing
12.30	Lunch break

THE ROLE OF DUALITY IN OPTIMIZATION

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The purpose of the talk is to convey the power and elegance of duality in obtaining interpretations, approximations, and computational schemes, for the optimal solutions of convex programs in finite and infinite dimension. The above is illustrated via examples in statistics, probability, civil and mechanical engineering, information/communication theory, and spectral estimation.

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Ben-Tal 2

NONSMOOTH OPTIMIZATION: ANALYSIS, METHODS AND APPLICATIONS

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The following subjects will be discussed:
1. The need for nonsmooth optimization: examples of problems which give rise to nondifferentiable mathematical programs;
2. Theoretical difficulties: the need for nonsmooth analysis;
3. Numerical difficulties;
4. A short review of the first order theory and some numerical algorithms;
5. Elements of a second order theory for nonsmooth optimization problems in topological vector spaces;
6. Special cases: twice differentiable problems, approximation with nondifferentiable norms, semi-infinite programming.

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SELF-ORGANIZING LISTS AND INDEPENDENT REFERENCES — A STATISTICAL SYNERGY

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A linear list is a data structure in which there is little structure. Thus it is simple to build and maintain, with the consequent penalty that it is relatively inefficient in use. Nevertheless, under quite common circumstances, it is the structure of choice.

We assume the following layout: a list L contains n records $\{R_i, 1 \leqslant i \leqslant n\}$, that are uniquely identified by their keys $\{K_i\}$. The content of the list is assumed to remain unchanged over time.

The list structure implies the following sequential search *access scheme:* in order to access R_i, the keys in positions 1, 2, $\cdots$ are compared successively against K_i until an equality is obtained. The number of these comparisons is defined as the *cost* of the access.

We assume that references of the list are generated according to the following *reference model:* each request is for the record R_i with a time-homogeneous probability $p_i > 0$, $1 \leqslant i \leqslant n$. This holds true independently of the list order and the history of past accesses. Such a scheme is known as the (stationary) independent-reference model (*irm*).

A natural objective is to have the smallest possible cost per access. If the reference-probability vector (*rpv*) $\{p_i\}$ were known, the optimal policy would be to keep the list sorted by the size of the record reference-probabilities, in non-increasing, static order.

The adjective "self-organizing" in the title refers to the situation where the rpv is *not known,* and the access mechanism activates periodically typically following each access a procedure that may change the order of the records, based on the reference history.

Various reorganization rules, or heuristics, have been proposed for this purpose. Their purpose is to try and decrease, using information produced by the reference string up to any given time, the expected cost of future references. Hester and Hirschberg (1985) survey most of the work in this area published by 1983.

The following are three of the better known reorganization- or update-schemes. They are activated each reference. To describe their operation we use the symbol $\sigma(i)$ to denote the location of R_i in the list. The state of the list is correspondingly said to be the permutation σ.

Move to the front method (MTF): Following an access to record R_i, in position $\sigma(i)$, it is placed at the head of the list, pushing the records in locations $1, 2, \cdots, \sigma(i) - 1$ one step back. This heuristic has been extensively studied. Most researchers referred to its asymptotic cost per access, under various rpv's. They show that the limiting expected cost per access is given by

$$E_{MTF}(C) = 1 + 2 \sum_{1 \leq i < j \leq n} \frac{p_i p_j}{p_i + p_j}. \tag{1}$$

Bitner (1979) considered also the number of requests required until the cost function converges to its asymptotic value. More precisely, he defined the "overwork" (OW) for any heuristic H as

$$OW_H \equiv \sum_{m \geq 0} (E_H^{(m)}(C) - E_H(C)), \tag{2}$$

where $E_H^{(m)}(C)$ is the expected cost of the $m+1$st access, assuming the initial state of the list is any of the $n!$ possible permutations with probability $1/n!$, and $E_H(C) = \lim_{m \to \infty} E_H^{(m)}(C)$. The expectation is evaluated with respect to the initial state and the history of references.

Recently, there have appeared some work [Sleator and Tarjan (1985), Bentley and McGeogh (1985)] that considers not the *expected* cost of MTF, but rather the worst possible cost, when averaged (or as called in that context amortized) over a long reference sequence. For one reason or another, MTF is the heuristic that has been most assiduously studied, and is commonly used as a yardstick by which other rules are measured.

Transposition rule (TR): A record R_i, when accessed at location $\sigma(i) > 1$ is transposed with its neighbor in location $\sigma(i) - 1$, getting thus one step closer to the head of the list.

Counter Scheme (CS): Each of the records R_i is associated with a frequency counter c_i, which is incremented whenever the record is accessed. The record is then moved forward (if needed) to keep the list records sorted in non-increasing order of their counters.

Let m be the total number of requests made to the list, then by the strong law of large numbers, for all $1 \leq i \leq n$

$$\frac{c_i}{m} \underset{m \to \infty}{\longrightarrow} p_i.$$

This implies that, in the limit, the list will be optimally ordered. Clearly, this is the best among the asymptotic costs of all possible heuristics.

There is a substantial amount of published statistical work on discriminating multinomial probabilities, under various requirements [a comprehensive account is provided in Gibbons *et al.* (1977)]. Here, however, the requirement is merely to minimize the expected value $E[\sigma(I)]$, where I is the index of the accessed record. The *irm* and this cost function interact and give rise to the following synergistic effects:

Broadly speaking, only the records with large probabilities make any significant contribution to the cost function. However, it is precisely those records for which the counters increase fast and provide good estimates early on.

Actually it is not even the probabilities proper we need to estimate, but only their sorted order. Now, when the difference between two probabilities, say $p_i - p_j > 0$ is substantial, again after a relatively short reference sequence there will be a very small probability for the event $\{c_i < c_j\}$. On the other hand, when $p_i - p_j > 0$ is very small, while it is true that the number of references required to order them correctly, with high probability, is huge, correspondingly the penalty of incorrect order is minute (even when p_i and p_j proper are *not* small since it is proportional to $p_i - p_j$).

We shall present two ways for limiting the space complexity of the counter scheme without damaging its superiority over other permutation rules.

One idea is to fix a maximal value $c_{\max}$ which the counters may not exceed. The other method stops the list-reorganization process after a finite, predetermined number of requests. This second approach not only bounds the space requirements, but also sets an upper bound on the time required to implement the counter scheme.

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Hofri 2

ANALYZING A PACKET SWITCH AS A NON-CONSERVATIVE EXHAUSTIVE-SERVICE POLLING SYSTEM

Cycle-Time and Capacity Analysis

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The system under consideration models a switching element that can be in N distinct states; in each of these states it handles two streams of requests in parallel. These streams can be viewed as representing the different routing requests put to the switch, and the element handles these requests at the address-bit level. The analysis will, for the most part, consider only two possible states. The restriction to two switch-states is for the purpose of the analysis only, but the methodology that we use can effect the same computations for a larger number of stages or switch-states, in each of which two streams are served side-by-side. The extension in entirely mechanical.
The description of the switch operation uses a time axis that is slotted into equal slots. Each switching activity requires precisely one complete slot.
One non-trivial and critical restriction of the mathematical model discussed is that all the input processes that describe the request-generation are assumed to be independent, each forming an i.i.d. sequence of random variables. These rv's specify the number of arrivals of new requests at successive slots to the switch queues. Queued requests are kept in buffers; there is a separate, unlimited buffer for each stream. A request that arrives at a slot may be serviced at the earliest during the next slot. Hence we choose to

represent the state of the buffers at "slot end"—or "slot beginning"—when all present requests are eligible for service.
To complete the description of the queueing model we need to specify the service regime yet: for the present discussion it does not matter, unless we examine the distribution of the waiting times, at which order requests are selected for service. We may assume that requests are handled on a FCFS basis at each stream. When the two-port server is at any switching-state, it will serve in each slot one request from each of the two streams that can be handled at that state (or position). If one stream is empty — only one request is handled. If at the beginning of a slot both buffers are empty, the switching-state is changed (instantaneously, at this time-scale) and one request from each non-empty buffer that is accessible in the new position is served. The new switching-state would persist for one slot even if both buffers were empty at its beginning, and of course, no service would be done then. If there are arrivals during such an idle slot the state would be maintained, to serve them until the first time both buffers are empty, when the server would continue to handle the other streams.
It is remarkable how similar in essence this model is to standard polling systems that have been studied extensively in the past, except that the service here is non-conservative: say the switch attends to the first position, at queues #1 and #2; if at a given time-slot only buffer #1 is non-empty, a single request is served, even if there may be requests queued at buffer #4, that the currently idle port could have served if the switch were at the next station. Another way to state the same characteristic is to say that the service rate is state-dependent, a well-known nemesis of queueing analyses. A trivial example of the difficulty: In a conservative polling system (with exhaustive service and infinite buffers, as we allow), the computation of the mean cycle time, the expected time between returns of the server to begin serving a given position, is immediate In our system however, determining this duration is a major effort. In fact, the part of the analysis that I plan to present, concerns only this variable.

STATIONARY REGENERATIVE SYSTEMS ON THE REAL LINE

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The theory of regenerative systems on the real line unifies the notions of strong Markov processes indexed by $\boldsymbol{R}$, recurrent events, regenerative processes, semi Markov processes etc. A regenerative system, as it will be defined in this talk, consists of a closed random set $M \subset \boldsymbol{R}$ and a right continuous process (X_t) such that

$$((M-T) \cap (0,\infty),(X_{T+s})_{s\geqslant 0}) \text{ and } (M \cap (-\infty,T],(X_{s\uparrow T})_s \quad \boldsymbol{R}$$

are conditionally independent given X_T, for all stopping times T falling in M. In the classical case of semi-regenerative processes, as treated, for example, by Cinlar (1975) $M = \{T_n : n\geqslant 0\}$ is the regeneration set. In our situation M needs not be discrete and hence the renewal counting process is meaningless. Nevertheless, under some mild regularity assumptions we can define a local time at M and exit laws that govern the excursions from M.

We establish a correspondence between stationary regenerative systems and invariant distributions for the Markov process (X_{τ_t}) where τ_t is the inverse of the local time at M. In particular we extend the notion of stationarity and the formulae for the stationary distribution of the backward and forward recurrence times, which appear in the context of classical renewal theory.

In the special case where the underlying process (X_t) is Markovian and M is the set of times that X visits a state (or a set), this theory provides a formula for the invariant distribution of X_t. Applications of this formula to the theory of Markovian storage systems will be discussed in my second talk.

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APPLICATIONS OF REGENERATIVE SYSTEMS TO MARKOVIAN STORAGE SYSTEMS

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In this talk we shall treat a general class of Markovian storage processes. The objective is to use the theory of regeneration for extending the results of Rubinovitch and Cohen (1980) on the connection between level crossing and the stationary distribution of the content process.

In the classical theory of dam processes the content is defined in terms of an input, which is usually a process with stationary independent increments and an output, which is assumed continuous and at a rate that may be state dependent. The resulting content process is then a Markov process. Here, we allow downwards jumps and make the rate of the jump appearance and the jump sizes state dependent.

We show that if the empty state is reached a.s. then there is a unique stationary distribution. It has a density for $x \geqslant 0$ which is equal to the long term average of the number of continuous downcrossings of level x devided by the continuous output rate $r(x)$. The set of times when the system is empty is normally a perfect set but it may have 0 Lebesgue measure. The tool for obtaining the above relation is a representation of the invariant distribution of the content process via the theory of regenerative systems and an ergodic theorem.

If time permits we shall apply the results to a specific example of a Markovian storage system with state dependent input and output. The system considered was constructed as a dual, of sorts, of a non

Markovian storage/production system, in such a way that the stationary distributions of the two systems are identical. Their density is obtained, as above, using level crossing analysis.

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SYMMETRIC, REFLEXIVE DUALITY FOR PROJECTIVELY-CONVEX SETS AND PRODUCTION THEORY

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Hackman and Passy (1988) introduced a generalization of convexity called Projective-convexity (or P-convexity). P-convexity is defined via a generalized path-property. It also posses a separation property: a point not in a well-behaved P-convex set can be separated from the set by a quadrant, which is the intersection of two closed halfspaces generated by orthogonal hyperplanes.

In this talk, we establish conditions under which P-convex sets possess a symmetric, reflexive duality. By symmetry we mean that a set is P-convex if and only if its dual (the set of its separators) is also P-convex. By reflexivity we mean that the bi-dual (the dual of the dual) of a P-convex set is P-convex, and embeds the original set. In addition, we provide conditions under which a set which possesses the "separation by quadrant" property is actually P-convex. Our results are necessary to generalize the existing duality of consumer/producer theory to the non-convex environment.

MATRIX CRITERIA FOR PSEUDO P-CONVEX QUADRATIC FORMS

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The general question of when the restriction of a quadratic form on $\boldsymbol{R}_k$ to the null space of a s by k matrix is positive semidefinite has been extensively investigated in O.R. literature. It is shown that this question is associated with characterization of generalized convex quadratic functions. Except for the family of quasiconvex quadratics, no characterization exists. In the present talk, we characterize pseudo P-convex quadratics.

A MODEL FOR MIXING BASED ON POISSON RANDOM MEASURES

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One of the most common elements in many chemical engineering processes is a vessel, or flow region, in which some "mixing process" takes place. This element is usually called a stirred tank and is a building block in modelling complex flow processes of chemical engineering.

In stirred tanks there are usually two related concepts of interest. One is the sojourn time of particles in the tank and the other is the spatial distribution of particles and the changes it undergoes with time. Both concepts are clearly probabilistic. Indeed a large body of literature has been devoted to the former and most of it centers on the distribution of the time that particles spend in stirred tanks, or in systems of interconnected stirred tanks. The connection between concentration and residence time has also been thoroughly investigated. Most of this work was based on the qualitative definitions of mixing that were given in the historical papers of Danckwerts (e.g. 1953a, 1953b).

On the other hand, as of today, there is no exact and agreed upon model for the second concept, the spatial distribution of particles in stirred tanks. In the present study we propose a model, based on Poisson random measures in three dimensional spaces, for the spatial distribution of particles in mixed flow regions. We investigate the elementary properties of this model, and its consistency with what is known today about stirred tanks. We show that this model agrees perfectly well with existing results on residence time distributions, on age distributions and on concentrations at different points in a stirred tank and in general flow systems.

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THE HUNTER PROBLEM AND OTHER APPLICATIONS OF THE THEORY OF COVERAGE

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A hunter tries to shoot a rabbit who is running at some distance in a wood. The rabbit's path is only partially seen by the hunter, because of randomly scattered trees. The number of shots depends on the length of the visible portion of the path, and every shot hits the target with a given probability. What is the rabbit's probability of survival? What are his chances if there are several hunters after him?

The problem is treated by a two dimensional model, where trees are represented by randomly distributed disks with randomly varying diameters. More specifically, we represent each tree by a point in a three dimensional point process, the coordinates of which defines the location of the center of the disk and its diameter.

Using the random field model, we developed an algorithm for the computation of moments of the total length of time in which the rabbit is visible to the hunter. The algorithm is based on coverage theory due to Robbins (1945 and 1946). Moments are applied to the approximation of the distribution of this time as well as the survival probability of the rabbit

A general survey of the methodology that was applied to the solution of this and similar problems, including an extension to three dimensions and to several hunters will be presented in the first lecture.

The distribution of the shadowed road segments covered by the running was studied by Chernoff and Daly (1957). They outlined the similarity between these segments and busy periods in certain queuing models. They constracted the differential equations for the conditional probabilities and outlined their solution by transformation.

The distribution of the total length of time in which the rabbit is visible to the hunter, as well as the distributions of the periods in which the rabbit is unseen can be approximated by discretization. An algorithm used for this approximation will be described in details in the second lecture.

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Yechiali 1

TIME-DEPENDENT STOPPING PROBLEMS WITH APPLICATION TO LIVE ORGAN (E.G. KIDNEY) TRANSPLANTS.

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A candidate is waiting for "offers" (e.g. kidneys for transplants) that become available from time to time. The values of the offers constitute a sequence of i.i.d. positive random variables. When an offer arrives, a decision is made whether to accept it or not. If it is accepted, the process terminates. Otherwise, the offer is lost and the process continues until the next arrival, or until a moment where it terminates by itself. Self-termination depends on an underlying lifetime distribution (which in applications corresponds to that of the candidate for transplant).

When the underlying process has an Increasing Failure Rate, and the arrivals form a renewal process, we show that the control-limit type policy that maximizes the expected reward is a non-increasing function of time. For non-homogeneous Poisson arrivals we derive a first-order differential equation for the control-limit function. The equation is explicitly solved for the case of discrete-valued offers, homogeneous Poisson arrivals and Gamma distributed lifetime. The solution is used to analyze a detailed numerical example based on actual kidney-transplant data.

STOCHASTIC SEQUENTIAL ASSIGNMENT BASED ON DISCRETE-MATCH LEVELS

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M "offers" (e.g., kidneys for transplants) arrive in a random stream and are to be sequentially assigned to N waiting candidates. Each candidate, as well as each arrival, is characterized by a random attribute drawn from a discrete-valued probability distribution function. An assignment of an offer to a candidate yields a reward $r(0)$ if they match, and a reward $r(1) \leqslant r(0)$ if not. We derive optimal sequential assigment policies which maximize the expected total reward for various cases of M versus N and various decay assumptions on the underlying life time distribution of the process. We give intuitive explanations for these optimal strategies and indicate applications.