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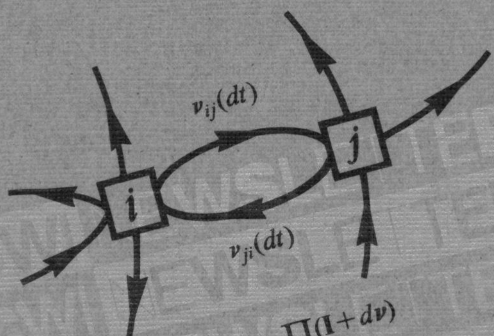

$$\mathbf{P}(s,t) = \prod_{]s,t]} (\mathbf{I} + d\nu)$$

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Product Integrals and Markov Processes

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The solution of the Kolmogorov differential equations for the transition probabilities of a finite state-space Markov process can be represented by a product-integral. For statistical applications it is useful to extend this representation to the general case when the transition probabilities are not absolutely continuous, indeed not even continuous. The correct definition of product integral for this purpose is given here, and some of its properties derived.

1. Introduction

One of the most widely used mathematical models for describing stochastic processes with dependence is that of a Markov chain.

An example where such models are applied is in the study of diseases, where a person can be in one of three states: healthy, sick or dead. The history of a patient is then described as a sequence of time points where transitions or jumps between the states healthy and sick occurs, possibly terminating with a transition to the state death.

The Markov assumption is that once the patient is healthy, say, the waiting time in that state is independent of the past history of the patient and only depends on the state he is in.

These models and some of their generalisations form the mathematical substance of the topic *Survival Analysis* which, briefly described, is the study of the life history of patients with respect to the occurence of certain events, see Andersen and Borgan [2] for a comprehensive review.

For another example from a totally different area take the following typical chemical reaction

$$A + B \rightleftarrows C.$$

If $A(t)$ denotes the number of molecules of type A in a solution, then if $A(t) = n$, a transition to $n-1$ means that an A molecule and a B molecule have collided in the solution and formed a C molecule. Similarly the transition to $n+1$ means that a C molecule has split into an A molecule and a B molecule. For a survey of the theory of Markov chains in physics and chemistry see van Kampen [11].

A finite state Markov chain is then a model for a stochastic process $X(t)$, $t \geqslant 0$ with values in the finite state space E. The probability measure $\mathbb{P}$, which describes the probabilistic properties of X, satisfies the Markov property

$$\mathbb{P}(X(t_{n+1})=i_{n+1}|X(t_1)=i_1,...,X(t_n)=i_n) =$$
$$\mathbb{P}(X(t_{n+1})=i_{n+1}|X(t_n)=i_n)$$

for any states $i_1, \ldots, i_n, i_{n+1}$ in E and time points $t_1<...<t_n<t_{n+1}$. This property is the mathematical formulation of the intuitive property that the process only remembers its immediate past.

This condition ensures that the probability measure is completely described by the initial distribution $p(i) = \mathbb{P}(X(0) = i)$ and the transition probabilities

$$p_{ij}(s,t) = \mathbb{P}(X(t) = j|X(s) = i), \; i \in E, \; j \in E, \; s<t.$$

It is not difficult to see that the matrix $\mathbf{P}(s,t)$ with elements $p_{ij}(s,t)$ satisfies the Chapman-Kolmogorov equation

$$\mathbf{P}(s,t) = \mathbf{P}(s,u)\mathbf{P}(u,t) \quad 0 \leqslant s \leqslant u \leqslant t < \infty$$

which under smoothness conditions gives rise to the Kolmogorov equations

$$\frac{\partial}{\partial t}\mathbf{P}(s,t) = \mathbf{P}(s,t)\mathbf{Q}(t)$$

and

$$\frac{\partial}{\partial s}\mathbf{P}(s,t) = -\mathbf{Q}(s)\mathbf{P}(s,t)$$

which together with the initial condition $\mathbf{P}(s,s) = \mathbf{I}$ (the identity matrix) determine the function $\mathbf{P}(s,t)$ uniquely. The function $\mathbf{Q}(t)$ is determined as

$$\mathbf{Q}(t) = \frac{\partial}{\partial t}\mathbf{P}(s,t)|_{s=t} = -\frac{\partial}{\partial s}\mathbf{P}(s,t)|_{s=t}$$

From the relation

$$1-p_{ii}(t,t+h) \approx h \cdot (-q_{ii}(t))$$

we see that $h \cdot (-q_{ii}(t))$ is the probability that in a short time interval a transition from state i will take place, thus $-q_{ii}(t)$ is the intensity with which the process leaves the state i at time t.

Similarly $q_{ij}(t)/(-q_{ii}(t))$ is the probability that the transition from i which takes place at time t goes to state j.

Note that whereas $\mathbf{P}(s,t)$ is a stochastic matrix, i.e. satisfies

$$p_{ij}(s,t) \geqslant 0, \; \sum_j p_{ij}(s,t) = 1,$$

$\mathbf{Q}(t)$ is an intensity matrix, i.e.

$$q_{ij}(t) \geqslant 0, \; i \neq j, \; q_{ii}(t) \leqslant 0, \; \sum_j q_{ij}(t) = 0.$$

Thus the study of the stochastic process $\mathbf{X}(t)$ is reduced to the study of the bilinear differential equation for the matrix valued function $\mathbf{P}(s,t)$. The solution to the Kolmogorov equations can be given by the subject of this note, the product integral

$$\mathbf{P}(s,t) = \prod_{s}^{t}(\mathbf{I}+\mathbf{Q}(u)du).$$

For reference to this, see SCHLESINGER [13] and DOBRUŠIN [3] or the monograph by DOLLARD and FRIEDMAN [4].

In the study of the estimation problem for a nonhomogeneous Markov Process (AALEN and JOHANSEN [1]) it turned out that we needed the product-integral representation of a transition probability which is piecewise constant between discrete jumps. This note contains a definition of the product integral, where the measure $\int_A \mathbf{Q}(u)du$ is replaced by an arbitrary matrix valued measure $\boldsymbol{\nu}$ on [0,1].

The monograph by DOLLARD and FRIEDMAN [4] gives a survey of various definitions and applications of product integrals but does not contain the definition that allows the Kolmogorov equations to describe the transition probabilities when these are only assumed to be right continuous and of bounded variation. In order to get a simple exposition of the basic properties of the product integral we shall define it by the Peano series, see Definition 2.1. This exploits the usual measure theory for matrix valued measures, in particular Fubini's theorem, and thus gives the existence and basic properties easily.

Section 2 contains the product integral and some of its properties and Section 3 then applies this integral to the representation of the transition probabilities in terms of its integrated intensities. We thus obtain a different approach to some of the results of JACOBSEN [9].

The present note was finished in 1977 as a technical report. The recent interest in the application of product integration in survival analysis and other areas of statistics justifies its publication now. A more comprehensive review of the application of product integrals in the statistical analysis of counting processes is forthcoming, see GILL and JOHANSEN [7].

2. THE PRODUCT INTEGRAL

Let $\mathfrak{B}$ denote the Borel sets of [0,1] and let $\boldsymbol{\nu}$ be a σ-additive finite signed measure with values in the set of $n \times n$ matrices, i.e. a matrix of n^2 real signed measures. We let $\boldsymbol{\nu}^{(n)}$ denote product measure on $[0,1]^n$ defined by

$$\boldsymbol{\nu}^{(n)}(B_1 \times \ldots \times B_n) = \boldsymbol{\nu}(B_1)\cdot\ldots\cdot\boldsymbol{\nu}(B_n), \ B_i \in \mathfrak{B}.$$

We shall use the notation $\|\cdot\|$ to denote the matrix norm $\|\boldsymbol{\nu}(B)\| = \sup_i \sum_j |\nu_{ij}(B)|$, and introduce the real positive measure

$$\nu_0 = \sum_i \sum_j |\nu_{ij}|$$

where $|\nu_{ij}| = \nu_{ij}^{+} + \nu_{ij}^{-}$.

One easily checks that $\|\boldsymbol{\nu}(B)\| \leq \nu_0(B)$ and that

$$\|\boldsymbol{\nu}^{(n)}(B_1 \times \ldots \times B_n)\| \leq \prod_{i=1}^{n} \nu_0(B_i).$$

We shall use the shorthand notation

$$\{u_1 < u_2 < \ldots < u_n\}$$

for the set

$$\{(u_1, u_2, \ldots, u_n) : u_1 < u_2 < \ldots < u_n\}.$$

We now give the basic definition:

DEFINITION 2.1. *For $B \in \mathcal{B}$ we define the product integral*

$$\prod_B (\mathbf{I} + d\boldsymbol{\nu}) = \mathbf{I} + \sum_{n=1}^{\infty} \boldsymbol{\nu}^{(n)}(B \times \ldots \times B \cap \{u_1 < \ldots < u_n\}).$$

Notice that the convergence of the series follows from the inequality

$$\|\boldsymbol{\nu}^{(n)}(B \times \ldots \times B \cap \{u_1 < \ldots < u_n\})\| \leq$$

$$\nu_0^{(n)}(B \times \ldots \times B \cap \{u_1 < \ldots < u_n\}) \leq \frac{1}{n!} \nu_0(B)^n.$$

The following inequalities follow easily from the definition.

PROPOSITION 2.1. *For $B \in \mathcal{B}$ we have*

$$\|\mathbf{I} + \boldsymbol{\nu}(B)\| \leq e^{\nu_0(B)}$$

$$\|\prod_B (\mathbf{I} + d\boldsymbol{\nu})\| \leq e^{\nu_0(B)}$$

$$\|\prod_B (\mathbf{I} + d\boldsymbol{\nu}) - \mathbf{I}\| \leq \nu_0(B) e^{\nu_0(B)}$$

$$\|\prod_B (\mathbf{I} + d\boldsymbol{\nu}) - \mathbf{I} - \boldsymbol{\nu}(B)\| \leq \frac{1}{2} \nu_0(B)^2 e^{\nu_0(B)}$$

Now we can immediately prove the basic multiplicativity property:

THEOREM 2.1. *For any $t \in [0,1]$ we have*

$$\prod_B (\mathbf{I} + d\boldsymbol{\nu}) = \prod_{B \cap [0,t]} (\mathbf{I} + d\boldsymbol{\nu}) \prod_{B \cap]t,1]} (\mathbf{I} + d\boldsymbol{\nu}).$$

Thus $\prod_B (\mathbf{I} + d\boldsymbol{\nu})$ is multiplicative over disjoint intervals, which is the reason for its name.

PROOF. For $i = 1, \cdots, n-1$ let

$$A(B,i,n) = B \times \ldots \times B \cap \{u_1 < \ldots < u_i \leq t < u_{i+1} < \ldots < u_n\}$$

with the obvious modification for $i = 0$ and n. We let

$$A(B,n) = B \times \ldots \times B \cap \{u_1 < \ldots < u_n\} = \bigcup_{i=0}^{n} A(B,i,n).$$

Now

$$\nu^{(n)}(A(B,n)) = \sum_{i=0}^{n} \nu^{(n)}(A(B,i,n))$$

$$= \sum_{i=0}^{n} \nu^{(i)}(A(B \cap [0,t],i))\nu^{(n-i)}(A(B \cap]t,1],n-i)).$$

Summing over n gives the result. □

EXAMPLES 2.4. If $d\nu = \mathbf{Q}dt$, where $\mathbf{Q}$ is a fixed matrix then

$$\nu^{(n)}(0 \leq u_1 < ... < u_n \leq t) = \frac{t^n \mathbf{Q}^n}{n!}$$

and hence

$$\prod_{[0,t]}(\mathbf{I}+d\nu) = e^{t\mathbf{Q}}.$$

If $d\nu = \mathbf{Q}_1 dt$ for $0 \leq t \leq t_1$ and $d\nu = \mathbf{Q}_2 dt$ for $t_1 < t \leq 1$, then using Theorem 2.1 and the previous example, we get

$$\Pi_{[0,t]}(\mathbf{I}+d\nu) = \begin{cases} e^{\mathbf{Q}_1 t} & 0 \leq t \leq t_1 \\ e^{\mathbf{Q}_1 t_1} e^{\mathbf{Q}_2 (t-t_1)}, & t_1 < t \leq 1. \end{cases}$$

As a final example we let $\nu = \mathbf{Q}\epsilon_{t_1}$ where $\mathbf{Q}$ is fixed and ϵ_{t_1} is a one point measure at t_1, then

$$\prod_{[0,t]}(\mathbf{I}+d\nu) = \begin{cases} \mathbf{I} & , 0 \leq t < t_1 , \\ \mathbf{I}+\mathbf{Q} & , t_1 \leq t \leq 1 . \end{cases}$$

Thus we get a piecewise constant function of t.

The following results give a different and perhaps more intuitive definition of the product integral. The definition we have chosen seems to give the basic results in a very efficient manner, since we can use existing integration and measure theory.

THEOREM 2.2. *Let* $0 = t_0 < t_1 < ... < t_n = 1$, *then*

$$\| \prod_{]0,1]}(\mathbf{I}+d\nu) - \prod_{i=0}^{n-1}(\mathbf{I}+\nu]t_i,t_{i+1}])\| \leq c \max_i \nu_0]t_i,t_{i+1}[.$$

PROOF. We split the product integral into the corresponding factors and define

$$\mathbf{M}_i = \prod_{]t_i,t_{i+1}]}(\mathbf{I}+d\nu),\ \mathbf{N}_i = \mathbf{I}+\nu]t_i,t_{i+1}]$$

then by Proposition 2.1, we get

$$\|\mathbf{M}_i\| \leq e^{\nu_0]t_i,t_{i+1}]},\quad \|\mathbf{N}_i\| \leq e^{\nu_0]t_i,t_{i+1}]}.$$

We also get

$$\|\mathbf{M}_i-\mathbf{N}_i\| = \|\prod_{]t_i,t_{i+1}[}(\mathbf{I}+d\boldsymbol{\nu})(\mathbf{I}+\boldsymbol{\nu}[t_{i+1}])-\mathbf{I}-\boldsymbol{\nu}]t_i,t_{i+1}[-\boldsymbol{\nu}[t_{i+1}]\|$$

$$\leqslant\|\prod_{]t_i,t_{i+1}[}(\mathbf{I}+d\boldsymbol{\nu})-\mathbf{I}-\boldsymbol{\nu}]t_i,t_{i+1}[\| + \|\prod_{]t_i,t_{i+1}[}(\mathbf{I}+d\boldsymbol{\nu})-\mathbf{I}\|\|\boldsymbol{\nu}[t_{i+1}]\|$$

$$\leqslant\frac{1}{2}(\nu_0]t_i,t_{i+1}[)^2 e^{\nu_0]t_i,t_{i+1}[}+\nu_0]t_i,t_{i+1}[e^{\nu_0]t_i,t_{i+1}[}.\nu_0[t_{i+1}]$$

$$\leqslant e^{\nu_0]t_i,t_{i+1}[}\ (\max_i \nu_0]t_i,t_{i+1}[)\nu_0]t_i,t_{i+1}].$$

Using these evaluations we get

$$\|\prod_{i=0}^{n-1}\mathbf{M}_i-\prod_{i=0}^{n-1}\mathbf{N}_i\|\leqslant\sum_{i=0}^{n-1}\|\mathbf{M}_0\ldots\mathbf{M}_{i-1}(\mathbf{M}_i-\mathbf{N}_i)\mathbf{N}_{i+1}\ldots\mathbf{N}_{n-1}\|$$

$$\leqslant e^{\nu_0[0,1]}(\sum_i\nu_0]t_i,t_{i+1}])\max_i \nu_0]t_i,t_{i+1}[$$

which gives the result we wanted to prove. □

Corollary 2.1. *Let t_{in} satisfy the conditions*

$$a)\ \ 0 = t_{0n}<t_{1n}<\ldots<t_{nn} = 1$$

and

$$b)\ \ \lim_{n\to\infty}\max_i \nu_0]t_{in},t_{(i+1)n}[= 0,$$

then the product integral can be computed as

$$\prod_{]0,1]}(\mathbf{I}+d\boldsymbol{\nu}) = \lim_{n\to\infty}\prod_{i=0}^{n-1}(\mathbf{I}+\boldsymbol{\nu}]t_{in},t_{(i+1)n}]).$$

Notice that condition b) can always be satisfied, since we can make sure that the atoms of $\boldsymbol{\nu}$(*i.e. of* ν_0) eventually are among the division points.
The next results are about differentiability of the product integral.

Theorem 2.3. *For $B\in\mathcal{B}$ we have*

$$\prod_B(\mathbf{I}+d\boldsymbol{\nu})-\mathbf{I} = \int_B \prod_{B\cap[0,u[}(\mathbf{I}+d\boldsymbol{\nu})\boldsymbol{\nu}(du)\,.$$

Proof. Using Fubini's theorem on the $(n+1)$-th term we get

$$\boldsymbol{\nu}^{(n+1)}(B\times\ldots\times B,\ 0\leqslant u_1<\ldots<u_{n+1}\leqslant 1)$$

$$= \int_B\boldsymbol{\nu}^{(n+1)}(B\times\ldots\times B,\ 0\leqslant u_1<\ldots<u_{n+1}\leqslant 1|u_{n+1} = u)\boldsymbol{\nu}(du)$$

$$= \int_B\boldsymbol{\nu}^{(n)}(B\times\ldots\times B,\ 0\leqslant u_1<\ldots<u_n<u)\boldsymbol{\nu}(du).$$

Summing over n gives the result. □

THEOREM 2.4. *For $B \in \mathcal{B}$ we have*

$$\prod_B(\mathbf{I}+d\boldsymbol{\nu})-\mathbf{I} = \int_B \boldsymbol{\nu}(du) \prod_{]u,1]\cap B}(\mathbf{I}+d\boldsymbol{\nu})$$

PROOF. Similar to that of Theorem 2.3.

If we define the function $\mathbf{F}$ by

$$t \to \prod_{[0,t]}(\mathbf{I}+d\boldsymbol{\nu})$$

then $\mathbf{F}$ is right continuous by Theorem 2.3 and it is of bounded variation. It thus determines a matrix valued measure, which by Theorem 2.3 is absolutely continuous with respect to ν_0. The integral relation can thus be reformulated as

$$\frac{d\mathbf{F}}{d\nu_0}(t) = \mathbf{F}(t-)\frac{d\boldsymbol{\nu}}{d\nu_0}(t), \text{ a.e.}[\nu_0].$$

If $\mathbf{H}: t \to \prod_{]t,1]}(\mathbf{I}+d\boldsymbol{\nu})$, then Theorem 2.4 can be reformulated as

$$\frac{d\mathbf{H}}{d\nu_0}(t) = -\frac{d\boldsymbol{\nu}}{d\nu_0}(t)\mathbf{H}(t), \text{ a.e.}[\nu_0].$$

3. MARKOV PROCESSES

In constructing a Markov Process one can start with the transition probabilities, satisfying the Chapman-Kolmogorov equations, then construct the process, i.e. the measure on a suitable function space, by the Kolmogorov consistency theorem, see DOOB [5], or via a general theorem of extension of continuous linear functionals, see NELSON [12], or GOODMAN and JOHANSEN [8]. In this case the discussion of the differential equations for the transition probabilities becomes a discussion of when a process is determined by its infinitesimal properties.

One can also start out with the waiting time distributions and the jump intensities and then construct the measure very directly and then prove that certain variables form a Markov Process and define the transition probabilities in terms of these. The differential equations can now be viewed as a convenient and different way of obtaining the transition probabilities, see JACOBSEN [9].

We shall here start with the intensities or rather the integrated intensities $\boldsymbol{\nu}$, i.e. we assume that

$$\nu_{ii} \leqslant 0, \; \nu_{ij} \geqslant 0, \; i \neq j \text{ and } \sum_j \nu_{ij} = 0.$$

From this measure we construct the transition probabilities by a product integral and this also gives us the differential equations for $\mathbf{P}$. Thus the approach is highly non-probabilistic as opposed to that of JACOBSEN [9]. The solution, however, is the same, as we shall show.

Thus, we let $\boldsymbol{\nu}$ be a finite signed measure on $[0,1]$ with values in the set of intensity matrices, then the following holds:

THEOREM 3.1. *If* $\boldsymbol{\nu}[t]+\mathbf{I}$ *is a stochastic matrix, i.e. if* $\nu_{ii}[t]\geqslant -1$, *then*

$$\mathbf{P}(B) = \prod_B(\mathbf{I}+d\boldsymbol{\nu})$$

is a stochastic matrix.

PROOF. Assume first that $\nu_{ii}[t] > -1$, $i = 1,\ldots,n$, i.e. no atoms of $\boldsymbol{\nu}$ are as large as -1. Then let us choose the partition t_{in} of $[0,1]$ so fine that $\mathbf{I}+\boldsymbol{\nu}]t_{in},t_{(i+1)n}]$ is a stochastic matrix. By Corollary 2.1 $P(B)$ is the limit of stochastic matrices, hence stochastic.

In general there can only be a finite number of points $t_1,\ldots,t_k$, such that some $\nu_{ii}[t_r] = -1$. By writing

$$\mathbf{P}(B) = (\mathbf{I}+\boldsymbol{\nu}(B\cap[0]))\cdot\prod_{i=1}^{k-1}\prod_{B\cap]t_i,t_{i+1}[}(\mathbf{I}+d\boldsymbol{\nu})(\mathbf{I}+\boldsymbol{\nu}(B\cap[t_{i+1}]))$$

we see that $\mathbf{P(B)}$ is stochastic.

For a given $\boldsymbol{\nu}$ we now define

$$\mathbf{P}(s,t) = \mathbf{P}(]s,t]) = \prod_{]s,t]}(\mathbf{I}+d\boldsymbol{\nu}),\ 0<s<t\leqslant 1,$$

$$\mathbf{P}(0,t) = \mathbf{P}([0,t]) = \prod_{[0,t]}(\mathbf{I}+d\boldsymbol{\nu}),\ 0\leqslant t\leqslant 1,$$

then it is seen that $\mathbf{P}(s,t)$ is right continuous in s and t (except for $s = 0$, $t\downarrow 0$), and that

$$\mathbf{P}(t-,t) = \mathbf{I}+\boldsymbol{\nu}[t],\ 0<t\leqslant 1$$

$$\mathbf{P}(t,t+) = \mathbf{I},\ 0<t<1$$

$$\mathbf{P}(0,0+) = \mathbf{I}+\boldsymbol{\nu}[0].$$

The multiplicativity of the product integral now immediately gives that $\mathbf{P}(s,t)$ satisfies the Chapman-Kolmogorov equations

$$\mathbf{P}(s,t) = \mathbf{P}(s,u)\mathbf{P}(u,t)\quad 0\leqslant s<u<t\leqslant 1,$$

and in this formulation, Theorem 2.3 gives the forward differential equation

$$\frac{\partial\mathbf{P}(s,t)}{\partial\nu_0(t)} = \mathbf{P}(s,t-)\frac{d\boldsymbol{\nu}}{d\nu_0}(t)\qquad \text{a.e.}[\nu_0]$$

whereas Theorem 2.4 gives the backward equation

$$\frac{\partial\mathbf{P}(s,t)}{\partial\nu_0(s)} = -\frac{d\boldsymbol{\nu}}{d\nu_0}(s)\mathbf{P}(s,t),\qquad \text{a.e.}[\nu_0]$$

which shows that $\mathbf{P}(s,t)$ does in fact have $\boldsymbol{\nu}$ as integrated intensity measure.

Using a result similar to Theorem 2.2 one can prove that for $s<t_{0n}<\ldots<t_{nn} = t$ such that $\max_j \nu_0]t_{jn},t_{(j+1)n}[\ \rightarrow 0$ we have

$$\prod_{]s,t]}(1+d\nu_{ii}) = \lim_{n\to\infty}\prod_{j=0}^{n-1} p_{ii}(t_{jn},t_{(j+1)n})$$

which is nothing but the waiting time distribution in state i, i.e.

$$\mathbb{P}\{X_u = i,\ s<u\leq t|X_s = i\} = \prod_{]s,t]}(1+d\nu_{ii}).$$

With the notation

$$G_i[0,t] = 1-\prod_{[0,t]}(1+d\nu_{ii})$$

and

$$\pi_{ij}(t) = -\frac{d\nu_{ij}}{d\nu_{ii}}(t),\ i\neq j,\ \pi_{ii}(t) = 0$$

we can now prove that the solution provided by JACOBSEN [9], starting with $\mathbf{G}$ and $\boldsymbol{\pi}$ is in fact the same as the solution provided here starting from $\boldsymbol{\nu}$.

We shall now assume that $\boldsymbol{\nu}$ satisfies the following extra conditions

1) $\boldsymbol{\nu}[0] = \mathbf{0}$

2) $\nu_i[t]>-1.$

i.e. $\mathbf{P}(s,t)$ becomes right continuous, also at 0, and no atom is as large as -1.

Notice that $\boldsymbol{\nu}$ can be recovered from $\mathbf{G}$ and $\boldsymbol{\pi}$, by the relations

$$\nu_{ii}(A) = -\int_A \frac{G_i(du)}{1-G_i[0,u[}$$

and

$$\nu_{ij}(A) = -\int_A \pi_{ij}(u)\nu_{ii}(du).$$

It is then seen that G_i is continuous at zero, and that $G_i[0,t] = 0$ if $\nu_{ii}[0,t] = 0$ and that $G_i[0,t]<1$, since $\nu_{ii}[0,1]$ is finite. In order to see the last result, where condition 2) is needed, we argue as follows: The largest atom of $|\nu_{ii}|$ is $1-\epsilon$ say. Now choose $0 = t_0<\ldots<t_n = 1$ such that $|\nu_{ii}]t_j,t_{j+1}[|<\frac{\epsilon}{2}$ then $1+\nu_{ii}]t_j,t_{j+1}]>\frac{\epsilon}{2}$ and

$$\log(1+\nu_{ii}]t_j,t_{j+1}])\geq\frac{\log\frac{\epsilon}{2}}{-1+\frac{\epsilon}{2}}\nu_{ii}]t_j,t_{j+1}].$$

Summing over j gives

$$\prod_j(1+\nu_{ii}]t_j,t_{j+1}])\geq c>0$$

which again implies that $\prod_B(1+d\nu_{ii})\geq c>0$.

Thus the functions $\mathbf{G}$ and $\boldsymbol{\pi}$ satisfy the conditions of Jacobsen and his solution $\tilde{\mathbf{P}}(s,t)$ is constructed to satisfy the integral equation:

$$\tilde{p}_{ij}(s,t) = \delta_{ij} 1-\delta_{ij}\frac{1-G_i[0,t]}{1-G_i[0,s]}+\sum_{k\neq i}\int_{]s,t]}\pi_{ik}(u)\tilde{p}_{kj}(u,t)\frac{G_i(du)}{1-G_i[0,s]}.$$

Using the definition of $(\mathbf{G},\pi)$ in terms of $\boldsymbol{\nu}$ this is

$$\tilde{p}_{ij}(s,t) = \delta_{ij}\prod_{]s,t]}(1+d\nu_{ii})-\sum_{k\neq i}\int_{]s,t]}\tilde{p}_{kj}(u,t)\prod_{]s,u[}(1+d\nu_{ii})\nu_{ik}(du) \qquad (3.1)$$

which is known to have a unique solution, see FELLER [6].

The function $\mathbf{P}(s,t) = \prod_{]s,t]}(\mathbf{I}+d\boldsymbol{\nu})$ is known to satisfy the equation

$$p_{ij}(s,t)-\delta_{ij} = \sum_{k}\int_{]s,t]}\nu_{ik}(du)p_{kj}(u,t). \qquad (3.2.)$$

In this equation we multiply by $\prod_{]a,s[}(1+d\nu_{ii})$ and integrate with respect to $\nu_{ii}(ds)$. If we then use the results:

$$\int_{]a,t]}\prod_{]a,s[}(1+d\nu_{ii})\nu_{ii}(ds) = \prod_{]a,t]}(1+d\nu_{ii})-1$$

and

$$\int_{]a,u[}\prod_{]a,s[}(1+d\nu_{ii})\nu_{ii}(ds) = \prod_{]a,u[}(1+d\nu_{ii})-1.$$

then we get after some reduction that $\mathbf{P}$ also satisfies equation (3.1.), hence $\mathbf{P} = \tilde{\mathbf{P}}$.

In fact the equations (3.1) and (3.2) are equivalent. If (3.1) is integrated with respect to $\nu_{ii}(ds)$ on $]a,t]$, we arrive at (3.2).

It should of course be pointed out that we are only dealing with a finite number of states, whereas Jacobsen treats the more general situation of a countable number of states.

Example

The results above are considerably simplified in case we want to describe a time homogeneous chain. The intuition behind the time homogenity is that the intensities $q_{ij}(t)$ do not depend on t. Thus in this case the intensity measure satisfies

$$\boldsymbol{\nu}(A) = \mathbf{Q}_0\lambda(A)$$

where λ is Lebesgue measure and $\mathbf{Q}_0$ is an intensity matrix. The transition probabilities are now given by

$$\mathbf{P}(s,t) = \prod_{s}^{t}(\mathbf{I}+\mathbf{Q}_0 du) = \exp((t-s)\mathbf{Q}_0)$$

The construction of the process from $\mathbf{G}$ and $\boldsymbol{\pi}$ is as before with the simplification that

$$G_i(0,t) = 1-\prod_{[0,t]}(1+q_{ii}du) = 1-\exp(tq_{ii})$$

showing that the waiting time distribution in each state is exponential with a state dependent parameter $-q_{ii}$.

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The Amoeba Distributed Operating System
(Part 2)

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The Amoeba Project is a distributed project on distributed operating systems. The project, which started as the author's PhD research project in 1978 [5], is now a joint project of CWI and Vrije Universiteit in Amsterdam. About a dozen people are working on the project, led by prof. dr. A. S. Tanenbaum (VU) and the author (CWI). Part 1 of this paper appeared in the previous Newsletter and discussed the interprocess communication facilities and the capability-based protection mechanisms of the Amoeba system. Here, in Part 2, some of the Amoeba services will be discussed.

1. Introduction

The basic entities of the Amoeba Distributed Operating System are *clients*, *services* and *objects*. A client process can request the service that manages an object to carry out an operation on it by sending a request to one of the service's server processes. For identification and protection, each object has a *capability*, a 128-bit string that contains enough sparseness that it cannot be forged. Inside the capability is a *port*, a 48-bit string that identifies the service managing the object. When a client makes a request, the system uses this port to find a server process to carry it out.

Traditional operating systems usually provide hundreds of *system calls*. These are 'procedure calls' into the operating system kernel. There are calls to open, read, write and close files, calls to create and destroy processes, to manipulate tape drives and terminals, and so forth. In Amoeba, there are only three* system calls, one for clients and two for servers: A client process makes a server request by calling *trans*; this sends a request to a service and waits for a reply. A server process receives a request by calling *getreq*, and sends a reply by a call to *putrep*. (See Part 1 for more details.)

* Actually, there are a few more for interrupting transactions and dealing with Gateways to other networks. Going into these would make this paper too technical, however.

There are no system calls for creating processes or reading files in Amoeba. When a new process has to be created, a request is sent to the process server, and when a file must be read, a request goes to the file server. Some of these services are *kernel services*, that is, they form part of the operating system kernel, others are implemented as user services, that is, the server runs as an ordinary user process. Amoeba makes no distinction, and clients perceive no difference.

The advantages of this method are obvious: Programmers have to deal with only one way of addressing services, and it allows the system designers to implement services outside the kernel for maintainability or inside it for speed. New versions of a service can always be tested outside the kernel, to be installed inside it only after thorough testing.

The next sections describe a few of the important services provided by the Amoeba system: Process Service, File Service, Directory Service, and Bank Service.

2. Process Service

Managing processes on Amoeba is a task that is carried out by a number of co-operating services. The central one is the service provided by the Amoeba operating system kernel. The kernel assigns processes to the processor so they can do their work. But there's more to process management: processes have to be assigned to the right processor; when a process crashes, or does something irregular, such as attempting to make a Unix system call on an Amoeba kernel, something has to be done about it; programs have to be fetched from a file before they can be run.

The design of the system is such that the Amoeba kernel implements a minimum of basic process management *mechanisms*, on top of which various *policies* can be implemented: If we ever decide to do process management in a different way, we want to run as little risk as possible that we have to change the kernel.

In the previous article, we explained that client processes block when they do transactions, and that server processes block when they wait for a client's request. It is very difficult to write programs using non-blocking transactions. It's much simpler using blocking ones. Additionally, blocking transactions can be implemented much more efficiently than non-blocking ones. To achieve parallelism, one uses parallel processes.

In traditional operating systems, each process runs in its own address space. In distributed systems, processes are created at such an enormous rate that the cost of making each one run in its own address space is prohibitive. Many distributed systems, therefore, provide *light-weight processes*, processes that share a single address space. Processes then have very little context, and process switching can be done very efficiently.

We call a light-weight process a *task*, and a group of tasks sharing an address space a *cluster*, although sometimes we'll use the term *process* for cluster as well. The address space is divided up into *segments*. A cluster can have a number of read-only segments (useful to hold the program's code), and a number of read-write segments, write-only segments (they can be useful, believe it or not), segments that can grow (for the stack), and so on.

Running a program on an Amoeba machine requires the following steps. First, the segments that the program will use must be created and filled with the proper contents. Then a cluster descriptor is given to the kernel, giving the mapping of the segments into the new cluster's address space, and the number and state of its tasks.

When a running cluster is stopped, the Amoeba Kernel hands over a cluster descriptor for it, describing the state of the cluster at the moment it was stopped.

The *process server* is a user-space server which runs on each Amoeba machine and acts as a sort of "agent on the spot" for remote execution. To run a process remotely, its cluster descriptor is sent to the process server on that machine. The process server there then fetches segments over the network, if necessary, and creates the cluster.

Co-operating process servers can use the kernel mechanisms to implement process migration for load balancing, for instance, or if a machine has to be brought down for maintenance.

Cluster descriptors also play an important role in exception handling. Each cluster descriptor contains a table that specifies for each kind of exception which server to call when it occurs. Special codes in the table can be used to ignore certain exceptions, or to kill the cluster.

When an exception occurs, the kernel sends the cluster descriptor to the specified server for handling. The handler can examine and manipulate the cluster using the information provided by the cluster descriptor, and access the cluster's memory through the segment capabilities in the cluster descriptor.

Operating system emulation can be viewed as a special case of debugging. A program, native to another operating system, can be run on Amoeba as if it ran on the operating system it was written for. Before the process is run, its environment is set up so that all system calls it does, all actions that cause exceptions are trapped to an operating system emulator server. The emulator can examine the state of the excepted process, determine what its original operating system would have done when this exception occurred and simulate that with the same mechanisms that the debugger uses.

The Amoeba process service mechanism thus provides a basic mechanism in the Amoeba kernel for process management (segments and cluster descriptors), which is used by user-space servers to augment this service with services for remote execution, load balancing through migration, local and remote debugging, checkpointing and operating system emulation.

3. File Service

The file system has been designed to be highly modular, both to enhance reliability and to provide a convenient testbed for doing research on distributed file systems. It consists of three completely independent pieces: the block service, the file service, and the directory service. In short, the block service provides commands to read and write raw disk blocks. As far as it is concerned, no two blocks are related in any way, that is, it has no concept of a file or other aggregation of blocks. The file service uses the block service to build up files with various properties. Finally, the directory service provides a mapping of symbolic names onto object capabilities.

The block service is responsible for managing raw disk storage. It provides an object-oriented interface to the outside world to relieve file servers from having to understand the details of how disks work. The principle operations it performs are:

- *allocate* a block, write data into it, and return a capability to the block
- given a capability for a block, *free* the block
- given a capability for a block, *read* and return the data contained in it
- given a capability for a block and some data, *write* the data into the block
- given a capability for a block and a key, *lock* or *unlock* the block

These primitives provide a convenient object-oriented interface for file servers to use. In fact, any client who is unsatisfied with the standard file system can use these operations to construct his own [8, 9].

The first four operations of *allocate*, *free*, *read*, and *write* hardly need much comment. The fifth one provides a way for clients to lock individual blocks. Although this mechanism is crude, it forms a sufficient basis for clients (e.g., file systems) to construct more elaborate locking schemes, should they so desire.

One other operation is worth noting. The data within a block is entirely under the control of the processes possessing capabilities for it, but we expect that most file servers will use a small portion of the data for redundancy purposes. For example, a file server might use the first 32 bits of data to contain a file number, and the next 32 bits to contain a relative block number within the file. The block server supports an operation *recovery*, in which the client provides the account number it uses in *allocate* operations and requests a list of all capabilities on the whole disk containing this account number. (The block server stores the account number for each block in a place not accessible to clients.) Although *recovery* is a very expensive operation, in effect requiring a search of the entire disk, armed with all the capabilities returned, a file server that lost all of its internal tables in a crash could use the first 64 bits of each block to rebuild its entire file list from scratch.

The purpose of splitting the block service and file service is to make it easy to provide a multiplicity of different file services for different applications. One such file service that we envision is one that supports flat files with no locking, in other words, the UNIX† model of a file as a linear sequence of bytes with no internal structure and essentially no concurrency control. This model is quite straightforward and will therefore not be discussed here further.

A more elaborate file service with explicit version and concurrency control for a multiuser environment will be described instead [4]. This file service is designed to support data base services, but it itself is just an ordinary, albeit slightly advanced, file service. The basic model behind this file service is that a file is a time-ordered sequence of versions, each version being a snapshot of the file made at a moment determined by a client [2, 7]. At any instant, exactly one version of the file is the *current version*. To use a file, a client sends a message to a file server process containing a file capability and a request to create a new, private version of the current version. The server returns a capability for this new version, which acts as if it is a block for block copy of the current version made at the instant of creation. In other words, no matter what other changes may happen to the file while the client is using his private version, none of them are visible to him. Only changes he makes himself are visible.

Of course, for implementation efficiency, the file is not really copied block for block. What actually happens is that when a version is created, a table of pointers (capabilities) to all the file's blocks is created. The capability granted to the client for the new version actually refers to this version table rather than the file itself. Whenever the client reads a block from the file, a bit is set in the version table to indicate that the corresponding block has been read. When a block is modified in the version, a new block is allocated using the block server, the new block replaces the original one, and its capability is inserted into the version table. A bit indicating that the block is a new one rather than an original is also set. This mechanism is sometimes called "copy on write."

Versions that have been created and modified by a client are called *uncommitted versions*. At a particular moment, the current version may have several (different) uncommitted versions derived from it in use by different clients. When a client is finished modifying his private version, he can ask the file server to *commit* his version, that is, make it the current version instead of the then current version. If the version from which the to-be-committed version was derived is still current at the time of the commit, the commit succeeds and becomes the new current version.

† UNIX is a Trademark of AT&T Bell Laboratories.

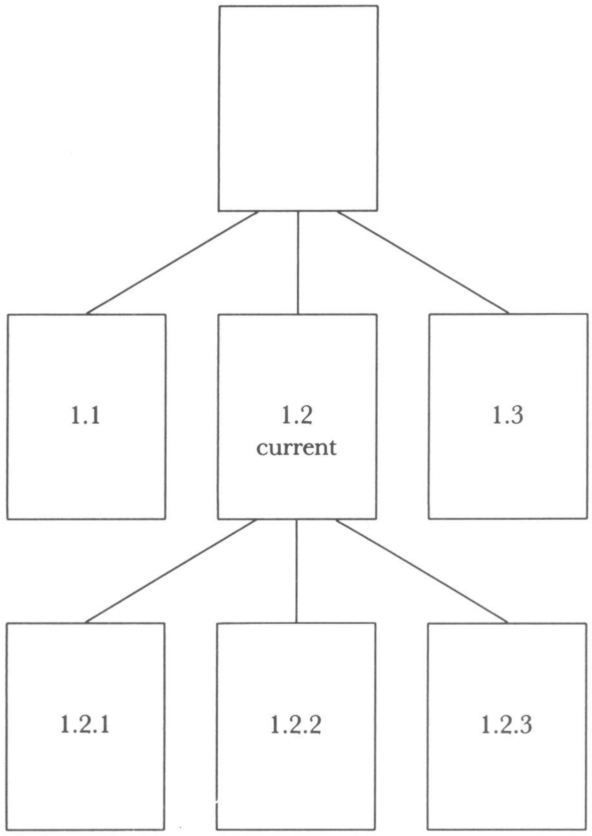

FIGURE 1.

As an example, suppose version 1 is initially the current version, with various clients creating private versions 1.1, 1.2, and 1.3 based on it. If version 1.2 is the first to commit, it wins and 1.2 becomes the new current version, as illustrated in FIGURE 1. Subsequent requests by other clients to create a version will result in versions 1.2.1, 1.2.2, and 1.2.3, all initially copies of 1.2.

The fun begins when the owner of version 1.3 now tries to commit. Version 1, on which it is based, is no longer the current version, so a problem arises. To see how this should be handled, we must introduce a concept from the data base world, *serializability* [1, 6]. Two updates to a file are said to be serializable if the net result is the same if they are run sequentially, in one or the other order. As a simple example, consider a two character file initially containing "ab." Client 1 wants to write a "c" into the first character, wait a while, and then write a "d" into the second character. Client 2 wants to write an "e" into the first character, wait a while, and then write an "f" into the second character. If 1 runs first we get "cd"; if 2 runs first we get "ef." Both of these are legal results, since the file server cannot dictate when the users run. However, its job is to prevent final configurations of "cf" or "de," both of which result from interleaving the requests. If a client locks the file before starting, does all its work, and then unlocks the file, the result will always be either "cd" or "ef," but never "cf" or "de." What we are trying to do is accomplish the same goal without using locking.

The idea behind not locking is that most updates, even on the same file, do not affect the same parts of the file, and hence do not conflict. For example, changes to an airline reservation data base for flights from San Francisco to Los Angeles do not conflict with changes for flights from Amsterdam to London. The strategy behind our commit mechanism is to let everyone make and modify versions at will, with a check for serializability when a commit is attempted. This mechanism has been proposed for data base systems [3], but as far as we know, not for file systems.

The serializability check is straightforward. If a version to be committed, *A*, is based on the version that is still current, *B*, it is serializable and the commit succeeds. If it is not, a check must be made to see if all of the blocks belonging to *A* that the client has read are the same in the current version as they were in the version from which *A* was derived. If so, the previous commit or commits only changed blocks that the client trying to commit *A* was not using, so there is no problem and the commit can succeed.

If, however, some blocks have been changed, modifications that *A*'s owner has made may be based on data that are now obsolete, so the commit must be refused, but a list is returned to *A*'s owner of blocks that caused conflicts, that is, blocks marked "read" in *A* and marked "written" in the current version (or any of its ancestors up to the version on which *A* is based). At this point, *A*'s owner can make a new version and start all over again. Our assumption is that this event is very unlikely, and that its occasional occurrence is a price worth paying for not having locking, deadlocks, and the delays associated with waiting for locks.

Because it is frequently inconvenient to deal with long binary bit strings such as capabilities, a directory service is needed to provide symbolic naming. The directory service's task is to manage directories, each of which contains a collection of (ASCII name, capability) pairs. The principal operation on a directory object is for a client to present a capability for a directory and an ASCII name, and request the directory service to look up and return the capability associated with the ASCII name. The inverse operation is to store an (ASCII name, capability) pair in a directory whose capability is presented.

4. Bank Service

The bank service is the heart of the resource management mechanism. It implements an object called a "bank account" with operations to transfer virtual money between accounts and to inspect the status of accounts. Bank accounts come in two varieties: individual and business. Most users of the system will just have one individual account containing all their virtual money.

This money is used to pay for CPU time, disk blocks, typesetter pages, and all other resources for which the service owning the resource decides to levy a charge.

Business accounts are used by services to keep track of who has paid them and how much. Each business account has a subaccount for each registered client. When a client transfers money from his individual account to the service's business account, the money transferred is kept in the subaccount for that client, so the service can later ascertain each client's balance. As an example of how this mechanism works, a file service could charge for each disk block written, deducting some amount from the client's balance. When the balance reached zero, no more blocks could be written. Large advance payments and simple caching strategies can reduce the number of messages sent to a small number.

Another aspect of the bank service is its maintenance of multiple currencies. It can keep track of say, virtual dollars, virtual yen, virtual guilders and other virtual currencies, with or without the possibility of conversion among them. This feature makes it easy for subsystem designers to create new currencies and control how they are allocated among the subsystems users.

The bank service described above allows different subsystems to have different accounting policies. For example, a file or block service could decide to use either a buy-sell or a rental model for accounting. In the former, whenever a block was allocated to a client, the client's account with the service would be debited by the cost of one block. When the block was freed, the account would be credited. This scheme provides a way to implement absolute limits (quotas) on resource use. In the latter model, the client is charged for rental of blocks at a rate of X units per kiloblock-second or block-month or something else. In this model, virtual money is constantly flowing from the clients to the servers, in which case clients need some form of income to keep them going. The policy about how income is generated and dispensed is determined by the owner of the currency in question, and is outside the scope of the bank server.

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Perspectives in Mathematical Sociology

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A strong tradition exists in the application of mathematics in physics. More and more, mathematical methods are applied in other disciplines such as biology and the humanities. It is noticed that each field demands its own type of mathematics. The question arises whether beside mathematical physics and mathematical biology a mathematical sociology can also be discerned.

1. Introduction

Recently, in a special issue of the *Journal of Mathematical Sociology*, attention was given to the role of mathematics in the social sciences. In particular the articles by Wilson [42] and Marsden and Laumann [30] give an impression of the progress that has been made in this direction. It would be inappropriate to write an article based on excerpts from this issue. An interested reader will certainly enjoy the original articles. In the present paper perspectives in mathematical sociology are discussed from the point of view of a mathematician, working on the modeling of physical and biological phenomena, with an eagerness to explore new fields.

Between mathematics and the applied disciplines an interaction exists which uncovers new areas of research: mathematics provides us with new methods to analyse problems in the sciences and the humanities. On the other hand problems in these disciplines may require a new type of mathematics. The study of nonlinear diffusion equations was strongly motivated from biology and chemistry. The discovery of the mechanism underlying chaotic behaviour of physical and biological systems (turbulence) required new mathematical tools such as the concept of fractional dimension and methods for analyzing discrete dynamical systems. The question arises whether sociological problems may also induce a new type of mathematics. The survey paper of Marsden and Laumann [30] shows how in quantitative sociology a set of mathematical techniques are brought together forming a coherent method for analyzing social

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structures. From the sociological point of view, a new theory was developed and for mathematics a new application was found. However, the mathematical techniques are not fundamentally new, as in the above example of chaotic dynamics.

In our exposé on the perspectives of mathematical sociology we focus our attention on the problem of relating individual action to collective phenomena. We will dwell upon cases for which no model can yet be formulated because of the fact that at a microscopic level structural changes are induced by macro-variables of the interaction process. In sociological problems, the complicated cause-effect chain between an individual action and a collective phenomenon is fairly well unraveled by a theory known as *structural individualism.* Reverse effects may also be described in this way. However, structural individualism is a qualitative explanatory theory, in which an explanation of a collective phenomenon is formulated in the context of sharply defined sociological problems. It is hoped that a mathematical formalism can be found, which quantifies the principles underlying this theory. Such a development would enhance the explanatory power of structural individualism and create the possibility of application in other disciplines like biology, see also LEVINS and LEWONTIN [26] for a discussion of the problem of analyzing ecological systems.

The principle of *self-organization* in physics, biology and sociology is an example of the type of interaction process described above. In the last ten years various theories of self-organization have been formulated. However, none of them describes micro-macro processes in a way that accounts adequately for structural changes at the microlevel induced by macro variables.

In Section 2 we review present, non-statistical, applications of mathematics in sociology. Two major types of applications can be distinguished: game theoretical problems and network analysis (based on several mathematical techniques, such as graph theory, optimization, and the theory of stochastic processes). In Section 3 sociological theories with a strong mathematical content are discussed. The work of Boudon is taken as a basis from where we make our excursions. Special attention is given to the work of LINDENBERG [27] about structural individualism. Parallels between modeling in physics, biology and sociology are made in Section 4, where we deal with the phenomenon of self-organization.

In this paper we will not discuss the role of statistical methods in sociology. We refer to FOX [12] for recent developments in regression analysis of sociological problems. FREEDMAN [13] has some critical remarks on the application of statistics in the social sciences. In the same volume responses are given by S.E. Fienberg and by K.G. Jöreskog and D. Sörenbom.

2. MATHEMATICS IN SOCIOLOGY TODAY

From the applications of mathematics in sociology we will discuss game theoretical problems and network analysis of social structures. The latter has been set forth as a widely accepted direction in sociology with its own journal (Social Networks). Furthermore, we will touch upon the mathematical and sociological aspects of voting.

In *game theory* a number of players all try to realize a maximal benefit for themselves in a game, being a set of rules. Each player has resources at his disposal and different choices of actions may be taken. A player will have preferences for a possible outcome of the game because of its pay off. In order to achieve such an outcome players may collaborate. Usually, in the game, players will have a conflict of interest and they may loose or win if the benefit is below or above an expected value. For a survey of game theory in the social sciences we refer to SHUBIK [38]. A game frequently cited in sociological studies is the 'prisoners dilemma', see HOFSTADTER [22].

In *social network analysis* a group of interacting individuals are represented by a graph. The nodes are the actors and a directed arc indicates the presence of a communication channel between two actors. This can be seen as a static structure in which one may analyse maximally connected subgraphs (cliques). It is also possible to make the graph time dependent. HOLLAND and LEINHARDT [23] construct such a dynamic social network model. The graph is given by a matrix with entries having a value zero (no communication) or one (directed communication). The entries change stochasticly at each time step with the parameters of the stochastic process depending on the current structure. The evolution of the process can be investigated by Monte Carlo simulations and stable stationary solutions may be interpreted in sociological terms. The sociological distance between two actors in a network can be measured from the minimal number of existing arcs that is needed to make a connection.

Social distance is also used in a different way. For each item of a set of n, an individual will have a score on a one-dimensional scale. SCHIFFMAN et al. [36] describe a method of multi-dimensional scaling in which they construct the smallest underlying space of dimension $m \leq n$, where the individuals take such positions that their mutual distances satisfy the requirements of a metric in this space.

In the study of *voting* a wide spectrum of applications of mathematics and sociology is found. First there is the problem of proportional representation. Let political party i have a fraction f_i of the votes. How many seats should it have in a parliament with N seats? That is, find the vector n/N such that it has, in some sense, a minimal distance to f, see TE RIELE [39]. In a second type of voting problem, members of a regional council are appointed by and from the local councils participating in the regional co-operation. The delegation from each council should reflect the political composition of this council, while in the regional council the political parties should have a proportional representation from all over the region. ANTHONISSE [1] studies this problem from a point of view of optimal flow in networks, see also PELEG [34]. Less mathematical is the question of voting behaviour. One may approach it as a problem of competition, as in biological population dynamics (COLLINS and KLEINER [9]) or as a sociological problem (LYPHART [29]), see also BERELSON and LAZARSFELD [4].

3. Sociological theories open to mathematical modeling

In this section we give a survey of sociological theories with a logical structure suited for mathematical modeling.

We start this overview with the founder of modern sociology Emile Durkheim. His investigations focus on social causes for the presence of collective phenomena. In his study on suicide he relates this act to social factors like religion and economical depression and revival. From the work of Durkheim one gets the impression that in society constants exist which rise above the acting of the individual. This idea is also met in the work of Quetelet [35] on his statistical description of the physiognomy of men.

At a later stage the idea evolved to connect acts of individuals to collective phenomena. First the attention went more in the direction of the interaction between some individuals (microsociology). This is seen in the work of Simmel, who studied the influence of the size of the group on social phenomena. Coleman [8] gives a quantitative mathematical description of the effect of group size. Another way of studying the behaviour of the individual is the stimulus-response theory of Skinner. Eventually, such theories of behaviour were put aside by theories of individual action as these are also applicable in a social context. The exchange theory of Homans [24] and the choice theory are examples of this new development.

In the last twenty years a new movement in sociology came into existence: *interpretive sociology*. Starting point for theories, brought together under this name, is their method of empirical research: the investigator should project himself in the social happening taking place in its natural environment. This attitude is in conflict with the classical ideas of observation and the use of a laboratory type of setting for doing experimental research. From these theories *symbolic interactionism* is the one with elements that are also found in self-organizing biological systems. It stresses the role of the acting individuals (actors) in the building of a social interaction pattern, see Mead [32] and Blumer [5]. Important in this theory is the meaning that is given to a social act. By this process the actor constructs images of himself, his co-actors and the environment. These images, in turn, control his activities in a social context.

Returning to analytical sociology we arrive at *methodological individualism*, in which the individual is the smallest logical unit of a social interaction system. An exponent of this theory is Boudon [6] with his analysis of social mobility. In his study on the relation between school career and social background, he analyses cohorts of students. In a stochastic model transition probability coefficients are determined and their dependence on the social parameters is measured. The coefficients are determined by flows at macrolevel and chance at microlevel is a reflection of these flows. Consequently, this is not a type of micro-macro system structure as we discussed in the A.

In his book *La Logique du Social* Boudon [7] gives a systems approach to social processes. He introduces this subject with a reference to the work of Hägerstrand [17] on the spread of an innovation. Hägerstrand's study concerns a Swedish governmental grant to farmers for fencing their land. It was noticed that in the five years after the start of subsidization in 1928, the process developed from spreading centers with personal contacts between farmers playing an important role, see fig. 1 for the comparable process of the spread of an infection. Hägerstrand had a sufficiently accurate picture of the microsociological structure to construct a model of the actual process. The premises of his model are:

a. At the start one actor has accepted the innovation.
b. The actors meet two by two.
c. The degree of acceptance of the innovation has a known distribution.
d. The willingness to accept increases with the number of meetings with a positive personal influence.
e. The meeting probability of two actors depends on their mutual distance.

Although the model is not cast in formula's, as one is used to in the exact sciences, the description is sufficiently accurate to understand the mechanism and to simulate the process.

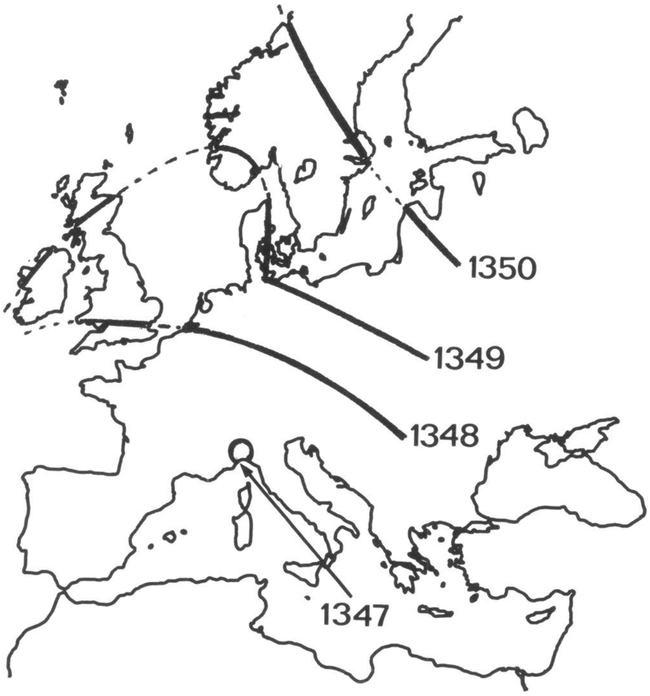

FIGURE 1. Spread of pestilence over Europe in the middle ages

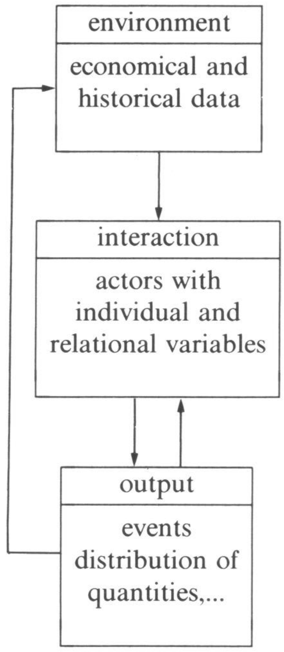

FIGURE 2. A systems approach to the interaction process; the arrows indicate the possible mutual influences of the components

In an interaction process, as described above, a number of components can be distinguished: the environment, the interaction system itself and the output of the process, see fig. 2. When all influences indicated by arrows are present we have a so-called transformation process containing different feedbacks. In fig.3 we sketch two reductions: the reproduction process and the accumulation process with one feedback. An example of the latter is the fluctuation of market prices in the process of supply and demand.

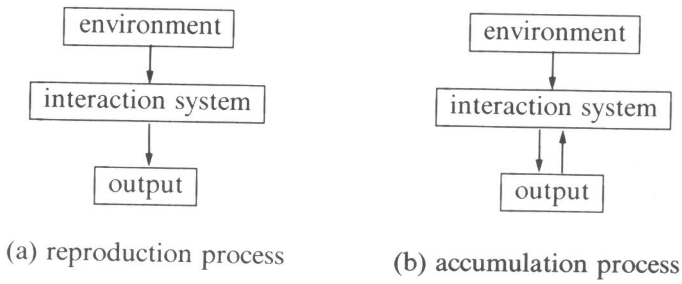

FIGURE 3. Two types of reduced transformation processes: farming can be seen as an example of (a) and a prey-predator system as an example of (b)

Last in our survey of sociological theories is *structural individualism.* This movement in Dutch sociology, which started in the seventies (WIPPLER [43]), links up with the methodological individualism of Boudon. The most important difference between the two is the quantitive approach of Boudon versus the qualitative explanatory theory of collective phenomena in the structural individualism. In a tight way individual laws (e.g. making profit or the benefit question) are coupled to individual activities by a clear bridge theory and an explanans and explanandum of a collective phenomenon are formulated. LINDENBERG [27] brought the composite explanation of a collective effect in the scheme of fig.4 (different definitions are used as Lindenberg's expressions have a meaning of their own in mathematics).

individualistic laws individualistic rules ——————— individual effect	⟶	transformation process collective rules individual effects ——————— collective effect

FIGURE 4. Scheme for an explanatory theory of collective effects (LINDENBERG [27])

We now come to the point of discussing the possible role of mathematics in this theory. In some problems a model can be used to describe a transformation process, such as Hägerstrand's diffusion model for the spread of an innovation. It is possible to analyse, in that case, the dynamics of the collective effect. It is remarked that then the microlevel is left completely: the dynamical system is studied in its macro-variables. In a similar way individual decisions are modeled stochasticly in Boudon's mobility theory and these decisions are reflected in the change of macro-variables (fractions of students). For those problems the models give a satisfying description. However, there are problems in which the collective effect influences the individualistic rules and makes it necessary to analyse simultaneously the micro- and macro structure. Such a type of modeling is not yet developed. Directions in research exist in which one tries to master this problem. The change of connections between individual nerve cells and the global behaviour of a neural network is an example of such a micro-macro structure. In the next section we will come back on this problem.

4. MATHEMATICAL PHYSICS, BIOLOGY AND SOCIOLOGY

The title of this section suggests the presence of a continuum of mathematical models of 'real world' phenomena. Such a continuation is found in the class of diffusion processes, as one may observe in the following five examples:

a. *Chemical reaction and diffusion.* In a medium there is a chemical reaction; the reactants diffuse in the medium and take part in the reaction process at neighbouring positions. The change of the concentration of the reactants as a function of time and position has the shape of a travelling wave. For this process a mathematical formulation exists, see ARIS [2].

b. *Pulse propagation in nerve cells.* In the membrane of a nerve cell an electro-chemical process takes place; the ions diffuse freely at both sides of the membrane. The process yields a propagating electric pulse wave along the membrane. HODGKIN and HUXLEY [21] formulated a mathematical model, which gives an excellent description of the phenomenon, although some details of the process (transport through the membrane) were not completely understood.

c. *Spread of a contagious disease.* In a biological population a contagious disease spreads out as a wave over the area. Also for this process a mathematical formulation exists, see DIEKMANN [10] (cf. figs 1 and 5).

d. *Spread of a new genotype.* In a biological population a new genotype spreads out over many generations, see for example FISHER [11].

e. *Spread of an innovation.* In a community an innovation is accepted in wider circles after periods of time, see HÄGERSTRAND [17].

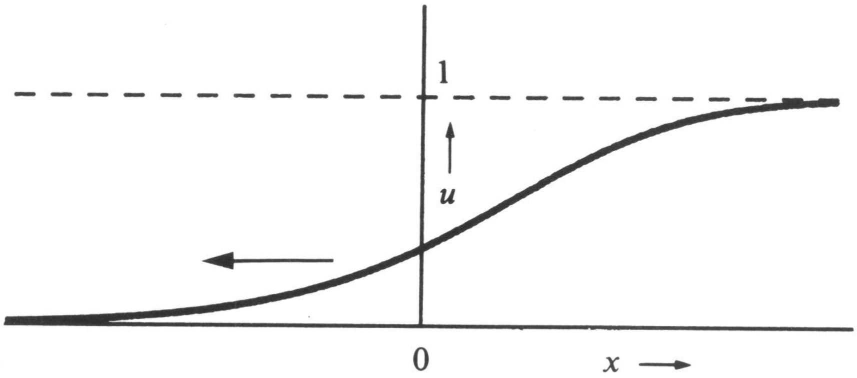

FIGURE 5. Travelling wave solution that describes the spread of a substance such as an infection over a population

Using a mathematical formulation we may construct a differential equation model that describes the underlying mechanism in the five diffusion processes. First we consider the case that within a closed box the carriers of a substance mix perfectly. Then the increase of the fraction of taken carriers is assumed to increase logisticly. That is it satisfies the differential equation

$$\frac{du}{dt}=\alpha u(1-u), \quad u(0)=\delta>0.$$

For a 1-dimensional continuum of connected boxes, we have to take into account the effect of diffusion between neighbouring boxes. The model equation is then a partial differential equation of parabolic type

$$\frac{\partial u}{\partial t}=\alpha u(1-u)+\beta\frac{\partial^2 u}{\partial x^2}\cdot$$

Let us assume that

$$u(x,t)=0 \quad \text{for} \quad x \to -\infty,$$

$$u(x,t)=1 \quad \text{for} \quad x \to \infty.$$

Using analytical methods one can prove that a travelling wave exists with a minimal velocity of $2\sqrt{\alpha\beta}$, see fig.5. FISHER [11] dealt with this problem in his analysis of the geographic spread of a genotype over a population.

A similar continuation of interrelated phenomena is found in the modeling of self-organization. Chemical reaction-diffusion processes may lead to stable inhomogeneous spatial concentration distributions. This, in turn, may explain the self-organized differentiation of cell functions in an embryo.

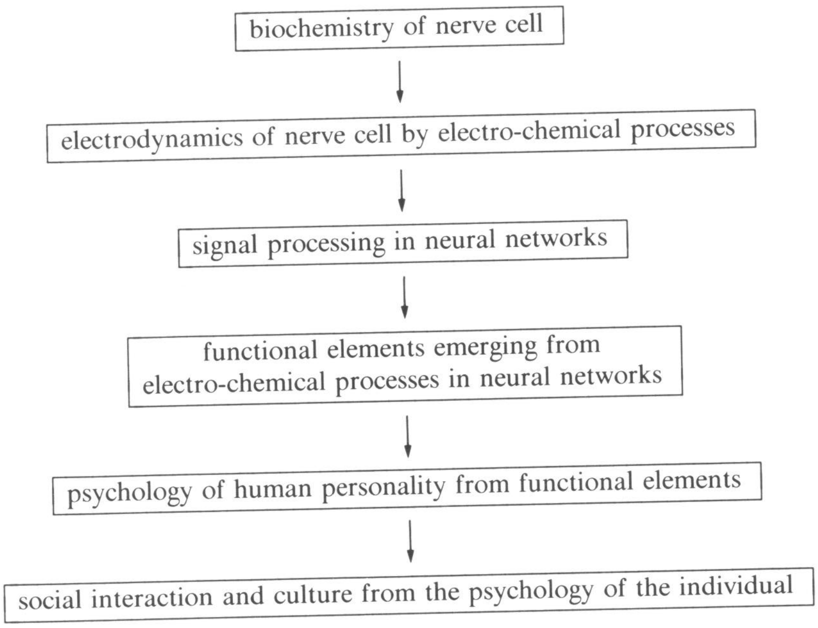

FIGURE 6. Scheme of levels in mental activity embedded in a wider framework (SCOTT [37])

Nerve cells (neurons) organize themselves. HEBB [19] introduced this element in neurodynamics through the concept of 'assembly'. An assembly is a group of neurons with mutual connections (synapses) that are strengthened if during a period of time the cells exhibit synchronous activity. As a result of this the probability, that the cells will be active at the same time in future, will increase. Of course the cells must have a certain freedom: they should not be tied up completely to the control of physiological functions. These unconstrained cells have the ability to organize their own structure within certain limitations. Consequently, this structure will differ from individual to individual; it carries thoughts and memories. The schematic representation of mental processes of fig.6 shows at which level self-organization enters in neurodynamics. Surprisingly the explanatory scheme of collective effects of Lindenberg applies quite well to mental activity, see figs. 4 and 7.

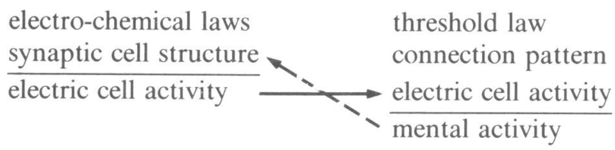

FIGURE 7. An explanatory scheme for mental activities in the manner of structural individualism

The change of the interaction structure in a network of neurons caused by the formation of cell assemblies is known as plasticity of the neuronal system. GRASMAN and JOHANNESMA [15] have indicated that social processes have properties comparable to plasticity of neural activities. The following three examples illustrate this relation, see the scheme of fig. 8 for a summary of the parallels.

a. *Hebb's cell assembles.* We already mentioned this model. In a neuronal network the activities consist of generation and conduction of electric pulses. This process yields structural changes in the network: strengthening and weakening of connections.

b. *Communication between elephant fishes (Mormyriden).* These fishes communicate through electric pulses comparable with the ones between nerve cells. Because of this activity (and environmental causes) the fish will change its position or will get in a different state of behaviour, which may make it react differently upon signals from other fishes, see HEILIGENBERG [20].

c. *A cocktail party.* At a party the most important activity is communication with one another. By this process one will change position and behaviour such that communication with certain others is advanced or prevented, e.g. by joining or leaving a group in conversation.

System	Activity	Structure
Cell assembly	electric pulses	synaptic connections
Elephant fishes	electric pulses	position, behaviour
Cocktail party	conversation	position, behaviour

FIGURE 8. Scheme of three transformation processes admitting plastic changes

The sociologist Weber described social interaction as the probability that social actions follow a certain pattern. Although it is not mentioned explicitly, the possibility exists that this probability changes in time. A pattern is culturally determined. Individuals may act differently and if they do it in the long term, the probability pattern (cultural system) will change, which indeed is a form of plasticity. In Mead's symbolic interactionism the role of the actors in the formation of the probability pattern is stressed. Not so clear in this theory is the presence of two time scales: the actual time scale for the social action of the individuals and the larger time scale with the change in interpretation that is given to social actions. Plastic changes are in the latter scale.

GROSSBERG [16] constructs a differential equation model of a neuronal network in which plastic changes are possible. Let x_i be the electric potential of the i^{th} neuronal cell and z_{ki} a measure for the number and strength of the synaptic connections from cell k to i (long-term memory trace), then the system of differential equations is of the form

$$\frac{dx_i}{dt} = f_i(x_i) + g_i(x_i)\sum_k A_{ki} z_{ki} e_k(x_k),$$

$$\frac{dz_{ki}}{dt} = -B_{ki} z_{ki} + C_{ki} h_i(x_i) d_k(x_k).$$

This network has learning properties. In simulation runs, macro-phenomena occur, that are easily perceived, but difficult to quantify.

The relation between micro- and macrolevel is of fundamental importance for the understanding of the dynamics of nonlinear systems with a large number of interacting components. In the literature dealing with the problem of self-organization we distinguish three theories which we discuss next.

The work of the Belgium group around Prigogine is based upon thermodynamic principles for *dissipative structures* in physical and chemical systems, see NICOLIS and PRIGOGINE [33]. A hypothetical chemical reaction, known as the Brusselator, plays a central role in their theory of self-organization, see fig. 9.

$$\begin{aligned} A &\rightleftarrows X, \\ B + X &\rightleftarrows Y + D, \\ 2X + Y &\rightleftarrows 3X, \\ X &\rightleftarrows E. \end{aligned}$$

FIGURE 9. The Brusselator; a hypothetical chemical reaction scheme admitting nonuniform concentration distributions over the medium

The reaction occurs in a medium where the reactants may diffuse freely. Stationary states exist for which the concentrations of the reactants have a nonuniform distribution over the medium. The nonlinearity in the third autocatalytic step of the scheme is responsible for this behaviour. The model can be used in developmental biology. The spontaneous change in the shape of the beginning embryo can be explained by this process (morphogenesis). It demonstrates the application of this principle of self-organization in biological problems.

A second theory of self-organization is the synergetics approach of HAKEN [18], who studies the occurrence of qualitative macroscopic changes caused by microscopic action principles of elementary subunits. The applications range from astrophysics over biology to sociology. In the series *Synergetics* edited by Haken, the mathematical techniques and their applications are brought together. We mention the application in sociology (WEIDLICH and HAAG [41]), in brain modeling (BASAR et al. [3]) and in various other fields (FREHLAND [14]). Haken was confronted with remarkable properties of nonlinear systems in his study on laser dynamics. Depending on the value of the parameters of a system a few state variables may change from stable to unstable. This subsystem determines the qualitative properties of the total system; the stable modes just follow (slaving principle).

Bifurcation theory is the branch of mathematical analysis that deals with the change in number, type and stability of stationary solutions of a nonlinear system as a function of the parameters. With the discovery of chaotic dynamics a new element entered the study of nonlinear systems: stable solutions may exhibit a seemingly random behaviour. LORENZ [28] was the first who noticed this phenomenon in a simple system of three coupled differential equations. This new concept in the theory of dynamical systems was rapidly admitted to existing theories of self-organization. However, this new development is drawing attention away from a competing problem. We can put this as follows: the insight in the complex dynamics of simple systems has increased dramatically in the last ten years, but on the other hand we still need to make a lot of progress in the analysis of simple dynamics of complex systems. Examples of such systems in physics, biology and sociology are easily found. We mention the well-known Ising problem dealing with the spin orientation of a ferro-magnetic object, the threshold phenomenon in epidemics and in nerve excitation and the brain as a complex system which handles questions having a yes/no outcome. In sociology we have the example of the race for the presidency between two candidates in a democratic electorial system.

Formal system theoretical aspects of self-organization are analyzed in the third approach, called *autopoiesis.* It deals with the structural coupling between systems and between a system and its environment. Autopoiesis is the realization through a closed organization of production processes such that the same organization of processes is generated through the interaction of their own products (components). Proliferation of biological cells is an illustrative example of autopoiesis. MATURANA and VARELA [31] formulate general principles which constitute a theory that is applicable in several disciplines, e.g. in the study of human organizations.

For a discussion of the principles of self-organization we refer to JANTSCH [25]. The above theories have in common that their scope ranges beyond the limitation of a discipline, as it is the case in Berthalanffy's general system theory, see VON BERTHALANFFY [40]. This can be seen as a strong point, but it also has its weakness. No authority in any of the accepted scientific disciplines can defend such a theory in all its elements spread out over the various fields of science. The switching from a metaphor in one field to another in a different field (as we did) indicates the missing of an appropriate mathematical formalism.

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Abstract: The present volume is one of the two tracts which are based on the dissertation 'Foundations and applications of Montague grammar'. The two volumes present an interdisciplinary study in mathematics, philosophy, computer science, logic, and linguistics. No knowledge of specific results in these fields is presupposed, although occasionally terminology or results from them are mentioned. Throughout the text it is assumed that the reader is acquainted with fundamental principles of logic, in particular of model theory, and that he is used to a mathematical kind of argumentation. The contents of the volumes have a linear structure: first the approach is motivated, next the theory is developed, and finally it is applied. Volume 1 contains an application to programming languages, whereas volume 2 is devoted completely to the consequences of the approach for natural languages. The volumes deal with many facets of syntax and semantics, discussing rather different kind of subjects from this interdisciplinary field. They range from abstract universal algebra to linguistic observations, from the history of philosophy to formal language theory, and from idealized computers to human psychology.

CS-R8618. P.M.B. Vitányi. *Non-sequential computation and laws of nature.*
AMS 68C05, 68C25, 68A05, 68B20, 94C99; CR B.7.0, C.2, D.4, F.2.2, F.2.3, G.2.2; 17 pp.; **key words:** sequential computation, parallel computation, distributed computation, VLSI, computational complexity, time, space, physics, communication, wires, limitations, laws of nature.
Abstract: Traditionally, computational complexity theory deals with sequential computations. In

the computational models the underlying physics is hardly accounted for. This attitude has persisted in common models for parallel computations. Wrongly, as we shall argue, since the laws of physics intrude forcefully when we want to obtain realistic estimates of the performance of parallel or distributed algorithms. First, we shall explain why it is reasonable to abstract away from the physical details in sequential computations. Second, we show why certain common approaches in the theory of parallel complexity do not give useful information about the actual complexity of the parallel computation. Third, we give some examples of the interplay between physical considerations and actual complexity of distributed computations.

CS-R8623. P.J.W. ten Hagen & M.M. de Ruiter. *Segment grouping, an extension to the graphical kernel system.*
AMS 69K32, 69K34, 69K30; 34 pp.; **key words:** computer graphics, graphics systems, standardization, window management.
Abstract: This paper introduces an extension in GKS, the ISO standard for 2D graphics software. Segment Grouping provides a device independent window manager functionally for application programmers. Segment Grouping allows for more efficient (given certain hardware restrictions) clipping and transformation of segments on both low and high function workstations.

CS-R8624. R.J. van Glabbeek. *Notes on the methodology of CCS and CSP.*
AMS 68B10, 68C01, 68D25, 68F20; CR F.1.2, F.3.2, F.4.3, D.3.1; 21 pp.; **key words:** concurrency, CCS, CSP, process algebra.
Abstract: In this paper the methodology of some theories of concurrency (mainly CCS (Milner's Calculus of Communicating Systems) and CSP (Hoare's theory of Communicating Sequential Processes)) is analysed, focusing on the following topics: the representation of processes, the identification issue, and the treatment of nondeterminism, communication, recursion, abstraction, divergence and deadlock behaviour. Process algebra turns out to be a useful instrument for comparing the various theories.

CS-R8625. J.A. Bergstra, J.W. Klop & E.-R. Olderog. *Failures without chaos: A new process semantics for fair abstraction.*
AMS 68B10, 68C01, 68D25, 68F20; CR F.1.1, F.1.2, F.3.2, F.4.3; 26 pp.; **key words:** process algebra, concurrency, failure semantics, bisimulation semantics.
Abstract: We propose a new process semantics that combines the advantages of fair abstraction from internal process activity with the simplicity of failure semantics. The new semantics is obtained by changing the way the original failure semantics of Brookes, Hoare and Roscoe or the equivalent acceptance semantics of de Nicola and Hennessy deal with infinite internal process activity, known as divergence. We work in an algebraic setting and develop the new semantics stepwise, thereby systematically comparing previous proposals.

CS-R8626. P. America, J.W. de Bakker, J.N. Kok & J.J.M.M. Rutten. *A denotational semantics of a parallel object-oriented language.*
AMS 68B10, 68C01; CR D.1.3, D.2.1, D.3.1, F.3.2; 43 pp.; **key words:** object-oriented programming, denotational semantics, parallelism, metric spaces, fixed points, fairness, domain equations, continuations.
Abstract: A denotational model is presented for the language POOL, a Parallel Object-Oriented Language designed by America. For this purpose we construct a mathematical domain of processes in the sense of De Bakker and Zucker. Their approach is extended to cover a wider class of reflexive domain equations, including function spaces. A category of metric spaces is considered, and the desired domain is obtained as a fixed point of a contracting functor. The domain is

sufficiently rich to allow a fully compositional definition of the language constructs in POOL, including advanced concepts such as process creation and method invocation by message. The semantic equations give a meaning to each syntactic construct depending on the POOL object executing the construct, the environment constituted by the declaration and a continuation. A preliminary discussion is provided on how to deal with fairness. Full mathematical details are supplied (in an appendix), with the exception of the general domain construction which is to be described elsewhere.

CS-R8627. N.W.P. van Diepen. *A study in algebraic specification: a language with goto-statements.*
AMS 68BXX; CR D.2.1, D.3.1, F.3.1, F.3.2; 27 pp.; **key words:** algebraic specification, goto-statement, modular specification, implementation of algebraic specifications, abstract programming.
Abstract: The algebraic specification of the semantics of Small - a programming language designed to illustrate specifications in denotational semantics - is given. Focus of attention are the specification of the semantics of goto-statements and the modular build-up of a language specification.

CS-R8628. P.M.B. Vitányi & B. Awerbuch. *Atomic shared register access by asynchronous hardware.* (detailed abstract)
AMS 68C05, 68C25, 68A05, 68B20; CR B.3.2, B.4.3, D.4.1, D.4.4; 16 pp.; **key words:** concurrency, atomicity, shared registers, asynchronous readers and writers.
Abstract: The contribution of this paper is two-fold. First, we describe two ways to construct multivalued atomic *n*-writer *n*-reader registers. The first solution uses atomic 1-writer 1-reader registers and unbounded tags. The other solution uses atomic 1-writer *n*-reader registers and bounded tags. The second part of the paper develops a general methodology to prove atomicity, by identifying a set of criteria which guarantee an effective construction for the required atomic mapping. We apply the method to prove atomicity of the two implementations for atomic multiwriter multireader registers.

CS-R8629. F.W. Vaandrager. *Process algebra semantics of POOL.*
AMS 68B10, 68C01, 68D25, 68F20; CR D.1.3, D.2.1, F.1.1, F.1.2, F.3.2, F.4.3; 64 pp.; **key words:** process algebra, concurrency, object-oriented programming, semantics of programming languages, attribute grammars, chaining operator.
Abstract: In this paper various semantics of the Parallel Object-Oriented Language (POOL) are described in the framework of ACP, the Algebra of Communicating Processes.

CS-R8630. M.L. Kersten & F.H. Schippers. *Towards an object-centered database language.*
AMS 69D42, 69H23, 69K14; CR D.3.2, H.2.3, I.2.4; 17 pp.; **key words:** object-oriented languages, database management, knowledge representation.
Abstract: In this report we discuss ongoing research in the area of object-oriented database systems at the CWI. The central theme of this paper is the friction encountered when using an Object-Oriented (O-O) language, such as Smalltalk, in the database arena. A series of (open) database issues is given for which the object-oriented paradigm does not provide an elegant solution. A refinement of the O-O concepts is given which emphasizes the dynamic classification of objects through its characteristic properties. Our approach is illustrated by a description of the *object-centered* database language *Godel* and its use. A central language concept is the *guardian*, which is

a high-level declarative description of a process which algorithmically reacts to states and to state changes of an object base. A prototype implementation of *Godel* has been implemented in C-Prolog.

CS-R8631. M.L. Kersten & F.H. Schippers. *Using the guardian programming paradigm to support database evolution.*
AMS 69D42, 69H23, 69K14; CR D.3.2, H.2.3, I.2.4; 14 pp.; **key words:** object-oriented languages, database management, knowledge representation.
Abstract: A guardian is a high-level declarative description of a process which algorithmically reacts to an observed state or state change of a database. In this paper the role of guardians in the object-centered programming language *Godel* is illustrated. This language has been designed to support the construction of knowledge based applications and adaptive information systems. It is shown that traditional problems such as maintaining the integrity of a database are easily resolved using guardians without constraining the evolutionary paths of the database definition. This is accomplished by relaxation of the class membership rule found in object-oriented languages. In *Godel* class membership is a dynamic property determined by the provable characteristics of the object, rather than the operator applied to create the object. Dynamic class membership separates the classification of the object from its time-dependent semantic behaviour. Therefore, integrity of the database can be preserved by a set of guardians which represent the integrity rules and react in an algorithmic fashion on any violation.

CS-R8632. J.C.M. Baeten, J.A. Bergstra & J.W. Klop. *Decidability of bisimulation equivalence for processes generating context-free languages.*
AMS 68C01, 68D25, 68F10, 68F20; CR F.1.1, F.3.2, F.4.3; 32 pp.; **key words:** process algebra, bisimulation semantics, context-free languages, context-free grammars, simple context-free languages.
Abstract: A context-free grammar (CFG) in Greibach Normal Form coincides, in another notation, with a system of guarded recursion equations in Basic Process Algebra. Hence to each CFG a process can be assigned as solution, which has as its set of finite traces the context-free language (CFL) determined by that CFG. While the equality problem for CFL's is unsolvable, the equality problem for the processes determined by CFG's turns out to be solvable. Here equality on processes is given by a model of process graphs modulo bisimulation equivalence. The proof is given by displaying a periodic structure of the process graphs determined by CFG's. As a corollary of the periodicity a short proof of the solvability of the equivalence problem for simple context-free languages is given.

CS-R8633. J. Heering & P. Klint. *A syntax definition formalism.*
AMS 68B99; CR D.2.1, D.3.1; 12 pp.; **key words:** user-definable syntax, abstract syntax, syntax definition formalism.
Abstract: The goal of the GIPE project is to generate interactive programming environments from formal language definitions. We currently envisage language definitions consisting of three parts: (a) a syntax section; (b) a static constraints section; (c) a dynamic semantics section. An initial version of the language definition formalism is being developed. In this paper we concentrate on the syntax section and introduce a new formalism which allows concrete and abstract syntax of specification (and other) languages to be defined simultaneously. The new formalism can be combined with a variety of specification languages. By doing so these obtain fully general user definable syntax.

CS-R8634. R.J. van Glabbeek. *Bounded nondeterminism and the approximation induction principle in process algebra.*

AMS 68Q10, 68Q45, 68Q55, 68N15; CR F.1.2, F.3.2, F.4.3, D.3.1; 23 pp.; **key words:** concurrency, process algebra, ACP, approximation induction principle, recursion, abstraction, fairness, liveness, consistency, bisimulation, bounded nondeterminism.

Abstract: This paper presents a new semantics of ACP_τ, the Algebra of Communicating Processes with abstraction. This leads to a term of ACP_τ, which is isomorphic to the model of process graphs modulo rooted $\tau\delta$-bisimulation of Baeten, Bergstra & Klop, but in which no special rootedness condition is needed. Bisimularity turns out to be a congruence in a natural way. In this model, the Recursive Definition Principle (RDP), the Commutativity of Abstraction (CA) and Koomen's Fair Abstraction Rule (KFAR) are satisfied, but the Approximation Induction Principle (AIP) is not. The combination of these four principles is proved to be inconsistent, while any combination of three of them is not. In Baeten, Bergstra & Klop a restricted version of AIP is proved valid in the graph model. This paper proposes a simpler and less restrictive version of AIP, not containing guarded recursive specifications as a parameter, which is still valid. This infinitary rule is formulated with the help of a family β_n of unary predicates, expressing bounded nondeterminism.

CS-R8635. P. Klint. *Modularization and reusability in current programming languages.*
AMS 68B10; CR D.2.2, D.3.3; 14 pp.; **key words:** software engineering, reusable software, modules, abstract data types, object-oriented programming.

Abstract: How well do modularization constructs in current programming languages allow the construction of truly reusable modules? This question is answered by examining the implementation of the datatype queues in Pascal, Modula-2, Ada and Smalltalk. The merits of object-oriented languages versus algorithmic languages are discussed from the perspective of reusability.

CS-N8607. J. Heering. *Program generators.* (In Dutch)
AMS 68B99; CR D.1.2, I.2.2, I.2.6; 52 pp.; **key words** program generator, specification language, fourth generation language, programming by example, induction, editing by example, partial evaluation.

Abstract: The rapid growth of the use of program generators and the associated specification languages ('fourth-generation languages') might easily lead to the impression that this is a new and revolutionary development. In fact, however, it is the natural and gradual continuation of a line of development almost as old as the computer itself. In the mean time the field has become very large. While many generators for every day usage prove their worth regularly in practice, more esoteric options are being developed in software laboratories, like the generation of programs from examples and the automatic derivation of efficient specialized versions from general programs using partial evaluations. (Copies of slides in Dutch)

OS-R8607. A. Schrijver. *Polyhedral combinatorics - some recent developments and results.*
AMS 05-XX, 05C70, 90C27; 15 pp.; **key words:** polyhedral, combinatorics, polynomial-time, combinatorial optimization, strongly polynomial, lattice reduction, cutting planes.

Abstract: Polyhedral combinatorics deals with characterizing convex hulls of vectors obtained from combinatorial structures, and with deriving min-max relations and algorithms for corresponding combinatorial optimization problems. In this paper, after an introduction discussing the matching polytope (Section 1) and some algorithmic consequences (Section 2), we give some illustrations of recent developments (viz. applications of lattice and decomposition techniques (Sections 3 and 4)),

we go into the relation to cutting planes (Section 5), and we describe some other recent results (Section 6).

OS-R8608. C. van Hoesel & A. Schrijver. *Edge-disjoint homotopic paths in a planar graph with one hole.*
AMS 05C38, 05C35, 05C10, 68R10; 14 pp.; **key words:** graphs, homotopy, edge-disjoint, routing, paths, multicommodity.
Abstract: We prove the following theorem, conjectured by K. Mehlhorn: Let $G=(V,E)$ be a planar graph, embedded in $\mathbb{R}^2$. Let O denote the interior of the unbounded face. Let I be the interior of some fixed bounded face. Let $C_1, \ldots, C_k$ be curves in $\mathbb{R}^2 \setminus (I \cup O)$, with end points in $V \cap \delta(I \cup O)$, so that for each vertex v of G the degree of v in G has the same parity (mod 2) as the number of curves C_i beginning or ending in v (counting a curve beginning and ending in v for two). Then there exist pairwise edge-disjoint paths $P_1, \ldots, P_k$ in G so that P_i is homotopic to C_i in the space $\mathbb{R}^2 \setminus (I \cup O)$ for $i=1,...,k$, if and only if for each dual path Q from $I \cup O$ to $I \cup O$ the number of edges in Q is not smaller than the number of times Q necessarily intersects the curves C_i. The theorem generalizes a theorem of Okamura and Seymour. Its proof yields a polynomial-time algorithm finding the paths as required.

OS-R8609. G.A.P. Kindervater & J.K. Lenstra. *The parallel complexity of TSP heuristics.*
AMS 90C27, 68Q15, 68Q25, 68R05; 14 pp.; **key words:** parallel computing, computational complexity, polylog parallel algorithm, log space completeness for $\mathcal{P}$, traveling salesman problem, heuristics.
Abstract: The problems of finding an (approximate) traveling salesman tour by the nearest neighbor, nearest merger, nearest insertion, cheapest insertion and farthest insertion heuristics are log space complete for $\mathcal{P}$. We show this by giving log space transformations from the circuit value problem. Hence, it is unlikely that such tours can be obtained in polylogarithmic work space on a sequential computer or, by the parallel computation thesis, in polylogarithmic time on a computer with unbounded parallelism. We also show that the double minimum spanning tree and nearest addition heuristics can be implemented in polylogarithmic time on a polynomial number of processors.

OS-R8610. A. Schrijver. *Distances and cuts in planar graphs.*
AMS 05C70, 05CXX, 90C35; 13 pp.; **key words:** multicommodity flows, distance function, planar graph.
Abstract: We prove the following theorem. Let $G=(V,E)$ be a planar bipartite graph, embedded in the euclidean plane. Let O and I be two of its faces. Then there exist pairwise edge-disjoint cuts $C_1, \ldots, C_t$ so that for each two vertices v,w with $v,w \in O$ or $v,w \in I$, the distance from v to w in G is equal to the number of cuts C_j separating v and w. This theorem is dual to a theorem of Okamura on plane multicommodity flows, in the same way as a theorem of Karzanov is dual to one of Lomonosov.

OS-R8611. C.A.J. Hurkens, A. Schrijver & E. Tardos. *On fractional multicommodity flows and distance functions.*
AMS 05C70, 05CXX, 90C35; 12 pp.; **key words:** multicommodity flows, distance function, planar graph.
Abstract: We give some results on the existence of fractional and integral solutions to multicommodity flow problems, and on the related problem of decomposing distance functions into cuts.

OS-R8612. J.K. Lenstra. *Interfaces between operations research and computer science.*
AMS 90BXX, 68-01; 16 pp.; **key words:** computation, efficiency, complexity, effectivity, analysis of algorithms, randomization, parallelism, interaction, computer aided routing, decision support system, expert system.
Abstract: This is the text of a plenary address at the Biennial Meeting of the Hellenic Operations Research Society, delivered in Athens on November 23, 1985.

OS-R8613. P.J.C. Spreij. *Selfexciting counting process systems with finite state space.*
AMS 93E03, 60G55, 60J75; 18 pp.; **key words:** counting process, Markov process, stochastic realization theory.
Abstract: Stochastic systems with counting process output and a finite state space are considered. This leads to studying processes with finite state space that are Markovian with respect to the flow of σ-algebras that is generated by the counting process. It appears that there is a close relationship between the transition intensities of the Markov process and the intensity of the counting process. Some consequences for stochastic realization problems are then studied.

NM-R8619. J.M. Sanz-Serna & J.G. Verwer. *Stability and convergence in the PDE/stiff ODE interphase.*
AMS 65X02, 65M10, 65M20; CR 5.17; 14 pp.; **key words:** numerical analysis, evolutionary partial differential equations, stiff ordinary differential equations, method of lines, stability, convergence.
Abstract: This is an expository paper showing the interplay between the analysis of numerical methods for evolutionary partial differential equations and some developments in the stiff ordinary differential equation literature. The notions of contractivity, one-sided Lipschitz conditions, logarithmic norms, B-convergence and order reduction are of particular importance.

NM-R8620. J.G. Blom, J.M. Sanz-Serna & J.G. Verwer. *On simple moving grid methods for one-dimensional evolutionary partial differential equations.*
AMS 65M50, 65M99; CR 5.17; 19 pp.; **key words:** numerical analysis, partial differential equations, time-dependent problems, moving grid methods, space-time finite elements.
Abstract: Two moving grid algorithms for the numerical integration of evolutionary one-dimensional partial differential equations are considered. Both algorithms discretize the equation on trapezoidal space-time elements and combine a prediction step with a remeshing technique in order to determine the orientation of the sides of the trapezoids joining nodes at consecutive time levels. The behaviour of the discretizations employed is forecast by means of the modified equation technique and then studied in a series of numerical experiments.

NM-R8621. P.W. Hemker & B. Koren. *A non-linear multigrid method for the steady Euler equations.*
AMS 65N30, 76G15, 76H05; 22 pp.; **key words:** Euler equations, second-order schemes, defect correction, multigrid methods.
Abstract: Higher-order accurate Euler-flow solutions are presented for some airfoil test cases. Second-order accurate solutions are computed by an iterative Defect Correction process. For two test cases even higher accuracy is obtained by the additional use of a τ-extrapolation technique. Finite volume Osher-type discretizations are applied throughout. Two interpolation schemes (one

with and one without a flux limiter) are used for the computation of the second-order defect. In each Defect Correction cycle, the solution is computed by non-linear multigrid iteration, in which Collective Symmetric Gauss-Seidel relaxation is used as the smoothing procedure. The computational method does not require tuning of parameters. The solutions show a good resolution of discontinuities, and they are obtained at low computational costs. The rate of convergence seems to be grid-independent.

NM-R8622. B. Koren & S.P. Spekreijse. *Multigrid and defect correction for the efficient solution of the steady Euler equations.*
AMS 35L65, 35L67, 65N30, 76G15, 76H05; 14 pp.; **key words:** steady Euler equations, multigrid methods, defect correction.

Abstract: An efficient iterative solution method for second-order accurate discretizations of the 2D Steady Euler equations is described and results are shown. The method is based on a nonlinear multigrid method and on the defect correction principle. Both first- and second-order accurate finite-volume upwind discretizations are considered. In the second-order discretization a limiter is used. An iterative Defect Correction process is used to approximately solve the system of second-order discretized equations. In each iteration of this process, a solution is computed of the first-order system with an appropriate right-hand side. This solution is computed by a nonlinear multigrid method, where Symmetric Gauss-Seidel relaxation is used as the smoothing procedure. The computational method does not require any tuning of parameters. Flow solutions are presented for an airfoil and a bi-airfoil with propeller disk. The solutions show good resolution of all flow phenomena and are obtained at low computational cost. Particularly with respect to efficiency, the method contributes to the state of the art in computing steady Euler flows with discontinuities.

NM-R8623. J.J. Rusch. *The use of defect-correction for the solution of the 2D compressible Navier-Stokes equations with large Reynolds number.*
AMS 65N30, 76G15, 76H05; 27 pp.; **key words:** steady Navier-Stokes equations, second-order schemes, defect correction, multigrid methods.

Abstract: We solve the 2-dimensional compressible Navier-Stokes equations by means of a defect-correction process. The approximate problems solved in the iteration are described by Euler equations with a source term. The discretization method applied is a finite volume method. The auxiliary problems are solved by a Multi-Grid technique. First, as a test problem, Poiseuille-flow is simulated at a low Mach number. The numerical solutions are compared with the solutions of the incompressible Navier-Stokes equations. Next, a channel model is used with a Mach number of approximately 0.85 at the inlet. Finally, the same model is considered, but now with a circular bump on its lower wall. No special grid refinements are made to resolve the boundary layers. Had more detailed solutions been desired, computer simulations should have been made with higher resolutions of the grid, especially near the wall. Nevertheless, the numerical solutions give a good qualitative impression of the global behaviour of the solution of the Navier-Stokes equations.

NM-N8603. H.J.J. te Riele, W. Borho, S. Battiato, H. Hoffmann & E.J. Lee. *Table of amicable pairs between* 10^{10} *and* 10^{52}.
AMS 10A20, 10A99; 187 pp.; **key words:** amicable number pairs.

Abstract: This report presents a (nonexhaustive) table of 10,445 amicable number pairs which have a smaller member between 10^{10} and 10^{52}. The majority of the pairs were found in the past five years by at least one of the five authors of this report, and are published here for the first time. Some tables of statistical information from the large table are also included.

NM-N8604. T.P. de Vries. *Space discretization of hyperbolic differential equations of the second order with periodic solutions.* (In Dutch)

AMS 65M20, 76B15; 26 pp.; **key words:** hyperbolic equations, periodic solution.

Abstract: We investigate the Cauchy-problem for hyperbolic differential equations of the second order, in which the frequencies of the Fourier components in the solution lie in a known interval.

MS-R8605. J.A. Smith. *Statistical modeling of daily rainfall occurrences.*
AMS 60G55, 62M99, 62F12; 16 pp.; **key words:** discrete point processes, Markov Bernoulli models, maximum likelihood estimators, likelihood ratio tests, daily rainfall data.

Abstract: In this paper likelihood-based inference procedures for discrete point process models are developed and a new family of discrete point process models for daily rainfall occurences is proposed. The model, which is termed a Markov Bernoulli process, can be viewed as a sequence of Bernoulli trials with randomized success probabilities. Contained within the family of Markov Bernoulli models are Markov chain and Bernoulli trial models. Asymptotic properties of maximum likelihood estimators of Markov Bernoulli model parameters are derived. These results provide the basis for assessing standard errors and correlation of parameter estimators and for developing likelihood ratio tests to choose among Markov Bernoulli, Markov chain, and Bernoulli trial models. Inference procedures are applied to a data set from Washinghton D.C.

MS-R8606. K. Dzhaparidze. *On LAN for counting processes.*
AMS 62M99; 18 pp.; **key words:** local asymptotic normality, binary experiments, likelihood ratio process, the Hellinger process, marked point process, counting process.

Abstract: Shiryayev's result concerning LAN binary experiments is discussed, with applications to counting processes. It is shown, in particular, that the convergence in probability of the Hellinger process to a continuous nondecreasing function ensures the LAN property of the likelihood ratio process, provided that a related Lindeberg-type condition also holds.

MS-R8607. J.A. Smith. *Estimating the upper tail of flood frequency distributions.*
AMS 62F10, 62F12, 62F25; 21 pp.; **key words:** quantile estimation, generalized Pareto distribution, extreme value distributions, flood frequency distribution, marked point processes.

Abstract: Procedures for estimating recurrence intervals of extreme floods are developed. Estimation procedures proposed in this paper differ from standard procedures in that only the largest 10-20% of flood peaks are explicitly used to estimate flood quantiles. Quantile estimation procedures are developed for both annual peak and seasonal flood frequency distributions. The underlying model of flood peaks is a marked point process $\{T_j^i, Z_j^i\}$ where T_j^i represents time of occurrence of the j^{th} flood during year i and the mark Z_j^i represents magnitude of the flood peak. Results from extreme value theory are used to parameterize the upper tail of flood peak distributions. Quantile estimation procedures are applied to the 95 year record of flood peaks from the Potomac River. Results suggest that Potomac flood peaks are bounded above. The estimated upper bound is only 20% larger than the flood of record.

MS-R8608. V.V. Slavova. *Berry-Esseen bound for Student's statistic.*
AMS 60F05, 62E20; 15 pp.; **key words:** Berry-Esseen bound, Student t-statistic.

Abstract: The Berry-Esseen bound for the distribution of Student's t-statistic is obtained under the sole condition that the underlying distribution has a finite 3rd moment.

MS-R8609. S.A. van de Geer. *A note on rates of convergence in least squares estimation.*

AMS 60B12, 60F99, 62F12, 62J02; 11 pp.; **key words:** rates of convergence, metric entropy.

Abstract: In the regression model $y_k = g(x_k) + \epsilon_k$, the regression function g is regarded as the unknown parameter. It is shown that entropy conditions on the class $\mathcal{G}$ of possible regression functions imply rates of convergence - in L^2-sense - of the least squares estimator. For infinite-dimensional models, this yields the well-known $\mathcal{O}_p(n^{-\frac{1}{2}})$-rate of convergence; for other models, a slower rate is obtained. In general, the rates cannot be improved. Some examples illustrate this.

MS-R8610. H.C.P. Berbee. *Uniqueness of Gibbs measures and absorption probabilities.*

AMS 82A25, 47D45, 60K35; 12 pp.; **key words:** uniqueness of Gibbs measures, positive operator, Markov operator, duality, absorbing state, inequality, Perron Frobenius theorem.

Abstract: Gibbs measures are studied using a Markov chain on the non-negative integers. Uniqueness of Gibbs measures follows from absorption of the chain at $\{0\}$. To this end we derive a certain inequality. For one-dimensional systems this improves a well-known uniqueness result of Ruelle and moreover for models near the $1/r^2$-interaction Ising model it is a natural improvement of some other results.

MS-R8611. R. Helmers & M. Hušková. *Berry-Esseen bounds for L-statistics with unbounded weight functions.*

AMS 60F05, 62E20; 18 pp.; **key words:** Berry-Esseen bounds, L-statistics, linear combinations of order statistics, unbounded discontinuous weight functions.

Abstract: Berry-Esseen bounds of order $n^{-\frac{1}{2}}$ are established for linear combinations of order statistics with unbounded weight functions. The weight functions are allowed to tend to infinity in neighbourhoods of zero and one at a logarithmic rate. A finite number of discontinuity points in the weight function is also permitted, provided a local smoothness condition is imposed on the inverse of the underlying distribution of the observations. The present report supplements Helmers and Hušková (1984), where (part of) Theorem 1, which deals with the case of a continuous unbounded weight function, was presented, together with an outline of its proof; Theorem 2 is a new result covering the case of a discontinuous unbounded weight function. The relation with recent work of Van Zwet (1984) and Friedrich (1985) is briefly pointed out.

MS-R8612. A. Sieders & K. Dzhaparidze. *A large deviation result for parameter estimators and its application to nonlinear regression analysis.*

AMS 60F10, 62F12, 62J02; 31 pp.; **key words:** M-estimators, large deviations, rate of convergence, least-squares, nonlinear regression, Michaelis Menten model.

Abstract: Elaborating on the work of Ibragimov and Has'minskii (1981) we prove a Law of Large Deviations (LLD) for M-estimators, i.e. those estimators which maximize a functional, continuous in the parameter, of the observations. This LLD is applied, using results of Petrov (1975), to the problem of parametric nonlinear regression in the situation of discrete time, independent errors and regression functions which are continuous in the parameter. This improves a result of Prakasa Rao (1984).

AM-R8605. Ph. Clément, O. Diekmann, M. Gyllenberg, H.J.A.M. Heijmans & H.R. Thieme. *Perturbation theory for dual semigroups. I. The sun-reflexive case.*

AMS 47D05; 15 pp.; **key words:** strongly continuous semigroup of bounded linear operators, weak * continuous semigroup, dual semigroup, bounded perturbation, variation-of-constants formula, age dependent population growth, dealy equations.

Abstract: We show how the theory of dual semigroups on non-reflexive Banach spaces can be used to obtain a natural generalization of the notion of a bounded perturbation of the generator and a new version of the variation-of-constants formula. As an application we discuss age-dependent population growth.

AM-R8606. S.M. Verduyn Lunel. *Complex analytical methods in RFDE theory.*
AMS 47D05, 34K05; 9 pp.; **key words:** C_0-semigroups, completeness, exponential series, generalized eigenfunctions, retarded functional differential equations, small solutions, Volterra convolution equation.

Abstract: Laplace transformation of a linear autonomous retarded functional differential equation (RFDE) with finite delay yields an analytic equation that has to be solved over a ring of meromorphic functions. This paper presents the techniques to study the equation and gives an overview of the recently obtained results related to completeness and convergence of the generalized eigenfunctions of the infinitesimal generator associated with a linear autonomous RFDE.

AM-R8607. H.J.A.M. Heijmans. *Semigroup theory for control on sun-reflexive Banach spaces.*
AMS 93B28, 47D05, 34K35; 12 pp.; **key words:** dual C_0-semigroup, sun-reflexive Banach space, weak * Riemann integral, variation-of-constants formula, abstract control system, (approximate) controllability, observability, delay equation, boundary control, stabilizability.

Abstract: We use the theory of dual C_0-semigroups, as developed by Phillips, to define and study a new class of control systems on nonreflexive Banach spaces. Our main results concern the (approximate) controllability and observability of such systems. We illustrate our abstract results with an application to a delay system.

AM-R8608. Jiang Furu. *Necessary conditions for resonance in turning point problems for ordinary differential equations.*
AMS 34E20; 9 pp.; **key words:** turning point problem, singular perturbation problem, Ackerberg-O'Malley resonance.

Abstract: In the theory of turning point problems for ordinary linear differential equations of second order necessary conditions for Ackerberg-O'Malley resonance have been studied by earlier writers. The present paper gives a sequence of necessary conditions for resonance, which is derived in an iterative way. Special cases are considered as illustrative examples.

AM-R8609. H.R. Thieme. *Well-posedness of physiologically structured population models for Daphnia magna.*
AMS 92A15; 35 pp.; **key words:** Daphnia magna, physiologically structured populations, energy allocation, age-dependent mortality, size-dependent reproduction, feed-back with variable time-delay, well-posedness, existence, uniqueness and continuous dependence on data of solutions, nonlinear partial functional-differential equations, measure-valued solutions.

Abstract: In this paper we heuristically discuss the well-posedness of three variants of the Kooijman/Metz model. Shortcomings concerning the uniqueness and continuous dependence of data of the solutions to one of the variants are traced back to an inconsistency in the biological concept of energy allocation in this model version. The conceptional consequences are discussed and an open question concerning energy allocation is pin-pointed. A rigorous mathematical treatment will be presented in a forthcoming publication.

AM-R8610. N.M. Temme. *Laguerre polynomials: asymptotics for large degree.*
AMS 41A60, 33A65, 34E05, 33A30; 13 pp.; **key words:** Laguerre polynomials, uniform asymptotic expansion, Whittaker functions, Bessel functions, saddle point method.

Abstract: The Laguerre polynomials $L_n^{(\alpha)}(x)$ are considered for large values of the degree n. The paper surveys a result of Erdélyi (1960), that gives for fixed $\alpha \geqslant 0$ and for $\alpha \in \mathbb{R}$ as uniformity parameter two asymptotic forms: the Bessel function case and the Airy function case. Next, more recent results of Baumgartner (1980) and Olver (1980) for Whittaker functions are interpreted for Laguerre polynomials; the parameter α can then be considered as a second uniformity parameter. A new method is given for obtaining similar asymptotic forms by using integral representations of $L_n^{(\alpha)}(x)$.

AM-N8602. H. Roozen. *Numerical construction of rays and confidence contours in stochastic population dynamics.*
AMS 60J70, 60J60; 16 pp.; **key words:** ray method, confidence contours.

Abstract: Consider a stochastic system with small stochastic perturbations in which the associated deterministic system has a stable equilibrium. The (quasi-)stationary forward Fokker-Planck equation is solved by the WKB-method, leading to a system of ray equations. This technical note deals with the numerical solution of the ray equations. The methods which are described here have been applied to stochastic birth-death models.

PM-R8604. M. Hušek & J. de Vries. *Preservation of products by functors close to reflectors.*
AMS 54B10, 18A40, 20M99; 16 pp.; **key words:** reflection, product, preservation of products, semigroup compactifications.

Abstract: It is shown that reflectors and similar functors in algebraic and topological-algebraic structures in many cases commute with products. In particular, reflectors of the category of (semi)topological semigroups into the subcategory of compact topological semigroups or groups have this property. The proofs are straightforward and avoid the use of almost periodic functions.

PM-R8605. J.C. van der Meer & R. Cushman. *Orbiting dust under radiation pressure.*
AMS 58F, 70F; 12 pp.; **key words:** constrained normal form, integrable system, reduction.

Abstract: In this paper we consider a perturbed Keplerian system describing orbiting dust under radiation pressure. We derive an integrable second order normal form for this Hamiltonian system. Finally, we analyze this integrable system by successive reduction to a one degree of freedom system.

CWI Activities

Autumn 1986

With each activity we mention its frequency and (between parentheses) a contact person at CWI. Sometimes some additional information is supplied, such as the location if the activity will not take place at CWI.

Study group on Analysis on Lie groups. Jointly with University of Leiden. Biweekly. (T.H. Koornwinder)

International mini-conference on Lie groups. Jointly with University of Leiden, Utrecht and Groningen. (T.H. Koornwinder)
During two days in the week April 21-15, 1987.

Seminar on Integrable Systems Theory. Once a month. (M. Hazewinkel)
A central object of study will be the work of Belavin and Drinfeld, especially the relation between simple Lie algebras and solutions of the so-called classical Yang-Baxter equation. Also, linearization aspects of nonlinear represenations and lattice KP, KdV will be discussed.

Seminar on Algebra and Geometry. Once a month. (A.M. Cohen)
The $SL_2(\mathbb{C})$ covariant algebra of the quintic and the sextic. (J. Brinkhuis, EUR) *Extremal Design Theory.* (A.E. Brouwer) *The fundamental theorem in invariant theory* (A.M. Cohen) *Groups with the property that each irreducible representation is primitive.* (N.S. Hekster) *A generalisation of Fischer spaces.* (F.G.M.T. Cuypers)

Cryptography working group. Monthly. (J.H. Evertse)

Colloquium 'STZ' on System Theory, Applied and Pure Mathematics. Twice a month. (J. de Vries)

Study group 'Biomathematics'. Lectures by visitors or members of the group. Jointly with University of Leiden. Bimonthly (O. Diekmann)
Topics for the next meetings are: *stochastic population dynamics, dynamics of structured populations.*

Study group on Nonlinear Analysis. Lectures by visitors or members of the group. Jointly with University of Leiden. Bimonthly (O. Diekmann)

The purpose is to follow and investigate recent developments on qualitative analysis of nonlinear equations.

Progress meetings of the Applied Mathematics Department. Weekly (N.M. Temme)

New results and open problems on the research topics of the department: biomathematics, mathematical physics, asymptotic and applied analysis, image analysis.

Study group on Statistical and Mathematical Image Analysis. Every three weeks. (R.D. Gill)

The group is presently studying J. Serra's approach to image analysis, 'mathematical morphology', and recent statistical contributions using Markov field modelling due to S. and G. Geman, J. Besag and B. Ripley.

Progress meetings of the Mathematical Statistics Department. Biweekly (R. Helmers)

Talks by members of the department on recent developments in research and consultation. Also talks by E. Valkeila (Helsinki) on counting processes.

Study group on Combinatorial Optimization. Biweekly. (B.J. Lageweg)

Twelfth Conference on the Mathematics of Operations Research and System Theory. 14,15,16 January at Lunteren. (B.J. Lageweg) Invited speakers: A. Bendell (Nothingham), W. Pulleyblank (Waterloo/Bonn), M. Desrochers (Montreal, Amsterdam), K. Melhorn (Saarbrücken), P. Toint (Namur).

System Theory Days. Irregular. (J.H. van Schuppen, J.M. Schumacher)

Study group on System Theory. Biweekly. (J.M. Schumacher)

Current topic: Discrete event dynamical systems.

Colloquium on Queueing Theory and Performance Evaluation. Irregular. (O.J. Boxma)

Progress meetings on Numerical Mathematics. Weekly. (H.J.J. te Riele)

Study group on Numerical Software for Vector Computers. Monthly. (H.J.J. te Riele)

Study group on Differential and Integral Equations. Lectures by visitors or group members. Irregular. (H.J.J. te Riele)

Study group on Graphics Standards. Monthly. (M. Bakker)

Study group on Dialogue Programming. (P.J.W. ten Hagen)

Study group on Logical Aspects of Artificial Intelligence. Biweekly. (M.L. Kersten & P.J.F. Lucas)

In this study group recent developments in formal theories in artificial intelligence are discussed. The main topics are: knowledge representation, inference methods, non-standard logics and plausible reasoning.

Process Algebra Meeting. Weekly. (J.W. Klop)

Visitors to CWI from Abroad

M.S. Berger (University of Massachusetts, Amherst, USA) 17-30 April. R. Bordewisch (Nixdorf Computers) 15 September. J.C. Butcher (University of Auckland, New Zealand) 1-13 September. C.I. Byrnes (Tempe, Arizona, USA) 15-17 September. D. Clément (SEMA, Sophia-Antipolis, France) 20, 21 October. T. Despeyroux (INRIA, Sophia-Antipolis, France) 20, 21 October. H. Dym (Weizman Institute, Rehovot, Israel) 4 September. P.P.B. Eggermont (University of Delaware, Newark, USA) 8 August. B.L. Fox (University of Montreal, Canada) 15 August. M. Gyllenberg (Helsinki, Finland) 6-10 September. B. Harsoyo (University of Jakarta, Indonesia) 23 June - 23 August. J. Hur (Korea Institute of Science and Technology, Seoul, South Korea) September. G. Kahn (INRIA, Sophia-Antipolis, France) 20, 21 October. B. Lang (INRIA, Rocquencourt, France) 20 October. R. Leadbetter (North Carolina University, USA) 22 July. R. Marino (Second University of Rome, Italy) 27 August. G. Meurant (Centre d'Etudes de Limeil-Valtenton, Villeneuve, France) 22-26 September. A.S. Morse (Yale University, New Haven, USA) 17 July. A.M. Odlyzko (AT&T Bell Laboratories, Murray Hill, USA) 19 September. C.M. Orange (Reed College, Portland, USA) May, June & July. V. Pascual (INRIA, Sophia-Antipolis, France) 20, 21 October. R. Platek (ORA, Ithaca, USA) 9 October. N. Sabadini (University of Milano, Italy) 6-31 October. L. Shrira (MIT/Technion, Haifa, Israel) 28 July. J. Sidi (SEMA, Rocquencourt, France) 20 October. E. Slud (University of Maryland) 20-21 November. J. Steinberg (Israel Institute of Technology, Israel) 1-15 August. E. Valkeila (University of Helsinki, Finland) October - December.

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