CWI NEWSLETTER

Centrum voor Wiskunde en Informatica
Centre for Mathematics and Computer Science

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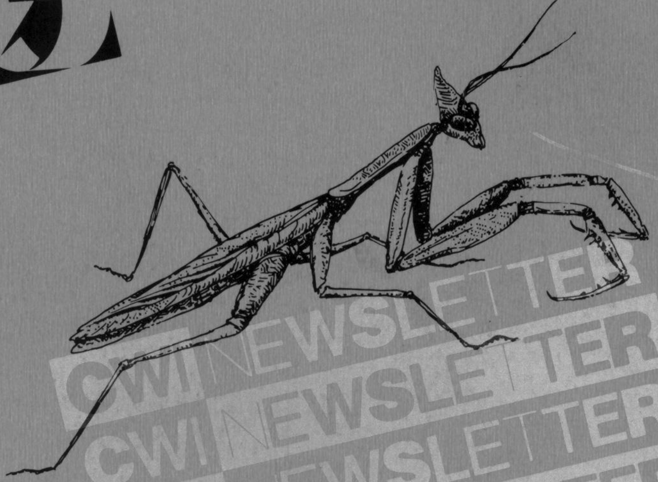
November 1983



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Number 1, November 1983

Editors

Arjeh M. Cohen Richard D. Gill Jan Heering

The CWI Newsletter is published quarterly by the Centre for Mathematics and Computer Science (Centrum voor Wiskunde en Informatica), Kruislaan 413, 1098 SJ Amsterdam, The Netherlands. The Newsletter will report on activities being conducted at the Centre and will also contain articles of general interest in the fields of Mathematics and Computer Science, including book reviews and mathematical entertainment. The editors encourage persons outside and in the Centre to contribute to the Newsletter. Normal referee procedures will apply to all articles submitted.

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Centrum voor Wiskunde en Informatica

Foreword

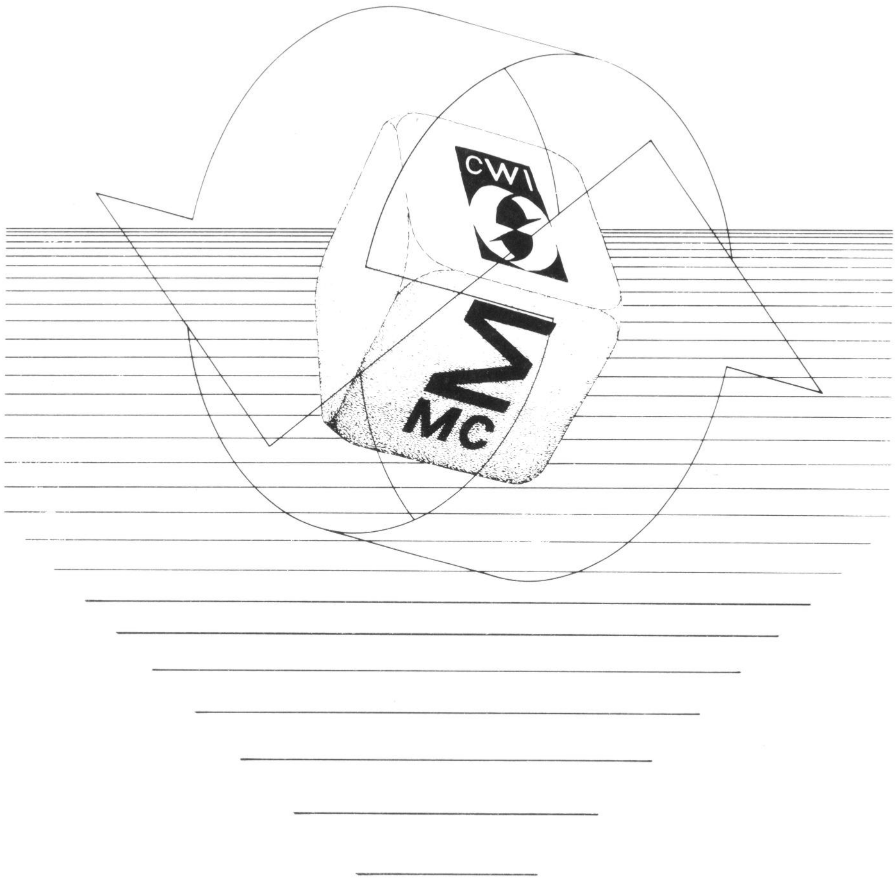
by P.C. Baayen (Scientific Director of CWI)

With pleasure and satisfaction we present the first issue of our new quarterly 'CWI Newsletter'. The CWI - Centrum voor Wiskunde en Informatica, that is, Centre for Mathematics and Computer Science - has been known under this name since September 1, 1983. However, as the Mathematical Centre it has been around since 1946, and has contributed to both mathematics and computer science since its inception. Its old name was eminently suitable in a period of history in which mathematics and mathematicians contributed significantly to the burgeoning new field of computers and computing. Now that Computer Science has come of age as an important and independent discipline, the Board of Trustees of the Centre have recognized this by changing the name of our institute.

The designation has changed - not so the designatum. Both Mathematics and Computer Science were taken to heart in the 'Mathematical Centre', and both will be given due attention under the banner of CWI, Centre for Mathematics and Computer Science. We are fully convinced that the practice of both computer science and mathematics in one and the same scientific institute can be benificial and stimulating for both sciences. In a sense, the new name of our institute is both a programme and a challenge. We want to do mathematical research in an environment where the exciting and powerful methods and tools developed by computer science are readily available. At the same time, we prefer to conduct research in computer science in a mathematical milieu, where mathematicians can contribute not only a point of view and a mental discipline, but also actual methods for developing and streamlining algorithms or for studying complexity, or logical foundations.

The old 'Mathematical Centre' maintained strong ties with computer scientists and mathematicians all over the world. Many hundreds visited the institute for a longer or shorter period. Many scientific workers of the Centre, in their turn, visited research institutes abroad. Sometimes these contacts led to cooperation in research, resulting in common papers; on other occasions the very real benefits of these scientific contacts were less directly traceable in print. Of course, the Centre publishes an 'Annual Report' in which everything of importance is duly reported; but this Annual Report is in Dutch and therefore enjoys only a limited circulation. For a long time we have felt the need of some form of regular communication with our many friends and colleagues, not only within but also outside the domain of the Dutch language.

So, it is indeed with pleasure and satisfaction that we present this first issue of the 'CWI Newsletter'. Using the recent name-change of our institute as a catalyst we now create a medium which, we hope. will reach all our many friends and will keep them informed about what is going on in and around our institute. We were lucky in having at hand dr. Arjeh M. Cohen and in being able to convince him of the need to act as editor of this Newsletter. We hope that this CWI Newsletter will contribute to international cooperation in Computer Science and in Mathematics, and also, equally important, to maintaining and cultivating friendship and appreciation between mathematicians and computer scientists.



Geometry of the Energy Momentum Mapping of the Spherical Pendulum

by R. Cushman

0. Introduction

Reconsideration of the spherical pendulum of classical mechanics with modern mathematical techniques has revealed the occurrence of monodromy in the way level sets of the energy momentum mapping fit together.

This phenomenon has been observed in Duistermaat [1], where the proof of its existence uses rather heavy results. Monodromy is a phenomenon which was not classically investigated because it deals with how certain families of energy momentum level sets fit together and hence can not be observed by looking at motions of a fixed energy and angular momentum. The main point of this article is to give a visual geometric argument which computes the monodromy in the spherical pendulum.

The first part of this article treats the spherical pendulum of classical mechanics from the point of view of Smale's program [4]. Specifically, we study the energy momentum mapping of the spherical pendulum which assigns to every point in phase space (the tangent bundle of the two sphere) the pair of real numbers given by the values of the total energy and angular momentum at that point. Because energy and angular momentum are conserved quantities [2,5], the spherical pendulum is completely integrable. Moreover the fibers of the energy momentum mapping are the sets of all positions and velocities with a given energy and angular momentum.

A well-known theorem of Arnold [1] seems to say that all the fibers, if connected, compact and smooth, are two dimensional tori. But this is only true if the derivative of the energy momentum mapping is surjective. Arnold's theorem does not apply to those motions of the spherical pendulum which are circles parallel to the equator of the two sphere. Thus we give here a careful geometric treatment of the topology of the fibers of the energy momentum mapping of the spherical pendulum. But this is not a complete qualitative description of the spherical pendulum, because the energy momentum mapping has monodromy. The monodromy will be treated in the second part of this article. Proofs are by pictures!

1. Smale's program

a. The energy momentum mapping.

We start with the construction of the energy momentum mapping of the spherical pendulum. For the basic physics of the spherical pendulum, see [5] p. 334 or [6]. Recall that the spherical pendulum is a particle of unit mass moving on a two sphere S^2 of unit radius under a constant vertical gravitational force of unit strength. Therefore phase space is the tangent bundle TS^2 of S^2, that is,

$$TS^2 = \left\{ (x,v)=(x_1,x_2,x_3,v_1,v_2,v_3)\in \mathbb{R}^3\times\mathbb{R}^3 \;\middle|\; \begin{array}{c} x_1^2+x_2^2+x_3^2=1 \\ x_1v_1+x_2v_2+x_3v_3=0 \end{array} \right\}$$

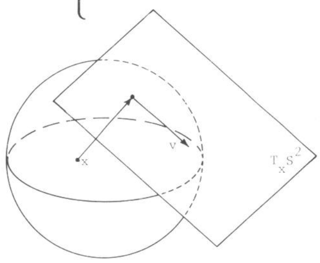


Figure 1.
A tangent plane $T_x S^2$ to the 2-sphere S^2 at the point x.

with projection $\pi : TS^2 \to S^2 : (x,v) \mapsto x$. Pictorially elements of the tangent space $T_x S^2$ to S^2 at x are represented by arrows with tail at x and perpendicular to x (Figure 1). TS^2 is the disjoint union of all tangent spaces. The Hamiltonian function of the spherical pendulum is the sum of the kinetic and potential energy of the particle and is the function

$$H:\ TS^2 \to \mathbb{R} : (x,v) \mapsto \tfrac{1}{2}\|v\|^2 + x_3 = \tfrac{1}{2}(v_1^2+v_2^2+v_3^2)+x_3.$$

Since a rotation of S^2 about the x_3-axis is a symmetry of the spherical pendulum, there is a corresponding conserved quantity (=integral) called the angular momentum, which is the function

$$L:\ TS^2 \to \mathbb{R} : (x,v) \mapsto x_2v_1 - x_1v_2.$$

Combining the energy and the angular momentum gives the energy momentum mapping

$$\mathcal{EM}:\ TS^2 \to \mathbb{R}^2 : (x,v) \mapsto (H(x,v), L(x,v)).$$

b. Critical points and critical values

To analyze the energy momentum mapping, the first order of business is to find its critical points, that is, points where its derivative $D\mathcal{EM}$ is not surjective. The rank of $D\mathcal{EM}$ is zero at the critical points of H (= critical points of L). These are precisely the equilibrium points of the spherical pendulum where the particle does not move, that is $(n,0)$, $(s,0)\in TS^2$ where n (respectively s) are the north (respectively south) pole of S^2. The corresponding critical values of $\mathcal{EM}$ are $(1,0)$ and $(-1,0)$.

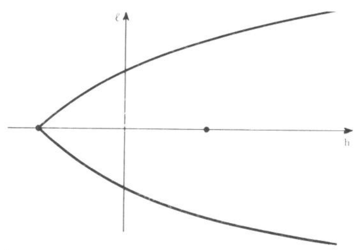


Figure 2.
Image of energy momentum mapping of spherical pendulum. Darkened curve is energy momentum values of relative equilibria. Large dots are energy momentum values of equilibria.

The rank of $D\mathcal{EM}$ is one when the derivatives of H and L are linearly dependent. This dependency occurs only on those orbits of the Hamiltonian system which are also orbits of the axial symmetry. Such orbits are called relative equilibria. In the spherical pendulum the relative equilibria are circles on S^2 which lie in a horizontal plane cutting the southern hemisphere. Calculations show that for a given noncritical value h of H, the angular momentum L attains its maximum and minimum values on the relative equilibria. Furthermore, calculations show that the darkened curves in Figure 2 are the critical values of $\mathcal{EM}$ corresponding to the relative equilibria. The image of $\mathcal{EM}$ is the region in Figure 2 bounded by the darkened curves.

c. *Regular fibers*

The set of regular values $\mathcal{R}$ of the energy momentum mapping consists of those points in the image which are not critical values. If $r=(h,l)\in\mathcal{R}$, then $\mathcal{T}_r=\mathcal{EM}^{-1}(r)$ is a regular fiber. Our next task is to determine the topological type of the regular fibers. Because r is a regular value of $\mathcal{EM}$, Arnold's theorem [1] implies that each connected component of $\mathcal{T}_r$ is a smooth two dimensional torus. The following discussion not only shows that $\mathcal{T}_r$ is connected but also shows how to visualize the torus. The basic idea is to describe $\mathcal{T}_r$ as some sort of bundle lying over $U_r=\pi(\mathcal{T}_r)\subseteq S^2$. The next argument shows that U_r is the closed region of S^2 which is shaded in Figure 5. Suppose $(x,v)\in\mathcal{T}_r$ and $x\neq n, s$. Then on T_xS^2, the inverse image $L^{-1}(l)$ is the affine line

$$l = x_2v_1 - x_1v_2$$

with $x_1\neq 0$ or $x_2\neq 0$. The closest point of $L^{-1}(l)$ to the origin of T_xS^2 is the vector

$$v^0 = \frac{l}{x_1^2+x_2^2}(x_2,-x_1,0).$$

Since $\|v\|^2 \geq \|v^0\|^2$ for all $v\in L^{-1}(l)\cap T_xS^2$, using $1=x_1^2+x_2^2+x_3^2$, we obtain

$$2(h-x_3) = 2(H(x,v)-x_3) \geq 2(H(x,v^0)-x_3) = \frac{l^2}{1-x_3^2} \qquad (*)$$

(see Figure 3). Thus U_r is the set of all $x\in S^2$ satisfying (*). Consider the cubic polynomial

$$V(x_3) = (1-x_3^2)(h-x_3)$$

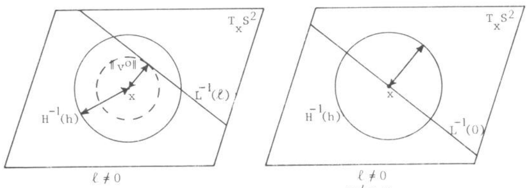


Figure 3.
Geometry of $H^{-1}(h)$ and $L^{-1}(l)$ in a fixed tangent space T_xS^2.

(see Figure 4). Then $V^{-1}([\frac{1}{2}l^2,\infty[)=[z_-,z_+]$ is the set of all $x_3 \in [-1,1]$ which

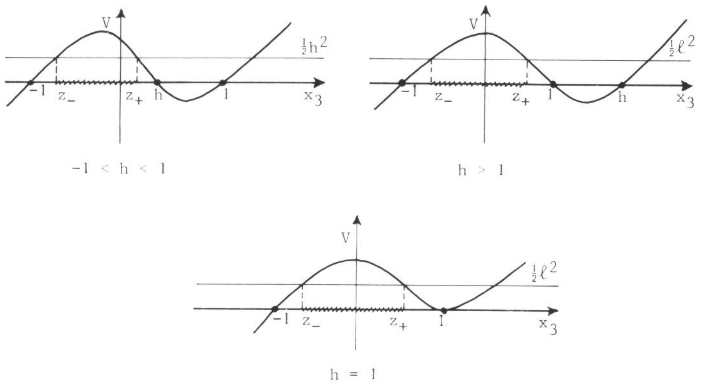


Figure 4.
Graph of V with $V^{-1}([\frac{1}{2}l^2,\infty[\,), \frac{1}{2}l^2 < \max_{x \in [-1,1]} V(x)$ indicated by hatched interval $[z_-,z_+]$.

satisfy (*). As long as

$$0 < \tfrac{1}{2}l^2 < V_m = \max_{x_3 \in [-1,1]} V(x_3)$$

the interval $[z_-,z_+]$ does not degenerate to a point and also is properly contained in $[-1,1]$. Thus U_r is the closed annular region of S^2 bounded by the two circles $C_n : x_3 = z_+$ and $C_s : x_3 = z_-$. When $l=0$ and $-1<h<1$, U_r is the closed region of S^2 containing the south pole s and bounded by $C_n : x_3 = h$.

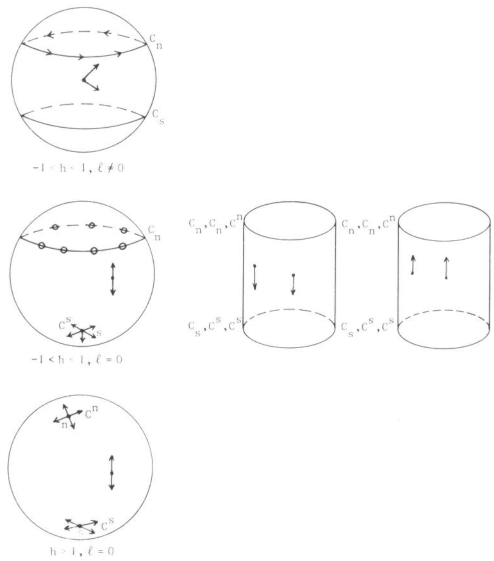


Figure 5.
Description of T_r as a bundle over U_r.

When $l=0$ and $h>1$, then $U_r = S^2$. To complete our picture of $\mathfrak{T}_r$ as a bundle over U_r we must determine the fiber $\pi^{-1}(x) \in \mathfrak{T}_r$ over $x \in U_r$. A close look at Figure 3 shows that if $l \neq 0$ and equality holds in (*), that is, x lies on the boundary ∂U_r of U_r, then v must be equal to v^0. Thus the fiber of $\mathfrak{T}_r$ over $x \in \partial U_r$ is a single vector v^0 with zero third component. If $l \neq 0$ and strict inequality holds in (*), that is, x lies in the interior of U_r, then the fiber $\pi^{-1}(x)$ of $\mathfrak{T}_r$ consist of two vectors with nonzero third component. Now suppose $l=0$ and $-1<h<1$. Then for $x \in \partial U_r$, that is, $x_3=h$, $\pi^{-1}(x)$ is the zero vector. If $x \in U_r - \{s\}$, $\pi^{-1}(x)$ consists of two nonzero vectors of opposite sign. If $x=s$, then $\pi^{-1}(x)$ is the circle $C^s: \|v\|^2=2(h+1)$ in $T_s S^2$, since $L^{-1}(0) \cap T_s S^2 = T_s S^2$. Finally suppose that $l=0$ and $h>1$. Then for $x \in U_r - \{s,n\}$, $\pi^{-1}(x)$ is two nonzero vectors of opposite sign; while $\pi^{-1}(s)$ is the circle $C^s: \|v\|^2=2(h+1)$ in $T_s S^2$ and $\pi^{-1}(n)$ is the circle $C^n: \|v\|^2=2(h-1)$ in $T_n S^s$ (see Figure 5). This completes the description of $\mathfrak{T}_r$ as a bundle over U_r. To see that $\mathfrak{T}_r$ is a two dimensional torus, we first split each of the circles C_n, C_s or C^s, C^n into two disjoint circles. Over the remaining points of U_r the fibers of $\mathfrak{T}_r$ consists of two vectors with nonzero third component. Taking those vectors with positive (negative) third component, we construct two cylinders, each with two labeled circles as boundary. Identifying the circles with the same label gives a two dimensional torus. (cf. Figure 6)

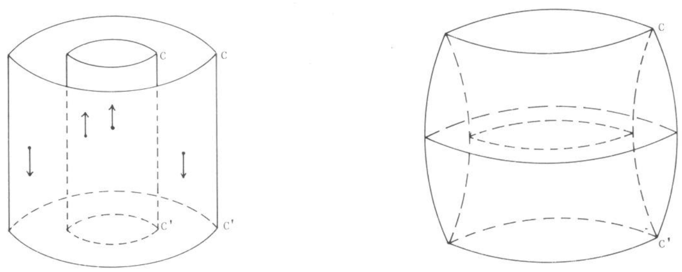


Figure 6.
Identification of regular fiber with a 2-torus

d. Critical fibers

Now we turn our attention to determining the topology of the fibers over the critical values. The fiber corresponding to the critical value $(-1,0)$ is the stable equilibrium $(s,0)$, that is, $\mathcal{E}\mathcal{M}^{-1}(-1,0)=(s,0)$. By construction the fiber corresponding to a relative equilibrium with critical value (l, h) is a circle in TS^2 corresponding to a positively (negatively) oriented horizontal circle in S^2 if $l>0$ $(l<0)$ and its oriented tangent vector of length h. Only the topology of

$\mathcal{T}=\mathcal{EM}^{-1}(1,0)$ corresponding to the critical value of the unstable equilibrium needs to be found. Clearly $\pi(\mathcal{T})=S^2$. The fiber of $\mathcal{T}$ over $x \in S^2-\{s, n\}$ consists of two vectors of opposite sign with nonzero third component; the fiber over s is a circle C^s; and the fiber over n is the zero vector (see Figure 7).

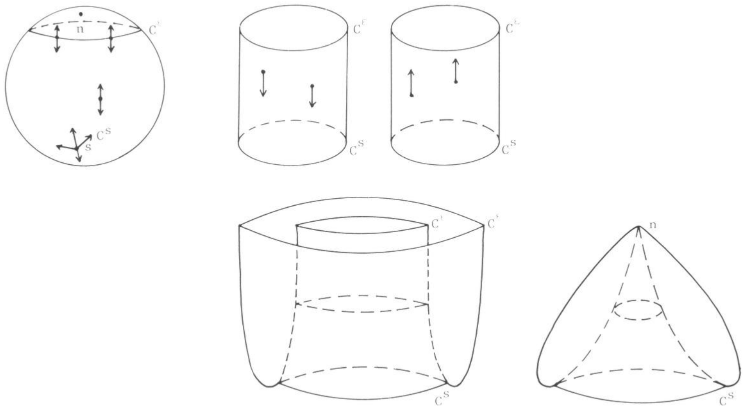


Figure 7.
Identification of singular fiber $EM^{-1}(1,0)$ as one point compactification of a cylinder.

Cut off a small circle C^ϵ near n. Split C^ϵ and C^s into two circles and form two cylinders from the vectors in the fiber of $\mathcal{T}$: one from the vectors with positive third component and the other from the vectors with negative third component. Join the circles labeled C^s together and collapse both C^ϵ circles to a single point. Thus $\mathcal{T}$ is a one point compactification of cylinder. In other words, $\mathcal{T}$ is a 2-sphere with its north and south pole pinched to a point.

e. Energy surface

Next we determine the topology of each energy surface $H^{-1}(h)$, $h>-1$. Let $\tilde{\pi}=\pi|H^{-1}(h)$. Suppose $-1<h<1$. Then $H^{-1}(h)$ is smooth. Since $2(h-x_3)=\|v\|^2\geqslant 0$, $\pi(H^{-1}(h))=\{x \in S^2 | x_3 \leqslant h\}$ which is topologically a closed 2-disc $\overline{D}^2$. Over $x \in \overline{D}^2$ the fiber $\tilde{\pi}^{-1}(x)$ of $H^{-1}(h)$ is the circle $\{v \in T_x S^2 | \|v\|^2=2(h-x_3)\}$; while over $x \in \partial \overline{D}^2$, $\tilde{\pi}^{-1}(x)$ is the zero vector. Split $\overline{D}^2$ along a diameter $\mathcal{L}$ into two disjoint half open discs $D_\pm$ and let $\mathcal{L}_u^\pm$, $u \in]-1,1[$, be a fibering of $D_\pm$ by parallel half open line segments perpendicular to $\mathcal{L}$ (see Figure 8). For each $u \in]-1,1[$, the inverse image $\tilde{\pi}^{-1}(\mathcal{L}_u^\pm)$ is topologically a 2-disc $\mathcal{D}_u^2$, being the union of circles which shrink to a point.

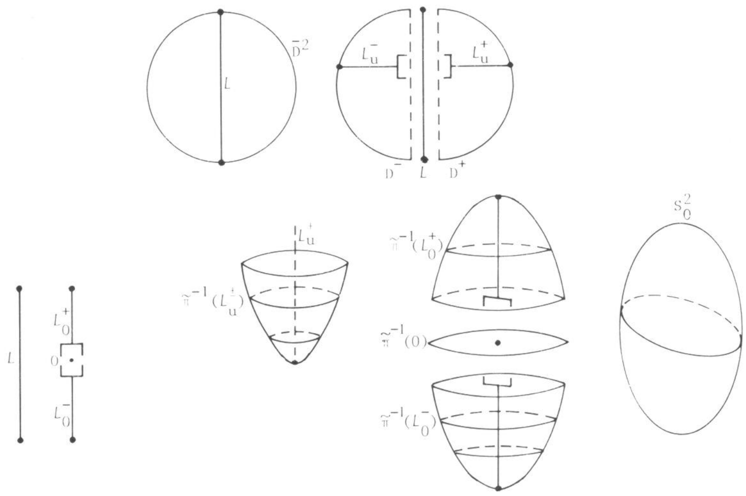


Figure 8.
Some of the building blocks of $H^{-1}(h)$, $-1<h<1$.

Therefore $\tilde{\pi}^{-1}(D_{\pm})$, being the union of 2-discs $\mathfrak{D}_u$ over $u \in]-1,1[$, topologically a 3-disc $D^3_{\pm}$. As shown in the second row of Figure 8, $\tilde{\pi}^{-1}(\mathfrak{L})$ is topologically a two sphere S^2_0. Therefore $H^{-1}(h)$ is the union of two 3-discs $D^3_{\pm}$ joined along a 2-sphere S^2_0. Hence, $H^{-1}(h)$ for $-1<h<1$ is topologically a three dimensional sphere S^3. Now suppose that $h>1$. Then $H^{-1}(h)$ is smooth and $\tilde{\pi}(H^{-1}(h))=S^2$. As before, over $x \in S^2$ the fiber $\tilde{\pi}^{-1}(x)$ is a circle in $T_x S^2$. Thus $H^{-1}(h)$ is topologically the tangent unit sphere bundle $T_1 S^2$ to S^2. But $T_1 S^2$ is the set of all pairs of orthonormal vectors in $\mathbb{R}^3$, which in turn is the set of all right handed orthonormal bases of $\mathbb{R}^3$. Therefore $H^{-1}(h)$ is the group $SO(3)$ of proper rotations of $\mathbb{R}^3$. Since $SO(3)$ is doubly covered by the special unitary group $SU(2)$, which is topologically a three dimensional sphere S^3, $SO(3)$ is topologically real projective three space $\mathbb{R}P^3$. When $h=1$, $H^{-1}(1)$ is not smooth, since it contains the critical point $(n,0)$. However $\tilde{\pi}(H^{-1}(1))=S^2$. Over $x \in S^2-\{n\}$, the fiber $\tilde{\pi}^{-1}(x)$ is a circle; while if $x=n$, $\tilde{\pi}^{-1}(n)=(n,0)$. Thus $H^{-1}(1)$ is $T_1 S^2$ with the fiber over n pinched to a point. All the information we have obtained about the topology of the fibers and energy level sets of the energy momentum map is summarized in Figure 9.

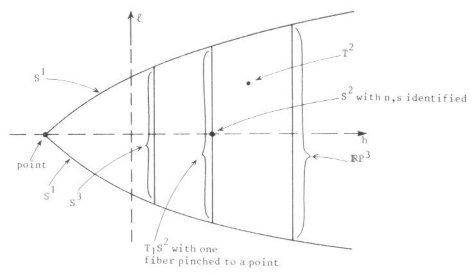


Figure 9.
Topology of fibers of energy momentum mapping.

f. Fitting together of fibers

We now show how the fibers of the energy momentum mapping fit together to form a fixed smooth energy surface. Fix $h \in]-1,1[\cup]1,\infty[$ and let $L_h^{\pm}$ be the intersection of the image of $\mathcal{EM}$ and a positive (negative) half line parallel with the l-axis. The fiber of $H^{-1}(h)$ over $L_h^{\pm}$ is a solid torus $ST^{\pm}$ which topologically is $S^1 \times D^2$. In view of the equation $H^{-1}(h) = \pi^{-1}(L_h^{+}) \cup \pi^{-1}(h,0) \cup \pi^{-1}(L_h^{-})$, each smooth energy surface is the union of two solid tori $ST^{\pm}$ joined together along a two dimensional torus $T^2 = \pi^{-1}(h,0)$. More precisely, ST^{+} and ST^{-} are glued together by an attaching mapping which is a diffeomorphism of T^2. The problem is: how to visualize this gluing mapping.

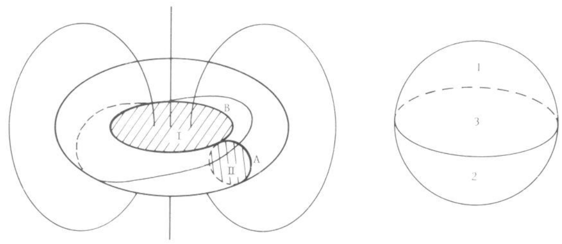
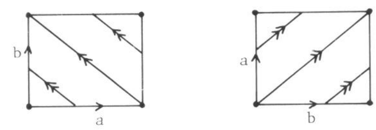


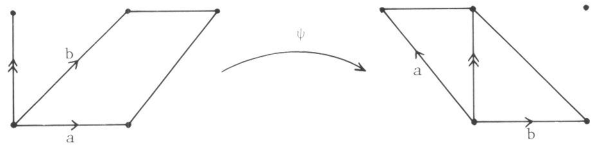
Figure 10.
S^3 as an S^1 bundle over S^2.

Suppose $-1<h<1$, then $H^{-1}(h)$ is S^3. The key observation is that S^3 is a fiber bundle over S^2 with fiber S^1 and group S^1 [7, p. 7]. Figure 10 gives a geometric realization of this bundle. Note that S^3 is thought of as the one point compactification of $\mathbb{R}^3$. Thus the vertical line in the figure is a circle. All the S^1 fibers pass transversally through either the 2-disc I or the 2-disc II, except the circles A and B which are the boundaries of I and II respectively. These 2-discs are identified with the hemispheres of S^2 with the same numbers; while the circles A and B are identified with the equator of S^2. The set of all fibers over each hemisphere of S^2 is a solid torus, being the union of all fibers which pass through either I or II; while the set of all fibers over the equator is T^2. Figure 11 shows the S^1 fibers on T^2 (oriented according to the action of the group S^1) from the point of view of the section "a" or "b" which corresponds to A or B respectively. All the S^1 fibers on T^2 can be throught of as the flow lines of a linear vectorfield on T^2. Taking the A circle as being horizontal, the flow lines are as depicted in the left hand side of Figure 11. The difference between the two left hand pictures is that in the first B is vertical while in the second the flow lines are vertical. This describes the "a" viewpoint. The "b" viewpoint is obtained similarly. The mapping ψ on T^2, given by $\begin{pmatrix} 1 & 0 \\ \pm 1 & 1 \end{pmatrix}$, is the change from the "a" viewpoint on T^2 to the "b"

viewpoint. Moreover, ψ shows how the solid torus ST^+ over



diagonal lines are S^1 fibers with arrows giving direction of S^1 action on fiber; dots joined by light lines give lattice defining T^2



S^1 fibers are vertical and viewpoint section horizontal; ψ maps a to a and b to b and is $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ with respect to standard basis.

Figure 11.
The gluing map for the solid torus decomposition of S^3.

the upper hemisphere is glued to the solid torus over the lower hemisphere. Given an orientation of T^2, the choice of sign in ψ is determined by the action of S^1 on the fibers [7, p. 135].

Suppose $h > 1$, then $H^{-1}(h)$ is $\mathbb{R}P^3$. Since every proper rotation of $\mathbb{R}^3$ is uniquely specified by giving an oriented axis of rotation and a right handed twist less than or equal to a half turn about this axis, $\mathbb{R}P^3 = SO(3)$ can be visualized as a closed 3-disc $\bar{D}^3$ in $\mathbb{R}^3$ of radius ½. (The length of the oriented axis, which is a vector in $\mathbb{R}^3$, gives the fraction of a turn about the axis). On the boundary $\partial\bar{D}^3$, which is a two dimensional sphere, diametrically opposite points are identified. Figure 12 gives the geometric realization of $\mathbb{R}P^3$ as an S^1 bundle over S^2. Again there are two 2-discs: I and II where II is the union of II' and II'' with diametrically opposite points on the heavy dashed line identified (see Figure 12).

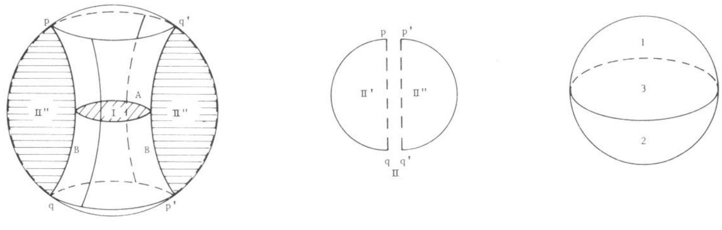


Figure 12.
$\mathbb{R}P^3$ as an S^1 bundle over S^2.

All the S^1 fibers except A and B, pass transversally through either I or II. The S^2 is assembled as in the S^3 case and also the solid torus decomposition. The S^1 fibers on T^2 are again drawn from the "a" and "b" section viewpoints (see Figure 13). Note that the fiber B wraps twice around the hole of T^2 while wrapping once around the meridian. The mapping $\tilde{\psi}$ from the "a" to the "b" viewpoint is $\begin{bmatrix} 1 & 0 \\ \pm 2 & 1 \end{bmatrix}$, which is the gluing map of the solid tori in $\mathbb{R}P^3$.

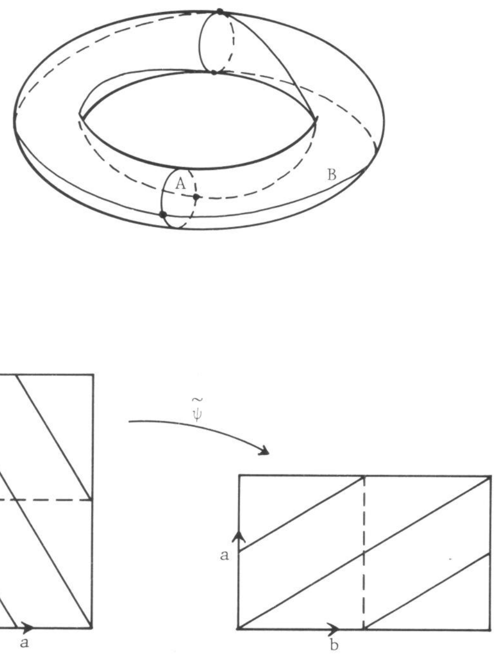
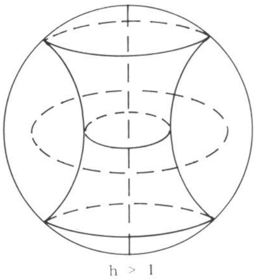


Figure 13.
The gluing map on T^2 for the solid torus decomposition of $R\,P^3$.

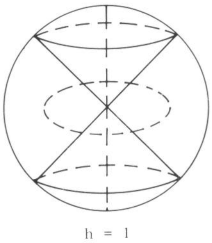
2. Bifurcation and monodromy

a. Existence of monodromy.

A glance at Figure 9 discloses that as h passes through 1 the topology of the energy surface $H^{-1}(h)$ changes from that of a three dimensional sphere to that of real projective three space. This bifurcation of $H^{-1}(h)$ (see Figure 14) is not due to a local bifurcation in the topology of the fibers of the energy momentum mapping, because over any open set U of regular values which does not contain (1,0), $\mathcal{EM}^{-1}(U)=U\times T^2$, that is, $\mathcal{EM}$ has two dimensional tori as fibers over U. In fact, the bifurcation of the energy surfaces signals the presence of monodromy in the energy momentum mapping, as the following discussion shows.



Antipodal points on S^2 identified



Antipodal points on S^2 identified.
Origin has conelike singularity.

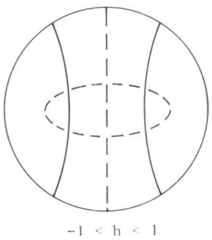


Figure 14.

S^2 identified to a point.
(Gives double cover of $h>1$ picture).

Let γ be a circle in the set $\mathcal{R}$ of regular values of $\mathcal{EM}$ with center at (1,0). Consider the bundle $\mathcal{B}=\mathcal{EM}^{-1}(\gamma)$ over γ with fiber T^2. Up to isomorphism $\mathcal{B}$ depends only on the homotopy class of γ in $\mathcal{R}$. Let $\Gamma^{\pm}$ be paths in the image of $\mathcal{EM}$ as drawn in Figure 15. Suppose that $\mathcal{B}$ is a trivial bundle, that is, $\mathcal{B}$ is diffeomorphic to $\gamma\times T^2$. Then $\mathcal{EM}^{-1}(\Gamma^-)$ is homeomorphic to $\mathcal{EM}^{-1}(\Gamma^+)$. But $\mathcal{EM}^{-1}(\Gamma^-)$ is homeomorphic to $H^{-1}(h')$ for some $h'\in]-1,1[$, which in turn is homeomorphic to S^3; while $\mathcal{EM}^{-1}(\Gamma^+)$ is homeomorphic to $H^{-1}(h'')$ for some $h''>1$ which in turn is homeomorphic to $\mathbb{R}P^3$.

But S^3 and $\mathcal{R}P^3$ are not topologically equivalent. Hence $\mathcal{B}$ is not a trivial bundle. Let $c\in\gamma$, then $\mathcal{B}'=\mathcal{EM}^{-1}(\gamma-\{c\})$ is a trivial T^2 bundle, since $\gamma-\{c\}$ is contractible. Therefore $\mathcal{B}$ is obtained from $\mathcal{B}'$ by a gluing diffeomorphism ϕ of $T^2=\mathcal{EM}^{-1}(c)$ into itself. The mapping ϕ is called the monodromy mapping of the bundle $\mathcal{B}$. Since isotopic monodromy mappings give rise to isomorphic bundles over γ, the fact that $\mathcal{B}$ is nontrivial implies that ϕ is not isotopic to the identity.

b. Calculation of monodromy.

A technical argument which is given below shows that the gluing maps of the S^1 bundles $\mathcal{EM}^{-1}(\Gamma^+)$ and $\mathcal{EM}(\Gamma^-)$ are respectively $\tilde{\psi}^- = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ and $\psi^- = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Therefore the monodromy is $\phi = (\tilde{\psi}^-)^{-1} \circ \psi^- = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. We now give the technical argument. The nonexpert reader is advised to skip the remainder of the subsection.

In order to compute the monodromy of $\mathcal{B}$ we introduce a certain Ehresmann connection [8] on $\mathcal{B}$ which allows us to parallel transport geometric objects from one fiber of $\mathcal{B}$ to another. Using a partition of unity it suffices to construct the connection locally. A nice local trivialization of $\mathcal{B}$ over γ is given by a choice of action angle coordinates [1]. For each (h, l) in some connected open subset V of γ the construction of action angle coordinates gives a Hamiltonian vectorfield X_F on the fiber $T^2_{h,l}$ of $\mathcal{B}$ over (h,l) such that

1) X_F is a linear combination of the Hamiltonian vectorfields X_L and X_H associated with the Hamiltonian functions L and H respectively;
2) X_F on $T^2_{h,l}$ has only periodic orbits of period one;
3) X_L and X_F generate a lattice $L_{h,l}$ which defines $T^2_{h,l}$ and depends smoothly on $(h,l) \in V$.

Thus $\mathcal{B}_V$, the piece of $\mathcal{B}$ over V, is diffeomorphic to $V \times T^2$. On $\mathcal{B}_V$ we define the vertical vectorfields of our Ehresmann connection to be vectorfields which are linear combinations of X_L and X_F while the horizontal vectorfields are nonzero vectorfields on V. Using this connection, parallel translation ϕ^- along the piece Γ^- of γ which joins P to Q transports the lattice L_P into the lattice L_Q (see Figure 15). Thus ϕ^- is a diffeomorphism of T^2_P onto T^2_Q which is the gluing map $\begin{bmatrix} 1 & 0 \\ \pm 1 & 1 \end{bmatrix}$ of the bundle $\mathcal{EM}^{-1}(\Gamma^-)$. Similarly, parallel translation ϕ^+ along the piece Γ^+ of γ joining P to Q is the gluing map $\begin{bmatrix} 1 & 0 \\ \pm 2 & 1 \end{bmatrix}$ of the bundle $\mathcal{EM}^{-1}(\Gamma^+)$.

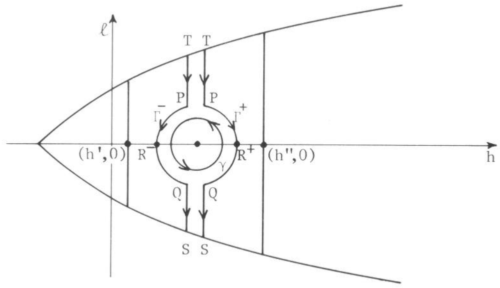


Figure 15.
Monodromy and bifurcation of the energy surfaces.

We now have to answer the delicate question of which sign to choose in ϕ^- and ϕ^+. As remarked earlier the sign choice is determined by the orientation of the bundle space (which induces an orientation on the two dimensional torus T^2) and the orientation induced by the action of the group S^1 on the oriented fiber S^1. In our case we determine the sign as follows. Let $p \in \Gamma^{\pm}$ and $\hat{p} \in T_p^2$. Suppose that $X_H(\hat{p})$ and $X_F(\hat{p})$ are linearly independent at $\hat{p}$. (This does not depend on the choice of $\hat{p} \in T_p^2$). We say that the Hamiltonian vectorfield X_H on T_p^2 has positive sense with respect to the ordered basis $\{X_L(\hat{p}), X_F(\hat{p})\}$ of the lattice L_p, if $X_H(\hat{p}) = \alpha X_L(\hat{p}) + \beta X_F(\hat{p})$ where either $\alpha > 0$ and $\beta > 0$ or $\alpha < 0$ and $\beta < 0$; otherwise X_H has negative sense. When parallel transporting L_p along $\Gamma^{\pm}$ from P to Q, X_H and X_F are linear dependent only at $R^{\pm}$ (see Figure 15) that is, when $l = 0$, because only then does X_H have a periodic flow on $T_{h,0}^2$. As l changes sign, X_L does also; more precisely, $X_L(p') = -X_L(\hat{p})$ where $p = (h, l) \in \Gamma^{\pm}$, $l > 0$ and $p' = (h, -l) \in \Gamma^{\pm}$. Since X_H and X_F are continuous, when l changes sign, the sense of X_H changes as p passes through $R^{\pm}$. Since the only sense change occurs at $R^{\pm}$ and by a suitable choice of X_F the sense of X_H at P can be made positive, the sense of X_H at Q, after being transported along Γ^- (Γ^+) from P to Q, is negative. Therefore the signs of both attaching maps $\phi^{\pm}$ are negative. Hence parallel translation along Γ^- from P to Q followed by parallel translation along the inverse of Γ^+ from Q to P defines a diffeomorphism ϕ of T_P^2 into itself which is given by

$$\phi = (\phi^+)^{-1} \circ \phi^- = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

ϕ is the monodromy mapping of the bundle $\mathcal{EM}^{-1}(\gamma)$ where $\gamma = (\Gamma^+)^{-1} \circ \Gamma^-$.

c. Other calculations of monodromy.

In addition to the above calculation, there are three other entirely different arguments which compute the monodromy of the spherical pendulum. The first is a physical geometric argument of Duistermaat [1] which will not be repeated here. The second one due to F. Ehlers in Bonn in essence shows that $\mathcal{EM}$ for values close to (1,0) is isomonodromic with its 2-jet at $(n,0)$. This result is nontrivial because $(n,0)$ is not a finitely determined singularity of $\mathcal{EM}$. On the other hand, the 2-jet of $\mathcal{EM}$ at $(n,0)$ is the energy momentum mapping of the two dimensional harmonic oscillator, for which the monodromy is easy to compute. For further details see [9]. In this same paper Min Oo in Bonn gives the following sequence of pictures (Figure 16) which geometrically computes the map $\phi_* : H_1(T_p^2) \rightarrow H_1(T_p^2)$ on homology induced by the monodromy ϕ. (Notice that the conventions in Figure 16 are the same as those in Figure 5). Here it is sketched how the two generators δ_0, ϵ_0 of the homology group are transformed when moved around the isolated singular value on γ.

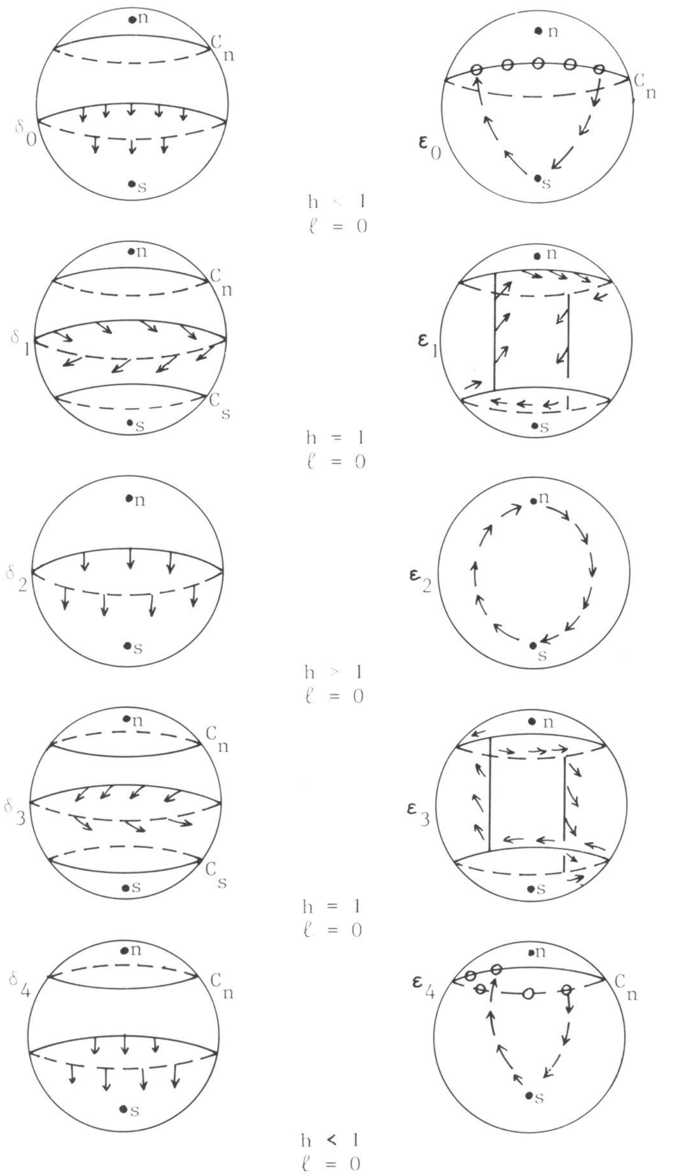


Figure 16.
Monodromy on Homology.
For $l = 1,2,3,4$ cycles δ_l, ϵ_l are basis of homology of $T^2_{h,l}$ obtained by homotopy from $\delta_{l-1}, \epsilon_{l-1}$, $l = 1,2,3,4$. ϵ_4 is homologous to $\epsilon_0 + \delta_0$ and δ_4 and δ_0. Thus $\psi_r = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ on homology.

3. Acknowledgements

To the expert reader my debt to Prof. J.J. Duistermaat in Utrecht is obvious. Also I would like to thank Drs. Min Oo and F. Ehlers for showing me their alternative methods for calculating the monodromy. Finally the geometric visualization of the gluing maps given in Figures 9-12 is due to Dr. Tim Poston of UCLA.

4. References

[1] J.J. Duistermaat, *On global action-angle coordinates,* Comm. Pure Appl. Math., **33** (1980), 687-706.

[2] H. Goldstein, *Classical mechanics,* 1st ed., Addison-Wesley, Reading, Mass., 1959.

[3] V.I. Arnold, *Mathematical methods of classical mechanics,* Springer Verlag, New York, 1978.

[4] S. Smale, *Topology and mechanics I,* Invent. Math., **10** (1970), 305-331.

[5] K. Symon, *Mechanics,* Addison-Wesley, Reading, Mass, 1961.

[6] A.G. Webster, *The dynamics of particles and rigid, elastic, and fluid bodies,* 2nd ed., Dover, 1959.

[7] N. Steenrod, *Topology of fiber bundles,* Princeton Univ. Press, Princeton, 1951.

[8] J. Wolf, *Differentiable fibre spaces and mappings compatible with Riemann metrics, Mich. Math. J.,* **11** (1964), 65-70.

[9] R. Cushman, F. Ehlers, and Min Oo, *Three proofs of monodromy in the spherical pendulum* (in preparation).

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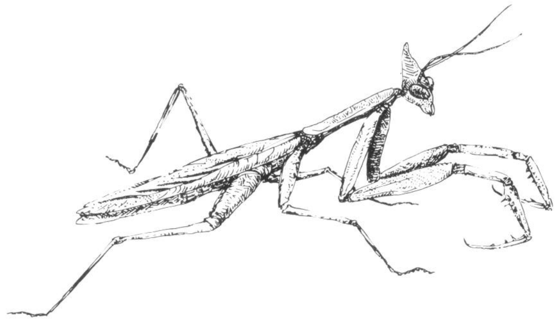


The Dynamics of Structured Populations

by Odo Diekmann

Physiological processes within individuals and behavioural patterns displayed by individuals are some of the subjects studied by biologists. Matters like growth, the succession of larval stages and reproduction are pieces of a sometimes remarkably complicated jig-saw puzzle called the life cycle.

On the other hand biologists also study the past and present state of large populations and try to predict their future development by calculating how the number of individuals changes as a consequence of reproduction and interaction (for example, competition for food).

Structured population models are intended to bridge the gap between the individual and the population level. The aim is to derive information about the dynamics of the population from information about the dynamics of the individuals or vice versa (cf. [1]).

The following three examples illustrate some of the main ideas.

1. If a predator eats (too) much prey he is not hungry any more and he will hunt with less zeal. Thus one expects that the functional response F (i.e., the number of prey eaten per predator per unit of time as a function of the prey density x) will be given by a graph as shown in Figure 1.

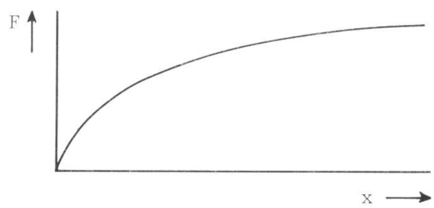


Figure 1.

Assuming that the state of each predator can be completely characterized by its satiation $s \geqslant 0$ (i.e., some measure for the contents of stomach and gut) the population can be described by the satiation-density function $s \rightarrow n(t, s)$ depending on time t. Thus $\int_{s_1}^{s_2} n(t, s) ds$ represents the number of predators with satiation between s_1 and s_2 at time t.

One can then derive the following equation:

$$\frac{\partial n}{\partial t}(t, s) = -\frac{\partial}{\partial s}(g(s)n(t, s)) - x(b(s)n(t, s) - b(s-w)n(t, s-w))$$

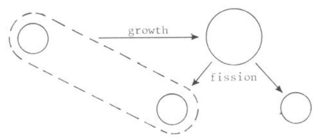
The first term at the right hand side describes the changes due to digestion (with rate $g(s)$), and the second term describes changes due to the consumption of preys of constant weight w which are caught with rate $xb(s)$. Handling times and changes in prey density x are neglected here because of differences in time-scale: prey capture and digestion are fast processes compared with reproduction, and slow processes compared with the actual handling of the prey.

As $t \rightarrow \infty$ the solution approaches a stable distribution $\hat{n}(s)$ and the functional response is explicitly given by

$$F(x) = x \int_0^\infty g(s)\hat{n}(s) ds.$$

Numerical calculations based on this formula confirm the qualitative form of Figure 1. Moreover, one can use experimental measurements of g and b to determine F quantitatively and subsequently use the result as an input for a prey-predator total population model at the time-scale of reproduction. We refer to [2] for further details.

2. Consider a population of unicellular organisms (bacteria or algae) and assume that the physiological state of an arbitrary cell is completely described by one quantity x which obeys a physical conservation law (for



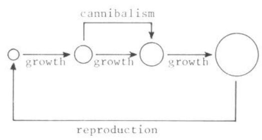
example, total mass or the amount of nitrogen atoms in the cell). We shall call x 'size'. Furthermore, assume that cells reproduce by binary fission into two exactly equal daughters. The balance of growth, death and division (with rates g, μ and b, respectively) leads to the equation

$$\frac{\partial n}{\partial t}(t, x) = -\frac{\partial}{\partial x}(g(x)n(t, x)) - \mu(x)n(t, x) - b(x)n(t, x) + 4b(2x)n(t, 2x),$$

which shows marked mathematical similarities to the equation in the first example. It turns out that the existence of a stable distribution hinges on the biologically interpretable condition that $x \rightarrow g(2x)$ and $x \rightarrow 2\,g(x)$ are not identical.

Density dependence (as a consequence of limited resources) can be incorporated by specifying how g depends on the available nutrients and how, conversely, the food supply is influenced by consumption. References [3] are an elaborate presentation of this example.

3. Predators may prefer mature prey above young prey or they may, on the contrary, eat only eggs. Individuals of many species change their diet at various stages in the life cycle and thus one may have to distinguish the



predator according to its maturity. Cannibalism seems to be a major regulating mechanism for many species. In all of these situations one needs a population structure (in terms of age, size, larval stadia ...) in order to describe the interaction properly. See [4] for some models and results.

The first step in building these models consists of finding a suitable explicit parametrization of the state of the individuals (satiation, size, age, ...). The state of the population is then given by the density function n describing the distribution in the individual state space.

In the course of time the state of each specific individual changes (owing to digestion, growth, aging, ...). Moreover, individuals are born and die. (These words have to be interpreted broadly: in the first example a predator which consumes a prey "dies" while at the same time a new predator with w added to the satiation "is born".) In the second step one draws up the balance of these processes to derive a (first order partial) differential equation for the infinitesimal change in the population state. The coefficients in the equation describe the functioning and the behaviour of the individuals but the solution describes (properties of) the population as a whole. Starting from biological knowledge one can incorporate the interaction of the population and its environment (including other populations) by specifying in detail how the birth, death and growth processes depend on environmental quantities. Thus, as a rule, the equations become nonlinear.

When suitable boundary conditions are added, an initial condition $n(0,x) = \phi(x)$ at $t=0$ singles out a unique solution $n(t,x;\phi)$. It is mathematically convenient to conceive of ϕ and $n(t,\cdot\,;\phi)$ as elements of a function space X (such as L_1 or C) and to write

$$n(t,\cdot\,;\phi) = S(t)\,\phi.$$

The family $\{S(t)\}_{t \geq 0}$ forms a semigroup of (continuous) mappings from X into X (i.e., $S(0)=I$ and $S(t_1)\ S(t_2)=S(t_1+t_2)$, $t_1,t_2 \geq 0$) such that the balance equation can be interpreted as

$$\frac{dn}{dt} = An$$

with A the infinitesimal generator of $\{S(t)\}_{t \geq 0}$. Thus, these problems fit into the general framework of dynamical systems on infinite dimensional spaces [5,6]. An important special feature of these population models is the occurrence of *non-local terms* (such as the ones with the transformed arguments $s-w$ and $2x$) and this gives the problems a certain flavour reminiscent of functional differential equations, see [8].

The linear theory of stable distributions is based on positivity (Krein-Rutman Theorem) and on compactness arguments [2,3]. The qualitative theory of *nonlinear age-structured* models has developed rather rapidly in recent years [4,7]. For the general case hardly any work on nonlinear problems has been done. The objective of the project 'Dynamics of structured populations' at CWI is to develop parts of a qualitative theory little by little, by applying general techniques, such as bifurcation theory [9], to concrete problems in this area.

A recent colloquium at CWI has brought about cooperation with several biologists. Team-work has produced a set of examples (such as the ones above) which are as simple as possible but yet biologically relevant. Their mathematical analysis is now in progress. Step by step complexity and realism will be built up in the hope that eventually a coherent general theory will arise.

References

[1] W. Streifer, *Realistic models in population ecology,* in: A. MacFadyen (ed.), Advances in Ecological Research, **8** (1974) 199-266.

[2] J.A.J. Metz & F.H.D. van Batenburg, *Holling's 'hungry mantid' model for the functional response considered as a Markov process. Part 0: A survey of the main ideas and results,* CWI Preprint TW 233, 1983; Parts I and II in preparation. Part III by H.J.A.M. Heijmans in preparation.

[3] O. Diekmann, H.J.A.M. Heijmans & H. Thieme, *On the stability of the cell size distribution,* CWI for Preprint TW 242, 1983 & O. Diekmann, H.A. Lauwerier, T. Aldenberg & J.A.J. Metz, *Growth, fission and the stable size distribution,* to appear in J. Math. Biol.

[4] D.S. Levine, *Some age-structure effects in predator-prey models,* in: H.I. Freedman & C. Strobeck (eds.), *Population Biology,* Springer LNiB, **52** (1983) 304-316.

[5] J.A. Walker, *Dynamical Systems and Evolution Equations, Theory and Applications,* Plenum Press, New York, 1980.

[6] A. Pazy, *Semigroups of linear operators and applications to partial differential equations,* Springer, Appl. Math. Sc., **44**, 1983.

[7] G.F. Webb, *Theory of nonlinear age-dependent population dynamics,* monograph, to appear.

[8] J.K. Hale, *Theory of functional differential equations,* Springer 1977.

[9] S.-N. Chow & J.K. Hale, *Methods of bifurcation theory,* Springer, 1982.

Mertens' Conjecture disproved

by Herman te Riele *

By means of a rather simple computer program it has been established that

$$\sigma_n(t) \approx 1.0615 ,$$

for $n = 2000$ and

$$t = t_0 = -\ 14045\ 28968\ 05929\ 98046\ 79036\ 16303\ 99781\ 12740\ 05919\ 99789\ 73803\ 99659\ 60762.52150\ 5$$

where

$$\sigma_n(t) = 2\sum_{j=1}^{n} \kappa\left[\frac{\gamma_j}{\gamma_n}\right] \frac{\cos(\gamma_j t - \pi\psi_j)}{|\rho_j \zeta'(\rho_j)|}, \quad -\infty < t < \infty,$$

$$\kappa(\tau) = (1-\tau)\cos(\pi\tau) + \pi^{-1}\sin(\pi\tau), \quad 0 \leq \tau \leq 1,$$

$\rho_j = \frac{1}{2} + i\gamma_j$, $1 \leq j \leq n$, is the j-th non-trivial simple zero of the Riemann zeta function,

and

$$\pi\psi_j = arg(\rho_j \zeta'(\rho_j)), \quad 1 \leq j \leq n,$$

where *arg* stands for the usual argument of a complex number. Consequently, the old conjecture of F. Mertens ([3], p.779) that $|M(x)| < \sqrt{x}$ for all $x > 1$, where $M(x) = \Sigma_{n \leq x} \mu(n)$ and μ is the Möbius function, is *false*, since every value of $\sigma_n(t)$ is a lower bound for $\limsup_{x \to \infty} M(x) \cdot x^{-\frac{1}{2}}$ ([1], p. 329). Mertens' conjecture would have implied the Riemann hypothesis [5], p. 320).

The major problem in this joint project of CWI and Bell Laboratories was to find a value of t for which $\sigma_n(t) > 1$. The number t_0, given above, was found by Andrew Odlyzko by using the so-called *lattice basis reduction algorithm* ([2], pp. 516-525). t_0 has the remarkable property that in 70 of the 2000 terms of $\sigma_{2000}(t_0)$ *all* cosine-values are very close to $+1$. In other words, in these 70 terms *all* numbers $\gamma_j t_0 - \pi\psi_j$ are very close to a multiple of 2π. The problem of finding such a candidate t_0 is known as the problem of *inhomogeneous diophantine approximation* or *Kronecker approximation.* That it is possible to solve this problem for 70 terms has been unthinkable until very recently.

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* This announcement reports on joint research with Andrew Odlyzko of Bell Laboratories (Murray Hill, New Jersey, USA)

It took the CRAY 1 computer of Bell Labs about three hours and a considerable amount of memory to find t_0.

A second problem was the high-precision computation of the imaginary parts γ_j of the first 2000 non-trivial zeros of the Riemann zeta function which was essential for the computation of

$$\cos(\gamma_j t_0 - \pi\psi_j), \quad 1 \leq j \leq n$$

in the above formula for $\sigma_n(t)$. This was carried out by the author on the CDC CYBER 175 - 750 computers of SARA. The zeros were computed with an accuracy of 105 decimal digits, with the help of a special multi-precision package of R.P. Brent of the Australian National University. The zeros were computed with the well-known Newton process from 28 digit approximations already known ([4]). The total amount of (nightly) computer time needed was about 40 hours.

The communication between CWI and Bell Labs was not maintained by letters, but by electronic mail transmitted via the VAX-computers of the two institutes. This enabled the participants to exchange their data and results with a very high speed, frequency and reliability. Without this facility the whole project would have taken at least six months. Now it was completed in less than two months.

Only five years ago the author still believed that Mertens' conjecture could not be disproved "using present day computers and current techniques" ([4], p. 356). Now, the disproof shows how rapidly new algorithms and super-fast computers have been developed in the past few years.

References

[1] W. Jurkat & A. Peyerimhoff, *A constructive approach to Kronecker approximations and its application to the Mertens conjecture,* J. reine angew. Math., **286/287** (1976) 322-340.

[2] A.K. Lenstra, H.W. Lenstra, Jr. & L. Lovàsz, *Factoring polynomials with rational coefficients,* Math. Ann., **261** (1982) 515-534.

[3] F. Mertens, *Ueber eine zahlentheoretische Funktion,* Sitzungsberichte Akad. Wien, **106**, Abt. 2a (1897) 761-830.

[4] H.J.J. te Riele, *Computations concerning the conjecture of Mertens,* J. reine angew. Math., **311/312** (1979) 356-360.

[5] E.C. Titchmarsh, *The theory of the Riemann zeta-function,* Oxford, 1951.

(received, October 21, 1983).

Special Functions and Group Theory

Oberwolfach Meeting from 13^th^ to 19^th^ March 1983

by Bob Hoogenboom

The purpose of this meeting was to discuss recent developments in the theory of special functions, with emphasis on the connections between special functions and group theory.

First of all, what is a special function? Many (ridiculous) definitions have been given, most of which exclude some very important examples of special functions. The one and only good definition I know is by Richard Askey, in [1]: A function is a special function if it occurs often enough that it gets a name. Important examples of special functions are the exponential, the gamma function, the Riemann zeta function, theta functions, Bessel functions, Jacobi polynomials, etc. The first three examples were not discussed at the meeting; the last three, and many others, were.

Secondly, how do special functions occur? The answer is: in many ways. Let me give three examples. Theta functions occur in physics, for instance as periodic solutions of Korteweg - de Vries type equations, and in connection with completely integrable systems, cf. [2]. Other special functions occur as solutions of certain second order differential equations, for instance the hypergeometric function ${}_2F_1(\alpha,\beta;\gamma;x)$, which is defined by

$$ {}_2F_1(\alpha,\beta;\gamma;x) := \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{(\gamma)_n n!} x^n, \tag{1}$$

where the shifted factorial $(a)_n$ is defined by $(a)_n := a(a+1)...(a+n-1)$ for $n = 1,2,...$. The hypergeometric function ${}_2F_1$ is a solution of the equation

$$ x(1-x)\frac{d^2}{dx^2}f + [\gamma-(\alpha+\beta+1)x]\frac{d}{dx}f = \alpha\beta f, \tag{2}$$

cf. [3, §2.1.1]. Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ are a special case of (1), namely

$$ P_n^{(\alpha,\beta)}(x) := \binom{n+\alpha}{n} {}_2F_1(-n, n+\alpha+\beta+1; \alpha+1; \tfrac{1}{2}(1-x)). \tag{3}$$

As a third example, I would like to mention representation theory. Some special functions appear in the study of linear representations of certain groups, for instance as matrix entries in irreducible representations of Lie groups. An example is the *nth* Bessel function

$$ J_n(x) := \frac{1}{2\pi}\int_0^{2\pi} e^{ix\sin\theta - in\theta} d\theta, \tag{4}$$

which arises as a matrix entry of the irreducible representations of the group of motions of the plane $\mathbf{R}^2$, cf. [5, Ch. IV]. I shall discuss some of the talks grouped around four different themes.

1. Semisimple Lie Groups

Under this category, functions on groups were studied, mostly spherical functions on semisimple Lie groups and generalizations. A spherical function on a semisimple Lie group G with respect to a maximal compact subgroup K is a K-invariant function on G/K which is a joint eigenfunction of all G-invariant differential operators on G/K.

New interpretations for addition formulas for functions of the second kind (that is, a second solution of a differential equation of type (2) which is regular at ∞) were given in terms of the pair $(G, K)=(SO(n,1), SO(n-1,1))$. Observe that the subgroup K is not compact. Here the space G/K can be interpreted as the hyperboloid of one sheet in $\mathbf{R}^n$ (Talks by Durand, Mizony). Matrix elements of the representations of $S0(n,1)$ were studied by a global approach, without using Lie algebra theory (Koornwinder : $n=3$, Takahashi: partial results for $n=4$). Other talks in this category were by Reimann, Terras and Hoogenboom.

2. Special functions and q-analogues

Under this category special functions were studied by analytic methods, without the use of group theory. The emphasis was put on the generalization of classical results to the so-called q-analogues of special functions. As someone told me lately, q-analogues are just the ordinary special functions, only with all the 1's replaced by a q. There remains only one problem: where are the 1's ? To give an example, the binominal theorem

$$(1-x)^{-\alpha}=\sum_{n=0}^{\infty}\frac{(\alpha)_n}{n!}x^n \tag{5}$$

has the following q-analogue

$$\frac{(\alpha x;q)_\infty}{(x;q)_\infty}=\sum_{n=0}^{\infty}\frac{(\alpha;q)_n}{(q;q)_n}x^n \tag{6}$$

$(|x|<1, |q|<1)$, where

$$(\alpha;q)_\infty = \prod_{n=0}^{\infty}(1-\alpha q^n),\ (\alpha;q)_n=(1-\alpha)(1-\alpha q)\ \cdots\ (1-\alpha q^{n-1}).$$

By writing $\alpha=q^a$, where a is a nonnegative integer, (6) formally leads to (5) if $q\uparrow 1$.

This phenomenon is inspired by the study of (parameters of) permutation representations of the Chevalley groups (i.e, the finite analogues of Lie groups), the theory of partitions, etc. (Talks by Gasper, Askey, Rahman).

Other talks in this section dealt with orthogonal polynomials (Dunkl), and solutions of differential equations (Sprinkhuizen-Kuyper, Trime'che)

3. Selberg integrals and Dyson conjectures

The classical Beta-integral reads as follows

$$\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \tag{7}$$

where Γ is the classical gamma function. As I learned at this meeting, this equation has been generalized in 1944 to the following equation

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^{p} u_i^{\alpha-1}(1-u_i)^{\beta-1}\prod_{i<j}(u_i-u_j)^{2\gamma}du_1 \cdots du_p = \tag{8}$$

$$= \prod_{n=1}^{p} \frac{\Gamma(1+n\gamma)\Gamma(\alpha+(n-1)\gamma)\Gamma(\beta+(n-1)\gamma)}{\Gamma(1+\gamma)\Gamma(\alpha+\beta+(p+n-2)\gamma)},$$

see Selberg [4]. Unfortunately, Selberg published his result in an obscure Norwegian journal, so that it remained unnoticed for some time. Formula (8) was used to prove some conjectures by Dyson about the constant term in a certain Laurent series expansion. The simplest instance of Dyson conjectures is as follows. Here *C.T.* denotes the constant term in the Laurent series expansion.

$$C.T.\prod_{i \neq j}^{n}(1-x_i x_j^{-1})^k = \frac{(nk)!}{(k!)^n}. \tag{9}$$

The talks in this category were devoted to generalizations of these Dyson conjectures, some of which are still open. (Talks by Macdonald, Stanton, Metha, Kadell).

4. Signal processing and the Heisenberg group.

The Radon transform plays an important role in the theory of computerized tomography. Also, in radar signal processing the Heisenberg group arises. Talks in this section were on radar tomography (Grünbaum, Schempp, Louis), and on the Heisenberg group (Greiner, Auslander).

The other talks (among a total of 37) dealt with Lie groups and physics (Louck, Milne, Kramer, Onofri), Combinatorics and Special functions (Seidel, Bannai, Foata), Gelfand pairs and hypergroups (Lasser, Letac), Seperation of variables (Miller), Theta functions (Hazewinkel), and various subjects (Ronveaux, Delvos, Calogero, Hermann).

The meeting was organized by R.A. Askey (Madison), T.H. Koornwinder (Amsterdam) and W. Schempp (Siegen). The proceedings will be published by Reidel, Dordrecht, the provisional title being: *'Special functions: group* *theoretic aspects and applications'* in the series 'Mathematics and its applications.'

References

[1] Askey, R.A., *Foreword,* in: *'Symmetry and separation of variables'* (W. Miller, jr.), Addison - Wesley, Reading, Mass., 1977.

[2] Dubrovin, D.A., *Theta functions and non-linear equations,* Russian Math. Surveys, **36**:2 (1982) 11-92.

[3] Erdélyi, A., W. Magnus, F. Oberhettinger & F. Tricomi, *Higher Trancedental Functions,* Vol. 1, McGraw-Hill, New York 1953.

[4] Selberg, A., *Bemerkninger om et multipelt integral,* Norsk Matematisk Tidskrift, **26** (1944) 71-78.

[5] Vilenkin, N. Ja., *Special functions and the theory of group representations,* Am. Math. Soc. Transl. of Math. Monographs, Vol. 22, Providence, R.I., 1968.

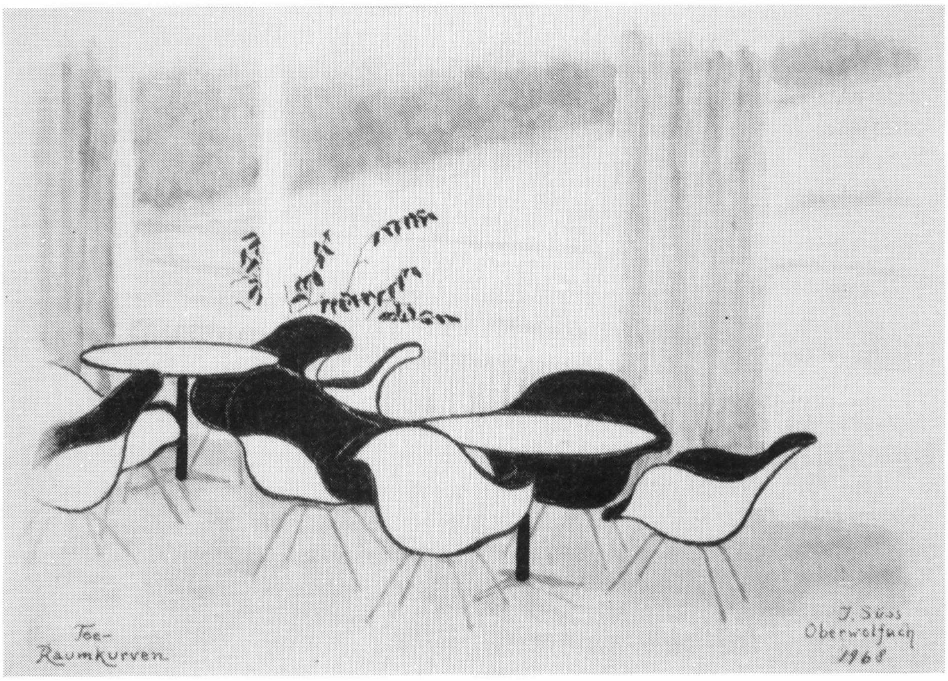

Illustration of 'Tee-Raumkurven' at Oberwolfach
by I. Süss, 1968.

Online Mathematical Literature Information

by S.I. Thé

Since the early seventies, online literature information systems have been successfully used in the Western scientific communities as a profitable supplementary tool to classical methods of literature search. For 'current awareness' and SDI (Selected Dissemination of Information) purposes, as well as for retrospective searching, this new (computerized) application has proven its usefulness and its cost-effectiveness and is still being expanded in several directions. Today many universities, research institutes, laboratories and libraries, especially in the US, have a terminal with the possibility of communicating with at least one of the more than 80 hosts, offering hundreds of databases/databanks searchable online.

Since 1980, the possibility of communicating with American and European hosts has also been available at CWI. Since last year most representative databases on mathematics - such as the Mathematical Reviews and the Zentralblatt für Mathematik - are searchable online. Furthermore - from this year on - the Citation Index on Computer Science and Mathematics (ISI-COMPUMATH) is also accessible online. To give a general view of the databases available online which cover mathematical subjects, we give a survey of the possibilities of finding mathematical literature in the following list:

COMPENDEX	(COMPuterised ENgineering inDEX) is the online version of the Engineering Index Monthly and covers mainly applied mathematics/mathematical statistics/computer science. (9,600,000 refs. from 1969 to date; monthly update 8400 refs.)
COMPUMATH	(COMPUter science and MATHematics citation index) corresponds to the COMPUMATH Citation Index and Current Contents and offers possibilities of online citation from more than 300 'core' journals on these subjects (275,000 refs. from 1976 to date; monthly update 3000 refs.)
INKA-MATH	is the online version of the 'Zentralblatt für Mathematik'. (434,200 refs. from 1972 to date; monthly update 4000 refs.)
INIS	(International Nuclear Information Service) - the online version of Atomindex - covers applied mathematics. (770,000 refs. from 1970 to date; monthly update 6000 refs.)
INSPEC	(International INformation Service for the Physical and Engineering Communities) is the online version of Physical Abstracts, Electrical and Electronics Abstracts and Computer and Control Abstracts. (2,026,000 refs. from 1969 to date; monthly update 14,000 refs.)

MATHFILE	is Mathematical Reviews online. (380,000 refs. from 1973 to date; monthly update 3000 refs.)
NASA	is the product of 2 bi-weekly abstract periodicals: the Scientific and Technical Aerospace Reports (STAR) and the International Aerospace Abstracts (IAA). (1,250,000 refs. from 1962 to date; monthly update 5000 refs.)
NTIS	(National Technical Information Service) of the US Dept. of Commerce covers all scientific and technical research reports sponsored by the US Government. (994,000 refs. from 1964 to date; biweekly update 2500 refs.)
PASCAL	(Programme Appl. à la Selection et à la Compilation Automatique de la Litterature) is the online version of the French abstract journal 'Bulletin Signalétique' - a multidiscipilinairy database of the Centre National de la Recherche Scientifique (CNRS). (3,100,000 refs. from 1973 to date; monthly update 40,000 refs.)
SCISEARCH	(Science Citation Index Online) covers scientific as well as technical disciplines by citation analysis. (Over 3,700,000 refs. from 1974 to date; monthly update 42,000 refs.)

Finally two databases - ERIC (Educational Resources Information Center) and MATHDI (MATHematische DIdaktik) - cover the subject Mathematics Education.

Furthermore there are many databases which are useful for searching other information in relation with mathematics, viz:

Conference Papers	: CPI (Conference Papers Index) & INKA-KONF,
Corporate authorities	: CORP database of ca. 50.000 names and addresses of Scientific Institutes all over the world,
Current Research	: SSIE (Smithsonian Sc. Inf. Exchange) Current Research (in the US) database,
Dissertations/Theses	: CDI (Comprehensive Dissertations Index),
Management & Control	: Management Contents,
Mathematical Biology	: BIOSIS (BIOSciences Information Services),
Mathematical Geology	: GEOREF of the American Geology Institute & PASCAL
Mathematical Physics	: SPIN (Searchable Phys. Inf. Notices) & INKA-PHYS,
Mathematical Software	: ISD (International Software Database) & PASCAL,
Microcomputers	: Microcomputer Index,
Report Literature	: SIGLE (System for Inf. on Grey Lit. in Europe)

and many other databases covering all kinds of information.

Abstracts of Recent CWI Publications

MC Tract 165. P.C.T. van der Hoeven, *On Point Processes.*
AMS 50G55, 126pp.
ABSTRACT: In this tract point processes are studied, i.e. probability mechanisms according to which a collection of points is chosen in some multi-dimensional space. We introduce in particular the notion of 'visibility' analogously to 'predictability', which is defined for random processes in time. Several smoothness conditions for point processes are considered in the context of visibility. So-called 'martingalelike measures' are defined. Some applications of this theory are indicated and the relevant quantities are determined in some examples of point processes.

MC Tract 166. H.B.M. Jonkers, *Abstraction, specification and implementation techniques.*
AMS 68B05, 317pp.
ABSTRACT: In this tract a number of ideas concerning abstraction, specification and implementation are developed (Chapter 1-3), which are then applied to the subject of garbage collection (Chapter 4-7).

On the basis of a simple model for abstraction, Chapter 1 discusses how abstraction can be systematically used in problem solving and in classifying large classes of problems and their solutions. In Chapter 2 the notion of a 'structure' is introduced, which allows arbitrary data structures to be modelled without the use of 'pointers'. It is then indicated how this concept could be used as the basis of a specification language. A loose version of this language is used in the other chapters for the description of algorithms and data structures. Chapter 3 describes a simple implementation method for both algorithms and data structures, based on a four-step technique of establishing a change of data representation. The method is demonstrated by means of a derivation of the Deutsch-Schorr-Waite marking algorithm.

The purpose of Chapter 4 is the introduction of a model of storage management, which is used in Chapter 5 to give a survey of the subject of garbage collection. Starting from two abstract algorithms, the main garbage collection and compaction algorithms are derived by means of 'correctness-preserving transformations'. The subject of Chapter 6 is the design of a storage management system for a machine-independent ALGOL 68 implementation. The method used is that of Chapter 3, where an abstract model is used to keep complexity under control. The design of the garbage collector, which is kept abstract in the system developed in Chapter 6, is described in Chapter 7. The approach is analogous to that of Chapter 6, except that the process of transformation of the garbage collector is carried through up to the level of machine code.

MC Tract 167. W.H.M. Zijm, *Nonnegative matrices in dynamic programming.*
AMS 90C39, 190pp.

ABSTRACT: In this monograph we study dynamic programming models in which the transition law is specified by a set of nonnegative matrices. These models include Markov decision processes with additive and multiplicative utility function, input-output systems with substitution, and controlled multitype branching processes. It is shown that all these models can be studied within one general matrix theoretical framework. This framework is built up by using dynamic programming methods and is based on the theory of sets of general nonnegative matrices.

In particular, we study asymptotic expansions of the vector function $x(n)$ defined by the dynamic programming recursion

$$x(n) = \max_{P \in \mathcal{K}} Px(n-1) \qquad n=1,2,...$$

(with $\mathcal{K}$ a set of nonnegative square matrices with the product property), and of its continuous-time counterpart

$$\frac{dx}{dt}(t) = \max_{Q \in \mathcal{M}} Qz(t) \qquad t \in [0,\infty)$$

(with $\mathcal{M}$ a set of M-matrices with the product property). The results cover many of well-known and also unknown results in e.g. Markov decision chains with several performance criteria. Both finite and infinite dimensional models are investigated.

MC Tract 168. J.H. Evertse, *Upper bounds for the numbers of solutions of diophantine equations.*
AMS 10B10, 124pp.
ABSTRACT: In this monograph we derive upper bounds for the numbers of solutions of diophantine equations taken from several classes. For example, we show that the so-called *Thue equation*

$$F(x,y) := a_0x^n + a_1x^{n-1}y + ... + a_{n-1}xy^{n-1} + a_ny^n = m \quad in \quad x,y \in \mathbb{Z} \quad (*)$$

has at most

$$n \left(7^{15(\binom{n}{3}+1)^2} + 6 \times 7^{2\binom{n}{3}(t+1)} \right)$$

solutions. (Here m is an integer such that $|m|$ is composed of t distinct primes and $F(x,y)$ is a binary form of degree $n \geqslant 3$ with $a_0,...,a_n \in \mathbb{Z}$ and $a_0 \neq 0$ such that the polynomial $F(x,1)$ has at least three distinct zeros in the field of algebraic numbers.) In the proof of this result we use an approximation technique due to Thue and Siegel. In certain special cases, much better results are derived. For instance if $n=3$ and $m=1$ then we show that (*) has at most *twelve* solutions. On the other hand, we also derive upper bounds for the number of solutions of more general equations such as the *Thue-Mahler equation*

$$|F(x,y)| = p_1^{k_1}...p_t^{k_t} \quad in \quad x,y,k_1,...,k_t \in \mathbb{Z} \quad \text{with} \quad gcd(x,y)=1,$$

where $p_1,\ldots,p_t$ are fixed, distinct primes.

MC Tract 169. H.R. Bennett, D.J. Lutzer, *Topology and order structures.* (part 2).
AMS 54Fxx, 107pp.
ABSTRACT: This volume contains papers growing out of a two week workshop on topology and linear orderings, sponsored jointly by NATO and Texas Tech. University, in August, 1980. The volume contains four papers on orderability, four papers on spaces related to linearly ordered spaces, e.g. via mappings (dendrons, Hahn-Mazurkiewicz theory), nine papers on the general topology of ordered spaces, plus a section on problems posed at the workshop.

IW232/83. J.A. Bergstra & J.W. Klop, *An algebraic specification method for processes over a finite action set.*
AMS 68B10, 13pp., KEY WORDS: concurrency, nondeterministic process, merge, process algebra, state transition system, algebraic specification, computable process.
ABSTRACT: We combine the techniques of abstract data type specification and of process algebra thus obtaining a flexible technique for process specification, provided a finite action set is used.

IW233/83. A.K. Lenstra, *Factoring multivariate polynomials over algebraic number fields.*
AMS 12A20, 15pp., KEY WORDS: polynomial algorithm, polynomial factorization.
ABSTRACT: We present an algorithm for factoring multivariate polynomials over algebraic number fields that is polynomial-time in the degrees of the polynomial to be factored. The algorithm is an immediate generalization of the polynomial-time algorithm for factoring univariate polynomials with rational coefficients.

IW234/83. J.A. Bergstra, J.W. Klop & J.V. Tucker, *Algebraic tools for system construction.*
AMS 68B10, 11pp., KEY WORDS: hierarchical and modular systems, composition tools, system architectures, concurrency, communicating processes, process algebra, fixed point equations, handshaking.
ABSTRACT: In this paper we consider a variety of computer systems and collect a set of informal principles concerning their hierarchical construction. These ideas are readily transformed into an elementary formal account of systems in which levels of abstraction are represented by algebras and the relationships between levels are represented by homeomorphisms. The algebraic approach to systems is then exemplified in an algebraic theory of concurrent systems based on a set of axioms called *ACP* - axioms for concurrent processes - in part modelled on the calculi of R. Milner. Several theorems concerning *ACP* are discussed.

IW235/83. J.A. Bergstra & J.W. Klop, *The algebra of recursively defined processes and the algebra of regular processes.*
AMS 68B10, 29pp., KEY WORDS: concurrency, nondeterministic process, merge, process algebra, regular processes, recursively defined processes, fixed point algebra.
ABSTRACT: We introduce recursively defined processes and regular processes, both in the presence and absence of communication. It is shown that both classes are process algebras. An interpretation of *CSP* in the regular processes is presented. As an example of recursively defined processes, bag and stack are discussed in detail. It is shown that the bag cannot be recursively defined without merge. We also introduce fixed point algebras which have interesting applications in several proofs. An example is presented of a fixed point algebra which has an undecidable word problem.

IW236/83. J.A. Bergstra & J.V. Tucker, *The axiomatic semantics of programs based on Hoare's logic.*
AMS 35D35, 32 pp., KEY WORDS: programming language definition; program specification; program verification; Hoare's logic; nondeterministic semantics.
ABSTRACT: This paper is about the Floyd-Hoare Principle which says that the semantics of a programming language can be formally specified by axioms and rules of inference for proving the correctness of programs written in the language. We study the simple language WP of while-programs and Hoare's system for partial correctness and we calculate the semantics of *WP* as this is determined by Hoare's logic. This calculation is possible by using relational semantics to build a completeness theorem for the logic. The resulting semantics *AX* we call the axiomatic semantics for *WP*. This *AX* is not the conventional semantics for *WP*: it need not be effectively computable or deterministic, for example. A large number of elegant properties of *AS* are proved and the Floyd-Hoare Principle is reconsidered.

IW237/83. J.A. Bergstra & J.-J.Ch. Meyer, *On specifying sets of integers.*
AMS 68B15, 14pp., KEY WORDS: set-theoretical data types, initial algebra semantics, equational and conditional specifications.
ABSTRACT: We consider the problem of deriving an algebraic specification for a rather simple set-theoretical data type called *SOI*. This is merely a collection of finite sets of integers equipped with an operator for inserting a number into a set and another for determining the cardinality of a set. We show that *SOI* has a finite conditional specification, but no finite equational specification, under the initial algebra semantics for specifications invented by the *ADJ* group.

IW238/83. J.W. de Bakker & J.I. Zucker, *Compactness in semantics for merge and fair merge.*
AMS 68B10, 15pp., KEY WORDS: concurrency, merge, recursion,

compactness, denotial semantics, fair merge, metric topology, Hausdorff metric, trace theory, hyperspace.
ABSTRACT: An analysis of the role of compactness in defining the semantics of the merge and fair merge operations is provided. In a suitable context of hyperspaces (sets of subsets) a set is compact iff it is the limit of a sequence of finite sets; hence, compactness generalises bounded nondeterminacy. The merge operation is investigated in the setting of a simple language with elementary actions, sequential composition, nondeterministic choice and recursion. Metric topology is used as a framework to assign both a linear time and a branching time semantics to this language. It is then shown that the resulting meanings are compact trace sets and compact processes, respectively. This result complements previous work by De Bakker, Bergstra, Klop & Meyer. For the fair merge, an approach using scheduling through random choice is adopted - since a direct definition precludes the use of closed, let alone of compact sets. In the indirect approach, a compactness condition is used to show that the fair merge of two fair processes yields a fair process.

BW186/83. E.A. van Doorn, *Connectivity of circulant digraphs.*
AMS 05C40, 14pp., KEY WORDS: circulant (digraph), connectivity.
ABSTRACT: An explicit expression is derived for the connectivity of circulant digraphs.

BW187/83, P.S. Krishnaprasad, S.I. Marcus & M. Hazewinkel, *Current algebras and the identification problem.*
AMS 93E11, 46pp., KEY WORDS: identification of linear systems; nonlinear filtering; estimation algebra; current algebra.
ABSTRACT: In this paper, we investigate the identification problem of linear system theory from the point of view of nonlinear filtering. Following the work of Brockett and Mitter, one associates with the problem in a natural way a certain (infinite dimensional) Lie algebra of differential operators known as the estimation algebra of the problem. For the identification problem the estimation algebra is a subalgebra of a current algebra. In this paper we study questions of representations and integrability of current algebras as they impinge upon the identification problem. A Wei-Norman type procedure for the associated Cauchy problem is developed which reveals a sequence of functionals of the observations that play the role of joint sufficient statistics for the identification problem.

BW188/83. J.K. Lenstra A.H.G. Rinnooy Kan, *Scheduling theory since 1981: an annotated bibliography.*
AMS 90B35, 27pp., KEY WORDS: deterministic scheduling, jobs, single machine, parallel machines, open shop, flow shop, job shop, stochastic scheduling, hierarchical scheduling, algorithm, optimization, approximation, worst case analysis, probabilistic analysis, computational complexity, classification.
ABSTRACT: This is an annotated bibliography of the literature on sequencing

and scheduling problems that was published in 1981 or later. The literature prior to 1981 is represented by some books and survey papers. This bibliography will appear in *Combinatorial Optimization: Annotated Bibliographies,* edited by M. O'hEigeartaigh, J.K. Lenstra and A.H.G. Rinnooy Kan, to be published by Wiley, Chichester, in 1984.

BW189/83. G.A.P. Kindervater, J.K. Lenstra, *Parallel algorithms in combinatorial optimization: an annotated bibliography.*
AMS 90Cxx, 22pp., KEY WORDS: parallel computer, computational complexity, parallel algorithm, evaluation of expressions, recurrence relation, numerical algebra, nonlinear optimization, sorting, graph, sequencing and scheduling, maximum flow, linear programming, knapsack, traveling salesman, dynamic programming, branch-and-bound.
ABSTRACT: This is an annotated bibliography of the literature on parallel computers and algorithms that is relevant for combinatorial optimization. We briefly survey the publications on machine models, computational complexity, and numerical problems, then deal with papers on discrete computer science and graph theory in more detail, and finally discuss the research reported so far on specific problems of combinatorial optimization.

NW157/83. P.J. van der Houwen and B.P. Sommeijer, *Linear Multistep methods with minimized truncation error for periodic initial value problems.*
AMS 65L05, 11pp., KEY WORDS: periodic initial value problems, linear multistep methods, accuracy.
ABSTRACT: A common feature of most methods for numerically solving ordinary differential equations is that they consider the problem as a standard one without exploiting specific properties the solution may have. Here we consider initial value problems the solution of which is a priori known to possess an oscillatory behaviour. The methods are of linear multistep type and special attention is paid to minimization of those terms in the local truncation error which correspond to the oscillatory solution components. Numerical results obtained by these methods are reported and compared with those obtained by the corresponding conventional linear multistep methods and by the methods developed by Gautschi.

NW158/83. P.W. Hemker, *Multigrid methods for problems with a small parameter in the highest derivative.*
AMS 65N20, 16pp., KEY WORDS: multigrid methods, convection diffusion equation, singular perturbation problem, relaxation method.
ABSTRACT: Problems related to the multigrid (*MG*-) solution of elliptic *PDE's* are discussed, when the coefficients of the highest derivative contains a small parameter. For the equation in two dimensions discretizations of finite element type are used, and for the solution of the resulting systems various variants of the *MG*- method are considered. As special cases the anisotropic diffusion and the convection diffussion *PDE's* are studied. For the

anisotropic diffusion equation it is shown that Incomplete *LU* (*ILU*) relaxation is often an efficient smoother but that it may fail in particular cases. Incomplete Line *LU* (*ILLU*) relaxation is reliable and always has a small smoothing factor. For the convection-diffussion equation the use of an asymmetric restriction is investigated, in particular in combination with the streamline-upwind Petrov-Galerkin discretization. A relation is given between the choice of the artificial streamline-upwind parameter and the choice of the asymmetric restriction in the *MG*-algorithm. An *MG*-algorithm with *ILLU*-relaxation and a coarse grid correction with a (possibly asymmetric) Galerkin coarse grid discretization appears to be a suitable choice for all problems considered.

NW159/83. E.J. van Asselt, *Termination strategies for Newton iteration in full multigrid methods.*
AMS 65H10, 16pp., KEY WORDS: full multigrid methods, Newton iteration, termination strategy, Van der Pol equation.
ABSTRACT: For the solution of nonlinear problems we consider full multigrid methods, in which each nonlinear discrete system is solved by the Newton method. A fixed and an adaptive strategy for terminating the Newton process on each grid are compared. For the adaptive strategy only the residuals outside the possible boundary and interior layers are used to terminate the Newton process, and the number of Newton iterations is much smaller than for the fixed strategy. Other advantages for the adaptive strategy are that no arbitrary termination criterion has to be selected in advance, and boundary and interior layers are detected automatically. Three numerical examples are given. These concern two $1-D$ singular perturbed nonlinear elliptic equations, and the Van der Pol equation, discretized with the Osher-Engquist difference scheme.

NW160/83. E.J. van Asselt, *On M-functions and nonlinear relaxation methods.*
AMS 65H10, 8pp., KEY WORDS: nonlinear relation methods, Newton-bisection method, *M*-functions.
ABSTRACT: Globally convergent nonlinear relaxation methods are considered for a class of BVPs, where the discretizations are continuous *M*-functions. It is shown that the equations with one variable occurring in the nonlinear relaxation methods can always be solved by Newton's method combined with the bisection method. The nonlinear relaxation methods are used to get an initial approximation in the domain of attraction of Newton's method. Numerical examples are given.

NW161/83. J.G. Verwer & K. Dekker, *Step-by-step stability in the numerical solutions of partial differential equations.*
AMS 65M10, 25pp., KEY WORDS: initial value problems, partial differential equations, stiff ordinary differential equations, nonlinear numerical stability, energy method, shallow water equations.

ABSTRACT: The subject of this paper is numerical stability in the time-integration of evolutionary problems in partial differential equations, primarily nonlinear problems. Following the method of lines approach, and supported by the strong developments which have taken place in the field of nonlinear stiff ordinary differential equations, the authors examine various useful numerical stability concepts for nonlinear partial differential equations, such as contractivity, monotonicity and conservation. The paper is of an expository nature. Its main objective is to illustrate the close connections between stiff problems and partial differential equations with respect to nonlinear stability. The well-known energy method plays an important role in this respect. Several examples of partial differential equations are treated so as to illustrate these connections. Now and then the authors embark upon applications which result from the nonlinear stability analysis, mainly in the two sections which deal with the well-known shallow water equations. For these equations a rigorous nonlinear stability analysis is presented.

SW96/83. A.W. Ambergen & W. Schaafsma, *Interval estimates for posterior probabilities in a multivariate normal classification model.*
AMS 62H30, 9pp., KEY WORDS: estimating posterior probabilities, classification, discriminant analysis, multivariate normal distributions.
ABSTRACT: This paper is devoted to the asymptotic distribution of estimators for the posterior probability that a p-dimensional observation vector originates from one of k normal distributions with identical covariance matrices. The estimators are based on training samples from the k distributions involved. Observation vector and prior probabilities are regarded as given constants. The validity of various estimators and approximate confidence intervals is investigated by simulation experiments.

SD115/83. B.F. Schriever, *Attitude Research on public transport in Rotterdam and surroundings: a statistical analysis of a survey on hypothetical journeys.* (in Dutch)
KEY WORDS: analysis of variance, general linear model, asymptotically efficient estimators.
ABSTRACT: In this report travelers' judgements about hypothetical journeys by public transport are analysed with an analysis of variance model. In the model a dependence structure is assumed between the different judgements. The model parameters are asymptotically efficiently estimated.

TW242/83. O. Diekmann, H.J.A.M. Heijmans & H.R. Thieme, *On the stability of the cell size distribution.*
AMS 92A15, 32pp., KEY WORDS: size-dependent growth, reproduction by fission, balance equation, first order partial differential equation, transformed arguments, stable size distribution.
ABSTRACT: A model for the growth of a size-structured cell population reproducing by fission into two identical daughters is formulated and analysed.

The model takes the form of a linear first order partial differential equation (balance law) in which one term has a transformed argument. Using semi-group theory and compactness arguments we establish the existence of a stable size distribution under a certain condition on the growth rate of the individuals. An example shows that one cannot dispense with this condition.

TW243/83. H.A. Lauwerier, *Chaos and order.*
AMS 34C35, 38pp., KEY WORDS: chaotic behaviour, dynamical systems, strange attractors, Feigenbaum's constant, *KAM* theorem.
ABSTRACT: The concept of 'chaos' considered here is the apparent stochastic behaviour of deterministic dynamical systems, such as for example a system of ordinary differential equations or an iterative planar map. During the last ten years there has been explosive progress in this field. Partly through the use of computer experiments, important results have been obtained by both mathematicians and physicists. An example is Feigenbaum's period doubling and the so-called *KAM* theorem. With examples from biology and experimental physics we give in an illustrative way an impression of the fascinating and complicated aspects of this field. Particular examples are turbulent flows of a fluid, the Lorenz attractor, celestial mechanics, the *KAM* theorem, self-similarity of strange attractors, the Hénon attractor, the map $x_{n+1}=ax_n(1-x_n)$, $0<a\leqslant 4$, Feigenbaum's constant, the Julia theory, chaotic behaviour of analytic functions, and finally some variations on a biological theme.

TW244/83. J.J.E. van der Meer, *Clines induced by a geographical barrier.*
AMS 35Bxx, 59pp., KEY WORDS: nonlinear diffusion equations, transmission condition.
ABSTRACT: The consequences of a geographical barrier in the habitat are studied in the context of a one-dimensional reaction-diffusion model. By the symmetry of the problem, each steady state solution generates three more - not necessarily different - solutions. It is proved that only monotone steady state solutions can be stable. We consider a special type of steady state solutions which occur in pairs of two by their symmetry. Necessary and sufficient conditions for these steady states to be stable are derived. A cline is a nonconstant stable steady state solution. It is proved that two is the maximum number of clines of the special type. Moreover, for large values of the parameter, i.e., for small penetrability of the barrier, it is proved that only steady state solutions of the special type can be stable. Finally, it is shown that the ω-limit set of any initial condition is a steady state solution.

TW245/83. H.A. Lauwerier, *Bifurcation of a map at resonance* 1:4
AMS 39Axx 48pp.,
ABSTRACT: The maps considered here find their origin in a discrete model of population dynamics of the logistic type and containing a delay term. The main theme is to obtain full understanding of the various computer plots in

the case that the multipliers of the equilibrium state are close to $\pm i$. It is shown how the theory of normal forms can be used here as an effective tool. For a discussion of the possible limit cycles the mapping is compared to a corresponding flow. A few new results have been obtained giving additional information for this hitherto incompletely known case.

TW246/83. H.A. Lauwerier, *Local bifurcation of a logistic delay map.*
AMS 58F14, 30pp.
ABSTRACT: The difference equation $x_{n+1}=ax_n(1-(1-b)x_n-bx_{n-1})$ is considered as an iterative Cremona transformation in the projective plane. Only local bifurcation phenomena are considered here. It is shown that the theory of normal forms can be used to explain or predict what can be seen on a personal computer with a visual display. In a set of Appendices the technique of the normal forms is given with an application to a quadratic map of the kind

$$x'=y, \quad y'=Ax+By+Cx^2+Dxy+Ey^2.$$

Explicit formulas are given for the shape and the size of the Hopf ellipse and for the axis of the Arnold forms at weak resonance points.

ZW197/83. Andries E. Brouwer and Arjeh M. Cohen, *Local recognition of Tits' geometries of classical type.*
AMS 51A05, 12pp., KEY WORDS: building, geometry of Lie type.
ABSTRACT: A method, based on Tits' work and involving an idea of M. Ronan, is developed for recognizing geometries which are locally buildings of classical type as quotients of buildings. Two applications are treated in detail showing that every finite nearly classical near polygon must be a dual polar space and that in the finite case of Cooperstein's theorem characterizing geometries of Lie type D_n the hypotheses can be weakened considerably.

ZW198/83. A.E. Brouwer & A.M. Cohen, *Computation of some parameters of Lie geometries.*
AMS 51B25, 21pp., KEY WORDS: Lie geometries, association schemes.
ABSTRACT: In this note we show how one may efficiently compute the parameters of a finite Lie geometry and we give the results of such computations in the most interesting cases. We also prove a lemma which is useful for showing that thick finite buildings do not have quotients which are (locally) Tits geometries of spherical type.

ZW199/83. T.H. Koornwinder & J.J. Lodder, *Generalised functions as linear functionals on generalized functions.*
AMS 46F05, 13pp., KEY WORDS: symmetrical theory of generalised functions, tempered distributions, products of generalised functions.
ABSTRACT: We give a sketch of a rigorous foundation of the model for a symmetrical theory of generalised functions introduced earlier by the second author. Starting with a suitable subspace PC of the space S' of tempered

distributions, we introduce a space SGF of 'new' generalised functions as a space of linear functionals on PC. Both on PC and SGF we have all the usual operations including a product. On PC this product operation is somewhat arbitrary but on SGF it is canonical and much nicer. Finally, PC and SGF are put together into a space GF of linear functionals on SGF.

ZW200/83. J. de Vries, *G-spaces: compactifications and pseudocompactness.*
AMS 54H15, 11pp., KEY WORDS: G-space, G-compactification, equivariant embedding, pseudocompactness.
ABSTRACT: This paper consists of two parts. In the first part, some of the existing theory on 'equivariant topology' is reviewed. It contains almost no new facts, but the material is used to explain the author's point of view. In the second part, some new results are proved. For example, if G is a locally compact topological group, then the concept of G-pseudocompactness for Tychonov G-spaces, as introduced by the author in an earlier paper, turns out to coincide with ordinary pseudocompactness. Also the relationship with G-pseudocompactness as introduced by Antonyan, namely, the equality of the maximal G-compactification and the Stone-Cech compactification, is investigated.

ZN105/83. A.E. Brouwer, *A note on the uniqueness of the Johnson Scheme.*
AMS 05C25, 6pp., KEY WORDS: Johnson scheme, Tetrahedral graph.
ABSTRACT: Given a graph with $\binom{n}{m}$ vertices, valency $m(n-m)$ such that each edge is in $n-2$ triangles and any two nonadjacent vertices have at most 4 common neighbours, Dowling proved that it is isomorphic with the graph of m-subsets of an n-set with Johnson distance 1 provided that $n>2m(m-1)+4$. Here we improve this bound sufficiently to obtain uniqueness in the desired case $m=4$, $n=24$. (Our lower bound for n is $n \geqslant \max(6m-1, m^2+2m-1)$.)

CWI Activities

Autumn 1983

With each activity we mention its frequency and (between parentheses) a contact person at CWI. Sometimes some additional information is supplied, such as the location if the activity will not take place at CWI.

- 'Mathematics and Computer Science' Symposium. On the occasion of the recent change of names from 'Mathematical Centre' to 'Centrum voor Wiskunde en Informatica' (= Centre for Mathematics and Computer Science). 25 Nov. 1983. Invited speakers:
  A.J. Baddeley (University of Bath, England)
  Stochastic geometry and image analysis.
  D.S. Scott (Carnegie Mellon University, Pittsburgh, U.S.A.)
  Infinite words.
  C.B. Jones (University of Manchester, England)
  Systematic program development.
  J.T. Schwartz (Courant Institute, New York, U.S.A.)
  Dextrous multifinger manipulation.
  L. Lovász (Eötvös University, Budapest, Hungary)
  Combinatorial algorithms.
- Introductory colloquium for teachers. Weekly. (J. de Vries)
- Study group on Analysis on Lie groups. Joint with University of Leiden. Biweekly. (T.H. Koornwinder)
- Seminar on Theta functions. Biweekly. (G.F. Helminck)
- Lecture series 'Heisenberg group and Weil representation'. Biweekly. (G.F. Helminck)
- Seminar on Algebra and Geometry. Biweekly. (A.M. Cohen).
- Study group on Cryptography. Biweekly. (A.E. Brouwer)
- Colloquium 'STZ' on System Theory, Applied and Pure Mathematics. Twice a month. (J. de Vries)
- Study group 'Biomathematics'. Joint with University of Leiden. (J. Grasman)
- Study group 'Nonlinear analysis'. Joint with University of Leiden. (S.A. van Gils)
- Progress meetings of the Applied Mathematics Department. Biweekly. (S.A. van Gils)
- Study group 'Semiparametric estimation theory'. Biweekly. (R.D. Gill)
- Study group 'Stochastic processes and their applications'. Joint with Technological University Delft. Biweekly. (P. Groeneboom)
- Lunteren meeting on Stochastics. 14,15,16 Nov. 1983 at 'De Blije Werelt', Lunteren. The following lecturers have been invited:
  A.J. Baddely (Bath), D.M. Mason (Newark), S. Johansen (Copenhagen), B. Ripley (Glasgow), F. Papangelou (Manchester), H.

Rost (Heidelberg). (R. Helmers)
- Progress meetings of the Mathematical Statistics Department. Monthly. (R.D. Gill)
- Ninth Conference on the Mathematics of Operations Research and Systems Theory. 11,12,13 Jan. 1984 at Lunteren. Part of this conference is the *Benelux Meeting on Systems and Control* 1984. Invited lecturers are R.E. Bixby (Evanston/Bonn), G.L. Nemhauser (Ithaca/Leuven), M.F. Neuts (Newark/Stuttgart), P.J. Schweitzer (Rochester) on Operation Research and K.J. Aström (Lund), A. Benveniste (Rennes) on System Theory. (E.A. van Doorn)
- National colloquium on Optimization. Twice a year. (J.K. Lenstra)
- System Theory Days. Irregular. (J.H. van Schuppen)
- Study group on System Theory. Biweekly. (J.H. van Schuppen)
- Colloquium on Parallel Computers and Computations. Joint with University of Utrecht. Biweekly, at Utrecht. (J.K. Lenstra)
- Colloquium 'Numerical Mathematics in Practice'. Biweekly. (J.G. Verwer)
- Study group on Differential and Integral Equations. Biweekly. (H.J.J. te Riele)
- Conference on Numerical Mathematics. 26,27,28 Sept. 1983 at Zeist.
  Invited speakers: K. Böhmer (Universität Marburg): Defect correction and/or a posteriori error estimates;
  I.S. Duff (AERE Harwell): (1) Basic aspects of numerical software; (2) Organization of numerical software libraries;
  T. Dupont (University of Chicago): A posteriori error estimation for evolution equations with time-dependent meshes;
  H.J. Stetter (Technische Universität Wien): (1) The role of defect correction in interval arithmetic, a general approach; (2) Details of algorithms;
  M.J. Kascic (Control Data Minnesota): (1) An introduction to vector processing with application to numerical methods; (2) Vorton dynamics: a case study of developing a fluid dynamics model for a vector processor.
  (J.G. Verwer)
- Colloquium 'From specification to implementation'. Biweekly. (J. Heering)
- Course on B. (L.J.M. Geurts)
- Study group on Graphic Standards. Monthly. (P.J.W. ten Hagen)
- Study group on Semantics of Programming. Triweekly. (J.W. Klop)
- Data Flow Club. Irregular. (A.H. Veen)
- Seminar on 'Adaptive Estimation', joint with University of Leiden, 7-9 Dec. 1983; lecturer P. Bickel from Berkeley. (R. Helmers)

Visitors to CWI from abroad.

R.L. Baber (Bad Homburg, West Germany) 18 November 1983. **A. Baddeley** (Bath, UK) 24-25 November 1983. **E. Badertscher** (University of Bern, Switzerland) 1983-1984. **Y. Benoist** (University of Paris VII, France) 4 November 1983. **E. Best** (GMD, Bonn, West Germany) 1-31 October 1983. **P.J. Bickel** (University of California, Berkeley, USA) 7-9 December 1983. **S.D. Brookes** (Carnegie Mellon University, Pittsburgh, USA) 13-16 July 1983. **H. Brunner** (University of Fribourg, Switzerland) 5 days in December 1983. **R. Byrne** (Brighton Polytechnics, UK) 20 June - 15 July 1983. **K.S. Chaudhuri** (Jadaupur University, Calcutta, India) 12 November 1983. **F. Cirello** (Brighton Polytechnics, UK) 20 June - 15 July 1983. **J. Darlington** (Imperial College, London, UK) 3-4 November 1983. **D.J. Evans** (Loughborough University of Technology, UK) 2 days in October 1983. **B.L. Fox** (University of Montreal, Canada) 8-9 August 1983. **S. Glasner** (Tel Aviv University, Israel) 4 months in 1983/1984. **Glowinski** (INRIA, Le Chesney, France) in 1983/1984. **R.L. Griess** Jr. (University of Michigan, Ann Arbor, USA) 4-6 August 1983. **F. Götze** (Cologne, West Germany) 19 October 1983. **M. Gyllenberg** (Helsinki, Finland) 11-12 July 1983. **J.I. Hall** (East Lansing, Michigan, USA) 27-28 October 1983. **J. Han** (Academica Sinica, Peking, Rep. of China) 23 October 1983 - 22 January 1984. **S. Hubbell** (University of Iowa, USA) 1 November 1983. **M. Ianelli** (Trento, Italy) 2 or 3 days in November 1983. **P. Janssen** (Diepenbeek, Belgium) 28-29 July 1983. **E.L. Lawler** (University of California, Berkeley, USA) in August 1983. **A. Lerat** (Ecole Nat. Superieure d'Arts et Métiers, Paris, France) 3 days in January/February 1984. **C.L. Liu** (University of Illinois, USA) 2 weeks in March 1984. **L. Lovász** (Eötvös University, Budapest, Hungary) 1 week in November 1983. **M.C. Mackay** (McGill University, Montreal, Canada) 3 days in September 1983. **F. Maffioli** (Politecnico di Milano, Italy) 8-26 August 1983. **D.M. Mason** (University of Delaware, USA) 21-22 November 1983. **G. Mason** (University of California, Santa Cruz, USA) 4-7 July 1984. **H. Matano** (Hiroshima University, Japan) 11-12 July 1983. **B. Mélèse** (INRIA, Le Chesnay, France) 21-23 November 1983. **S.K. Mitter** (MIT, Cambridge, Massachusets, USA) 15-31 August 1983. **Th. Neusmann** (Darmstadt, West Germany) 8 July 1983. **R. Nisbet** (Glasgow University, UK) 1 week in autumn 1983. **T.J. Ott** (Bell Lab., Holmdel, USA) 5 July 1983. **J.L. Palacios** (Universidad Simon Bolivar, Caracas, Venezuela) 3 August 1983. **H.O. Peitgen** (University of Bremen, West Germany) 17 October 1983. **G. Picci** (University of Padua, Italy) in August 1983. **A.J. Pritchard** (University of Warwick, Coventry, UK) 6 September 1983. **J.M. Sanz-Serna** (Univ. de Vallodolid, Spain) 1-6 November 1983. **G. Schiffner** (University of Bremen, West Germany) 3 November 1983. **M. Schumacher** (Dortmund, West Germany) 28 November - 2 December 1983. **Mrs. J. Scott** (Oxford University Computing Lab., UK) 5 days in December 1983. **F. Timmesfeld** (Justus-

Liebig-Universität, Giessen, West Germany) 31 October - 4 November 1983. **J. Tiuryn** (Warssaw, Poland) 16 July - 31 August 1983. **J.V. Tucker** (Leeds University, UK) 15 July - 31 August 1983. **Zhen-Dong Yuan** (East China Normal University, Shanghai, Rep. China) 2 September 1983. **J.I. Zucker** (SUNY at Buffalo, USA) 15 July - 31 August 1983.