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## q-SPECIAL FUNCTIONS AND THEIR OCCURRENCE IN QUANTUM GROUPS

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## SUMMARY

In classical Lie group theory irreducible representation spaces are often decomposed into subspaces belonging to irreducible representations of a subgroup (or a chain of subgroups). Matrix elements of the representation w.r.t. a basis obtained in this way can often be expressed in terms of special functions. In particular, spherical functions are the matrix elements being left and right invariant w.r.t. a subgroup forming a Gelfand pair with the big group. An elementary example is the group SU(2) with diagonal subgroup isomorphic to U(1). Then the matrix elements of the irreducible representations of SU(2) w.r.t. a U(1)-basis can be expressed in terms of Jacobi polynomials and the spherical functions are Legendre polynomials. Nothing essential changes when we take the basis w.r.t. a subgroup conjugate to U(1), for instance SO(2).

For quantum groups the situation is different. Only few quantum subgroups are available, in general, and we cannot take conjugates of quantum subgroups. For instance, quantum SU(2) has a quantum subgroup U(1) (an ordinary group), but no other nontrivial quantum subgroups are known and no conjugates of U(1)can be taken. The matrix elements of the irreducible representations of quantum SU(2) have been computed (cf. [5], [4], [1]) as little q-Jacobi polynomials and the spherical matrix elements as little q-Legendre polynomials. (See Koelink & Koornwinder's q-special functions tutorial in these Proceedings.)

In order to say something about the missing conjugates of a quantum subgroup, we use the quantized universal enveloping algebra. Let  $\mathcal{U} = \mathcal{U}_q(sl(2))$ be the algebra generated by A,  $A^{-1}$ , B, C with relations  $AA^{-1} = 1 = A^{-1}A$ , AB = qBA, AC = qCA,  $[B, C] = (A^2 - A^{-2})/(q - q^{-1})$ .  $\mathcal{U}$  becomes a Hopf

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algebra with comultiplication  $\Delta(A) = A \otimes A$ ,  $\Delta(B) = A \otimes B + B \otimes A^{-1}$ ,  $\Delta(C) = A \otimes C + C \otimes A^{-1}$ . Call  $X \in \mathcal{U}$  twisted primitive if  $\Delta(X) = A \otimes X + X \otimes A^{-1}$ . Then X has this property iff X is a linear combination of  $A - A^{-1}$ , B and C. This three-dimensional space is a kind of quantum analogue of the Lie algebra sl(2).

The Hopf \*-algebra  $\mathcal{A}$  of "polynomial functions" on quantum SU(2) is in doubly non-degenerate Hopf algebra duality with  $\mathcal{U}$ . Call  $a \in \mathcal{A}$  left (right) invariant w.r.t. a twisted primitive element  $X \in \mathcal{U}$  if  $(X \otimes id)(\Delta(a)) = 0$  (respectively  $(id \otimes X)(\Delta(a)) = 0$ ).

Theorem ([2], [3]). Let  $X_1$  and  $X_2$  be twisted primitive elements in  $\mathcal{U}$ . Then the elements of  $\mathcal{A}$  being left invariant w.r.t.  $X_1$  and right invariant w.r.t.  $X_2$ form a subalgebra generated by a single element  $\rho$  of  $\mathcal{A}$  which is quadratic in the generators of  $\mathcal{A}$ . The intersection of this subalgebra with the subspace of  $\mathcal{A}$ belonging to an irreducible representation of quantum SU(2) of odd dimension 2l+1 is one-dimensional and spanned by a certain Askey-Wilson polynomial of degree l and argument  $\rho$ .

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144