

SIXTEENTH CONFERENCE
ON THE
MATHEMATICS OF OPERATIONS RESEARCH



CONFERENCE CENTER 'DE BLIJE WERELT'
LUNTEREN, THE NETHERLANDS

JANUARY 16-18, 1991

Organized by
CWI (Centrum voor Wiskunde en Informatica),
Amsterdam, The Netherlands

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AIM AND SCOPE

The aim of the conference is to promote the research activities and the cooperation between researchers in the mathematics of operations research.

An important theme of the conference will be stochastic financial models. There will be a minicourse on this subject, which will be given by A.C.F. Vorst (EU Rotterdam), M.C.A. van Zuylen (KU Nijmegen) and A. Eikeboom (Massachusetts Institute of Technology, Cambridge USA). Eikeboom will also deliver a lecture on this subject in the main program, and one of the lectures of S. Zenios will also tie in with this subject. The second lecture of Zenios will be about the use of parallel and supercomputers in Operations Research.

Furthermore, three other non-Dutch specialists have been invited to give two lectures on recent developments in their field of interest. They have been asked to present a tutorial survey of their area in the first talk and discuss their own recent work in the second lecture. I. Mitrani (Newcastle) and Ph. Nain (INRIA) will discuss subjects from the stochastic side of Operations Research, more in particular about queueing theory. Y. Greenberg (Montreal) will speak about the theory of social situations, a game-theoretic subject.

The program should give ample opportunity for informal discussions.

ORGANIZATION

Organizers

The conference is organized by CWI (Centrum voor Wiskunde en Informatica / Centre for Mathematics and Computer Science) in Amsterdam, in particular by O.J. Boxma and B.J. Lageweg.

Sponsors

The conference is organized under the auspices of the Dutch Research Community in the Mathematics of Operations Research and System Theory, with financial support of the Dutch Mathematical Society and the Netherlands Society of Operations Research.

PROGRAM

Invited speakers

A.M. Eikeboom (Massachusetts Institute of Technology, Cambridge, USA):

1. Order imbalance and order dependence on the Paris Bourse

Y. Greenberg (McGill University, Montreal, Canada):

1. The theory of social situations: concepts
2. The theory of social situations: applications to game theory

I. Mitrani (University of Newcastle, Newcastle, U.K.):

1. Tandem queueing models of manufacturing systems
2. An asymptotically optimal protocol for high speed data networks with small buffers

Ph. Nain (INRIA, Paris, France):

1. On the Erlang traffic model: equilibrium and transient analysis, stochastic comparisons, optimization issues and computational aspects
2. Recent results for the optimal control of simple queueing systems

S. Zenios (Wharton School, Philadelphia, USA):

1. Advances in parallel and supercomputing applications in Operations Research
2. Planning under uncertainty with mortgage-backed securities

Minicourse: Option pricing and optimal portfolio selection

M.C.A. van Zuijlen (Catholic University, Nijmegen):

Option pricing and Brownian motion

A.C. F. Vorst (Erasmus University, Rotterdam):

Optimal consumption and portfolio selection

A.M. Eikeboom (Massachusetts Institute of Technology, Cambridge, USA):

Recent developments in the theory of optimal consumption and portfolio selection

TIME SCHEDULE

Wednesday January 16, 1991

11.30 Opening
11.40 Mitrani (1)
12.30 *Lunch*
15.00 Minicourse (1): van Zuijlen
15.50 *Tea break*
16.20 Greenberg (1)
17.10 Minicourse (2): Vorst
18.30 *Dinner*

Thursday January 17, 1991

8.00 *Breakfast*
9.00 Nain (1)
9.50 *Coffee break*
10.20 Minicourse (3): Eikeboom
11.10 Greenberg (2)
12.30 *Lunch*
15.30 *Tea break*
16.00 Zenios (1)
16.50 Mitrani (2)
18.30 *Dinner*

Friday January 18, 1991

8.00 *Breakfast*
9.00 Eikeboom (1)
9.50 *Coffee break*
10.20 Nain (2)
11.10 Zenios (2)
12.00 Closing
12.30 *Lunch*

RECENT DEVELOPMENTS IN THE THEORY OF
OPTIMAL CONSUMPTION AND PORTFOLIO SELECTION

Arnout M. Eikeboom

Sloan School of Management
Massachusetts Institute of Technology
Cambridge
USA

We present the Cox-Huang approach to finding the optimal consumption and portfolio-selection rules in the continuous-time model. This method is based on martingale theory rather than dynamic programming. The formulation explicitly includes the restrictions that consumption and wealth must be non-negative. The derivation of the Cox-Huang technique uses a particular optimal portfolio called the 'growth-optimum' portfolio.

We present the properties of this portfolio and we apply the method to finding the optimal portfolio rules when the nonnegativity constraint on consumption is binding.

ORDER IMBALANCE AND ORDER DEPENDENCE
ON THE PARIS BOURSE

Arnout M. Eikeboom

Sloan School of Management
Massachusetts Institute of Technology
Cambridge
USA

We investigate the balance and independence of sequences of buy and sell orders which result in transactions of individual stocks on the Paris Stock Exchange. Tests of binomial proportions demonstrate that imbalance in the daily flows of buy and sell orders is often highly statistically significant. Also, using exact distribution theory for runs we find highly significant *positive* dependence in many daily sequences of buy and sell orders for individual stocks. Likelihood-ratio tests based on Markov chain models confirm these conclusions. Implications for market efficiency are briefly discussed.

THE THEORY OF SOCIAL SITUATIONS:
CONCEPTS

Yossi Greenberg

McGill University
Montreal
Canada

The theory of social situations is a new and integrative approach, in the spirit of game theory, to the study of formal models in the social and behavioral sciences. The theory unifies both the representation of a social environment - as a *situation*, and the equilibrium concept - requiring that the 'standard of behavior be stable'. A *situation* specifies all the relevant aspects for rational decision making, such as the existing institutions and the negotiation process. It comprises the two notions: *position* and *inducement correspondence*. A *position* describes 'the current state'. It specifies the set of players, the set of feasible *outcomes* and the preferences of the individuals over this set. A (social) *situation* is a pair (g, G) , where G is a collection of positions and g is the inducement correspondence which specifies for each group of players the positions it can induce from any given position in G , when a particular outcome in that position is recommended. Moreover, all of the induced positions must also belong to G .

Stability requires that the recommended outcomes made to the players be both 'internally and externally stable'; the recommendations cannot be self-defeating and, at the same time, should not be arbitrary - they should account for alternatives that were not recommended.

Thus, the unified description of the social environment as a situation is completely detached from the unified stability criterion for the recommended outcomes. In addition, the theory of social situations:

- naturally accommodates coalition formation;
 - sheds new light on and relates many of the currently disparate game-theoretic solution concepts;
 - suggests new and interesting solutions;
 - takes most of the "rationality requirements" away from the players and puts them on the recommendations;
 - allows for bounded rationality concerning both the perception and the computation abilities of the players; and, finally,
 - the theory is simple and intuitively appealing.
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THE THEORY OF SOCIAL SITUATIONS:
APPLICATIONS TO GAME THEORY

Yossi Greenberg

McGill University
Montreal
Canada

The following are some of the results obtained from applying the theory of social situations to game theory.

I. Extensive form (tree) games:

Result 1: The set of subgame perfect equilibria paths is the unique maximal stable standard of behavior (*SB*). Moreover, stability may refine perfection, (and all of the other refinements), in an appealing way.

II. Games in coalitional (characteristic function) form:

Result 1: The core is the unique stable *SB* if the negotiation process is such that dissatisfied players 'leave the rest of the players never to negotiate with them again'.

Result 2: A set K is a von Neumann and Morgenstern solution (stable set) if and only if it is a stable *SB* when coalitions can counterobject, (using feasible payoffs).

Result 3: The unique stable *SB* when players 'update their reservation payoffs', yields the *stable bargaining set*, which is always nonempty.

III. Normal (strategic) form games:

Result 1: The set of Nash equilibria is the unique stable *SB* when the beliefs of each player is that all other players will adhere to their strategies. [Beliefs are part of the description; the equilibrium is the stable *SB*.]

Result 2: The set of coalition-proof Nash equilibria is the unique stable *SB* when the beliefs of each coalition is that nonmembers will adhere to their strategies.

New solution concepts arise from the stable *SB* that results from *open negotiations*, where players can either commit to playing, unconditionally, a given strategy, or make *contingent threats* of the form 'if nonmembers follow the proposed strategies, then we shall...'

IV. Implementation:

Result 1: A social choice rule is strongly monotonic if and only if it can be implemented by a situation where only 'credible objections' can be made.

TANDEM QUEUEING MODELS OF MANUFACTURING SYSTEMS

Isi Mitrani

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Production lines consisting of a number of service cells in sequence are examined in the steady-state. A cell receives its input from its left-hand neighbour (or from the outside world if it is first in the line), and passes its output to the right-hand one (or to the outside world if it is last). Moreover, the storage capacity of each cell is finite, so that jobs may be blocked when attempting to enter it. The first cell cannot be starved of raw materials, but finished products may or may not be blocked in the last cell. Several control strategies are described and compared. Special emphasis is placed on the 'kanban' policy, which is an exponent of the 'just-in-time' philosophy in manufacturing.

A stochastic model of the production line is constructed and analysed. An approximate evaluation method is developed, based on examining first a single cell in isolation and then combining the isolated cells through fixed-point equations. Both 'closed' and 'open' versions of the model are considered, distinguished by the treatment of the output from the last cell. When there is never any shortage of demands for finished products, i.e. when the latter are removed from the system as soon as they are ready, the model is closed and its state space is finite. On the other hand, when demands arrive at random and may be queued, the model is open and the question of ergodicity arises. The open cell has to be analysed separately.

The problem of optimising the allocation of space among the cells is also discussed.

REFERENCES

- Mitra, D. and Mitrani, I., Analysis of a Kanban Discipline for Cell Coordination in Production Lines, I. *Management Science*, (to appear).
Mitra, D. and Mitrani, I., Analysis of a Kanban Discipline for Cell Coordination in Production Lines, II: Stochastic Demands. *Operations Research*, (to appear).
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AN ASYMPTOTICALLY OPTIMAL PROTOCOL
FOR HIGH SPEED DATA NETWORKS
WITH SMALL BUFFERS

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As the transmission speeds of modern data networks increase, the propagation delays, which do not scale, become quite important for the design of flow control protocols. It has been shown that, with a sliding window control policy, the optimal window size increases linearly with the transmission speed. This makes the cost of memory for buffers a major factor in the protocol design. However, it has also been shown that the mean and standard deviation of the number of packets in a buffer increase only as the square root of the transmission speed. This fact suggests that the network can be implemented using small buffers, and still achieve a high ratio between the realised throughput and the ideal one. In other words, small but appropriately sized buffers overflow so rarely that even with a simple-minded protocol like go-back-n, the loss of throughput due to retransmissions is negligible.

The above result is established by an asymptotic analysis of the tail of the unbounded buffer occupancy distribution, as the transmission speed approaches infinity. That tail distribution is then used to estimate the finite buffer overflow probability and hence the realised throughput. Numerical and simulation results are also presented.

REFERENCES

Mitra, D. and Mitrani, I., Asymptotic Optimality of the Go-Back-N Protocol in High Speed Data Networks with Small Buffers. *Procs., 4th International Conf. on Data Comm. Systems and Their Performance*, Barcelona, June 1990, pp. 17-31.

ON THE ERLANG TRAFFIC MODEL:
EQUILIBRIUM AND TRANSIENT ANALYSIS,
STOCHASTIC COMPARISONS, OPTIMIZATION ISSUES AND
COMPUTATIONAL ASPECTS

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The Erlang traffic model - studied by A. K. Erlang in 1917 - is one of the oldest, most basic models in teletraffic. In its simplest version, it is an $M/M/N/N$ queueing system: the offered traffic is Poisson and the service is provided by N trunks, holding times are i.i.d. exponentially distributed, and blocked calls are lost (no queueing). Over 70 years later and in spite of its apparent simplicity, easy extensions of this model are still the object of ongoing research!

This research is of increasing importance for the performance evaluation studies of computer communication networks (e.g., ISDN). In this lecture, classical and recent results on circuit-switched networks will be reviewed, including equilibrium (steady-state blocking probabilities) and asymptotic results (blocking probabilities at time t for large N), stochastic comparisons (monotonicity properties), optimization issues (stochastic knapsack problem, routing schemes) and computational aspects (computation of the partition function). The discussion will emphasize the mathematical tools involved in this research, including reversibility and insensitivity theory, stochastic orderings, pathwise arguments and large deviation theory.

RECENT RESULTS FOR THE OPTIMAL CONTROL
OF SIMPLE QUEUEING SYSTEMS

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The goal of a typical stochastic control problem is to locate a policy which optimizes (maximizes or minimizes) the expectation of a global objective. The global function is a function of the state of the system, as it evolves stochastically in time, and of the chosen policy. Many stochastic control problems (inventory control, routing, flow control, priority assignment, etc.) in a variety of fields (operation researches, computer networks, etc.) actually reduce to Markov Decision Problems (MDP's) or Semi-Markov Decision Problems (SMDP's). If so, the Dynamic Programming (DP) equation or Bellman's equation provides a general tool for determining the optimal control. However, in most cases of interest the solution to the DP equation cannot be obtained analytically, which in turn implies that the optimal policy cannot be determined from the explicit solution of the DP equation. An alternative approach - besides numerical methods that are not feasible in general due to computational problems - is to show that expected properties of the objective function (monotonicity, convexity, concavity, submodularity, etc.) propagate under the DP operator. If so, the objective function inherits these properties, which may yield the optimal control (e.g., threshold policies or switching curves in queueing systems). However, this method - known as the value iteration algorithm - fails to work (at least from an analytic point of view) when the objective function does not possess 'nice properties', or when these properties cannot be used to determine the optimal control.

Recently, J.P.C. Blanc, P. de Waal, P. Nain and D. Towsley have proposed an alternative approach to the value iteration algorithm. This approach, which is based on a partial construction of the solution of the DP equation, has already been fruitfully applied to several stochastic control problems, including flow control problems and the optimal control of vacation schemes in simple queueing systems. This lecture will discuss the basic features of this new device through a number of examples.

OPTIMAL CONSUMPTION AND PORTFOLIO SELECTION

Ton Vorst

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The selection of optimal security portfolios is a longstanding problem in financial theory and goes back to the work of Markowitz for one period models. Merton, in the early seventies, derived optimal portfolio selection rules in a continuous-time framework. Assuming that the asset prices follow geometric Brownian motions, as was also assumed in the derivation of the Black-Scholes formula for option prices, he also derives mutual fund theorems, as Markowitz had in the one period case. The main technique that is used in deriving the optimal selection rules and the mutual fund theorem is stochastic dynamic programming.

In this talk we will elaborate on the assumptions in the portfolio selection models and the interpretation of the results.

ADVANCES IN PARALLEL AND SUPERCOMPUTING
APPLICATIONS IN OPERATIONS RESEARCH

Stavros A. Zenios

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Parallel and supercomputing designs are having a significant impact on the size and scope of operations research models. Models that were considered intractable at the beginning of the decade are now solved within seconds of real time. We have seen the solution of linear and nonlinear models with millions of variables.

In this tutorial we target three objectives.

- (1) Survey diverse OR applications where parallel computing is already playing - or could potentially play - a key role. Such applications include financial modeling under uncertainty, production and distribution, transportation, and the estimation of social accounting tables.
- (2) Discuss the major issues in designing and analyzing parallel solution techniques, like decompositions and relaxations.
- (3) Illustrate the potential impact of the technology with current experiences on the use of vector supercomputers like the CRAY Y-MP or the massively parallel Connection Machine CM-2 with 65536 processing elements.

A general reference on parallel optimization methods is: S.A. Zenios, *Parallel Numerical Optimization: Current Status and an Annotated Bibliography*. *ORSA J. on Computing*, 1, 1989, pp. 20-43. Additional references will be distributed at the conference.

PLANNING UNDER UNCERTAINTY WITH
MORTGAGE-BACKED SECURITIES

Stavros A. Zenios

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and Simulation
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We will discuss three aspects of a model for the valuation and portfolio management of mortgage-backed securities:

- (1) a statistical model for the estimation of prepayments,
- (2) option adjusted spread methodology for valuation, and
- (3) cash-flow matching under uncertain interest rate scenarios via stochastic programming.

The option adjusted spread methodology and the stochastic programming models are extremely compute-intensive. We will discuss the role of parallel computing technology in building and solving these models. Progress made at the HERMES laboratory using a distributed system of DEC workstations and a massively parallel Connection Machine will be highlighted.

Two technical reports are available and will be distributed at the conference.

OPTION PRICING AND BROWNIAN MOTION

Martien C.A. van Zuijlen

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Uncertainty and its modeling with the aid of probability theory have played an important role in financial economics. Already Bachelier (1900) introduced in his thesis a stochastic process (now known as Brownian motion) in order to describe the erratic behavior of stock prices over time and to derive theoretical values of various options, for example the European call option. Samuelson (1965) replaced the ordinary Brownian motion by the geometric Brownian motion, which led to the first rigorous formulation of the option pricing problem and to the celebrated option pricing formula of Black and Scholes (1973) and Merton (1973).

We will:

- a. discuss how Harrison and Kreps (1979) and, in particular, Harrison and Pliska (1981) have made Black and Scholes' somewhat heuristic arguments rigorous by introducing the theory of semimartingales and stochastic integration (Ito) as a good setting for modeling continuous trading;
- b. indicate how in a finite securities market model the concepts of *no arbitrage* and *completeness* can be studied and characterized;
- c. show how the Black and Scholes pricing formula can be obtained as a limit of the pricing formula in the binomial finite security market model.

Finally, some recent developments in the approximation theory of continuous securities market models will be mentioned briefly.

