CWI NEWSLETTER

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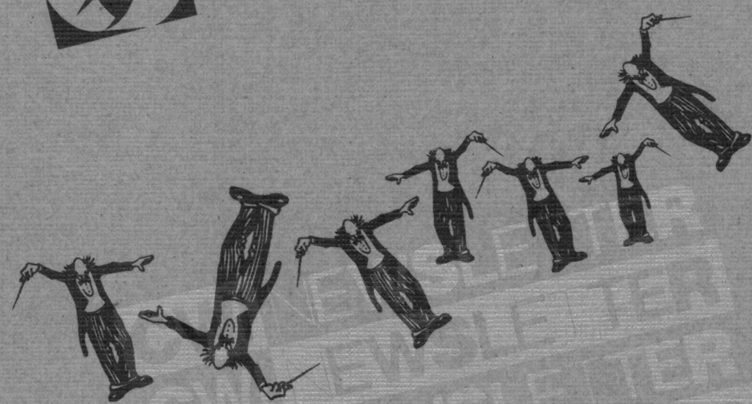

NEWSLETTER CWI NEWSLETTER CWI NEW

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The CWI Newsletter is published quarterly by the Centre for Mathematics and Computer Science (Centrum voor Wiskunde en Informatica), Kruislaan 413, 1098 SJ Amsterdam, The Netherlands. The Newsletter will report on activities being conducted at the Centre and will also contain articles of general interest in the fields of Mathematics and Computer Science, including book reviews and mathematical entertainment. The editors encourage persons outside and in the Centre to contribute to the Newsletter. Normal referee procedures will apply to all articles submitted.

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CWI

Centre for Mathematics and Computer Science

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The Centre for Mathematics and Computer Science (CWI) is the research institute of the Stichting Mathematisch Centrum (SMC), which was founded on 11 February 1946.

The goal of CWI is to carry out fundamental and advanced research in mathematics and computer science, with special attention to those areas in which the research may have important applications.

Research at CWI is organized in eight scientific departments:

- Pure Mathematics;
- Applied Mathematics;
- Mathematical Statistics;
- Operations Research and System Theory;
- Numerical Mathematics;
- Software Technology;
- Algorithms and Architecture;
- Interactive Systems

There are also a number of supporting service divisions, in particular the Computer Systems and Telematics Division, and an extensive Library.

The subdivision of the research into eight departments is less rigid than it appears, for there exists considerable collaboration between the departments. This is a matter of deliberate policy, not only in the selection of research topics, but also in the selection of the permanent scientific staff.

In Memoriam A. van Wijngaarden

Director Mathematical Centre 1961-1980

P.C. Baayen & J. Nuis
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On February 7th 1987, Prof. A. van Wijngaarden passed away. Practically since its foundation Van Wijngaarden had been connected with the MC, first as head of the Computing Department, from 1961 till 1980 as director, and after that for another year as advisor to the Board of Trustees and Board of Directors. During these 35 years he determined to a high extent the image of the MC. A pioneer in computer science with his roots in (numerical) mathematics, he also embodied one of the principles on which the MC has always set great store: mathematics going hand in hand with computer science. But also outside the MC, which during these years grew from a handful of people into an institute with well over 150 employees, Van Wijngaarden put his stamp on a number of developments. The birth of computer science in the Netherlands, and its development into a fully-grown discipline, is due for an important part to him. Internationally, his contribution to the development of the programming languages ALGOL 60 and ALGOL 68, and his role in such organisations as IFIP, have been of lasting significance.

Van Wijngaarden was born in Rotterdam on November 2nd, 1916. He studied mechanical engineering at the Technical University of Delft where he took his doctor's degree in 1945 on a thesis entitled 'Some Applications of Fourier Integrals to Problems of Elasticity'. Before he joined the Mathematical Centre on January 1st, 1947, he had held positions at the Technical University of Delft and at the National Aviation Laboratory (NLL). Immediately after his appointment as head of the nascent Computing Department, Van Wijngaarden went on a prolonged fact-finding mission to England and the U.S.A. in order to orientate himself about the newly developed electronic computer, and its use. In the course of this trip he came into contact with such computer pioneers as Wilkes, Turing, Wilkinson, Goldstine and Von Neumann. Upon his return, one of his conclusions was that the MC would do best to design and construct itself a computer. In 1952, under the leadership of Van Wijngaarden, the ARRA (Automatische Relais-Rekenmachine Amsterdam) — the first electronic computer machine in the Netherlands and one of the first on the continent of Europe — was completed at the MC. Members of the development team were B.J. Loopstra and C.S. Scholten, for a shorter period G.A. Blaauw, and at a later stage E.W. Dijkstra and W.L. van der Poel. The latter was at that moment employed by the Laboratory of the PTT (Post and Telecommunications Office), but worked in close collaboration with the

MC. Of the total of five computers which the Netherlands had in 1955, three (ARRA, FERTA and ARMAC) had been built under Van Wijngaarden's leadership at the MC, while the constructor of PTERA of the PTT — Van der Poel — was his first Ph.D. student, and later conferred Van Wijngaarden's own honorary doctor's degree at the Delft Technical University. (The fifth machine was a Ferranti computer at Shell.) In 1959 commercial interest for computer construction led, in cooperation with the life insurance company Nillmij, to the founding of an independent company: Electrologica (later incorporated in Philips Data Systems).

In the meantime Van Wijngaarden had already actually build up a computing service, which under his direct leadership — each morning he gave the 'calculator girls' a one hour lesson, and he was personally involved in setting up the calculation schemes — contributed considerably to the realisation of the goals of the MC. In this way, in a contract for the NLL, for those days difficult and extensive calculations were made of the vibration (flutter) of aeroplane wings in subsonic streams.

During these 'calculating years' Van Wijngaarden was also active as a mathematician. He published many articles on various subjects in applied and numerical mathematics, and some in the area of the theory of numbers. His name lives on for numerical mathematicians in the Van Wijngaarden Transformation. Appreciation for his scientific work followed quickly: in 1952 Van Wijngaarden became 'special professor' at the University of Amsterdam and in 1958 'professor extraordinary' in a chair on 'Numerical Mathematics and Computer Methodology'. In 1959 he was elected to the Royal Dutch Academy of Sciences. Many of his pupils have leading functions in research and industry. At the moment 10 of his 15 former Ph.D. students hold themselves a professorship.

Already in the period when he was involved in the construction of computers, but even more after the actual production process had been passed on to commercial interests, Van Wijngaarden's scientific interest focussed on the mathematics of programming. Together with his co-worker E.W. Dijkstra he made essential contributions to the development of ALGOL 60 and he was the great pioneer of ALGOL 68. The contribution of each of the four authors of the ALGOL 68 report were characterized concisely by Van Wijngaarden as follows: 'Koster: transputter; Peck: syntax; Mailloux: implementation; Van Wijngaarden: party ideologist'. The report itself, translated into Bulgarian, Chinese, German, French and Russian, stands from a scientific point of view, in its severity and sharpness of definition, at a lonely peak of mathematical elegance and thoroughness. As part of the ALGOL 68 project Van Wijngaarden developed his elegant and forceful concept of two-level grammars (called after him W-grammars). The design of ALGOL 68 has had an enormous influence on the development of later programming languages and on programming theory and practice in general.

Van Wijngaarden's interest, for that matter, was not limited to algorithmic languages. He was fascinated by language in general — and in particular by the interface between computer and natural language. He was very keen on the correct and cultivated use of language, and had a special feeling for puns and etymology. He also contributed actively as a member of the Working Group on Frequency Research of the Dutch Language. His love for language, however, did not prevent him from concluding his scientific career in 1981 with a lecture entitled 'Languageless programming'.

Already at the end of the fifties Van Wijngaarden was actively devoting his organisational talents to both national and international matters. Because also of his courteous and forthcoming way of dealing with people he played an important part in the founding of the Dutch Computer Society in 1959 and of the International Federation for Information Processing (IFIP) in 1960. He held important functions in both organisations for many years.

During the fifties and sixties the computer work of the two Amsterdam universities (the Free University and the University of Amsterdam, VU and UvA) was done at the MC. When, about 1970, this started to form too great a burden for the institute, it was decided to found SARA, a joint computer centre for VU, UvA and the Mathematical Centre. Van Wijngaarden, who was one of those who took the initiative for this unique form of cooperation, was a member of the board of SARA until the end of 1980.

Appreciation for the scientific and organisational contributions which Van Wijngaarden has given to the development of computers, information technology and electronic information processing, is apparent from the many invited professorships he has held and from the decorations bestowed upon him. He spent various periods as visiting professor at the University of New York, the University of California at Berkeley, and the University of Chicago. He was honoured nationally in 1973 when he was appointed Knight of the Order of the Dutch Lion, in 1979 he received an honorary doctorate at the Technical University of Delft (when his first Ph.D. student Van der Poel conferred his degree) and in 1981 upon leaving the MC the award of the 'Silver Medal' of the City of Amsterdam. Foreign awards were the 'Medaille d'argent de la ville de Paris' (1959), an honorary doctorate of the Institut National Polytechnique in Grenoble (1979), the Wilhelm Exler Medaille of the Österreichische Gewerbeverein in Vienna (1981) and the Computer Pioneer Award of the IEEE (1986).

Van Wijngaarden had a charming and lovable personality. At the same time he was a perfectionist. He characterized his working method as that of an engineer: 'Engineers don't start by talking: what on earth would they be supposed to say? They make the design for a project and then say: do you want it?'. These were the lines on which Van Wijngaarden lived and worked: with great style, aiming at perfection, but at the same time with attention and personal interest for the people around him. Here we would like to express our sincere gratitude for everything that was done for the Mathematical Centre and the scientific world by Adriaan van Wijngaarden, Dutch mathematician and computer scientist.

A. van Wijngaarden, director Mathematical Centre 1961-1980

Mathematical Morphology: an Algebraic Approach

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Mathematical morphology is a theory on morphological transformations which form the basic components for a number of algorithms in quantitative image analysis. In this paper we present an overview of the basic principles of mathematical morphology, and initiate a generalization of the theory by taking the object space to be an arbitrary complete lattice.

1. Principles of mathematical morphology

1.1. Introduction

A person who comes into touch with image processing for the very first time will probably be overwhelmed by the enormous amount of literature that appears every year, and it is not unlikely that he or she will be deterred by the dispersion which characterizes the field. A first branch, which is beyond the scope of this paper, originates from classical signal analysis, and its basic tools are convolution and (Fourier, Karhunen-Loeve) filtering methods. Most of the operations are linear and sometimes even reversible, which means that its performance is not attended with loss of information. For a rather complete account of this approach we refer to Rosenfeld and Kak [11]. A second branch in image processing is formed by mathematical morphology, a somewhat axiomatic theory containing elements of integral geometry, stereometry and stochastic geometry.

Essentially, mathematical morphology is a theory on morphological transformations and functionals, which, if chosen properly, make it possible to measure useful geometric features of images. The main body of the theory was developed at the Centre of the Paris School of Mines at Fontainebleau in France, and its success is due in part to the fact that the theoretical research kept pace with the development of an image analysis system, called the 'texture analyser'. The books of Matheron [9] and Serra [12] (see also [1,3]) provide a complete overview of the theory of mathematical morphology, the main idea of which is captured by the following quotation from the Preface of [12]:

'The notion of a geometrical structure, or texture, is not purely objective. It does not exist in the phenomenon itself, nor in the observer, but somewhere in between the two. Mathematical morphology quantifies this intuition by introducing the concept of structuring elements. Chosen by the morphologist, they interact with the object under study, modifying its shape and reducing it to a sort of caricature which is more expressive than the actual initial phenomenon...'

A morphological transformation of an image (a subset of $\mathbb{R}^n$ or $\mathbb{Z}^n$) is obtained by taking in a prescribed manner unions and intersections of a number of translates of this set and its complement. The collection of translation vectors involved constitutes the so-called structuring element. In practice one can reveal certain geometrical information about objects by sequential application of morphological transformations involving cleverly chosen structuring elements: it is clear that the number of possibilities is unlimited.

An important feature of (nontrivial) morphological transformations is their irreversibility: the transformed image contains less information than the original one. Or in mathematical terms: morphological transformations are not injective.

In the discrete case morphological transformations bear much resemblance to cellular automata (or cellular logic) transformations. Such transformations are performed by giving each pixel a new state depending on its old state and the old state of its neighbours: see [4,10]. An implicit consequence of the specific structure of a morphological transformation, which is of great practical value, is that one can use the build-in parallelism of the computer.

This paper consists of two parts. In the first part we survey some of the basic theory, whereas in the second part we indicate how this theory can be generalized to complete lattices. In the following section we present the basic transformations of mathematical morphology, namely dilation, erosion, closing and opening. The step from the continuous to the discrete space, involving the digitalization of images, can be justified if one can supply the continuous object space (whose elements are sets) with a topology. The introduction of a topology also enables one to prove robustness of transformations. In Section 1.3 we present such a topology. At that place we also discuss the basic principles, which, according to Serra's philosophy, define the morphological transformations. These principles include translation invariance and semi-continuity. At the end of Section 1.3, we formulate some mathematical questions raised by these principles. Together with the inborn impulse of any mathematician to generalize whatever he can lay hands on, these questions have been our main motivation to strive for a more axiomatic algebraic approach.

Such an algebraic approach is initiated in Part 2. There the basic assumption is that the underlying object space forms a complete lattice. In Section 2.1 we survey the relevant results of lattice theory. In Section 2.2 we give an abstract definition of dilation and erosion: this definition depends on the choice of a commutative automorphism group on the lattice (being the generalization of the translation group on $\mathbb{R}^n$ or $\mathbb{Z}^n$). Under some extra assumptions we can give a complete characterization of dilations and erosions. MATHERON [9] has proved that every increasing translation invariant transformation can be written as an intersection of dilations, or equivalently, as a union of erosions. In Section 2.3 we prove an abstract version of this theorem. Finally, in Section 2.4, we speculate about what has to be done in the future.

1.2. Dilation and erosion, closing and opening

Throughout this section, let E be the Euclidean space $\mathbb{R}^n$ or the discrete space $\mathbb{Z}^n$. Essential is that E is a commutative group. Let $\mathcal{P}(E)$ be the space of all subsets of E. A binary image can be represented by a subset X of E. We call X the object and $\mathcal{P}(E)$ the object space. If $X \subset E$ and $h \in E$ then we denote by X_h the translate of X along h:

$$X_h = \{x+h : x \in X\}.$$

If $X, Y \subset E$ then we say that X hits Y, $X \Uparrow Y$, if $X \cap Y \neq \varnothing$. Let A be an arbitrary subset of E. The *dilation* of a set X by the element A is defined by

$$X \check{\oplus} A = \{h \in E : A_h \Uparrow X\}.$$

The *erosion* of X by A is defined by

$$X \check{\ominus} A = \{h \in E : A_h \subset X\}.$$

We call A the *structuring element*. It is an easy exercise to show that the dilation of an image gives the same result as the erosion of its background, i.e.

$$(X \check{\oplus} A)^c = X^c \check{\ominus} A.$$

Here X^c denotes the complement of X. We say that dilation and erosion are complementary (or dual) operations. Let the Minkowski addition $\oplus$ and subtraction $\ominus$ of two sets $X, Y \subset E$ respectively be given by

$$X \oplus Y = \bigcup_{y \in Y} X_y$$

$$X \ominus Y = \bigcap_{y \in Y} X_y.$$

Then we have the following relationships:

$$X \check{\oplus} A = X \oplus \check{A}$$

$$X \check{\ominus} A = X \ominus \check{A},$$

where $\check{A} = -A = \{-a : a \in A\}$. The incredulous reader may verify this. Typical properties of dilation are

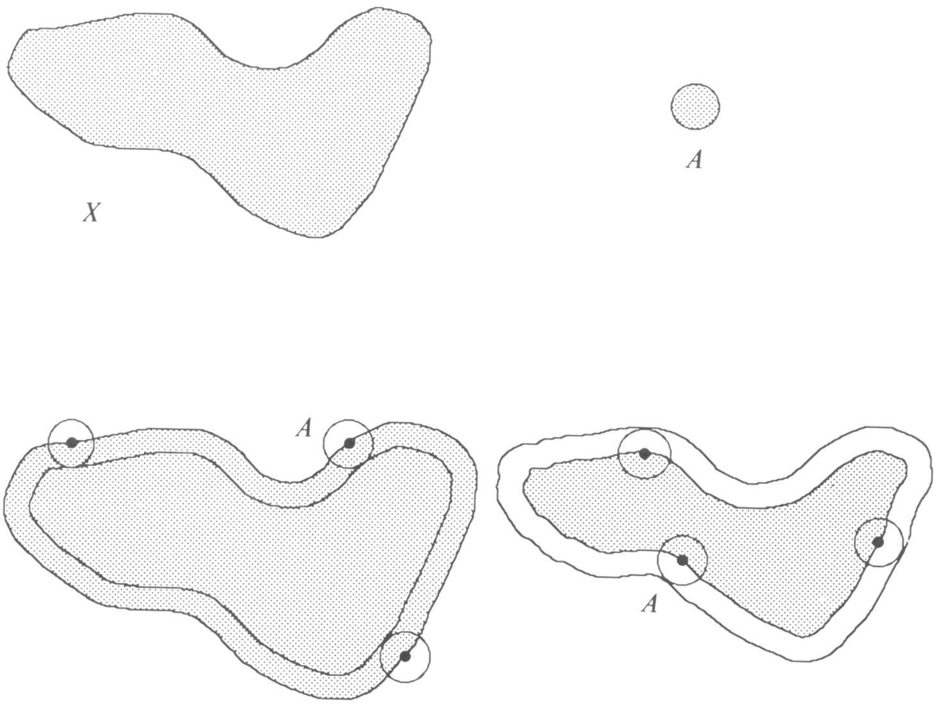
(i) $(X \oplus \check{A})_h = X_h \oplus \check{A}$,

(ii) $(\bigcup_{i \in I} X_i) \oplus \check{A} = \bigcup_{i \in I} (X_i \oplus \check{A})$

where I is an arbitrary finite or infinite index set, and $X_i \subset E$, $i \in I$. Thus dilation is distributive with respect to union and invariant under translation. Similar properties hold for erosion

(i) $(X \ominus \check{A})_h = X_h \ominus \check{A}$.

(ii) $(\bigcap_{i \in I} X_i) \ominus \check{A} = \bigcap_{i \in I} (X_i \ominus \check{A})$.



dilation of X by A erosion of X by A

FIGURE 1. Dilation and erosion in the continuous case

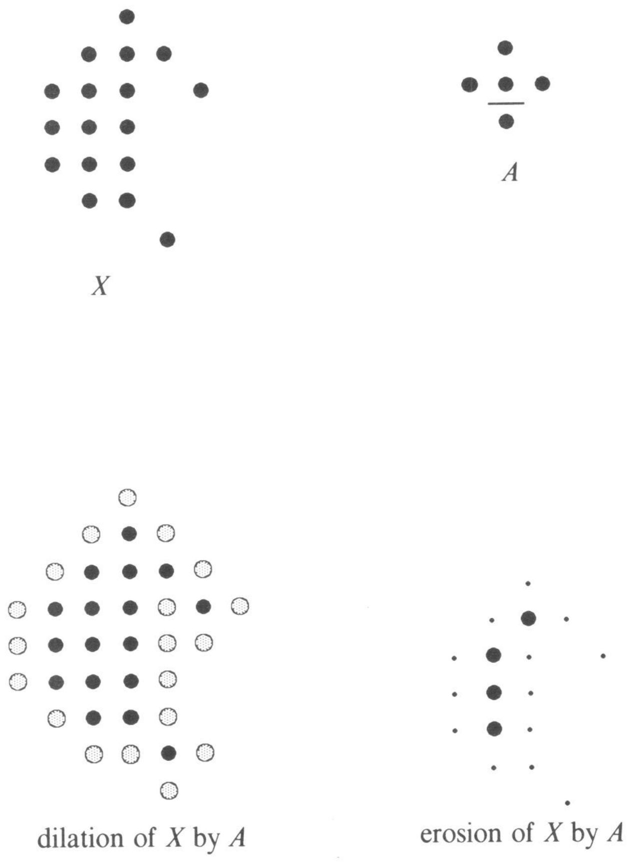


FIGURE 2. Dilation and erosion in the discrete case

● points which belong to X

◎ points which belong to $X \oplus \check{A}$ but not to X

· points which belong to X but not to $X \ominus \check{A}$

The underlining in A denotes the location of the origin.

One can easily prove the following algebraic relations:

$$(X\check{\oplus}A)\check{\oplus}B = X\check{\oplus}(A\oplus B)$$

$$(X\check{\ominus}A)\check{\ominus}B = X\check{\ominus}(A\oplus B)$$

$$X\check{\oplus}(A\cup B) = (X\check{\oplus}A)\cup(X\check{\oplus}B)$$

$$X\check{\ominus}(A\cup B) = (X\check{\ominus}A)\cap(X\check{\ominus}B).$$

These relations have the important practical implication that dilations and erosions with a structuring element which is too large to be handled by the hardware at one stage can be decomposed. Although it is true that dilation and erosion have a very simple algebraic structure, their importance is enormous. Perhaps this is most clearly illustrated by a theorem of MATHERON [9] which we state below. But first we give some definitions.

Let Ψ be a mapping from the object space $\mathcal{P}(E)$ into itself. We say that Ψ is *increasing* if

$$X\subset Y\Rightarrow\Psi(X)\subset\Psi(Y).$$

Note that dilation and erosion are increasing transformations. We call Ψ *translation invariant* if

$$\Psi(X_h) = (\Psi(X))_h,$$

for every $X\subset E$ and $h\in E$. The complementary (or dual) mapping Ψ^* of Ψ is defined by

$$\Psi^*(X) = (\Psi(X^c))^c.$$

The *kernel* $\mathcal{V}$ of a mapping Ψ is defined by

$$\mathcal{V} = \{A\subset E : 0\in\Psi(A)\}.$$

The kernel of the dual mapping Ψ^* is denoted by $\mathcal{V}^*$.

MATHERON'S THEOREM. *Let Ψ: $\mathcal{P}(E)\rightarrow\mathcal{P}(E)$ be an increasing, translation invariant mapping. Then*

$$\Psi(X) = \bigcup_{A\in\mathcal{V}}(X\check{\ominus}A) = \bigcap_{A\in\mathcal{V}^*}(X\check{\oplus}A),$$

for every $X\subset E$.

Note that the second equality follows from the first by duality. In Section 2.3 we shall prove an abstract version of Matheron's Theorem.

Two important increasing, translation invariant transformations on $\mathcal{P}(E)$ are the *closing* and the *opening*. The closing and the opening of a set X by a structuring element A are respectively defined by

$$X^A = (X \oplus \check{A}) \ominus A$$

$$X_A = (X \ominus \check{A}) \oplus A.$$

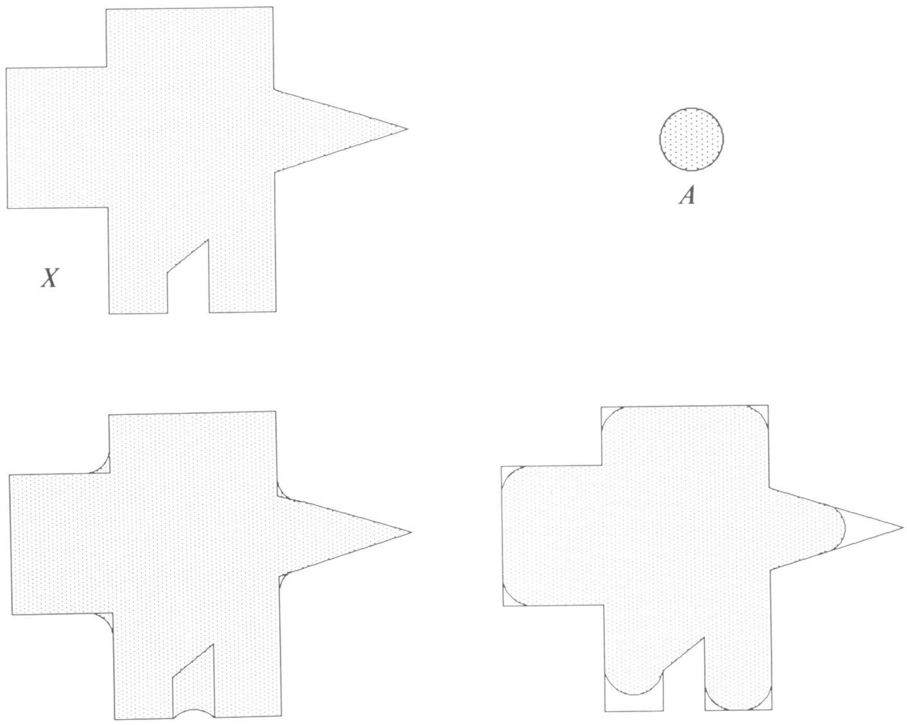
Closing and opening are complementary transformations. Some straightforward manipulations show that for every $X \subset E$:

$$X_A \subset X \subset X^A,$$

i.e., closing is an *extensive* operation whereas opening is *anti-extensive*. Furthermore, both operations are *idempotent*:

$$(X^A)^A = X^A, \ (X_A)_A = X_A.$$

Morphological transformations which are increasing and idempotent are sometimes called *morphological filters* or *M-filters*. Note the analogy with the ideal band-pass filter from classical signal analysis.



closing of X by A — opening of X by A

FIGURE 3. Closing and opening in the continuous case

We conclude this section by indicating an application of the opening. This operation makes it possible to define *size distributions*. This goes roughly as follows. An object built up of several smaller and larger isolated grains is put through a sequence of smaller and smaller sieves. Then a size distribution of X is given by the function $r \rightarrow \text{area}(X_{rA})$, where A is a compact convex structuring element (its shape may be chosen according to the shape of the grains), and $r > 0$ is a measure for the width of the sieve.

So far, the objects under study are considered as subsets of the continuous Euclidean space $\mathbb{R}^n$, or the discrete space $\mathbb{Z}^n$. Eventually, one is also interested in grey-valued images. Although such objects do not a priori fit into the framework, it is possible to extend the theory to account for them as well. There are at least two ways to do this. The first way is to represent each grey-valued image by a continuum of sets, the so-called cross sections. To every cross section one can apply the original morphological transformation, thus obtaining a new continuum of sets representing the transformed grey-valued image. The second way is to represent an image by its umbra (the graph together with all points in its shadow) which is a set again. To this set one can apply a morphological transformation yielding an umbra again, and from this the transformed grey-valued image is easily obtained. This is all we are going to say about this matter, and we refer the interested reader to Serra's book [12] and to a paper by STERNBERG [13]. For the rest of this paper we shall restrict to binary (i.e. black-white) images.

1.3. Morphological transformations

From Matheron's theorem we learned that dilation and erosion are very important transformations, since they are the building blocks of all increasing translation invariant transformations on $\mathcal{P}(E)$. A moment of reflection tells us that they also constitute the basis for all decreasing, translation invariant mappings. Namely, if $X \rightarrow \Phi(X)$ is decreasing (i.e., $X \subset Y \Rightarrow \Phi(Y) \subset \Phi(X)$), then the mapping $X \rightarrow \Phi(X^c)$ is increasing and Matheron's theorem yields that

$$\Phi(X) = \bigcup_{A \in \mathcal{W}} (X^c \ominus \check{A}),$$

where $\mathcal{W} = \{A \subset E : 0 \in \Phi(A^c)\}$.

An example of a transformation which needs to be neither increasing nor decreasing is the *hit-or-miss transformation*, which can e.g. be used to detect corner points of objects. Here the structuring element consists of two components A and B. Its definition goes as follows:

$$X \circledast [A, B] = \{h \in E : A_h \subset X \text{ and } B_h \subset X^c\} = (X \oplus \check{A}) \cap (X \oplus \check{B})^c.$$

We present an example in Figure 4.

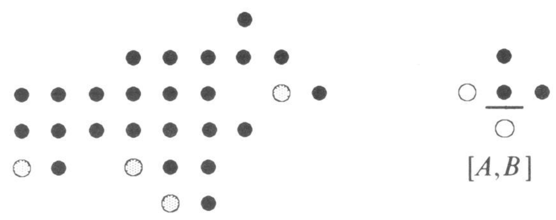


FIGURE 4. The hit-or-miss transformation can be used to detect corner points. ● and ◎ belong to X and ○ belongs to $X \circledast [A,B]$. The structuring element consists of a component A given by ● and B given by ○.

As a next step one can define the *thinning* $X \to X \setminus X \circledast [A,B]$ and the *thickening* $X \to X \cup (X \circledast [A,B])$. The thinning and thickening operation form the basis for a whole collection of algorithms which transform sets into figures with exotic names like *skeleton, homotopic pruning, skiz*, and *pseudo-convex hull.* We refer the inquisitive reader to chapter XI of Serra's book. At this place it is important to mention that Serra works on the hexagonal grid, and that he chooses the structuring elements accordingly.

The hit-or-miss transformation also forms the foundation for the definition of a topology on a space of subsets of E. A topology is indispensable to estimate errors committed in digitalizing images and to prove (or disprove) robustness of certain image transformations. Around 1974, G. MATHERON [9] and D.G. KENDALL [7], independently of each other, laid the foundations for a general theory of random sets, and it is not too surprising that these breakthroughs have had a strong impact on the development of mathematical morphology. We give a very short outline of Matheron's approach. Also see [1].

Let E be a topological space which is locally compact, Hausdorff, and separable (i.e., E admits a countable base). Of course, the example we have at the back of our mind is $E = \mathbb{R}^n$. We shall introduce a topology, the so-called hit-or-miss topology, on $\mathcal{F}(E)$, the space of all closed subsets of E, but we do not refrain from noting that we might as well have chosen the open subsets. Let $K \subset E$ be compact and $G \subset E$ open. We define

$$\mathcal{F}^K = \{X \in \mathcal{F}(E) \colon X \cap K = \varnothing\}$$

$$\mathcal{F}_G = \{X \in \mathcal{F}(E) \colon X \cap G \neq \varnothing\}.$$

The hit-or-miss topology on $\mathcal{F}(E)$ is defined by the base elements $\mathcal{F}^K \cap \mathcal{F}_{G_1} \cap \cdots \cap \mathcal{F}_{G_m}$, where K is compact, and G_i is open, $i = 1,...,m$. In other words, the sets $\mathcal{F}^K$ and $\mathcal{F}_G$ form a subbase for the hit-or-miss topology.

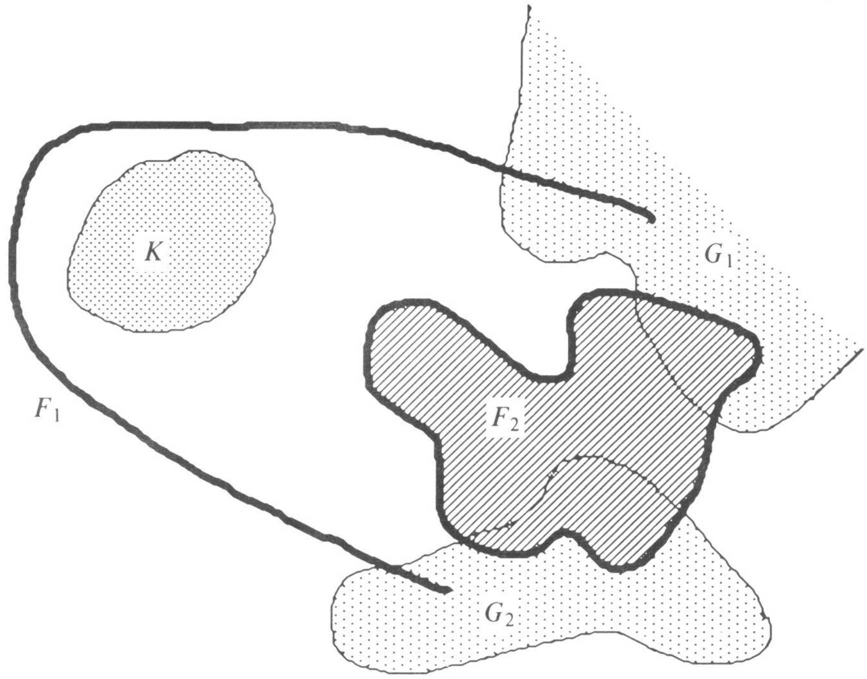


FIGURE 5. F_1 and F_2 both belong to the base element $\mathfrak{F}^K \cap \mathfrak{F}_{G_1} \cap \mathfrak{F}_{G_2}$

The space $\mathfrak{F}(E)$ equipped with the hit-or-miss topology is compact, Hausdorff, and separable. Note that E is not required to be compact but only locally compact: see MATHERON [9, Theorem 1.2.1]. A random closed set is by definition a random element of $\mathfrak{F}(E)$ with the Borel σ-algebra. In fact, every random closed set is specified by the probability distribution $p[K \cap X = \emptyset]$ where K ranges over all compact subsets of E.

Let ψ be a mapping from an arbitrary topological space S into $\mathfrak{F}(E)$. Then ψ is upper-semi-continuous (u.s.c.) if for any compact set $K \subset E$, the set $\psi^{-1}(\mathfrak{F}^K)$ is open in S. Analogously, ψ is lower-semi-continuous (l.s.c.) if for any open set $G \subset E$, the set $\psi^{-1}(\mathfrak{F}_G)$ is open in S. If the topological space S admits a countable base, in particular if $S = \mathfrak{F}(E)$, then there exist some easy criteria for upper- and lower-semi-continuity: see [9], [12]. For the basic transformations of mathematical morphology, MATHERON [9] has obtained the following continuity results:

(i) $X \to X \oplus \check{A}$ is continuous on $\mathfrak{F}(E)$ if A is compact

(ii) $X \to X \ominus \check{A}$, $X \to X^A$, and $X \to X_A$ are upper-semi-continuous if A is compact.

Actually, Matheron proved a much stronger result.

In Chapter I of his book [12], Serra treats at length four principles which, according to his philosophy, every transformation has to satisfy in order to get the predicate 'morphological'. These principles, which we discuss below, are unmistakably inspired by practical considerations.

The first principle, concerning translation invariance, excludes transformations which require knowledge of the position of the object of interest. In mathematical terms:

$$\Psi(X_h) = (\Psi(X))_h. \tag{I}$$

Frequently, an object has to be magnified or reduced before one can work with it. For transformations one wants to apply, this means that they have to be compatible under change of scale. Denoting the transformation by Ψ_λ, where λ is the scale parameter, we can write the second principle abstractly as

$$\Psi_\lambda(\lambda X) = \lambda\Psi_1(X), \tag{II}$$

where $\lambda X = \{\lambda x : x \in X\}$.

The third principle says that local knowledge of the object is sufficient to obtain local knowledge about the transformed image:

$$\forall_X \forall_{\text{bounded } Z'} \exists_{\text{bounded } Z} : [\Psi(X \cap Z)] \cap Z' = \Psi(X) \cap Z'. \tag{III}$$

Note that this definition allows that Z depends on X: in practical cases this will almost never occur.

The last principle says something about stability of the transformation:

$$\Psi \text{ is semi-continuous with respect to the hit-or-miss topology.} \tag{IV}$$

Note that this last principle implicitly assumes that Ψ maps closed sets on closed sets.

The basic transformations dilation, erosion, closing and opening indeed satisfy the four principles if the structuring element is compact and nonempty. As far as the applications are concerned, these principles are quite satisfactory. But they also evoke a number of questions in a theoretician's mind. Let us state some of them. (1) Is it possible to give a complete characterization of all morphological transformations? Matheron's theorem only gives a partial answer to this question. (2) As we already mentioned, the fourth principle includes the assumption that the object space should be $\mathcal{F}(E)$ instead of $\mathcal{P}(E)$. But the algebraic structure of these two spaces are completely different (see Part 2 below). For example, we do not have a natural complement on $\mathcal{F}(E)$, which means in particular that the definition of the hit-or-miss transformation needs to be adapted. (3) The specific structure of dilation and erosion shows that translation plays a very special role. Why is this? Can this role be assigned to another group operation on E, rotation for instance? Note that rotation invariance is not included in the four principles.

These and other questions have motivated us to look for a more abstract approach, not so much because we expect new applications, but merely because we hope that such an approach gives a better understanding.

2. TOWARDS AN ALGEBRAIC APPROACH

2.1. *Some basic results from lattice theory*

In this preliminary section we survey some of the basic results on lattices. For a complete account of the theory we refer to the monographs of BIRKHOFF [2] and GRÄTZER [6].

A set L with a partial ordering relation $\leqslant$ is called a *lattice* if for any finite nonempty subset K of L the *supremum* $\vee K$ and *infimum* $\wedge K$ exist. Recall that $a \in L$ is called the supremum of K if $x \leqslant a$ for every $x \in K$ and if $a \leqslant a'$ for any other such element a'. A similar definition holds for the infimum. We shall write $x \vee y$ instead of $\vee\{x,y\}$ and $x \wedge y$ instead of $\wedge\{x,y\}$. It is easily seen that

$$x \leqslant y \Leftrightarrow x \wedge y = x \Leftrightarrow x \vee y = y. \tag{2.1}$$

We write $x < y$ if $x \leqslant y$ and $x \neq y$. A lattice L can contain at most one element a which satisfies $a \leqslant x$ for all $x \in L$. If such an element exists we denote it by 0 and call it the *zero* of L. Similarly, there can exist at most one element b such that $x \leqslant b$ for all $x \in L$. Such an element, if present, is called the *unit* of L and is denoted by 1. A lattice with a zero and a unit is called *bounded*. The lattice is called *distributive* if

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) \tag{2.2a}$$

$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z), \tag{2.2.b}$$

for every $x,y,z \in L$. Let L be a bounded lattice. We say that x possesses a *complement* y if

$$x \wedge y = 0, \quad x \vee y = 1. \tag{2.3}$$

The bounded lattice L is called *complemented* if any of its elements has a complement. It is an easy exercise to show that in a bounded distributive lattice an element x can have at most one complement which is then denoted by x^*.

DE MORGAN'S IDENTITIES. *Let x,y be elements of the bounded, distributive lattice L with complements x^* and y^* respectively. Then $x \vee y$ and $x \wedge y$ possess complements as well, and*

$$(x \vee y)^* = x^* \wedge y^*$$

$$(x \wedge y)^* = x^* \vee y^*.$$

A complemented, distributive lattice is called a *Boolean lattice*. Every Boolean lattice can be considered as an algebra with the binary operations $\vee$ and $\wedge$, and the unary operation $*$. Considered this way, L is called a *Boolean algebra*. In a number of cases the lattice L is only 'half-complemented' in the sense that only one of the relations in (2.3) is satisfied. A *Brouwerian lattice* is a lattice L in which for every couple, $a,b \in L$ the set $\{x : a \wedge x \leqslant b\}$ contains a greatest element $b:a$, the *relative pseudo-complement of a in b*: below we shall present an example. If L is a Boolean lattice, then, of course, $b:a = b \vee a^*$. In a

Brouwerian lattice with a zero, the element $x^* = 0{:}x$ is called the *pseudo-complement* of x. Note that, by definition, x^* is uniquely defined. A theorem in [2] says that every Brouwerian lattice is distributive. It is not hard to figure out how dual Brouwerian lattices should be defined.

A lattice L is called *complete* if any subset K (finite or infinite) has a supremum and an infimum. If L is a nonempty complete lattice then one gets, by taking $K = L$, that L has a zero and a unit. It is easily deduced from (2.2) that in any distributive lattice L the relations

$$x \wedge (\vee_{i \in I} x_i) = \vee_{i \in I} (x \wedge x_i) \tag{2.4a}$$

$$x \vee (\wedge_{i \in I} x_i) = \wedge_{i \in I} (x \vee x_i) \tag{2.4b}$$

are valid for any finite index set I. In a complete Boolean lattice these relations hold for any infinite index set as well. A lattice in which (2.4a) is valid for an arbitrary index set is called *infinite-supremum-distributive*, whereas the lattice is called *infinite-infimum-distributive* if (2.4b) holds. It is relatively easy to show that a complete lattice is Brouwerian if and only if it is infinite-supremum-distributive, and in that case $b{:}a = \vee\{x : a \wedge x \leqslant b\}$.

Let L be a lattice with a zero. An element $\xi \in L$ is called an *atom* if $x < \xi$ implies that $x = 0$. Analogously, an element ξ' of a lattice with a unit is called a *dual atom* if $\xi' < x$ implies that $x = 1$. Atoms are denoted by Greek symbols and dual atoms by Greek symbols with a prime. We denote the set of all atoms by Λ. An *atomic lattice* is a lattice in which every element is the supremum of the atoms it dominates, i.e.,

$$x = \vee_{\xi \leqslant x} \xi.$$

Similarly, we define dually atomic lattices.

The reader who wishes to know more about lattices and the relation with set theory may consult [2,5,6,8]. For those who had enough, we present some examples. It goes without saying that our choice is highly influenced by the application we have in mind.

EXAMPLES

(a) Let E be some nonempty set. Then $\mathcal{P}(E)$ is a complete lattice with the partial ordering: $X \leqslant Y$ if $X \subset Y$, i.e., X is included in Y. The supremum and infimum correspond to the union and intersection respectively. With the set complement $\mathcal{P}(E)$ becomes a Boolean lattice. Moreover, $\mathcal{P}(E)$ is atomic where the atoms are of the form $\{e\}$, where $e \in E$. At this point we mention the following important general result. Every complete, atomic Boolean lattice L is isomorphic to the field $\mathcal{P}(\Lambda)$, where Λ is the set of all atoms of L.

(b) If E is a nonempty topological space, then we denote by $\mathcal{F}(E)$ the space of all closed subsets of E. If we define (a 'bar' denoting closure)

$$\wedge_{i \in I} X_i = \bigcap_{i \in I} X_i$$

$$\bigvee_{i \in I} X_i = \overline{\bigcup_{i \in I} X_i},$$

for an arbitrary index set I and arbitrary elements $X_i \in \mathfrak{F}(E)$, then $\mathfrak{F}(E)$ is a complete, distributive lattice which is infinite-infimum-distributive. Moreover, if E is a T_1-space (i.e., every singleton $\{e\}$ with $e \in E$ is closed), then $\mathfrak{F}(E)$ is atomic. In this case, BIRKHOFF [2] calls $\mathfrak{F}(E)$ a T_1-lattice.

The space $\mathfrak{G}(E)$ of all open subsets of the topological space E is a complete, distributive lattice which is infinite-supremum-distributive, hence $\mathfrak{G}(E)$ is a Brouwerian lattice with pseudo-complement $X^* = (\overline{X})^c$. If $X = X^{**}$, then we call X a *regular open set*. We leave it as an exercise to the reader to verify that the space of all regular open subsets forms a complete Boolean lattice.

(c) As a final example we mention the lattice consisting of all functions f mapping a set E into the closed interval $[0,1]$, with the pointwise ordering:

$$f \leqslant g \Leftrightarrow f(x) \leqslant g(x), \quad \forall_{x \in E}.$$

Note that this lattice is relevant in the context of grey-value images. The supremum and infimum are respectively defined by $(f \vee g)(x) = \max\{f(x), g(x)\}$ and $(f \wedge g)(x) = \min\{f(x), g(x)\}$. It is obvious that this lattice is complete and distributive. Furthermore, it is worth noticing that the lattice of example (a) lies embedded in the present one, where the embedding operation is given by $X \rightarrow \mathbf{1}_X$, $X \subset E$. Here $\mathbf{1}_X$ is the characteristic function corresponding to the set X.

The remainder of this section is devoted to lattice morphisms. Let L be a lattice. A mapping f from L into L is called a *(lattice) endomorphism* if f preserves finite infima and suprema, i.e.,

$$f(x \vee y) = f(x) \vee f(y) \tag{2.5a}$$

$$f(x \wedge y) = f(x) \wedge f(y) \tag{2.5b}$$

for every $x, y \in L$. If, in addition, f is a bijection, then f is called an *automorphism.* In that case f^{-1}, the inverse mapping, also satisfies (2.5). Suppose that f is an automorphism. If L is a bounded lattice then $f(0) = 0$ and $f(1) = 1$. If, moreover, L is a Boolean lattice, then f also preserves complements:

$$f(x^*) = (f(x))^*. \tag{2.6}$$

Finally, if L is a complete lattice then the relations (2.5) remain valid for infinite suprema and infima:

$$f(\bigvee_{i \in I} x_i) = \bigvee_{i \in I} f(x_i)$$

$$f(\bigwedge_{i \in I} x_i) = \bigwedge_{i \in I} f(x_i).$$

Every mapping $f: L \rightarrow L$ satisfying at least one of the relations (2.5a), (2.5b) is increasing, i.e.,

$$x \leqslant y \Rightarrow f(x) \leqslant f(y). \tag{2.7}$$

The converse does not hold.

For future use we state the following lemma.

LEMMA 1. *Let L be a lattice with a zero, and let Λ be the (possibly empty) set of atoms. If f is an automorphism on L, then f leaves Λ invariant.*

PROOF. If Λ is empty then the lemma is trivially satisfied. So assume that $\Lambda \neq \varnothing$, and take $\xi \in \Lambda$. We must show that $f(\xi) \in \Lambda$. Assume that there is a $y \in L$ such that $y < f(\xi)$. Then $f^{-1}(y) < \xi$, hence $f^{-1}(y) = 0$. But this implies that $y = 0$. Thus $f(\xi)$ is an atom. □

2.2. Dilation and erosion

In this section we shall give an abstract definition of dilation and erosion on an arbitrary complete lattice. In Section 1.2 we have considered dilation and erosion on the complete Boolean lattice $\mathcal{P}(E)$, where E was $\mathbb{R}^n$ or $\mathbb{Z}^n$. We recall that two basic properties of dilation were:

(i) $(T_h X) \check{\oplus} A = T_h(X \check{\oplus} A)$

(ii) $(\bigcup_{i \in I} X_i) \check{\oplus} A = \bigcup_{i \in I} (X_i \check{\oplus} A)$,

where $T_h X = X_h$, i.e., T_h is translation along a vector $h \in E$. We note that the family of translations $\mathcal{T} = \{T_h : h \in E\}$ forms a commutative group of automorphisms on the lattice $\mathcal{P}(E)$. Erosion is characterized by similar properties, the only difference being that in (ii) union has to be replaced by intersection. These two properties of dilation and erosion are used as the premises for an abstract definition. Assume for the remainder of this section that L is a complete lattice. Let $\mathcal{T}$ be a commutative group of automorphisms on L. For notational convenience we shall write Tx instead of $T(x)$ if $T \in \mathcal{T}$. A mapping $\psi : L \to L$ is called a *$\mathcal{T}$-mapping* if ψ commutes with every T:

$$\psi(Tx) = T\psi(x), \quad T \in \mathcal{T},\ x \in L.$$

We say that ψ is a *$\mathcal{T}$-dilation* if

(i) ψ is a $\mathcal{T}$-mapping

(ii) $\psi(\bigvee_{i \in I} x_i) = \bigvee_{i \in I} \psi(x_i)$,

for every index set I. Similarly, a mapping $\phi : L \to L$ is called a *$\mathcal{T}$-erosion* if in (ii) the supremum is replaced by the infimum.

If L is a Boolean lattice, then the dual of a $\mathcal{T}$-dilation is a $\mathcal{T}$-erosion and conversely (the dual f^* of a mapping f on a complemented lattice is defined by $f^*(x) = (f(x^*))^*$). Let $\mathcal{A}$ be an arbitrary subset of $\mathcal{T}$. It is easy to check that the mapping

$$\psi(x) = \bigvee_{T \in \mathcal{A}} Tx \tag{2.8}$$

is a $\mathcal{T}$-dilation and that

$$\phi(x) = \bigwedge_{T \in \mathcal{A}} Tx \tag{2.9}$$

is a $\mathfrak{T}$-erosion. Note that these expressions are nothing but straightforward generalizations of the original definitions: see Section 1.2. For the rest of this section we will restrict our attention to $\mathfrak{T}$-dilations. It should be clear by now that dilation and erosion are just complementary notions. We address ourselves to the following question: is every $\mathfrak{T}$-dilation of the form (2.8)? It turns out that we can give an affirmative answer to this question under two extra assumptions.

ASSUMPTION. L is atomic.

ASSUMPTION. For every couple $\xi, \eta \in \Lambda$ there is a $T \in \mathfrak{T}$ such that $T\xi = \eta$.

If the latter assumption is satisfied, we call the automorphism group *total*. Now let $\psi: L \rightarrow L$ be a $\mathfrak{T}$-dilation. Define $\mathfrak{A} \subset \mathfrak{T}$ by

$$T \in \mathfrak{A} \Leftrightarrow T\xi \leqslant \psi(\xi),$$

where ξ is an arbitrary atom of L. By using that ψ is a $\mathfrak{T}$-mapping and that $\mathfrak{T}$ is total, one easily obtains that $\mathfrak{A}$ is independent of the choice of ξ. Furthermore, one gets immediately that

$$\bigvee_{T \in \mathfrak{A}} T\xi \leqslant \psi(\xi), \quad \xi \in \Lambda.$$

We can even show equality. Suppose, namely, that we have strict inequality. Then, since L is atomic, there exists an atom η such that $\eta \leqslant \psi(\xi)$ but not $\eta \leqslant \bigvee_{T \in \mathfrak{A}} T\xi$. From the fact that $\mathfrak{T}$ is total we know that $\eta = T'\xi$ for some $T' \in \mathfrak{T}$. Hence $T'\xi \leqslant \psi(\xi)$, yielding that $T' \in \mathfrak{A}$. But this implies that $\eta \leqslant \bigvee_{T \in \mathfrak{A}} T\xi$, a contradiction. Thus we have proved that

$$\bigvee_{T \in \mathfrak{A}} T\xi = \psi(\xi), \quad \xi \in \Lambda.$$

But now we are almost done. Consider namely an arbitrary element x of L. Then $x = \bigvee_{\xi \leqslant x} \xi$. Thus

$$\begin{aligned} \psi(x) &= \psi(\bigvee_{\xi \leqslant x} \xi) = \bigvee_{\xi \leqslant x} \psi(\xi) = \bigvee_{\xi \leqslant x} \bigvee_{T \in \mathfrak{A}} T\xi \\ &= \bigvee_{T \in \mathfrak{A}} \bigvee_{\xi \leqslant x} T\xi = \bigvee_{T \in \mathfrak{A}} T(\bigvee_{\xi \leqslant x} \xi) = \bigvee_{T \in \mathfrak{A}} Tx. \end{aligned}$$

This proves the main result of this section.

THEOREM 1. *Let L be a complete, atomic lattice and let $\mathfrak{T}$ be a total commutative group of automorphisms on L. Then every $\mathfrak{T}$-dilation ψ is of the form*

$$\psi(x) = \bigvee_{T \in \mathfrak{A}} Tx.$$

We can state a similar result for erosions on dually atomic lattices. Notice that $\mathfrak{A}$ is the analogue of the structuring element of Section 1.2. It is time to give some examples.

EXAMPLES

(a) Consider the complete atomic (and dually atomic) Boolean lattice $\mathcal{P}(E)$, where E is $\mathbb{R}^n$ or $\mathbb{Z}^n$. Let $\mathcal{T}$ be the group of all translations T_h, $h \in E$. Then every $\mathcal{T}$-dilation is of the form

$$\Psi(X) = \vee_{T \in \mathcal{A}} TX,$$

or equivalently,

$$\Psi(X) = \bigcup_{h \in A} T_{-h} X = \bigcup_{h \in A} X_{-h} = X \oplus \check{A},$$

where $A \subset E$ is given by: $h \in A$ if and only if $T_{-h} \in \mathcal{A}$. So in this case the class of all $\mathcal{T}$-dilations (and of course of $\mathcal{T}$-erosions) coincides with the original class.

In exactly the same way we obtain a complete characterization of $\mathcal{T}$-dilations on the complete, atomic lattice $\mathcal{F}(\mathbb{R}^n)$, where $\mathcal{T}$ is again the translation group. In this case every $\mathcal{T}$-dilation Ψ is of the form

$$\Psi(X) = \overline{\bigcup_{h \in A} X_{-h}}.$$

By duality, we also get a complete characterization of $\mathcal{T}$-erosions on the complete, dually atomic lattice $\mathcal{G}(\mathbb{R}^n)$.

(b) An advantage of our approach is that we are free to choose any automorphism group we want to: it is only required that this group is commutative and total. An interesting example is provided by the rotation-multiplication group. Consider the complete, atomic Boolean lattice $\mathcal{P}(\mathbb{C} \setminus \{0\})$. Let '$\cdot$' be the complex multiplication on $\mathbb{C}$. Let T_z be the automorphism given by

$$T_z X = \{x \cdot z : x \in X\}.$$

If $z = re^{i\theta}$ in polar coordinates, then T_z can be interpreted as a combination of a rotation by an angle θ and a multiplication with factor r. Then $\mathcal{T} = \{T_z : z \in \mathbb{C} \setminus \{0\}\}$ forms a commutative automorphism group which is total. Needless to say that the performance of dilation and erosion in this example is completely different from the classical situation. In the discrete case a polar grid is required: see Figure 6 below.

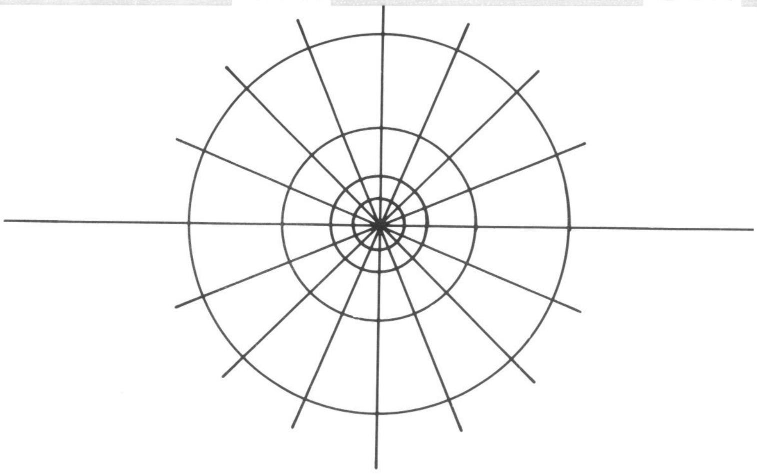


FIGURE 6. The polar grid of example (b)

(c) As a final example, we mention the following variant of example (b). Consider the Boolean lattice $\mathcal{P}(\mathbb{C})$ and the total commutative automorphism group $\mathcal{T}=\{T_z: z\in\mathbb{C}\}$ defined by

$$T_zX = \{x \circ z: x\in X\},$$

where $z_1 \circ z_2 = (r_1+r_2)e^{i(\theta_1+\theta_2)}$, if $z_j = r_je^{i\theta_j}$.

2.3. Increasing transformations and Matheron's theorem

From Matheron's theorem we learned that every increasing translation invariant transformation on $\mathcal{P}(E)$, where E is $\mathbb{R}^n$ or $\mathbb{Z}^n$, can be written as an intersection of dilations, or, alternatively, as a union of erosions. In the present section we will show that this result can be established within our framework. But before stating and proving this generalization, we present an alternative formulation of the results obtained in the previous section. Actually, this reformulation is suggested by the examples above. In these examples the automorphism group $\mathcal{T}$ is isomorphic with a group structure on Λ, the set of all atoms. This is no coincidence but just an alternative formulation of our second assumption. To see this, assume that L is a complete atomic lattice, and that $\mathcal{T}$ is a total commutative automorphism group on L. First we note that, if for some $T_0\in\mathcal{T}$ and some $\xi\in\Lambda$ we have $T_0\xi=\xi$, then this holds for every $\eta\in\Lambda$, which amounts to saying that T_0 is the identity mapping. Suppose namely that $\eta\in\Lambda$. Then there is a $T\in\mathcal{T}$ so that $\eta=T\xi$. Hence $T_0\eta=T_0T\xi=TT_0\xi=T\xi=\eta$.

Now fix an arbitrary $\omega\in\Lambda$. We call ω the origin. For every $\xi\in\Lambda$ there exists a $T_\xi\in\mathcal{T}$ such that $T_\xi\omega=\xi$. Thus we can define an operation $+$ on Λ as follows:

$$\xi+\eta = T_\xi T_\eta\omega, \quad \xi,\eta\in\Lambda.$$

This definition makes sense because it is independent of the particular choice of T_ξ. It is easy to see that $(\Lambda, +)$ is a commutative group with identity ω. Conversely, every commutative group operation $+$ on Λ 'generates' a total commutative automorphism group on L. Let $-\xi$ denote the inverse of ξ with respect to the group operation $+$.

It should be clear by now how one can rewrite (2.8) and (2.9) in terms of the group operation $+$. Let ψ be given by (2.8) and define $a \in L$ by: $a = \bigvee_{T \in \mathcal{T}} T^{-1}\omega$. Then

$$\psi(x) = x \check{\oplus} a := \bigvee_{\alpha \leqslant a} x_{-\alpha} = \bigvee\{\xi\colon a_\xi \wedge x \neq 0\}, \quad x \in L.$$

Here $x_\alpha = \{\xi + \alpha\colon \xi \leqslant x\}$. Similarly the $\mathcal{T}$-erosion of (2.9) can be written as:

$$\phi(x) = x \check{\ominus} a := \bigwedge_{\alpha \leqslant a} x_{-\alpha} = \bigvee\{\xi\colon a_\xi \leqslant x\}, \quad x \in L.$$

Before we give the abstract version of Matheron's theorem, we recall that a mapping $f\colon L \to L$ is increasing if $x \leqslant y$ implies that $f(x) \leqslant f(y)$.

THEOREM 2. *Let L be a complete, atomic lattice, and let $f\colon L \to L$ be an increasing $\mathcal{T}$-mapping, then*

$$f(x) = \bigvee_{\alpha \in \mathcal{V}} (x \check{\ominus} a),$$

where $\mathcal{V} = \{a \in L\colon \omega \leqslant f(a)\}$ is the kernel of f.

PROOF. We show that $\xi \leqslant f(x)$ if and only if $\xi \leqslant \bigvee_{a \in \mathcal{V}} (x \check{\ominus} a)$.

(i) Let $\xi \leqslant f(x)$. Then $\omega \leqslant T_{-\xi} f(x) = f(T_{-\xi} x)$. Hence $y := T_{-\xi} x \in \mathcal{V}$ by definition. Thus

$$\bigvee_{a \in \mathcal{V}} (x \check{\ominus} a) \geqslant x \check{\ominus} y = y_\xi \check{\ominus} y = \bigwedge_{\eta \leqslant y} (y_\xi)_{-\eta}$$
$$= \bigwedge_{\eta \leqslant y} (y_{-\eta})_\xi = T_\xi(\bigwedge_{\eta \leqslant y} y_{-\eta}) \geqslant T_\xi \omega = \xi.$$

(ii) Conversely, assume that $\xi \leqslant \bigvee_{a \in \mathcal{V}} (x \check{\ominus} a)$. So there is an element $a \in \mathcal{V}$ such that

$$\xi \leqslant x \check{\ominus} a = \bigwedge_{\eta \leqslant a} x_{-\eta}.$$

Therefore $\xi \leqslant x_{-\eta}$ for every η satisfying $\eta \leqslant a$. But this implies that $\eta \leqslant x_{-\xi}$, for every $\eta \leqslant a$. Thus

$$a = \bigvee_{\eta \leqslant a} \eta \leqslant x_{-\xi},$$

and by the increasingness of the mapping f, $f(a) \leqslant f(x_{-\xi})$. Since $a \in \mathcal{V}$, we find that $\omega \leqslant f(a) \leqslant f(x_{-\xi})$, hence $\xi = T_\xi \omega \leqslant T_\xi f(x_{-\xi}) = f(x)$, which proves the result. □

Similarly one can show that on a complete, dually atomic lattice every increasing transformation can be written as the infimum of $\mathcal{T}$-dilations. On a complete, atomic Boolean lattice both characterizations hold.

2.4. Concluding remarks

As a first step towards an abstract algebraic approach, the results obtained so far may seem quite satisfactory. However, as we will indicate below, a lot remains to be done. But let us first give a brief summary of our results.

If L is a complete, atomic lattice, e.g. $L=\mathcal{P}(\mathbb{R}^n)$, then every $\mathcal{T}$-dilation ψ is of the form

$$\psi(x) = x \oplus a, \quad x \in L,$$

for some $a \in L$. Furthermore, every increasing $\mathcal{T}$-mapping is a supremum of $\mathcal{T}$-erosions. Similarly, if L is a complete, dually atomic lattice, e.g. $L=\mathcal{G}(\mathbb{R}^n)$, then every $\mathcal{T}$-erosion takes the form

$$\phi(x) = x \ominus a, \quad x \in L,$$

for some $a \in L$, and every increasing $\mathcal{T}$-mapping is an infimum of $\mathcal{T}$-dilations. These results become more transparent if L is a complete Boolean lattice. In that case, the assumption that L is atomic is equivalent to the assumption that L is dually atomic, and $\mathcal{T}$-dilations and $\mathcal{T}$-erosions are dual mappings. We recall that a complete, atomic Boolean lattice L is isomorphic with the field $\mathcal{P}(\Lambda)$, where Λ is the set of all atoms, and that every total, commutative automorphism group on L induces a group structure on Λ. Thus, algebraically speaking, there is no distinction between this case and the original case described in Part 1 where $L=\mathcal{P}(\mathbb{R}^n)$: see also Section 2.2, Example (a).

Let L be a complete lattice and let $\mathcal{T}$ be a total, commutative automorphism group on L. We define $M_{\mathcal{T}}^{+}(L)$ as the set of all increasing $\mathcal{T}$-mappings on L. Besides $\mathcal{T}$-dilations and $\mathcal{T}$-erosions, this set also contains compositions of these transformations such as $\mathcal{T}$-closings and $\mathcal{T}$-openings. On the set $M_{\mathcal{T}}^{+}(L)$ we can define the partial order $\leqslant$ by:

$$f \leqslant g \iff \forall_{x \in L}: f(x) \leqslant g(x).$$

Then $M_{\mathcal{T}}^{+}(L)$ becomes a complete lattice with supremum and infimum respectively given by

$$(f \vee g)(x) = f(x) \vee g(x), \quad x \in L$$

$$(f \wedge g)(x) = f(x) \wedge g(x), \quad x \in L.$$

These observations imply that every $\mathcal{T}$-mapping which is obtained from $\mathcal{T}$-dilations and $\mathcal{T}$-erosions by means of suprema, infima, and compositions is increasing: no such thing as the hit-or-miss transformation can be obtained in this way. If, however, L is a Boolean lattice then the mapping $x \rightarrow f(x^*)$ is a decreasing $\mathcal{T}$-mapping if $f \in M_{\mathcal{T}}^{+}(L)$. Replacing the complement by the pseudo-complement, we can do the same trick if L is a Brouwerian lattice.

In Section 1.3 we have argued, following Serra, that a theory of transformations is not very meaningful if one cannot give shape to the notion of (semi-) continuity, which requires a topology on the lattice L. In the second part of this paper we have consistently omitted to speak about topological aspects. Here we shall somewhat retrieve this omission by mentioning in a few lines an important result that can be found in the literature. We will certainly come back to this point in the future. It needs no explanation that a topology on L has to be related to the ordering relation, and that the automorphism group $\mathcal{T}$ should have the right continuity properties with respect to this topology.

The lattice $\mathcal{F}(\mathbb{R}^n)$ of all closed subsets of $\mathbb{R}^n$ with the opposite ordering ($X \leqslant Y$ if $Y \subset X$) is a so-called *continuous lattice*. On a continuous lattice one can define the so-called *Lawson topology*. On the lattice $\mathcal{F}(\mathbb{R}^n)$ this topology coincides with the hit-or-miss topology (see [5] for more details). This observation which we consider to be an extra justification of our approach, may serve as an underlining of the assertion that thinking about mathematical generalizations is not only a pleasant pass-time (it is, of course), but may also give a deeper understanding of the original theory.

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frontiers in Computing

An International Conference, December 9-11-1987
Amsterdam, The Netherlands

During this conference various trends in Parallel Computing as well as in Optical, Molecular and Bio-Computing will be introduced. The impacts of these developments in the medium and long term will be assessed. Trends such as these pose questions as to the usefulness of the increased processing power generated, e.g.: *'Who is producing the environments and software systems for making 100 MIPS workstations and 100 GigaFLOPS Supercomputers useful?'* Discussions of such problems will be given appropriate priority at the conference.

TOPICS OF THE CONFERENCE
- Non-Conventional Architectures
- Architectures for Neuro- and Supercomputing
  - Bio- and Molecular Components
  - Optical Components
- New Architectures for large systems: Networks
- Computing needs of Society and the impact on software
- Comparison of IT programmes

MOTO-OKA MEMORIAL SESSIONS
Two special sessions will be organized to commemorate the late Professor Tohru Moto-oka (1929-1985). His work was of crucial importance for developing new computer architectures, in particular the Fifth Generation Computer. He had the foresight of a new computer system in which new hardware techniques (VLSI) and sophisticated software developments (Logic Programming) are genuinely combined into intelligent systems.

The programme includes two panels, on Comparison of IT Programmes and on Future Networks for Research, as well as tutorials, workshops and a commercial exhibition.

FIC SCIENTIFIC SECRETARIAT
Centrum voor Wiskunde en Informatica (CWI), attention Ms. J. Clement, P.O. Box 4079, 1009 AB Amsterdam, The Netherlands; telephone: +31 (20) 592 4171, telex: 12571 mactr nl.

Concurrency

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The Concurrency Project is devoted to the study of mathematical models of *parallelism.* Whereas a *sequential* computation runs on one processor, a parallel computation assumes a number of processors which interact in a variety of ways depending upon the particular situation. At the macroscopic level, one often speaks of *distributed systems*, and examples one may think of are networks of computers or distributed databases. At the microscopic level, parallel architectures are the prime example, where sometimes thousands of, mostly identical, processors cooperate in computations the organization of which is a major focus of current algorithmic research.

The emphasis in the project in our department (Department of Software Technology, CWI) is on *languages* for parallelism and, more specifically, on formal models which are exploited in the mathematical analysis of parallel programming concepts. The project participates in two collaborative efforts, viz. in ESPRIT project 415 and in the Dutch National Concurrency Project. Moreover, part of its foundational work is performed in a fruitful cooperation with the Free University of Amsterdam, the University of Kiel (FRG) and the State University of New York at Buffalo.

1. ESPRIT PROJECT 415

ESPRIT is the European Strategic Program for Research and Development in Information Technology. In project 415, six major European industries, with Philips as prime contractor, work together on 'Parallel Architectures and Languages for Advanced Information Processing: a VLSI Directed Approach'. Six subprojects investigate parallel architectures based on a variety of parallel programming styles such as object oriented, functional and dataflow, and logic programming. Moreover, the project has two working groups which provide the subprojects with a forum for integrating their activities in the areas of

Semantics and Proof Techniques, and of Architecture and Applications.

CWI is involved in project 415 as subcontractor of Philips. We firstly participate in the research concerned with the mathematical modelling of POOL. This is an acronym for Parallel Object Oriented Language, the language developed by Philips as conceptual starting point for the architectural design of its DOOM (Distributed Object Oriented Machine) system. So far, we have jointly designed an operational and denotational semantics for POOL [2,3], and we have made initial steps towards the design of a proof theory, i.e. a method for formally deriving properties of programs, for POOL [9]. Secondly, the Concurrency project participates in the activities of the Working Group on Semantics and Proof Techniques, and we coordinate the production of the yearly deliverable of the Working Group [10].

It is an important part of the policy of project 415 to spend substantial effort on educational and promotional activities, and the CWI has contributed to the organization of such events. The proceedings of the Advanced School on Current Trends in Concurrency (organized for ESPRIT 415 by the LPC, see Section 2 below) have appeared in [8]. Project 415 also sponsored the highly successful PARLE conference - Parallel Architectures and Languages Europe, June 1987. The PARLE proceedings were published in [6,7].

2. The Dutch National Concurrency Project

Under the joint direction of J.W. de Bakker, W.P. de Roever (Technological University Eindhoven) and G. Rozenberg (Leiden University), the Dutch National Concurrency Project (LPC abbreviating its Dutch name Landelijk Project Concurrency) has been operative since the beginning of 1984. The project is primarily sponsored by SION, the Research Foundation for Computer Science which is part of the Netherlands Organization for the Advancement of Pure Research (ZWO). The project concentrates on syntactic, semantic and proof theoretic aspects of Concurrency, and it has from its inception given great weight to effective cooperation with industrial research (in particular Philips and Shell laboratories). The section which concentrates on semantics of concurrency is located at CWI. J.N. Kok performs research on the semantics of nondeterministic dataflow, a computational model which underlies an important branch of contemporary research in parallel architectures. A recent achievement was the design of a 'fully abstract' mathematical model: briefly, this qualification amounts to the fact that the model has the right level of detail seen from the point of view of an observer of the behaviour of a dataflow net.

LPC has gradually become involved in a number of affiliated activities. A list of these activities includes: monthly 'concurrency days', devoted to tutorial and specialized presentations of research within the scope of LPC, visiting professors supported by the Dutch National Facility for Informatics, cooperation with ESPRIT projects 415 and 937 (Descartes: Debugging and Specification of ADA Real Time Systems), and formalized contacts with two other national programs: the French C^3 program (Cooperation, Concurrency and Communication) and the British *Alvey* Program, chapter Formal Aspects of Computer

Science. One example of an activity in the latter category is the C^3/LPC Concurrency Colloquium, held May 15, 16 at the CWI.

Our experience with LPC up to now has been most favorable: the cooperation has proven fruitful for our own concurrency research, both for scientific crossfertilization and as framework for a multitude of organizational activities.

The logo of the Dutch National Concurrency Project

3. Foundational research

Besides the activities which have been performed (mainly) under the auspices of the projects described above, we have also continued our ongoing research on the mathematical foundations of concurrency semantics, to a large extent embedded in joint subprojects with John Meyer (Free University of Amsterdam), Ernst-Rüdiger Olderog (University of Kiel) and Jeff Zucker (SUNY at Buffalo). We restrict ourselves to two examples: The *metric process theory* developed jointly with Zucker has proven a very useful tool for our denotational semantics for POOL, and comparative insights collected in [4,5] have found application in work in progress to assess various models for the new programming notion of *process creation,* abstracted from a key concept of the above mentioned language POOL (cf. [1]).

Finally, we mention our work on the semantics of the language *Occam,* a language gaining increased popularity due to the success of the *Transputer,* and our attempts to unify and interrelate a number of the semantic models for maybe parallel, maybe infinite, logic programming. Lastly, we want to point out that in the arrangements of instruments tuned for concurrency semantics, a key part is also due to the *process algebra* movement, ably conducted by Bergstra and Klop (CWI, Department of Software Technology).

4. A small case study

A sequential program S may, semantically, be seen as a function ϕ which transforms a given input state σ to an output state σ'. For example, the assignment $x := x+1$ will change a state where x equals zero to a state where x equals one. Notationally, we express this by writing $[\![S]\!] = \phi \in \Sigma \to \Sigma$: *the meaning* $[\![S]\!]$ of S is a function ϕ from domain Σ to codomain Σ. Sequential composition $S_1; S_2$ is modelled in a straightforward manner by *function composition.* In a context where *nondeterminism* is present, we enlarge the codomain Σ to the *power set* $\mathcal{P}(\Sigma)$. It is then also meaningful to model the meaning $[\![S_1 [\,] S_2]\!]$ of the nondeterministic choice construct $S_1 [\,] S_2$ by the function $\lambda\sigma.([\![S_1]\!](\sigma) \cup [\![S_2]\!](\sigma))$, where the two operands of the set union '$\cup$' in this function are elements of $\mathcal{P}(\Sigma)$. (The notation $\lambda\sigma. \cdots \sigma \cdots$, where $\cdots \sigma \cdots$ is some expression involving σ, defines a function which maps the argument σ to the value $\cdots \sigma \cdots$.) There is an essential difficulty we encounter when we want to extend this approach to the language construct $S_1 \| S_2$ standing for the parallel execution of S_1 and S_2 - here and often elsewhere taken in the sense of the arbitrary interleaving of the elementary ('atomic') actions making up S_1 and S_2. There is no immediate mathematical operator modelling the programming notion of parallel execution, and in the semantic research one has resorted to a number of ways out of this problem. One method yielding only partial solace is to abstract from programs as *state transformations* and to use models which consist of structured entities - sets of sequences or tree-like structures - consisting of atomic or uninterpreted actions only.

For example, instead of working with building blocks such as the assignment $x := x+1$, one now views this as nothing but an atomic action a. This gives rise to models which have a strong flavor of *formal language theory*: there is little surprise here, since sequences of elementary actions are eminently suitable to undergo the operation of *merge*, contrary to what is the case for (state transforming) functions. A second method, which we have in fact been advocating for some time, following Scott's *general domain theory* and Plotkin's introduction of *'resumptions'*, is to introduce a process domain P as solution of the 'equation'

$$P \cong \{p_0\} \cup (\Sigma \rightarrow \mathcal{P}_{\text{closed}}(\Sigma \times P))$$

(the work involved in solving the equation takes place in a metric setting, $\cong$ denotes isometry and p_0 the nul process). Now assume $p \in P$. Then either p equals p_0, or p is a function, and it is meaningful to write $<\sigma', p'> \in p(\sigma)$: this expresses that process p, for input σ, yields output σ' *together with* the resumption p'. Processes have now a structure which combines features of sequences $(\Sigma \times P)$, sets $(\mathcal{P}_{\text{closed}}(\cdots))$ and functions $(\Sigma \rightarrow \cdots)$, and the semantic operators '$\circ$', '$\cup$' and '$\|$' can all be defined satisfactorily. This allows typical denotational equations such as

$$[\![S_1 \| S_2]\!] = [\![S_1]\!] \| [\![S_2]\!].$$

Many more details of this style of semantic definitions may be found in [3].

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Teaching Computer Science in Nicaragua

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The second half of 1986 we spent in Managua, the capital of Nicaragua, teaching computer science at the *Universidad Nacional de Ingenieria*. In the following we relate some of our experiences.

Computers in a developing country

Nicaragua is a poor, underdeveloped country with an economy severely crippled by a never ending war. Why would a country like that need computers? The person who is going to give a meaningful answer to that question should know about computers, their possibilities, their limitations, their pitfalls, their costs. He should also know about Nicaragua, the structure of its society, the mentality of its people. In short, it should be a Nicaraguan computer specialist. These, however, hardly exist.

Why is the question important? Why can't it just be left unanswered until more pressing problems have been solved? Twenty years ago in the Netherlands we did not have much of an idea what to do with computers (some would argue that we still don't) nor did we have sufficient specialists to manage their introduction and yet with trial and error but without any major disasters an information society has slowly developed. Why couldn't Nicaragua follow a similar course? There are two important differences: poverty and the low price of microcomputers.

Poverty makes a developing society very fragile: its resources are so scarce that no extras are available when anything goes wrong. The mishaps that inevitably accompany the introduction of computers can therefore have devastating effects on the society as a whole. If, for instance, the only refinery in the country suspends production for a week the effects can be felt all over the country.

On the other hand, the low price of micro computers and the presence of many western 'internacionalistas' accustomed to the convenience of a computer on their desk top, have caused an explosive growth in the number of computers in the country. (It more than doubled in the last two years.) Like people here some twenty years ago, Nicaraguans cannot wait to have the computer solve all their problems. Local know-how is urgently needed to prevent unrealistic assumptions, misuse of scarce resources and a growing dependence on foreign technology and expertise.

The university

A similar shortage of trained specialists can be felt in other technical disciplines as well. This is usually the case in a developing country, but it is even more acute in Nicaragua, because after the revolution in 1979 ('El Trionfo' as they say in Nicaragua) most of the technical specialists chose to look for a well paying job in the U.S.A. rather than await their fate under a socialist-leaning government. This government soon decided to concentrate all higher technical training in a new technical university called the *Universidad Nacional de Ingenieria* or *UNI* for short. Some departments (e.g. architecture or industrial engineering) came from one of the other two universities, others (e.g. the computer science department) were newly created.

Working at the UNI makes one accustomed to scarcity: there is a shortage of everything except students. Most apparent to the casual visitor is the shortage of space. The university is housed in a former high school that was for the greater part destroyed by the 1972 earthquake, which wiped out most of Managua's center. Most of the university grounds are ruins. The parts that have been rebuilt serve several functions simultaneously. For instance, the room of twenty square meter we worked in is the office for 8 teachers, the library for students and staff, the room for meetings and sometimes serves as class room. A main attraction of this room is that it has a functioning air conditioner (if there is no power failure), which adds to the noise level as well as to the number of casual visitors. If a discussion becomes too heated (this is Latin America) people are sometimes asked to continue their discussion outside. There, in the corridors, students hold their meetings and study, usually in groups.

Classes are sometimes taught in the corridors or in make-shift class rooms in the ruins. The government has recently allocated some construction materials (very hard to come by) to the university to build class rooms: two rows of concrete bricks, a roof of corrugated iron and partitions of plywood. Part of the partitions are painted so they can serve as some sort of blackboard. There are neither window panes nor doors; the wind blows in freely (a good thing in such a hot country) and passers-by often stop and listen for a few minutes.

Furniture and supplies are hard to get. It is normal to share your desk: if you find yours occupied in the morning you try and find an empty chair and work from your lap. A blackboard eraser is a treasure that you do not easily lend out. There is no money to make copies or print course notes. An imported text book costs the equivalent of a monthly salary, far beyond what the average student can afford.

The most pressing shortage in the university is that of competent teachers. After the revolution many of the university's teachers either left the country or were called to higher posts. The present mayor of Managua, for example, was a highly esteemed physics professor. These vacancies had to be filled by people who lack the necessary background to have a full grasp of the material they are teaching. Consequently, many of these teachers do not like questions and their classes are akin to a copying machine: the teacher merely dictates the course notes to the students. The average exam only tests the ability to reproduce these notes by heart. When we announced that ours would be open-book exams we caused some panic among the students: understanding would be tested and that requires a way of studying they were not accustomed to.

The official goal is that eight years from now all classes are taught by competent Nicaraguan teachers. In the mean time, most teachers are foreigners of many different nationalities. Most prominent are the Latin Americans, especially Cubans, but also sympathizers from Chile, Peru, Guatemala etc. A few are North Americans and the rest comes from a West European or a socialist country (mostly Russians and East Germans).

The Computer Science Department

The Computer Science Department at the UNI is virtually a creation of its current director, a West German formerly affiliated at GMD. He selected the students that were going to be the future teachers, organized the construction material to house the department, and most important, he organized an extensive support network in Germany, Switzerland, and the U.K., that has kept up a steady, although never sufficient, inflow of equipment and supplies. So far five microcomputers, three printers, 600 books and a small, but reasonably equipped, hardware repair shop have been donated.

The main task of the department is teaching. In the first two years the computer science majors follow a general curriculum, attaining a level roughly comparable to that of a European high school. In the next four years they follow a somewhat simplified version of the average computer science curriculum in Western Europe. The department also teaches an introductory computing course to students of other careers. These programming courses used to be taught exclusively from the blackboard. Now students flock around one of the donated micros in groups of eight. It is fun to witness their excitement when they have succeeded in running their first program, but, with some 500 students every year, it constitutes a major demand on the department's resources.

An important task of the department is consultancy. It happens more and more often that a foreign aid project donates a microcomputer to some Nicaraguan institution. The people there often run into problems after the foreign experts have left. Lately, these people are finding their way to the department for advice. This gives the students an invaluable insight into the kind of problems they will likely have to deal with in their careers.

The students

Admission to the university is based on the results of an exam, but favors students from agricultural areas and from poor families. The tuition fee (US dollar 0.40 a year) is affordable for even the poorest people. Financial support, however, scarcely exists and few families can afford to pay the living expenses of their children when they go to university. Most students, therefore, still come from the urban middle class and combine their study with a full time job. Consequently, courses are usually taught at night. The admission policy does not favor women, nor does it need to: they already constitute more than half of the students. This high proportion of women (remember that this is a technical university) is largely due to the war: many men of this age are in the army or have left the country to avoid being drafted. The latter seems to occur rather frequently in the middle class.

Eventually, the annual inflow of computer science majors will be some 75 students. Now 25 are in their third or fourth year and half of them are employed by the university: if they resist the opportunities offered by the private sector they will be the teachers of the next generations. The students we had were bright and in theory they knew a lot; we have seen course notes of mathematics courses from the first two years and the range of subjects was impressive. The level of understanding they had attained was, however, very low: things we had to explain ranged from the meaning of 'if-and-only-if' to how to calculate the number of microseconds in a millisecond.

Nearly all of the students worked very hard. Some came in at 8 a.m. worked until 6 p.m. and then followed classes till 9:30. In the weekends they often came together to study in groups. Many of them already have a considerable teaching load: they teach the introductory course or one of the courses they received the year before. The brightest student even taught a brand new course, learning the material from a book while teaching it to the other students.

The shack where the university staff can get a hot lunch consisting of rice, beans and sometimes meat. The political slogan commemorates the foundation of the Sandinista party 25 years ago by Carlos Fonseca. It reads: 'Carlos, 25 years later: here surrenders nobody'.

Our work

Our main task was designing and teaching computer science courses for the third and fourth year. The courses we worked on were called (to give you a flavor of Spanish computer jargon): *Sistemas Operativos*, *Electronica Digital*, *Lenguajes Formales* and *Organizaciön de Computadoras*. We wrote course notes in close cooperation with the student who is going to teach the course next year: we wrote a first version that resembled Spanish, she wrote a second version in real Spanish, which we finally checked to see if she had correctly understood the material.

Usually, students can read some English, but practically none of them can speak or understand it. Consequently, we had to teach in Spanish. We had studied Spanish for about a year before going to Nicaragua, and that turned out to be barely sufficient. In the first month we often had to fumble for words; later this language barrier became much less severe. Preparing the course notes beforehand helped enormously to gather the necessary vocabulary. However, inventing a nice example during the lecture continued to be hazardous and often we had trouble understanding a question.

Despite this language handicap, we felt highly appreciated and we have enjoyed our work tremendously. The scarcity created many problems but also a pioneer-like quality, that combined with all the youthful enthusiasm gave a unique atmosphere of hope, progress, and a strong sense of a new beginning. In Nicaragua they like to call it the revolutionary spirit.

Politics

Nicaragua does not have a democracy of the kind we are accustomed to in Western Europe. Political parties with strong ties to the former Somoza regime are not allowed. There is no freedom of press: television and radio is government controlled and the only opposition newspaper was closed shortly after we arrived. The Sandinista party is in power and there is no doubt they intend to keep it that way. Yet, within these limits functions a democracy healthier than in most of Central America: the elections were widely considered fair and in the parliament there is a genuine debate. We have not seen any sign of political repression. The police was the friendliest and most polite we have ever experienced. Nobody seemed to be afraid to express discontent: people complain as openly and as frequently as in any western country. Nicaragua claims to have one of the best functioning and most liberal correctional systems in Latin America. We visited prisons and were impressed by their facilities and the relaxed atmosphere.

The war takes a terrible toll. Dozens of people are killed every day. The army has been able to confine the war to the mountainous regions in the North and the East, so people in Managua are not in danger. But even there the war makes itself felt every day: a machine gun is as normal as a bicycle in Amsterdam. In the streets you see quite a few eighteen year old boys in wheel chairs. Quite a few people we talked to, among them some of our students, had lost a close relative in the war. The victims are shown on television and in the newspapers.

Support for the Sandinistas is a lot less than it was directly after the revolution, but hardly anybody sees the contras as a viable alternative. Unless the U.S.A. sends its marines to Nicaragua for a full-scale invasion (as they have done sixty years ago) a military victory for the contras is out of the question. The war and the US boycott, however, slowly bleed the country to death. The inflation rate is 700 %. Prices rise every week, but in the 6 months that we were there the government salaries were not adjusted. Consequently, our salary dropped from 100 to 40 US dollar a month. Not a major problem for us, but for Nicaraguans it is disastrous. Basic commodities like rice, beans, oil or soap are hard to come by. Since salaries in the private sector are two to five times higher than those of the state, staying in a government job is a sacrifice that less and less people can afford. Many of them start spending part of their time marketeering on the black market, which brings productivity down even further.

Conclusion

In spite of these problems, most Nicaraguans have not given up. They are proud to have thrown out Somoza and they are determined to defend what they have gained. The blessings of the revolution may not come as easily as they once thought, they keep working on it with an incredible commitment. If you would like to share in that and you are able to teach computer science courses in Spanish for a minimum of one semester, we can heartily recommend the UNI as a most inspiring and gratifying work environment. We will be glad to act as intermediaries.

Abstracts

of Recent CWI Publications

When ordering any of the publications listed below please use the order form at the back of this issue.

CWI Tract 29. H.L. Trentelman. *Almost Invariant Subspaces and High Gain Feedback.*
AMS 93-02, 93B27, 93B50, 93B55, 93C05, 93C15, 93C35, 93C45, 93C60, 93D15, 93D20; 239 pp.
Abstract: In this monograph a theory is developed around the notion of 'almost controlled invariance'. The theory presented mainly falls within the area of research commonly known as the 'geometric approach' to linear systems, but also more 'frequency domain oriented' methods are used. The work consists of five chapters. In chapter one the basic concepts of controlled invariance and almost controlled invariance are introduced. Chapter two provides characterizations in terms of distributional inputs. As a first application, the L_p/L_q-almost disturbance decoupling problem is discussed. In chapter three the notion of L_p-almost invariance is treated. The L_p-almost disturbance decoupling problem is discussed and results are provided in the context of pole placement by low-order dynamic output feedback. Chapter four treats the L_p-almost disturbance decoupling problem with bounded peaking. Finally, in chapter five, a theory concerning the dual concept of almost conditionally invariant subspaces is developed.

CWI Tract 30. A.G. de Kok. *Production-Inventory Control Models: Approximations and Algorithms.*
AMS 90B05; 214 pp.
Abstract: This monograph concerns the probabilistic analysis of a variety of one-product production-inventory models in which the central problem is to coordinate the production rate with the inventory level in order to cope with random fluctuations in demand. Here the main goal is to meet service level constraints corresponding to service measures such as the long-run fraction

of the demand to be met directly from stock on hand, while minimizing the average costs. Here we account for linear holding costs and a fixed cost for changing the production rate. The control of the inventory is governed by a so-called (m, M)-rule. Using results from random walk theory and renewal theory accurate and practically useful approximations are obtained for all performance characteristics, from which the optimal control levels m and M can be computed.

CWI Tract 31. E.E.M. van Berkum. *Optimal Paired Comparison Design for Factorial Experiments.*
AMS 62J15, 62K05; 153 pp.

Abstract: In paired comparison experiments observations are made by presenting pairs of objects to one or more judges. This method is used extensively in experimental situations where objects can be judged only subjectively, that is to say, when it is impossible or impracticable to make relevant measurements in order to decide which of two objects is preferable. When all pairs are presented to each of n judges (round robin), then the number of paired comparison is $n\binom{t}{2}$, where t is the number of objects. This number is often too large for practical purposes. Bradley and Terry postulate the existence of parameters, π_i for T_i, where T_i is the i-th object or treatment. In many cases these parameters are functions of quantities determining the objects and a linear model can be formulated. The information from this model can be used to construct designs, that are more efficient than the round robin design, i.e., less comparisons are needed to measure the parameters of the linear model with the same accuracy as the round robin design. The aim of this book is to construct such designs.

CWI Tract 32. J.H.J. Einmahl. *Multivariate Empirical Processes.*
AMS 60F15, 60F05, 60E15, 60G17, 62E20, 60G30; 102 pp.

Abstract: In this monograph multivariate empirical processes based on a sequence of independent and identically distributed random vectors are considered. As indexing sets either quadrants, which are identified with points, or rectangles with sides parallel to the coordinate axes are used. With the aid of sharp probability inequalities a number of optimal results is obtained in this setup, especially for weighted (uniform) empirical processes. These results concern weak convergence and strong limit theorems (in particular laws of the iterated logarithm) for the weighted case and the behaviour of two oscillation moduli of the empirical processes for the unweighted case. Some of the results are new in any (finite) dimension, whereas others generalize univariate theorems to arbitrary dimension; it turns out that in some situations there is a remarkable difference between dimension one and higher dimensions. The monograph is concluded by some generalizations and applications.

CWI Tract 33. O.J. Vrieze. *Stochastic Games with Finite State and Action Spaces.*
AMS 90D05, 90D15, 93E05, 93E20; 221 pp.

Abstract: In this monograph two-person zero sum stochastic games with finite state and action spaces are treated. It consists of two parts: one in which the discounted reward criterion stands central, and a second, which is concerned with the average reward criterion. In the 'discounted' part a new proof of the existence of the value is given, based on mathematical programming techniques. Structural properties of the solution sets are elaborated, such as an axiomatic characterization of the value function, perturbation theory and topological properties of the solution sets. Besides a review of existing algorithms, a new algorithm, built up by 'fictitious' play is presented. In the 'average' part of this monograph the emphasis lies on structural properties of this type of stochastic games. Games where the value is independent of the initial state are treated rigorously. Furthermore it is shown that for every stochastic game both players possess easy states, i.e. states for which the player can assure himself the average value by playing stationarily. Also for this

class of games a review of existing algorithms is given and further for two subclasses (one-player-control and switching control) new finite step algorithms are presented. The monograph terminates with an appendix in which preliminary facts on matrix games and Markov decision problems are stated.

CWI Tract 34. P.H.M. Kersten. *Infinitesimal Symmetries: A Computational Approach.*
AMS 53B50, 35Q20, 68B99; 155 pp.
Abstract: This monograph is concerned with computational aspects in the determination of infinitesimal symmetries and Lie-Bäcklund transformations of differential equations. Moreover some problems are calculated explicitly. A brief introduction to some concepts in the theory of symmetries and Lie-Bäcklund transformations, relevant for this book, are given. The mathematical formalism is shortly reviewed. The jet bundle formulation is chosen, in which, by its algebraic nature, objects can be described very precisely. Consequently it is appropriate for implementation. A number of procedures are discussed, which enable one to carry through computations with the help of a computer. These computations are very extensive in practice. The Lie algebras of infinitesimal symmetries of a number of differential equations in Mathematical Physics are established and some of their applications are discussed, i.e., Maxwell equations, nonlinear diffusion equation, nonlinear Schrödinger equation, nonlinear Dirac equations and self dual $SU(2)$ Yang-Mills equations. Lie-Bäcklund transformations of Burgers' equation, Classical Boussinesq equation and the Massive Thirring Model are determined. Furthermore non-local Lie-Bäcklund transformations of the last equation are derived.

CWI Syllabus 11. P.W.H. Lemmens (ed.). *Discrete Wiskunde - Tellen, Grafen, Spelen en Codes.*
162 pp.
Abstract: This syllabus (in Dutch) contains (corrected) reprints of lectures about discrete mathematics and graph theory, presented at Vacation Courses during the period 1963-1980. They cover interesting aspects of the theory and its applications, without aiming at completeness.

CWI Syllabus 12. J. van de Lune. *Introduction to Tauberian Theory: From A. Tauber to N. Wiener.*
AMS 40E05; 102 pp.
Abstract: This booklet contains a fairly detailed 'continuous mathematical history' of the early development of Tauberian theory. From Tauber's elementary theorems, along Pitt's work, it gradually works its way up to the Fourier analytic approach of Wiener. The final chapters deal with Ikehara's theorem and (after the necessary preparations) the prime number theorem. Each chapter contains a separate list of relevant references.

CS-R8638. J.W. de Bakker & J.-J.Ch. Meyer. *Order and metric in the stream semantics of elemental concurrency.*
AMS 68B10, 68C01; CR D.3.1, F.3.2, F.3.3; 18 pp.; **key words:** concurrency, denotational semantics, Smyth order, metric spaces, merge, infinitary languages.
Abstract: Two denotational semantics for a language with simple concurrency are presented. The language has parallel composition in the form of the shuffle operation, in addition to the usual sequential concepts including full recursion. Two linear time models, both involving sets of finite and infinite streams, are given. The first model is order-theoretic and is based on the Smyth order. The second model employs complete metric spaces. Various technical results are obtained relating the order-theoretic and metric notions. The paper culminates in the proof that the two semantics

for the language considered coincide. The paper completes previous investigations of the same language, establishing the equivalence of altogether four semantic models for it.

CS-R8701. J.C.M. Baeten & R.J. van Glabbeek. *Another look at abstraction in process algebra.*
AMS 68Q55, 68Q45, 68Q10, 68N15; CR F.3.2, F.4.3, F.1.2, D.3.1; 32 pp.; **key words:** concurrency, process algebra, hidden step, hiding, abstraction, silent step, internal action.
Abstract: Central to theories of concurrency is the notion of abstraction. Abstraction from internal actions is the most important tool for system verification. In this paper, we look at abstraction in the framework of the Algebra of Communicating Processes. We introduce a hidden step η, and construct a model for the resulting theory ACP_η. We briefly look at recursive specifications, fairness and protocol verification in this theory, and discuss the relations with Milner's silent step τ.

CS-R8702. S. van Egmond, F.C. Heeman & J.C. van Vliet. *INFORM: an interactive syntax-directed formulae-editor.*
CR I.7.2; 17 pp.; **key words:** syntax-directed editing, typesetting, mathematical formulae, interactive.
Abstract: An article or book does not simply exist of a collection of characters, but it is structured. A book consists of chapters, which in return consist of subchapters, paragraphs, pictures, etc. The difference in approach between text-editing and document-editing is that the former regards the text as a string of characters while the latter takes the structure of the text into account. As a subproject of the development of an interactive document-editor we have designed and implemented a prototype of an interactive grammar-driven editor for mathematical formulae.

CS-R8703. R. van Liere & P.J.W. ten Hagen. *Introduction to dialogue cells.*
AMS 69K34, 69K36; CR I.3.4, I.3.6; 20 pp.; **key words:** user interface and management systems, man-machine interaction, dialogue design tools.
Abstract: General-purpose tools for specifying user interfaces are currently of wide interest. Of particular interest is the research done on the precise functionality of such tools. The tools presented here are based on the concept of dialogue cells. The dialogue cell system provides a number of features seldom found in other systems, such as: (i) various forms of user freedom; (ii) abstraction mechanisms for input devices; (iii) advanced application communication mechanisms; (iv) low level workstation control; (v) high degree of portability. This report introduces various concepts related to dialogue cells and dialogue programming environments.

CS-R8704. L.M. Kirousis, E. Kranakis & P.M.B. Vitányi. *Atomic multireader register.*
AMS 68C05, 68C25, 68A05, 68B20; CR B.3.2, B.4.3, D.4.1, D.4.4; 11 pp.; **key words:** register, run, safe, regular, atomic, boolean, flip-flop, reader, writer.
Abstract: We present: (i) A new implementation of an atomic, 1-writer, 1-reader, m-valued register from O (log m) safe, boolean registers (i.e., from scratch). The solution uses neither copying (of the values to be written) nor repeated reading. (ii) An implementation of an atomic, 1-writer, n-reader, multivalued register from $O(n^2)$ atomic, 1-writer, 1-reader, multivalued registers. Both constructions rely on the same idea. In a sense (ii) is a generalization of (i). This closes the last gap in the atomic shared register area. Together with some earlier constructions these results show how to construct atomic, multireader, multiwriter registers from - basically - elementary hardware like flip-flops.

CS-R8705. J.A. Bergstra, J. Heering & P. Klint. *ASF-An algebraic specification formalism.*
AMS 68B99; CR D.2.1, F.3.2; 45 pp.; **key words:** software engineering, specification languages, algebraic specification formalism, modularization, origin rule, module expression, normalization of module expressions.
Abstract: The algebraic specification formalism ASF supports modularized (first-order) equational specifications. Among its features are (a) import and parameterization of modules; (b) hidden (auxiliary) sorts and functions; (c) positive conditional equations; (d) overloaded functions; (e) infix operators. Most of the context-dependent errors in ASF specifications are violations of the so-called origin rule. Besides catching errors, this rule enforces a certain modularization of ASF specifications. The meaning of the modularization constructs of ASF is defined by means of a syntactical normalization procedure. Numerous examples of both correct and incorrect ASF specifications are given in an appendix.

CS-R8706. J.A. Kaandorp. *Interactive generation of fractal objects.*
AMS 58F13. 69K30, 69K34; 17 pp.; **key words:** fractals, self-similar sets, computer graphics, graphics utilities.
Abstract: In this paper a description is given of the development of data structures by which a large class of fractal objects can be represented using production rules, together with an algorithm to generate these objects. These data structures and the algorithm can be used to produce many of the classical examples of self-similar sets, mentioned for example by Mandelbrot. The algorithm and data structures are part of a fractal system by which production rules can be designed; production rules can be retrieved from and stored in a fractal library.

CS-R8708. P.M.B. Vitányi. *Locality, communication and interconnect length in multicomputers.*
AMS 68C05, 68C25, 68A05, 68B20, 94C99; CR B.7.0, C.2, D.4, F.2.2, F.2.3, G.2.2; 11 pp.; **key words:** multicomputers, distributed computation, complexity of computation, locality, communication, wires, optical computing, symmetric circuits, *n*-cube, scalability.
Abstract: The lower bound on the average interconnected (wire) length in *d*-dimensional embeddings of 'symmetric' circuits is proved to be within a small constant multiplicative factor of the obvious worst-case upper bound. The proof is simple, geometrical, and works for wires with zero volume, thus increasing the range of applicability of the results; for instance, to optical (fiber) or photonic (fiberless, laser) communication. While getting rid of the 'von Neumann' bottleneck in the shift from sequential to non-sequential computation, a new communication bottleneck arises. This results from the interplay between locality of computation, communication, and the number of dimensions of physical space.

CS-R8709. P. America & J.J.M.M. Rutten. *Solving reflexive domain equations in a category of complete metric spaces.*
AMS 68B10, 68C01; CR D.1.3, D.3.1, F.3.2; 33 pp.; **key words:** domain equations, complete metric spaces, category theory, converging towers.
Abstract: This paper presents a technique by which solutions to reflexive domain equations can be found in a certain category of complete metric spaces. The objects in this category are the (nonempty) metric spaces and the arrows consist of two maps: an isometric embedding and a non-distance-increasing left inverse to it. The solution of the equation is constructed as a fixed point of a functor over this category associated with the equation. The fixed point obtained is the direct limit (co-limit) of a convergent tower. This construction works if the functor is contracting, which

roughly amounts to the condition that it maps every embedding to an even denser one. We also present two additional conditions, each of which is sufficient to ensure that the functor has a unique fixed point (up to isomorphism). Finally, for a large class of functors, including function space constructions, we show that these conditions are satisfied, so that they are guaranteed to have a unique fixed point. The techniques we use are so reminiscent of Banach's fixed-point theorem that we feel justified in speaking of a category-theoretic version of it.

CS-R8710. F.S. de Boer. *A proof rule for process-creation.*
AMS 70A05; CR F.3.1; 42 pp.; **key words:** proof theory, pre- and post-conditions, concurrency, dynamic process creation, cooperation test, bracketed section, global invariant.
Abstract: A Hoare-style proof system for partial correctness is defined for a language which embodies the kind of parallelism which stems from process-creation. We make use of the following proof theoretic concepts: cooperation test, global invariant, bracketed section and auxiliary variables. These concepts have previously been applied to CSP, DP and to a subset of ADA containing the ADA-rendez-vous. We shall study the proof theory of a language with a generalization of the synchronous message-passing mechanism of CSP. The syntactic construct of process creation and its semantic definition is taken from the language POOL. One of the main characteristics of proof-theoretical interest of this language is that the communication partner referred to by an output statement (or input statement) is not syntactically identifiable, in contrast with the previously mentioned languages. Basically our proof system is built upon the way the semantic mechanism of process identification is brought to the syntactic level. We have proven the proof system to be sound and relatively complete.

CS-R8711. A.P.J.M. Siebes & M.L. Kersten. *Using design axioms and topology to model database semantics.*
AMS 69H21, 22A26, 69K14; CR H.2.1, I.2.4; 14 pp.; **key words:** semantic data models, database theory.
Abstract: The freedom to combine information stored in a database using the operators provided by its datamodel introduces many caveats, such as with view-updates and integrity preservation, for the database designer. To alleviate these problems we define a formal model that explicates the database semantics through entity definitions and limits their use along well-defined paths. Our approach is based on six design axioms and concepts borrowed from topology. This way we achieve a unified description of both the database intension and its extension. In particular, we show that generalization / specification hierarchies are naturally cast into proper subset hierarchies in the entity type topology. Moreover, the limitations posed on the construction of entity types preserve the Armstrong axioms for functional dependencies. This way our model captures much of the real-world semantic constraints and remains sound and complete.

CS-R8712. J. Rekers. *A parser generator for finitely ambiguous context-free grammars.*
AMS 68F25, 68B99; CR D.3.4, F.4.2; 17 pp.; **key words:** parser generator, context-free grammar, ambiguity, Tomita's parsing algorithm, syntax definition formalism, user-defined syntax.
Abstract: An implementation of the syntax definition formalism SDF is described. SDF combines the specification of lexical syntax, context-free syntax, and abstract syntax in a single formalism. From an SDF definition a lexical scanner and parse tables are generated. These parse tables together with a universal parser form a parser for the defined language. The algorithm used in the universal parser allows finitely ambiguous context-free grammars.

CS-N8608. R.C. van Soest. *Consistency and completeness of a knowledge base.* AMS 69K11; CR I.2.1; 19 pp.; **key words:** expert systems, consistency and completeness of a knowledge base.

Abstract: This paper introduces a method for detecting rules which can cause inconsistency or incompleteness in a knowledge base containing production rules. For the greater part this method is based on the structure of a knowledge base as used together with the DELFI-2 system, an expert system shell which has been developed at the Delft University of Technology. However, this method can be used for similar rule-based systems as well. The main idea of this method is to make use of declarative knowledge before considering the production rules.

OS-R8615. S.A. Smulders. *Modelling and simulation of freeway traffic flow.* AMS 60K30, 90B20, 93E03; 55 pp.; **key words:** freeway traffic, stochastic systems, modelling.

Abstract: In the Netherlands a freeway control and signalling system has been installed on several freeways some years ago. One purpose of the system is to improve traffic flow and avoid the development of congestion. In this report the first step towards this aim is made by the development of a traffic model. The proposed model is simulated for various traffic situations and modified to achieve realistic performance.

OS-R8701. A. Schrijver. *Polyhedral combinatorics.* AMS 05CXX, 05C35, 90C10; 67 pp.; **key words:** polyhedra, linear programming, total unimodularity, total dual integrality, matching polytope, blocking polyhedra, anti-blocking polyhedra, cutting planes, integer hull.

Abstract: This paper gives a survey of the field of 'polyhedral combinatorics', and is destined to become a chapter of the Handbook of Combinatorics.

OS-R8702. J.L. van den Berg & O.J. Boxma. *Throughput analysis of a flow-controlled communication network with buffer space limitations.* AMS 60K25, 68M20; 16 pp.; **key words:** computer communication network, flow control, virtual circuit, overflow, (negative) acknowledgement, closed queueing network, throughput.

Abstract: This paper studies the traffic flow in a virtual circuit of a computer communication network with window flow control. Due to finite buffer capacity, overflow of data packets is possible. Lost packets have to be retransmitted. To maintain the original order of the packets, subsequently sent packets also have to be retransmitted, causing a deteriorIation of throughput. An approximation method, based on a queueing network model, is developed to analyse the throughput behaviour. This method leads to exact results for the single-hop network. For the multi-hop circuit, it yields very accurate approximations, as is illustrated by simulation.

OS-R8703. M. Desrochers. *A note on the partitioning shortest path algorithm.* AMS 90C27, 90C35, 90C39, 68R05, 05C38; 5 pp.; **key words:** shortest path, computational complexity, negative length cycles detection.

Abstract: Recently Glover, Klingman and Philips proposed the Partitioning Shortest Path (PSP) algorithm. The PSP algorithm includes as variants most of the known algorithms for the shortest path problem. In a subsequent paper, together with Schneider, they proposed several variants on the PSP and conducted computational tests. Three of the variants were the first polynomially bounded shortest path algorithms to maintain sharp labels as defined by Shier and Witzgall. Two of these variants had computational complexity $O(|N|^2|A|)$, the other $O(|N|^3)$. In this note, we add a new step to the PSP algorithm resulting in new variants also scanning from sharp labels and having computational complexity $O(|N|^3)$ for two of them and $O(|N|^2)$ for the other. This new step also provides a test for the early detection of negative length cycles.

OS-R8704. J.W. Polderman. *A state space approach to the problem of adaptive pole assignment.*

AMS 93C40; 26 pp.; **key words:** adaptive pole-placement, self-tuning, certainty-equivalence.

Abstract: An algorithm for adaptive pole-placement for the class of single-input/single-output systems of order n is proposed. The asymptotic properties of the algorithm do not depend on persistently exciting signals. Excitation is used only initially to avoid pole-zero cancellation of the parameter estimates. The main result is that the asymptotic behaviour of the system equals the behaviour one would have obtained on the basis of the true system. Since this does not imply full identification of the desired control law, we propose the term weak self-tuning. The reason that such a result can be obtained without identification of the true system is the following: Suppose we have a wrong estimate of the system, and that based on that estimate we generate the controls, and that the incorrectness of the estimate is not revealed by the resulting closed-loop behaviour of the system, then the inputs are exactly equal to the ones we would have applied if we had known the system.

OS-R8705. J.W. Polderman. *Adaptive exponential stabilization of a first order continuous-time system.*

AMS 93C40, 8 pp.; **key words:** adaptive pole-placement, self-tuning, certainty-equivalence.

Abstract: An algorithm for adaptive pole-placement for the class of first order continuous-time systems is proposed. The asymptotic properties of the algorithm do not depend on the presence of persistently exciting signals. Excitation is used only initially to avoid that parameter estimates become non-controllable. The main result is that the asymptotic behaviour of the system equals the behaviour one would have obtained on the basis of the true system. Identification of the system-parameters, however, does not necessarily take place. The reason that such a result can be obtained without identification of the true system is the following: Suppose we have a wrong estimate of the system, and that based on that estimate we generate the controls, and that the incorrectness of the estimate is not revealed by the resulting closed-loop behaviour of the system, then the inputs are exactly equal to the ones we would have applied if we had known the system.

OS-R8706. S.A. Smulders. *Modelling and filtering of freeway traffic flow.*

AMS 90B20, 93E03, 93E11; 17 pp.; **key words:** freeway traffic, modelling, filtering.

Abstract: In the Netherlands a freeway control and signalling system has been installed on several freeways some years ago. One purpose of the system is to improve traffic flow and avoid the development of congestion. In this paper the second step towards this aim is taken by the development of a filter that estimates the state of traffic at every time instant. The proposed model (see OS-R8615) is simulated for various traffic situations and modified to achieve realistic performance. The filter is presented and its performance when applied to simulated traffic data shown. The investigations are currently being continued with the application of the filter to real data.

OS-R8707. O.J. Boxma & W.P. Groenendijk. *Waiting times in discrete-time cyclic-service systems.*

AMS 60K25, 68M20; 13 pp.; **key words:** discrete-time queueing system, cyclic service, (pseudo-) conservation law.

Abstract: This paper considers single-server, multi-queue systems with cyclic service in discrete time. Non-zero switch-over times between consecutive queues are assumed; the service strategies at the various queues may differ. A decomposition for the amount of work in such systems is obtained, leading to an exact expression for a weighted sum of the mean waiting times at the various queues. The present paper is the companion paper of Boxma & Groenendijk where the continuous-time case is treated.

OS-N8701. J.M. Anthonisse, J.K. Lenstra & M.W.P. Savelsbergh. *Functional description of CAR, an interactive system for 'computer aided routing'.* (In Dutch)
AMS 90B05, 90C27, 90C50, 68U05; 19 pp.; **key words:** physical distribution, clustering, routing, man-machine interaction, color graphics.

Abstract: CAR is an interactive software package which can be used to support operational distribution management. It has been developed at the Centre for Mathematics and Computer Science (CWI) in the period 1983-1986. This document contains a general description of CAR, a detailed description of the interface between CAR and software in its environment (data entry and report generation), and a user manual in the form of a functional description of all available commands.

NM-R8701. J.H.M. ten Thije Boonkkamp. *The odd-even hopscotch pressure correction scheme for the computation of free convection in a square cavity.*
AMS 65M20, 65N05, 76D05; 17 pp.; **key words:** free convection, Navier-Stokes equations in Boussinesq approximation, odd-even hopscotch method, pressure correction method.

Abstract: The odd-even hopscotch scheme is an integration scheme applicable to large classes of time-dependent partial differential equations. In this paper we examine it for the computation of free convection in a square cavity. The odd-even hopscotch scheme is combined with the pressure correction method in order to decouple the computation of the pressure from that of the velocity and temperature. The resulting scheme is called the odd-even hopscotch pressure correction scheme, and when combined with a suitable space discretization this scheme proves to be efficient regarding computing time and storage requirements. In order to test the accuracy of our scheme, the solution of the free convection problem computed with this scheme is compared with a very accurate reference solution computed by de Vahl Davis.

NM-R8702. M. Louter-Nool. *Translation of algorithm 539: Basic linear algebra subprograms for Fortran usage in FORTRAN 200 for the Cyber 205.*
AMS 65V05, 65FXX; 14 pp.; **key words:** vectorization, basic linear algebra subprograms, Fortran 200, stride problems, operations on contiguously and non-contiguously stored real and complex vectors, operations on vectors stored in reverse order.

Abstract: This paper describes the vectorization of the BLAS, a set of linear algebra subprograms for Fortran usage. In reference [5] of this report the efficiency of BLAS, as standard available on the CDC Cyber 205, was examined and suggestions for improvements were given. This examination has led to the vectorized BLAS as presented here. Moreover, this version admits negative increment values; i.e. vectors can also be treated in reverse order. The number of data movements has been kept to a minimum. This BLAS version has been written in CDC Fortran 200. It has been optimized for a 1-pipe Cyber 205, but it is also appropriate for the 2- and 4-pipe versions.

NM-R8703. E.D. de Goede & F.W. Wubs. *Explicit-implicit methods for time-dependent partial differential equations.*

AMS 65M05, 65M10, 65M20; 22 pp.; **key words:** partial differential equations, explicit-implicit methods, method of lines, tridiagonal equations, incomplete cyclic reduction, stability.

Abstract: For the integration of partial differential equations we distinguish explicit and implicit integrators. Implicit methods allow large integration steps, but require more storage and are more difficult to implement than explicit methods. However, explicit methods are subject to a restriction on the integration step. In this paper, we introduce explicit-implicit methods, which form a combination of explicit and implicit calculations. For these methods, the impact of varying the explicitness, and thus the implicitness, on the stability is examined.

NM-R8704. P.J. van der Houwen & B.P. Sommeijer. *Diagonally implicit Runge-Kutta-Nyström methods for oscillatory problems.*
AMS 65L05, CR G.1.7; G.1.8; 17 pp.; **key words:** numerical analysis, ordinary differential equations, periodic solutions, phase-lag analysis, Runge-Kutta-Nyström methods.

Abstract: Implicit Runge-Kutta-Nyström (RKN) methods are constructed for the integration of second-order differential equations possessing an oscillatory solution. Based on a linear homogeneous test model we analyse the phase errors (or dispersion) introduced by these methods and derive so-called dispersion relations. Diagonally implicit RKN methods of relatively low algebraic order are constructed, which have a high order of dispersion (up to 10). Application of these methods to a number of test examples (linear as well as non-linear) yields a greatly reduced phase error when compared with 'conventional' DIRKN methods.

NM-R8705. P.J. van der Houwen, C. Boon & F.W. Wubs. *Analysis of smoothing matrices for the preconditioning of elliptic difference equations.*
AMS 65N10; 13 pp.; **key words:** numerical analysis, elliptic boundary value problems, preconditioning, smoothing.

Abstract: Smoothing techniques have been used for stabilizing explicit time integration of parabolic and hyperbolic initial-boundary value problems. Similar techniques can be used for the preconditioning of elliptic difference equations. Such techniques are analysed in this paper. It is shown that the spectral radius of the Jacobian matrix associated with the system of equations can be reduced considerably by this type of preconditioners, without much computational effort. Theoretically, this results in a much more rapid convergence of function iteration methods like the Jacobi type methods. The use of smoothing techniques is illustrated for a few one-dimensional and two-dimensional problems, both of linear and nonlinear type. The numerical results show that the use of rather simple smoothing matrices reduces the number of iterations by at least a factor 10.

NM-R8706. B.P. Sommeijer. *A note on a diagonally implicit Runge-Kutta-Nyström method.*
AMS 65L05; CR G.1.7; G.1.8; 4 pp.; **key words:** numerical analysis, ordinary differential equations of second-order, diagonally implicit Runge-Kutta-Nyström methods, interval of periodicity.

Abstract: It is shown that it is possible to obtain fourth-order accurate diagonally implicit Runge-Kutta-Nyström methods with only 2 stages. The scheme with the largest interval of periodicity, i.e. (0,12), is given. Furthermore, the requirement of P-stability decreases the order to 2.

NM-R8707. P.J. van der Houwen & B.P. Sommeijer. *Improving the stability of predictor-corrector methods by residue smoothing.*
AMS 65M20, 65L20; 15 pp.; **key words:** numerical analysis, predictor-corrector

methods, parabolic differential equations, stability, residue smoothing.

Abstract: Residue smoothing is usually applied in order to accelerate the convergence of iteration processes. Here, we show that residue smoothing can also be used in order to increase the stability region of predictor-corrector methods. We shall concentrate on increasing the real stability boundary. The iteration parameters and the smoothing operators are chosen such that the stability boundary becomes as large as $c(m,q)m^2 4^q$ where m is the number of right-hand side evaluations per step, q the number of smoothing operations applied to each right-hand side evaluation, and $c(m,q)$ a slowly varying function of m and q, of magnitude 1.3 in a typical case. Numerical results show that, for a variety of linear and nonlinear parabolic equations in one and two spatial dimensions, these smoothed predictor-corrector methods are at least competitive with conventional implicit methods.

NM-N8605. J.H.M. ten Thije Boonkkamp. *A note on the behaviour of the odd-even hopscotch scheme for a convection-diffusion problem with discontinuous initial data.*
AMS 65M99; 5 pp.; **key words:** odd-even hopscotch scheme, convection-diffusion problem, discontinuous initial data.

Abstract: In this note we study the behaviour of the odd-even hopscotch scheme, when applied to a convection-diffusion problem with discontinuous initial data. It turns out that the odd-even hopscotch solution is dependent upon the implementation of the scheme.

NM-N8701. J.J. Rusch. *Some results obtained with the Roe and Steger & Warming schemes for transsonic flows in a MG/finite volume method.* (In Dutch)
AMS 35L65, 65N30; 29 pp.; **key words:** multigrid method, stationary Euler equations, transsonic flow.

Abstract: In this report three schemes are compared for the multigrid finite-scheme solution of the stationary Euler equations in a channel with a 'bump'. These are the schemes of Osher, Steger and Warming, and Roe. The schemes of Osher and Roe turn out to be very similar while that of Steger and Warming yields completely different results. Roe's scheme is used here in a Newton method for which the Jacobian of the numeric flux has to be calculated. Because of the complex structure of Roe's method this is very time consuming compared with the other methods. In particular the behaviour of a solution with transsonic flow was studied.

MS-R8701. E. Valkeila. *A Prohorov bound for a Poisson process and a Bernoulli process.*
AMS 60F14, 60G55, 60K05; 10 pp.; **key words:** counting process, compensator, Prohorov distance, weak convergence, Bernoulli process.

Abstract: Let M be a counting process with the compensator A and let N be the Poisson process with the compensator B. We give an upper bound for the Prohorov distance between the two processes in terms of the compensators, when the first process is a Bernoulli process with dependency.

MS-R8702. N. Keiding & R.D. Gill. *Random truncation models and Markov processes.*
AMS 62G05, 62M05; 55 pp.; **key words:** counting processes, delayed entry, inference in stochastic processes, intensity function, left truncation, non-parametric maximum likelihood, product integral, survival analysis.

Abstract: The life table with delayed entry is a concept as old as the life table itself. The product-limit estimator is the continuous-time version of the life table, and its generalization to allow for delayed entry, or left truncation, is currently being repeatedly rediscovered in the mathematical-statistical literature even though it is a well-established biostatistical tool. The present note shows how the derivation of a nonparametric estimator of a distribution function under random truncation is a special case of results on the statistical theory of counting processes by Aalen and Johansen. This framework also clarifies the status of the estimator as nonparametric maximum likelihood estimator, and consistency, asymptotic normality and efficiency may be derived directly as special cases of Aalen and Johansen's general theorems and later work.

MS-N8602. S.A. van de Geer & M.M. Voors. *A computer program for the broken-plane two-phase regression model.* (In Dutch)
AMS 62J02; 41 pp.; **key words:** generating tree, least squares, two-phase regression.

Abstract: The model considered is $y = \min(\alpha_0 + \alpha_1 x_1 + \alpha_2 x_2, \beta_0 + \beta_1 x_1 + \beta_2 x_2) + \epsilon$. The paper presents an algorithm for least-squares estimation of the parameters and a computer program in Pascal.

AM-R8701. Jiang Furu. *Boundary value problems for ordinary differential equations with multiple turning points.*
AMS 34E20; 10 pp.; **key words:** turning point problem, asymptotic expansion.

Abstract: In this paper we consider the boundary value problems for ordinary differential equations with multiple turning points by means of the method of multiple scales. The uniformly valid asymptotic expansions of solutions have been constructed both for resonant case and non-resonant case under certain conditions. The results have been generalized.

PM-R8701. M. Hušek & J. de Vries. *A note on compactifications of products of semigroups.*
AMS 22A15, 54D35; 8 pp.; **key words:** semitopological semigroups, compactification, cartesian product.

Abstract: Junghenn, generalizing results of De Leeuw and Glicksberg and of Berglund and Milnes, has shown that the almost periodic (AP) compactification of an arbitrary cartesian product of semitopological semigroups with identity is (canonically) isomorphic to the corresponding product of the AP compactifications of the factors. He also showed that the analogue of this result holds for the strongly almost periodic (SAP) case. The proofs lean heavily on the characterizations of these compactifications in terms of algebras of functions. In this note we give an 'intrinsic' proof, using only the object and morphisms of the categories at hand.

PM-R8702. S.N.M. Ruijsenaars. *Relativistic Calogero-Moser systems and solitons.*
AMS 35Q20, 70F10, 70H40; 9 pp.; **key words:** relativistic particle dynamics, solitons.

Abstract: In this paper we survey a number of results pertaining to a novel class of completely integrable N-particle systems, which generalize the well-known Calogero-Moser (CM) systems, and which were discovered in collaboration with H. Schneider. The new systems are relativistically invariant, and the CM systems result upon taking a parameter to ∞ which may be regarded as the speed of light. They can be quantized in such a fashion that integrability is preserved. However, we here restrict ourselves to the classical level, and present our earlier results with a bias towards relations with soliton PDE. In Section 2 we describe our systems and their relation to the CM

systems, and discuss their relativistic invariance and complete integrability. Section 3 is concerned with a matrix whose symmetric functions yield the N independent commuting Hamiltonians. This matrix is an essential tool in the construction of the action-angle map, which is discussed in Section 4. This section also prepares the ground for Section 5, which deals with the relation of the particle systems to various well-known soliton equations. It turns out that the solutions consisting of solitons, antisolitons, breathers,... may all be viewed as manifestations of underlying point particle dynamics associated with our systems. The relation leads in particular to a natural concept of soliton space-time trajectory, which is presented and discussed in Section 6.

CWI Activities

Spring 1987

With each activity we mention its frequency and (between parentheses) a contact person at CWI. Sometimes some additional information is supplied, such as the location if the activity will not take place at CWI.

Seminar on Integrable Systems. Once a month. (M. Hazewinkel)
A central object of study is the work of Belavin and Drinfeld, especially the relation between simple Lie algebras and solutions of the so-called classical Yang-Baxter equation. Also, linearization aspects of nonlinear representations and lattice KP, KdV will be discussed.

Seminar on Algebra and Geometry. Once a month. (A.M. Cohen)
The Cohomology of the Schubert variety and Coxeter groups.

Cryptography working group. Monthly. (J.H. Evertse)

Colloquium 'STZ' on System Theory, Applied and Pure Mathematics. Twice a month. (J. de Vries)

Study group 'Biomathematics'. Lectures by visitors or members of the group. Jointly with University of Leiden. Bimonthly. (O. Diekmann)
Topics for the next meetings are: stochastic population dynamics, dynamics of structured populations.

Study group on Nonlinear Analysis. Lectures by visitors or members of the group. Jointly with University of Leiden. Bimonthly. (O. Diekmann)
The purpose is to follow and investigate recent developments on qualitative analysis of nonlinear equations.

Progress meetings of the Applied Mathematics Department. Weekly. (N.M. Temme)
New results and open problems on the research topics of the department: biomathematics, mathematical physics, asymptotic and applied analysis, image analysis.

Study group on Statistical and Mathematical Image Analysis. Every three weeks. (R.D. Gill)

The group is presently studying J. Serra's approach to image analysis, 'mathematical morphology', and recent statistical contributions using Markov field modelling due to S. and D. Geman, J. Besag and B. Ripley.

Progress meetings of the Mathematical Statistics Department. Biweekly. (H.C.P. Berbee)

Talks by members of the department on recent developments in research and consultation.

Study group on Empirical Processes. Jointly with University of Amsterdam. Biweekly. (S. van de Geer)

The group is studying the recent book *Convergence of Stochastic Processes* by D. Pollard, and related literature.

System Theory Days. Irregular. (J.H. van Schuppen, J.M. Schumacher)

Study group on System Theory. Biweekly. (J.M. Schumacher)

Current topic: Discrete event dynamical systems.

Colloquium on Queueing Theory and Performance Evaluation. Irregular. (O.J. Boxma)

Progress meetings on Numerical Mathematics. Weekly. (H.J.J. te Riele)

Study group on Numerical Software for Vector Computers. Monthly. (H.J.J. te Riele)

Study group on Differential and Integral Equations. Lectures by visitors or group members. Irregular. (H.J.J. te Riele)

Study group on Graphics Standards. Monthly. (M. Bakker)

National Study Group on Concurrency. Jointly with Universities of Leiden & Eindhoven and several industrial research establishments. 30 January, 27 February, 20 March, 22 May. (J.W. de Bakker)

Study group on Dialogue Programming. (P.J.W. ten Hagen)

Process Algebra Meeting. Weekly. (J.W. Klop)

Visitors to CWI from Abroad

S. Bayerl (University of München, FRG) 11-13 February. M. Bellia (University of Pisa, Italy) 11-13 February. Binggen Zhang (Shandong College of Oceanography, China) 27 February - 4 March. H.A. Blair (Syracuse University, USA) 18-20 March. P. Borras (INRIA, Rocquencourt, France) 12 February. P.G. Bosco (CSELT, Torino, Italy) 11-13 February. M. Bruynooghe (Catholic University, Leuven, Belgium) 19-20 March. G. Burn (GEC, London, UK) 11-13 February. R. Caferra (LIFIA, Grenoble, France) 11-13 February. M. Chabrier (University of Dyon, France) 12 February. Chu Duc (University of Hanoi, Vietnam) 2 March. D. Clément (SEMA, Sophia-Antipolis, France) 12 February. W. Cook (Columbia University, USA) 12-13 March. W. Damm (RWTH, Aachen, FRG) 11-13 February. I. Darmgaard (University of Arhus, Denmark) 7-10 January. P. Degano (University of Pisa, Italy) 11-13 February. B. Fuchssteiner (University of Paderborn, FRG) 25-28 January. E. Glück-Hiltrop (Stollmann, Hamburg, FRG) 11-13 February. A. Gombani (LADSEB-CNR, Padua, Italy) 6-7 April. L. Guibas (DEC Systems Research Center & Stanford University, USA) 26-28 March. C.V. Jones (Wharton School, Pennsylvania, USA) 13 March. G. Kahn (INRIA, Sophia-Antipolis, France) 12 February. B. Lang (INRIA, Rocquencourt, France) 12 February. J. Lasseter (Animation Research and Development Pixar Inc., San Rafael, USA) 10 February. A.K. Lenstra (University of Chicago, USA) January - February. A. Lucena (National Laboratory for Scientific Computing, Rio de Janeiro, Brazil). 20 March. F. Maffioli (Politecnico di Milano, Italy) 11-13 February. M.J. Metthey (C.E.C., Brussels, Belgium) 12 February. E. Ostby (Animation Research and Development Pixar Inc., San Rafael, USA) 10 February. J. Paixao (University of Lisboa, Portugal) 19-20 March. C. Palamidessi (University of Pisa, Italy) 11-13 February. V. Pascual (INRIA, Sophia-Antipolis, France) 9-12 February. J.M. Pereira (LIFIA, Grenoble, France)

11-13 February. L. Praly (Ecoles de Mines, Paris, France) 14-16 January. H.N. Psaraftis (M.I.T., Cambridge, USA) 27 February. W.R. Pulleyblank (University of Waterloo, USA) 12-13 March. L. Rideau (INRIA, Sophia-Antipolis, France) 9-12 February. N. Sabadino (University of Milano, Italy) 26 March. P. Schäfer (AEG, Berlin, FRG) 11-13 February. P. Schnoebelen (LIFIA, Grenoble, France) 11-13 February. D.B. Shmoys (M.I.T. Cambridge, USA) 19 January - 2 February. J. Sidi (SEMA, Montrouge, France) 12 February. M. Sievers (Stollmann, Hamburg, FRG) 11-13 February. K. Soni (University of Tennessee, Knoxville, USA) March - May. R.P. Soni (University of Tennessee, Knoxville, USA) March - May. M.J. Todd (Cornell University, Ithaca, USA) 21-22 January. P. Toth (University of Bologna, Italy) 18 February. S. Tsur (M.C.C. Austin, Texas, USA) 22 January. W. Wiwianka (University of Paderborn, FRG) 25-28 January.

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