Autocrine inhibition of cell motility can drive epithelial branching morphogenesis in the absence of growth

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1. Introduction

During the embryonic development of organs such as lungs, kidneys and the mammary gland, epithelial tissues undergo shape changes resulting in a tree-like structure of branches [1,2]. The function of branched organs is to optimize the exchange of chemicals with the surrounding tissue by maximizing the interfacial area. The dynamics of branching from an initially tube-shaped epithelial tissue, called the duct, into the surrounding mesenchymal tissue involves cellular mechanisms such as directed cell migration, oriented cell division, cell shape changes, cell differentiation and cell competition (see reviews [3–5]). The specific process of branching morphogenesis varies per organ, but the key mechanisms are believed to be conserved [6,7]. Although the dynamics of branching in various organs have been characterized well (see for instance, lung [8], kidney [9], mammary gland [10], pancreas [11]), it is still poorly understood what mechanisms drive branching morphogenesis, which of these mechanisms are fundamental, and which ones act on top of the fundamental mechanisms for ‘fine tuning’.

Mathematical modelling is a helpful tool to analyse branching morphogenesis. A first class of models asks how biological rules operating on single branches and branch tips can lead to an observed branching pattern. For example, to explain the anatomy of the urinary collecting duct tree, Davies et al. [12] have
proposed that the ureteric tubules secrete a hypothetical repulsive factor. The tips of tubules grow towards lower, local concentrations of the repulsive factor at a speed inversely proportional to its concentration. Tubule tips bifurcate once the concentration of the repulsive factor drops below a threshold. The model was used to help explain observed repulsive branch interactions in explants of the collecting urinary duct trees of the mouse kidney, and to plan follow-up experiments. Scheele et al. [13] analysed the morphogenesis of the murine mammary gland using a statistical branching model. They constructed trees of which the branches bifurcate or terminate with a near equal probability. This growth process accurately reproduced the distribution of the number of branching levels in the murine mammary gland and the kidney, in support of the potential homology of epithelial branching processes [13]. In a spatially extended variant of this model, growing branches were assumed to terminate as soon as they approached an existing duct, possibly due to TGF-β signalling [14]. This model was able to reproduce observed tissue architecture, such as local densities of branches and directional biases of branch growth. An additional rule stating that approaching branches were repelled by adjacent branches produced better fits with the observed branch density in the kidney.

A second class of mathematical models, which includes the one proposed in this paper, focuses on the cellular and molecular mechanisms responsible for branch tip initiation, branch progression, tip splitting and tip termination. For a long time, it was thought that localized, differential cell proliferation is the main driving factor of branching, but this may not always be true [4]. In the chicken and mouse lung, the buds that initiate new branches form prior to the first appearance of cell proliferation [15,16]. Signalling factors from the mesenchyme have been proposed to drive branching [17]. However, the mesenchyme is not required either, as epithelial tissues can branch in the absence of a surrounding mesenchyme in vitro [18–21]. Thus, it is still poorly understood how epithelial tissues branch autonomously in the absence of cell proliferation and mesenchyme. Here we propose a cellular mechanism for such autonomous branching of epithelial tissues.

Epithelial branching has been proposed to be analogous to Laplacian growth, a process that underlies branching in many non-biological systems, including crystal growth [22] and viscous fingering [23]. In a mathematical modelling study of lung morphogenesis, it was proposed that the epithelium branches into the surrounding mesenchyme if the mesenchyme is less viscous than the luminal fluid in the epithelium [24]. In such Laplacian growth processes, the interface of a domain advances with a velocity proportional to the gradient of a field that obeys the Laplacian equation ($V^2u = 0$), i.e. a field dominated by the diffusion equation, aka the heat equation [25], with $u = 0$ at the interface. Thus points of the morphology located at an interface of positive curvature, which may arise from random deviations from a initially homogeneous boundary, experience a higher gradient of the Laplacian field and will advance faster than the points at flat or concave locations of the interface. This effect is known as the Mullins–Sekerka instability. Instead of pressure and viscosity fields, Laplacian growth dynamics of tissues could also be governed by molecular concentration fields. For example, in the context of tumour branching, it was proposed that cell proliferation depends on the availability of oxygen [26]. Analogously, in a Laplacian growth model of epithelial branching [27], it was proposed that growth is proportional to the local flux of fibroblastic growth factors (FGF).

Other mathematical models have studied in detail how the regulatory interactions between the epithelium and mesenchyme can drive branching. Such epithelial–mesenchymal cross-talk may regulate the highly stereotypic branching patterns of the lung [8] and the kidney [28]. Hirashima & Iwasa [29] studied epithelial–mesenchymal cross-talk in a mathematical model based on the cellular Potts model. They assumed that a deformable epithelial layer is chemoattracted to localized sources of growth factor such as FGF10 or GDNF. This chemotactic mechanism together with cell proliferation produces branches through a buckling mechanism, where the number of branches depended on the ratio between the proliferation and the chemotaxis speed [29]. In a further paper, Hirashima et al. [30] showed how secretion of SHH by progressing buds can regulate the required localized expression of FGF10 in the mesenchyme. Inhibition of FGF10 expression at high concentrations of SHH, combined with activation of SHH expression at lower concentrations of SHH produce peaks of FGF10 at a small distance from the bud tip. As the tip approaches the lung border, the SHH locally accumulates, leading to a split expression pattern of FGF10. Menshykau et al. [31,32] have proposed a model with additional interactions between SHH and FGF10 that could lead to a Turing-type reaction–diffusion mechanism for branching morphogenesis: a positive feedback loop is closed if apart from regulation of FGF10 by SHH, FGF10 from the mesenchyme also induces SHH production in the epithelial cells. The model suggests that the SHH ligand–receptor interactions allow the localized spots to stabilize. By letting the growth rate of the tissue domain depend on the level of ligand–receptor signalling, it was shown that the tissue branches out [33]. Similar Turing mechanisms are thought to be at work in the kidney [34]. All in all this work suggests that an intricate signalling network between the epithelium and mesenchyme generates a pattern of growth factors that drives branching by locally upregulating tissue growth.

Alongside the growth factor interaction network discussed above, evidence has accumulated over the last fifteen years that autocrine inhibitory signals, such as TGF-β provide a robust mechanism for epithelial branching morphogenesis [35–37]. The epithelial cells secrete a diffusive signal which, upon binding, inhibits their own proliferation or their own motility. At convexly curved locations, the inhibitory signal dissipates more easily than at flat or concave locations, much like heat radiates out more rapidly from a mountain peak whereas it gets ‘trapped’ within valleys. Using tissue-engineered configurations of cells, Nelson et al. [35] have shown that murine mammary epithelial cells exhibit such geometry-dependent sprouting activity. Using an image-based model of murine kidney morphogenesis, the Iber group [38] found that a model in which the epithelium secreted an inhibitory signal more robustly and more accurately predicted the future sites of branching than alternative scenarios, such as the Turing-type system discussed above. The Sneppen team [39] showed that the branching growth of in vitro cultures of murine pancreatic cells is well explained using a cell-based model in which the autocrine signal inhibits cell proliferation. These mechanisms are analogous to those based on Laplacian growth proposed previously for branching morphogenesis of cell agglomerates [24–27]. However, these models based on proliferation do not explain how epithelia can form branching configurations in the absence of growth, as observed in cell cultures [18–21] and in vivo [15,16].
Here we introduce a hybrid model based on the cellular Potts model (CPM) and partial-differential equations (PDE) to study how such autocrine inhibition of stochastic cell motility suffices for branching morphogenesis. We assume that the local concentration of an autocrine signal inhibits cell protrusion activity at the boundary of the tissue. We first show that this simple mechanism is a sufficient explanation for the *in vitro* observations by Nelson et al. [35]. Then we show that the same mechanism also suffices to reproduce branching morphogenesis in the absence of cross-talk with mesenchymal tissues and in the absence of cell proliferation. Finally we study the behaviour of the model in the presence of proliferation, and show that it suffices to reproduce previously reported behaviour of branch epithelia such as self-avoidance.

2. Results

*In vitro* observations suggest that cell protrusions are inhibited by the local concentration of TGF-β [35], leading to the hypothesis that diffusion of autocrine TGF-β drives curvature-dependent sites of branching [35]. To test if this mechanism suffices to drive epithelial branching, we developed a hybrid cell based and continuum model (figure 1). A particularly well-suited modelling framework for this purpose is the cellular Potts model (CPM) [40,41]. The CPM naturally represents the stochastic protrusion and retraction of cells as they were observed in mammary epithelial cell cultures [35]. The CPM also naturally represents the collective migration of cells during branch extension. The CPM (figure 1a) simulates the random motility of cells by mimicking iterative attempts to extend and retract pseudopods, e.g. filopodia and lamellipodia. We assume that these cellular extensions and retractions are regulated by a chemoinhibitor, e.g. TGF-β (figure 1b). Following Nelson et al. [35], we assume that this chemoinhibitor is secreted by the cells, that it diffuses through the extracellular matrix (ECM) in which the cells are embedded, and that it is gradually broken down, e.g. through enzymatic degradation or binding and inactivation in the matrix. The chemoinhibitor inhibits the formation of cellular protrusions in a concentration-dependent manner.

(a) Model description

The CPM describes cells as a collection of lattice sites on a two-dimensional, regular square lattice $\Lambda \subset \mathbb{Z}^2$. Each lattice site $x \in \Lambda$ is assigned a spin or cell identifier $\sigma(x) \in \mathbb{Z}^{(0,+)}$, an index of the cell that occupies this lattice site, such that a cell $C(s) = \{ x \in \Lambda : \sigma(x) = s \}$, i.e. the set of lattice sites with the same cell identifier $s$. $C(0)$ represents the medium, i.e. the lattices sites $x$ for which $\sigma(x) = 0$. The system evolves by minimizing a Hamiltonian

$$H = \sum_{\langle x,x' \rangle} f(\sigma(x), \sigma(x'))I_{\sigma(x') \neq \sigma(x)} + \lambda \sum_{1 \leq s \leq r} (\sigma(s) - A_{\text{target}})^2,$$

(2.1)

where $(x, x')$ is a pair of adjacent lattice sites, $f(\sigma(x), \sigma(x'))$ is an interfacial energy, describing cell adhesion and interfacial tensions [42], $I_{\text{expression}}$ is the indicator function, where $I_{\text{expression}} = 1$ if [expression] is true and $I_{\text{expression}} = 0$ otherwise. The second right-hand-side term introduces a volume constraint, with $\sigma(s) = C(s)$, the area of cell $s$ and $A_{\text{target}}(s)$, the target area of cell $s$ and parameter $\lambda$ is a Lagrange multiplier.

The CPM simulates cell motility through random attempts to retract or extend the cell boundaries. To simulate a random cell extension or retraction, the algorithm iteratively picks a random lattice site $x$, and calculates the energy change $\Delta H$ resulting from a copy of $\sigma(x)$ into an adjacent lattice site $x'$. It then accepts this copy depending on the change in energy, $\Delta H$, resulting from it, with probability

$$P(\Delta H + H_{\text{work}}) = \begin{cases} e^{-\Delta H + H_{\text{work}}} & , \Delta H + H_{\text{work}} \geq 0 \\ 1 & , \Delta H + H_{\text{work}} < 0 \end{cases}$$

(2.2)

$T$ is a motility parameter, aka ‘cellular temperature’, and represents the amount and magnitude of active, random cell fluctuations, which may act against the passive forces given by the energy $H$.

$H_{\text{work}}$ indicates the energy that is dissipated (e.g. due to friction or viscosity) during a move. It is used here to model chemoinhibition by a field of a secreted chemical, $c$,

$$H_{\text{work}} = c(x')I_{\sigma(x') > 0}I_{\sigma(x) = 0}$$

(2.3)

where $\chi$ regulates the strength of the inhibition, and $c(x')$ is the chemoinhibitor concentration at the target location $x'$. In the CPM, time is measured in Monte Carlo Steps (MCS), i.e. the number of movement attempts as there are sites in the lattice.

The dynamics of the chemoinhibitory signal is given by a PDE,

$$\frac{\partial c(x,t)}{\partial t} = D \nabla^2 c(x,t) + a_1 I_{\sigma(x)>0} - \epsilon c(x,t) I_{\sigma(x)=0},$$

(2.4)

where the secretion and decay terms depend on the current state of the CPM. A simulation consists of consecutive steps.
of the CPM and the PDE, where one timestep of the CPM is followed by 3s of inhibitor dynamics. This time scale was chosen in accordance to our previous work [43]. The parameter values are given in table 1.

### (b) Model mimics experimental observation of branching at convex sites

Nelson et al. [35] have reported that mammary epithelial cells show geometry-dependent sprouting. They cultured murine mammary epithelial cells inside small micropatterned cavities stamped into collagen gels. After induction with growth factors, the cells formed multicellular sprouts preferentially at the positively curved (convex) parts of the cell clusters (figure 2a,b). At crevices between two cell clusters, where a secreted growth factor would accumulate, no sprouts formed (figure 2c,d). To test if our mathematical model suffices for explaining these observations, we initialized our model simulations with the shapes used in the experiments by Nelson et al. [35] (figure 2c–h). The size of the geometry matched those used in the in vitro experiments [35] and we used the parameters in table 1, for which there are on average five cells across the diameter, corresponding to nuclei counts in Nelson et al. [35].

We first ran the CPM model for 750 MCS during which time cell shapes could equilibrate. For the next 750 MCS, we simulated only the PDE such that the chemical field approached a steady state (figure 2i). The shapes and lengths of these simulation steady-state gradients matched well with the experimentally observed TGF-β (compare fig. 4A in [35] with figure 2i). We then simulated the CPM and the PDE concurrently for a further 4000 MCS, allowing for some cell expansion (1 μm² of additional target area per 100 MCS), but disabling cell division. Frequency maps show the percentage of the last 4000 MCS that a cell is present at the site. Similar to the in vitro observations [35], in silico multicellular sprouts appeared preferentially at the convex parts of the geometry. For the ‘C’ shape, branching occurs at the tips and the ‘belly’ of the ‘C’, which are the convex parts (figure 2e and electronic supplementary material, video S1). Similarly, for the ‘Y’ shape, sprouts appeared around the convex tips (figure 2f and electronic supplementary material, video S2). Our model could also reproduce the setup shown in figure 2c,d. Here cells were placed into two rectangular wells positioned at 90° angles or adjacent to each other. As in the C- and Y-shaped geometries, the cells sprouted at the convex regions of the morphologies away from the other morphologies (figure 2g,h and electronic supplementary material, videos S3 and S4). No sprouts were formed at the convex regions near the crevice between the two geometries due to accumulation of the inhibitory signal (red curve in figure 2i). This observation is correctly reproduced in the simulations.

A discrepancy between the predictions of our simulations and the experimental observations is that we needed to keep a bit of pressure on the cell boundaries through a slow cell expansion term. In the absence of this term, the concave cell boundaries moved inwards, as can still be observed in figure 2g,h; a more refined representation in the CPM of the ‘cavities’ that contained the cells in the experiments will likely prevent this inward motion and hence improve the predictions. Possibly this also solves a further discrepancy with the experiments: the cell expansion term. In the absence of this term, the concave cell boundaries moved inwards, as can still be observed in figure 2g,h; a more refined representation in the CPM of the ‘cavities’ that contained the cells in the experiments will likely prevent this inward motion and hence improve the predictions. Possibly this also solves a further discrepancy with the experiments: the cell expansion term.

### Table 1. Default parameter settings.

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
<th>unit</th>
<th>motivation</th>
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<tbody>
<tr>
<td>$T$</td>
<td>cellular temperature</td>
<td>50</td>
<td>—</td>
<td>chosen followed by sensitivity analysis (figure 6)</td>
</tr>
<tr>
<td>$A_{\text{target}}$</td>
<td>target area</td>
<td>50</td>
<td>μm²</td>
<td>estimated based on nuclear stainings in Nelson et al. [35]</td>
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<tr>
<td>$\chi$</td>
<td>chemoinhibition parameter</td>
<td>25</td>
<td>—</td>
<td>chosen followed by sensitivity analysis (figure 5)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>area constraint strength</td>
<td>50</td>
<td>—</td>
<td>chosen</td>
</tr>
<tr>
<td>$D$</td>
<td>diffusion coefficient</td>
<td>$3.75 \times 10^{-12}$</td>
<td>μm² s⁻¹</td>
<td>chosen together with value of $e$ based on diffusion length reported in Nelson et al. [35]</td>
</tr>
<tr>
<td>$e$</td>
<td>decay rate</td>
<td>$5 \times 10^{-3}$</td>
<td>s⁻¹</td>
<td>based on half-lives reported in Wakefield et al. [44], decay rate in plasma is in range $10^{-4} - 10^{-2}$; diffusion length $\sqrt{D/e}$ matches observations in Nelson et al. [35]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>secretion rate</td>
<td>$5 \times 10^{-4}$</td>
<td>s⁻¹</td>
<td>estimated</td>
</tr>
<tr>
<td>$J_{01}$</td>
<td>cell–ECM adhesion energy</td>
<td>50</td>
<td>—</td>
<td>estimated to give slightly adhesive cells</td>
</tr>
<tr>
<td>$J_{11}$</td>
<td>cell–cell adhesion energy</td>
<td>20</td>
<td>—</td>
<td>estimated to give slightly adhesive cells</td>
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<td>$d_{\text{timestep}}$</td>
<td>timestep in PDE integrator</td>
<td>0.2</td>
<td>s</td>
<td>numerical stability versus efficiency</td>
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<td>$d_{\text{x}}$</td>
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<td>$1 \times 10^{-6}$</td>
<td>m</td>
<td>numerical stability versus efficiency</td>
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<tr>
<td>$p$</td>
<td>PDE iterations per MCS</td>
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<td>—</td>
<td>numerical stability versus efficiency</td>
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<td>$t_{\text{p}}$</td>
<td>time of increase of area by one lattice site</td>
<td>250</td>
<td>MCS</td>
<td>chosen arbitrarily</td>
</tr>
<tr>
<td>$A_{\text{p}}$</td>
<td>area of division</td>
<td>100</td>
<td>μm²</td>
<td>estimated</td>
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</table>
We next asked if this mechanism proposed in Nelson et al. [35] also suffices for branching morphogenesis. We initiated the model simulation with a disc-shaped structure of radius 0.225 mm in a lattice of 0.9 mm by 0.9 mm containing approximately 1000 cells. Figure 3 and electronic supplementary material, video S5 show a model simulation for the first 9000 MCS. A first look at the time series of the simulation shows that after approximately 1000 MCS the boundary of the disc becomes bumpy. Then around 3000 MCS, many droplet-like extensions appear. The length of these extensions increase and, as a result, a fully branched structure, with evenly thick branches is formed, that stabilizes around 8000 MCS. To test if and to what extent this result depends on the numerical resolution (Δx), we have repeated it for Δx = 5 × 10⁻⁷ m (electronic supplementary material, video S6) and for Δx = 2.5 × 10⁻⁷ m (electronic supplementary material, video S7), i.e. twice and four times the original resolution. At these refined resolutions, the simulations progressed more slowly due to the reduced length scale of the cellular extensions and retractions, but otherwise the results did not depend on the spatial resolution. We thus performed our parameter studies at Δx = 1 × 10⁻⁶ m for computational efficiency.

To quantify branching, we define the compactness of the morphology as $C = A_{\text{tissue}} / A_{\text{hull}}$ [43], i.e. the ratio between the area of the largest connected component of the tissue and the area of its convex hull, the smallest convex polygon that contains the tissue [45]. A compactness of 1 implies a perfectly circular tissue shape, whereas a low value of compactness indicated a high degree of branching or high degree of cell scattering. The compactness of the morphology rapidly drops during the first 5000 MCS and slowly decreases after that (figure 3b) indicating slow thinning of the branches.

Initially, the boundary of the circular tissue roughens due to random cell motility. While the secreted inhibitor accumulates at the concave locations of the morphology, it diffuses away more easily at the convex locations (figure 3a). As a result, cell motility is inhibited more strongly at the concave and flat locations. This concentration profile results in a lower compactness at the concave locations of the morphology, which further inhibits cell motility. This negative feedback loop leads to the formation of a branched structure that is stabilized by the accumulation of the secreted inhibitor at the concave locations of the morphology. The resulting pattern is characterized by evenly thick branches, which are formed due to the balance between cell motility and inhibitor accumulation. The compactness of the morphology is defined as $C = A_{\text{tissue}} / A_{\text{hull}}$, where $A_{\text{tissue}}$ is the area of the largest connected component of the tissue and $A_{\text{hull}}$ is the area of its convex hull. A compactness of 1 implies a perfectly circular tissue shape, whereas a low value of compactness indicates a high degree of branching or high degree of cell scattering. The compactness of the morphology rapidly drops during the first 5000 MCS and slowly decreases after that (figure 3b) indicating slow thinning of the branches.
regions than at the convex regions of the morphology, such as the branch tips. Thus the secreted inhibitor leads to a geometry-dependent rate of cell extension. Cell motility is strongly inhibited at the ‘valleys’ between the branches, but is more frequent at the sprout tips. This leads to a ‘ratchet’-type, dissipative branching mechanism: cells attempt to extend and retract randomly, and at sufficiently high temperatures they can do so even against the local energy gradient generated by the inhibitor (equation (2.2)). Because the secreted inhibitor dissipates at branch tips, such extensions against the energy gradients are more frequent at branch tips than in the ‘valleys’. To test if indeed branching is due to such a ‘ratchet’-like, dissipative mechanism, we measured the cumulative energy of the system $H_{\text{cum}}(t) = H(t) - H(0) = \sum_{i=0}^{t} \sum_{\text{MCS}} (\Delta H + H_{\text{work}})(i)$, where $\sum_{\text{MCS}}$ is the sum over all the accepted copy attempts with a Monte Carlo Step. Indeed $H_{\text{cum}}$ increases as a function of time (figure 3c), showing that for many of the moves $\Delta H > 0$ holds. Thus many of the moves act against a large positive energy contribution $H_{\text{work}}$, which is due to the inhibition of cell motility (equation (2.4)).

(d) Random motility regulates branch initiation and branching speed

The previous section showed that the proposed branching mechanism is driven by random cell motility. Interestingly, Btbd7, a positive regulator of epithelial cell motility, is required for branching morphogenesis of salivary glands and the lung [46], and is expressed in branching end buds in a variety of branching epithelial organs [47]. Knock-out of Btbd7 reduces epithelial cell motility in the end buds of submandibular salivary glands, leading to incomplete branching [47]. We thus hypothesized that the level of random cell motility should have similar effects in our mathematical model. The distribution of the magnitudes of the random forces exerted by the cell extensions and retractions is given by parameter $T$, 

\[
\text{compactness} = \frac{H(t) - H(0)}{C_0}
\]

\[
\text{energy spend by the system} \quad (H - H_0 = \sum \Delta H)
\]

Figure 3. Simulation of branching by autocrine inhibition. (a) Timelapse of a model realization—see also electronic supplementary material, video S5; (b) compactness as a function of time; shaded regions, standard deviations of 100 simulations; (c) energy spend by the system $(H - H_0 = \sum \Delta H)$ as a function of time; shaded regions, standard deviations of 100 simulations. (Online version in colour.)
the motility parameter, aka cellular temperature (equation (2.2)). Figure 4a shows morphologies for increasing values of T. Consistent with the inhibition or knockout of Btbd7 [46,47], at low values of T the tissue did not branch, because only a few random cell protrusions were strong enough to overcome the effect of the chemoinhibitor. For slightly higher cell motility, at cellular temperatures of around $T = 20$, the tissue developed droplet-like extensions: as soon as one or a few protrusions locally overcame the effect of the inhibitor the curvature locally increased leading to reduced levels of chemoinhibition. For elevated values of T, the tissue branched normally and the branches became longer and thinner than for lower values of the cellular temperature. Indeed the compactness of the morphologies formed after 20 000 MCS declined sharply for increasing values of T up to around $T = 20$ (figure 4b), reflecting that branching occurs from around $T = 20$. Figure 4c shows the compactness as a function of time for increasing values of T. For low values of T ($T = 18$ and $T = 20$), the compactness decreased slowly over time, but it did not reach a low compactness before the end of the simulation, while for higher values of T branching accelerates. We quantified speed of branching by measuring $t(C = 0.8)$, the time required for the tissue to drop below a compactness of 0.8 (dashed line in figure 4c). Figure 4d shows that the speed of branching quickly increases with the value of T and then saturates. In conclusion, consistent with experimental observation, the motility parameter T regulates the initiation and the speed of branching and has a small effect on branching morphology.

(e) Strength of autocrine inhibition has a biphasic effect on branching

The previous section showed how the cellular temperature affected branching dynamics. We next studied the effect of
the chemoinhibition strength $\chi$. Because the value of $\chi$ and $T$ both determine the probability that a cellular protrusion is accepted (see equations (2.2) and (2.3)), the effect of $\chi$ and $T$ are likely interrelated. Interestingly, the chemoinhibition strength (figure 5(a, b)) has a biphasic effect on branching. For relatively low values of $\chi = 50$, the morphology retained its circular shape and no branches formed (figure 5(c); the increased values of the branch length for low values of $\chi$ in figure 5(c) are due to artefacts of the skeletonization algorithm). For these low values of $\chi$ the impact of the autocrine signal was negligible, such that the dynamics was dominated by the ‘standard’ Hamiltonian (equation (2.1)). At higher values of $\chi$, the response to the autocrine inhibitory signal differed to a sufficiently large extent between concave and convex regions, such that the curvature effect set in and branches formed, as shown by the reduced compactness (figure 5(b)) and increased branch length (figure 5(c)). At even higher values of $\chi$, the branches became thinner and longer. As the chemoinhibition is active only at cell–ECM interfaces, at higher values of $\chi$ the chemoinhibition term became dominant over the other components of the Hamiltonian including the surface tension. This also allowed droplets to break off from the spheroid; this is an irreversible process, because the chemoinhibitory field made it energetically costly for such droplets to join the morphology again. For the largest values of $\chi$ tested, branches did not form, because cell protrusions became very costly. In conclusion, the chemoinhibition strength $\chi$ regulates the degree of branching in a biphasic manner.

(f) Decreasing surface tension promotes branching
In a study where lung epithelium was isolated in vitro, Hartmann & Miura [27] showed that a decrease in surface tension, by disruption of the cytoskeleton using cytochalasin D, results in more but smaller branches. Similarly, inhibiting cell contractility, thus reducing surface tension, promotes branching morphogenesis of the pancreas [19]. We therefore studied how the value of surface tension affected branching morphogenesis. In the CPM, surface tension $\gamma_0$ is defined as $\gamma_0 = \gamma_{01} - (J_{11}/2)$ [40], where $\gamma_{01}$ is the interfacial energy between cells and medium and $J_{11}$ is the interfacial energy between two cells. Figure 6 shows the effect of $\gamma_0$ on branching morphogenesis. In agreement with the experimental data, increasing the surface tension reduced the number of branches and yielded thicker branches. For very low values of $\gamma_0$ many thin branches appeared, which occasionally merged with one another. In simulations with increased surface tension the branches became thicker (figure 6(b)) and shorter (figure 6(c)). For the highest values of $\gamma_0$ tested, the branches became droplet-shaped and had more variable thickness than for lower values of $\gamma_0$. In agreement with Hartmann & Miura [27], these results illustrate that the surface tension $\gamma_0$ acts as a restoring force that counteracts the curvature effect due to chemoinhibition.

(g) Space-filling branching growth and branch avoidance in, presence of cell proliferation
The results above suggest that autocrine inhibition of random cell protrusion is sufficient for branching morphogenesis of epithelial cells, even in the absence of cell proliferation and the of
regulatory interactions with the mesenchyme. However, in vivo branching morphogenesis usually requires cell proliferation [18]. Therefore, we next investigated how the proposed branching morphogenesis mechanism behaved in the presence of cell proliferation. To mimic cell proliferation, the target area was incremented by 1 once every 100 MCS. Cells divided over their short axis after the actual area had reached a threshold value; thus pressure from adjacent cells inhibited proliferation. Figure 7 and electronic supplementary material, video S8 show a simulation initiated from a circular blob of proliferative cells. The insets in the final configuration highlight two proliferation events (purple and cyan dots). With proliferation, the mechanism produced a space-filling branching structure. The branches did not merge: branch tips that grew towards each other were repelled by one another (figure 7; black circle) or they were terminated. Such ‘self-loathing’ [48] of branches is due to the accumulation of the autocrine inhibitor between branches (figure 2g,h) and has been observed in ex vivo cultures of murine urinary collecting ducts [12] and mammary gland tissue [14]. Note that in our model, the auto-inhibition is responsible for the branching morphogenesis itself, and also gives rise to ‘self-loathing’ [48] of adjacent branches, leading to branch avoidance.

3. Discussion

Using mathematical modelling we have shown that the mechanism for geometry-dependent sprouting in tissue-engineered constructs of mammary epithelial cells proposed by Nelson et al. [35] suffices for autonomous branching morphogenesis of epithelial tissues, in the absence of cell proliferation and interaction with the surrounding mesenchymal tissues. Importantly, and in contrast to related models based on Laplacian growth principles, the present model produces sprouts and branching structures in the absence of proliferation. The model suggests that branching morphogenesis can occur due to a ‘ratcheting’ mechanism, which favours random cellular protrusions at convex locations of the morphology over protrusions at concave locations. The present model derives conceptually and methodologically from our previous work on angiogenesis [43], in particular from the model variant based on ‘extension-only chemotaxis’ (see § ‘A Dissipative Sprouting Mechanism’ in Merks et al. [43]). The key difference with this previous work is that the autocrine chemoinhibition mechanism studied here is based on the concentration of \( c(x) \) (equation 2.3), whereas the chemotactic mechanisms studied previously relied on chemical gradients \( \nabla c(x) \). This small difference has a large effect on the patterns predicted by the model: the present mechanism explains the formation of self-avoiding branching patterns, as observed in many epithelial-derived branching organs, whereas the previous work predicted the formation of network-like patterns, such as those found in microvasculature. In the presence of proliferation our model is similar to previous models based on Laplacian growth. In these models, tissues branch due to a Mullins–Sekerka instability, where positive curvatures experience higher gradients of a pressure field [24], a growth-promoting [27] or a growth-inhibiting field [39]. In particular, the latter model by the Sneppen group [39], which is based...
on growth-inhibition, resembles ours. However, our model differs from it in an essential aspect: our model can explain branching morphogenesis in absence of proliferation.

The assumptions of the model are simple, but plausibly based on experimental observation [35], and its predictions agree surprisingly well with some observations of branching epithelial organs. At the same time, there are of course many observations that our model cannot explain and that will open up new perspectives for future modelling studies. In particular, observations of renal epithelial cells challenge the hypothesis of autocrine inhibition of motility studied in this work [49]. Renal epithelial cells grown on micropatterns of specific curved geometry exhibit curvature-dependent protrusions. These are potentially regulated by autocrinically secreted BMP7, a member of the TGF-β superfamily. To test this hypothesis, Martin et al. [49] applied a flow to the culture medium that should flush away any diffusive signals potentially secreted by the cells. Surprisingly, this treatment did not affect the curvature-dependent protrusions. The authors suggested that membrane tension might be responsible for the curvature effect instead. Pavlovich et al. [36] argued that the autocrine inhibition mechanism may uniquely apply to mammary epithelial patterning. An alternative explanation for the observations by Martin et al. [49] may be that autocrinically secreted signals are bound to secreted ECM proteins, which would prevent them from being flushed away. Such pericellular retention of signalling molecules has been observed for VEGF in endothelial cells [50]. Indeed the activity of TGF-β activity is likely regulated by the chemistry and the mechanics of the ECM. TGF-β is bound to the ECM in a latent form. The active moiety can be released from the ECM through proteolysis [51] or through mechanical stretching of TGF-β [52]. Further supporting the importance of the ECM in branching morphogenesis, reducing the cytoskeletal tension (likely reducing the cellular traction forces exerted on the ECM) reduces the number of branches formed in embryonic lung explants [53,54]. In our previous work, we have modelled how TGF-β release from fibrin matrices can regulate angiogenic sprouting [55], and how mechanical cell-ECM interactions can coordinate pattern formation and sprouting in endothelial cell cultures [56,57]. In our ongoing studies, we are incorporating these two approaches into our models of epithelial branching. These model extensions may provide deeper insight into the role of the ECM in branching morphogenesis. Apart from the leads provided by the observations of Martin et al. [49], another direction for further research may consider TGF-β release through ECM proteolysis and mechanical straining of the ECM by the epithelial cells. Possibly such models of the mechanochemical cross-talks in TGF-β signalling can help reconcile some of the experimental observations on the role of membrane tension and mechanical force generation in branching morphogenesis.

Another lead that may provide new insights into branching morphogenesis concerns the analogy with scratch assays for wound healing. In such assays, epithelial cells migrate into a free surface left open by the scratch [58,59]. During closing of the scratch, the epithelial cells organize into finger-like structures, which will eventually merge to close the free space.
The finger-like structures extending from the boundary look similar to initial branching structures, and like in branching, proliferation does not seem to be the driving factor in finger-like structures extending from the boundary. In a model by Mark et al. [62], the Mullins–Sekerka instability arises from a curvature-dependent cell motility. In a follow-up paper which included velocity alignment of cells [61], it was shown that the thickness of the fingers depends on the length scale on which the cells in the bulk align their velocities. Indeed, epithelial cells in fingers are observed to move together in a highly coordinated fashion [59]. The relationship between motility and curvature was based on observations where epithelial cells have a higher protrusion rate at convexly curved surfaces [63].

For computational efficiency, the analyses in this work are based on two-dimensional simulations. However, it is straightforward to extend the model to three dimensions (see electronic supplementary material, video S9 for a three-dimensional example with proliferation). Our two-dimensional simulations represent quasi-two-dimensional cultures, e.g. mammary epithelial in thin fibrin gels [64] or kidney rudiments cultured on filters supported by Trowell screens [12]. To represent such quasi-two-dimensional situations, we have assumed that the decay of the autocrine signal only takes place outside of the cells and the signal only affects cell motility at the periphery. An alternative interpretation of a two-dimensional model could be the projection of the three-dimensional case in two dimensions. In this interpretation of the two-dimensional model, we should also consider the degradation of the signal underneath the cells.

The present model is of course a great simplification of epithelial branching morphogenesis in vivo. Epithelial morphogenesis involves interactions of many signalling molecules and receptors from the epithelium and mesenchyme and, despite some similarities, there are large differences between organs of epithelial origin. The model generates variable branching patterns such as in mammary epithelial tissues [13,14], whereas other organs such as lung and kidney display highly stereotypic, reproducible patterns of branching [8,28]. Future work could explore the hypothesis that additional signalling molecules and the interaction with the surrounding mesenchyme and the ECM could fine-tune generic branching mechanisms such as the one presented here, leading to more stereotypical branching.

4. Methods

(a) Model implementation

The model was implemented using the Tissue Simulation Toolkit (http://sourceforge.net/projects/tst). The PDE is solved by using a forward Euler method on a regular square lattice matching that of the CPM, i.e. $\Delta x = 2\,\mu m$. The CPM and PDE are coupled using an operator splitting approach: after running the CPM for one Monte Carlo step, 15 of the numerical integration steps are performed with $\Delta t = 0.2\,s$, such that the PDE runs for $t_c = 3\,s$ per MCS. As initial conditions for the PDE, we assume $c(x,0) = 0$ for all $x$. We used zero boundary conditions, i.e. $c(x, t) = 0$ at all boundaries.

(b) Morphological measures

The morphologies were characterized using the following morphometric measures.

(i) Compactness

The compactness is defined as the ratio between $A$, the domain covered by the tissue, and the area of its convex hull $A_{\text{hull}}$ [45]:

$$C_{\text{comp}} = \frac{A_{\text{basic}}}{A_{\text{hull}}}.$$  \hspace{1cm} (4.1)

The convex hull is the smallest convex polygon that encloses the object of interest. A compactness of $C_{\text{comp}} = 1$ indicates a convex tissue shape, whereas a lower value of the compactness indicates branching or cell scattering.

(ii) Branch length

In order to find the branches of the structure, we generate the morphological skeleton of the tissue [65,66]. Using this skeleton image, we calculate the length of the branches as follows. For every edge, the two nodes of the edge are removed from the skeleton image by removing all lattice sites around the nodes with increasing radius, until a radius $w$ is found such that the skeleton image is divided into at least two separate components, of which one is the edge of interest. The length of a branch is then determined by counting the pixels that make up the branch and adding twice the radius $w$ to the final result.

(iii) Branch thickness

To calculate branch thickness, we adopted an approach by Filipov et al. [67]. We take $a$ to be the image of the tissue and let $b(r)$ be a disc $b(r) = (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq r$ with variable radius $r$. The branch thickness can now be defined as the value of $r$ for which branches disappear out of the morphological opening $a \ast b(r)$.

The area of the morphological opening decreases with the radius $r$ (figure 8), because more branches disappear from the image with increasing $r$. We approximate the branch thickness by finding a point where this graph decreases sufficiently fast and then becomes flat, indicating that most branches have disappeared. At some point the graph becomes more or less horizontal. This region corresponds to the circular part of the tissue, in which many circles $b(r)$ fit. So, the value for $r$ for which the graph becomes horizontal indicates the thickness of the branches. We detect this horizontal region by first finding a region where the graph decreases sufficiently fast and then a region where it decreases slowly. Let $M_a(r)$ be the area of $a \ast b(r)$. We find an $0 < r_1 \leq r_{\text{max}}$ for which $M_a(r_1) - M_a(r_1 - 1) < a_1$ and then the value $r_1 < r_2 \leq r_{\text{max}}$ for

![Figure 8. Example of the radius $r$ of a branching morphology as a function of the area of the morphological opening $a \ast b(r)$. (Online version in colour.)](image-url)
which $M_{r_2}(r) - M_{r_1}(r_1 - 1) > a_2 (r_2 \text{ is set to } r_{\text{max}} \text{ if such a value does not exist}).$ The branch thickness is then found by taking the value of $r$ for which $M_{r_1}(r) \text{ is closest to } \frac{1}{2} (M_{r_1}(0) + M_{r_1}(2 - 1)).$ The values of $a_1$ and $a_2$ are determined empirically. The value of $r_{\text{max}}$ is set to 30 to reduce computation time.

In case no branches or only very small branches are present ($M_{r_1}(r) - M_{r_1}(r - 1) \leq a_1$ for all $0 < r \leq r_{\text{max}}$) we apply the following algorithm. When the decrease in $M_{r_1}(r)$ is not larger than $-a_1$ in the entire graph we simply take the distance from the centre of mass of the tissue to an ECM point in four different directions and select the lowest distance as the radius. In this case, the radius represents the radius of the unbranched cell aggregate but we take this as the branch thickness. We follow this approach, because increasing the radius to the width of the initial circular tissue (typically more than twice as large as $r_{\text{max}} = 30$) and repeatedly computing $M_{r_1}(r)$ would require excessive computation time.

Data accessibility. The simulations have been performed with a version of the Tissue Simulation Toolkit (https://sourceforge.net/projects/tst/).

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