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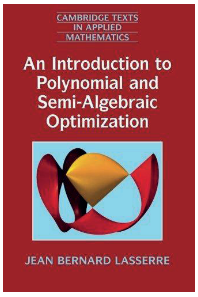

Jean Bernard Lasserre
An Introduction to Polynomial and Semi-Algebraic Optimization
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Reviewed by Monique Laurent

Polynomial optimisation revolves around the problem of minimising a polynomial function over a set defined by polynomial inequalities. While this problem belongs to the well established field of nonlinear optimisation, the accent here is on finding the global minimum rather than a local minimum only. A new approach for polynomial optimisation was introduced around 2000, exploiting real algebraic properties of positive polynomials and classical results from moment theory. Jean Bernard Lasserre, a main driving force behind a whole wealth of developments in this very active research field, presents in this book a comprehensive introduction to this approach. The main objective is exposing hierarchies of tractable relaxations for the polynomial optimisation problem and discussing their properties, applications and extensions. The book is written at a level aiming to be accessible to students, engineers and non-expert researchers. It includes selected exercises and a guide to the software GloptiPoly implementing the method, and could be used as graduate text. As some of the foundational results are presented without proof the reader will need to refer to the existing literature in order to complete his/her understanding of this fascinating theory.

The book has three parts. Part I exposes relevant results from real algebraic geometry and moment theory. These are rooted in classical works by Hilbert, characterising when multivariate polynomials are sums of squares of polynomials, and by Stieltjes, characterising univariate sequences of moments of measures. The author reviews these and many recent results, giving sums-of-squares type representations for polynomials positive over basic closed semi-algebraic sets. He presents the general approach for designing hierarchies of convex relaxations, which give lower bounds converging to the global minimum of the polynomial optimisation problem. A key feature is a concrete optimality criterion (based on some rank stability condition for moment sequences) which permits to conclude that a given relaxation is exact and to compute global minimisers of the original problem. The author concludes Part I with a hierarchy of outer approximations for the cone of positive polynomials, based on the moment approach.

Part II is dedicated to the study of these hierarchies of convex relaxations for the polynomial optimisation problem. Depending on the type of positivity certificate that is used the relaxations boil down to linear or semidefinite programs, for which efficient numerical solvers exist. The author explains and analyses the advantages of the various approaches. He also draws an interesting parallel between the classical KKT conditions in continuous optimisation and the sums-of-squares certificates used in polynomial optimisation. The author goes on with addressing how to exploit sparsity or symmetry structure of the problem, how to extend the approach to the minimisation of rational or semi-algebraic functions, and concludes with another use of the moment approach to get converging upper bounds that can be computed as general eigenvalue problems.

Part III presents several specialisations and extensions. In particular the author discusses properties for convex polynomial optimisation and how the approach can be applied to parametric polynomial optimisation, for finding convex under-estimators, and to problems whose feasibility region is defined by quantifiers. The book closes with a beautiful generalisation of the Löwner–John theorem, stating that for any degree d there is a unique homogeneous polynomial of degree d whose level set contains a given set K and has minimum volume. For degree 2 this level set is the celebrated minimum volume Löwner–John ellipsoid.