→ de Groot ARCHIEF (behorende bij [illegible] tafelkleedje)

Table cloth showing the „Gaussian" primes

The table cloth shows the Gaussian primes in the complex plane. The origin O is at the centre, and the x- and y-axes are parallel to the sides of the cloth. The Gaussian complex integers are defined by $m + i\,n$, where both **m** and **n** run independently through all positive and negative ordinary integers (zero included) and $i = \sqrt{-1}$. Thus $-7+2\,i$, $0-5\,i$, $21 + 7\,i$, $12 + 0\,i$ are all complex integers. Clearly the product of two complex integers is again a complex integer as follows from

$$(m_1 + in_1)\ (m_2 + in_2) = m_3 + in_3$$

where

$$m_3 = m_1\,m_2 - n_1\,n_2$$

$$n_3 = n_1\,m_2 + m_1\,n_2$$

As is well known, in the field of the ordinary (rational) integers, not every integer can be decomposed into smaller factors; the ordinary integers which cannot be decomposed are the ordinary primes. The same is true in the field of the complex integers. Thus the complex integers which cannot be written as the product of two other complex integers (ignoring the factors 1, —1, i and —i) are called the complex, or Gaussian primes. The latter are marked as little squares on the table cloth.

It can be shown that ordinary primes such as 3, 7, 11 which are of the form $4k - 1$ (k integer), remain primes in the Gaussian field and they are therefore marked as such on the real and complex axes. However, the ordinary primes of the form $4k + 1$, such as 5, 13, 17 ...... are no longer primes in the Gaussian field, because they can be decomposed as follows

$$5 = (1 + 2\,i) \,.\, (1-2\,i),$$
$$13 = (3 + 2\,i) \,.\, (3-2\,i),$$
$$17 = (4 + i) \,.\, (4-i),$$

and therefore the ordinary primes such as 5, 13, or their associates, 5i, 13i, —5i, —13i, —5, —13 & are not marked on the axes.

Further, numbers like $3+2i$, $5+4i$ are also complex primes and are marked as such on the diagram. These complex primes have the property that their "norm", i.e. the square of their modulus, is an ordinary prime of the form $4k + 1$, e.g.

$$3^2 + 2^2 = 13,$$
$$5^2 + 4^2 = 41,$$

BALTH. VAN DER POL

GENEVA,
March, 1954

Cloth made by
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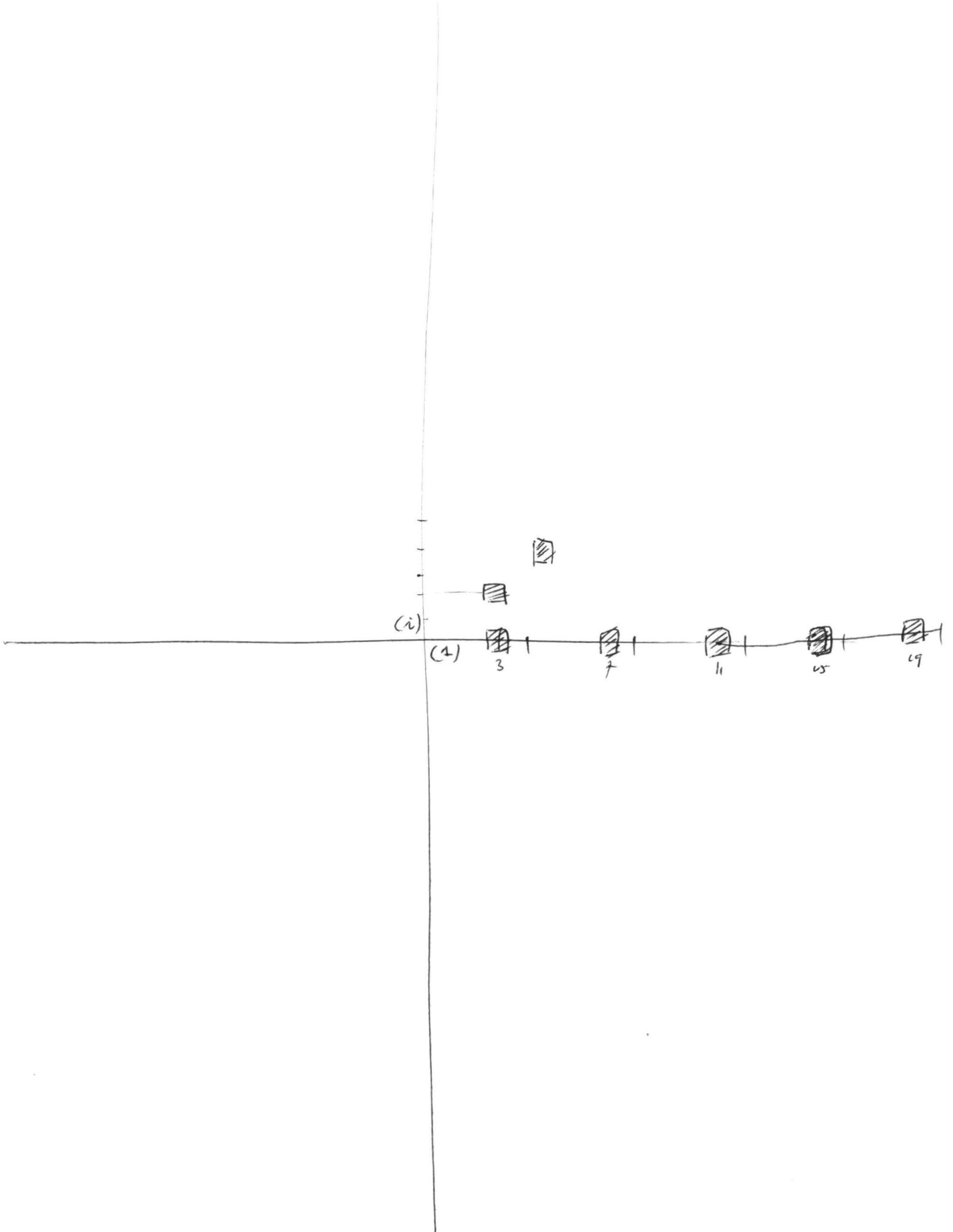
(i)
(1)
3
7
11
15
19