Corrigendum


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A R T I C L E   I N F O

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In section 4 of the paper mentioned in the title it is assumed that, for all \( x \in \{0,1\}^* \), the string \( x^* \) is the only incompressible string such that \( U(x^*) = x \). However, this assumption is wrong in that for many \( x \) there may be an incompressible string \( p \) with \( |x| > |p| > |x^*| \) such that \( U(p) = x \). Moreover, the computation of \( U(p) = x \) may be faster than that of \( U(x^*) = x \). For example, the function from \( x \in \{0,1\}^* \) to the least number of steps in a computation \( U(p) = x \) for an incompressible string \( p \) may be computable. The argument in the paper is correct if the function from \( n \) to the least number of steps in the computation of \( U(p) = x \) with \( |x| = n \) for an incompressible string \( p \) is shown to be incomputable. This is the case as shown below.

The length of a shortest program is \( |x^*| = K(x) \). Define the set of incompressible strings by \( P = \{ p : |p| = K(p) \} \). Let \( \mathcal{N} \) denote the natural numbers. Replace Lemma 1 in the paper under consideration by

**Lemma 1.** Let \( \phi \) be defined by \( \phi(n) = \max_{|x|=n} \min_{d \in \mathcal{N}, p \in P} |d : U^d(p) = x |. \) Then \( \phi \) is not computable and grows faster than any computable function.

**Proof.** For every \( n \) and string \( x \) of length \( n \) there is an incompressible string \( s \) with \( |s| \leq n \) such that \( U^d(s) = x \) with \( d \) least. Namely, since \( U(x^*) = x \) for all \( x \) there is such a \( d \) for all \( x \).

Towards a contradiction: assume that the function \( \phi \) is computable and for each length \( n \) we can compute the number of steps \( d_n = \min_{d \in \mathcal{N}, |x|=n} \min_{p \in P} |d : U^d(p) = x |. \)

Define for each string \( x \) of length \( n \) the set \( P_x = \{ p : |p| \leq n, U^{d_x}(p) = x \}. \) There are by assumption incompressible strings in \( P_n \), denoted by \( s_x \) (\( K(s_x) = |s_x| \)). We can describe such incompressible \( s_x \) using \( P_x \) by \( \log |P_x| + K(x) \geq K(s_x) \). Therefore \( |P_x| \geq 2^{K(s_x) - K(x)} \). For all \( n \) and \( x \) of length \( n \) we have \( U(p) = x \) for all \( p \in P_x \), and therefore \( P_x \cap P_y = \emptyset \) for \( x, y \) are strings of length \( n \) and \( x \neq y \). Therefore,

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\[
\sum_{x:|x|\leq n} |P_x| \geq \sum_{x:|x|\leq n} 2^{K(s_x) - K(x)} \geq \sum_{p:|p|\leq n} 2^{|p|} = 2^{n+1} - 1.
\]

If all \(s_x\) above satisfy \(K(s_x) = K(x)\) then we have equality throughout in the above inequality. However, \(K(s_x|x) \geq \log |x| \) \([1,2]\) and hence the time function to compute \(s_x\) from \(x\) is incomputable contrary to the contradictory assumption. Suppose that some \(s_x\) satisfies \(K(s_x) > K(x)\), that is \(|s_x| > |x^*|\), which means that there are more strings of length at most \(n\) which are programs for some \(x\) of length \(n\) than there are strings of length at most \(n\). This is impossible. Therefore the contradictory assumption is false in this case as well and the function \(\phi\) can not be computable. For every \(n\) the argument holds for every number of steps \(D(n) \geq d_n\) where \(D\) is a computable function. Hence any function majorizing \(\phi\) can not be computable. Therefore \(\phi\) grows faster than any computable function. \(\square\)

As a further correction: In the first line of the proof of Theorem 2 the string \(x_n\) is more properly denoted by \(x\); the role of \(x^*_n\) is served by the string \(p\) defining \(\phi(n)\) in Lemma 1; and the remaining changes are self-evident.

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References