Laurent van den Bos, Benjamin Sanderse

Uncertainty quantification for wind energy applications

Literature review

Report SC-1701
August 2017

Centrum Wiskunde & Informatica

Abstract

Uncertainties are omni-present in wind energy applications, both in external conditions (such as wind and waves) as well as in the models that are used to predict key quantities such as costs, energy yield, and fatigue loads. This report summarizes and reviews the application of uncertainty quantification techniques to wind energy problems. In the wind industry, including uncertainties in predictions has classically been done by using Monte Carlo methods. Recently, more advanced methods have been considered (e.g. polynomial chaos expansion, stochastic collocation, and Gaussian process regression), which are based on smartly sampling the model (e.g. a complex aerodynamic blade model). These methods generally have a greater efficiency compared to Monte Carlo (depending on model properties) and additionally yield computationally cheap surrogate models. Furthermore, surrogate models purely based on data (e.g. via proper orthogonal decomposition) have received significant interest, especially for the representation of turbulent wind turbine wakes. Both model-driven and data-driven surrogate models play a crucial role in making control and optimization studies feasible. In the near future, we expect that recent trends in uncertainty quantification, namely Bayesian model calibration and optimization under uncertainty, will become increasingly popular in wind energy applications.

Contents

1
Introduction

1.1 Background

The size of the annual wind energy production is growing each year. At the end of 2016, the global cumulative installed capacity was more than 480 GW, of which approximately 55 GW was installed during the year 2016 [37]. The size of the wind turbines has significantly increased over the years, with the largest turbines now reaching rotor diameters of 150 m. These turbines are operating in the lower part of the atmospheric boundary layer, with large wind shear and turbulence levels, and uncertain, continuously varying meteorological conditions. These weather conditions—such as atmospheric stability, wave height, and of course wind—severely affect the service life of the turbines and the power yield of the farm as a whole. It is common practice in industry that this uncertain behaviour is captured using rather conservative safety factors due to a lack of detailed knowledge of uncertainties and how they propagate [103]. For example, a rigorous characterisation of wind conditions in the IEC guidelines is missing [43, 58]. However, at the same time wind turbines still suffer premature failures and reduced lifetimes compared to design predictions, mainly caused by our limited understanding of how turbines operate and interact with the atmosphere [94]. The presence of uncertainties in wind farm output, turbine lifetime, and failure probability leads to an increased financial investment risk. Consequently, there is room for reducing the costs of offshore wind energy by using a probabilistic approach for the design of wind turbines and wind farms in which the effect of uncertainties is quantified and reduced.

Currently, the use of uncertainty assessments in the wind energy industry is growing. In the research agenda of the European Academy of Wind Energy (EAWE) [58] the need for quantification of uncertainties is recognized, especially in the areas of aerodynamics, reliability, and design[1]. A prototypical example of uncertainty quantificiation in wind farm models is the computation of the annual energy production (AEP) and the associated confidence in the prediction [113]. Simple statistical models (using the turbine power curve and a Weibull distribution for the wind field) can quickly give a rough idea about the mean and variance of the power production, but have a rather low accuracy because important physics, such as wake effects, are missing. Higher-fidelity models can make more accurate power predictions, but the quantification of

[1] *'A further additional approach is wind turbine and wind farm uncertainty modelling, which is still in its infancy to date but forms an intrinsic aspect for minimising unexpected failures and downtime.' [58]*

uncertainties becomes cumbersome due to the computational expense associated with these models. One of the main topics in uncertainty quantification, also addressed in this report, is to make uncertainties computable for complex models in which multiple source of uncertainties are involved.

1.2 Uncertainty Quantification

The growing awareness in the wind energy community on the relevance of computing with uncertainties is in parallel with significant advancements in the field of uncertainty quantification (UQ). The field of UQ provides tools and methods to include statistics and randomness into model predictions. It entails many different topics, such as sensitivity analysis, parameter estimation, data assimilation, and surrogate modelling. A short summary of some main concepts in UQ are included in this report.

1.3 Goal and report outline

This literature review partially continues from the conclusions from a previous review by one of the authors[2]. Similar to that review, the main target audience in this report is again the wind energy community. The main goal is to summarize and review the application of uncertainty quantification techniques to wind energy problems. The focus is mostly on the type of UQ techniques, but we will show practical examples of uncertainties in models and external conditions, such as wind, waves, blade and wake aerodynamics, and power production (chapter 2). Furthermore, to make the report self-contained, we have included a chapter in which we explain the main concepts in UQ (chapter 3). Chapter 4 forms the core of this work and reviews the UQ techniques that have been used for wind energy applications, mainly focusing on fluid dynamics applications. Structural mechanics and fluid-structure interaction applications are not directly covered, but the UQ techniques discussed in this report can in principle also be applied in those areas. In chapter 5 we give a summary and an outlook towards expected future developments.

[2] *'... a possible next step is the quantification of uncertainties in the computations. These uncertainties originate not only from discretization and turbulence modelling errors but also from the description of the inflow, terrain geometry, rotor geometry, etc. A quantification of uncertainties would make the comparison with experimental data more fair and will give a guideline in which areas the CFD of wind turbine wakes has to be improved.' [89]*

2
Examples of uncertainties in models and external conditions

In this chapter we give examples of uncertainties in models and external conditions encountered in wind turbine design. In sections 2.1-2.2, we discuss uncertain external conditions relevant for wind turbine operation, focusing on wind and waves. For a more exhaustive list of uncertainty sources, see for example Veldkamp [103]. A key distinction we make in this chapter is between a *statistical* approach, where predictions are made based on long-term measurement data, and a *physical modelling* approach, where predictions are made based on conservation laws. In sections 2.3-2.4 we give examples of uncertainties in models used for blade and wake aerodynamics. A graphical representation is given in figure 2.1.

A large amount of literature on (deterministic) models for wind turbines, wind farm aerodynamics, wake effects, and power production, is available. The interested reader can consult the general introduction in wind energy provided by the book of Burton et al. [12].

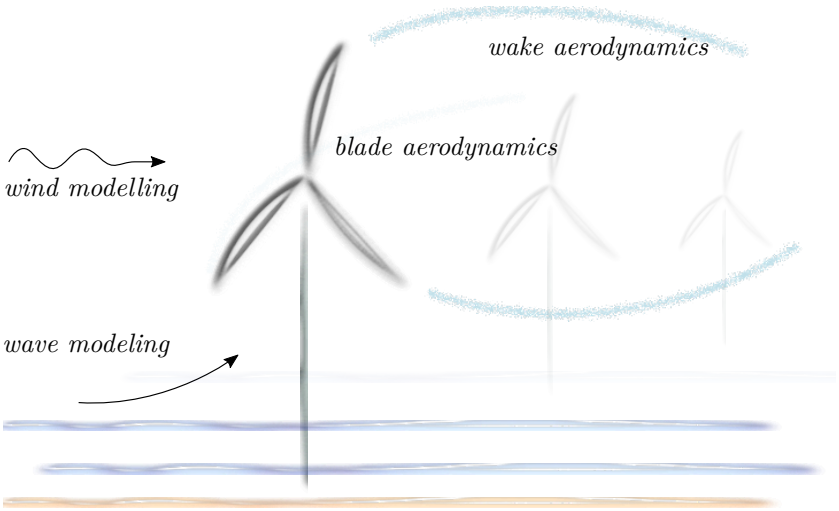

Figure 2.1: This chapter discusses models for external conditions (wind, waves) and for blade and wake aerodynamics.

2.1 Wind modelling

2.1.1 Mean wind speed and annual energy production

One of the main purposes in wind modelling is to accurately estimate the annual energy production (AEP), defined as

$$\text{AEP} = T \cdot \mathbb{E}[P(U)], \qquad \mathbb{E}[P(U)] = \int P(U)\rho(U)\mathrm{d}U,$$

where T is the time interval (i.e. 1 year), $\mathbb{E}[P]$ is the expected power, $P(U)$ is the power as function of wind speed U, and $\rho(U)$ is the probability density function (pdf) of the wind speed (discussed below). This integral is split into several bins or sectors, associated to different wind directions, and numerically evaluated for each bin. This is a statistical approach, in which uncertainties in the wind field are captured via a probability density function that is constructed based on long term wind measurements.

The canonical model for $\rho(U)$ is the Weibull distribution, a pdf with two parameters, constructed based on 10-minute averaged wind speed data. For different bins the sector-wise Weibull distribution is used [24], although joint pdfs for wind speed and direction can also be considered [14, 15]. Other distributions can be employed to fit wind data, and sometimes perform better than Weibull [73], for example a bimodal Weibull distribution [47] or a maximum entropy distribution [4]. The fitting method used to determine the parameters of the distribution is important; for example, Dorvlo [18] noted that different results are obtained if different fitting methods are used. He compared a regression method, moments method, and Chi-square method and large differences were obtained for the two parameters of the Weibull distribution. It is clear that in this simple wind model large uncertainties are present.

A special case of the Weibull distribution is the Rayleigh distribution, which is used in the IEC standard [43, 44]. The mean wind speed at hub height V_{hub} according to the Rayleigh probability distribution is given by

$$\rho_R(U_{\text{hub}}) = 1 - \exp\left(\pi\left(\frac{U_{\text{hub}}}{2U_{\text{ave}}}\right)^2\right), \tag{2.1}$$

where $U_{\text{ave}} = 0.2U_{\text{ref}}$, and U_{ref} depending on the class[1] of the wind turbine.

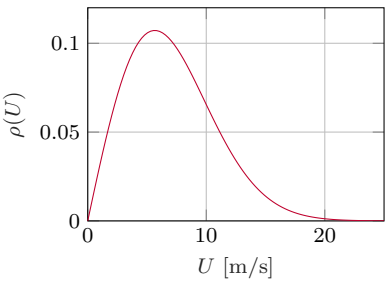

Figure 2.2: Example of a Weibull distribution for wind speed.

[1] Wind turbines are put in classes depending on their properties. For example, different classes are formed by wind turbines of different sizes and whether they are offshore or onshore turbines.

2.1.2 Power spectrum

The mean wind speed at hub height is not sufficient to simulate load cases on a wind turbine. For such simulations, the wind speed over the swept area is required, typically at a number of points that form a grid. A spectrum of the turbulent wind field at these points is chosen according to e.g. the von Kármán or Kaimal theory [12, 101]. This gives a continuous, one-sided power spectrum density at a point j on the disk, depending on the frequency f, called $G_{j,j}(f)$. A discrete representation of the spectrum is obtained by dividing the spectrum into bands with width Δf and center frequency of each band designated as f_m. The

correct variance is maintained by defining this discrete spectrum $S_{j,j}(f)$ by

$$S_{j,j}(f_m) = \frac{1}{2}\Delta f G_{j,j}(f_m).$$

The full spectral matrix S also requires cross-spectral densities (off-diagonal terms). The power spectra on each point across the rotor disc have to correlate, which is accomplished by using a coherence function $\mathrm{Coh}_{j,k}$:

$$|S_{j,k}(f_m)| = \mathrm{Coh}_{j,k}(f_m, \Delta r_{j,k}, U_j, U_k)\sqrt{S_{j,j}(f_m)S_{k,k}(f_m)},$$

where $\Delta r_{j,k}$ denotes the distance between points j and k and U_j is the mean wind speed at point j. Together, all power and cross spectral densities are put in the matrix S. Using a decomposition $S = HH^{\mathrm{T}}$, the Fourier coefficients of the wind field can be written as

$$V_j(f_m) = \sum_{k=1}^{j} H_{j,k}(f_m)\exp(i\vartheta_{k,m}),$$

where $\vartheta_{k,m}$ is the phase angle associated with the k^{th} input point and m^{th} frequency component. Typically, it is taken randomly distributed in the interval $[0, 2\pi)$. Time series of the wind speed follow by taking an inverse Fourier transform of the V_j coefficients. Examples of coherence functions and power spectra can be found in [101]. This idea has been further improved by Mann [67], who also incorporated the atmospheric shear layer of the flow of the von Kármán wind turbulence model. His results are in some respects more physically reasonable and the algorithm is generally faster.

2.1.3 Weather and wind forecasting

An alternative statistical approach is the use of autoregressive models, often used in weather forecasting [50, 112], and wind speed prediction. An example of an autoregressive moving-average (ARMA) model for the mean wind speed U is the Gaussian process

$$U_t = c + \varepsilon_t + \sum_{i=1}^{p}\phi_i U_{t-i} + \sum_{i=1}^{q}\vartheta_i\varepsilon_{t-i},$$

where ϕ_i is the autoregressive parameter, ϑ_i is the moving average parameter, c is a constant, and ε_t is an error term, for example modelled as a normally distributed random variable. The ARMA model can be seen as a fitting procedure based on past data with the aim to predict a future state.

2.1.4 Precursor simulations

An alternative to the mainly statistical approaches mentioned above is a physical modelling approach in which the wind field is the result of a simulation based on computational fluid dynamics (CFD) techniques. For blade loading calculations, see section 2.3, such an approach is not used because it is too expensive. However, for wake modelling (section

2.4), this is a common approach and is known as a precursor simulation. In this technique the atmosphere, including the atmospheric boundary layer, is simulated without the presence of wind turbines. The turbulent wind field that is generated is then used as boundary condition for a simulation that includes the wind turbines. Recently, coupling wake simulations to atmospheric mesoscale models have made this approach to wind field simulation even more advanced [90]. Such mesoscale models are the realm of Numerical Weather Prediction (NWP) and will not be discussed here.

2.2 Wave modelling

Many results exist about modelling wind for the simulation of wind turbines, due to the advanced knowledge of onshore wind turbines. The modelling of sea waves with the aim of evaluating offshore turbine loading is a less developed field [1], but nevertheless very important [2]. Like in wind modelling, the existing approaches will be categorized in statistical approaches and physical modelling approaches. An excellent introduction to sea wave modelling is the book of Holthuijsen [41], which is used frequently in the coming sections.

2.2.1 Statistical approaches

Most key concepts in sea wave modelling are based on long-term statistics of sea waves: the significant wave height (i.e. the mean of the highest one-third of waves in the wave record), the significant wave period, and the wave spectrum. These concepts are used in the IEC standard [44] for offshore wind turbines, which describes that a design sea state has to be described by a wave spectrum, together with the significant wave height, a peak spectral period, and a mean wave direction. Examples of wave spectra are the Pierson-Moskowitz spectrum (based on the concept of fully developed sea) and the JONSWAP spectrum. The IEC standard also prescribes the normal, severe, and extreme sea states which need to be taken into account in turbine design.

The wave spectrum can be used to derive a probability density function for the wave height. In case of a narrow spectrum, the wave height follows a Rayleigh distribution (similar to the wind speed, equation (2.1)). The free parameter in the Rayleigh distribution can be expressed directly in terms of the moments of the wave spectrum. In a way quite similar to the statistical approach in wind modelling, a sea state can then be obtained by superposition of harmonic waves of different random amplitudes and phases (the so-called random-phase/amplitude model); these are so-called irregular waves. The amplitudes and phases are sampled based on their respective probability density functions.

2.2.2 Physical modelling approaches

The harmonic waves mentioned in the last section are solutions of linearised conservation equations (mass and momentum conservation), describing surface gravity waves (also known as Airy wave theory). The

random-phase/amplitude model is thus in fact a mix between statistical and physical modelling approaches. The assumption of linearity is a good approximation for waves of small height in deep water. If waves are steep or the water is shallow, the wave dynamics becomes non-linear and more advanced theories must be used, for example Stokes' wave theory, cnoidal wave theory, or streamfunction theory. Figure 2.3 summarizes many existing wave theories.

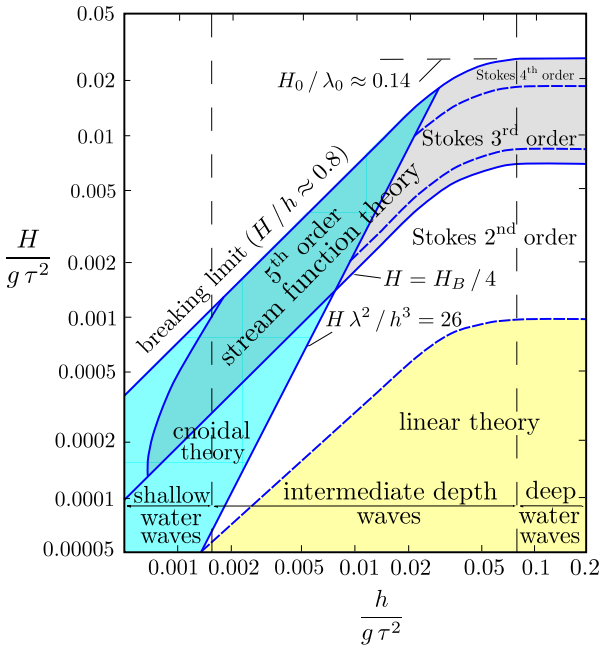

Figure 2.3: The ranges of applicability of various wave theories (figure from Wikipedia–Stokes wave; see also [41]); h is the mean water depth, H the wave height and τ the wave period.

Stokes' theory, which is applicable to intermediate depth waves, successively adds corrections to existing wave models. Repeatedly applying it, starting with the linear wave model, the n^{th}-order Stokes wave model is obtained. It is based on perturbation theory and expands the wave in terms of the wave steepness $\varepsilon = ka$, with k the wave number and a the amplitude. The wave height η is then written as:

$$\eta = \varepsilon\eta_1 + \varepsilon^2\eta_2 + \varepsilon^3\eta_3 + \dots.$$

The principle is to first solve for η_1, then for η_2 (given η_1) and so on. For example, the first solution is the solution from linear wave theory:

$$\eta_1 = k^{-1}\cos(\omega t - kx),$$

and the second solution is the so-called second order Stokes correction:

$$\eta_2 = k^{-1}\frac{\cosh kd}{4\sinh^3 kd}(2 + \cosh 2kd)\cos(2\omega t - 2kx),$$

with d the water depth. The equations quickly become very complicated for higher orders.

It is common practice in wind turbine industry to use Airy linear wave theory for calculating fatigue loads [105]. However, offshore wind turbines are generally sited in shallow water and non-linear effects and

steeper waves are important [72]. Differences in fatigue predictions are indeed observed depending on whether first-order, second-order, or fully non-linear waves are used [105], and computed loads on turbines are generally larger with non-linear wave models [1]. In [69, 70] potential theory is used with a boundary element method to solve the non-linear free surface problem and used to study extreme loading conditions on offshore wind turbines. An alternative is to model fully non-linear waves with Boussinesq theory [66] or, even more advanced, with Navier-Stokes type methods (which can include wave breaking), but this technique is in general computationally too expensive for calculating wave impacts on offshore turbines.

2.3 Blade models

Aerodynamic blade models are crucial to determine the loads on wind turbine blades and therefore their lifetime (for more information on aerodynamics of wind turbines see e.g. [40, 97]). We distinguish between two approaches: the engineering methods, based on Blade Element Momentum (BEM) theory, and methods based on CFD techniques. Both approaches are 'physics based', and apply conservation laws (conservation of mass and momentum) to arrive at predictive models.

2.3.1 Blade-Element-Momentum theory

BEM is based on equating solutions for forces obtained from blade element theory to those obtained in momentum theory. It is the industrial workhorse in designing wind turbine blades and assessing loading scenarios. Although BEM is applied in many cases and shows acceptable results, it still can have an error up to 20% (see [97], and for a recent review see [7]). Some shortcomings are that the theory assumes an infinite number of blades, that yaw and stall are not properly taken into account, and that accurate airfoil lift and drag coefficients are required. The latter are typically obtained with more advanced methods such as CFD models. Several improvements and corrections have been suggested, which compensate for the limitations in the theory, but the largest uncertainties are still in the model constants associated with these corrections.

2.3.2 Potential flow and CFD models

To determine the relevant aerodynamic coefficients of airfoil sections, as needed in BEM, one can solve the Navier-Stokes equations or use accurate measurements, such as wind tunnel data. For design purposes, solving the Navier-Stokes equations may be too expensive, and instead the incompressible potential equations, coupled with a boundary layer model, are often used. The potential flow equations arise from the assumptions of incompressible, irrotational flow. This means that a velocity potential exists (such that $\mathbf{v} = \nabla\phi$), which satisfies the Laplace equation $\Delta\phi = 0$. The incompressibility and irrotationality assumptions are accurate away from the boundary of the blade. The boundary layer is modelled differently, namely using the boundary layer equations. The

potential flow and boundary layer equations are coupled through an interaction scheme, as for example done in Xfoil [20].

The potential flow methods can be classified as CFD methods since they solve a partial differential equation, although they do not require the generation of a body-fitted mesh. More advanced methods, which take into account vorticity, viscosity and compressibility in the entire flow fields, are based on discretization of the Navier-Stokes equations on body-fitted grids [89]. Recently, also immersed boundary methods have been applied to wind turbine computations [31, 51]. Such methods solve the full Navier-Stokes equations by locally changing the discretization method in the neighbourhood of the blade, avoiding the construction of a body-fitted grid. In all cases, a turbulence model is required to model the effect of the smallest scales in the flow.

2.4 Wake modelling

The placement of offshore wind turbines in wind farms results in wake effects that negatively influence power production and fatigue life of downstream turbines, due to a reduced wind speed and increased turbulence intensity [89, 97]. Many different wake models exist; most of them are *not* based on statistical approaches but rather on physical arguments. As in case of the blade models, we mention here two distinct approaches: the low-fidelity algebraic models, used for wind farm optimization and design, and the high-fidelity turbulence-resolving CFD models, used for detailed computations.

2.4.1 Algebraic models

One of the simplest algebraic wake models is the model of Jensen [48]. It is a combination of momentum theory (applied to the near wake) with conservation of mass (applied to the far wake), assuming a linear wake expansion:

$$U_{\text{wake}}(x) = U_{\text{in}} \left(1 - 2a \left(\frac{r_0}{r_0 + \alpha x} \right)^2 \right),$$

where a is the axial induction factor, x the distance behind the rotor, r_0 the initial radius, α the wake expansion ratio, and U_{in} the incoming wind speed. Many improvements and extensions of the Jensen model have been made. For example, Larsen's model [62] does not use the linear wake expansion form, but uses Prandtl's turbulent boundary layer equations to describe the wake region. The Frandsen model [32] includes interaction with neighbouring rows and with the atmospheric boundary layer. These simplified and fast wake models are the work horse in industry and keep on being improved. Recently, Bastankhah and Porté-Agel [8] and Keane et al. [52] developed analytical wake models based on a Gaussian velocity profile in the wake instead of the top-hat profile used by Jensen.

2.4.2 CFD models

Instead of analytical modelling based on simplified versions of the equations for conservation of mass and momentum it is also possible to use

CFD techniques to solve the full incompressible Navier-Stokes equations:

$$\nabla \cdot \mathbf{u} = 0,$$
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \Delta \mathbf{u}.$$

Here, $\mathbf{u}$ is the velocity field, p is the pressure and ρ is the density. To incorporate turbulence, Reynolds Averaged Navier-Stokes (RANS) or Large Eddy Simulation (LES) models are employed. LES is the state-of-the-art, but computationally expensive and mainly used to develop, test, and improve less expensive models, and to parametrize wind farm effects in weather and climate models; see [11, 31, 71, 89]. A recent review that discusses the different scales and the associated methods in mesoscale to microscale wind farm flow modelling is given in [90], and also includes a section on the relevance of uncertainty quantification.

2.5 Summary and conclusions

In this chapter we have given examples of uncertainties in models and external conditions. We distinguished between a *statistical* approach, such as the characterization and generation of a wind field by a power spectrum, and a *physical modelling* approach, such as the solution of the Navier-Stokes equations to predict wake effects. In the former case, the uncertainty is captured in the statistics, for example in the Weibull distribution of the mean wind speed. With more measurements this distribution will get better characterized. In the latter case, the uncertainty is present in the model assumptions, for example the use of an LES model to resolve turbulent flow. With better understanding of the physics (or more computer power), the uncertainty in simulation results can be reduced. The irreducible uncertainties are called *aleatory*, while the reducible uncertainties are called *epistemic*. The distinction between the two is the choice of the modeller; for example, wind and wave fields can be generated with a statistical approach, but also with computational fluid dynamics techniques. This distinction will be further discussed in chapter 3.

3
A crash course in UQ

3.1 Introduction and definitions

In the field of uncertainty quantification (UQ) the problem of making predictions under uncertainties is addressed, by studying how uncertainties in parameters and models affect the uncertainty in quantities of interest. Several books and review articles have appeared on the topic of UQ, e.g. [36, 63, 75, 95, 99, 110, 111]. The field of UQ is large and covers many topics, such as optimization, parameter estimation and calibration, model reduction, and sensitivity analysis [99]. A main distinction that we use in this report is between uncertainty characterization and uncertainty propagation [95].

Characterization is determining the nature of the uncertainties. These can be roughly categorized in input (parametric) uncertainties and model-form uncertainties. A further distinction is between *aleatory* (irreducible, stochastic) and *epistemic* (reducible, systematic, due to a lack of knowledge) uncertainties. Aleatory uncertainties get better characterized with an increase in knowledge, whereas epistemic uncertainties get reduced with an increase in knowledge. The distinction between the two is the modeller's choice.[1] The atmospheric wind speed, for example, has typically been treated as aleatory uncertainty, being represented by probability density functions (such as the Weibull distribution) and turbulence spectra, from which wind fields can be generated - see chapter 2. When more data becomes available, for example from meteorological masts, the probability density functions can be better characterized. However, in coupled mesoscale-microscale models [90], the wind field is modelled with meteorological models, and then the uncertainty in wind speed prediction can be seen as an epistemic uncertainty which reduces upon improved model efforts. For describing aleatory uncertainties consensus exists on the use of probability theory (probability density functions), but for epistemic uncertainties several approaches are being used, such as interval analysis and evidence theory [76, 114]. Interval analysis is used when only lower and upper bounds are available. Evidence theory, a generalization of classical probability theory, uses belief and plausibility as measures.

[1] *'Aleatory or epistemic? Does it matter?'* [56].

Propagation is relating uncertainties in inputs and models to uncertainties in output quantities, the quantities of interest (QoI). In wind energy studies, the ultimate quantity of interest is typically the levelized cost

of energy (LCOE). The LCOE is in itself influenced by other quantities of interest, such as the wind farm power production, the fatigue loading on the turbine blades, or sound level produced by the turbine. Related to the propagation question is the analysis of sensitivities: which input quantities have most influence on the QoI? For example, an outcome of a sensitivity analysis could be that there is a large uncertainty in the variation of insect contamination on turbine blades, but that the effect on the produced sound is marginal [82].

A last distinction we make is between *intrusive* and *non-intrusive* methods. Non-intrusive methods rely on existing models that are employed as a black box, see figure 3.1. Since these black box models can be computationally very expensive (e.g. CFD codes), the art is to smartly sample the input parameters and get accurate output predictions with as few samples as possible. In intrusive methods on the other hand the mathematical model is changed and existing codes have to be modified. Depending on the model properties, this can lead to efficient methods, but requires generally considerable coding effort that requires revision once the model changes. The main focus in this report is on non-intrusive methods, since these are most widely applied in the wind energy community (an exception is the work of Fluck and Crawford [28]).

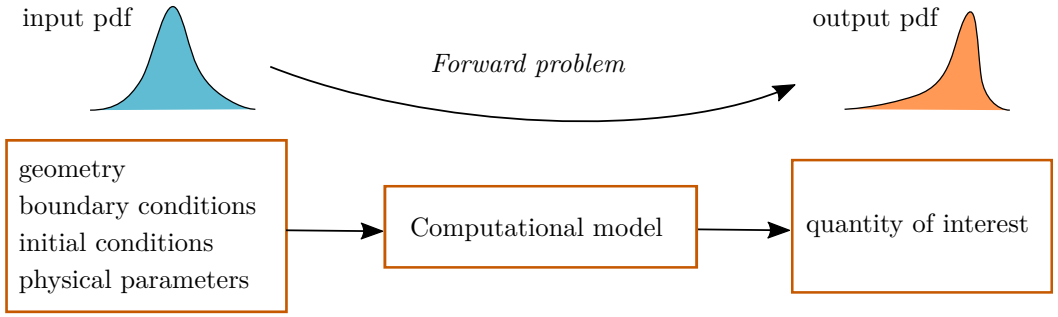

Figure 3.1: Uncertainty propagation with black box models.

3.2 Uncertainty propagation via sampling-based methods

In this report we focus on uncertainty propagation via sampling-based, non-intrusive methods. We will discuss Monte Carlo methods, polynomial-based methods, and Gaussian process regression. We are mainly interested in the pdf associated with a QoI given the pdfs for input parameters (figure 3.1). Methods which perform well for constructing moments of the solution (such as moment methods and Taylor expansion methods), are therefore less applicable or have shown to perform less well [64].

Both the input parameters Q and the QoI Y are random variables, which are related through the model via

$$Y = f(x, Q),$$

where x represents other (deterministic) variables, such as the independent variables space and time. Realizations of the random variable Q are denoted by $q = Q(\omega)$, where ω is an event from the sample space Ω.[2]

[2] Example: counting number of heads when tossing two coins [95]. Q is then the random variable representing the number of heads in the toss, Ω the sample space with possible outcomes, $(H,H),(T,H),(H,T),(T,T)$, and an example event is $\omega = (H,H)$ for which $Q(\omega) = 2$.

3.2.1 Monte Carlo methods

Monte Carlo is the simplest and most well-known sampling technique. It is based on drawing *random* samples from the probability density function $\rho_Q(q)$ of the random variable(s) Q (e.g. via inverse transform sampling). The mean of the quantity of interest Y is approximated via

$$\mu_Y = \mathbb{E}[Y] \approx \frac{1}{N}\sum_{i=1}^{N} f(x, q_i),$$

with N the number of samples. The error in μ_Y decreases with $1/\sqrt{N}$, independent of the dimensionality of the problem (i.e., the number of random variables). Improved sampling techniques are for example Quasi Monte Carlo and Latin Hypercube Sampling (LHS) [77]. Although Monte Carlo methods are versatile, robust and easy to implement and have a strong mathematical theory, they typically require many individual model runs. This can yield a prohibitively large computational expense.

3.2.2 Polynomial chaos expansion

In contrast to Monte Carlo methods, where samples are random, and carry the same weight, polynomial chaos expansions are pseudospectral methods that use a smarter choice of sampling points. This choice is based on the fact that the pdf $\rho_Q(q)$ associated to the random variable Q defines a set of orthogonal polynomials $\phi_i(q)$ [36]:

$$\int \phi_i(q)\phi_j(q)\rho_Q(q)\mathrm{d}q = \begin{cases} \gamma_i & i = j, \\ 0 & i \neq j. \end{cases}$$

An example of a normally distributed (Gaussian) random variable with mean 2 and standard deviation 1 is shown in figure 3.2. The associated orthogonal polynomials are the *Hermite* polynomials shown in figure 3.3.

The quantity of interest Y can be represented in terms of these orthogonal polynomials as

$$Y = f(x, q) \approx f_{\mathrm{PCE}}(x, q) = \sum_{i=0}^{N} \hat{f}_i(x)\phi_i(q), \tag{3.1}$$

the so-called generalized Polynomial Chaos (gPC) expansion [109]. The problem has shifted to the determination of the coefficients $\hat{f}_i(x)$:

$$\hat{f}_i(x) = \frac{1}{\gamma_i}\int f(x, q)\phi_i(q)\rho_Q(q)\mathrm{d}q.$$

This integral is typically estimated using a quadrature rule:

$$\int f(x, q)\phi_i(q)\rho_Q(q)\mathrm{d}q \approx \sum_{k=1}^{K} f(x, q_k)\phi_i(q_k)w_k,$$

where q_k and w_k are the nodes and weights of the quadrature rule, respectively, representing the nodes where the model $f(x, Q)$ has to be evaluated (sampled). The quadrature rule can for example be based on the zeros of the polynomial basis. The nodes can be reused for

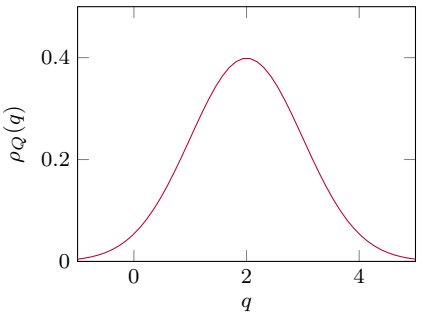

Figure 3.2: Normally distributed random variable.

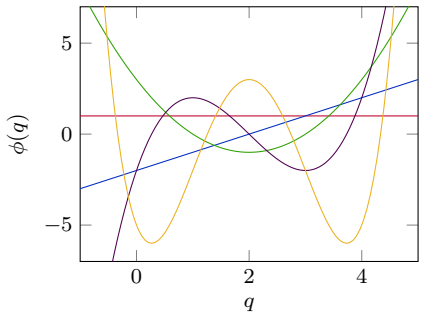

Figure 3.3: First 5 Hermite polynomials orthogonal to the random variable in 3.2.

computing each $\hat{f}_i$, so the number of model evaluations is K. If the error in the quadrature rule is negligible, the expansion (3.1) possesses spectral convergence. As an alternative to determining the coefficients with quadrature methods, regression methods can be used [111].

The resulting polynomial $f_{\text{PCE}}(x,q)$ is an analytical approximation of the real model $f(x,q)$, from which estimates of the statistics of interest can be obtained. The polynomial approximation can also serve as a cheap 'surrogate' model for solving inverse or optimization problems.

3.2.3 Stochastic collocation (interpolation)

Another polynomial method, also based on smart sampling and constructing an approximating polynomial, is the Stochastic Collocation (SC) method[3]. Instead of using the gPC expansion (3.1), an interpolation polynomial is directly built based on the model evaluations $f(x,q_i)$:

$$f(x,q) \approx f_{\text{SC}}(x,q) = \sum_{i=1}^{N} L_i(q) f(x,q_i), \tag{3.2}$$

where $L_i(q)$ are Lagrange interpolation polynomials,

$$L_i(q) = \prod_{\substack{k=1 \\ k\neq i}}^{N} \frac{q-q_k}{q_i-q_k}.$$

The interpolant is exact in the nodes, i.e. $f_{\text{SC}}(x,q_i) = f(x,q_i)$. An important choice is where to place the nodes q_i; typical examples are Clenshaw-Curtis nodes or Gauss nodes. Equidistant nodes should generally not be used as they can give unstable interpolants (see figure 3.4).

Similar to the PCE method, the interpolant can be used instead of the full model to get relevant statistical properties. Smith [95] gives a comparison between stochastic collocation and polynomial chaos expansions. One advantage of SC over PCE is that it does not rely on construction of orthogonal polynomials and hence is simpler to apply to general parameter distributions, including dependent variables. This makes it also well suitable for model calibration. An advantage of PCE over SC is that from the PCE coefficients one can directly obtain the sensitivity indices (see section 3.3). A disadvantage of both SC and PCE is that the accuracy of the interpolation polynomial or quadrature rule degrades as the dimension of the problem increases. Furthermore, the model under consideration should have sufficient smoothness to prevent over- and undershoots (see figure 3.5). A comparison study [21] concluded that both methods perform very similar, although when differences are present, SC is a consistent winner.

[3] Some authors reserve the term PCE for intrusive methods [109], and instead denote non-intrusive PCE methods as collocation methods, distinguishing between 'interpolation' (what we call SC) and 'discrete projection' (what we call PCE).

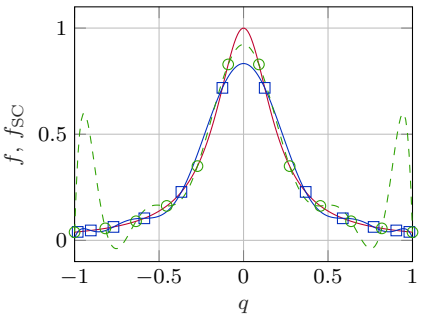

Figure 3.4: Stochastic collocation applied to Runge function. In red original model, in blue interpolation with Clenshaw-Curtis nodes, in green with equidistant nodes.

3.2.4 Gaussian process regression (Kriging)

An alternative to the polynomial methods from sections 3.2.2-3.2.3 is Gaussian process regression, also known as Kriging. The idea is to first construct a Gaussian process assuming a certain covariance matrix and then to regress it using model evaluations (or measurement data) with

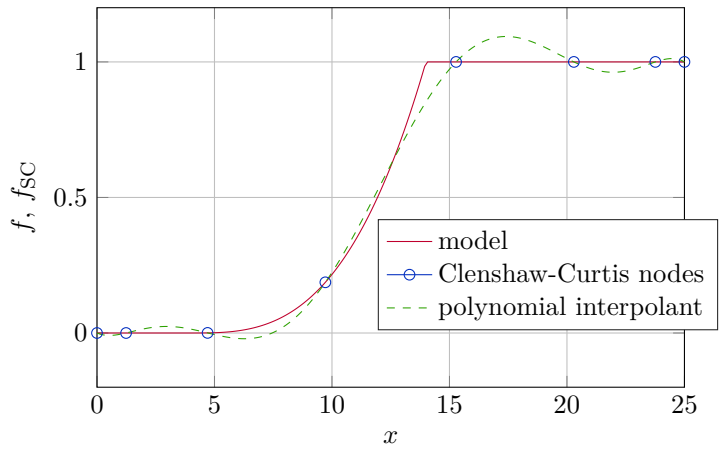

Figure 3.5: Example of stochastic collocation for polynomial reconstruction of a model $f(x)$. In a typical wind energy application, this could represent the power curve with x the wind speed and f the normalized power.

a Gaussian noise structure. In a Bayesian context (further explained in section 3.4.1) these are the prior and likelihood, respectively, and the resulting Kriging model is the posterior. A main advantage of Kriging is that not only a surrogate is constructed, but that also an error estimate is obtained (the covariance of the posterior). Furthermore, the extension to multiple dimensions is straightforward and it is relatively easy to incorporate additional knowledge. Disadvantages are the computational cost associated with the inversion of the covariance matrix and the choice of the so-called hyperparameters: correlation length in the likelihood model and variance of the noise. An example of Kriging is shown in figure 3.6, where the model $f(x)$ is evaluated at a number of points (the training data) and a Gaussian process f_{GP} with mean and covariance is constructed through the training data points.

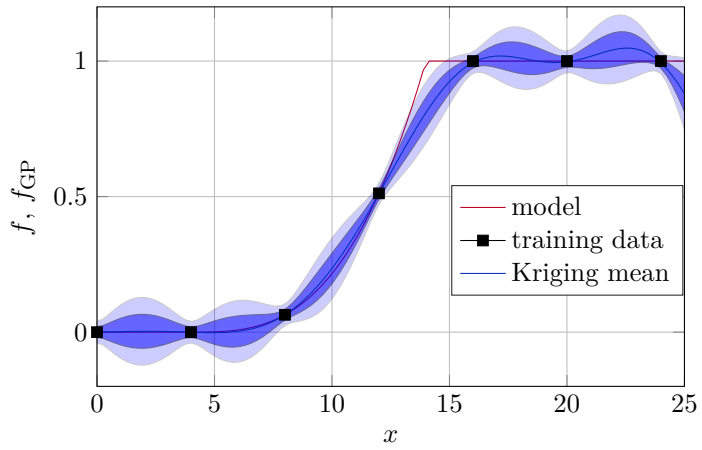

Figure 3.6: Example of Gaussian process regression applied to the example of figure 3.5, for specific choice of hyperparameters. Shaded regions are one and two standard deviations from the mean.

3.2.5 Multivariate extensions and the curse of dimensionality

In many applications, the number of uncertain parameters is larger than one, and the PCE and SC methods require multivariate extension. The most straightforward extension is to use tensor products (employing one-dimensional rules in each dimension), but this leads to exponential growth of the required number of samples (figure 3.7). For example, with 5 random variables and 10 samples for each variable, 10^5 samples are

necessary; this is the *curse of dimensionality*. Monte Carlo methods do not suffer from this. A widely used alternative for tensor product grids is to use so-called sparse grids (also called Smolyak grids [96]), which have the same polynomial accuracy as tensor product grids, but a significantly lower cost (figure 3.8). The multi-dimensional case is much more difficult and involved than the one-dimensional case, and consequently the formulation of high-dimensional interpolation and integration methods is an active field of research.

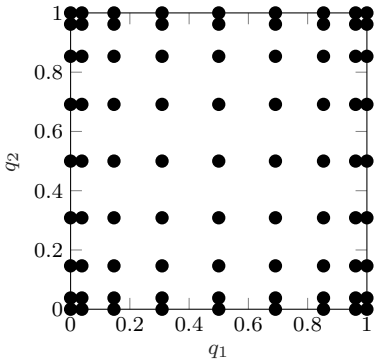

Figure 3.7: Two-dimensional tensor grid based on one-dimensional Clenshaw-Curtis nodes.

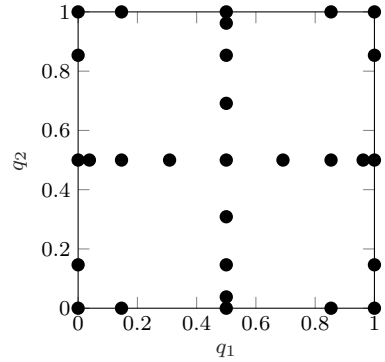

Figure 3.8: Two-dimensional sparse grid based on one-dimensional Clenshaw-Curtis nodes.

3.3 Sensitivity analysis

Sensitivity analysis is a central topic in the field of uncertainty quantification. It can be used, for example, to determine what parameters have a large influence on the model output and what parameters have little effect and can potentially be fixed at their nominal values in uncertainty propagation studies. A main distinction is between *local* and *global* sensitivity analysis. Local sensitivities are typically defined via partial derivatives: the change of the model output with respect to a change in inputs at a given condition. This can be computed for example with finite difference approximations and adjoint methods. Global sensitivity analysis, on the other hand, is concerned with determining what part of the model output can be attributed to the model input, over the entire range of input values. A widely used technique in global sensitivity analysis is based on an analysis of variance (ANOVA) decomposition of the model. This decomposition naturally leads to the definition of sensitivity indices, the so-called Sobol' indices [98]. The first order indices measure which part of the variance of the model output is related to a certain input, while higher order indices also measure interaction effects. For models that can be written as a sum of submodels, each depending on a single Gaussian random variable, the ANOVA decomposition yields the same results as summation of variances [95].

3.4 Calibration

The uncertainty propagation and sensitivity techniques from section 3.2-3.3 allow the study of parametric uncertainties on quantities of interest. In principle, this can be done with the model only, i.e. without using experimental data or field data. However, when such data is also taken into account, one can calibrate the models by estimating parameters in order to obtain more accurate predictions under uncertainties. Calibration can be seen as an inverse or *backward problem*, see figure 3.9, in contrast to the forward approach from figure 3.1. A common way to solve calibration problems is by using regression analysis, for example with ordinary or non-linear least squares methods [95]. An alternative approach, which we discuss here, is to use Bayesian model calibration [53, 85, 86, 91].

3.4.1 Bayesian model calibration

Bayesian model calibration has been applied often in uncertainty quantification problems with parametric uncertainties [55]. The Bayesian framework models both aleatoric and epistemic uncertainties as probabilistic, i.e. the parameters to be calibrated are treated as random variables. The following ingredients are necessary: (i) *prior* knowledge, (ii) measurement data, (iii) a computational model, (iv) a statistical model, and (v) Bayes' formula. Using these ingredients, the parameters of the model are calibrated as random variables, resulting in the *posterior*. The posterior can be used to assess the uncertainty in the model and to make future predictions. The advantage of the Bayesian framework is that prior knowledge and the statistical model are naturally included in the formulation.

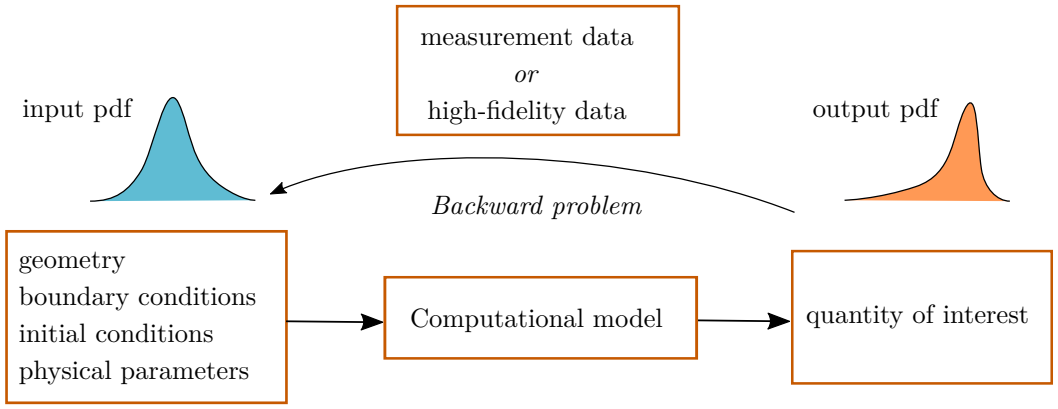

Figure 3.9: Uncertainty calibration with black box models.

This statistical model describes the relation between the data and the model in a probabilistic fashion. Kennedy and O'Hagan [53] introduced the following basic statistical model:

$$z_i = f(x, \boldsymbol{\theta}) + \epsilon_i,$$

where z is measurement data for observation i, f is again the model, $\boldsymbol{\theta}$ the parameters to be calibrated (similar to the random variables ξ used in section 3.2), x are known inputs, and ϵ_i encodes model and observation errors. A commonly used error term, $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$, assumes normally distributed errors and assumes that the model on average returns the correct data. It does not take spatial model errors into account (i.e., an error term depending on x).

The fifth ingredient is the application of the rule of Bayes (the law of conditional probabilities):

$$p(\boldsymbol{\theta}|\mathbf{z}) = \frac{p(\mathbf{z}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{z})}, \tag{3.3}$$

where $p(\boldsymbol{\theta})$ is prior information, $p(\mathbf{z}|\boldsymbol{\theta})$ the likelihood, $p(\boldsymbol{\theta}|\mathbf{z})$ the posterior and $p(\mathbf{z})$ the evidence. For the simple case that the error is assumed normally distributed and assuming that σ is given, the likelihood is given by:

$$p(\mathbf{z}|\boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2}\frac{(\mathbf{z} - f(\boldsymbol{\theta}))^2}{\sigma^2}\right], \tag{3.4}$$

where $\mathbf{z} = (z_1, \ldots, z_N)^{\mathrm{T}}$. Combining (3.3) and (3.4), evaluating $p(\boldsymbol{\theta}|\mathbf{z})$ up to a scaling constant is possible. It is however non-trivial to determine statistical moments, samples, and other statistical quantities, because only certain values of $p(\boldsymbol{\theta}|\mathbf{z})$ can be determined. Since the posterior does not have an explicit form, it is difficult to sample from it. Typically Markov chain Monte Carlo methods (MCMC) are used, but these methods can require thousands of sampling points before convergence of the posterior is achieved. This is prohibitively expensive given that each sampling point corresponds to an evaluation of the model. The downside of the Bayesian model calibration approach is therefore its computational expense. A possible solution is to use a surrogate of the full model f, for example the polynomial models f_{PCE} or f_{SC} introduced in equations (3.2) and (3.2).

3.4.2 Data assimilation

Data assimilation can be seen as the integration of mathematical-physical models with measurement data, typically to be performed in real-time. The classical example is the one of weather prediction, where weather simulation models are updated once new observations are made. Data assimilation can be seen as a Bayesian calibration method with a certain choice of prior and likelihood. The most basic data assimilation method, the linear Kálmán filter, is the best unbiased estimator assuming a linear model and linear observation operator, and Gaussian distributions for the errors in both the model and the data. In the extended Kálmán filter the restriction of linearity is relaxed by a linearisation of the model and the observation operator. In the ensemble Kálmán filter the aim is to reduce the computational cost of Kálmán filtering (associated with inverting the covariance operator) by Monte Carlo sampling [23].

3.5 Input parametrization and modal decompositions

In many practical applications, the number of random variables can be very large. For example, a turbulent inflow field can be modelled as a high-dimensional correlated random field with a certain power spectrum and covariance function (section 2.1.2). In a UQ context, such correlated random fields are typically approximated with a lower dimensional expansion, such as a Karhunen-Loève (KL) expansion. This expansion is determined by the eigenvalues and eigenvectors of the covariance function. The coefficients in the expansion constitute the random variables that can be used for uncertainty quantification studies.

The KL expansion is closely related to proper orthogonal decomposition (POD), principal component analysis (PCA), and singular value decomposition (SVD). In POD, for example, the covariance matrix is computed based on snapshot data. A matrix is created which consists of snapshots from a discrete velocity field u at different time instances t^i ($u^i \in \mathbb{R}^M$):

$$A = \begin{pmatrix} u^1 & \ldots & u^N \end{pmatrix},$$

and the covariance matrix is formed as $K = \frac{1}{N} A^T A$. An eigenvalue

problem is solved to find the eigenfunctions ϕ and eigenvalues λ of K, and a reduced model $\tilde{u}$ for the velocity field is formed by selecting the J most energetic components (corresponding to the largest singular values of A):

$$\tilde{u} = \sum_{j=1}^{J} a_j \phi_j.$$

Another, recently developed, modal decomposition technique is dynamic mode decomposition (DMD [93]). It has similarities with POD, but in contrast to POD (which is a purely statistical method), DMD assumes that the snapshots are generated by a dynamical system. The advantage over POD is that DMD can distinguish between different time scales, even if they have comparable energy content. A comparison between POD and DMD for studying the stability of tip vortices in wakes is given in [92].

3.6 Summary

In this chapter we have discussed some of the main components in uncertainty quantification for black box models. For *propagation* of uncertainty, Monte Carlo methods are robust and simple to implement, but (depending on the model properties) can be computationally inefficient compared to surrogate methods such as polynomial chaos expansion, stochastic collocation, and Gaussian process regression. These surrogate methods are useful also for solving the backward *calibration* problem, in which uncertainties are characterized based on the combination of model results and experimental data. A possible calibration technique is Bayesian model calibration, which has the advantage that prior information can be included and that the resulting parameters are characterized by a probability distribution, although it is generally computationally expensive. For characterization of high-dimensional problems modal decompositions such as a Karhunen-Loève expansion can be used to reduce the dimension of the random space.

4
UQ techniques for wind turbine applications

4.1 Introduction

Many wind energy studies have been performed in which the effect of input parameter variations on outputs has been assessed. For example, Porté-Agel et al. [83] studied the influence of the wind direction on the power output of a wind farm by simulating the Horns Rev wind farm with an LES model at many different inflow angles. It was shown that the power output is highly sensitive to wind direction: a change in wind direction of just 10° from the worst case (full wake) conditions, can result in a power output change of 43%. Although this is an important result, this type of parameter variation study is often not denoted as uncertainty quantification, because the probability distribution associated with inputs is not taken into account. In UQ, the wind direction would be seen as a random variable with an associated probability density function. For example, in [33], the wind direction is assumed to be normally distributed and wake effects are studied based on this distribution. With a weighted average over several simulations, the measurement data could be much better represented than in the case of single wind direction simulations. It is concluded that earlier reported discrepancies between models and data were not just caused by inaccuracy of the models, but also by large uncertainty in the experimental datasets. The benefit of using UQ, in which probabilities are included in simulations, is therefore to significantly increase the accuracy and confidence in simulation outcomes. However, this comes at an increased computational cost - in this study, 'cheap' engineering wake models were used, and a large number of model evaluations could be easily performed. For more complex RANS or LES models a similar analysis might not be tractable.

The first studies in the wind community in which the probability distribution of uncertain parameters is explicitly taken into account, use a relatively simple technique: summation of individual variances of the uncertainties (the root-sum-square method). This assumes that all uncertainties have a Gaussian distribution (see section 3.3). Lackner et al. [60, 61] use this method for estimating the uncertainty in wind resource, in wind turbine power output and energy production, and in overall AEP. The assumptions of independent, Gaussian uncertainties is however limiting in many applications. In this chapter we give an overview of more advanced methods that have been applied recently in

wind turbine applications.

4.2 Monte Carlo techniques

A straightforward improvement of the root-sum-square method of [60], which allows for correlated and non-Gaussian inputs, is to use Monte Carlo sampling. For example, Engelen [22] use a Monte Carlo approach to study the non-linear effect of (correlated) uncertainties in array efficiency and availability on the energy yield of a wind farm. Multiple non-Gaussian uncertainty sources are also addressed by [59], who study the effect of wind velocity, surface roughness, air density on the wind energy potential with a Monte Carlo method. Uncertainties in the model itself, namely in the power performance curve, are also included in their work, and a good comparison between measured and computed standard deviation of the AEP was found. Similarly, a probabilistic description of the power curve in combination with a Monte Carlo technique is used in [49], where it is assumed that the power curve follows a normal distribution with a varying mean and a constant standard deviation.

The construction of a joint distribution function for the correlated variables wind direction and wind speed is discussed in Feng and Shen [24]. A bivariate distribution (a joint distribution function of wind speed and wind direction) is constructed based on interpolation of histogram data, and used for layout optimization with a random search algorithm.

4.3 Approaches from reliability engineering

In the field of reliability engineering the question arises what the probability is of a wind turbine failing, given uncertainty in external conditions, components, etc. This is a large field of research which has similarities to uncertainty quantification and we will therefore mention a few approaches. An early account of reliability and failure of components in wind farms is given in [102], where the difference between common and independent failure causes is addressed. More recently, Veldkamp [103, 104] gives a comprehensive overview of the effect of several uncertainties on wind turbine loads, specifically on fatigue loads. Five different groups of uncertainties are described and characterized with probability density functions: wind climate, sea climate, aerodynamics, structural model, and material fatigue properties. These uncertainties are used to compute failure probabilities by both a Monte Carlo method and a First Order Reliability Method (FORM). The FORM consists of transforming the random variables to standard normal distributions and linearising the objective function, and is shown to be faster than the Monte Carlo method. Similarly, Hu et al. [42] perform a reliability analysis for wind turbine blades under wind load uncertainty, and create an optimal fatigue-reliable design with minimum costs. Monte Carlo sampling is used to calculate the probability of failure and a Kriging-based surrogate model is built to perform the optimisation.

4.4 Propagation with surrogate models

The Monte Carlo techniques mentioned in sections 4.2 and 4.3 can be computationally very expensive, because many model evaluations are often required to obtain accurate results. Consequently, potentially more efficient techniques, such as the surrogate model techniques described in sections 3.2.2-3.2.4, have recently gained interest in the wind energy community.

The stochastic collocation (SC) method is applied by Petrone et al. [82] to study the effect of uncertainties in meteorological conditions, insect contamination and manufacturing tolerances on sound pressure level and turbine power coefficient. The curse of dimensionality is alleviated by using a so-called simplex stochastic collocation method, which extends the interpolation idea of stochastic collocation to higher dimensions by using a triangulation of the random space [107, 108], which has the advantage of being able to adaptively refine in regions of interest. This method is also used to optimize turbine design [80, 81]. Another approach for reducing the curse of dimensionality in stochastic collocation is to use an adaptive sparse (Smolyak) grid, as done by Guo [39]. He studies the effect of wind speed on turbine response and the effect of uncertainties in blade material properties on deformation and stresses of turbine blades and on failure probabilities. The input random variables are based on a parametrization of the turbulent wind field (see sections 3.5 and 4.6). A stochastic collocation method is also used by Rinker [87], who constructs a four-dimensional polynomial interpolant to be able to calculate the sensitivity of turbine load response to turbulence parameters (called response surface in [87]). Monte Carlo methods are applied to this response surface model to calculate the global sensitivity (Sobol') indices. It is found that the majority of the variance in the turbine loading can be attributed to the variance in the free stream wind speed and turbulence intensity. In this case the computational costs are manageable since the number of samples used for each random variable is relatively small (5-10). The polynomial degree of the interpolant is limited to five to prevent overfitting.

Similar to SC, the polynomial chaos expansion (PCE) has been used in wind energy applications to study uncertainty propagation. The basic ingredient in PCE, which also forms the main difference with SC, is building the orthogonal polynomial basis. A non-trivial example of building the polynomial basis for a sector-wise Weibull distribution for the wind speed and for a uniform distribution for the wind direction is given in [74]. In order to handle the different wind sectors, a multi-element PCE method is used, in which the random space is divided in several elements, and in each element locally a polynomial expansion is constructed. Murcia et al. [74] use this PCE to predict uncertainty in the AEP of a wind farm. The main role of PCE in this work is as an integration rule with respect to a probability density function, and it is indeed much more efficient than existing methods (which are based on the trapezoidal method). This is confirmed by the results of Padrón et al. [78], who take a very similar approach as [74], and use the analytic

polynomial expression to optimize the layout of an offshore wind farm. Issues with PCE may appear near discontinuities, where a polynomial method can show under- and overshoots. Murcia et al. [74] show how oscillations appear in the polynomial approximation of the power curve, especially in the region of rated power or close to discontinuities (similar to figure 3.5). Limiting the polynomial order such as in [87] can alleviate this issue. Liu et al. [65] employ a very low order (second order) PCE, which compares well to a coarse Monte Carlo simulation (only 129 samples are used). They investigate the effect of uncertainties in pressure coefficients and force coefficients of turbine blades due to uncertainties in angle of attack and wind speed, by using 2D and 3D CFD simulations. Foti [30, 31] also uses a relatively low-order (fourth order) PCE in two random variables and shows that this already gives more accurate results than a Monte Carlo approach that uses thousands of samples. He finds a significant effect of uncertainties in ground roughness and induction factor on the power production of a wind farm.

Ashuri et al. [6] take an approach related to the polynomial methods just mentioned. In their uncertainty quantification technique they expand the probability density function of the quantity of interest (in their case the LCOE) in terms of a Gram-Charlier series, whose coefficients are determined via numerical integration. To reduce computational costs, the multi-dimensional model describing the power output is assumed to consist of a sum of univariate models, reducing the multi-dimensional integrals to one-dimensional integration over each uncertain parameter. These integrals are approximated with Gaussian quadrature methods, with the nodes and weights chosen based on the probability density functions of the random input variables (similar to PCE). With this technique, they investigate the influence of meteorological conditions (wind shear, air density, average wind speed, Weibull shape parameter) on the LCOE.

Kriging (Gaussian process regression, section 3.2.4), has been used to a lesser extent than the polynomial methods. Wang et al. [106] consider Kriging to combine low- and high-fidelity wake models into an accurate surrogate model for predicting power output and AEP. The low-fidelity model is the semi-analytical Larsen model, and the high-fidelity model is a RANS model. Co-Kriging, in which data from both the low- and high-fidelity model is used, is shown to give better predictions than normal Kriging. It also turns out that building a surrogate model based on wind speed is easier than wind direction, since the model is smoother with respect to wind speed than to wind direction. A continuous pdf of wind direction is therefore recommended in order to improve the analysis. Some of the common issues with Kriging, such as the expensive inversion of the covariance matrix, and determination of the hyperparameters, are not explicitly mentioned in this article.

4.5 Intrusive methods

Fluck and Crawford [28] are one of the few sources who develop an intrusive UQ method: a stochastic Galerkin method is used to create a

stochastic blade loading model. This offers the significant advantage that the entire stochastic solution is found at once without having to resort to multiple deterministic realizations. However, in case of nonlinear models the coupled system of equations that has to be solved can become large and intractable. In later work, Fluck and Crawford [29] focus on the construction of a reduced order model that generates a turbulent wind field based on fewer random variables than the commonly used Veers model, and that can be used for example as input for the stochastic aerodynamic model in [28].

4.6 Data-driven models and model calibration

In the previous sections, uncertainty quantification was mainly model-driven. The data-driven approach, in which data is incorporated to calibrate or tune models has recently gained significant interest in wind and wake modelling.

One approach, still mainly based on physical modelling, is to assume a physical-mathematical model with certain free parameters and to tune the model parameters based on experimental data (or high-fidelity simulations). For example, Iungo et al. [46] tune the mixing length parameter of a RANS turbulence model by using LES data and obtain a computationally efficient wake model. Similarly, Gebraad et al. [35] extend the Jensen wake model to include yaw effects and calibrate the parameters of the model based on LES data. This parametric model is used in an optimization algorithm, and the resulting optimal yaw settings are subsequently tested in an LES study. Van Buren et al. [100] are one of the few who perform the calibration based on Bayesian inference (section 3.4.1). They develop a simplified finite element model for a turbine blade. A sensitivity analysis based on ANOVA is carried out first to identify the most important parameters influencing blade vibrations. Subsequently, the parameters in the simplified model are calibrated with Bayesian inference techniques (using MCMC). To alleviate the computational costs associated with MCMC a Gaussian process is used to emulate the finite element model.

A completely different, purely data-driven, approach is to use snapshots of turbulent velocity fields from experiments or CFD simulations and to decompose these into lower dimensional models with methods such as POD (section 3.5). These lower dimensional models can then be evaluated at a fraction of the cost of the original model. For example, Bastine et al. [9] employ POD to construct stochastic wake models based on LES data and show that with only six modes (and a homogeneous turbulence field for small scales) the wake and the turbine load dynamics on large time scales can be reproduced. Doubrawa et al. [19] also construct a stochastic wake model based on LES data. The stochastic model is formed by a combination of a Fourier transform of the LES data for the mean flow, and an autoregressive model (see section 2.1.3) for temporal perturbations. In contrast to many engineering wake models, this synthetic wake has the advantage that it reproduces the mean characteristics of the original LES wake, and is asymmetric and unsteady. Guo [39] uses

a bi-orthogonal decomposition (to separate temporal components from coupled spatial-stochastic modes) and Karhunen-Loève decomposition (to decouple the spatial and stochastic components) to construct a low-fidelity wind model, based on wind snapshots from experimental data. A synthetic wind field is then constructed by randomly sampling from the low-fidelity model.

A third approach, roughly in between the previous two, is to only assume that the underlying physics can be modelled as a dynamical system, so that the dynamic mode decomposition (DMD) technique can be applied [31, 45]. Foti [31] uses an LES database to create a DMD and studies the effect of the nacelle and of the operating regime on wake meandering. Iungo et al. [45] use DMD to build a reduced order model for wakes based on high fidelity LES results. The resulting model is updated with (synthetic) data using a Kálmán filter. These models have the potential to be used for real-time control and optimization of operational wind farms.

4.7 Optimization and control under uncertainty

Many techniques outlined in the previous sections have as ultimate goal to obtain a turbine or farm design, or a control strategy, that has an optimal energy yield or minimal costs. This leads to an optimization problem. Most optimization techniques currently used are gradient-based [16, 27, 34, 38, 54], or involve approaches from game theory [68], Bayesian optimization [79], particle swarm optimization [10], or evolutionary (genetic) algorithms. We will not go into details of optimization algorithms, but rather discuss the recent trend of including uncertainty in the optimization process. OUU, optimization under uncertainty [88], is a relatively new field with only a few applications in the wind energy community. The goal of OUU is to obtain *robust designs*, i.e. designs which not only perform well at a specific condition, but also when conditions are uncertain.

Petrone [80, 81] develops several methods for optimization under uncertainty and applies them to turbine blade optimization given insect contamination. The resulting design is more robust towards uncertainties than the blade shape that resulted from a deterministic optimal design. Campobasso et al. [13] similarly obtain an optimal blade design (in terms of aerodynamic performance) with respect to uncertainties in the blade geometry, by using Monte Carlo sampling and univariate reduced quadrature methods, and a BEM method for the aerodynamic analysis. Quick et al. [84] employed OUU with a different goal, namely optimization of rated power, rotor diameter, and hub height of a wind farm with respect to LCOE, given uncertainty in the wind resource. For the case under consideration, the probabilistic optimum design is not very different from the deterministic design, except when assuming unusually large uncertainties. These optimization studies, especially when including uncertainties, are computationally very expensive and therefore are mainly performed with engineering models. Instead of engineering models, one can rely on reduced order models that are based on high fidelity simulations or

accurate experimental data. Examples are the surrogate models based on polynomial techniques or on Kriging mentioned in section 4.4, or the data-driven methods like POD or DMD mentioned in section 4.6.

Besides optimization, the development of fast and accurate models is also important for real-time control applications [5, 57]. Most farm controllers use open-loop control [25, 26] with simplified models of the aerodynamic interactions between turbines: for example, the static models mentioned in section 4.6 [35, 46], or simple dynamic models (e.g. dynamic Park [5]). The wake interaction models need to contain a parameter variation methodology to be applicable in a control setting. Furthermore, real-time updates of the model state based on measurements are necessary to prevent the model predictions from drifting. This is typically done using the data assimilation techniques mentioned in section 3.4.2 [17, 45]. The control objective is often a reduction in fatigue loading or power optimization and is performed by axial induction control or yaw control [57]; typical controller parameters are the blade pitch, the thrust or axial induction factor, and the yaw angle of the turbines. An alternative objective is to follow so-called power set points ('active power control') [3] that are determined by the grid operator. In any case, combining control and optimization with uncertainty predictions is an ongoing and challenging research area with a large potential impact on turbine operation and design.

5
Summary and outlook

In this report we have reviewed the existing literature for uncertainty quantification (UQ) for wind energy applications. Uncertainties are omnipresent in the wind industry, both in the external conditions (such as wind and waves) as well as in the models that are used to predict key quantities such as costs, energy yield and fatigue loads. The models currently in use are often based on a statistical approach or on a physical modelling approach. The statistical approach has classically been the basis for many wind and wave models. For example, time series of wind or waves are often generated by randomly sampling from measured power spectra. Physics-based models, on the other hand, are increasing in fidelity and popularity due to continuing advances in computational methods and computational power. However, even though such models can reduce the epistemic (systematic) uncertainties in predictions, stochastic (aleatoric) uncertainties inevitably remain.

The field of UQ offers several approaches for dealing with both types of uncertainties. A commonly used approach is the Monte Carlo method, which is robust but often too expensive due to its slow convergence, requiring many model evaluations. More advanced UQ methods, such as polynomial chaos expansion or Gaussian process regression, are becoming more popular and are being applied to estimate the effect of parametric uncertainties. An example is the uncertainty in the annual energy prediction of a wind farm given uncertainties in wind speed and direction. Generally, only a few uncertain parameters have been taken into account in most studies. This is due to the curse of dimensionality: when many uncertain parameters are present, the number of required model evaluations grows exponentially. Sparse grid techniques and surrogate models are employed to significantly reduce this cost. Global sensitivity studies are very useful to find which input parameters contribute most to a quantity of interest and which parameters can be left out of the analysis.

Next to the model-driven uncertainty studies, interest in the wind community is growing in the use of data-driven techniques for the derivation of simplified models. This data is often obtained from high-fidelity models, although experimental data can in principle be used as well. An example is the generation of a synthetic wind field by random sampling from a modal decomposition of LES data. In general, however, the approach of using data to calibrate models is an inverse problem for which

the issue of computational costs and of problem dimensionality becomes even more pressing than for the forward problem. This has received little attention so far. Bayesian model calibration, an elegant method for solving inverse problems which can take into account prior information, is a possible candidate but needs acceleration to be applicable for cases of practical interest.

The eventual goal behind many uncertainty quantification studies is to understand and/or reduce uncertainties with the aim of obtaining an optimal design that is robust under uncertainties. The field of optimization under uncertainty is still in its infancy in the wind energy sector, and is expected to grow significantly. The optimization algorithm adds an additional layer of complexity on top of the high costs already associated with UQ. The major challenge is therefore, again, to make this computationally tractable. Consequently, the development of accurate but cheap surrogate models is a field of active research, with strong benefits to the forward, the backward and the optimization problem.

5.1 Acknowledgements

This report is part of the Dutch research program EUROS (Excellence in Uncertainty Reduction of Offshore wind Systems), which is supported by NWO domain Applied and Engineering Sciences and partly funded by the Dutch Ministry of Economic Affairs. The main goal of EUROS is to assess the influence of uncertainties on wind turbine design and planning in order to achieve significant cost reductions. A key concept in this program, highlighted also in this report, is a paradigm shift to a probabilistic approach in wind turbine modelling.

Bibliography

[1] P. Agarwal and L. Manuel. On the modeling of nonlinear waves for prediction of long-term offshore wind turbine loads. *Journal of Offshore Mechanics and Arctic Engineering*, 131, 2009. doi:10.1115/1.3160647.

[2] P. Agarwal and L. Manuel. Wave models for offshore wind turbines. In *46th AIAA Aerospace Sciences Meeting and Exhibit*. American Institute of Aeronautics and Astronautics (AIAA), 2008. doi:10.2514/6.2008-1336.

[3] J. Aho, L. Pao, and P. Fleming. An active power control system for wind turbines capable of primary and secondary frequency control for supporting grid reliability. In *51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*. AIAA 2013-0456, 2013. doi:10.2514/6.2013-456.

[4] S. Akpinar and E. K. Akpinar. Estimation of wind energy potential using finite mixture distribution models. *Energy Conversion and Management*, 50(4):877–884, 2009. doi:10.1016/j.enconman.2009.01.007.

[5] J. Annoni. *Modeling for Wind Farm Control*. PhD thesis, University of Minnesota, 2016.

[6] T. Ashuri, T. Zhang, D. Qian, and M. Rotea. Uncertainty quantification of the levelized cost of energy for a 20 MW research wind turbine model. In *34th Wind Energy Symposium*, number January, pages 1–11, Reston, Virginia, 2016. American Institute of Aeronautics and Astronautics. doi:10.2514/6.2016-1998.

[7] C. J. Bai and W. C. Wang. Review of computational and experimental approaches to analysis of aerodynamic performance in horizontal-axis wind turbines (HAWTs). *Renewable and Sustainable Energy Reviews*, 63:506–519, 2016. doi:10.1016/j.rser.2016.05.078.

[8] M. Bastankhah and F. Porté-Agel. A new analytical model for wind-turbine wakes. *Renewable Energy*, 70:116–123, 2014. doi:10.1016/j.renene.2014.01.002.

[9] D. Bastine, L. Vollmer, M. Wächter, and J. Peinke. Stochastic wake modeling based on POD analysis. *Wind Energy Science Discussions*, (November):1–36, 2016. doi:10.5194/wes-2016-38.

[10] A. Behnood, H. Gharavi, B. Vahidi, and G. H. Riahy. Optimal output power of not properly designed wind farms, considering wake effects. *Electrical Power and Energy Systems*, 63:44–50, 2014. doi:10.1016/j.ijepes.2014.05.052.

[11] S.-P. Breton, J. Sumner, J. N. Sørensen, K. S. Hansen, S. Sarmast, and S. Ivanell. A survey of modelling methods for high-fidelity wind farm simulations using large eddy simulation. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 375, 2017. doi:10.1098/rsta.2016.0097.

[12] T. Burton, N. Jenkins, D. Sharpe, and E. Bossanyi. *Wind Energy Handbook*. John Wiley & Sons, Ltd., 2001.

[13] M. S. Campobasso, E. Minisci, and M. Caboni. Aerodynamic design optimization of wind turbine rotors under geometric uncertainty. *Wind Energy*, 19(1):51–65, 2016. doi:10.1002/we.1820.

[14] J. Carta, P. Ramírez, and S. Velázquez. A review of wind speed probability distributions used in wind energy analysis: Case studies in the Canary Islands. *Renewable and Sustainable Energy Reviews*, 13(5):933–955, 2009. doi:10.1016/j.rser.2008.05.005.

[15] J. A. Carta, P. Ramirez, and C. Bueno. A joint probability density function of wind speed and direction for wind energy analysis. *Energy Conversion and Management*, 49(6):1309–1320, 2008. doi:10.1016/j.enconman.2008.01.010.

[16] M. T. van Dijk, J.-W. van Wingerden, T. Ashuri, and Y. Li. Wind farm multi-objective wake redirection for optimizing power production and loads. *Energy*, 121:561–569, 2017. doi:10.1016/j.energy.2017.01.051.

[17] B. M. Doekemeijer, S. Boersma, L. Y. Pao, and J. W. van Wingerden. Ensemble Kalman filtering for wind field estimation in wind farms. In *2017 American Control Conference (ACC)*, pages 19–24. IEEE, 2017. doi:10.23919/ACC.2017.7962924.

[18] A. S. S. Dorvlo. Estimating wind speed distribution. *Energy Conversion and Management*, 43(17):2311–2318, 2002. doi:10.1016/S0196-8904(01)00182-0.

[19] P. Doubrawa, R. J. Barthelmie, H. Wang, and M. J. Churchfield. A stochastic wind turbine wake model based on new metrics for wake characterization. *Wind Energy*, 20(3):449–463, 2017. doi:10.1002/we.2015.

[20] M. Drela and M. B. Giles. Viscous-inviscid analysis of transonic and low Reynolds number airfoils. *AIAA Journal*, 25(10):1347–1355, 1987. doi:10.2514/3.9789.

[21] M. Eldred and J. Burkardt. Comparison of non-intrusive polynomial chaos and stochastic collocation methods for uncertainty quantification. In *47th AIAA Aerospace Sciences Meeting including*

The New Horizons Forum and Aerospace Exposition, volume 976. American Institute of Aeronautics and Astronautics (AIAA), 2009. doi:10.2514/6.2009-976.

[22] C. M. Engelen. The nonlinear effect of combining uncertainties on the energy yield of an offshore wind farm. Master's thesis, Delft University of Technology, 2015.

[23] G. Evensen. *Data Assimilation: The Ensemble Kalman Filter*. Springer, 2009. doi:10.1007/978-3-642-03711-5.

[24] J. Feng and W. Shen. Modelling wind for wind farm layout optimization using joint distribution of wind speed and wind direction. *Energies*, 8(4):3075–3092, 2015. doi:10.3390/en8043075.

[25] P. Fleming, P. M. Gebraad, S. Lee, J.-W. van Wingerden, K. Johnson, M. Churchfield, J. Michalakes, P. Spalart, and P. Moriarty. Simulation comparison of wake mitigation control strategies for a two-turbine case. *Wind Energy*, 18(12):2135–2143, 2015. doi:10.1002/we.1810.

[26] P. Fleming, J. Aho, P. Gebraad, L. Pao, and Y. Zhang. Computational fluid dynamics simulation study of active power control in wind plants. *American Control Conference*, pages 1413–1420, 2016. doi:10.1109/ACC.2016.7525115.

[27] P. A. Fleming, A. Ning, P. M. O. Gebraad, and K. Dykes. Wind plant system engineering through optimization of layout and yaw control. *Wind Energy*, 19(2):329–344, 2016. doi:10.1002/we.1836.

[28] M. Fluck and C. Crawford. A stochastic aerodynamic model for stationary blades in unsteady 3D wind fields. *Journal of Physics: Conference Series*, 753, 2016. doi:10.1088/1742-6596/753/8/082009.

[29] M. Fluck and C. Crawford. An engineering model for 3D turbulent wind inflow based on a limited set of random variables. *Wind Energy Science Discussions*, (March):1–20, 2017. doi:10.5194/wes-2017-7.

[30] D. Foti, X. Yang, and F. Sotiropoulos. Uncertainty quantification of infinite aligned wind farm performance using non-intrusive polynomial chaos and a distributed roughness model. *Wind Energy*, 20 (6):945–958, 2017. doi:10.1002/we.2072.

[31] D. V. Foti. *Quantification and reduction of uncertainty of model predictions of wind turbines and plants via high-fidelity simulations*. PhD thesis, University of Minnesota, 2016.

[32] S. Frandsen, R. Barthelmie, S. Pryor, O. Rathmann, S. Larsen, J. Højstrup, and M. Thøgersen. Analytical modelling of wind speed deficit in large offshore wind farms. *Wind Energy*, 9(1-2):39–53, 2006. doi:10.1002/we.189.

[33] M. Gaumond, P.-E. Réthoré, S. Ott, A. Peña, A. Bechmann, and K. S. Hansen. Evaluation of the wind direction uncertainty and its impact on wake modeling at the Horns Rev offshore wind farm. *Wind Energy*, 17(8):1169–1178, 2013. doi:10.1002/we.1625.

[34] P. M. O. Gebraad and J. W. van Wingerden. Maximum power-point tracking control for wind farms. *Wind Energy*, 18(3):429–447, 2015. doi:10.1002/we.1706.

[35] P. M. O. Gebraad, F. W. Teeuwisse, J. W. Van Wingerden, P. A. Fleming, S. D. Ruben, J. R. Marden, and L. Y. Pao. A data-driven model for wind plant power optimization by yaw control. *Proceedings of the American Control Conference*, pages 3128–3134, 2014. doi:10.1109/ACC.2014.6859118.

[36] R. G. Ghanem and P. D. Spanos. *Stochastic Finite Elements: A Spectral Approach*. Springer Science + Business Media, 1991. doi:10.1007/978-1-4612-3094-6.

[37] Global Wind Energy Council. Global Wind Report - Annual Market Update 2016. Technical report, 2017.

[38] J. P. Goit and J. Meyers. Optimal control of energy extraction in wind-farm boundary layers. *Journal of Fluid Mechanics*, 768:5–50, 2015. doi:10.1017/jfm.2015.70.

[39] Q. Guo. *Incorporating Stochastic Analysis in Wind Turbine Design: Data-Driven Random Temporal-Spatial Parameterization and Uncertainty Quantification*. PhD thesis, Iowa State University, 2013.

[40] M. O. L. Hansen and H. A. Madsen. Review paper on wind turbine aerodynamics. *Journal of Fluids Engineering*, 133, 2011. doi:10.1115/1.4005031.

[41] L. H. Holthuijsen. *Waves in Oceanic and Coastal Waters*. Cambridge University Press, 2007. doi:10.1017/cbo9780511618536.

[42] W. Hu, K. K. Choi, H. Cho, N. J. Gaul, and O. I. Zhupanska. Reliability-based design optimization of wind turbine blades for fatigue life under wind load uncertainty. In L. Qing, P. Grand, and L. Zhongpu, editors, *Advances in Structural and Multidisciplinary Optimization, Proceedings of the 11th World Congress on Structural and Multidisciplinary Optimization*, pages 547–555. International Society for Structural and Multidisciplinary Optimization (ISSMO), 2015.

[43] IEC. Wind turbines – part 1: Design requirements. Technical Report 61400-1 Ed. 3, International Electrotechnical Commission, 2005.

[44] IEC. Wind turbines – part 3: Design requirements for offshore wind turbines. Technical Report 61400-3 Ed. 1, International Electrotechnical Commission, 2009.

[45] G. V. Iungo, C. Santoni-Ortiz, M. Abkar, F. Porté-Agel, M. A. Rotea, and S. Leonardi. Data-driven Reduced Order Model for prediction of wind turbine wakes. *Journal of Physics: Conference Series*, 625:012009, 2015. doi:10.1088/1742-6596/625/1/012009.

[46] G. V. Iungo, F. Viola, U. Ciri, M. a. Rotea, and S. Leonardi. Data-driven RANS for simulations of large wind farms. *Journal of Physics: Conference Series*, 625:012025, 2015. doi:10.1088/1742-6596/625/1/012025.

[47] O. A. Jaramillo and M. A. Borja. Wind speed analysis in La Ventosa, Mexico: a bimodal probability distribution case. *Renewable Energy*, 29(10):1613–1630, 2004. doi:10.1016/j.renene.2004.02.001.

[48] N. O. Jensen. A note on wind generator interaction. Technical Report 2411, Risø National Laboratory Roskilde, Denmark, 1983.

[49] T. Jin and Z. Tian. Uncertainty analysis for wind energy production with dynamic power curves. *2010 IEEE 11th International Conference on Probabilistic Methods Applied to Power Systems*, pages 745–750, 2010. doi:10.1109/PMAPS.2010.5528405.

[50] J. Jung and R. P. Broadwater. Current status and future advances for wind speed and power forecasting. *Renewable and Sustainable Energy Reviews*, 31:762–777, 2014. doi:10.1016/j.rser.2013.12.054.

[51] S. Kang, X. Yang, and F. Sotiropoulos. On the onset of wake meandering for an axial flow turbine in a turbulent open channel flow. *Journal of Fluid Mechanics*, 744:376–403, 2014. doi:10.1017/jfm.2014.82.

[52] A. Keane, P. E. O. Aguirre, H. Ferchland, P. Clive, and D. Gallacher. An analytical model for a full wind turbine wake. *Journal of Physics: Conference Series*, 753:032039, 2016. doi:10.1088/1742-6596/753/3/032039.

[53] M. C. Kennedy and A. O'Hagan. Bayesian calibration of computer models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 63(3):425–464, 2001. doi:10.1111/1467-9868.00294.

[54] R. N. King, K. Dykes, P. Graf, and P. E. Hamlington. Optimization of wind plant layouts using an adjoint approach. *Wind Energy Science*, 2(1):115–131, 2017. doi:10.5194/wes-2-115-2017.

[55] P. K. Kitanidis. Parameter uncertainty in estimation of spatial functions: Bayesian analysis. *Water Resources Research*, 22(4):499–507, 1986. doi:10.1029/wr022i004p00499.

[56] A. D. Kiureghian and O. Ditlevsen. Aleatory or epistemic? Does it matter? *Structural Safety*, 31(2):105–112, 2009. doi:10.1016/j.strusafe.2008.06.020.

[57] T. Knudsen, T. Bak, and M. Svenstrup. Survey of wind farm control-power and fatigue optimization. *Wind Energy*, 18(8):1333–1351, 2015. doi:10.1002/we.1760.

[58] G. A. M. van Kuik, J. Peinke, R. Nijssen, D. Lekou, J. Mann, J. N. Sørensen, C. Ferreira, J. W. van Wingerden, D. Schlipf, P. Gebraad, H. Polinder, A. Abrahamsen, G. J. W. van Bussel, J. D. Sørensen, P. Tavner, C. L. Bottasso, M. Muskulus, D. Matha, H. J. Lindeboom, S. Degraer, O. Kramer, S. Lehnhoff, M. Sonnenschein, P. E. Sørensen, R. W. Künneke, P. E. Morthorst, and K. Skytte. Long-term research challenges in wind energy - a research agenda by the European Academy of Wind Energy. *Wind Energy Science*, 1(1):1–39, 2016. doi:10.5194/wes-1-1-2016.

[59] S.-D. Kwon. Uncertainty analysis of wind energy potential assessment. *Applied Energy*, 87(3):856–865, 2010. doi:10.1016/j.apenergy.2009.08.038.

[60] M. Lackner, A. Rogers, and J. Manwell. Uncertainty analysis in wind resource assessment and wind energy production estimation. In *45th AIAA Aerospace Sciences Meeting and Exhibit*. American Institute of Aeronautics and Astronautics (AIAA), 2007. doi:10.2514/6.2007-1222.

[61] M. A. Lackner, A. L. Rogers, and J. F. Manwell. Uncertainty Analysis in MCP-Based Wind Resource Assessment and Energy Production Estimation. *Journal of Solar Energy Engineering*, 130 (3):031006, 2008. doi:10.1115/1.2931499.

[62] G. C. Larsen. A simple wake calculation procedure. Technical Report 2760, Risø National Laboratory Roskilde, Denmark, 1988.

[63] O. Le Maître and O. Knio. *Spectral Methods for Uncertainty Quantification*. Springer, 2010. doi:10.1007/978-90-481-3520-2.

[64] S. H. Lee and W. Chen. A comparative study of uncertainty propagation methods for black-box-type problems. *Structural and Multidisciplinary Optimization*, 37(3):239–253, 2009. doi:10.1007/s00158-008-0234-7.

[65] Z. Liu, X. Wang, and S. Kang. Stochastic performance evaluation of horizontal axis wind turbine blades using non-deterministic CFD simulations. *Energy*, 73:126–136, 2014. doi:10.1016/j.energy.2014.05.107.

[66] P. Madsen, H. Bingham, and H. Liu. A new Boussinesq method for fully nonlinear waves from shallow to deep water. *Journal of Fluid Mechanics*, 462:1–30, 2002. doi:10.1017/S0022112002008467.

[67] J. Mann. Wind field simulation. *Probabilistic Engineering Mechanics*, 13(4):269–282, 1998. doi:10.1016/s0266-8920(97)00036-2.

[68] J. R. Marden, S. D. Ruben, and L. Y. Pao. A model-free approach to wind farm control using game theoretic methods. *IEEE Transactions on Control Systems Technology*, 21(4):1207–1214, 2013. doi:10.1109/TCST.2013.2257780.

[69] E. Marino, C. Borri, and U. Peil. A fully nonlinear wave model to account for breaking wave impact loads on offshore wind turbines. *Journal of Wind Engineering and Industrial Aerodynamics*, 99(4): 483–490, 2011. doi:10.1016/j.jweia.2010.12.015.

[70] E. Marino, C. Lugni, and C. Borri. A novel numerical strategy for the simulation of irregular nonlinear waves and their effects on the dynamic response of offshore wind turbines. *Computer Methods in Applied Mechanics and Engineering*, 255:275–288, 2013. doi:10.1016/j.cma.2012.12.005.

[71] D. Mehta, A. van Zuijlen, B. Koren, J. Holierhoek, and H. Bijl. Large Eddy Simulation of wind farm aerodynamics: A review. *Journal of Wind Engineering and Industrial Aerodynamics*, 133: 1–17, 2014. doi:10.1016/j.jweia.2014.07.002.

[72] M. B. van der Meulen, T. Ashuri, G. J. W. van Bussel, and D. P. Molenaar. Influence of nonlinear irregular waves on the fatigue loads of an offshore wind turbine. *The Science of Making Torque from Wind*, pages 1–10, 2012. doi:10.13140/2.1.3034.5606.

[73] E. C. Morgan, M. Lackner, R. M. Vogel, and L. G. Baise. Probability distributions for offshore wind speeds. *Energy Conversion and Management*, 52(1):15–26, 2011. doi:10.1016/j.enconman.2010.06.015.

[74] J. P. Murcia, P. E. Réthoré, A. Natarajan, and J. D. Sørensen. How many model evaluations are required to predict the AEP of a wind power plant? *Journal of Physics: Conference Series*, 625, 2015. doi:10.1088/1742-6596/625/1/012030.

[75] H. N. Najm. Uncertainty quantification and polynomial chaos techniques in computational fluid dynamics. *Annual Review of Fluid Mechanics*, 41(1):35–52, 2009. doi:10.1146/annurev.fluid.010908.165248.

[76] W. L. Oberkampf, J. C. Helton, and K. Sentz. Mathematical representation of uncertainty. *American Institute of Aeronautics and Astronautics Non-Deterministic Approaches Forum*, (April): 1–23, 2001. doi:http://dx.doi.org/10.2514/6.2001-1645.

[77] A. B. Owen. *Monte Carlo Theory, Methods, and Examples.* http://statweb.stanford.edu/~owen/mc/, 2013.

[78] A. S. Padrón, A. P. J. Stanley, J. J. Thomas, J. J. Alonso, and A. Ning. Polynomial chaos for the computation of annual energy production in wind farm layout optimization. *Journal of Physics: Conference Series*, 753:032021, 2016. doi:10.1088/1742-6596/753/3/032021.

[79] J. Park and K. H. Law. A Bayesian optimization approach for wind farm power maximization. volume 9436, page 943608, 2015. doi:10.1117/12.2084184.

[80] G. Petrone. *Optimization under Uncertainty*. PhD thesis, Università degli studi di Napoli Federico II, 2012.

[81] G. Petrone, C. de Nicola, D. Quagliarella, J. Witteveen, J. Axerio-Cilies, and G. Iaccarino. Wind turbine optimization under uncertainty with high performance computing. *41st AIAA Fluid Dynamics Conference, Honolulu, Hawaii*, (June):1–16, 2011.

[82] G. Petrone, C. de Nicola, D. Quagliarella, J. Witteveen, and G. Iaccarino. Wind turbine performance analysis under uncertainty. In *49th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*. American Institute of Aeronautics and Astronautics (AIAA), 2011. doi:10.2514/6.2011-544.

[83] F. Porté-Agel, Y.-T. Wu, and C.-H. Chen. A numerical study of the effects of wind direction on turbine wakes and power losses in a large wind farm. *Energies*, 6(10):5297–5313, 2013. doi:10.3390/en6105297.

[84] J. Quick, K. Dykes, P. Graf, and F. Zahle. Optimization under uncertainty of site-specific turbine configurations. *Journal of Physics: Conference Series*, 753:062012, 2016. doi:10.1088/1742-6596/753/6/062012.

[85] J. Ray, Z. Hou, M. Huang, and L. P. Swiler. Bayesian calibration of the community land model using surrogates. Technical report, Sandia National Laboratories, 2014.

[86] J. Ray, S. Lefantzi, S. Arunajatesan, and L. J. DeChant. Bayesian calibration of a RANS model with a complex response surface - a case study with jet-in-crossflow configuration. In *45th AIAA Fluid Dynamics Conference*. American Institute of Aeronautics and Astronautics (AIAA), 2015. doi:10.2514/6.2015-2784.

[87] J. M. Rinker. Calculating the sensitivity of wind turbine loads to wind inputs using response surfaces. *Journal of Physics: Conference Series*, 753:032057, 2016. doi:10.1088/1742-6596/753/3/032057.

[88] N. V. Sahinidis. Optimization under uncertainty: state-of-the-art and opportunities. *Computers & Chemical Engineering*, 28(6-7): 971–983, 2004. doi:10.1016/j.compchemeng.2003.09.017.

[89] B. Sanderse, S. P. van der Pijl, and B. Koren. Review of computational fluid dynamics for wind turbine wake aerodynamics. *Wind Energy*, 14(7):799–819, 2011. doi:10.1002/we.458.

[90] J. Sanz Rodrigo, R. A. Chávez Arroyo, P. Moriarty, M. Churchfield, B. Kosović, P.-E. Réthoré, K. S. Hansen, A. Hahmann, J. D. Mirocha, and D. Rife. Mesoscale to microscale wind farm flow

modeling and evaluation. *Wiley Interdisciplinary Reviews: Energy and Environment*, 6(2):e214, 2017. doi:10.1002/wene.214.

[91] K. Sargsyan, H. N. Najm, and R. Ghanem. On the statistical calibration of physical models. *International Journal of Chemical Kinetics*, 47(4):246–276, 2015. doi:10.1002/kin.20906.

[92] S. Sarmast, R. Dadfar, R. F. Mikkelsen, and P. Schlatter. Mutual inductance instability of the tip vortices behind a wind turbine. *Journal of Fluid Mechanics*, 755:705–731, 2014. doi:10.1017/jfm.2014.326.

[93] P. J. Schmid. Dynamic mode decomposition of numerical and experimental data. *Journal of Fluid Mechanics*, 656:5–28, 2010. doi:10.1017/S0022112010001217.

[94] S. Schreck, J. Lundquist, and W. Shaw. U.S. Department of Energy Workshop Report: Research Needs for Wind Resource Characterization. Technical report, NREL/TP-500-43521, 2008.

[95] R. Smith. *Uncertainty Quantification, Theory, Implementation, and Applications*. SIAM, 2014.

[96] S. Smolyak. Quadrature and interpolation formulas for tensor products of certain classes of functions. *Soviet Mathematics, Doklady*, 4:240–243, 1963.

[97] H. Snel. Review of aerodynamics for wind turbines. *Wind Energy*, 6(3):203–211, 2003. doi:10.1002/we.97.

[98] I. M. Sobol'. Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. *Mathematics and Computers in Simulation*, 55:271–280, 2001.

[99] T. J. Sullivan. *Introduction to Uncertainty Quantification*. Springer International Publishing, 2015. doi:10.1007/978-3-319-23395-6.

[100] K. L. Van Buren, M. G. Mollineaux, F. M. Hemez, and S. Atamturktur. Simulating the dynamics of wind turbine blades: part II, model validation and uncertainty quantification. *Wind Energy*, 16(5):741–758, 2013. doi:10.1002/we.1522.

[101] P. S. Veers. Three-dimensional wind simulation. Technical Report SAND88–0152 UC–261, Sandia National Laboratories, 1988.

[102] P. Veers. All wind farm uncertainty is not the same: the economics of common versus independent causes. *Windpower*, (May 2000), 1995.

[103] D. Veldkamp. *Chances in Wind Energy, A Probabilistic Approach to Wind Turbine Fatigue Design*. PhD thesis, Delft University of Technology, 2006.

[104] D. Veldkamp. A probabilistic evaluation of wind turbine fatigue design rules. *Wind Energy*, 11(6):655–672, 2008. doi:10.1002/we.287.

[105] H. F. Veldkamp and J. van der Tempel. Influence of wave modelling on the prediction of fatigue for offshore wind turbines. *Wind Energy*, 8(1):49–65, 2005. doi:10.1002/we.138.

[106] Y. M. Wang, P.-E. Réthoré, M. P. van der Laan, J. P. M. Leon, Y. Q. Liu, and L. Li. Multi-fidelity wake modelling based on Co-Kriging method. *Journal of Physics: Conference Series*, 753: 032065, 2016. doi:10.1088/1742-6596/753/3/032065.

[107] J. A. S. Witteveen and G. Iaccarino. Simplex elements stochastic collocation for uncertainty propagation in robust design optimization. In *48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*. American Institute of Aeronautics and Astronautics (AIAA), 2010. doi:10.2514/6.2010-1313.

[108] J. A. S. Witteveen and G. Iaccarino. Simplex stochastic collocation with random sampling and extrapolation for nonhypercube probability spaces. *SIAM J. Sci. Comput.*, 34(2):A814–A838, 2012. doi:10.1137/100817504.

[109] D. Xiu and G. Karniadakis. The Wiener-Askey polynomial chaos for stochastic differential equations. *SIAM Journal on Scientific Computing*, 24(2):619–644, 2002.

[110] D. Xiu. Fast numerical methods for stochastic computations: a review. *Communications in computational physics*, 5(2-4):242–272, 2009.

[111] D. Xiu. *Numerical Methods for Stochastic Computations*. Princeton University Press, 2010.

[112] Y. Zhang, J. Wang, and X. Wang. Review on probabilistic forecasting of wind power generation. *Renewable and Sustainable Energy Reviews*, 32:255–270, 2014. doi:10.1016/j.rser.2014.01.033.

[113] Y. Zhang, J. Wang, and X. Wang. Review on probabilistic forecasting of wind power generation. *Renewable and Sustainable Energy Reviews*, 32(March 2016):255–270, 2014. doi:10.1016/j.rser.2014.01.033.

[114] E. Zio and N. Pedroni. Methods for representing uncertainty - a literature review. Technical report, numéro 2013-03 des Cahiers de la Sécurité Industrielle, Fondation pour une Culture de Sécurité Industrielle, Toulouse, France, 2013.