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Optimizing barge utilization in hinterland container transportation

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Abstract

In hinterland container transportation the use of barges is getting more and more important. We propose a real-life operational planning problem model from an inland terminal operating company, in which the number of containers shipped per barge is maximized and the number of terminals visited per barge is minimized. This problem is solved with an integer linear program (ILP), yielding strong cost reductions, about 20%, compared to the method used currently in practice. Besides, we develop a heuristic that solves the ILP in two stages. First, it decides for each barge which terminals to visit and second it assigns containers to the barges. This heuristic produces almost always optimal solutions and otherwise near-optimal solutions. Moreover, the heuristic runs much faster than the ILP, especially for large-sized instances.

KEYWORDS

heuristic, hinterland transportation, integer linear programming, multimodal transportation

1 | INTRODUCTION

In container logistics, we encounter two major trends: (a) the increasing size of container vessels and (b) decreasing freight rates (*World container and general shipping*, 2016). The first trend leads to an increasing number of containers that are delivered at once at a deep-sea terminal. This puts a large pressure on the terminal operations, but also the hinterland transport needs to be optimized in order to guarantee an efficient dispatch of all containers. The second trend is putting pressure on the cost side of container transportation. Rough estimates are that between 40 and 80% of the total costs for the transport of a container are made in the hinterland (Notteboom, 2004), whereas the inland transportation usually covers only a small fraction of the distance of the total trip. To reduce the costs of transportation, the use of barges is extremely important. Barges are a cheap and eco-friendly way of transporting containers, especially compared to trucks. On top of that, road congestion is a serious problem in the densely populated areas where deep-sea ports are usually located. For instance, 40% of the vehicles in the area of the port of Rotterdam are heavily delayed (Behdani, Fan, Wiegmans, & Zuidwijk, 2016). Combining all the aspects above, both the European Commission (2011) and the Port of Rotterdam Authority (2011) have the ambition to achieve a modal shift from truck to barge and train. Drawbacks of barges are that they are slow and less flexible in visiting terminals. A barge operator has to make an appointment for visiting a terminal a few days in advance. On the other hand, the more containers are on a barge, the lower the average transportation costs per container are. Combining these two aspects, it is clear that for barge transportation, the planning of the containers is a difficult and important problem.

In this paper, we consider a real-life operational barge planning problem from the perspective of an inland terminal. This terminal has contracts with barge operating companies which offer barge services and with trucking companies. The inland terminal may choose the mode and day of transportation of a container, as long as the container arrives on time at the customer. From the inland terminal, the containers are transported to their final destination. The customers of the terminal are located close to the inland terminal, so the last mile transportation will always be done by a truck on the desired day. We refer to Caris and Janssens (2013) for an operational planning problem for this last-mile transportation. Next to the mode of transportation from the deep-sea terminal to the inland port, also the day of the hinterland transportation needs to be decided. The difference between the arrival and the departure of a container is called *dwell time* (Steenken, Vo, & Stahlbock, 2004). An important factor in the decision for the day of transportation are the storage costs at the inland and deep-sea terminal. The storage costs differ per day and are not the same for the deep-sea terminal and the inland terminal. Since the storage costs are quite substantial, optimizing the dwell time at a terminal is an important aspect of hinterland container logistics (Iannone, 2012).

Usually, the appointments a barge operator needs to make are restrictive and not negotiable so that they indirectly imply the route the barge needs to sail. Moreover, as pointed out by Fazi, Fansoo, and Woensel (2015), the deep-sea terminals are usually densely clustered, thus a good route is not hard to construct. For this reason, the routing of the barges is not considered in our problem. Nevertheless, mooring at a terminal is a time-consuming process, thus if a barge moors at a terminal, it is beneficial to load many containers. On top of that, at many terminals there is a big chance of incurring a delay. At the port of Rotterdam, delays of a few hours are not uncommon for barges. Hence, it is desired to visit a terminal as rarely as possible, in order to reduce the delay of a barge. A disadvantage of our approach is that our time interval is set to a day, so we cannot take time into consideration. Time can, for instance, be important if shipments and vehicle moves have to be synchronized, or if terminals or locks have certain opening times (Sharyapova, 2014). In our problem setting, the barge operator is responsible for ensuring that the timing of the barge is such that all terminals can be visited. Moreover, the only synchronization is at the inland terminal and we can include in our formulation a constraint that our container should arrive on time at the inland terminal.

Next to making a barge planning that does not visit many terminals, the capacity of the barges should also be used as much as possible. If there are more containers on a barge, the fixed costs such as the wage of the skipper can be divided over more containers. Consequently, the cost per container will decrease. Therefore, one part of the objective of our barge planning is to minimize the empty container spots of the barges. The combination of maximizing the number of containers shipped by barge, minimizing the storage costs and minimizing the number of terminals visited by barge is a complicated and nontrivial problem. Sometimes, it might be better to have more storage costs in order to visit fewer terminals or being able to ship more containers by barge. Making a planning that deals with all these aspects is now done by hand at the inland terminal, but we will propose an integer linear program (ILP) model to solve this problem.

The contribution of this paper is threefold. First, we discuss a problem that is faced by an inland terminal and that has not been studied in the literature before. The uniqueness of this problem lies in the fact that not only the assignment of containers to services is considered and that the routes of the barges do not need to be decided. The goal is to minimize the costs of container hinterland transportation and minimize the number of terminals visited by barge. This problem is richer than problems in which only containers are assigned to services and easier than problems in which the routes also have to be decided. The latter fact makes that the problem can be solved much faster, while still being relevant for practice. Second, we present an ILP-model that can be used to solve this problem. We test our model on real-life data from an inland terminal based in the Netherlands and we achieve a cost reduction of about 20% compared to current practice. For large problem instances the computation time of the ILP might be too long to be used in practice. Hence, our third contribution is that we propose a method that solves the ILP in two steps. This method reduces the computation time of difficult instances immensely, while still producing solutions that are extremely close to the optimal solution.

This article is organized as follows. We start with giving a review of the existing literature for operational problems in container hinterland transportation in Section 2. Afterward, we give a detailed description of the problem in Section 3. In Section 4, we present three methods to solve the problem. In Section 5, the three methods are used to solve medium-sized and large-sized instances and the results are compared. Finally, we will draw some conclusions and indicate further research directions in Section 6.

2 | LITERATURE REVIEW

In the literature there are three types of definitions that are closely connected to container hinterland transportation: *intermodal*, *multimodal*, and *synchromodal* transportation. Intermodal transportation is a form of transportation in which the goods are shipped in only one transportation unit (container) during the entire shipment. Multimodal transportation means that the goods are shipped by at least two different types of modes. In synchromodal transportation there is synchronization between multiple different modes of transportation, meaning that at any time the best mode of transportation based on the circumstances is chosen (SteadieSeifi, Dellaert, Nuijten, van Woensel, & Raou, 2014). There is much overlap between these definitions and all of them could be applied to our problem. Recently, quite a few review papers have been published in the area of these three types of transportation problems (Carris, Macharis, & Janssens, 2013; SteadieSeifi et al., 2014; Van Riessen, Negenborn, & Dekker, 2015). In the literature, there are also papers on interterminal transport by barges that are related to our problem, see, for example, Heilig and Voß (2017), Schroer, Corman, Duinkerken, Negenborn, and Lodewijks (2015), and Tierney, Vo, and Stahlbock (2014). Interterminal transportation deals with the transportation of containers from one terminal at a deep-sea port to another terminal in the same port.

In the multimodal literature, most attention has been paid to strategic problems, such as network design, but little focus has been on operational problems (Mes & Iacob, 2016). Nevertheless, there are a few recent papers that explicitly deal with operational decision making in synchromodal transportation. Based on the decision made, the problems in the existing literature are divided into three categories: (a) problems in which containers are assigned to existing barge services, (b) problems in which the routes of barges are determined for a given demand, and (c) problems in which both the assignment of containers to barges and the route of barges are decided upon.

In the first category, the assignment of containers to services can be done offline and online. In an offline planning problem, the assignment of containers to services is done once the information of all containers is known, whereas in an online problem, the planner needs to decide the service of a container immediately once the booking is done. In Baykasoglu and Subulan (2016), a multiobjective planning problem of the loads in an intermodal network is presented. In their model, the transportation costs, the service level, and CO_2-emissions are optimized for both the import and export flow of containers. Another offline problem is discussed in Tierney et al. (2014), in which the interterminal transportation in a deep-sea terminal is analyzed. The goal of this problem is not to minimize costs, but to minimize the delay from containers being transshipped within the port. Perez Rivera and Mes (2016) study an offline problem in which containers have to be assigned to modes of transportation in a synchromodal network in order to minimize the costs. They formulate this problem as a Markov decision process in order to also take into account uncertain future costs. In Van Riessen, Negenborn, and Dekker (2016), optimal offline solutions are obtained for an intermodal planning problem in which the objective is to minimize the transportation costs and penalties of being late. These offline solutions are used to infer a decision tree that is then used to make online decisions. Mes and Iacob (2016) also consider an online planning problem. They propose a k-shortest path approach in which the planner receives for each order the k best paths in the network in terms of costs, delays, and CO_2-emissions. Finally, the work of Wang, Bilegan, and Crainic (2016) incorporates revenue management for barge transportation. Given a fixed barge schedule, the goal is to decide for every incoming order whether to accept the order or not and which service to use for the container.

The paper of Li, Negenborn, and Lodewijks (2016) falls under the second category of the literature. In this paper, a distributed constraint optimization problem is formulated to decide upon the route of vessels in a port. In this problem, it is known for each vessel which terminals it should visit and how many containers to load at these terminals and the decision to be made is the route of the vessels. In Karlaftis, Kepaptsoglou, and Sambracos (2009), a case study is presented in which containers have to be transported between ports on islands in the Aegean Archipelago and mainland Greece. If a ship is visiting a port, all containers have to be transported, so this problem is formulated as a capacitated vehicle routing problem.

Finally, we discuss three studies that fall into the third category. In Fazi et al. (2015), a decision tool is developed for planning the hinterland transportation. Similar to our problem, Fazi et al. (2015) maximize barge utility and penalize a barge visit at a terminal. Nevertheless, they account for sailing time and the transportation costs are per hour. Therefore, they need to calculate the actual route the barge is sailing. Moreover, they are not considering any storage costs at the deep-sea or inland terminal. The model of Behdani et al. (2016) decides which containers to assign to a service and when a service should leave a terminal. Their goal is to minimize both the transportation costs and the waiting time of containers at terminals. Finally, in the work of Sharyapova (2014) a scheduled service network design is introduced with continuous time synchronization and transshipment constraints. The goal here is to minimize the total operational costs by selecting which services to use, determining the timing of the vehicles and deciding which containers to transport with which service. Since the time of a service is incorporated in the decision making, we have chosen to group this work in the third category and not in the first.

3 | PROBLEM DESCRIPTION

Our problem focuses on the transportation of containers from multiple deep-sea terminals to a single inland terminal. Each container arrives with a deep-sea vessel at a deep-sea terminal. If a container is already at the deep-sea terminal, the day of arrival is naturally known. In case a container arrives in the future, an estimated time of arrival (ETA) is known. We will refer in both cases to ETA as the day of arrival. Moreover, for each container a certain day is known, the so-called *call date*, at which it has to be present at the customer. Customers are located in the direct neighborhood of the inland terminal, thus if the container arrives at the inland terminal exactly at the call date, it can still be shipped on time to the customer. Therefore, the call date can also be seen as the day the container has to arrive at the inland terminal. As unloading a large deep-sea vessel may take hours, we assume that the container is available for hinterland transportation a day after the ETA. In our problem, transportation of the containers by barge takes 1 day, so the day before the call date is the last day a container can be shipped by barge. As transportation by truck is much faster, the container can still be shipped by truck on the call date. Our approach can easily be adjusted to a situation in which transportation or unloading takes a different time length.

The ETA and the call date of the container impose hard constraints on the possible days of shipments. On top of that, the ideal day of shipment is also influenced by *demurrage costs* and *storage costs*. Demurrage costs are costs paid to the carrier of the container if the container stays too long at the deep-sea terminal. Storage costs are the costs incurred when the container is at the inland terminal. Usually, a container has a certain demurrage free period for which no demurrage costs have to be paid. After that demurrage free period, demurrage costs are paid per day that a container is located at a deep-sea terminal. Storage costs are the costs associated with the number of days the container is located at the inland terminal. Generally, the storage costs per day are much lower than the demurrage costs per day because space at an inland terminal is less scarce than at a deep-sea terminal. In other words, before the demurrage free period has ended, it is cheaper to store the container at the deep-sea terminal than at the inland terminal and vice versa after the demurrage free period has ended. For each day it is straightforward to calculate the demurrage and storage costs that are incurred with transporting a container on that day. Given these costs, finding the day for which the minimum demurrage and storage costs have to be paid is easy.

A barge schedule that specifies which barge is present at the deep-sea port on which day is made by the barge operator and thus input for our model. We assume that each barge has only 1 day on which it can load containers at the deep-sea port. This assumption is reasonable because all barges have a tight schedule in order to ship as many containers as possible. Consequently, if we assign a container to a barge, then the day of transportation is also known. Each barge has a maximum capacity that cannot be exceeded. The size of a container is measured in twenty-foot equivalent unit (TEU) and so is the maximum capacity of the barge. For each barge, there are fixed costs to use that barge. Moreover, transporting a unit of TEU on a barge has certain costs.

Besides a barge, it is also possible to ship a container by truck. We assume that a truck can only transport a single container, irrespective of the size of a container. As a result, the costs of shipping a container by truck are much higher than the shipping costs by barge. An advantage of trucking a container is that from a practical perspective it is reasonable to assume that there is always a truck available. If we decide to ship a container by truck, it can thus be shipped on any day. Consequently, the day of truck shipment is the day with the minimum storage and demurrage costs. As a result, the transportation day of a truck does not need to be determined. All in all, for each container it has to be decided on which barge it is shipped or if it is shipped by truck.

We consider a planning problem with a finite horizon. In determining the length of the planning horizon a trade-off has to be made between reliable information and planning flexibility. If a short planning horizon is chosen, the information of the containers is rather reliable and unlikely to be subject to changes. On the other hand, only a few barges might be available for the transportation of the containers. Hence, the assignment possibilities for each container are restrictive. In case a long planning horizon is chosen, the flexibility of assigning the containers to barges increases, but the available information becomes more unreliable. In practice, a planning horizon of a week is used. A week ahead all information of the containers and barges is often rather reliable. If one takes a longer planning horizon it might happen that a customer has not put in an order yet or that the ETA of the sea vessel changes, for instance, because of bad weather conditions. Recent works have also focused on the ETA of a container being stochastic, see for instance Perez Rivera and Mes (2017) and Zuidwijk and Veenstra (2015).

The finite planning horizon results in the fact that some containers have a call date after the end of the planning horizon. We will call these containers *low-priority* containers and containers that need to be shipped during the planning period will be called *high-priority* containers. In Section 4, a more formal definition of low and high-priority containers will be given. When time progresses, each low-priority container will eventually become a high-priority container. These low-priority containers do not necessarily need to be transported in the planning period. Two standard approaches for dealing with low-priority containers are either ignoring them for the current planning period or forcing them to be transported anyway. These approaches work fine if the number of arriving containers at the deep-sea port and the available barge capacity is almost constant for each day. However, in practice, these numbers are far from constant over the days.

To illustrate why these two methods might fail, consider a situation in which a large batch of containers is available for transportation on a specific day, but the call date of these containers is a day after the end of the planning horizon. If there is a barge in the planning period with some unused capacity and the low-priority containers are ignored, we face the risk that there is insufficient barge capacity after the current planning period and we need to transport (part of) the batch per truck, whereas it was possible to ship at least part of the batch per barge. On the other hand, if there is undercapacity on the barges in the current planning period and one has decided that low-priority containers have to be transported, some containers will be transported by truck in the current planning period. However, it might be possible that there is a large amount of barge capacity available on the day after the end of our planning horizon. So in hindsight, it would have been possible to ship our batch of containers by barge.

Ideally, we would like to ship the low-priority containers only if there is capacity left on the barges in the current planning period and wait with transportation of the low-priority containers if no barge capacity is left. Therefore, it is not possible to take minimizing the transportation costs as an objective, because not transporting low-priority containers is always cheaper than transporting the low-priority containers. To ensure that low-priority containers also have the possibility of being transported, we have chosen to maximize the containers transported by barge as the objective, instead of minimizing the transportation costs. Since transportation by barge is much cheaper than by truck, a planning in which the number of containers that is shipped per barge is maximal is also likely to be a planning in which the transportation costs are low.

The containers are located at multiple deep-sea terminals and in order to visit as few terminals as possible by barge, we impose penalty costs for each visit. The penalty costs result in a situation in which a terminal is only visited if there are enough containers that can benefit from the fact that they do not have to be transported by truck. To achieve a situation in which there is a large set of containers available for transportation, some containers have to be shipped on another day than their ideal shipment day. Instead of penalizing a terminal visit by barge, it would also have been possible to impose a constraint on the number of terminals visited per barge. The reason why we have not chosen for this option is twofold. First, if visiting one more terminal than the "maximum" number of terminal visits reduces the transportation costs by a large amount, we would like to visit that terminal anyway. Second, if there are two schedules with the same total transportation costs and one is visiting fewer terminals than the other, the schedule with fewer terminal visits is preferred. In summary, a trade-off has to be made between the transportation costs, the demurrage and storage costs and the number of terminals that are visited by barge. To find the optimal solution for this problem is all but trivial if the instances become of realistic size.

3.1 | Problem instance: Running example

Throughout this paper, the same problem instance will be used as an example to illustrate the problem. First in Figure 1, we explain the different characteristics of containers and afterward in Figures 2 and 3, we illustrate how containers can be assigned to barges and trucks. In Figure 1, an example of possible classes of containers at a deep-sea terminal is given. In this example, there are 10 containers, which all have a size of two TEU, and the planning horizon is 3 days. A container can only be transported on a day if the rectangle representing the container is in the column representing that day. So for instance, container 2 can be transported on all 3 days, but container 10 only on day 3. As container 2 is available for transportation on day 1 and needs to be transported at the latest on day 3, it means that its ETA is day 0 and its call date is day 4. The fact that container 2 is light gray on day 3 means that demurrage costs have to be paid if it is transported on that day. Containers 3 and 4 are colored dark gray on day 3 because demurrage costs for 2 days have to be paid when they are shipped on day 3. Containers 1–4 are rather flexible because we can ship them on any of the 3 days, whereas containers 8–10 can only be transported on one specific day. Besides, also container 5 can only be transported on day 1 because it was not assigned to a truck or barge before day 1. Container 7 is a low-priority container because its call date is after the end of the planning period.

In the running example we will consider a situation with two terminals: terminal R and S. The 10 containers that are given in Figure 1 are located at both terminal R and S. Moreover, barges are available on day 1 and day 3 and both of these barges have a maximum capacity of 15 TEU. As there are only containers of two TEU, each barge can ship at most seven containers. The fixed costs of using the barges are set low enough to ensure that both barges will always be used. Container 9 from Figure 1 is ignored in the remainder of the running example because it can never be transported by a barge. In Figures 2 and 3, two different examples are given of how the containers can be allocated to barges and trucks. At the top of Figure 2, the available containers are shown for each terminal for each day. On day 1 in the situation in Figure 2, barge 1 is visiting terminal R, as indicated by the arrow pointing from barge 1 to the box with all available containers at terminal R on day 1. The available containers that are transported by a barge are indicated with a circle within the square of the container. So in Figure 2, all containers at terminal R that are available for transportation are transported by barge 1. At terminal S on day 1 in Figure 2, there are containers with a diamond inside their box. These containers, namely S5, S6 and S8, will be transported by a truck. We need to ship these containers with a truck because on day 3 when the next barge is available they cannot be transported anymore. If there is neither a circle nor a diamond inside the square of a container, it means that a container is not transported on that day. For example, containers S1–S4 are not transported on day 1 in Figure 2. On day 3, in Figure 2, container S2 has a light gray box and containers S3 and S4 have a dark gray box. Similar to Figure 1, the light gray box represents a container for which 1 day of demurrage costs have to be paid and the dark gray box represents a container for which 2 days of demurrage costs have to be paid.

In contrast to visiting only one terminal, it is also possible that a barge visits two terminals, as is illustrated in Figure 3. If we compare the situations of Figures 2 and 3, we see that in Figure 2 four containers are shipped per truck and in Figure 3 only two. Besides, in Figure 3 seven containers are shipped on the barge of day 3 and in Figure 2 only six. Moreover, in the situation illustrated in Figure 2, there are in total 5 days with demurrage costs and in Figure 3 only four. All in all, the barges in Figure 2 are visiting fewer terminals than in Figure 3, but in Figure 2 more demurrage days occur, fewer containers are shipped per barge and more per truck. It depends on how severe a visit of a terminal is penalized if the situation of Figures 2 or 3 is preferred.

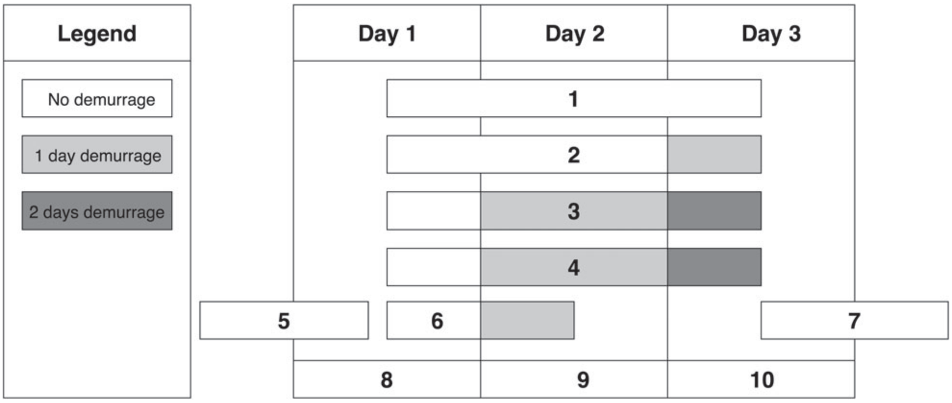

FIGURE 1 Example of different container classes at a deep-sea terminal

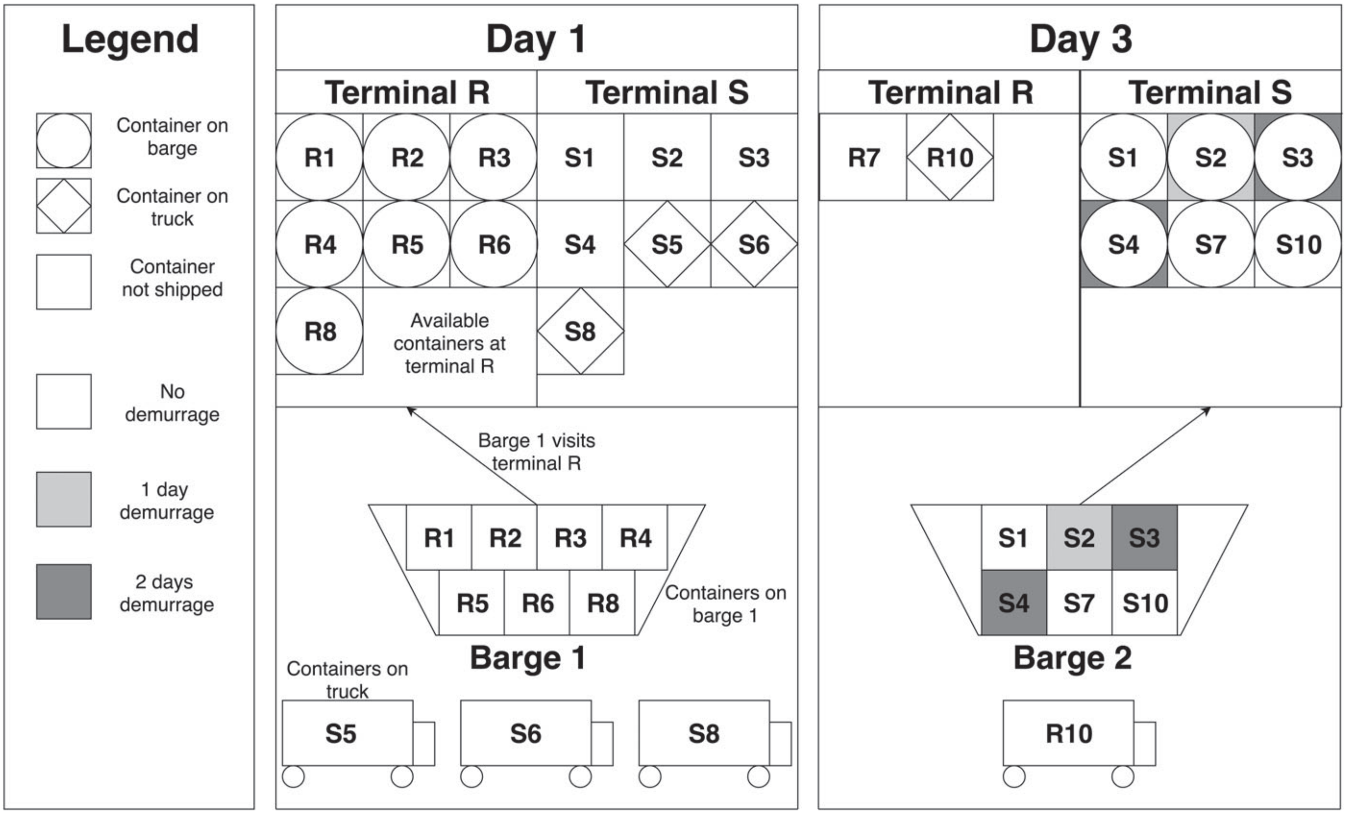

FIGURE 2 Example of container assignment with one terminal visit per barge

4 | MATHEMATICAL MODELS

To solve the problem discussed in Section 3, we present three different algorithms in this section. First of all, we present an optimal ILP-formulation in Section 4.1. Afterward, in Section 4.2 a two-stage heuristic that is based on the ILP-formulation from Section 4.1 is formulated. Finally, in Section 4.3 we present an algorithm that mimics the behavior of experienced planners who plan the containers to barges manually.

4.1 | Optimal ILP formulation

To formulate the ILP, first some notation is introduced. We consider an instance with n containers that are located at R deep-sea terminals. In the entire planning period there are b barges available. Each container could be assigned to these b barges or to a truck, thus in total there are $b+1$ vehicles: $v=0$ is a truck and $v=1, \dots, b$ are the barges. Let the barges be numbered in increasing order of their day at the deep-sea port. In other words, barge 1 is the first barge to be in the deep-sea port and barge b the last barge. The remainder of the input data is as follows:

- c^T are the shipping costs per truck;
- c_v^B are the shipping costs for a unit of TEU on barge $v=1, \dots, b$;
- c_{iv}^D are the demurrage costs for container i if it is transported by vehicle v;
- c_{iv}^S are the storage costs for container i if it is transported by vehicle v;

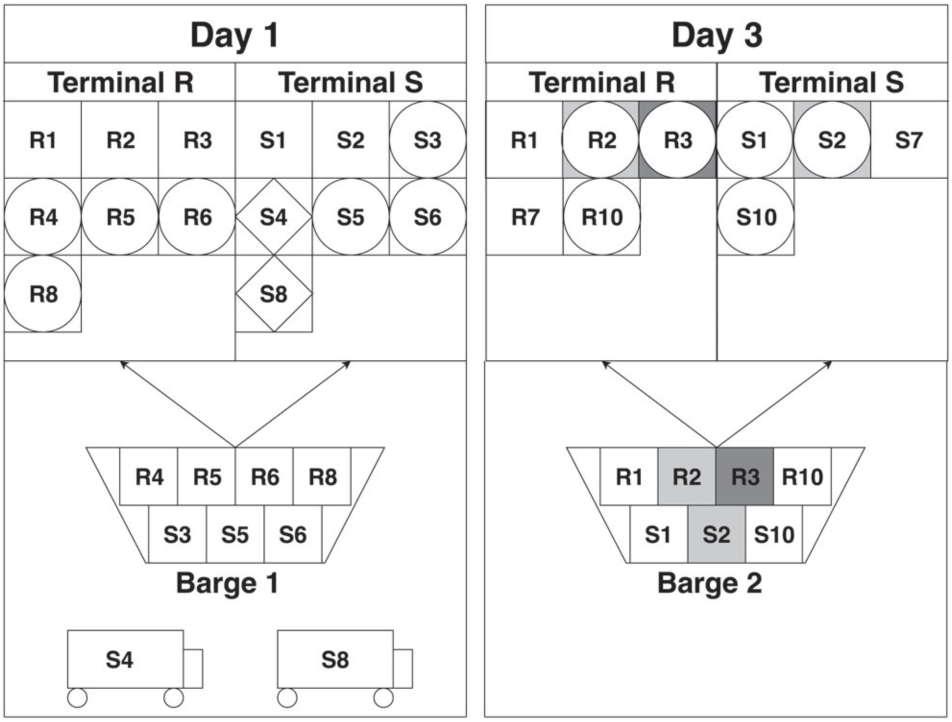

FIGURE 3 Example of container assignment with two terminal visits per barge

- π_{rv} are the penalty costs if barge v is visiting terminal r;
- ρ_v are the fixed costs for barge v;
- u_v is the maximum capacity of barge v in TEU;
- d_v is the day that barge v is at the deep-sea port;
- t_i is the size in TEU of container i;
- a_i is the estimated arrival date of container i at the deep-sea terminal;
- b_i is the call date of container i;
- m_i is the deep-sea terminal where container i is located;
- τ is the last day of the planning horizon;
- T is the largest size in TEU of a container, that is, $T = \max_{i=1,\ldots,n} t_i$
- $\mathscr{B}$ is the set of all available barges;
- $\mathscr{C}$ is the set of all containers;
- $\mathscr{H}$ is the set of all high-priority containers;
- $\mathscr{L}$ is the set of all low-priority containers.

In Section 3, high-priority containers were defined as containers that had to be transported in the planning period. Now, a more formal definition will be given. There are two criteria that determine the priority of a container. First, if the call date of a container is before the end of the planning period, the container is a high-priority container. Second, if the total costs of shipping the container on the last barge of the planning period are higher than the total costs of shipping a container by truck, the priority of the container is also high. The reasoning behind the second criterion is based on the fact that the demurrage costs are higher than the storage costs at the inland terminal. So after the end of the demurrage free period, the total storage and demurrage costs will increase for each day. Hence, if it is cheaper to transport a container per truck in the current planning period than on the last barge, it is also cheaper to transport the container per truck in the current planning period than on a barge in the next planning period. Using the notation just introduced, the two criteria will be formalized in Definition 1. If a container is not a high-priority container, it is automatically a low-priority container, that is, $\mathscr{L} = \mathscr{C} \setminus \mathscr{H}$.

Definition 1 Container $i \in \mathscr{C}$ is in the set of high-priority containers $\mathscr{H}$, if at least one of the two inequalities holds:

- $b_i \leq \tau$;

$$c_b^B + c_{ib}^D + c_{ib}^S \geq c^T + c_{i0}^D + c_{i0}^S.$$

Besides the input parameters, three types of binary decision variables are used:

$$X_{iv} = \begin{cases} 1 & \text{if container } i \text{ is transported by vehicle } v; \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_{rv} = \begin{cases} 1 & \text{if terminal } r \text{ is visited by barge } v; \\ 0 & \text{otherwise.} \end{cases}$$

$$Z_v = \begin{cases} 1 & \text{if barge } v \text{ is used;} \\ 0 & \text{otherwise.} \end{cases}$$

Using these variables we can define the following ILP:

$$\min \sum_{v=1}^{b} \left(c_v^B \left(u_v - \sum_{i=1}^{n} t_i X_{iv} \right) \right) + \sum_{i=1}^{n} c^T X_{i0} + \sum_{v=0}^{b} \sum_{i=1}^{n} (c_{iv}^D + c_{iv}^S) X_{iv}$$

$$+\sum_{v=1}^{b}\sum_{r=1}^{R}\pi_{rv}Y_{rv}+\sum_{v=1}^{b}\rho_v Z_v, \tag{1}$$

subject to:

$$\sum_{i=1}^{n} t_i X_{iv} \leq u_v \quad \text{for } v = 1, \dots, b; \tag{2}$$

$$\sum_{v=0}^{b} X_{iv} = 1 \quad \text{for } i \in \mathscr{H}; \tag{3}$$

$$\sum_{v=0}^{b} X_{iv} \leq 1 \quad \text{for } i \in \mathscr{L}; \tag{4}$$

$$X_{iv} \leq Y_{rv} \quad \text{for } i = 1, \dots, n \wedge v = 1, \dots, b \wedge r = m_i; \tag{5}$$

$$Y_{rv} \leq Z_v \quad \text{for } r = 1, \dots, R \wedge v = 1, \dots, v; \tag{6}$$

$$X_{iv} = 0 \text{ if } d_v \leq a_i \vee d_v \geq b_i \quad \text{for } i = 1, \dots, n \wedge v = 1, \dots, b; \tag{7}$$

$$X_{iv} \in \{0,1\} \quad \text{for } i = 1, \dots, n \wedge v = 0, \dots, b; \tag{8}$$

$$Y_{rv} \in \{0,1\} \quad \text{for } r = 1, \dots, R \wedge v = 1, \dots, b; \tag{9}$$

$$Z_v \in \{0,1\} \quad \text{for } v = 1, \dots, b. \tag{10}$$

The objective function in (1) consists of five sums. The first sum makes that for each unit of TEU that is not used on barge v $\left(u_v - \sum_{i=1}^{n} t_i X_{iv}\right)$, a penalty of c_v^B has to be paid. As a result, this sum ensures that the capacities of the barges are utilized as much as possible. The first sum is equivalent to a situation in which at first $u_v c_v^B$ is paid to the barge operator and the barge operator refunds c_v^B for each TEU that is actually transported on barge v. The second sum contains all shipping costs that are made by truck shipment. The third sum is the total of all demurrage and storage costs. Furthermore, the fourth sum contains the penalties that are paid for visiting a terminal by barge. Finally, the fifth sum is all the fixed costs that need to be paid to use the barges. Constraint (2) makes that each barge ships at most its maximum capacity in TEU. Constraint (3) forces all high-priority containers to be shipped exactly once, and constraint (4) ensures that all low-priority containers are shipped at most once. Constraint (5) connects the X_{iv} and Y_{rv} variables. If a container is picked up by a barge, then that barge also needs to visit the terminal where the container is located. Similarly, constraint (6) connects the Y_{rv} and Z_v variables. Terminal r can only be visited by barge v if barge v is used. Constraint (7) ensures that a container is not transported before its arrival at the deep-sea port or after its call date. Finally, Constraints (8)–(10) are the integrality constraints.

The value of π_{rv} obviously has a big influence on the outcome of the ILP-model. As π_{rv} is an artificial penalty, it could be set to any value. The value of π_{rv} indirectly imposes certain constraints on the optimal solution. In Lemmas 1–3 below, we show a relation between the value of π_{rv} and respectively, the minimum number of containers needed to visit terminal r by barge v, the maximum number of terminals visited by barge v and the minimum number of TEU on barge v.

Lemma 1 *If $\pi_{rv} \geq \lambda(c^T + Tc_v^B)$ and barge v is visiting r, then at least λ containers from terminal r are loaded on barge v.*

Proof For the sake of contradiction, let $\mathscr{K}$ be a set consisting of $k < \lambda$ containers from terminal r which are loaded on barge v in the optimal solution. The total costs of shipping the containers in set $\mathscr{K}$ are equal to:

$$\pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B). \tag{11}$$

Using the fact that the size of a container is at most T TEU ($t_i \leq T$), we can derive the following lower bound on the costs of shipping the k containers from set $\mathscr{K}$ with barge v:

$$\begin{aligned}
&\pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) \\
&\geq \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - T c_v^B) \\
&= \pi_{rv} - kTc_v^B + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S).
\end{aligned} \tag{12}$$

The fact that each container could be transported by a truck is used below for an upper bound for (11).

$$\begin{aligned}
\pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) &\leq \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S + c^T) \\
&\leq \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{iv}^S + c^T) \\
&= kc^T + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S).
\end{aligned} \tag{13}$$

The first inequality uses the fact that the total costs for trucking the containers in the set $\mathscr{K}$ are an upper bound for the optimal costs of transporting the containers in the set $\mathscr{K}$. The second inequality follows from the property that container i can be transported by a truck on any day, so we know that the sum of the demurrage and storage costs when transporting the container by truck are not larger than when the container is shipped on barge v: $c_{i0}^D + c_{i0}^S \leq c_{iv}^D + c_{iv}^S$. Combining the lower bound from (12) and the upper bound from (13) leads to the following relation:

$$\begin{aligned}
&\pi_{rv} - kTc_v^B + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S) \leq kc^T + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S) \\
&\Rightarrow \pi_{rv} - kTc_v^B \leq kc^T \\
&\Rightarrow \pi_{rv} \leq k(c^T + Tc_v^B) < \lambda(c^T + Tc_v^B).
\end{aligned}$$

The last inequality follows from the assumption that $k < \lambda$ and leads to a contradiction with the assumption of this lemma, namely $\pi_{rv} \geq \lambda(c^T + Tc_v^B)$. ■

Lemma 2 *If $\pi_{rv} \geq \frac{u_v(c_v^B+c^T)-\rho_v}{\lambda}$ for all terminals r for barge v, then barge v will not visit more than λ terminals.*

Proof Let the set of terminals visited by barge v and the set of containers transported on barge v be denoted as $\mathscr{R}$ and $\mathscr{K}$, respectively. For the sake of contradiction, assume that barge v is visiting $m = |\mathscr{R}| > \lambda$ terminals. The costs of shipping the containers from $\mathscr{K}$ on barge v are equal to:

$$\sum_{r\in\mathscr{R}} \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) + \rho_v.$$

First, a lower bound on these total costs will be derived:

$$\begin{aligned}
&\sum_{r\in\mathscr{R}} \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) + \rho_v \\
&\geq \sum_{r\in\mathscr{R}} \pi_{rv} - u_v c_v^B + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S) + \rho_v \\
&\geq \sum_{r\in\mathscr{R}} \pi_{rv} - u_v c_v^B + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) + \rho_v \\
&\frac{m}{\lambda}(u_v(c_v^B + c^T) - \rho_v) - u_v c_v^B + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) + \rho_v.
\end{aligned} \tag{14}$$

The first inequality follows from the fact that the total number of TEU assigned to a barge can never exceed the capacity. The second inequality holds because the storage and demurrage costs for shipping a container by truck are always lower than shipping a container by barge. The final inequality is a direct consequence of the assumption of this lemma. Similarly to Lemma 1, an upper bound can be derived using the costs of transporting the containers per truck:

$$\begin{aligned}
&\sum_{r\in\mathscr{R}} \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) + \rho_v \\
&\leq \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S + c^T) \\
&= |\mathscr{K}| c^T + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) \\
&\leq u_v c^T + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S).
\end{aligned} \tag{15}$$

The final inequality follows from the fact that the smallest size of a container is 1 TEU, so the cardinality of the set $\mathscr{K}$ is at most u_v. Combining the lower bound from (14) and the upper bound from (15), we get the relation:

$$\begin{aligned}
&\frac{m}{\lambda}(u_v(c_v^B + c^T) - \rho_v) - u_v c_v^B + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) + \rho_v \\
&\leq u_v c^T + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) \\
&\Rightarrow \frac{m}{\lambda}(u_v(c_v^B + c^T) - \rho_v) - u_v c_v^B + \rho_v \leq u_v c^T \\
&\Rightarrow \frac{m}{\lambda} \leq 1,
\end{aligned}$$

which is a contradiction to our assumption that $m > \lambda$. ∎

Lemma 3 *If $\pi_{rv} \geq \alpha u_v(c^T + c_v^B) - \rho_v$ for every terminal r for barge v, then barge v is filled with at least αu_v if barge v is used for transportation.*

Proof Let the set of terminals visited by barge v and the set of containers transported on barge v be denoted as $\mathscr{R}$ and $\mathscr{K}$, respectively. For the sake of contradiction, assume that barge is filled with βu_v TEU with $\beta < \alpha$. The costs of shipping the containers from $\mathscr{K}$ on barge v are equal to:

$$\sum_{r\in\mathscr{R}} \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) + \rho_v.$$

Similarly to the proofs of Lemmas 1 and 2, a lower bound and upper bound for these costs will be derived. The lower bound is equal to:

$$\begin{aligned}
&\sum_{r\in\mathscr{R}} \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) + \rho_v \\
&\geq \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) + \rho_v \\
&\geq \pi_{rv} - \beta u_v c_v^B + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) + \rho_v.
\end{aligned} \tag{16}$$

The final inequality uses the fact that we know that barge v is filled with βu_v TEU. The upper bound uses again the fact that all containers could be shipped per truck and is as follows:

$$\begin{aligned}
&\sum_{r\in\mathscr{R}} \pi_{rv} + \sum_{i\in\mathscr{K}} (c_{iv}^D + c_{iv}^S - t_i c_v^B) + \rho_v \\
&\leq \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S + c^T) \\
&\leq \beta u_v c^T + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S).
\end{aligned} \tag{17}$$

Combining the lower and upper bound from (16) and (17), we get:

$$\begin{aligned}
&\pi_{rv} - \beta u_v c_v^B + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) + \rho_v \\
&\leq \beta u_v c^T + \sum_{i\in\mathscr{K}} (c_{i0}^D + c_{i0}^S) \\
&\Rightarrow \pi_{rv} - \beta u_v c_v^B + \rho_v \leq \beta u_v c^T \\
&\Rightarrow \pi_{rv} \leq \beta u_v (c^T + c_v^B) - \rho_v < \alpha u_v (c^T + c_v^B) - \rho_v,
\end{aligned}$$

which is a contradiction to the assumption of this lemma. ∎

Which of the bounds provided by the three lemmas is the tightest depends on the parameters used. The bounds from Lemmas 2 and 3 are tight if the fixed costs of using a barge are high. However, in the upper bounds in the proof of both these lemmas all containers that were originally assigned to the barge are transported per truck, which could be a weak upper bound. In the upper bound of Lemma 1, a smaller number of containers are transported per truck.

We will close this section with three remarks that give easy adjustments for the ILP-model in Equations (1)–(10).

Remark 1 It might be desirable to scale the costs of visiting a terminal exponentially with the number of visits. Since the more terminals are visited, the more likely it is that a barge is delayed. It is still possible to model this problem as an ILP. To this end, an extra type of binary decision variable is needed, namely $\Gamma_{vj} \in \{0, 1\}$, which indicates whether barge v visits exactly j terminals or not. Moreover, let δ_{vj} be the total costs for visiting j terminals with barge v. The sum $\sum_{v=1}^{b} \sum_{r=1}^{R} \pi_{rv} Y_{rv}$ in the objective function in (1) will have to be replaced by $\sum_{v=1}^{b} \sum_{j=0}^{R} \delta_{vj} \Gamma_{vj}$. Besides, an extra constraint of the type:

$$\sum_{r=1}^{R} Y_{rv} = j\Gamma_{vj} \quad \text{for } v = 1, \ldots, b \wedge j = 0, \ldots, R,$$

is needed. An advantage of the exponential costs for visiting a terminal is that the terminal visits will be more equally divided among the barges. For example, with linear costs visiting eight terminals with one barge and two with the other is equally expensive as visiting five terminals with both barges. In the case of exponential costs, the latter scenario is cheaper. An obvious disadvantage is the need to introduce more decision variables and constraints. Moreover, with exponential costs it is not possible to distinguish between the terminals: each terminal has the same costs for visiting that terminal as jth terminal. In practice, there are terminals for which it is harder to get an appointment or which impose a higher chance of delay, so we would like to penalize a visit to these terminals more severely.

Remark 2 Instead of using binary variables for each container, it could also be possible to use integer variables for a group of containers. A group of containers consists of all containers with the same ETA, call date, size in TEU, terminal, demurrage free period, demurrage costs and storage costs. An advantage of using these groups of containers is that the number of variables is reduced. However, the number of characteristics containers need to share to be in a group is substantial, so it is likely that the decrease in the number of variables will not be large. We have chosen to implement the binary variable version because it gives a planner using the model more possibilities in manually making decisions on a container level. For example, a planner could decide to ship a specific container on a truck.

Remark 3 A terminal might require a minimum number of containers to be picked up by barge on a visit. This requirement is caused by the fact that the number of available berths at a deep-sea terminal is limited. If only a few containers are picked up at a single visit, the time the ship is occupying the berth in relation to the number of loaded containers is relatively high, because mooring is a time-consuming activity. On the other hand, a constraint to maximize the number of containers to be picked up by a barge visit might also be needed because it often happens that only a limited number of containers can be handled by a terminal. Let us denote the maximum number of containers that can be picked by barge v at terminal r as M_{rv} and the minimum number as μ_{rv}. For any combination of r and v the following constraints could be added:

$$\sum_{i:m_i=r} X_{iv} \geq \mu_{rv}, \tag{18}$$

$$\sum_{i:m_i=r} X_{iv} \leq M_{rv}. \tag{19}$$

Running example continued

The running example introduced in Section 3 is continued. We define the cost parameters as follows: $c^T = 150$, $c_1^B = c_2^B = 25$, the demurrage costs for each container are 60 per day and the storage costs 2 per day. Let the fixed costs for the barges be $\rho_1 = \rho_2 = 500$. Figure 2 is the outcome if a penalty of 100 is given to a terminal visit, and Figure 3 corresponds to a solution if for each terminal visit 250 has to be paid. For this problem instance there are many solutions that are all optimal. For example, in Figure 3, switching one of the containers on barge 1 with a container on the truck yields the same costs. Moreover, container R3 which is transported on barge 2 can be switched with R4, S3, or S4 yielding the same solution.

4.2 | Two-stage ILP-based heuristic

An important aspect of an operational planning for synchromodal transportation is that it can easily be adjusted (Van Riessen et al., 2015). If new information becomes available or if certain data changes, the planning should be recalculated. One could think of the arrival of new containers or, as discussed in Remark 3 in Section 4.1, limitations imposed by the container terminals. The problem formulation from Section 4.1 is a generalization of the generalized assignment problem (GAP) (Fisher, Jaikumar, & Wassenhove, 1986). In the GAP there is a set of jobs that needs to be assigned for minimum costs to a set of agents, which have a maximum capacity. In our generalization, the containers are the jobs and the barges and trucks are the agents. The generalization can be done by setting π_{rv} and ρ_v to zero and setting infinite costs for assigning a container to a barge that is either before the ETA or after the call date. On top of that, all containers are high-priority so have to be transported. The GAP is an NP-hard problem (Fisher et al., 1986) and as we will show in Section 5 for larger instances the running time of our problem is indeed too long for practical purposes. Therefore, in this section a heuristic based on the ILP-model from the previous section is developed.

The main advantage of the problem formulated in Section 4.1, compared to other models in the literature, is that the route is not calculated, which reduces the number of decision variables by a factor R. Our formulation requires only a Y_{rv}-variable to indicate if terminal r is visited by a barge v. If the route is also to be decided, a variable indicates also which terminal is visited after the other terminal. In real-life instances, containers are located at a relatively small number of deep-sea terminals. In the ILP-model described in the previous section, there is an X-variable for each container-barge combination, a Y-variable for each barge-terminal combination, and a Z-variable for each barge. As there are many more containers than terminals and barges, the number of Y-variables and Z-variables is small compared to the number of X-variables. To give some idea about this difference, in the instances we consider the number of containers is about 50–100 times larger than the number of terminals and about 100 times larger than the number of barges. Therefore, the number of integrality constraints decreases significantly if the X-variables are relaxed. This gives rise to the following two-stage heuristic:

Heuristic 1 Two-stage ILP-based heuristic:

Step 1: Solve the ILP with relaxing the constraints in (8), that is, $0 \leq X_{iv} \leq 1$. Let $\overline{Y}$ and $\overline{Z}$ be the optimal outcome of the Y-variables and Z-variables, respectively.

Step 2: Fix $\overline{Y}$ and $\overline{Z}$, set all X_{iv}-variables to be binary and solve the remaining ILP.

In other words, this heuristic determines in Step 1 which barges are used and which terminals are visited, and in Step 2 it finds the optimal allocation of the containers to barges and trucks, given the set of terminals that are visited. For every potential set of visited terminals in Step 1, it is possible to find a feasible allocation of the containers in Step 2 because each container can be assigned to a truck. The solution of the ILP from Section 4.1 is denoted by X^*, Y^* and Z^* and the solution from Heuristic 1 by $\overline{X}$, $\overline{Y}$, and $\overline{Z}$. The value for the X-variables after Step 1 of Heuristic 1 is denoted as $\widetilde{X}$. Moreover, let $v(X, Y, Z)$ be the value of the objective function for X, Y, and Z. After the Y-variables and Z-variables are fixed, it is decided in the second step of Heuristic 1 how to assign the containers to the barges and trucks, which is equivalent to the GAP.

Corollary 1 *There exists an optimal solution for Step 1 of Heuristic 1 for which the number of containers that are not completely assigned to one vehicle is at most the number of barges for which the total capacity is used.*

Proof Given that $\overline{Y}$ and $\overline{Z}$ are fixed, the value for $\widetilde{X}$ is the LP-relaxation of the GAP. In Benders and van Nunen (1983), it is shown that for a linear relaxation of the GAP, the number of fractional assignments is at most the number of machines scheduled to the maximum capacity. Since in our problem the number of trucks is unlimited, the number of containers which is fractionally assigned is at most the number of barges. ∎

Since the number of barges is small compared to the number of containers, the solution after Step 1 is almost feasible, which has two consequences. First, from the solution after Step 1, it is easy to find a feasible solution in Step 2. For instance, by assigning the fractional assigned containers to a truck. Moreover, the value of the objective function after Step 1 is likely to be close to the optimal value, so it is probably a tight lower bound. Combining these two properties, Step 2 is likely to perform fast. Furthermore, if Heuristic 1 selects in Step 1 the optimal terminals to visit, then the heuristic will produce an optimal solution, as is shown in Lemma 4. In Lemma 5, a bound on the difference between the solution from Heuristic 1 and the optimal solution is provided.

Lemma 4 *If* $\overline{Y} = Y^*$, *then* $v(\overline{X}, \overline{Y}, \overline{Z}) = v(X^*, Y^*, Z^*)$.

Proof If $\overline{Y} = Y^*$, then it must also be that $\overline{Z} = Z^*$. In case at least one $\overline{Y}_{rv}$-variable for barge v is equal to one, then constraint (6) implies that $\overline{Z}_v = 1$. If all $\overline{Y}_{rv}$-variables for barge v are zero, the variable $\overline{Z}_v$ could take value zero or one, but since the objective is to minimize costs the $\overline{Z}_v$ will always be zero. The same arguments hold for Y^* and Z^*, so $\overline{Y} = Y^*$ implies that $\overline{Z} = Z^*$. By the optimality of the ILP, we have that $v(\overline{X}, \overline{Y}, \overline{Z}) \geq v(X^*, Y^*, Z^*)$. Since the $\overline{X}$-variables are the optimal variables, given the variables $\overline{Y}$ and $\overline{Z}$, it must hold that:

$$v(\overline{X}, \overline{Y}, \overline{Z}) = v(\overline{X}, Y^*, Z^*) \leq v(X^*, Y^*, Z^*).$$

All in all, the values $v(X^*, Y^*, Z^*)$ and $v(\overline{X}, \overline{Y}, \overline{Z})$ should be the same. ∎

Lemma 5 *The value of the objective function of the solution of Heuristic 1 is bounded by:*

$$v(\overline{X},\overline{Y},\overline{Z}) \le v(X^*,Y^*,Z^*) + (T-1)\left(\frac{1}{T}bc^T + \sum_{v=1}^{b} c_v^B\right).$$

Proof In Step 1 of Heuristic 1, the integrality of the X_{iv}-variables is relaxed, so the objective function after Step 1 is a lower bound for the optimal solution and the solution of the heuristic: $v(\widetilde{X},\overline{Y},\overline{Z}) \le v(X^*,Y^*,Z^*) \le v(\overline{X},\overline{Y},\overline{Z})$. Let $\mathscr{F}$ be the set of containers which is fractionally assigned to a barge, that is, $\mathscr{F} := \{i \in \mathscr{C} \mid \exists v \in \mathscr{B} : 0 < X_{iv} < 1\}$. By Corollary 1, we know that $|\mathscr{F}| \le b$. We will construct a feasible solution $(X', \overline{Y}, \overline{Z})$ from the solution after Step 1 from the heuristic, in the following way:

- If $i \in \mathscr{F} \cap \mathscr{H}$, then $X'_{i0} = 1$ and $X'_{iv} = 0$ for $v = 1, \dots b$;
- If $i \in \mathscr{F} \cap \mathscr{L}$, then $X'_{iv} = 0$ for $v = 0, \dots, b$;
- If $i \notin \mathscr{F}$, then $X'_{iv} = \widetilde{X}_{iv}$ for $v = 0, \dots, b$.

In other words, in the solution X' none of the containers in $\mathscr{F}$ are assigned to a barge. All high-priority containers in $\mathscr{F}$ are assigned to a truck and all low-priority containers in $\mathscr{F}$ are not transported. We will first show that the sum of the demurrage and storage costs in X' is not higher than in $\widetilde{X}$. We will only focus on the containers in the set $\mathscr{F}$, because the other containers have the same demurrage and storage costs in X' and $\widetilde{X}$. The difference between the storage and demurrage costs in X' and $\widetilde{X}$ is given by:

$$\begin{aligned}
&\sum_{i\in\mathscr{F}\cap\mathscr{H}} (c_{i0}^D + c_{i0}^S)(1-\widetilde{X}_{i0}) - \sum_{i\in\mathscr{F}}\sum_{v=1}^{b}(c_{iv}^D + c_{iv}^S)\widetilde{X}_{iv} \\
&= \sum_{i\in\mathscr{F}\cap\mathscr{H}}\left(c_{i0}^D + c_{i0}^S - \sum_{v=0}^{b}(c_{iv}^D + c_{iv}^S)\widetilde{X}_{iv}\right) \\
&\quad - \sum_{i\in\mathscr{F}\cap\mathscr{L}}\sum_{v=1}^{b}(c_{iv}^D + c_{iv}^S)\widetilde{X}_{iv} \\
&\le \sum_{i\in\mathscr{F}\cap\mathscr{H}}\left(c_{i0}^D + c_{i0}^S - \sum_{v=0}^{b}(c_{iv}^D + c_{iv}^S)\widetilde{X}_{iv}\right) \\
&\le \sum_{i\in\mathscr{F}\cap\mathscr{H}}\left(c_{i0}^D + c_{i0}^S - \min_{v=0,\dots,b}\{c_{iv}^D + c_{iv}^S\}\sum_{v=0}^{b}\widetilde{X}_{iv}\right) \\
&= \sum_{i\in\mathscr{F}\cap\mathscr{H}}\left(c_{i0}^D + c_{i0}^S - (c_{i0}^D + c_{i0}^S)\sum_{v=0}^{b}\widetilde{X}_{iv}\right) = 0.
\end{aligned} \tag{20}$$

In the final equality, the property from constraint (3) is used, which implies that $\sum_{v=0}^{b}\widetilde{X}_{iv} = 1$, for $i \in \mathscr{F} \cap \mathscr{H}$. Second, we will look at the increase in barge and truck shipping costs in X' compared to $\widetilde{X}$, which is equal to:

$$\sum_{i\in\mathscr{F}\cap\mathscr{H}} c^T(1-\widetilde{X}_{i0}) + \sum_{i\in\mathscr{F}}\sum_{v=1}^{b} t_i c_v^B \widetilde{X}_{iv}. \tag{21}$$

We divide the set $\mathscr{F} \cap \mathscr{H}$ into the following two subsets:

$$\mathscr{J} := \{i \in \mathscr{F} \cap \mathscr{H} : \widetilde{X}_{i0} > 0\};$$

$$\mathscr{K} := \{i \in \mathscr{F} \cap \mathscr{H} : \widetilde{X}_{i0} = 0\}.$$

The containers in set $\mathscr{J}$ are partially assigned to a truck after the first step of Heuristic 1. The containers in set $\mathscr{K}$ are fractionally assigned to at least two barges. Without loss of generality, we assume that u_v is integral, so if a fraction of a container is assigned to a truck it should always be at least one TEU. Hence, for all $i \in \mathscr{J}$ the value $\widetilde{X}_{iv} \ge \frac{1}{T}$. Using a similar argument, the fraction of TEU from container i on barge v is always at most $t_i - 1$, which gives us the following upper bound for Equation (21):

$$\begin{aligned}
&\sum_{i\in\mathscr{J}} c^T(1-\widetilde{X}_{i0}) + \sum_{i\in\mathscr{K}} c^T + \sum_{i\in\mathscr{F}}\sum_{v=1}^{b} t_i c_v^B \widetilde{X}_{iv} \\
&\le \sum_{i\in\mathscr{J}} c^T\left(\frac{T-1}{T}\right) + \sum_{i\in\mathscr{K}} c^T + \sum_{i\in\mathscr{F}}\sum_{v=1}^{b}(t_i - 1)c_v^B \\
&\le \left(\frac{T-1}{T}|\mathscr{J}| + |\mathscr{K}|\right)c^T + (T-1)\sum_{v=1}^{b} c_v^B.
\end{aligned} \tag{22}$$

To derive a bound regardless the sizes of $\mathscr{J}$ and $\mathscr{K}$, the maximum of $\left(\frac{T-1}{T}|\mathscr{J}| + |\mathscr{K}|\right)$ has to be calculated. From Corollary 1 it follows that $|\mathscr{F}| \le b$ and since every container $i \in \mathscr{K}$ is at least assigned to two different barges, we know that $|\mathscr{J}| + 2|\mathscr{K}| \le b$. In case $T = 1$, the set $\mathscr{F}$ is empty because u_v is integral and each barge v has at most one fractionally assigned container. Hence, T should be at least two, which implies that $\frac{T-1}{T} \ge \frac{1}{2}$. Given this fact and the constraint $|\mathscr{J}| + 2|\mathscr{K}| \le b$, the maximum of $\left(\frac{T-1}{T}|\mathscr{J}| + |\mathscr{K}|\right)$ is attained at $|\mathscr{J}| = b$ and $|\mathscr{K}| = 0$. Thus Equation (22) can be further bounded by:

$$\begin{aligned}
&\left(\frac{T-1}{T}|\mathscr{J}| + |\mathscr{K}|\right)c^T + (T-1)\sum_{v=1}^{b} c_v^B \\
&\le \frac{T-1}{T}bc^T + (T-1)\sum_{v=1}^{b} c_v^B.
\end{aligned} \tag{23}$$

Combining the two results from Equations (20) and (23), the following bound for the solution of Heuristic 1 can be derived:

$$v(\overline{X},\overline{Y},\overline{Z}) \le v(X',\overline{Y},\overline{Z})$$

$$\leq v(\widetilde{X},\overline{Y},\overline{Z}) + \frac{T-1}{T}bc^T + (T-1)\sum_{v=1}^{b} c_v^B$$
$$\leq v(X^*,Y^*,Z^*) + (T-1)\left(\frac{1}{T}bc^T + \sum_{v=1}^{b} c_v^B\right).$$

■

The bound derived in Lemma 5 is tight, as the following example illustrates. Consider a small example with one barge with $u_1 = 3$ and two terminals. Let the planning period be a single day. At terminal 1, two containers with a size of two TEU are located and at terminal 2 there is one container of size one TEU. All those three containers are high-priority and since the planning period is 1 day, the demurrage and storage costs for shipping them per truck or per barge are the same. Therefore, we ignore those costs for the rest of the example. Let the penalty costs for visiting a terminal be $\pi_{11} = \pi_{21} = \frac{1}{2}c^T + \varepsilon$ for $\varepsilon > 0$. The optimal solution for this problem instance is to visit both terminals and ship one container from terminal 1 and the container from terminal 2 per barge and the other container from terminal 1 per truck. The total costs of this solution are: $2c^T + 2\varepsilon - 3c_1^B$. On the other hand, the first step of the heuristic only visits terminal 1. At this terminal, one container is assigned integrally to the barge and only half of the other container is assigned to the barge. The other half of the container and the container at terminal 2 are transported by truck. Hence, the barge in the second step of the heuristic can only visit terminal 1 and load one of the two containers at that terminal. The other two containers are shipped per truck, resulting in the following costs: $2\frac{1}{2}c^T + \varepsilon - 2c_1^B$. All in all, the difference between the solution produced by the two-stage heuristic and the optimal solution is: $\frac{1}{2}c^T - \varepsilon + c_1^B$. As we can take ε arbitrarily small, this is equivalent to the difference $(T-1)\left(\frac{1}{T}bc^T + \sum_{v=1}^{b} c_v^B\right)$ from Lemma 5.

Running example continued

Similar as in Section 4.1, the running example instance is solved for both $\pi_{rv} = 100$ and $\pi_{rv} = 250$. Contrary to the ILP-solution, the two-stage heuristic produces the same solution for the two penalties. For both penalties, the solution after Step 1 of the heuristic is the same and is given in Figure 4.

After the first stage of Heuristic 1, all containers but container S4 are assigned in the same way as in Figure 3. Half of container S4 is assigned to barge 1 and half of container S4 is assigned to barge 2. With this assignment, the full capacity of 15 TEU of the two barges is used. In the second step of Heuristic 1, container S4 is assigned to a truck, because it is not possible to assign eight containers with a size of two TEU integrally to one of the two barges. All in all, the heuristic produces for both $\pi_{rv} = 100$ and $\pi_{rv} = 250$ the same solution as in Figure 3. So for $\pi_{rv} = 100$, the terminal visits after Step 1 are the same as the optimal visits and thus by Lemma 4 the heuristic produces an optimal solution. In the setting that $\pi_{rv} = 250$, the terminal visits after Step 1 are not optimal and the final solution of the heuristic is not optimal, either. The optimal value of the objective function is 2,668, whereas the two-stage heuristic produces a solution with a value of 2,754. The difference between those two solutions, namely 86, is significantly smaller than the bound given by Lemma 5, which is $(2-1)\left(\frac{1}{2}\times 2\times 150 + 25 + 25\right) = 200$.

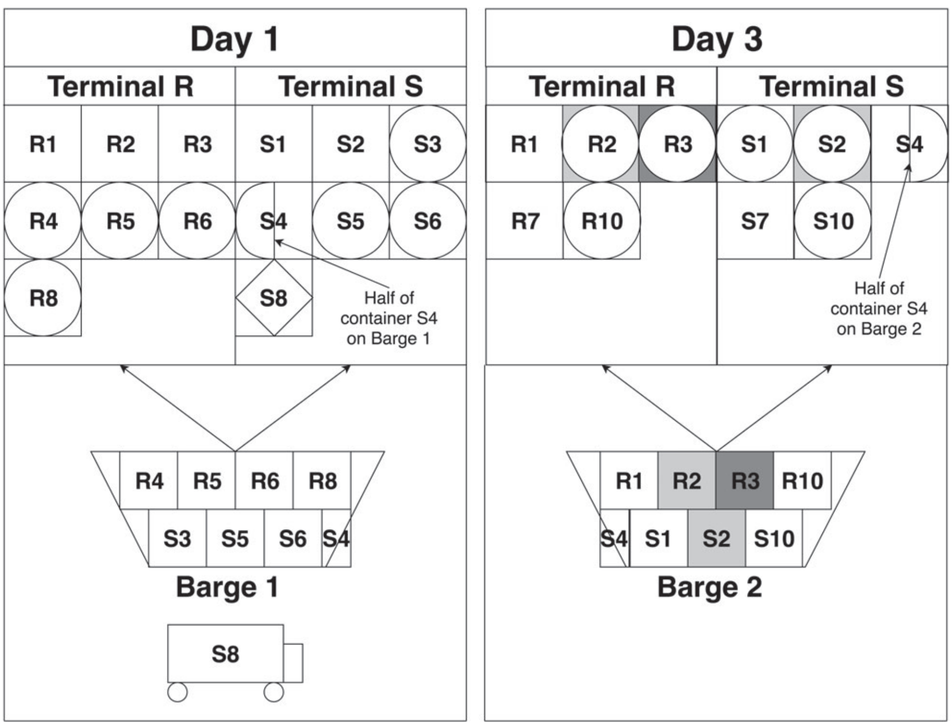

FIGURE 4 Assignment of the containers from the running example after the first step of Heuristic 1

4.3 | Planner algorithm

In current practice, the transportation planning of the containers is made by experienced planners from the inland terminals. Based on interviews with practitioners we have developed an algorithm that imitates the behavior of that planner. The algorithm is a greedy algorithm that uses more or less a first come first serve approach. It was also pointed out by Van Riessen et al. (2016) and Wang et al. (2016) that these kinds of methods are often used in real-life planning. This planner algorithm is described in pseudocode in Algorithms 1 and 2. In Algorithm 1, it is described how the planner decides which terminals to visit and which barges to use. Algorithm 2 selects for a specific barge the containers to assign to that barge.

Algorithm 1: Algorithm to select barges and terminal visits.

```
   Sort the barges in non-decreasing order of the day they
     are present at the deep-sea port.
1  for All barges do
2      Assign unassigned containers to barge according to
         Algorithm 2
3      for All terminals do
4          if Barge loads sufficient containers at terminal
             with respect to Lemma 1 then
5          Visit terminal;
       end
6      Assign all unassigned containers located at visited
         terminals according to Algorithm 2
7      if Costs of shipping containers assigned to barge
         plus the barge rent is smaller than costs of trucking
         all high-priority containers assigned to barge then
8      Use barge;
   end
9  for All unassigned high-priority containers do
10     Transport container by truck
   end
```

Algorithm 2: Algorithm to select containers.

```
Input: Specifications of barge
Select containers which have ETA and call date such
  that they could be transported on barge
Sort the containers in non-decreasing order of their
  end of demurrage free period, ETA and terminal.
for All unscheduled containers do
    if Call date of container is between arrival of
      current barge and next barge & current barge has
      capacity to fit container then
    Assign container to barge;
end
for All unscheduled containers do
    if Demurrage free period of container ends before
      next barge arrives & current barge has capacity to
      fit container then
    Assign container to barge;
end
for All unscheduled high-priority containers do
    if Current barge has capacity to fit container then
    Assign container to barge;
end
for All unscheduled low-priority containers do
    if Current barge has capacity to fit container then
    Assign container to barge;
end
```

Assign all unassigned containers located at visited terminals according to Algorithm 2.

In Algorithm 1, the barges are considered in chronological order of the day they are at the deep-sea port. It uses Algorithm 2 as a subroutine to determine the containers that will be assigned to the current barge. In lines 4 and 5, it is decided which terminals are visited. A terminal is visited only if sufficient containers are loaded at that terminal. We use Lemma 1 to decide what is sufficient in order to make a comparison with the ILP and the two-stage heuristic. If there are terminals at which insufficient containers are loaded, the barge is not visiting them, so it might be that some capacity on the barge becomes available. Therefore, Algorithm 2 is run again in line 6 to see if there are unassigned containers available on terminals that are already visited. In line 7, the planner algorithm decides if it is going to use the current barge. If transporting the high-priority containers that are assigned to the barge per truck is more expensive than shipping them with the barge, the barge is used. Otherwise, we do not use the barge. After all barges are considered, it might be that there are still some unassigned containers. If these containers have a high-priority, they are shipped by a truck.

Algorithm 2 uses a barge as input and selects only the containers that could be transported on that barge. These containers are sorted based on the end of their demurrage free period, ETA and deep-sea terminal. Afterward, the algorithm goes four times through all containers and checks if a container can be assigned to the barge. In the first for-loop of Algorithm 2, containers which have a call date such that they can be transported by this barge, but not with the next barge, are added to the barge, unless the remaining capacity of the barge is not sufficient. In the second for-loop, the algorithm goes again through all unscheduled containers and if the demurrage free period ends before the next barge arrives, it is scheduled on the current barge if it has free capacity. The third part of Algorithm 2 goes through all unscheduled high-priority containers. These containers are added to the barge if it is not full yet. If there is still capacity left on the barge, in the fourth for-loop low-priority containers are assigned to the barge.

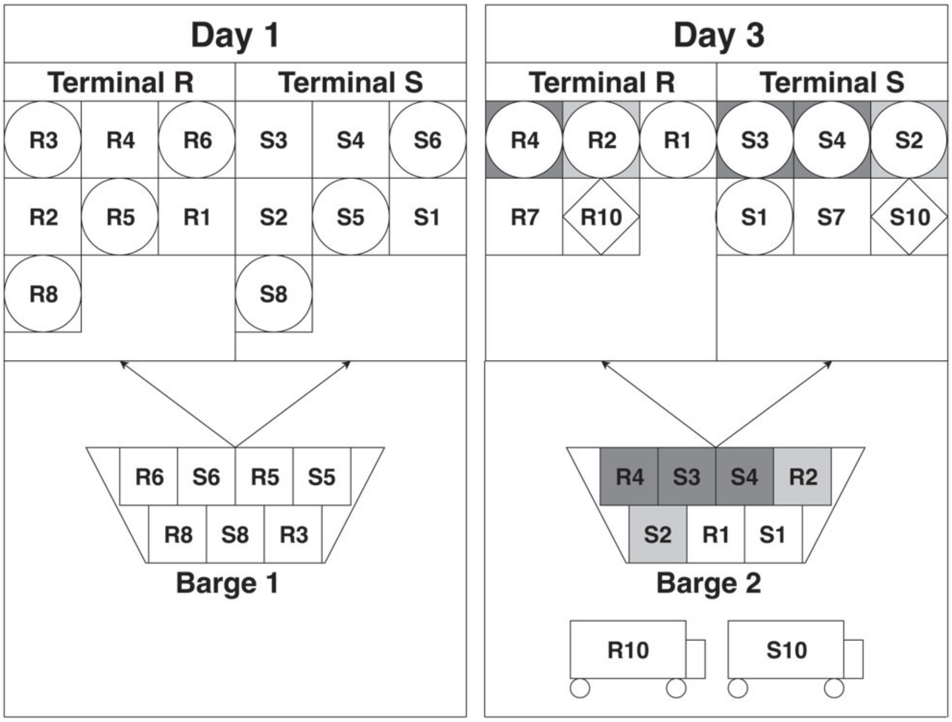

FIGURE 5 Assignment of containers from the running example via the planner algorithm

Running example continued

In Figure 5, containers from the running example are assigned according to the planner algorithm. The order in which the containers are displayed at the terminal from left to right and top to bottom is the way they are sorted at the beginning of Algorithm 2. The demurrage free period of containers 3, 4 and 6 ends on day 1 and they have the same ETA, so for these containers the ties are just broken by their container number. After the containers for which demurrage is relevant, container 5 is the first container because its ETA is the lowest. In the barges, the containers are shown from left to right and top to bottom in the order they are assigned to that barge. Container R6 is the first container to be assigned to barge 1 because it is the first container at Terminal R whose call date is before the arrival day of the next barge. Container S6 is the next container because it has the same characteristics as R6, but it is only located at another terminal. All containers in Figure 5 that are assigned to a barge are assigned in line 2 of Algorithm 1. Container R3 is the only container assigned to a barge in the second for-loop of Algorithm 2 in which the demurrage free period is decisive, all other containers are already assigned in the first for-loop of Algorithm 2.

The planner algorithm produces a solution in which four terminals are visited. The minimum containers picked up at a terminal by barge is three. According to Lemma 1 with previously defined cost parameters, three is higher than the minimum containers to be picked up given $\pi_{rv} = 100$ or $\pi_{rv} = 250$. Hence, in lines 4 and 5 of Algorithm 1 it is decided to visit all terminals and thus there is no capacity to assign containers in line 6. We have assumed that in the running example the fixed costs are set low enough that both barges are used. In lines 9 and 10 containers R10 and S10 are assigned to a truck. As the planner algorithm is visiting four terminals in total, it is interesting to see how it performs in comparison to the outcome of Figure 3, which is the optimal solution for visiting four terminals. In both scenarios 14 containers are shipped per barge and two per truck. However, in Figure 3 demurrage costs had to be paid for 4 days and in Figure 5 there are eight demurrage days. The planner algorithm is a greedy algorithm, which is not able to look into the future. In this example, that results in the fact that it is not able to detect that it already has to ship container by truck on the first day. In the end, in the planner algorithm it is decided to transport containers per truck on day 3 and thus more demurrage has to be paid.

5 | COMPUTATIONAL RESULTS

In this section, the performance of the three methods introduced in Section 4 is compared. We are interested in how much improvement can be made by implementing the ILP-method from Section 4.1 compared to the planner algorithm from Section 4.3 that models the current practice. Second, both the running time and the solution quality of the two-stage heuristic are compared with the ILP.

5.1 | Medium-sized instances

To evaluate the performance of the three methods, we have used 12 instances based on real-life data. Each instance consists of all containers that are available for transportation and the barges that can be used for transportation. The planning horizon for each instance is set to a week. In Table 1, some key properties of the different instances are given. In

TABLE 1 Summary of properties of 12 medium-sized instances

Instance	Number containers (n)	Total TEU	High-priority containers (%)	Number barges (b)	Total capacity barges
M1	670	957	35	4	886
M2	1,163	1,636	38	6	1,329
M3	549	843	91	4	886
M4	651	990	90	4	941
M5	753	1,083	60	4	850
M6	863	1,279	95	4	1,004
M7	892	1,221	91	4	945
M8	596	877	96	4	945
M9	503	770	91	4	945
M10	1,064	1,584	91	5	1,027
M11	855	1,177	72	4	945
M12	924	1,300	83	4	945

Abbreviation: TEU, twenty-foot equivalent unit.

the second column of Table 1, the number of containers for each instance is given. This number varies roughly between 500 and 1,000. In the third column, the total number of TEU of these containers is shown. The percentage of high-priority containers, that is, the containers that need to be shipped, differs between 35 and 96%. In column 5, the number of barges is given, which is most of the time equal to four, but because the capacity of the barges is not always the same, the maximum capacity in TEU shown in the last column differs.

Besides the container and barge characteristics, the costs that are used need to be defined. For transporting a container we use $c^T = 150$ and $c_v^B = 25$ for every barge. The demurrage costs are 40 for every day after the end of the demurrage free period for containers of one TEU and 60 for larger containers. The storage costs are 1 per TEU per day that a container is stored at the inland terminal. Moreover, the fixed costs of using any barge are $\rho_v = 4,500$. Finally, the value for π_{rv} is chosen in such a way that according to Lemma 1 at least five containers are picked up at every terminal, so $\pi_{rv} = 1{,}000$ for every barge and terminal.

In Table 2, the optimal value of the objective function of the ILP-model is compared to the outcome of the two-stage heuristic and the planner algorithm. In the second column, the optimal solution of the ILP-model is given. In the third and fourth column, the objective function of the two-stage heuristic and the percentage difference with the optimal solution is given. Finally, in the last two columns, the costs of the solution from the planner algorithm are given and the percentage difference with the optimal solution. The first thing to note from Table 2 is that the two-stage heuristic produces for all but one of the instances the optimal solution. For the instance for which the solution of the two-stage heuristic is not optimal, it is only 0.2% more expensive than the optimal solution. Second, the value of the planner algorithm is on average 20% higher than the optimal solution. Nevertheless, the solution quality of the planner algorithm differs substantially per instance. For example, for M2 and M9 it produces a solution that is within 2% of the optimal solution, but for instance M10 the difference between the optimal solution and the solution of the planner algorithm is almost 45%.

TABLE 2 The solution of the planner heuristic and two-stage heuristic compared with the optimal solution

Instance	Optimal solution	Two-stage heuristic	Δ%	Planner algorithm	Δ%
M1	58,213	58,213	0	69,002	18.5
M2	57,785	57,785	0	58,651	1.5
M3	48,429	48,429	0	54,954	13.5
M4	41,812	41,884	0.2	45,942	9.9
M5	47,575	47,575	0	55,679	17.0
M6	68,628	68,628	0	90,075	31.3
M7	63,833	63,833	0	76,198	19.4
M8	52,611	52,611	0	63,034	19.8
M9	42,476	42,476	0	43,320	1.8
M10	82,399	82,399	0	119,431	44.9
M11	71,428	71,428	0	96,970	35.8
M12	65,263	65,263	0	85,814	31.5

TABLE 3 Costs for the optimal ILP and the planner algorithm split out per category

Type costs	Optimal solution	Planner algorithm	Δ
	Total costs	Total costs	
Unused TEU barge penalty	77,250	56,475	20,775
Truck costs	211,800	173,850	37,950
Demurrage costs	47,780	176,340	−128,560
Storage costs	52,622	52,314	308
Terminal visit penalty	131,000	211,000	−80,000
Fixed barge costs	180,000	189,000	−9,000

Abbreviations: ILP, integer linear program; TEU, twenty-foot equivalent unit.

To understand in which aspect of the planning the ILP outperforms the planner algorithm, the total costs are split out into different categories in Table 3. In the second column of Table 3, the total costs for the planner solution for each category are given. In the third column, the planner's total costs are shown and in the fourth column, the difference between the optimistic planner and the optimal solution is calculated. In general, the planner algorithm ships more containers per barge and fewer containers per truck. However in doing that, the barges visit in the planner algorithm more than 60% more terminals than in the optimal solution. On top of that, the planner algorithm results in almost four times as many demurrage costs. An intuitive explanation is that in the optimal solution a container is shipped more often per truck in order not to visit a terminal or to reduce the demurrage costs than in the solution from the planner algorithm. All in all, the ILP-method potentially yields a great amount of costs savings.

The running time for the ILP for all 12 medium-sized instances is less than 3 s. The two-stage heuristic is on average about 1.4 times faster. As the running time for the ILP is not that long, one could argue for these instances a heuristic solution is not really needed. In the next section, we will consider larger instances to see if the ILP is still a good solution method.

5.2 | Large-sized instance

The medium-sized instances from the previous section could be solved by the ILP in reasonable time. In this section, it will be investigated how well the running time of the ILP scales with the input sizes. We have constructed randomly 10 instances with 1,500 containers. Moreover, the barge schedule consists of six barges and is the same for every instance. Since a large number of containers are selected randomly for each instance, the characteristics of the containers in the large-sized instances are quite similar. These large-sized instances are solved by the ILP, the two-stage heuristic and the planner algorithm using the same cost parameters as for the medium-sized instances.

The results for these large-sized instances are given in Table 4 below. In the second, third and fourth column of the table the running times of the, respectively, ILP, two-stage heuristic and planner algorithm are given. In the fifth, sixth and seventh column the objective function of the solution from these three methods is given. Although the instances are rather similar, the running time of the ILP differs substantially. For instances L7 and L10, the ILP did not find the optimal solution after 3 hr, when the algorithm was stopped. On top of that, also instances L5 and L9 took almost 5 min to produce the optimal solution, which might already be too long if a planner needs to recalculate the consequences of a change to the schedule. On the other hand, the running time of the two-stage heuristic is for all 10 instances about 10 s, even for the instances for which the ILP could not find the optimal solution in 3 hr. The planner algorithm produces for all instances in a fraction of a second a solution. However, the value of the objective function of the planner algorithm is about 50% higher than for the other two algorithms. Similarly, as was shown in Table 3, the solution of the planner algorithm for the large-sized instances has especially more demurrage costs and terminal visits.

The objective function for all 10 instances is almost the same, which is not surprising as the instances are similar. Out

TABLE 4 Running times and objective function of the ILP, the two-stage heuristic and the planner algorithm for 10 large-sized instances

	Running time (s)			Objective function		
Instance	ILP	Two-stage	Planner	ILP	Two-stage	Planner
L1	13	8	≪1	96,595	96,595	152,703
L2	32	7	≪1	99,096	99,096	161,385
L3	24	8	≪1	98,167	98,167	157,132
L4	13	7	≪1	98,788	98,788	145,083
L5	297	11	≪1	101,777	101,777	162,332
L6	31	7	≪1	100,406	100,406	158,824
L7	—	7	≪1	—	102,378	162,919
L8	68	8	≪1	101,777	101,808	167,846
L9	280	10	≪1	101,055	101,055	153,598
L10	—	10	≪1	—	99,712	161,736

Abbreviation: ILP, integer linear program.

of the eight instances for which the optimal solution is known, the two-stage heuristic produces the optimal solution seven times. The two-stage heuristic does not produce the optimal solution for instance L8, but its solution is only 0.03% worse. Moreover, the two-stage heuristic produces the solutions for these eight instances on average almost 10 times faster than the ILP. Concluding, the ILP-method does not work well for the large-sized instances. For two instances, it could not produce the optimal solution. Furthermore, for all instances the two-stage heuristic produces in a fraction of the time of the ILP a solution that is almost always optimal and otherwise it is almost optimal.

6 | CONCLUSION AND FURTHER RESEARCH

We have proposed an ILP to minimize the storage and demurrage costs, the truck transportation costs, the empty space on barges and the number of terminals visited by barge. In order to fill the barges as good as possible, we have introduced the concept of high-priority and low-priority containers. The high-priority containers need to be transported, whereas the low-priority containers can be used to fill the barges.

To evaluate the benefits of the ILP, we have introduced the planner algorithm that produces a solution in a similar way as an experienced planner. The potential cost savings for the medium-sized instances is about 20% and for large-sized instances it is about 50%. The fact that the number of variables for assigning a container to a barge is much larger than the number of variables indicating whether a barge is used and a terminal is visited by a barge, is used in the two-stage heuristic. The theoretical difference between the optimal solution and the solution of the heuristic is given. Nevertheless, computational experiments show that the two-stage heuristic almost always finds the optimal solution. Moreover, for the large-sized instances the two-stage heuristic finds the solution much faster than the ILP, which could not find the optimal solution within 3 hr for some instances.

We have only included the import flow of containers because that is the dominant flow in most of Europe (Fazi et al., 2015). A natural extension would be to include the export flow of containers in this model. It would be interesting to see how much can be gained by combining the two flows in one model, instead of planning the two flows separately of each other. A second topic that might be interesting to investigate if it is possible to make a planning that is more robust for the feedback of the terminal. The current model could possibly make a completely different schedule once one constraint of the type (18) and (19) is added. In practice, one would not want to communicate with many terminals to change the planning, because there is only one terminal that makes a small adjustment to the planning. A planning in which the number of containers picked up by a barge per terminal does not change that much if one extra constraint is added, is likely to be preferable. Finally, we have assumed that the barge schedule was given as input, but this schedule could be suboptimal. It would be interesting to see how much could be gained by only having a set of barges available and to decide on the day a barge is at the deep-sea port. With this setting, one can investigate the benefits of stronger collaboration between the inland terminal and the barge operator. The question will also be if our heuristic is still reasonably applicable to the changed setting.

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