A Conceptual Model for Inexact Reasoning in Rule-Based Systems

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ABSTRACT

Most expert knowledge is ill-defined and heuristic. Therefore, many present-day rule-based expert systems include a mechanism for modeling and manipulating imprecise knowledge. For a long time, probability theory has been the primary quantitative approach for handling uncertainty. Other (mathematical) models of uncertainty have been proposed during the last decade, several of which depart from probability theory. In this paper, so-called inference networks are introduced to demonstrate the application of such a model for inexact reasoning in a rule-based top-down reasoning expert system. This approach enables the formulation of a conceptual model for inexact reasoning in rule-based systems. This conceptual model is used to show some inadequacies in the certainty factor model, a model that has been proposed by the authors of the MYCIN system and that has actually been applied in expert systems. A syntactically correct reformulation of the certainty factor model is proposed, and this new formalism is used to discuss some of the model's properties.

KEYWORDS: ***expert systems, inexact reasoning, inference networks, certainty factor model***

1. INTRODUCTION

When building expert systems one finds that in many real-life domains expert knowledge is not precisely defined but of an imprecise nature. To be useful in an environment in which such imprecise knowledge has to be employed, an expert system has to capture the uncertainties that go with the represented pieces of

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knowledge. In the recent past, researchers in artificial intelligence have sought methods for representing uncertainties and have developed reasoning procedures for manipulating uncertain knowledge. The terms *plausible* and *inexact reasoning* are often used in relation to this field of research.

For a long time, Bayesian probability theory has been the only quantitative approach to modeling and handling uncertainty. Bayesian probability theory, however, cannot be applied in a straightforward manner in rule-based expert systems. Several other mathematical models of uncertainty have been proposed, such as Shafer's belief theory [1] and Zadeh's possibility theory [2, 3]. Quite a different approach that has been recently presented is Cohen's theory of endorsement [4]. Unfortunately, most of these mathematical models are computationally demanding. Many researchers have therefore proposed and used empirical models. An example of such an ad hoc model is the certainty factor model developed by E. H. Shortliffe and B. G. Buchanan, the authors of the MYCIN system [5]. Lee et al. present an introduction to inexact reasoning and discuss most of the models mentioned above [6].

In this paper, we introduce a conceptual model for plausible reasoning that can be used to investigate the suitability of an actual model for application in a rule-based expert system. The conceptual model can also be used as a framework for comparing different actual models. Here, we use this conceptual model to examine the certainty factor model; this latter model is our example throughout the paper. Since its introduction in the 1970s, it has enjoyed widespread use in rule-based top-down reasoning expert systems, such as MYCIN and similar systems. Part of the success of the certainty factor model can be accounted for by its computational simplicity. Although the certainty factor model is frequently employed in practical situations, it has been subject to severe criticism from theoreticians. In this paper we do not address the theoretical aspects of the model in detail; the theoretical foundation of the certainty factor model is discussed elsewhere (Van der Gaag [7]). Here, we use so-called inference networks to demonstrate its application in a rule-based expert system using top-down inference as a reasoning technique. This approach enables us to show some syntactical inadequacies in the notation by Shortliffe and Buchanan. From these observations we arrive at a syntactically correct formalism without affecting the intended meaning of the model.

2. BASIC NOTIONS

Shortliffe and Buchanan have developed an empirical method for modeling and handling uncertainty in MYCIN, a rule-based expert system using top-down inference as a reasoning technique; their method is called the *certainty factor model.* A slightly modified version of this model has been implemented in the expert system shell EMYCIN as has been used in many similar expert system shells. We will discuss only this latter version.

Although we assume that the reader is acquainted with production rules and top-down inference, a short description of these notions is provided in order to introduce some terminology. In a rule-based top-down reasoning expert system applying the certainty factor model for the manipulation of uncertainty, there are three major components:

1. *Production rules and associated certainty factors.* Basically, an expert in the domain in which the expert system is to be used models his or her knowledge of the field in a set of production rules of the form $e \rightarrow h$. The left-hand side e of a production rule is a positive Boolean combination of conditions; that is, e does not contain any negations. Without loss of generality we assume that e is a conjunction of disjunctions of conditions. Throughout this paper, e as well as its constituting parts will be called *(pieces of) evidence.* In general, the right-hand side h of a production rule is a conjunction of conclusions. In this paper, we restrict ourselves to single-conclusion production rules; note that this restriction is not an essential one. Henceforth, a conclusion will be called a *hypothesis.*

   An expert associates with the hypothesis h in a production rule $e \rightarrow h$ a (real) number $\mathrm{CF}(h, e, e \rightarrow h)$, quantifying the degree to which the observation of evidence e confirms the hypothesis h. The values $\mathrm{CF}(x, y, z)$ of the (partial) function CF are called *certainty factors;* $\mathrm{CF}(x, y, z)$ should be read as "the certainty factor of x, given y and the derivation z of x from y." From here on we use the more suggestive notation $\mathrm{CF}(x \dashv y, z)$. Certainty factors range from -1 to $+1$. A positive certainty factor is associated with a hypothesis h given some evidence e if the hypothesis is confirmed to some degree by the observation of this evidence; the certainty factor $+1$ indicates that the occurrence of evidence e completely proves the hypothesis h. A negative certainty factor is suggested if the observation of evidence e disconfirms the hypothesis h. A certainty factor equal to zero is suggested by the expert if the observation of evidence e does not influence the confidence in the hypothesis h.

   Shortliffe and Buchanan use the two-argument notation $\mathrm{CF}(h, e)$; as will be discussed shortly, it is necessary to introduce the derivation of h from e in the notational convention.

2. *User-supplied data and associated certainty factors.* During a consultation of the expert system, the user is asked to supply actual case data. The user attaches a certainty factor $\mathrm{CF}(e \dashv u, u \rightarrow e)$ to every piece of evidence e he supplies the system with. In order to be able to treat production rules and user-supplied data uniformly, we assume that the set of production rules supplied by the expert is augmented with a set of fictitious production rules $u \rightarrow e$, where u represents the user's de facto knowledge and e a piece of user-supplied evidence.

3. *A (top-down) inference engine and a (bottom-up) scheme for propagating uncertainty.* Top-down inference is a goal-directed reasoning technique in which the production rules are applied exhaustively to prove one or more goal hypotheses. A production rule is said to *succeed* if each of its conditions is fulfilled; otherwise, the rule is said to *fail*. Due to the application of production rules, during the inference process several intermediary hypotheses are confirmed to some degree. The certainty factor to be associated with an intermediary hypothesis h is calculated from the certainty factors associated with the production rules that were used in deriving h. For the purpose of thus propagating uncertainty, several functions for combining certainty factors are defined. The remainder of this paper provides a thorough treatment of the propagation of uncertainty prescribed by the model.

3. RULE-BASED DERIVATIONS AND DERIVATION TREES

In the foregoing section the basic notions of the certainty factor model have been discussed in an informal manner. In this section some formal definitions are provided.

DEFINITION 3.1. *Let $\mathcal{A}$ denote a set of atomic propositions. Let $\mathcal{E}$ denote the set of conjunctions of disjunctions of elements of $\mathcal{A}$; that is, $\mathcal{E}$ contains elements of the form*

$$\bigwedge_{i=1}^{n}\left(\bigvee_{j=1}^{m_i} a_{ij}\right), \quad a_{ij} \in \mathcal{A}, \qquad n, m_i \geqslant 1.$$

A hypothesis is an element $h \in \mathcal{A}$. A piece of evidence is an element $e \in \mathcal{E}$. Let u be a fixed element of $\mathcal{A}$ representing the user's de facto knowledge. A production rule is an expression $e \rightarrow h$, where e is a piece of evidence and h is a hypothesis. Let $\mathcal{P}$ denote a fixed, finite set of production rules.

In Section 2 the notion of a derivation with respect to a set of production rules was introduced. This notion is now defined.

DEFINITION 3.2. *Let $\mathcal{P}$ be defined as above. A derivation $D^{i,j}$ of j from i with respect to $\mathcal{P}$ is defined recursively as follows.*

1. *$e \rightarrow h$ is a derivation of h from e with respect to $\mathcal{P}$ if $e \rightarrow h \in \mathcal{P}$.*
2. *If $D^{u,e}$ is a derivation of e from u with respect to $\mathcal{P}$ and $D^{e,h}$ is a derivation of h from e with respect to $\mathcal{P}$, then $((D^{u,e}) \circ (D^{e,h}))$ is a derivation of h from u with respect to $\mathcal{P}$. $((D^{u,e}) \circ (D^{e,h}))$ is called the (sequential) composition of the derivations $D^{u,e}$ and $D^{e,h}$.*
3. *If D^{u,e_1} is a derivation of e_1 from u with respect to $\mathcal{P}$ and D^{u,e_2} is a*

derivation of e_2 from u with respect to $\mathcal{P}$, then $((D^{u,e_1}) \& (D^{u,e_2}))$ is a derivation of $(e_1 \wedge e_2)$ from u with respect to $\mathcal{P}$. $((D^{u,e_1}) \& (D^{u,e_2}))$ is called the conjunction of the derivations D^{u,e_1} and D^{u,e_2}.

4. *If D^{u,e_1} is a derivation of e_1 from 8 with respect to $\mathcal{P}$ and D^{u,e_2} is a derivation of e_2 from u with respect to $\mathcal{P}$, then $((D^{u,e_i}))|(D^{u,e_2}))$ is a derivation of $(e_1 \vee e_2)$ from u with respect to $\mathcal{P}$. $((D^{u,e_1})|(D^{u,e2}))$ is called the disjunction of the derivations D^{u,e_1} and D^{u,e_2}.*
5. *If $D_1^{u,h}$ and $D_2^{u,h}$ are derivations of h from u with respect to $\mathcal{P}$, then $((D_1^{u,h})\|(D_2^{u,h}))$ is a derivation of h from u with respect to $\mathcal{P}$. $((D_1^{u,h})|(D_2^{u,h}))$ is called the parallel composition of the derivations $D_1^{u,h}$ and $D_2^{u,h}$.*

The set of all derivations with respect to $\mathcal{P}$ is denoted by $\mathfrak{D}$.

In what follows, we will omit parentheses from elements of $\mathcal{E}$ and $\mathfrak{D}$ as long as ambiguity cannot occur.

EXAMPLE 3.1. Let $\mathcal{P}$ be the set consisting of the following production rules:

$$d \wedge f \rightarrow b$$

$$a \rightarrow d$$

$$b \rightarrow i$$

$$u \rightarrow a$$

$$u \rightarrow b$$

$$u \rightarrow f$$

Then $D^{u,d} = (u \rightarrow a) \circ (a \rightarrow d)$ is a derivation of d from u, and

$$D^{u,i} = (((u \rightarrow b)\|((((u \rightarrow a) \circ (a \rightarrow d)) \& (u \rightarrow f)) \circ (d \wedge f \rightarrow b))) \circ (b \rightarrow i))$$

is a derivation of i from u.

We conclude this section by presenting a graphical representation of derivations. A graphical representation of a derivation is called a *derivation tree.* As the notion of derivation tree is rather straightforward, we will confine ourselves to loosely introducing the building blocks for derivation trees. The derivation tree corresponding to a derivation D is built beginning at the right end and using these basic representations. Let $\rho(D)$ denote the graphical representation of the derivation D. Then

1. For the representation of a production rule $u \rightarrow h$,

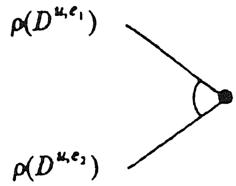
$$\rho(u \rightarrow h) = u \rightarrow h$$

2. For the representation of the composition of two derivations,

$$\rho((D^{u,e}) \circ (e \rightarrow h)) = \rho(D^{u,e}) \rightarrow h$$

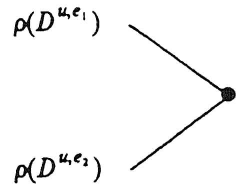
3. For the representation of the conjunction of two derivations,

$$\rho((D^{u,e_1})\ \&\ (D^{u,e_2})) =$$



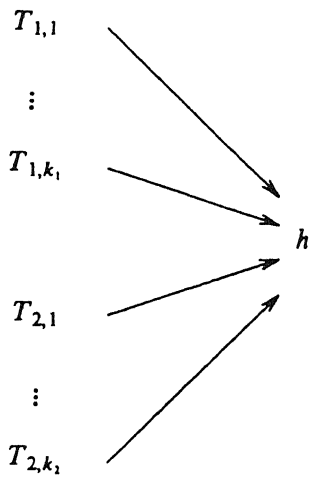
4. For the representation of the disjunction of two derivations,

$$\rho((D^{u,e_1})\ |\ (D^{u,e_2})) =$$



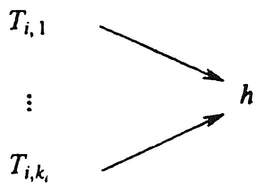
5. For the representation of the parallel composition of two derivations of the same hypothesis h,

$$\rho((D_1^{u,h})\ \|\ (D_2^{u,h})) =$$



where

$$\rho(D_i^{u,h}) =$$



So we simply join the two derivation trees of h.

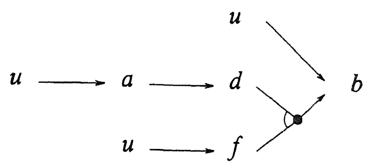


Figure 1. A Derivation Tree.

EXAMPLE 3.2. Consider the set of production rules of Example 3.1. The derivation tree of the derivation

$$D^{u,b} = ((u \to b) \| ((((u \to a) \circ (a \to d)) \,\&\, (u \to f)) \circ (d \wedge f \to b))),$$

that is, $\rho(D^{u,b})$, is shown in Figure 1.

4. RULE-BASED INFERENCE AND INFERENCE NETWORKS

In this section we see with the help of an example that an expert system with a fixed set of production rules applying the EMYCIN top-down reasoning strategy determines a unique derivation in the set of all derivations with respect to this set of production rules. We assume that a backward-chaining strategy is used, that is, that the production rules are applied in the order in which they have been specified. Equally, the conditions in a production rule are evaluated in the specified order. Furthermore, the actual inference process is simplified by assuming that the user is asked to confirm or disconfirm to some degree each piece of evidence that cannot be derived from the production rules. It is left to the reader to verify that this simplification is not an essential one.

EXAMPLE 4.1. Consider the set of production rules consisting of the following six elements. Note that the set of rules is not yet supplemented with the fictitious production rules representing the user-supplied evidence discussed in Section 2.

$$e \to h$$
$$a \wedge (b \vee c) \to h$$
$$d \wedge f \to b$$
$$f \vee g \to h$$
$$a \to d$$
$$b \to i$$

Suppose for the moment that h is the goal hypothesis of the consultation. First the rule $e \to h$ is selected to be applied; e now becomes the next goal hypothesis to be confirmed. As there are no production rules concluding on e, the user is asked to confirm or disconfirm e. Assume that he disconfirms e; in this case the production rule $e \to h$ fails. Subsequently, the user is asked to confirm or

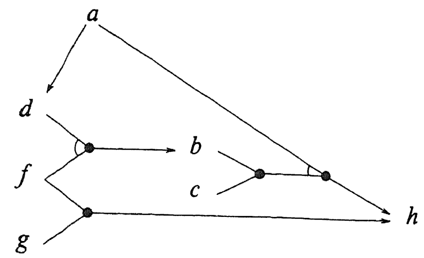


Figure 2. An Inference Network.

disconfirm a. When a is confirmed, f will be asked, etc. We assume that a, c, f, and g are confirmed by the user. Therefore, the production rules $a \wedge (b \vee c) \rightarrow h$, $d \wedge f \rightarrow b$, $f \vee g \rightarrow h$, and $a \rightarrow d$ succeed. Note that the production rule $b \rightarrow i$ is not used in the derivation of h.

A top-down inference process as discussed in the foregoing example is often depicted in a so-called *inference network*. An inference network is built from the representations of those production rules that actually succeeded during the inference process. In this paper, in depicting inference networks we use building blocks similar to the ones introduced in Section 3 for the graphical representation of derivations.

EXAMPLE 4.2. Consider the inference process described in Example 4.1. The inference network corresponding to this process is shown in Fig. 2.

An inference network is extended with the production rules $u \rightarrow e$, where e is a piece of user-supplied evidence relevant to the production rules that actually succeeded during the consultation of the system in deriving one or more of the goal hypotheses. Recall that u represents the user's de facto knowledge.

EXAMPLE 4.3. The inference network of Figure 2 is extended with the production rules $u \rightarrow a$, $u \rightarrow c$, $u \rightarrow f$, and $u \rightarrow g$. The thus extended inference network is depicted in Figure 3. Consider once more the production rule $a \wedge (b \vee c) \rightarrow h$. Up to now we have assumed that a and c were both confirmed by the user and that b was derived. The reader can easily verify that this rule also succeeds in the case that b has been derived, and the user has confirmed a and has *dis*confirmed c. In this case, the inference network is exactly the same as the one shown in Figure 2. Although in this case the user has supplied negative information on c, the network is extended in the same way.

Note that each production rule may be applied at most once during an inference process. Furthermore, the networks composed of only those production rules that actually succeeded are guaranteed to be acyclic, since the EMYCIN reasoning mechanism prevents cyclic reasoning chains. From this latter observation, we have that each extended inference network can be transformed

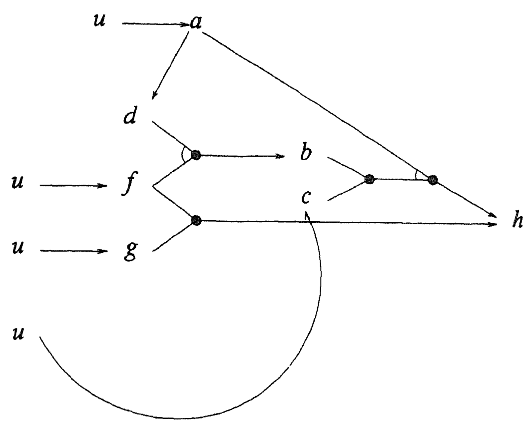


Figure 3. The Extended Inference Network.

in such a way that from each node either one arrow of type —• or one arrow of type → departs by duplicating nodes and arrows if necessary; that is, an inference network is transformed into a tree.

EXAMPLE 4.4. Figure 4 shows the inference network resulting from the transformation of the inference network depicted in Figure 3. Notice the duplication of nodes a and f.

Such a transformed inference network equals exactly one derivation tree, corresponding to a unique element of the set of all derivations with respect to the set of production rules.

5. MODELING THE PROPAGATION OF UNCERTAINTY

In the foregoing sections it has been shown that a rule-based inference process can be graphically represented as an inference network corresponding to a unique derivation tree. In this section, such treelike inference networks are used to demonstrate the propagation of uncertainty in inference processes. Henceforth, the phase "inference network" denotes a transformed inference network corresponding to a derivation tree.

Recall that an expert has attached a certainty factor $\mathrm{CF}(h \dashv e, e \rightarrow h)$ to the conclusion h of the production rule $e \rightarrow h$ and that the user has attached a certainty factor $\mathrm{CF}(e \dashv u, u \rightarrow e)$ to the conclusion e of the production rule $u \rightarrow e$, representing the fact that he has supplied the system with the actual information e. In an inference network, a certainty factor assigned to a hypothesis in a production rule is attached to the arrow in the representation of the rule. Therefore, if an expert has assigned the certainty factor

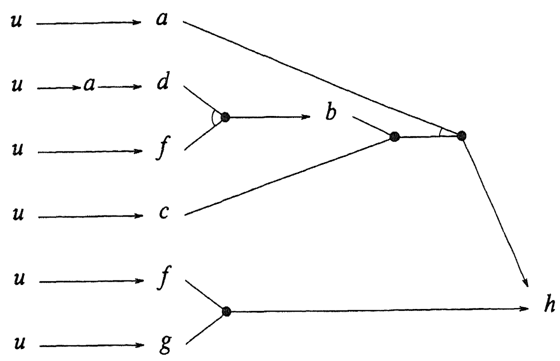
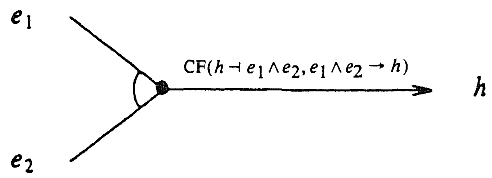


Figure 4. The Transformed Inference Network.

CF($h \dashv e_1 \wedge e_2, e_1 \wedge e_2 \rightarrow h$) to the hypothesis h in the production rule $e_1 \wedge e_2 \rightarrow h$, this is represented as shown below:



The aim of the certainty factor model is to calculate a certainty factor CF($h \dashv u, D^{u,h}$) for each goal hypothesis h, where $D^{u,h}$ is the derivation of h from u with respect to a fixed set of production rules exhaustively applied in a top-down reasoning fashion; it is obvious that such a certainty factor is dependent upon the certainty factors attached to the arrows in the inference network as well as on the structure of the inference network itself.

The way the certainty factor CF($h \dashv u, D^{u,h}$) for each goal hypothesis h is calculated from other certainty factors is discussed with the help of the inference network. We define a number of basic compression steps that are used to compress an inference network in a finite number of steps to

$$u \xrightarrow{\mathrm{CF}(h \dashv u, D^{u,h})} h$$

for each goal hypothesis h. As we will see shortly, in each compression step the number of arrows (and certainty factors) in the network is diminished. The certainty factors that disappear in a compression step are combined into a new certainty factor. For that purpose a combination function is associated with each compression step. There are four basic compression steps:

1. The inference network

$$u \xrightarrow{\mathrm{CF}(e \dashv u,\, D^{u,e})} e \xrightarrow{\mathrm{CF}(h \dashv e,\, e \rightarrow h)} h$$

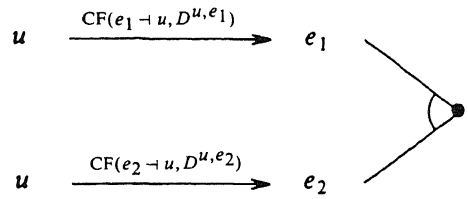
representing the composition of the derivations $D^{u,e}$ and $e \rightarrow h$, is compressed to yield

$$u \xrightarrow{\mathrm{CF}(h \dashv u,\, (D^{u,e}) \circ (e \rightarrow h))} h$$

With this compression step, a combination function $f_\circ$ is associated such that

$$\mathrm{CF}(h \dashv u,\ (D^{u,e}) \circ (e \rightarrow h)) = f_\circ(\mathrm{CF}(e \dashv u,\ D^{u,e}),\ CF(h \dashv e,\ e \rightarrow h))$$

2. The inference network



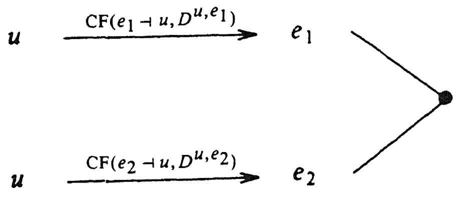
representing the conjunction of the derivations D^{u,e_1} and D^{u,e_2}, is compressed to yield

$$u \xrightarrow{\mathrm{CF}(e_1 \wedge e_2 \dashv u,\, (D^{u,e_1}) \& (D^{u,e_2}))} e_1 \wedge e_2$$

With this compression step, a combination function $f_\&$ is associated such that

$$\mathrm{CF}(e_1 \wedge e_2 \dashv u,\ (D^{u,e_1})\ \&\ (D^{u,e_2})) = \\ = f_\&(\mathrm{CF}(e_1 \dashv u,\ D^{u,e_1}),\ \mathrm{CF}(e_2 \dashv u,\ D^{u,e_2}))$$

3. The inference network



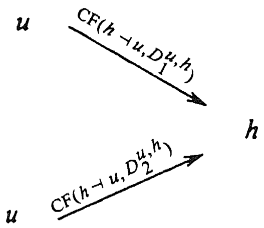
representing the disjunction of the derivations D^{u,e_1} and D^{u,e_2}, is compressed to yield

$$u \xrightarrow{\mathrm{CF}(e_1 \vee e_2 \dashv u,\, (D^{u,e_1}) | (D^{u,e_2}))} e_1 \vee e_2$$

With this compression step, a combination function $f_|$ is associated such that

$$\mathrm{CF}(e \vee e_2 \dashv u, (D^{u,e_1})|(D^{u,e_2})) =$$
$$= f_|(\mathrm{CF}(e_1 \dashv u, D^{u,e_1}), \mathrm{CF}(e_2 \dashv u, D^{u,e_2}))$$

4. The inference network



representing the parallel composition of the derivations $D_1^{u,h}$ and $D_2^{u,h}$, is compressed to yield

$$u \xrightarrow{\mathrm{CF}(h \dashv u, (D_1^{u,h})\|(D_2^{u,h}))} h$$

With this compression step, a combination function $f_\|$ is associated such that

$$\mathrm{CF}(h \vdash u, (D_1^{u,h})\|(D_2^{u,h})) = f_\|(\mathrm{CF}(h \vdash u, D_1^{u,h}), \mathrm{CF}(h \vdash u, D_2^{u,h}))$$

Since the application of each of these four compression steps reduces the number of arrows in an inference network, termination of the compression is guaranteed.

EXAMPLE 5.1. The inference network of Figure 4 can be compressed to

$$u \xrightarrow{\mathrm{CF}(h \dashv u, D^{u,h})} h$$

where

$$D^{u,h} = (((u \to a) \ \& \ (((((u \to a) \circ (a \to d)) \ \& \ (u \to f)) \circ (d \wedge f \to b))|$$
$$|(u \to c))) \circ (a \wedge (b \vee c) \to h))\|(((u \to f)|(u \to g)) \circ (f \vee g \to h))$$

6. SOME DESIRABLE PROPERTIES OF THE COMBINATION FUNCTIONS

In the foregoing section the propagation of uncertainty during an inference process has been modeled by the compression of the corresponding inference network. We have defined four basic compression steps and have introduced combination functions corresponding with these compression steps. In this section we discuss some desirable properties for each of these combination functions.

Recall that the certainty factor CF($h \dashv e$, $e \rightarrow h$) quantifies the degree to which the occurrence of evidence e confirms the hypothesis h. However, the truth of a piece of evidence e (i.e., whether e has actually occurred) may not always be determined with absolute certainty; with every piece of evidence e supplied by the user, a certainty factor is associated that is not necessarily equal to $+1$. Furthermore, in using production rules, intermediary hypotheses are confirmed to some degree and may in turn be used as evidence in other production rules concluding on new hypotheses. Basic compression step 1, describing the composition of derivations and its associated combination function $f_\circ$, deal with this situation. From now on, we will call the function $f_\circ$ the *combination function for (propagating) uncertain evidence.*

We have seen that the evidence e in the production rule $e \rightarrow h$ can be an intermediary hypothesis that has been confirmed to some degree. If the certainty factor CF($e \dashv u$, $D^{u,e}$) of the evidence e given some derivation of e and u is known, the combination function for uncertain evidence can handle this situation. As we discussed in Section 2, however, the evidence e in a rule $e \rightarrow h$ can be a conjunction of disjunctions of pieces of evidence. In order to be able to apply the combination function $f_\circ$ for uncertainty evidence, the certainty factor CF($e \dashv u$, $D^{u,e}$) of the Boolean combination e has to be computed from the certainty factors of its constituent parts. Basic compression steps 2 and 3, dealing with the conjunction and disjunction of derivations and their associated combination functions $f_\&$ and $f_|$, refer to this situation. From now on, the function $f_\&$ will be called the *combination function for conjunctions of hypotheses,* and the function $f_|$ the *combination function for disjunctions of hypotheses.* When referring to the two functions, we will call them the *combination functions for composite hypotheses.*

It will be obvious that it is desirable that the application of the combination functions for composite hypotheses render the resulting certainty factor of a conjunction of disjunctions of pieces of evidence independent of the order in which the constituent parts of each of the disjunctions and the constituent parts of each of the conjunctions are specified. For example, the production rules $e_1 \wedge e_2 \rightarrow h$ and $e_2 \wedge e_1 \rightarrow h$ should yield the same result. Furthermore, the resulting certainty factor of a positive Boolean combination of pieces of evidence has to be independent of the way in which the constituent parts of each of the disjunctions and the constituent parts of the conjunction are taken together to be combined. Therefore, the combination functions for composite hypotheses $f_\&$ and $f_|$ have to respect the property of commutativity,

$$f_\&(x, y) = f_\&(y, x) \quad \text{and} \quad f_|(x, y) = f_|(y, x)$$

for all certainty factors x and y, and the property of associativity,

$$f_\&(f_\&(x, y), z) = f_\&(x, f_\&(y, z)) \quad \text{and} \quad f_|(f_|(x, y), z) = f_|(x, f_|(y, z))$$

for all certainty factors x, y, and z.

When different successful production rules $e_i \rightarrow h$ conclude on the same hypothesis h, a certainty factor $\mathrm{CF}(h \dashv u, D_i^{u,h})$ is derived from the application of each of them. The net certainty factor of h is dependent upon each of these partial certainty factors. Basic compression step 4, describing the parallel composition of derivations and its associated combination function $f_{\|}$, deal with multiple production rules. From now on, we will call the function $f_{\|}$ the *combination function for (combining the results of) multiple production rules concluding on the same hypothesis.*

Again, it is desirable that the application of the function $f_{\|}$ render the resulting certainty factor of a hypothesis h independent of the order in which the different production rules concluding on h are applied. Furthermore, it is desirable that the resulting certainty factor be independent of the way in which the results of the different rules are taken together to be combined. Therefore, the combination function $f_{\|}$ has to respect the property of commutativity,

$$f_{\|}(x, y) = f_{\|}(y, x)$$

for all certainty factors x and y, and the property of associativity,

$$f_{\|}(f_{\|}(x, y), z) = f_{\|}(x, f_{\|}(y, z))$$

for all certainty factors x, y, and z.

Finally, we want the four combination functions to be monotonic increasing. Therefore, the combination functions $f_{\circ}$, $f_{\&}$, $f_{|}$, and $f_{\|}$ have to respect the following property:

If $x \geqslant x'$ and $y \geqslant y'$, then

$$f_{\circ}(x, y) \geqslant f_{\circ}(x', y'),$$

$$f_{\&}(x, y) \geqslant f_{\&}(x', y'),$$

$$f_{|}(x, y) \geqslant f_{|}(x', y'),$$

and

$$f_{\|}(x, y) \geqslant f_{\|}(x', y')$$

7. THE ACTUAL COMBINATION FUNCTIONS

Shortliffe and Buchanan introduced four combination functions for combining certainty factors in their original paper [5]. In this section we discuss these combination functions of Shortliffe and Buchanan and show the correspondence to our combination functions $f_{\circ}$, $f_{\&}$, $f_{|}$, and $f_{\|}$.

The Combination Function for Propagating Uncertain Evidence

In the situation in which the evidence e in a production rule $e \rightarrow h$ is an intermediary hypothesis confirmed to some degree, the certainty factor $\mathrm{CF}(e \dashv u, D^{u,e})$ of the intermediary hypothesis e given some derivation of e

from u is used as a weighting factor for the certainty factor CF($h \vdash e, e \rightarrow h$) associated with the rule. Adapted to our notational convention, the combination function for uncertain evidence described by Shortliffe and Buchanan reads as follows:

$$\mathrm{CF}(h \dashv u, (D^{u,e}) \circ (e \rightarrow h)) =$$
$$= \mathrm{CF}(h \dashv e, e \rightarrow h) \cdot \max\{0, \mathrm{CF}(e \dashv u, D^{u,e})\}$$

or using the function $f_\circ$,

$$f_\circ(x, y) = y \cdot \max\{0, x\}$$

where x denotes the certainty factor CF($e \dashv u, D^{u,e}$) of the intermediary hypothesis e, and y denotes the certainty factor CF($h \dashv e, e \rightarrow h$) associated with the production rule $e \rightarrow h$. From this latter formulation the reader can easily verify that the combination function respects the property of monotony.

Shortliffe and Buchanan propose the following formulation of the combination function for propagating uncertain evidence (although the function is not stated explicitly in the original work, it is the straightforward analog of the corresponding functions for their basic measures of uncertainty):

$$\mathrm{CF}(h, i) = CF'(h, i) \cdot \max\{0, \mathrm{CF}(i, e)\}$$

where CF$'(h, i)$ is the certainty factor associated with h given that evidence i is observed with absolute certainty, that is, the certainty factor the expert has assigned to the hypothesis h in the (single-conclusion) production rule $i \rightarrow h$. The certainty factor CF(i, e) denotes the actual certainty factor of i given some prior evidence e; similarly, CF(h, i) is the actual certainty factor of h after the application of the rule $i \rightarrow h$. In our opinion, the actual certainty factor of h after the application of the production rule, expressed on the left-hand side of the formulation given above, is dependent not only upon h and i, but upon e as well. The dependency on e is not expressed in the original formulation of the combination function. This inadequacy has caused the need to introduce the seemingly strange quoted function CF$'$. The observation that the actual certainty factor of the hypothesis h is dependent upon all intermediary hypotheses that were used in deriving h has led to the introduction of the notion of derivation with respect to a fixed set of production rules in our formulation of the uncertainty factor function. Notice that we have not affected the intended meaning of the original formulation of the combination function for propagating uncertain evidence.

The Combination Functions for Composite Hypotheses

If the evidence e in the production rule $e \rightarrow h$ is a conjunction of disjunctions of pieces of evidence, the certainty factors of each of the separate pieces of evidence are combined into a single certainty factor of e. For this purpose we have introduced the combination functions $f_\&$ and $f_|$. Shortliffe and Buchanan argue that the belief in a conjunction of hypotheses is only as good as the belief in the hypothesis that is believed less strongly. A complementary observation is made for the belief in a disjunction of hypotheses. Pursuing his observation, their combination function for a conjunction of hypotheses reads as follows:

$$\mathrm{CF}(e_1 \wedge e_2 \dashv u, (D^{u,e_1}) \,\&\, (D^{u,e_2})) =$$
$$= \min\{CF(e_1 \dashv u, D^{u,e_1}), \mathrm{CF}(e_2 \dashv u, D^{u,e_2})\}$$

For a combination function for a disjunction of hypotheses they have chosen

$$\mathrm{CF}(e_1 \vee e_2 \dashv u, (D^{u,e_1})|(D^{u,e_2})) =$$
$$= \max\{\mathrm{CF}(e_1 \dashv u, D^{u,e_1}), \mathrm{CF}(e_2 \dashv u, D^{u,e_2})\}$$

or, using the functions $f_\&$ and $f_|$,

$$f_\&(x, y) = \min\{x, y\} \qquad \text{and} \qquad f_|(x, y) = \max\{x, y\}$$

where x denotes the certainty factor $\mathrm{CF}(e_1 \dashv u, D^{u,e_1})$ and y denotes the certainty factor $\mathrm{CF}(e_2 \dashv u, D^{u,e_2})$. From this formulation it should be obvious that these combination functions respect the properties of commutativity, associativity, and monotony.

Shortliffe and Buchanan propose the following formulation of these combination functions:

$$CF(h_1 \wedge h_2, e) = \min\{CF(h_1, e), CF(h_2, e)\}$$
$$CF(h_1 \vee h_2, e) = \max\{CF(h_1, e), CF(h_2, e)\}$$

These combination functions can be used to combine the certainty factors of several hypotheses given the *same* evidence. In practice, however, the certainty factors of the hypotheses to be combined are generally derived along different inference paths and differ in the second argument due to the original formulation of the combination function for propagating uncertain evidence. The reader can verify that the reformulation of the combination functions for composite hypotheses has the same meaning as the original formulation.

The Combination Function for Multiple Production Rules

The combination function still to be discussed concerns multiple production rules concluding on the same hypothesis, that is, our function $f_\|$. The following combination function is given by Shortliffe and Buchanan to deal with this situation:

$$\text{'F}(h \dashv u, (D_1^{u,h})\|(D_2^{u,h})) =$$

$$= \begin{cases} \text{CF}(h \dashv u, (D_1^{u,h}) + \text{CF}(h \dashv u, D_2^{u,h})(1 - \text{CF}(h \dashv u, D_1^{u,h})) \\ \quad \text{if CF}(h \dashv u, D_1^{u,h}),\ \text{CF}(h \dashv u, D_2^{u,h}) > 0 \\ \dfrac{\text{CF}(h \dashv u, D_1^{u,h}) + \text{CF}(h \dashv u, D_2^{u,h})}{1 - \min\{|\text{CF}(h \dashv u, D_1^{u,h})|, |\text{CF}(h \dashv u, D_2^{u,h})|\}} \\ \quad \text{if } -1 < \text{CF}(h \dashv u, D_1^{u,h}) \cdot \text{CF}(h \dashv u, D_2^{u,h}) \leqslant 0 \\ \text{CF}(h \dashv u, D_1^{u,h}) + \text{CF}(h \dashv u, D_2^{u,h})(1 + \text{CF}(h \dashv u, D_2^{u,h})) \\ \quad \text{if CF}(h \dashv u, D_1^{u,h}),\ \text{CF}(h \dashv u, D_2^{u,h}) < 0 \end{cases}$$

ng the function $f_\|$ renders a more perspicuous formulation:

$$f_\|(x, y) = \begin{cases} x + y - xy & \text{if } x, y > 0 \\ \dfrac{x + y}{1 - \min\{|x|, |y|\}} & \text{if } -1 < xy \leqslant 0 \\ x + y + xy & \text{if } x, y < 0 \end{cases}$$

:re x denotes the certainty factor $\text{CF}(h \dashv u, D_1^{u,h})$ and y denotes the certainty or $\text{CF}(h \dashv u, D_2^{u,h})$. This combination function respects the properties of ımutativity and associativity, as shown by Spiegelhalter [8].

hortliffe and Buchanan [5] give the following formulation of this combina- function:

$$\text{CF}(h, e_1 \wedge e_2) = \begin{cases} \text{CF}(h, e_1) + \text{CF}(h, e_2)(1 - \text{CF}(h, e_1)) \\ \quad \text{if CF}(h, e_i) > 0, \quad i = 1, 2 \\ \dfrac{\text{CF}(h, e_1) + \text{CF}(h, e_2)}{1 - \min\{|\text{CF}(h, e_1)|, |\text{CF}(h, e_2)|\}} \\ \quad \text{if } \textit{one of } \text{CF}(h, e_i) < 0, \quad i = 1, 2 \\ \text{CF}(h, e_1) + \text{CF}(h, e_2)(1 + \text{CF}(h, e_1)) \\ \quad \text{if CF}(h, e_i) < 0, \quad i = 1, 2 \end{cases}$$

ıs noted that this function is mistakenly not defined if at least one of (h, e_1) and $\text{CF}(h, e_2)$ equals zero. Furthermore, the case in which $(h, e_1) \cdot \text{CF}(h, e_2) = -1$ should be excluded explicitly since the combination ıction is undefined in this case. A more serious criticism is that in this mulation of combining the results of multiple production rules concluding on the same hypothesis the same symbol $\wedge$ is used as in describing a conjunction of two hypotheses or pieces of evidence. Shortliffe and Buchanan seem to assume that the success of a production rule $e_1 \wedge e_2 \rightarrow h$ is equivalent to the success of the two production rules $e_1 \rightarrow h$ and $e_2 \rightarrow h$. As such an equivalence is apt to be violated due to inconsistent function values given by the expert (and the user), we have introduced another notational convention. Again, the reformulation does not change the original meaning of the combination function.

A Numerical Example

To conclude, the application of the combination functions is demonstrated by means of a numerical example.

EXAMPLE 7.1. Consider the following three production rules:

$$d \wedge f \rightarrow b$$
$$a \rightarrow h$$
$$b \wedge c \rightarrow h$$

The expert has provided the following certainty factors:

$$\mathrm{CF}(b \dashv d \wedge f, d \wedge f \rightarrow b) = 0.80$$
$$\mathrm{CF}(h \vdash a, a \rightarrow h) = 0.70$$
$$\mathrm{CF}(h \dashv b \wedge c, b \wedge c \rightarrow h) = 0.50$$

We assume that h is the goal hypothesis. The user of the system supplies the following information during the consultation:

$$\mathrm{CF}(a \dashv u, u \rightarrow a) = 0.50$$
$$\mathrm{CF}(c \dashv u, u \rightarrow c) = 0.40$$
$$\mathrm{CF}(d \dashv u, u \rightarrow d) = 1.00$$
$$\mathrm{CF}(f \dashv u, u \rightarrow f) = 0.90$$

Then it takes the following six computations to arrive at a certainty factor of h:

1. $\mathrm{CF}(h \dashv u, (u \rightarrow a) \circ (a \rightarrow h)) = 0.70 \times 0.50 = 0.35$
2. $\mathrm{CF}(d \wedge f \dashv u, (u \rightarrow d) \,\&\, (u \rightarrow f)) = \min\{1.00, 090\} = 0.90$
3. $\mathrm{CF}(b \dashv u, ((u \rightarrow d) \,\&\, (u \rightarrow f)) \circ (d \wedge f \rightarrow b)) = 0.80 \times 0.90 = 0.72$
4. $\mathrm{CF}(b \wedge c \dashv u, (((u \rightarrow d) \,\&\, (u \rightarrow f)) \circ (d \wedge f \rightarrow b)) \,\&\, (u \rightarrow c)) =$ $= \min\{0.40, 0.72\} = 0.40$
5. $\mathrm{CF}(h \dashv u, ((((u \rightarrow d) \,\&\, (u \rightarrow f)) \circ (d \wedge f \rightarrow b)) \,\&\, (u \rightarrow c)) \circ$ $\circ (b \wedge c \rightarrow h)) = 0.50 \times 0.40 = 0.20$
6. $\mathrm{CF}(h \dashv u, (((u \rightarrow a) \circ (a \rightarrow h)) \| ((((u \rightarrow d) \,\&\, (u \rightarrow f)) \circ (d \wedge f \rightarrow b)) \,\&$ $\& \,(u \rightarrow c)) \circ (b \wedge c \rightarrow h))) = 0.35 + 0.20 \times 0.65 = 0.48$

8. CONCLUSION

We have discussed a conceptual model for inexact reasoning in rule-based top-down reasoning expert systems. This conceptual model has been used to examine the certainty factor model proposed by Shortliffe and Buchanan. In this discussion we have abstracted from several implementation issues, such as the discontinuity in the evaluation of the left-hand side of a production rule, that is, the 0.2 threshold added to the model for pragmatic reasons. We have modeled the scheme of propagating uncertainty by the compression of the inference network corresponding to the actual inference process. For this we have defined four basic compression steps on an inference network and have associated a function with each of these compression steps. The correspondence of these functions of the actual combination functions of the certainty factor model has been highlighted.

Furthermore formal definitions of the certainty factor function and its combination functions have been introduced. These formal definitions have been the point of departure for a discussion of some of the theoretical issues involved in the certainty factor model in a subsequent paper (Van der Gaag [7]). In their paper Shortliffe and Buchanan suggested a theoretical foundation for the model in Bayesian probability theory but did not provide a thorough justification for this basis or for the combination functions given the probabilistic definitions. The probabilistic basis of the model as well as the combination functions have been severely criticized, largely because of the ad hoc character of these parts of the model. In [7], we address the question of whether the combination functions can be accounted for by the probabilistic basis suggested by Shortliffe and Buchanan.

Although I have used the conceptual model in this paper only in examining the certainty factor model, the model has been successfully applied in investigating other actual models that have been proposed for inexact reasoning and in comparing several actual models.

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