

## Computation of All the Amicable Pairs Below $10^{10}$

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**Abstract.** An efficient exhaustive numerical search method for amicable pairs is described. With the aid of this method all 1427 amicable pairs with smaller member below  $10^{10}$  have been computed, more than 800 pairs being new. This extends previous exhaustive work below  $10^8$  by H. Cohen. In three appendices (contained in the supplements section of this issue), various statistics are given, including an ordered list of all the gcd's of the 1427 amicable pairs below  $10^{10}$  (which may be useful in further amicable pair research). Suggested by the numerical results, a theorem of Borho and Hoffmann for constructing APs has been extended.

**1. Introduction.** Let  $\sigma(m)$  denote the sum of all the divisors of  $m$ , including 1 and  $m$ . An *amicable pair* (AP) is a pair of positive integers  $(m, n)$ ,  $m < n$ , such that  $\sigma(m) = \sigma(n) = m + n$ . We note that  $m$  is *abundant* (since  $\sigma(m) > 2m$ ) and that  $n$  is *deficient* (since  $\sigma(n) < 2n$ ). The smallest AP is

$$(220, 284) = (2^{25} \cdot 11, 2^2 \cdot 71).$$

In order to check whether or not a given positive integer  $m$  is the smaller member of an amicable pair, it seems necessary, at first sight, to compute  $\sigma(m)$  and  $n := \sigma(m) - m$ , to check whether  $n > m$  (i.e., whether  $m$  is abundant), and, if so, to compute  $\sigma(n)$  and compare  $\sigma(m)$  with  $\sigma(n)$ . This involves one or two complete factorizations, in case  $m$  is deficient or abundant, respectively. However, a closer look reveals that it is often possible to find out whether a given number  $m$  is deficient (hence cannot be the smaller member of an AP) without the need to factorize it completely. Moreover, once  $\sigma(m)$  and  $n (= \sigma(m) - m)$  have been computed, it is often possible to discover that  $\sigma(n) \neq \sigma(m)$  without the need to factorize  $n$  completely.

These considerations have guided the design of an efficient exhaustive numerical AP search algorithm, the details of which are given in Section 2. With the aid of this algorithm we have extended Cohen's exhaustive list of all 236 APs with smaller member below  $10^8$  [4] to all 1427 APs with smaller member below  $10^{10}$ . Of these, 601 have been published earlier [6], [7]. The other 826 seem to be new, and are published here for the first time (9 of them have been communicated to the author already in 1983 and 1984 by Woods (2), Borho (2) and Lee (5)). Section 3 presents details of the computations together with several tables collected from this search. Moreover, a result of Borho and Hoffmann for constructing APs is extended, as was suggested by the numerical tables.

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Three appendices to this paper appear in the supplements section of this issue. These may also be obtained by writing to the author.

In Appendix I, we present the complete list of all 1427 APs with smaller member below  $10^{10}$  ordered according to the size of the smaller members of the pairs. Appendix II displays the same list with a different ordering, viz., according to the various occurring types (defined in Section 3). Finally, Appendix III tabulates all the greatest common divisors of the 1427 APs, in increasing order, together with their frequencies of occurrence, and, for each gcd  $g$ , the rank numbers of all the APs  $(m, n)$  for which  $\gcd(m, n) = g$ .

**2. Check Whether a Given  $m$  is the Smaller Member of an AP.** Let  $p_i$  be the  $i$ th prime,  $P_{i,j} := \prod_{k=i}^{i+j-1} p_k$ ,  $Q_{i,j} := \prod_{k=i}^{i+j-1} p_k / (p_k - 1)$ . We start with the following lemma which gives an upper bound for  $\sigma(m)/m$ .

**LEMMA 2.1.** *If  $m$  only has prime divisors  $\geq p_i$  ( $i \geq 1$ ) and if  $m < P_{i,j+1}$  ( $j \geq 1$ ) then  $\sigma(m)/m < Q_{i,j}$ .*

*Proof.* Since  $m < P_{i,j+1} = p_i p_{i+1} \cdots p_{i+j}$ , and since any prime divisor of  $m$  is  $\geq p_i$ , it follows that  $m$  has at most  $j$  different prime divisors  $\geq p_i$  (otherwise we would have  $m \geq p_i p_{i+1} \cdots p_{i+j} = P_{i,j+1}$ ). This implies that

$$\frac{\sigma(m)}{m} = \prod_{p^e \parallel m} \frac{p^{e+1} - 1}{p^e(p-1)} = \prod_{p^e \parallel m} \frac{p - p^{-e}}{p-1} < \prod_{p \mid m} \frac{p}{p-1} \leq \prod_{k=i}^{i+j-1} \frac{p_k}{p_k-1} = Q_{i,j}. \quad \square$$

In the algorithm below, this lemma is invoked very frequently. Therefore, we require a precomputed table of  $P$ - and  $Q$ -values, large enough so that the values needed can be found quickly by simple table look-ups.

Now we describe an efficient algorithm to check whether a given positive integer  $m$  belongs to an AP  $(m, n)$  with  $m < n$ . This algorithm is based on the observation that when, for given  $\gamma$  and  $N$ , we want to verify one of the relations  $\sigma(N)/N > \gamma$ ,  $= \gamma$ ,  $< \gamma$ , and when the primes  $2, 3, \dots, p$  have been tried as divisors of  $N$ , it may be possible

(i) to detect, with Lemma 2.1, whether  $\sigma(N)/N < \gamma$  by using the information that the *unfactored* portion of  $N$  only has prime divisors  $> p$ , and

(ii) to detect whether  $\sigma(N)/N > \gamma$  by using the *factored* portion of  $N$ .

In this way, much unnecessary factorization time may be avoided. The price to pay for this gain lies in the time needed to consult the  $P$ - and  $Q$ -tables used in Lemma 2.1. In the algorithm, the index  $i_{\max}$  is the maximum value of  $i$  for which Lemma 2.1 is invoked. In order to restrict this table look-up time,  $i_{\max}$  should not be chosen too large. The optimal value of  $i_{\max}$  also depends on the actual implementation of the algorithm (cf. Section 3).

**Algorithm to Check Whether  $m$  is the Smaller Member of an AP.**

*Step 1.* (Find out whether  $m$  is abundant; in this step, keep  $m = m_1 m_2$  where  $\gcd(m_1, m_2) = 1$ ,  $m_1$  is the factored and  $m_2$  is the unfactored portion of  $m$ ,  $\alpha := \sigma(m_1)/m_1$ ; start with  $m_1 := 1$ ,  $m_2 := m$ ,  $\alpha := 1$ .)

Start factoring  $m$  by trial dividing  $m_2$  by the primes  $p_1, p_2, \dots \leq m_2^{1/2}$ . In case a prime power divisor  $p_{i-1}^e$  ( $e \geq 1$ ) of  $m_2$  has been found, update  $m_1$ ,  $m_2$  and  $\alpha$  ( $m_1 := m_1 p_{i-1}^e$ ,  $m_2 := m/m_1$ ,  $\alpha := \alpha \cdot \sigma(p_{i-1}^e)/p_{i-1}^e$ ). After the trial division with  $p_{i-1}$  (whether or not  $p_{i-1}$  divides  $m_2$ ): if  $\alpha < 2$  and  $4 \leq i \leq i_{\max}$ , check whether  $m$

is possibly deficient as follows: by inspecting the  $P$ -table find the smallest value of  $j$  ( $=:j^*$ ) such that  $m_2 < P_{i,j+1}$ ; if  $\alpha Q_{i,j^*} < 2$ , then STOP (because, in that case,  $m$  is deficient: by Lemma 2.1 we have  $\sigma(m_2)/m_2 < Q_{i,j^*}$  so that

$$\frac{\sigma(m)}{m} = \frac{\sigma(m_1)}{m_1} \cdot \frac{\sigma(m_2)}{m_2} = \alpha \frac{\sigma(m_2)}{m_2} < \alpha Q_{i,j^*} < 2).$$

If  $\alpha \geq 2$ , or  $i < 4$  or  $i > i_{\max}$ , the deficiency check on  $m$  is left out. After the complete factorization of  $m$  (and simultaneous computation of  $\sigma(m)$ ): if  $m < \sigma(m) - m =: n$  (i.e.,  $m$  is abundant), go to Step 2, otherwise STOP.

End of Step 1

*Step 2.* (Given  $m$ ,  $\sigma(m)$  and  $n = \sigma(m) - m$ , check whether  $\sigma(n) = \sigma(m)$ ; during the factorization of  $n$  try to exclude those  $m$  for which  $\sigma(n) \neq \sigma(m)$  as early as possible by testing whether  $\sigma(n)/n \neq \beta$  where  $\beta = \sigma(m)/n$ ; in this step, keep  $n = n_1 n_2$ , where  $\gcd(n_1, n_2) = 1$ ,  $n_1$  is the factored and  $n_2$  the unfactored portion of  $n$ ,  $\alpha := \sigma(n_1)/n_1$ ; start with  $n_1 := 1$ ,  $n_2 := n$ ,  $\alpha := 1$ .)

Start factoring  $n$  by trial dividing  $n_2$  by the primes  $p_1, p_2, \dots \leq n_2^{1/2}$ . In case a prime power divisor  $p_{i-1}^e$  ( $e \geq 1$ ) of  $n_2$  has been found, update  $n_1, n_2$  and  $\alpha$ : if the updated  $\alpha$  satisfies  $\alpha > \beta$ , then STOP (because, in that case, we have

$$\frac{\sigma(n)}{n} = \frac{\sigma(n_1)}{n_1} \frac{\sigma(n_2)}{n_2} \geq \frac{\sigma(n_1)}{n_1} = \alpha > \beta = \frac{\sigma(m)}{n},$$

so that  $\sigma(n) \neq \sigma(m)$ ). After the trial division with  $p_{i-1}$  (whether or not  $p_{i-1}$  divides  $n_2$ ): if  $4 \leq i \leq i_{\max}$  check whether  $\sigma(n)/n < \beta$  as follows: by inspecting the  $P$ -table find the smallest value of  $j$  ( $=:j^*$ ) such that  $n_2 < P_{i,j+1}$ . If  $\alpha Q_{i,j^*} < \beta$ , then STOP (because, in that case,  $\sigma(n)/n < \beta$ : by Lemma 2.1 we have  $\sigma(n_2)/n_2 < Q_{i,j^*}$  so that

$$\frac{\sigma(n)}{n} = \frac{\sigma(n_1)}{n_1} \cdot \frac{\sigma(n_2)}{n_2} = \alpha \frac{\sigma(n_2)}{n_2} < \alpha Q_{i,j^*} < \beta).$$

If  $i < 4$  or  $i > i_{\max}$ , the check on  $\sigma(n)/n < \beta$  is omitted. After the complete factorization of  $n$  (and simultaneous computation of  $\sigma(n)$ ): check whether  $\sigma(n) = \sigma(m)$ . If so,  $(m, n)$  is an AP.

End of Step 2

**3. Computing All the APs Below 10<sup>10</sup>.** In order to compute all the APs  $(m, n)$  with  $m < n$  and  $10^8 < m \leq 10^{10}$  (thus extending H. Cohen's computations reported in [4]), we distinguish between  $m \equiv 0 \pmod{6}$  (the easy case), and  $m \not\equiv 0 \pmod{6}$  (the hard case).

If  $m \equiv 0 \pmod{6}$  and  $n = \sigma(m) - m$  is even, then  $(m, n)$  cannot be an AP [5]. Therefore,  $n$  should be odd. In that case, we have [6]  $m = 2^\mu M^2$ ,  $n = N^2$ , with  $\mu \in \mathbb{N}$ ,  $M$  and  $N$  being odd. For all the numbers  $m = 2^\mu M^2$  with  $3 \mid M$  and  $10^8 < m \leq 10^{10}$ , we computed  $n := \sigma(m) - m$  and checked whether  $n$  was a perfect square. Not a single such case was found. Computer time was about 6 CPU seconds.

For all  $m \not\equiv 0 \pmod{6}$  with  $10^8 < m \leq 10^{10}$  we used the algorithm of Section 2 to find all APs in this range. The optimal choice of  $i_{\max}$  for our FORTRAN-implementation on a CYBER 750 was about 75. This value was chosen to be fixed for the whole range. The speed-up factor of our program was about 15, compared with a

straightforward program which, given  $m$ , computes  $\sigma(m)$  and, if  $n := \sigma(m) - m > m$ , computes  $\sigma(n)$ . A slight increase of the speed was obtained as follows. In Step 1, in case a prime (power) factor of  $m_2$  was found and  $m_1$  and  $\sigma(m_1)$  (among others) were updated, it was checked whether *both*  $m_1$  and  $\sigma(m_1)$  were divisible by one of the primitive abundant numbers  $20 = 2^2 \cdot 5$ ,  $28 = 2^2 \cdot 7$ ,  $70 = 2 \cdot 5 \cdot 7$  and  $88 = 2^3 \cdot 11$ . If so, the algorithm was stopped since this implied that also  $m$  and  $\sigma(m)$ , hence also  $n = \sigma(m) - m$  were divisible by this abundant number, so that both  $m$  and  $n$  were abundant. This is impossible for an AP  $(m, n)$ .

The total time to cover the range  $10^8 < m \leq 10^{10}$  was about 1000 (low priority) CPU hours, spent in the last seven months of 1984.

The total number of APs  $(m, n)$  found with  $m < n$  and  $10^8 < m \leq 10^{10}$  was 1191. In Appendix I (of the supplements section) all the APs with smaller member  $\leq 10^{10}$  are given (including the 236 APs with smaller member  $\leq 10^8$ ). For each pair we list the decimal representation and the prime factorization of the members, a rank number, a code (letter plus digit) referring to the discoverer, and the type of the pair (defined below). For example, pair #1427 reads as follows:

1427	9967523980	2E2.257.5.17.37.3083
R9 42	12890541236	2E2.257.107.117191.

Table 1 gives the meaning of the codes, and their frequencies of occurrence. Extensive information about the sources of the pairs with code L1 is given in the survey paper [6].

There are 1015 pairs with even members and 412 with odd members. The minimal and maximal values of  $m/n$  are 0.6979 and 0.999858 for the APs #567 and #1010, respectively.

Let  $A(x)$  be the number of APs  $(m, n)$  with  $m < n$  and  $m \leq x$ . From the list of APs with  $m \leq 10^8$ , Bratley et al. [3] concluded that for  $x \leq 10^8$ ,  $A(x)$  is approximately proportional to  $x^{1/2}/\ln(x)$ . In Table 2 we give, for  $x = k \cdot 10^9$  ( $1 \leq k \leq 10$ ):  $A(x)$ ,  $A(x)\ln(x)/x^{1/2}$ ,  $A(x)(\ln(x))^2/x^{1/2}$  and  $A(x)(\ln(x))^3/x^{1/2}$ . From these figures we may draw the conclusion that for  $x \leq 10^{10}$ ,  $A(x)$  is approximately proportional to  $x^{1/2}/(\ln(x))^3$ .

TABLE 1

*Status list of the first 1427 APs  $(m, n)$ ,  $m < n$ , with  $m \leq 10^{10}$*

code	# APs	references and remarks
L1	508	[6]
R2	1	[9] (#1056)
W1	73	sent to the author by D. Woods on June 29, 1982 and published in [7]
R3	19	found by the author with the methods described in [8], and published in [7]
W2	1	sent in by D. Woods on Feb. 16, 1983 (#330)
R6	1	found by the author in May, 1983 (#1375)
W3	1	sent in by D. Woods on July 11, 1983 (#1050)
L2	5	sent in by E. J. Lee in July, 1984 (## 778, 860, 894, 1241, 1261)
B4	2	sent in by W. Borho on Nov. 2, 1984 (## 809, 1393)
R9	816	found by the author during the systematic search described in this paper

TABLE 2

Comparison of  $A(x)$  with  $x^{1/2}/(\ln(x))^i, i = 1, 2, 3$

$x/10^9$	$A(x)$	$A(x)\ln(x)/x^{1/2}$	$A(x)(\ln(x))^2/x^{1/2}$	$A(x)(\ln(x))^3/x^{1/2}$
1	586	0.3840	7.958	164.9
2	762	0.3649	7.815	167.4
3	898	0.3578	7.807	170.4
4	1009	0.3527	7.799	172.4
5	1100	0.3474	7.759	173.3
6	1185	0.3444	7.755	174.6
7	1256	0.3403	7.715	174.9
8	1317	0.3358	7.656	174.6
9	1377	0.3327	7.625	174.8
10	1427	0.3286	7.566	174.2

We define an AP  $(m, n), m < n$ , to be a *regular amicable pair of type  $(i, j)$* , if  $(m, n) = (gM, gN)$ , where  $g = \gcd(m, n), \gcd(g, M) = \gcd(g, N) = 1, M$  and  $N$  are squarefree, and the numbers of prime factors of  $M$  and  $N$  are  $i$  and  $j$ , respectively. Other pairs are called *irregular* or *exotic*. There are 1082 regular and 345 irregular APs with smaller member  $\leq 10^{10}$ . It is easy to see that there are no regular pairs of type  $(1, j), j \geq 1$ : let  $g$  be the gcd of such an AP, so that  $(m, n) = (gp, gN)$  where  $p$  is a prime and  $\gcd(g, p) = \gcd(g, N) = 1$ . We have  $m < n$ , hence  $p < N$ . By definition,  $\sigma(gp) = \sigma(gN)$ , implying that  $p + 1 = \sigma(N)$ . Since, for any  $N \in \mathbb{N}, \sigma(N) > N$ , this implies that  $p + 1 > N$ , a contradiction. We note that in this argument  $N$  need not be squarefree.

In Table 3 we give the frequency distribution of the various types among the first 1082 regular APs. We note that there are relatively few regular APs of type  $(i, 1), i \geq 2$ , and of type  $(i, j)$  with  $i < j$ .

In [7] the total number of known APs with smaller member  $\leq 10^{10}$  was 601 (these are the APs belonging to the first four codes in Table 1). Among them were 104 irregular APs, i.e., 17.3%. Comparing this figure with the 345 irregular APs in our *complete* list of APs with smaller member  $\leq 10^{10}$ , i.e., 24.2%, we see that relatively many irregular APs were found in our systematic search.

In Appendix II (of the supplements section) we present lists of all the 1082 regular APs arranged according to their types, together with a list of the 345 exotic APs. This appendix may be useful for searches of APs of a special type.

The regular pairs of type  $(i, 1), i \geq 2$ , play an important role as “mother” pairs in methods to generate new APs from given pairs. In [8] a substantial part of the new APs found there was constructed from such mother pairs. In [1], Borho and Hoffmann have partially generalized the methods from [8] by introducing the concept of a *breeder*: a breeder is a pair of positive integers  $(a_1, a_2)$  such that the equations

$$a_1 + a_2x = \sigma(a_1) = \sigma(a_2)(x + 1)$$

TABLE 3  
 Frequency distribution of the first 1082 regular APs  
 of type  $(i, j)$ ,  $i \geq 2$ ,  $j \geq 1$

$i =$	$j =$	1	2	3	4	5	row totals
2		20	67	21	4	0	112
3		16	271	280	24	0	591
4		1	78	201	63	2	345
5		0	6	18	7	3	34
column totals		37	422	520	98	5	1082

have a positive integer solution  $x$ . If  $x$  is a prime, then  $(a_1, a_2x)$  is an amicable pair. For certain breeders, called "special" breeders, Borho and Hoffmann formulate the following

**THEOREM 1** [1]. *Let  $(a_1, a_2)$  be a special breeder, i.e.,  $a_1 = au$ ,  $a_2 = a$ , with  $\gcd(a, u) = 1$ . Take any factorization of  $C := \sigma(u)(u + \sigma(u) - 1)$  into two different factors  $D_1, D_2$  ( $C = D_1D_2$ ). Then, if the numbers  $s_i = D_i + \sigma(u) - 1$ , for  $i = 1, 2$ , and also  $q = u + s_1 + s_2$  are primes not dividing  $a$ , then  $(auq, as_1s_2)$  is an amicable pair.  $\square$*

Regular APs of type  $(i, 1)$ ,  $i \geq 2$ , are of the form  $(au, ap)$ ,  $p$  prime, and the numbers  $(au, a)$  are special breeders which generally produce many APs with the above theorem.

In our list of 1427 APs we found a few APs, e.g., #647 and #955, which suggested that the condition  $\gcd(a, u) = 1$  in Theorem 1 may be dropped. In fact, we have

**THEOREM 2.** *Let  $(au, a)$  be a breeder, i.e., there exists a positive integer  $x$  such that  $au + ax = \sigma(au) = \sigma(a)(x + 1)$ . Take any factorization of  $C := (x + 1)(x + u)$  into two different factors  $D_1, D_2$  ( $C = D_1D_2$ ). Then, if the numbers  $s_i = D_i + x$ , for  $i = 1, 2$ , and also  $q = u + s_1 + s_2$  are primes not dividing  $a$ , then  $(auq, as_1s_2)$  is an amicable pair.  $\square$*

The proof of this theorem is left to the reader.

If  $\gcd(a, u) = 1$ , then  $\sigma(au) = \sigma(a)\sigma(u)$ , so that  $x = \sigma(u) - 1$  and Theorem 2 reduces to Theorem 1. As an example, AP #955 gives the breeder  $(au, a)$  with  $a = 3.5.7.19$  and  $u = 7.29.47.181$ . Theorem 2 yields 16 new APs with this breeder as input.

It is known [5] that most even APs have a pair sum which is  $\equiv 0 \pmod{9}$ . Our search proves that indeed Poulet's pair #503:  $(2^4331.19.6619, 2^4331.199.661)$  is the smallest exceptional pair. All known exceptional pairs had members  $\equiv 7 \pmod{9}$  and a pair sum  $\equiv 5 \pmod{9}$ . In our search, we found two even APs with pair sum  $\equiv 3 \pmod{9}$ , viz., the (irregular) pairs:

$$\#577: 2^4 \begin{cases} 19^2 103.1627 \\ 3847.16763 \end{cases} \quad \text{and} \quad \#874: 2^{21} 9 \begin{cases} 13^2 37.43.139 \\ 41.151.6709. \end{cases}$$

TABLE 4  
*The 17 APs among the first 1427, whose pair sum is  $\not\equiv 0 \pmod{9}$*

	even members	odd members
regular	# 503, type (2,2)	# 899, type (3,2)
	# 1031, type (2,2)	# 1057, type (2,2)
	# 1081, type (2,2)	# 1158, type (3,2)
irregular	# # 577, 874	# # 7, 38, 78, 113, 256, 440, 1083, 1175, 1380

TABLE 5  
*All (37) pairs from the first 1427 APs having the same pair sum*

rank numbers	pair sum	prime decomposition of the pair sum, i.e., exponents belonging to the primes												
		2	3	5	7	11	13	17	19	23	29	31	37	
32 35	1296000	7	4	3										
105 109	20528640	9	6	1		1								
137 138	37739520	10	4	1	1			1						
172 173	75479040	11	4	1	1			1						
272 276	321408000	10	4	3									1	
282 286	348364800	13	5	2	1									
350 351	556839360	6	6	1	1	1								1
347 355	579156480	9	5	1	2				1					
373 375	638668800	12	4	2	1	1								
368 377	661893120	12	5	1	1				1					
395 399	761177088	10	5		1				1	1				
411 415	796340160	6	5	1	2	1			1					
427 433	883872000	8	4	3		1								1
462 476	1181174400	7	5	2	2									1
486 491	1282417920	8	5	1	1				1					1
574 582	2068416000	9	5	3	1				1					
626 630	2395008000	10	5	3	1	1								
653 665	2682408960	12	5	1	2	1								
695 697	3155023872	11	4		1	1	1		1					
717 730	3599769600	13	4	2	1									1
751 753	4049740800	10	6	2	1									1
798 807	4606156800	13	3	2	2			1						
786 787	4716601344	13	2		1		1		1					1
824 840	5094835200	10	7	2	1			1						
940 941	6824563200	9	3	2	2			1						1
926 952	6897623040	13	7	1	1	1								
997 998	7925299200	11	5	2	2		1							
1012 1019	8273664000	11	5	3	1				1					
1069 1097	10027929600	12	5	2			1							1
1124 1142	11195712000	9	3	3		1			1					1
1147 1150	11416204800	9	4	2	1	2	1							
1143 1181	12098211840	12	5	1		1	1	1						
1232 1233	13473008640	10	5	1	2		1	1						
1254 1265	14341017600	12	4	2	1		1		1					
1249 1255	14478912000	9	5	3	2					1				
1272 1278	15058068480	10	5	1	2		1		1					
1410 1425	19926466560	14	5	1	1	1	1							

These are the first two examples of APs of the form described in [5, Theorem I, case (b)] (also cf. the remarks immediately following Table I in [5]). Table 4 gives the rank numbers of the 17 APs with smaller member  $\leq 10^{10}$  whose pair sum is  $\not\equiv 0 \pmod{9}$ , divided into even and odd pairs, and regular and irregular pairs.

Another question, suggested by Professor C. Pomerance, is whether pairs, triples, quadruples, etc. of APs exist having the *same pair sum*. Among the first 1427 APs, we found 37 such pairs of APs, but no such triples, quadruples, etc. Table 5 gives the rank numbers of these pairs of APs, and the prime factorization of their pair sums. The pair sums only have prime divisors  $\leq 37$ . In 30 of the 37 cases at least one member of the pair was found during the exhaustive search described in the present paper.

In Appendix III (of the supplements section) we tabulate all the greatest common divisors of the first 1427 APs, ordered according to their size, with frequencies, and with the rank numbers of all the APs corresponding to a given gcd. This might be useful in further searches for special APs, and in searches for so-called *isotopic* APs (cf., [6, p. 83]). For example, new APs, isotopic with APs from the list of 1427 APs, are obtained by replacing the common factor  $3^3 5$  in # #882 and 1087 by  $3^2 7 \cdot 13$ , by replacing the common factor  $3^3 5^3$  in #1205 by  $3^2 5^2 31$ , and by replacing the common factor  $3^3 5^2 31$  in # #717 and 1228 by  $3^6 5 \cdot 23 \cdot 137 \cdot 547 \cdot 1093$ , and by  $3^{10} 5 \cdot 23 \cdot 107 \cdot 3851$ .

In [8], we have presented methods to find new APs from known APs. By applying these methods to the new APs among the first 1427 APs, we have found 117 new APs (with smaller member  $> 10^{10}$ ). The new APs were found mainly from mother pairs having a relatively simple structure, like those of type  $(i, 1)$ ,  $i > 1$ . They will be published in a forthcoming report [2], together with many other new amicable pairs.

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# Supplement to Computation of All the Amicable Pairs Below $10^{10}$

By H. J. J. te Riele

## Appendix I

The first 1427 APs

1	220	2E2.5.11	3D	31	600392	2E3.13.23.251	6D	
L1	21	284	2E2.71	L1	32	669688	2E3.97.863	
2	1184	2E5.37		32	609928	2E3.11.29.239		
L1	X	1210	2.5.11E2	L1	32	686072	2E3.191.449	
3	2620	2E2.5.131		33	624184	2E3.11.41.173		
L1	22	2924	2E2.17.43	L1	32	691256	2E3.71.1217	
4	5020	2E2.5.251		34	635624	2E3.11.31.233		
L1	22	5564	2E2.13.107	L1	32	712216	2E3.127.701	
5	6232	2E3.19.41		35	643336	2E3.29.47.59		
L1	X	6368	2E5.199	L1	32	652664	2E3.17.4799	
6	10744	2E3.17.79		36	667964	2E2.11.17.19.47		
L1	22	10856	2E3.23.59	L1	43	783556	2E2.31.71.89	
7	12285	3E3.5.7.13		37	726104	2E3.17.19.281		
L1	X	14595	3.5.7.139	L1	32	796696	2E3.53.1879	
8	17296	2E4.23.47		38	802725	3.5E2.7.11.139		
L1	21	18416	2E4.1151	L1	X	863835	3.5.7.19.433	
9	63020	2E2.23.5.137		39	879712	2E5.37.743		
L1	21	76084	2E2.23.827	L1	X	901424	2E4.53.1063	
10	66920	2E4.47.89		40	898216	2E3.11.59.173		
L1	22	66992	2E4.53.79	L1	32	980984	2E3.47.2609	
11	67095	3E3.5.7.71		41	947835	3E3.5.7.17.59		
L1	22	71145	3E3.5.17.31	L1	32	1125765	3E3.5.31.269	
12	69615	3E2.7.13.5.17		42	998104	2E3.17.41.179		
L1	21	87633	3E2.7.13.107	L1	32	1043096	2E3.23.5669	
13	79750	2.5E3.11.29		43	1077890	2.5.11.41.239		
L1	X	88730	2.5.19.467	L1	33	1099390	2.5.17.29.223	
14	100405	3E2.5.7.11.29		44	1154450	2.5E2.11.2099		
L1	32	124155	3E2.5.31.89	L1	22	1189150	2.5E2.17.1399	
15	122265	3E2.5.13.11.19		45	1156870	2.5.11.13.809		
L1	21	139815	3E2.5.13.239	L1	32	1292570	2.5.19.6803	
16	122368	2E9.239		46	1175265	3E2.7E2.13.5.41		
L1	X	123152	2E4.43.179	L1	21	1438983	3E2.7E2.13.251	
17	141664	2E5.19.233		47	1185376	2E5.17.2179		
L1	X	153176	2E3.41.467	L1	X	1286744	2E3.41.3923	
18	142310	2.5.7.19.107		48	1280565	3E2.5.13.11.199		
L1	32	160730	2.5.47.359	L1	22	1340235	3E2.5.13.29.79	
19	171856	2E4.23.467		49	1328470	2.5.11.13.929		
L1	22	176336	2E4.103.107	L1	X	1483850	2.5E2.59.503	
20	176272	2E4.23.479		50	1358595	3E2.5.19.7.227		
L1	22	180848	2E4.89.127	L1	22	1486845	3E2.5.19.37.47	
21	185368	2E3.17.29.47		51	1392368	2E4.17.5119		
L1	32	203432	2E3.59.431	L1	22	1464592	2E4.239.383	
22	196724	2E2.11.17.263		52	1466150	2.5E2.7.59.71		
L1	22	202444	2E2.11.43.107	L1	X	1747930	2.5.47.3719	
23	280540	2E2.5.13E2.83		53	1468324	2E2.11.13.17.151		
L1	X	365084	2E2.107.053	L1	43	1749212	2E2.37.53.223	
24	308620	2E2.5.13.1187		54	1511930	2.5.7.21599		
L1	32	389924	2E2.43.2267	L1	23	1598470	2.5.19.47.179	
25	319550	2.7.5E2.11.83		55	1669910	2.5.11.17.19.47		
L1	X	430402	2.7.71.433	L1	42	2062570	2.5.239.863	
26	356408	2E3.13.23.149		56	1798875	3E3.5E3.13.41		
L1	32	399592	2E3.199.251	L1	X	1870245	3E2.5.13.23.139	
27	437456	2E4.19.1439		57	2062464	2E5.59.1103		
L1	22	455344	2E4.149.191	L1	22	2090656	2E5.79.827	
28	469028	2E2.7E2.2393		58	2236570	2.5.7.89.359		
L1	X	486178	2.7E2.11E2.41	L1	33	2429030	2.5.23.59.179	
29	503056	2E4.23.1367		59	2652728	2E3.13.23.1109		
L1	22	514736	2E4.53.607	L1	32	2941672	2E3.71.5179	
30	522405	3E2.5.13.19.47		60	2723792	2E4.37.43.107		
L1	22	525915	3E2.5.13.29.31	L1	32	2874064	2E4.263.683	

























1381 9029724795 3E2.7.13.5.31.83.857  
 R9 43 11118146949 3E2.7.13.71.263.727  
 1382 9054843164 2E5.67.383.11027  
 L1 32 9086834464 2E5.71.3999407  
 1383 9070949152 2E5.53.83.64439  
 R9 32 9343940768 2E5.971.300719  
 1384 9075291470 2.7E2.5.23.43.61.307  
 R9 X 11614384306 2.7.41.113.241.743  
 1385 9083125275 3.5E2.7.19.139.6551  
 R9 33 9115709925 3.5E2.7.29.47.12739  
 1386 9086970310 2.5.53.11.83.89.211  
 R9 43 9607155770 2.5.53.17.719.1483  
 1387 9097736655 3.5.7E2.19.47.83.167  
 R9 32 9435270705 3.5.7E2.19.587.1151  
 1388 9100969376 2E5.79.113.31859  
 R9 33 9204512224 2E5.179.227.7079  
 1389 9136521225 3E3.5E2.13.17.73.839  
 R9 43 10287235575 3E3.5E2.59.97.2663  
 1390 9153086085 3E2.5.13.11.37E2.1039  
 R9 X 10021735035 3E2.5.13.47.389.937  
 1391 9159365024 2E5.53.241.22409  
 L1 32 9290429416 2E3.17.4157.16433  
 1392 9160950225 3.5E2.19.7.17.89.607  
 R9 44 10308056175 3.5E2.19.37.47.59.71  
 1393 9173012056 2E3.13.8689.10151  
 B4 33 9353372744 2E3.19.201.218987  
 1394 9205801010 2.5.7E3.109.24623  
 R9 X 10256406990 2.5.37.53.191.2749  
 1395 9225491168 2E5.83.151.23063  
 R9 33 9270558368 2E5.113.251.10223  
 1396 9251688032 2E5.67.179.24187  
 R9 33 9338472928 2E5.101.1229.2351  
 1397 9257732715 3.5.7.11.347.23099  
 L1 22 9263662485 3.5.7.11.449.17863  
 1398 9262239250 2.5E3.11.1783.1809  
 R9 X 9673564910 2.5.17.47.181.6689  
 1399 9271316500 2E2.5E3.53.271.1291  
 R9 X 11451453932 2E2.47.101.683.883  
 1400 9286443650 2.5E2.11.59.419.603  
 R9 44 9949825150 2.5E2.53.113.149.223  
 1401 9294561770 2.5.13.29.53.181.257  
 R9 42 9874755670 2.5.13.701.108359  
 1402 9303632390 2.5.13.29.113.21839  
 R9 33 9510953210 2.5.13.59.359.3457  
 1403 9307840630 2.11.5.43.313.6287  
 R9 44 9457161098 2.11.7.17.523.6907  
 1404 9344815064 2E3.13.59.659.2311  
 R9 44 9881776936 2E3.67.79.109.2141  
 1405 9357224877 3E2.7E3.13.11E2.41.47  
 R9 X 10162493523 3E5.7E3.13.83.113  
 1406 9374110636 2E2.11.19.52.83.2549  
 R9 54 10058113364 2E2.13.47.269.15299  
 1407 9408690824 2E3.13.41.59.149.251  
 R9 53 10595069176 2E3.167.1259.6299  
 1408 9449054312 2E3.19.71.139.6299  
 R9 44 9602145688 2E3.29.59.179.3919  
 1409 9490622048 2E5.61.4861999  
 L1 X 9500349952 2E9.4079.4549  
 1410 9535950765 3E2.5.11.23.31.41.659  
 R9 43 10390515795 3E2.5.11.127.197.839  
 1411 9549021568 2E7.37.101.19963  
 R9 X 10182996752 2E4.9043.70379  
 1412 9581473976 2E3.19.59.1068407  
 W1 23 9649870024 2E3.19.151.593.709  
 1413 9616775744 2E6.83.317.5711  
 W1 32 9760750144 2E6.2351.64871  
 1414 9703910930 2.7.17.5.47.173501  
 L1 32 11882513902 2.7.17.1223.40823  
 1415 9723053488 2E4.23.89.307.967  
 R9 43 10240668752 2E4.263.461.5279  
 1416 9766111856 2E4.17.137.262079  
 L1 32 10415096464 2E4.12959.50231  
 1417 9818506568 2E3.13E2.29.179.1399  
 R9 X 10933693432 2E3.89.1049.14639  
 1418 9834780205 3E2.5.11.19.139.7523  
 L1 22 980418915 3E2.5.11.19.569.1847  
 1419 9836011130 2.5.11.17.79.139.479  
 R9 54 11065876870 2.5.31.107.149.2239  
 1420 9844469775 3E2.5E2.7.23.29.9371  
 R9 X 11910566385 3E2.5.19.991.14057  
 1421 9852159848 2E3.31.23.433.3989  
 R9 33 10096563352 2E3.31.61.251.2659  
 1422 9875558210 2.5.29.7.233.20879  
 R9 33 11231616190 2.5.29.239.347.467  
 1423 9880655085 3E2.5.7.17.233.7919  
 R9 X 10935385875 3E2.5E3.19.431.1187  
 1424 9881488304 2E4.29.47.109.4157  
 R9 42 10535954096 2E4.5279.124739  
 1425 9883587230 2.5.11.47.191.10009  
 R9 33 10042879330 2.5.11.127.571.1259  
 1426 9958985128 2E3.19.103.197.3229  
 R9 33 9994662872 2E3.19.113.659.883  
 1427 9967523980 2E2.257.5.17.37.3083  
 R9 42 12890541236 2E2.257.107.117191

## Appendix II

The first 1427 APs ordered according  
to the various occurring types

AMICABLE PAIRS OF TYPE (2,1):

1 220 2E2.5.11  
 L1 21 284 2E2.71  
 8 17296 2E4.23.47  
 L1 21 18416 2E4.1151  
 9 63020 2E2.23.5.137  
 L1 21 76084 2E2.23.827  
 12 69615 3E2.7.13.5.17  
 L1 21 87633 3E2.7.13.107  
 15 122265 3E2.5.13.11.19  
 L1 21 139815 3E2.5.13.239  
 46 1175265 3E2.7E2.13.5.41  
 L1 21 1438983 3E2.7E2.13.251  
 104 9363584 2E7.191.383  
 L1 21 9437056 2E7.73727  
 117 11498355 3E4.5.11.29.89  
 L1 21 12024045 3E4.5.11.2699  
 162 31536855 3E2.5.7.53.1889  
 L1 21 32148585 3E2.5.7.102059  
 291 175032884 2E2.13.17.389.509  
 L1 21 175826716 2E2.13.17.198899  
 297 183408615 3E2.5.13.19.29.569  
 L1 21 190055385 3E2.5.13.19.17099  
 303 196421715 3E2.5.19.37.7.887  
 L1 21 224703405 3E2.5.19.37.7103  
 460 536637465 3E2.7E2.13.97.5.193  
 L1 21 646745463 3E2.7E2.13.97.1163  
 629 1191953763 3E2.7E2.11.13.41.461  
 L1 21 1223611389 3E2.7E2.11.13.19403  
 640 1225052829 3E4.7.11.29.13.521  
 L1 21 1321639011 3E4.7.11.29.7307  
 792 2172649216 2E8.257.33023  
 L1 21 2181168896 2E8.8520191  
 888 2935281375 3E3.5E3.13.149.449  
 L1 21 2961518625 3E3.5E3.13.67499  
 1030 4149106335 3E4.5.11E3.43.179  
 L1 21 4268776545 3E4.5.11E3.7919  
 1191 6066248175 3E2.5E2.13.31.149.449  
 L1 21 6120471825 3E2.5E2.13.31.67499  
 1219 6370495978 2.7E2.19.23.11.13523  
 L1 21 6950103062 2.7E2.19.23.162287

TOTAL NUMBER: 20

AMICABLE PAIRS OF TYPE (3,1):

86 6955216 2E4.19.137.167  
 L1 31 7418864 2E4.463679  
 151 23358248 2E3.37.23.47.73  
 L1 31 25233112 2E3.37.85247  
 164 32205616 2E4.17.167.709  
 L1 31 34352624 2E4.2147039  
 196 52695376 2E4.17.151.1283  
 L1 31 56208368 2E4.3513023  
 270 147366765 3E2.7E2.13.5.53.97  
 L1 31 182028483 3E2.7E2.13.31751  
 312 205843365 3E2.7E2.13.5.43.167  
 L1 31 254264283 3E2.7E2.13.44351  
 390 347263216 2E4.17.137.9319  
 L1 31 370414064 2E4.23150879  
 446 492275992 2E3.131.13.23.1571  
 R9 31 553544168 2E3.131.528191  
 648 1254255550 2.5E2.23.19.137.419  
 R9 31 1333078850 2.5E2.23.1159199  
 661 1309651310 2.5.11.29.571.719  
 R9 31 1359071800 2.5.11.12355199  
 753 1957374968 2E3.31.17.107.4339  
 L1 31 2092365832 2E3.31.8436959  
 782 2115211995 3E3.5.13.17.31.2287  
 R9 31 2312891685 3E3.5.13.1317887  
 979 3693013664 2E5.41.131.21487  
 L1 31 3812143072 2E5.119129471  
 1009 3986534090 2.5.929.7.11.5573  
 L1 31 4971106870 2.5.929.535103  
 1228 6562770525 3E3.5E2.31.17.19.971  
 R9 31 7322055075 3E3.5E2.31.349919  
 1300 7696871576 2E3.19.53.127.7523  
 R9 31 7904894824 2E3.19.52005887

TOTAL NUMBER: 16

AMICABLE PAIRS OF TYPE (4,1):

779 2099442345 3.5.7.11.13.37.3779  
 L1 41 2533809495 3.5.7.24131519

TOTAL NUMBER: 1

AMICABLE PAIRS OF TYPE (2,2):

3 2620 2E2.5.131  
 L1 22 2924 2E2.17.43  
 4 5620 2E2.5.251  
 L1 22 5544 2E2.13.187  
 6 18744 2E3.17.79  
 L1 22 18856 2E3.23.59  
 10 66928 2E4.47.89  
 L1 22 66992 2E4.53.79  
 11 67095 3E3.5.7.71  
 L1 22 71145 3E3.5.17.31  
 19 171856 2E4.23.467  
 L1 22 176336 2E4.103.187  
 20 176272 2E4.23.479  
 L1 22 188048 2E4.89.127  
 22 196724 2E2.11.17.263  
 L1 22 282444 2E2.11.43.187  
 27 437456 2E4.19.1439  
 L1 22 455344 2E4.149.191  
 29 583056 2E4.23.1367  
 L1 22 514736 2E4.53.607  
 30 522405 3E2.5.13.19.47  
 L1 22 525915 3E2.5.13.29.31  
 44 1154450 2.5E2.11.2099  
 L1 22 1189150 2.5E2.11.1399  
 48 1280565 3E2.5.13.11.199  
 L1 22 1348235 3E2.5.13.29.79  
 50 1358595 3E2.5.19.7.227  
 L1 22 1486845 3E2.5.19.37.47  
 51 1392368 2E4.17.5119  
 L1 22 1464592 2E4.239.383  
 57 2082464 2E5.59.1103  
 L1 22 2090656 2E5.79.827  
 63 2802416 2E4.17.10303  
 L1 22 2947216 2E4.167.1103  
 85 6377175 3E2.5E2.7.4049  
 L1 22 6680025 3E2.5E2.11.2699  
 87 6993610 2.5.13.23.2399  
 L1 22 7158710 2.5.13.53.1039  
 92 7677248 2E6.139.863  
 L1 22 7684672 2E6.167.719  
 100 9071685 3E2.5.31.7.929  
 L1 22 9498555 3E2.5.31.11.619  
 101 9199496 2E3.29.19.2087  
 L1 22 9592504 2E3.29.173.239  
 109 10254970 2.5.11.53.1759  
 L1 22 10273670 2.5.11.59.1583  
 111 10572550 2.5E2.31.19.359  
 L1 22 10854650 2.5E2.31.47.149  
 125 13921528 2E3.19.67.1367  
 L1 22 13965672 2E3.19.101.911  
 130 15002464 2E5.37.12671  
 L1 22 15334304 2E5.227.2111  
 139 17908064 2E5.53.10559  
 L1 22 18017056 2E5.79.7127  
 149 22508145 3E3.5.11.23.659  
 L1 22 23111055 3E3.5.11.79.197  
 197 56055872 2E6.79.11087  
 L1 22 56598208 2E6.383.2309

204 66595130 2.5.31.7.30689  
 L1 22 74624390 2.5.31.59.4091  
 230 90437150 2.5E2.23.19.4139  
 L1 22 94372450 2.5E2.23.137.599  
 233 95629904 2E4.67.37.2411  
 L1 22 97580944 2E4.67.227.401  
 236 97841735 3E2.5.7.71.4339  
 L1 22 97945705 3E2.5.7.239.1381  
 242 109410345 3E4.5.11.41.599  
 L1 22 110132055 3E4.5.11.59.419  
 254 131483835 3E2.5.7.71.5879  
 L1 22 132692805 3E2.5.7.223.1889  
 260 136549413 3E2.7.11.13.23.659  
 L1 22 140287067 3E2.7.11.13.79.197  
 337 256948065 3E3.5.11.23.7523  
 L1 22 263110815 3E3.5.11.53.3343  
 341 265192208 2E4.131.23.5501  
 L1 22 275148200 2E4.131.251.523  
 369 305363984 2E4.43.89.4987  
 L1 22 306962896 2E4.43.173.2579  
 478 595858064 2E4.37.179.5623  
 L1 22 596654896 2E4.37.239.4217  
 495 651016665 3E2.5.13.19.37.1583  
 L1 22 663576615 3E2.5.13.19.227.263  
 503 666030256 2E4.331.19.6619  
 L1 22 696630544 2E4.331.199.661  
 520 749300864 2E8.383.7643  
 L1 22 750555392 2E8.1567.1871  
 528 766292835 3.5.7.11.503.1319  
 L1 22 766512285 3.5.7.11.769.863  
 538 770805945 3E2.7.13.41.5.4591  
 L1 22 914053959 3E2.7.13.41.163.167  
 556 902335744 2E8.383.9203  
 L1 22 903709952 2E8.1151.3067  
 641 1237880448 2E6.73.264959  
 L1 22 1252205632 2E6.479.40047  
 609 1525222575 3E3.5E2.17.23.5779  
 L1 22 1571007825 3E3.5E2.17.79.1733  
 695 1558818261 3E2.7.11.13.23.7523  
 L1 22 1596205611 3E2.7.11.13.53.3343  
 708 1666611045 3E4.5.11E2.71.479  
 L1 22 1670433435 3E4.5.11E2.89.383  
 718 1750776704 2E7.137.99839  
 L1 22 1762592896 2E7.2879.4783  
 729 1786492785 3.5.7.11.293.5279  
 L1 22 1790052495 3.5.7.11.1231.1259  
 755 1974754485 3E4.5.11E2.59.683  
 L1 22 1987985835 3E4.5.11E2.113.359  
 787 2152573605 3E2.7.13.37.5.14207  
 L1 22 2564027739 3E2.7.13.37.191.443  
 934 3865930695 3E2.5.11.29.43.6263  
 L1 22 3873366805 3E2.5.11.29.47.5741  
 1031 4150593232 2E4.43.61.9809  
 L1 22 4213181968 2E4.43.859.7129  
 1052 4377991936 2E8.293.58367  
 L1 22 4390866176 2E8.3583.4787  
 1057 4429428675 3.5E2.7.19.167.2659  
 L1 22 4436670525 3.5E2.7.19.239.1861  
 1081 4796703664 2E4.43.67.104059  
 L1 22 4855069456 2E4.43.373.18919

1090 4893008692 2E2.11.109.13.78479  
 L1 22 5259164108 2E2.11.109.839.1307  
 1183 5983596512 2E5.79.227.10427  
 L1 22 5999426848 2E5.79.631.3761  
 1199 6143533695 3E4.5.13.17.68639  
 L1 22 6414291585 3E4.5.13.71.17159  
 1224 6470496595 3E2.5.17.19.23.19379  
 L1 22 6582073005 3E2.5.17.19.37.12239  
 1236 6722173575 3E2.5E2.13E2.17.10399  
 L1 22 7083639225 3E2.5E2.13E2.311.599  
 1259 7074650624 2E9.947.14591  
 L1 22 7076729344 2E9.1367.10111  
 1397 9257732715 3.5.7.11.347.23099  
 L1 22 9263662485 3.5.7.11.449.17863  
 1418 9834780285 3E2.5.11.19.139.7523  
 L1 22 9884118915 3E2.5.11.19.569.1847

TOTAL NUMBER: 67

AMICABLE PAIRS OF TYPE (3,2):

14 100485 3E2.5.7.11.29  
 L1 32 124155 3E2.5.31.89  
 18 142310 2.5.7.19.187  
 L1 32 168730 2.5.47.359  
 21 185368 2E3.17.29.47  
 L1 32 203432 2E3.59.431  
 24 308620 2E2.5.13.1187  
 L1 32 389924 2E2.43.2267  
 26 356408 2E3.13.23.149  
 L1 32 399592 2E3.199.251  
 31 600392 2E3.13.23.251  
 L1 32 669688 2E3.97.863  
 32 609928 2E3.11.29.239  
 L1 32 686072 2E3.191.443  
 33 624184 2E3.11.41.173  
 L1 32 691256 2E3.71.1217  
 34 635624 2E3.11.31.233  
 L1 32 712216 2E3.127.701  
 35 643336 2E3.29.47.59  
 L1 32 652664 2E3.17.4799  
 37 726104 2E3.17.19.281  
 L1 32 796696 2E3.53.1879  
 40 898216 2E3.11.59.173  
 L1 32 980984 2E3.47.2609  
 41 947835 3E3.5.7.17.59  
 L1 32 1125765 3E3.5.31.269  
 42 998104 2E3.17.41.179  
 L1 32 1043096 2E3.23.5669  
 45 1156870 2.5.11.13.809  
 L1 32 1292570 2.5.19.6803  
 59 2652728 2E3.13.23.1109  
 L1 32 2941672 2E3.71.5179  
 60 2723792 2E4.37.43.107  
 L1 32 2874064 2E4.263.683  
 62 2739704 2E3.11.163.191  
 L1 32 2928136 2E3.31.11807  
 65 3276856 2E3.11.23.1619  
 L1 32 3721544 2E3.647.719  
 67 3786904 2E3.11.23.1871  
 L1 32 4300136 2E3.467.1151  
 68 3805264 2E4.29.59.139  
 L1 32 4006736 2E4.179.1399  
 76 5147032 2E3.11.23.2543  
 L1 32 5843048 2E3.383.1907  
 81 5726072 2E3.11.31.2099  
 L1 32 6369928 2E3.79.10079  
 91 7577350 2.5E2.11.23.599  
 L1 32 8493050 2.5E2.59.2079  
 98 8754130 2.5.7.11.11369  
 L1 32 10893230 2.5.757.1439  
 114 10992735 3E2.5.13.19.23.43  
 L1 32 12070305 3E2.5.13.47.439  
 118 11545616 2E4.19.163.233  
 L1 32 12247504 2E4.491.1559  
 121 12397552 2E4.23.59.571  
 L1 32 13136528 2E4.359.2287  
 129 14654150 2.5E2.7.149.281  
 L1 32 16817050 2.5E2.179.1879









AMICABLE PAIRS OF TYPE (3,3):

Table listing amicable pairs of type (3,3) with columns for pair ID, number, type, and value. Includes entries like 43 1077898 2.5.11.41.239 and 169 34364912 2E4.43.199.251.

Table listing amicable pairs of type (3,3) with columns for pair ID, number, type, and value. Includes entries like 326 238238864 2E4.41.47.7727 and 328 242182930 2.5.11.19.115077.

Table listing amicable pairs of type (3,3) with columns for pair ID, number, type, and value. Includes entries like 451 512412550 2.5E2.13.73.10799 and 452 51443924 2E3.13.89.55579.

SUPPLEMENT

















1273	7319982604	2E2.11E2.17.389.2287	1370	8924047490	2.5.17.19.521.5303
R9 X	7633515956	2E2.11.41.659.6421	R9 X	9017050750	2.5E3.17.271.7829
1275	7336299285	3E2.5.7.19.443.2767	1371	8935581375	3E2.5E3.7.53.79.271
R9 X	8001520875	3E3.5E3.47.73.691	R9 X	10128267585	3E2.5.19.47.103.2447
1277	7347995392	2E8.269.106703	1374	8971549100	2E2.5.11.2963.13763
L1 X	7373955488	2E5.71.3245579	R9 X	11589884804	2E2.151.21E2.431
1281	7430796945	3E2.5.13.11E2.113.929	1380	9018517725	3E4.5E2.7.349.1823
R9 X	7967123775	3E2.5E2.13.569.4787	R9 X	10138589475	3.5E2.7.19311599
1284	7482410325	3E2.5E2.13.31.179.461	1384	9075291470	2.7E2.5.23.43.61.307
R9 X	7531628715	3E2.5.13.31.29.14321	R9 X	11614384306	2.7.41.113.241.743
1287	7523483030	2.5.7E2.53.271.1069	1390	9153086085	3E2.5.13.11.37E2.1039
R9 X	8601297130	2.5.37.71.327419	R9 X	10021735035	3E2.5.13.47.389.937
1288	7534469228	2E2.19.11.13.761.911	1391	9159365024	2E5.53.241.22409
R9 X	8810613652	2E2.19E2.227.26879	L1 X	9290429416	2E3.17.4157.16433
1289	7562801950	2.5E2.11.2099.6551	1394	9205801010	2.5.7E3.109.24623
L1 X	7792465250	2.5E3.19.1640519	R9 X	10296406990	2.5.37.53.191.2749
1291	7579753125	3.5E5.7.19.6079	1398	9262239250	2.5E3.11.1783.1889
R9 X	7619274875	3.5E2.7.19.419.1823	R9 X	9673564910	2.5.17.47.181.6689
1293	7592910975	3E3.5E2.13.43.20123	1399	9271316500	2E2.5E3.53.271.1291
R9 X	7778605185	3E2.5.13.43.29.10663	R9 X	11451453932	2E2.47.101.663.883
1305	7780806075	3.5E2.7.17.149.5851	1405	9357224877	3E2.7E3.13.11E2.41.47
R9 X	7893190725	3.5E2.7E2.43.199.251	R9 X	10162493523	3E5.7E3.13.83.113
1309	7822669778	2.7E2.11.19.167.2287	1409	9490622048	2E5.61.4861999
R9 X	7952449582	2.7.11.19.131.20747	L1 X	9500349952	2E9.4079.4549
1311	7900303190	2.5.31.13.619.3167	1411	9549021568	2E7.37.101.19963
R9 X	7938683050	2.5E2.31.47.59.1847	R9 X	10182996752	2E4.9043.70379
1318	8006313199	3E3.7E2.19.11.23.1259	1417	9818506568	2E3.13E2.29.179.1399
R9 X	8548514801	3E3.7.19.13.113.1619	R9 X	10933693432	2E3.89.1049.14639
1320	8052147896	2E3.13.509.152111	1420	9844469775	3E2.5E2.7.23.29.9371
R9 X	8239047304	2E3.23.97E2.4759	R9 X	11910566385	3E2.5.19.991.14057
1323	8119394450	2.5E2.31.19.23.11907	1423	9880655085	3E2.5.7.17.233.7919
R9 X	9005223790	2.5.31.73.107.3719	R9 X	10935385875	3E2.5E3.19.431.1187
1333	8269106625	3E2.13E2.31.61.5E3.23			
L1 X	9402333759	3E2.13E2.31.61.7.467			
1335	8279312030	2.5.11.37.881.2309			
R9 X	8443831330	2.5.11E2.1107.5879			
1337	8293896650	2.5E2.13.89.307.467			
R9 X	8596897270	2.5.13.131.181.2789			
1342	8376676490	2.7.5.13.19E2.43.593			
R9 X	11698280566	2.7.967.864107			
1344	8455838230	2.7.5.97.911.1367			
R9 X	9150518762	2.7.11E2.83.151.431			
1345	8459517832	2E3.17E3.31.53.131			
R9 X	9400398968	2E3.79.2087.7127			
1346	8467262505	3E4.5.13E2.17.19.383			
R9 X	9899027415	3E4.5.31.487.1619			
1348	8491739828	2E2.13.11E2.103.13103			
R9 X	9271203916	2E2.13.83.227.9463			
1349	8502526305	3E2.7E2.13.5.47.6311			
R9 X	10355911839	3E2.7.13.53.227.1051			
1355	8557778500	2E2.5E3.197.283.307			
R9 X	10355067452	2E2.23.9371.12011			
1361	8676652320	2E3.19E2.47.97.659			
R9 X	9066365272	2E3.43.59.587.761			
1362	8717951385	3E2.5.13.11E2.79.1559			
R9 X	9407501415	3E2.5.13.37.223.1949			
1366	8850661358	2.7E2.19.11.601.719			
R9 X	8937716242	2.7.19.13.89.113.257			
1367	8851379072	2E7.29.433.5587			
R9 X	9435731728	2E4.3467.170099			
				TOTAL NUMBER:	345

# Appendix III

## The gcd's of the first 1427 APs

GCD	FREQ	RANK NUMBER(S) OF AP'S WITH THIS GCD									
2	2	2	278								
4	67	1	3	4	23	24	36	53	64	83	97
		115	153	154	165	209	226	238	255	267	274
		294	313	323	345	347	348	502	505	570	586
		632	645	721	738	748	750	763	776	800	807
		811	822	842	846	852	867	877	909	920	937
		975	976	1060	1119	1132	1160	1251	1307	1316	1322
		1350	1355	1365	1368	1374	1399	1406			
8	208	5	6	17	21	26	31	32	33	34	35
		37	40	42	47	59	62	65	67	69	74
		76	80	81	84	90	93	94	95	103	110
		112	116	120	122	126	131	140	143	144	148
		150	159	161	163	174	180	182	184	186	192
		199	201	206	208	212	217	218	239	240	244
		258	266	279	280	293	296	298	300	305	311
		314	315	317	318	330	333	339	353	357	361
		363	374	375	376	382	387	400	405	417	432
		438	447	452	455	458	459	463	467	475	498
		513	523	525	546	552	553	558	571	574	579
		580	587	591	599	606	631	633	639	644	646
		652	658	665	668	690	702	713	716	720	722
		727	737	762	771	788	806	808	814	836	845
		848	854	855	858	863	869	871	876	881	912
		913	919	925	942	946	953	972	973	974	978
		981	982	992	999	1011	1026	1028	1043	1063	1077
		1078	1079	1113	1117	1120	1126	1130	1141	1152	1153
		1164	1170	1182	1184	1213	1218	1222	1235	1243	1250
		1252	1253	1279	1302	1320	1329	1345	1354	1356	1361
		1376	1378	1391	1393	1404	1407	1408	1417		
10	94	13	18	43	45	49	52	54	55	58	70
		75	79	98	99	105	107	128	146	190	210
		219	227	228	264	275	283	286	289	299	328
		332	343	378	420	437	441	448	476	479	507
		519	534	544	589	598	609	617	620	627	664
		670	674	676	723	759	769	826	834	835	843
		859	897	904	1001	1007	1023	1027	1042	1053	1062
		1064	1066	1074	1093	1103	1104	1105	1108	1111	1123
		1148	1159	1167	1209	1221	1240	1257	1266	1285	1287
		1373	1394	1398	1419						
14	30	25	61	73	77	119	127	135	177	187	198
		249	377	395	399	412	470	480	651	701	756
		790	809	825	857	873	1070	1110	1342	1344	1384
15	1	900									

16	150	8	10	16	19	20	27	29	39	51	60
		63	68	86	118	121	156	158	164	169	175
		179	185	193	196	213	231	232	248	257	281
		284	285	288	307	320	322	326	327	336	351
		365	366	381	385	390	397	407	429	433	434
		435	445	450	453	454	474	486	487	509	514
		527	536	545	550	573	575	577	578	583	625
		643	654	669	671	672	673	682	692	696	699
		706	735	742	744	778	799	828	850	866	887
		894	903	906	918	930	945	951	958	962	966
		970	1006	1010	1016	1020	1021	1035	1036	1041	1059
		1075	1086	1095	1106	1115	1116	1125	1127	1128	1129
		1139	1146	1154	1180	1187	1192	1194	1198	1202	1211
		1214	1227	1238	1241	1244	1256	1261	1268	1274	1292
		1297	1326	1328	1336	1364	1367	1411	1415	1416	1424
21	1	316									
22	3	89	567	1403							
32	76	57	130	139	142	155	216	229	246	247	268
		287	324	335	344	346	352	371	380	411	413
		419	439	456	457	468	471	477	535	540	568
		595	616	655	686	694	710	757	761	827	856
		868	878	883	916	923	948	956	957	960	979
		991	1003	1058	1067	1072	1118	1122	1163	1166	1178
		1210	1223	1242	1246	1277	1308	1310	1341	1343	1360
		1382	1383	1388	1395	1396	1409				
44	37	22	88	211	262	304	388	398	428	566	572
		596	603	608	624	662	697	752	758	770	823
		829	868	892	911	914	915	932	964	989	990
		1032	1092	1101	1273	1303	1321	1330			
45	26	14	100	132	141	214	220	252	265	301	331
		342	465	497	618	607	775	777	885	1094	1100
		1135	1258	1275	1371	1420	1423				
50	43	44	71	91	124	129	181	223	263	272	358
		404	423	451	500	512	551	739	751	773	797
		821	837	864	880	901	947	967	985	988	1008
		1038	1096	1107	1131	1190	1204	1207	1208	1230	1289
		1295	1340	1400							
52	13	133	195	271	422	473	499	504	562	634	997
		1319	1340	1357							
63	2	746	931								
64	22	92	197	464	526	594	611	638	641	801	886
		891	902	907	1015	1076	1156	1245	1283	1286	1315
		1331	1413								
68	3	362	402	485							

76	11	207	269	543	565	569	816	874	1071	1255	1272
		1288									
92	4	9	200	515	612						
98	6	28	273	585	711	1089	1165				
105	24	7	38	102	136	282	295	310	489	490	508
		555	601	679	683	693	733	779	798	917	969
		1022	1040	1203	1379						
110	20	189	292	349	350	483	494	623	661	802	922
		995	1112	1172	1181	1215	1226	1247	1260	1335	1425
124	2	442	983								
128	2	104	718								
130	16	87	166	277	449	466	592	691	736	784	865
		1017	1150	1237	1337	1401	1402				
135	47	11	41	82	145	152	171	189	215	221	224
		225	319	354	372	391	409	418	481	484	493
		518	539	590	604	613	656	666	675	728	754
		772	795	921	959	963	1004	1005	1034	1037	1085
		1174	1176	1185	1189	1347	1363	1372			
136	5	996	1212	1233	1294	1298					
148	2	444	987								
152	16	125	360	436	559	600	805	812	831	879	895
		1002	1091	1300	1324	1412	1426				
154	16	168	394	421	610	626	630	647	649	653	804
		872	898	986	1002	1157	1359				
165	2	96	414								
170	13	205	340	393	426	511	650	732	783	844	1216
		1262	1264	1370							
182	4	160	172	338	1171						
184	6	431	704	774	781	943	1136				
190	10	680	740	861	938	1012	1060	1098	1151	1276	1280
212	2	548	619								
225	10	85	157	167	510	538	549	1014	1186	1188	1201
230	1	905									
231	1	1144									
232	3	101	537	1044							

236	1																		
238	2	1301																	
248	4	234	1414																
250	1	276	392	753	1421														
255	1	588																	
256	4	705																	
266	8	520	556	792	1052														
273	1	308	368	491	582	602	734	749	1366										
285	2	78																	
286	1	123	968																
290	2	764																	
296	3	191	1422																
296	3	151	183	1220															
310	11	66	204	730	794	818	839	853	1025	1099	1311								
315	13	1323																	
322	1	162	236	254	472	584	700	813	817	833	1140								
328	1	1145	1197	1351															
376	1	1073																	
405	5	936																	
424	2	259	379	663	724	1346													
434	5	747	1234																
465	1	403	496	709	714	1024													
484	2	622																	
488	2	712	884																
495	4	576	890																
512	1	516	533	593	1410														
518	1	1259																	
525	17	517																	
		410	521	628	657	688	789	878	939	1000	1083								
		1158	1173	1175	1225	1305	1380	1385											

530	1																				
548	1	1386																			
574	1	1304																			
585	41	1047																			
		15	30	48	56	106	114	137	138	173	178										
		188	203	251	261	290	302	334	560	607	667										
		703	707	726	820	824	862	924	927	950	984										
		1061	1084	1147	1177	1206	1269	1281	1325	1362	1375										
		1390																			
592	1	478																			
632	1	1018																			
664	1	564																			
670	1	243																			
675	15	147	202	462	506	554	614	642	684	841	889										
		1013	1069	1195	1217	1389															
682	1	1168																			
688	3	369	1031	1081																	
692	1	944																			
693	5	176	321	424	715	1239															
735	4	235	355	415	1029																
752	1	965																			
765	3	1143	1200	1317																	
776	1	1314																			
790	1	677																			
808	1	977																			
819	19	12	170	222	389	698	719	767	780	786	849										
		949	971	998	1045	1051	1114	1162	1349	1381											
825	2	1050	1124																		
848	2	384	1306																		
850	1	635																			
855	7	50	72	329	396	482	1155	1169													
884	1	291																			
891	3	416	935	1161																	



5296	1	503																	
5535	2	488	766																
5733	10	46	270	312	501	532	908	928	1039	1046	1109								
6237	1	1290																	
6622	1	1121																	
6885	3	386	791	1327															
6975	1	1088																	
7030	1	1271																	
7564	1	367																	
7605	1	524																	
8775	2	425	940																
9009	3	260	695	743															
9290	1	1009																	
9405	1	1418																	
9975	2	1057	1291																
11115	2	297	495																
11475	1	689																	
13041	1	785																	
13330	1	581																	
13923	1	1232																	
13965	2	1196	1387																
14355	1	994																	
14445	1	741																	
14535	1	1224																	
14553	1	768																	
15561	2	1254	1312																
15795	1	1358																	
16245	1	364																	
16275	1	256																	

18135	2	893	1284																
18837	1	678																	
20925	6	717	760	810	1097	1228	1263												
21068	1	910																	
21879	1	1048																	
23095	1	1296																	
25155	1	1293																	
30303	1	787																	
31635	1	303																	
33579	1	530																	
34749	1	531																	
38025	1	1236																	
38493	1	1299																	
40131	1	1405																	
42026	1	1219																	
43075	1	888																	
44732	1	1033																	
48735	1	896																	
49005	3	708	755	993															
52725	1	899																	
63063	1	629																	
68607	1	796																	
90675	1	1191																	
108927	4	847	1056	1134	1334														
100873	1	640																	
463905	1	1102																	
539055	1	1030																	
556101	1	460																	
2876211	1	1333																	

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