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FOUR MOLS OF ORDER 10 WITH A HOLE OF ORDER 2

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Abstract: A transversal design with hole $T[6;10]-T[6;2]$ is constructed from a separable group divisible design GD[5, 1, 8; 48].

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A long time ago I constructed the design $T[6;10]-T[6;2]$. This is a partial transversal design with six groups of size 10 and $10^2-2^2=96$ blocks such that two points from different groups are joined by a unique block unless they both are in the subset Y of the point set, where Y is a set meeting each group in precisely two points - or, in other words, four mutually orthogonal partial Latin squares of order 10 where in each of the squares the upper left hand 2×2 corner is empty.

Since it was buried in an internal note (Brouwer, 1978) containing obsolete tables of MOLS, and was not formulated in the language of MOLS with holes, and since people keep asking for the construction, let me give the construction explicitly.

0. Notation

I_m denotes a set of cardinality m. $\mathbb{Z}_m$ denotes the cyclic group of order m. A group divisible design $\mathrm{GD}[k,\lambda,m;v]$ (Hanani's notation) is a group divisible PBIB design with blocks of size k, groups of size m, v vertices (varieties), such that $\lambda_1=0$, $\lambda_2=\lambda$ (cf. Raghavarao (1971), p. 127). A transversal design $\mathrm{TD}[r;n]$ is a group divisible design $\mathrm{GD}[r,1,n;rn]$. It is well known that such a design is equivalent to $r-2$ mutually orthogonal Latin squares of order n. $T(r,1)$ is the set of natural numbers n such that there exists a $\mathrm{TD}[r;n]$.

1. Construction of a GD[5, 1, 8; 48]

Let $X=(I_2\times\mathbb{Z}_3)\times(\mathbb{Z}_2\times\mathbb{Z}_4)$. Take the groups

$$\{g\}\times(\mathbb{Z}_2\times\mathbb{Z}_4)$$

for $g\in I_2\times\mathbb{Z}_3$. Take the blocks

$$\{0000, 0102, 0210, 1011, 1112\}, \qquad \{0000, 0100, 0211, 1000, 1103\},$$

$$\{0000, 0101, 1002, 1102, 1211\}, \qquad \{0000, 0103, 1013, 1111, 1203\},$$

all mod $(-, 3, 2, 4)$ (where we write $ijkl$ instead of (i, j, k, l)). This yields the desired design.

2. Construction of a T[6, 1; 48]

In the previous section we constructed a pairwise balanced design $(X, \mathcal{B})$ on 48 points and with block sizes 5 and 8 in such a way that we have a tactical decomposition $\mathcal{B}=\mathcal{B}_0\cup\mathcal{B}_1\cup\mathcal{B}_2$ where $\mathcal{B}_1$ and $\mathcal{B}_2$ are symmetric 1-designs (with blocks of size 5) and $\mathcal{B}_0$ is a parallel class (consisting of blocks of size 8). (For: take for $\mathcal{B}_1$ the 1st and the 3rd orbits given, etc.) Since $5, 8\in T(6, 1)$ it follows that $48\in T(6, 1)$.

In the terminology of Raghavarao (1971) this design is separable (note that he calls our parallel classes 'Classes of type II', p. 36); now apply his Theorem 3.4.2.

Therefore we have 4 mutually orthogonal latin squares of order 48.

3. Construction of a T[6; 10] − T[6; 2]

Each of the blocks of the group divisible design constructed above misses a unique group. Enlarge each group $G_g=\{g\}\times(\mathbb{Z}_2\times\mathbb{Z}_4)$ with two new points (g, ∞_i) $(i=1, 2)$, and enlarge each block in $\mathcal{B}_i$ missing G_g with the point (g, ∞_i) $(i=1, 2,\ g\in I_2\times\mathbb{Z}_3)$. This produces the required design, where the hole is $Y=I_2\times\mathbb{Z}_3\times\{\infty_1, \infty_2\}$.

5039	8225	4568	9753	6812	0390	2487	7976	3141	1604
9460	6109	8336	5678	4947	7823	1091	3584	0252	2715
6748	9571	7219	8007	0685	5954	4830	2192	1363	3426
8114	7458	9642	4329	3293	1786	6965	5801	2070	0537
2856	4794	6083	3932	1479	8661	0128	9317	7505	5240
0903	3867	5495	7180	9024	2549	8772	1238	4616	6351
4281	1910	0874	6596	2308	9135	3659	8443	5727	7062
7697	5382	2921	1845	8550	3018	9206	0769	6434	4173
1522	2633	3700	0411	5166	6277	7344	4055	—	—
3375	0046	1157	2264	7731	4402	5513	6620	—	—

Fig. 1. Four MOLS of order 10 with a hole of size 2.

References

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