# FIFTEENTH CONFERENCE ON THE MATHEMATICS OF OPERATIONS RESEARCH

CONFERENCE CENTER 'DE BRON'
DALFSEN, THE NETHERLANDS

JANUARY 15-17, 1990

Organized by the
Centre for Mathematics and Computer Science,
Amsterdam, The Netherlands

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## AIM AND SCOPE

The aim of the conference is to promote the research activities and the cooperation between researchers in the mathematics of operations

The main theme of the conference will be interior point methods. There will be a minicourse on this subject, which will be given by J.F. Ballintijn (KSLA Amsterdam), C. Roos (TU Delft) and A. Schrijver (CWI Amsterdam). Four specialists from abroad will also tie in with this subject. From them, K.M. Anstreicher (USA) and C.C. Gonzaga (Brasil) will present overviews of interior point methods, M.J. Todd (USA) will speak about a variant of Karmarkar's algorithm and J.Ph. Vial (Switzerland) will show an application.

Furthermore, three non-Dutch specialists have been invited to give two lectures on recent developments in their field of interest. They have been asked to present a tutorial survey of their area in the first talk and discuss their own recent work in the second lecture. Ph. Flajolet (Paris) will discuss stochastic analysis of algorithms, D. Towsley (Amherst) will speak about queueing networks and W.J. Cook (Morristown) will give lectures about combinatorial optimization subjects.

The program should give ample opportunity for informal discussions.

## ORGANIZATION

*Organizers*

The conference is organized by the Centre for Mathematics and Computer

Science in Amsterdam, in particular by O.J. Boxma, H.C. Tijms and B.J. Lageweg.

*Sponsors*
The conference is organized under the auspices of the Dutch Research Community in the Mathematics of Operations Research and System Theory, with financial support of the Dutch Mathematical Society and the Netherlands Society of Operations Research.

PROGRAM

*Invited speakers*

K.M. Anstreicher (Yale School of Organization and Management, USA):
Interior algorithms for linear programming since 1984

W.J. Cook (BellCore, USA):
1. Polyhedral methods in combinatorial optimization
2. Solving general integer programming problems

Ph. Flajolet (INRIA, France):
1. Recent trends in the average-case analysis of data structures (1)
2. Recent trends in the average-case analysis of data structures (2)

C.C. Gonzaga (COPPE-Federal University of Rio de Janeiro, Brasil):
An overview of $O(\sqrt{n}L)$-iteration algorithms for linear programming

M.J. Todd (Cornell University, USA):
A Dantzig-Wolfe-like variant of Karmarkar's interior-point linear programming algorithm

D. Towsley (University of Massachusetts, USA):
1. An introduction to optimization and control of queueing networks
2. Scheduling policies for real-time and parallel processing systems

J-Ph. Vial (University of Geneva, Switzerland):
Central planners should use central prices

*Minicourse: Interior point methods*

A. Schrijver (CWI, Amsterdam):
The algorithm of N. Karmarkar for linear programming

C. Roos (University of Technology, Delft):
Polynomial-time algorithms for linear programming based on the use of the logarithmic barrier penalty function

J.F. Ballintijn (KSLA, Amsterdam):
Implemenentation aspects and performance results of the dual affine algorithm

TIME SCHEDULE

*Monday January 15, 1990*

| | |
|---|---|
| 11.30 | Opening |
| 11.40 | Minicourse (1): Schrijver |
| 12.30 | *Lunch break* |
| 15.00 | Towsley (1) |
| 15.50 | *Tea break* |
| 16.20 | Flajolet (1) |
| 17.10 | Anstreicher |
| 18.30 | *Dinner* |

*Tuesday January 16, 1990*

| | |
|---|---|
| 9.00 | Minicourse (2): Roos |
| 9.50 | *Coffee break* |
| 10.20 | Cook (1) |
| 11.10 | Minicourse (3): Ballintijn |
| 12.30 | *Lunch break* |
| 15.00 | Flajolet (2) |
| 15.50 | *Tea break* |
| 16.20 | Towsley (2) |
| 17.10 | Gonzaga |
| 18.30 | *Dinner* |

*Wednesday January 17, 1990*

| | |
|---|---|
| 9.00 | Todd |
| 9.50 | *Coffee break* |
| 10.20 | Cook (2) |
| 11.10 | Vial |
| 12.00 | Closing |
| 12.30 | *Lunch break* |

# INTERIOR ALGORITHMS FOR LINEAR PROGRAMMING SINCE 1984

Kurt M. Anstreicher

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This talk presents an overview of the recent development of interior algorithms for linear programming, beginning with the announcement of Karmarkar's algorithm in 1984. Historical and mathematical relationships between various algorithms, including projective, affine scaling, path following and scaled potential methods, are described. Issues of theoretical complexity versus practical efficiency, and ease of implementation, are also considered. The talk ends with the mention of some promising directions for future work.

# IMPLEMENTATION ASPECTS AND PERFORMANCE RESULTS OF THE DUAL AFFINE ALGORITHM

Koos Ballintijn

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After a global discussion of the dual affine algorithm, the lecture will focus on the number of steps in the method, which will play an important role in the correct functioning of the algorithm. Topics which will be discussed are:

a) Problem reduction;
b) Symbolic $AD^2A^T$ calculations;
c) Ordering algorithms and symbolic Cholesky factorization;
d) Calculation of search directions and primal variables.

Extra attention will be given to the treatment of ranges and bounds. Also the use of artificial bounds to create a feasible starting point will be discussed.

In the final part of the talk numerical results obtained so far will be compared with the performance of commercial Simplex LP solvers like XMP and MPSX.

This is joint work with T.M. Doup (KSLA) and G.S. Pierce (SIPM).

# POLYHEDRAL METHODS IN COMBINATORIAL OPTIMIZATION

Bill Cook

Bell Communications Research
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Morristown, NJ 07960
USA

The basic goal of polyhedral combinatorics is to apply the linear programming dual equation

$$\max\{wx: Ax \leq b\} = \min\{yb: yA = w, \ y \geq 0\}$$

to combinatorial problems. We illustrate the basic techniques with geometric versions of matching and travelling salesman problems, and discuss some recent work on the use of polyhedral methods on large-scale optimization problems.

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Cook 2

# SOLVING GENERAL INTEGER PROGRAMMING PROBLEMS

Bill Cook

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Many advances have been made in solution techniques for specially structured integer programming problems, such as travelling salesman, max-cut, and fixed-charge problems. With respect to these advances, very little progress has been made on solving general (mixed) integer problems. This, of course, is not true when viewed from the theorectical side: H.W. Lenstra (1981) made a major breakthrough, obtaining a polynomial-time algorithm when the number of integer variables is fixed. We discuss a practical implementation of a Lenstra-like algorithm, based on a general basis reduction method of Lovasz and Scarf (1988), and report on the solution of a number of small (but difficult) examples. For problems with up to 100 integer variables, the method seems to compare favorably with existing branch and bound codes. This talk is based on joint work with Herb Scarf.

# RECENT TRENDS IN THE AVERAGE CASE ANALYSIS OF DATA STRUCTURES

Philippe Flajolet

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Knuth has shown how a large number of algorithms and data structures of fundamental use in computer science can be analyzed in the average case. A typical paradigm decomposes an analysis into setting a collection of related combinatorial counting problems, finding the solutions to recurrence relations that express such counting problems, and last performing asymptotic evaluations. The method is especially fruitful when applied to problems of *low* (polynomial) computational complexity, for instance sorting and searching algorithms.

These two talks will discuss recent mathematical approaches to these problems of average case analysis in the domain of *decomposable* data structures.

First, the emergence of symbolic methods in combinatorial counting avoids recurrences and permits us to derive directly generating function equations from specifications, for a large class of combinatorial counting problems. In this way, we can compile specifications into functional equations of various forms.

Second, we know from classical analytic number theory that singularities of functions bear strong relations to the asymptotic form of their coefficients. A similar *transfer* can be achieved here in many cases of interest for functional equations arising in the analysis of many computer algorithms.

Mathematically, this chain allows us to bypass the stage of explicit solutions to recurrences (we may not have closed forms), and it leads naturally to general and synthetic observations on the probabilistic properties of many classical combinatorial structures.

Computationally, it is also systematic enough. We will show how to use the facilities offered by computer algebra systems in order to develop an automatic analyzer that implements several of the combinatorial and asymptotic tools described above.

# AN OVERVIEW OF $O(\sqrt{n}L)$-ITERATION ALGORITHMS FOR LINEAR PROGRAMMING

Clovis C. Gonzaga

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Karmarkar's method is a primal algorithm that solves the linear programming problem in $O(nL)$ iterations, each of them including the projection of a vector on the null space of a scaled constraint matrix. This complexity was reduced to $O(\sqrt{n}L)$ by Renegar in 1986, who devised the first method based on following the central trajectory for the problem. A large number of path-following methods were later developed, all with this low complexity, but unfortunately also all dependent on following the path by short steps.

Finally now there is a new batch of methods that achieve the low complexity without the restriction to short steps. These methods either rely on a parameter whose updates depend on approaching the central path (and can thus be considered as path-following algorithms), or solve the primal-dual problem and do not care for the central path.

In this talk we discuss the interpretation of all methods as primal-dual: if the complete dual information is carried by the algorithm, the low complexity is naturally obtained. If the algorithm works only with primal variables, then it needs to approach the central path to obtain dual information, which is easily available near the path.

# POLYNOMIAL-TIME ALGORITHMS FOR LINEAR PROGRAMMING BASED ON THE USE OF THE LOGARITHMIC BARRIER PENALTY FUNCTION

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After the publication of Karmarkar's projective method for solving linear programming problems many other interesting polynomial-time interior point approaches were proposed. These variants can be devided in three large groups:

- Projective methods
- Path-following methods
- Potential-Reduction methods.

We shall deal with approaches from the second and the third category.

A path-following method approximately follows the *central path* of the problem; this is a smooth curve from the *analytic center* of the feasible region to an optimal point. Any such method uses a parametrized *potential function*, which in most cases is strictly convex. The minimizing point for the potential function lies on the central path. Conversely, each point of the central path is obtained in this way. Thus, the central path becomes parametrized as well. The goal then becomes to find a good approximation of the optimal point on the central path. This goal is reached by an iterative process as follows.

Given a feasible point, close to the central path, for some value of the parameter, one updates the parameter with a small amount in the direction of the optimal value. Then, doing a Newton step with respect to the potential function corresponding to the new value of the parameter, one obtains a feasible point which lies in the vicinity of the point on the central path corresponding to the new value of the parameter. And so on.

In our talk we start with a path following algorithm, which uses the logarithmic barrier function, with penalty parameter $\mu$, as potential function. It has a surprisingly simple and elegant proof of its polynomial behaviour. The algorithm requires $O(\sqrt{n}L)$ iterations. This is typical for path-following methods. Here $n$ denotes the dimension of the feasible space, and $L$ the size of the problem. Per iteration $O(n^3)$ arithmetic operations are needed to realize the Newton step. So the overall complexity bound becomes $O(n^{3.5}L)$ arithmetic operations. We shall indicate how this bound can be reduced to $O(n^3L)$ arithmetic operations.

Path-following methods have two serious disadvantages: the starting point has to be close to the central path, and in each iteration only small reductions in the penalty parameter $\mu$ are realized. We finally will deal with a variant of the algorithm which does not have these drawbacks. It belongs to the third category and can be considered as a polynomial-time implementation of the classical logarithmic barrier function approach. A series of 'long' Newton steps are taken between successive large reductions of the penalty parameter. The long steps aim at getting back to the vicinity of the central path. The algorithm requires at most $O(nL)$ iterations, which yields an overall complexity bound of $O(n^4L)$ arithmetic operations. This bound can be improved to $O(n^{3.5}L)$.

# THE ALGORITHM OF N. KARMARKAR FOR LINEAR PROGRAMMING

Lex Schrijver

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We give a description of the linear programming algorithm published by Narendra Karmarkar in 1984. The method is based on selecting a sequence of interior points that converges to an optimum solution. Features are the termination in polynomially bounded time and an excellent performance in practice.

**REFERENCES**

N. Karmarkar, A new polynomial-time algorithm for linear programming. *Combinatorica 4,* 1984, pp. 373-395.

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# A DANTZIG-WOLFE-LIKE VARIANT OF KARMARKAR'S INTERIOR-POINT LINEAR PROGRAMMING ALGORITHM

Mike J. Todd

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and Industrial Engineering
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We show that a variant of Karmarkar's projective algorithm for linear programming can be viewed as following the approach of Dantzig-Wolfe decomposition. At each iteration, the current primal feasible solution generates *prices* which are used to form a simple subproblem. The solution to the subproblem is then incorporated into the current feasible solution. With a suitable choice of stepsize a constant reduction in potential function is achieved at each iteration. We also use our analysis to motivate a new primal simplex pivot rule, closely related to rules used by Klotz and Schage.

# AN INTRODUCTION TO OPTIMIZATION AND CONTROL OF QUEUEING NETWORKS

Don Towsley

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USA

The purpose of this talk is to provide a survey of the area of optimization and control of queueing networks. We will be concerned primarily with the problem of REAL-TIME control of such systems, i.e., techniques that can be used on real-world queueing systems as found in communication networks, computer systems, production systems, etc...

The first part of the talk will consider the problem of determining the structure of policies that optimize various performance measures in queueing systems. We will survey some of the techniques commonly used for determining the structure of optimum policies for a wide variety of problems. These techniques fall broadly into two categories:

1) methods based on dynamic programming [Baras, Dorsey & Makowski, 1985; Ross, 1970; Walrand, 1988];
2) methods based on sample path dominance [Buyukkuc, Varaiya & Walrand, 1985; Walrand, 1988].

When applicable, the latter methods are simpler to apply than the former.

For a specific problem, the policy determined using the above techniques typically includes several parameters whose values must be chosen in order to optimize the performance of the system in question. For example, many optimum policies are of a threshold type where the value of the threshold must be carefully chosen. This is particularly important when the policy is implemented on a real system where the workload may change over time, necessitating a recalculation of the optimum values of the policy parameters (e.g. threshold). In the second part of the talk we address the problem of on-line determination of the optimum values of the parameters associated with optimal control policies. This is similar to the problem of estimating gradients within a Monte Carlo simulation and we will survey ideas from perturbation analysis [Cassandras & Strickland, 1989; Suri, 1989] and the method of likelihood ratios [Glynn, 1989; Heidelberger & Towsley, 1989]. We will conclude this part of the talk with a simple application.

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# SCHEDULING POLICIES FOR REAL-TIME AND PARALLEL PROCESSING SYSTEMS

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This talk will consist of two parts. In the first part we treat the problem of scheduling jobs in a real-time system and in the second the problem of scheduling jobs in a parallel processing system. The common theme throughout the talk will be the use of sample path techniques for proving the optimality of various policies.

In the first part we consider multiple servers serving a stream of jobs with deadlines. Here a job is not served if it misses its deadline. We illustrate how sample path techniques can be used to show that the policies that maximize the fraction of jobs that complete by their deadlines always schedule the job closest to its deadline. This result is established for several classes of policies that do not use service time information, for general arrivals, arbitrary deadlines and identically and independently distributed service times. Details of these results can be found in [Panwar, 1985; Panwar, Towsley & Wolf, 1988; Bhattacharya & Ephremides, 1989; Panwar & Towsley, 1989].

We also consider a system where all jobs are served, regardless of whether they complete before their deadline. Define the job lag time to be the difference between the completion time and the deadline. Using sample path arguments [Shantikumar & Sumita, 1987], it is possible to obtain convex orderings among the lag times under different scheduling policies. For example, from among the class of non-preemptive policies that do not use service time information but do use deadline information, the earliest deadline first policy minimizes and the longest deadline first policy maximizes the lag time in the sense of convex ordering under fairly general assumptions. Similar results hold for other classes of policies such as the class of preemptive policies and the class of policies that do not use deadline information. These results are easily extended to tandem queueing systems. Details can be found in [Towsley & Baccelli, to appear].

In the second part of the talk we consider a model of a parallel processing system first introduced by Baccelli & Liu [1988]. Briefly a job is represented by a directed acyclic graph where the nodes represent tasks that must be executed and the edges represent precedence constraints between the tasks. An arrival stream of jobs having the same precedence graph is executed by a set of processors. Each processor is responsible for executing a subset of tasks within the task graph. This pairing of tasks and processors is determined by a static allocation and because of the stochastic nature of the arrival process and the service times, a queue of tasks from the same job and different jobs may develop at each server. Consequently, a scheduling policy is required at each server to determine the order in which tasks are served. This policy must answer two questions:

1) If there are two or more tasks in the queue belonging to the same job which task to serve first;
2) If there are tasks belonging to different jobs, which task to serve first.

Using sample path arguments, it is possible to answer the second question. FIFO maximizes the job throughput and minimizes the response time in the sense of convex increasing order. Sample path arguments can also be used to extend some of the results from the first part of the talk to parallel systems serving jobs with deadlines. Details of this work can be found in [Baccelli, Liu & Towsley, 1989].

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# CENTRAL PLANNERS SHOULD USE CENTRAL PRICES

Jean-Philippe Vial

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The aim of this presentation is to give an application of the new interior point methods for linear programming to the decomposition approach of Dantzig and Wolfe.

The celebrated decomposition algorithm can be given the following economic interpretation. A central planner collects activity proposals from decentralized agencies. The planner combines the proposals in an optimal way to match the constraints on some common resources. The prices associated to this optimal use of the resources are sent back to the agencies. In response, the agencies return new proposals, which are computed on the price information basis.

From a computational point of view, the original Dantzig-Wolfe method has never really met its promises. The observed lack of efficiency is due to an inappropriate choice of the price system. Selecting prices on the basis of an optimal use of the resources leads the planner to disregard most of the past proposals of the agencies. We advocate the use of more balanced prices. The concept of central point, which underlies most of the new interior point methods, provides a possible alternative choice for the price system.

Our claim is justified on heuristic grounds, through the (known) equivalence between the decomposition scheme and the problem of minimizing of a piecewise linear function. We report on numerical experiments which sustain the claim.