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G-SPACES: COMPACTIFICATIONS AND PSEUDOCOMPACTNESS

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ABSTRACT

This paper consists of two parts. In the first part, some of the existing theory on "equivariant topology" is reviewed. It contains almost no new facts, but the material is used to explain the author's point of view, In the second part, some new results are proved. For example, if G is a locally compact topological group, then the concept of G-pseudocompactness for Tychonov G-spaces, as introduced by the author in an earlier paper, turns out to coincide with ordinary pseudocompactness. Also the relation with G-pseudocompactness as introduced by Antonyan, namely, the equality of the maximal G-compactification and the Stone–Čech compactification, is investigated.

1. INTRODUCTION

In this paper I want to report about my recent investigations in the category of G-spaces. First, let me briefly describe this category. Let G be a topological group, arbitrary, but fixed in the discussions. The objects in the category TOP^G are the *G-spaces,* i.e. pairs $\langle X, \pi \rangle$ where X is a topological space and π is a continuous action of G on X, that is,

$\pi\colon G \times X \to X$ is a continuous mapping such that for all $x \in X$ and all $s, t \in G$ the following identities hold:

$$ex = x; \quad (st)x = s(tx)$$

(we shall mostly write tx instead of $\pi(t, x)$; e denotes the unit element of G). This implies, that for each $t \in G$ the mapping $\pi^t\colon x \mapsto tx\colon X \to X$ is a homeomorphism of X, and that the mapping $\bar{\pi}\colon t \mapsto \pi^t$ is a homomorphism of groups from G into the full homeomorphism group of X. The morphisms in TOP^G are the *continuous equivariant mappings*, i.e. mappings that commute with actions. Thus, if $\langle X, \pi \rangle$ and $\langle Y, \sigma \rangle$ are G-spaces, then a continuous mapping $f\colon X \to Y$ is a morphism of G-spaces whenever $f \circ \pi^t = \sigma^t \circ f$ for all $t \in G$.

Examples

(a) For every topological space X there is the *trivial action* of G on X defined by $tx := x$ for all $t \in G$, $x \in X$. If $\langle X, \pi \rangle$ and $\langle Y, \sigma \rangle$ are G-spaces with trivial actions π and σ, then every continuous function $f\colon X \to Y$ is a morphism of G-spaces.

(b) G acts on itself by multiplication from the left, i.e. $\langle G, \omega \rangle$ is a G-space with action $\omega(t, s) := ts$ for $t, s \in G$. Note, that if $\langle X, \pi \rangle$ is an arbitrary G-space, then for every $x \in X$ the continuous mapping $\pi_x\colon t \mapsto tx\colon G \to X$ is a morphism of G-spaces.

(c) Assume that G is locally compact. Let $C_c(G)$ denote the space of all continous real-valued functions on G, endowed with the compact – open topology. Let ρ be the action of G on $C_c(G)$, defined by $\rho(t, f)(s) := f(st)$ for $(t, f) \in G \times C_c(G)$ and $s \in G$ (continuity of ρ is guaranteed by local compactness of G). If $\langle X, \pi \rangle$ is an arbitrary G-space and $f \in C(X)$, then for every $x \in X$ one has $f \circ \pi_x \in C_c(G)$, and it is not difficult to show that the mapping $x \mapsto f \circ \pi_x\colon X \to C_c(G)$ is a morphism of G-spaces from $\langle X, \pi \rangle$ to $\langle C_c(G), \rho \rangle$. For details, cf. [9], 2.1.3 and 2.1.13.

For a study of the category TOP^G from a categorical point of view, see [9]. In the present paper, I want to pay attention to the study of this category from a topological point of view. Roughly, this means to re-do topology with G-spaces instead of ordinary spaces and with morphisms of G-spaces instead of continuous maps. For general references of this program, see [3], [7], [8] and [12].

One problem to resolve beforehand is, which objects in TOP^G should play the role which in ordinary topology is played by the real line **R** and the interval $[0; 1]$, and, similarly, which objects should replace $C(X)$ and $C^*(X)$ (here the asterisk means: *bounded* functions). I shall sketch two approaches (plus a variation of the second one).

(i) In general, the only "natural" action of G on **R** (or on a bounded subset of **R** like the interval $[0; 1]$) is the trivial one. *So consider* **R** *as a trivial G-space.* If $\langle X, \pi \rangle$ is an arbitrary G-space, then a continuous mapping $f: X \to \mathbf{R}$ is equivariant iff f is constant on G-orbits of X, iff there is a continuous mapping $f': X/G \to \mathbf{R}$ such that $f = f' \circ q_G$ (here X/G is the *orbit space* of $\langle X, \pi \rangle$, i.e. the quotient space of X, defined by the partition of X into *orbits* (sets Gx with $x \in X$); the mapping $q_G: X \to X/G$ is the corresponding quotient mapping). These considerations lead to an approach in which the role of **R** is played by **R** (with trivial action of G), but where the role of $C(X)$ is played by $C(X/G)$. An example of this approach can be found in the papers [5] and [6]. This approach might be characterized by saying, that properties of the category TOP are lifted to the category TOP^G by means of the functor $S_1^G: \mathrm{TOP}^G \to \mathrm{TOP}$ which assigns to every G-space its orbit space (cf. [9], 3.3.13 (iii) and 3.4.7). The disadvantage of this method is, that usually X/G has bad properties. Often one has to restrict oneself to the full subcategory of TOP^G, defined by the property that X/G is Hausdorff. Another possibility is to consider only the case that G is compact or, at least, that G acts *properly* on certain "crucial" spaces.

(ii) For the next approach, it is necessary that *the group G is locally compact.* Starting point is the observation, that TOP can be considered as $\mathrm{TOP}^{\{e\}}$, where $\{e\}$ is the trivial one-element group. In doing so, the space **R** may be considered as $C_c(\{e\})$. So in trying to generalize to arbitrary groups G, one is led to the idea of replacing **R** by the space $C_c(G)$. As was remarked in Example (c) above, there is a natural action ρ of G on $C_c(G)$, provided G is locally compact. In this approach, for a G-space

$\langle X, \pi \rangle$, instead of $C(X)$ one should consider $\mathrm{Mor}_G(\langle X, \pi \rangle, \langle C_c(G), \rho \rangle)$, the space of all morphisms of G-spaces from $\langle X, \pi \rangle$ into $\langle C_c(G), \rho \rangle$, and instead of $C^*(X)$ one should consider the space of all $f \in \mathrm{Mor}_G(\langle X, \pi \rangle, \langle C_c(G), \rho \rangle)$ such that $f[X]$ has compact closure in $C_c(G)$. This approach can be found e.g. in [1]. For a categorical interpretation of the G-space $\langle C_c(G), \rho \rangle$, see [9], 6.3.6. (also 6.3.5) and, in much more detail but from a slightly different viewpoint, [13].

(iii) The following approach is equivalent with (ii). As is explained in [9], 6.3.5, there is a natural way to identify, for an arbitrary G-space $\langle X, \pi \rangle$, the set $\mathrm{Mor}_G(\langle X, \pi \rangle, \langle C_c(G), \rho \rangle)$ with $C(X)$: if $f: \langle X, \pi \rangle \to \langle C_c(G), \rho \rangle$ is a morphism of G-spaces, then $\tilde{f}: x \mapsto f(x)(e)$ is an element of $C(X)$. and if $g \in C(X)$, then the mapping $g^*: x \mapsto g \circ \pi_x: X \to C_c(G)$ is a morphism of G-spaces. Since $\tilde{f}^* = f$ and $\tilde{g}^* = g$, this defines a one-to-one correspondence between $\mathrm{Mor}_G(\langle X, \pi \rangle, \langle C_c(G), \rho \rangle)$ and $C(X)$ (with the topologies of uniform convergence, this correspondence is even a homeomorphism). Under this correspondence, the subset of all $f \in \mathrm{Mor}_G(\langle X, \pi \rangle, \langle C_c(G), \rho \rangle)$ such that $f[X]$ has compact closure in $C_c(G)$ corresponds with the set of all functions $g \in C(X)$ which are bounded and have the property that $\{g \circ \pi_x\}_{x \in X}$ is equicontinuous on G (Ascoli's theorem), or, equivalently,

$$(*) \qquad \forall \epsilon > 0 \ \exists U \in \mathscr{V}_e \mid |g(tx) - g(x)| < \epsilon \quad \text{for all } t \in U \text{ and all } x \in X.$$

The set of all continuous real-valued functions on X satisfying this condition will be denoted by $UC\langle X, \pi \rangle$, and its elements are called *π-uniformly continuous functions*. The set of all bounded members of $UC\langle X, \pi \rangle$ will be denoted by $UC^*\langle X, \pi \rangle$. The above suggests, that the role of $C^*(X)$ in general topology should be played by $UC^*\langle X, \pi \rangle$. This approach can be found in [11] and [14]. It will be examplified in the "Facts" below.

Examples.

(d) Consider the G-space $\langle G, \omega \rangle$ of Example (b) above. Then $UC\langle G, \omega \rangle = \mathrm{RUC}(G)$, the space of right uniformly continuous functions on G.

(e) Consider a G-space $\langle X, \pi \rangle$ with X compact. Then $UC\langle X, \pi \rangle = UC^*\langle X, \pi \rangle = C^*(X) = C(X)$, i.e. every continuous real-valued function on X is π-uniformly continuous. It follows, that if $\varphi\colon \langle Y, \sigma \rangle \to \langle X, \pi \rangle$ is a morphism of G-spaces, then $\widetilde{\varphi}[C(X)] := \{f \circ \varphi \mid f \in C(X)\} \subseteq UC^*\langle Y, \sigma \rangle$.

(f) If G is discrete, then for every G-space $\langle X, \pi \rangle$ one has $UC\langle X, \pi \rangle = C(X)$ (take $U := \{e\}$ in the definition above).

It follows from the Examples (e) and (f), that $UC^*\langle X, \pi \rangle$ might be a good substitute for $C^*(X)$, in particular in connection with compactifications. This belief is strengthened by the following facts.

Facts.

(1) Let $\langle X, \pi \rangle$ be a G-space. Then $UC^*\langle X, \pi \rangle$ is a closed subalgebra of the Banach algebra $C_u^*(X)$, containing all constant functions. In addition, it is *invariant* in the sense that $f \circ \pi^t \in UC^*\langle X, \pi \rangle$ for all $f \in UC^*\langle X, \pi \rangle$ and $t \in G$. In [11] it is proved, that the morphism of G-spaces $\varphi\colon \langle X, \pi \rangle \to \langle X', \pi' \rangle$ with X' a compact Hausdorff space (so-called *G-compactifications*) correspond exactly to the closed invariant subalgebras A of $UC^*\langle X, \pi \rangle$, containing the constants; the correspondence is by the rule $A = \widetilde{\varphi}[C(X')]$ (compare Example (e) above). It follows immediately, that the G-compactification

$$\varphi_{\langle X, \pi \rangle}\colon \langle X, \pi \rangle \to \beta_G \langle X, \pi \rangle$$

corresponding to the whole algebra $UC^*\langle X, \pi \rangle$ is *maximal*, or *universal*, in the sense that every morphism of G-spaces from $\langle X, \pi \rangle$ to a compact Hausdorff G-space factorizes over it. Moreover, by definition, every $f \in UC^*\langle X, \pi \rangle$ factorizes over it. Thus, this universal G-compactification behaves like the Stone–Čech compactification in ordinary topology, with $C^*(X)$ replaced by $UC^*\langle X, \pi \rangle$, (For details, cf. [11].)

(2) Call a G-space $\langle X, \pi \rangle$ *G-Tychonov* whenever X is a Hausdorff space and $\langle X, \pi \rangle$ is *G-completely regular*, i.e. $UC^*\langle X, \pi \rangle$ separates points and closed subsets of X; cf. [3]. Clearly, the universal G-compactification $\varphi_{\langle X, \pi \rangle}\colon \langle X, \pi \rangle \to \beta_G \langle X, \pi \rangle$ is an equivariant embedding iff $\langle X, \pi \rangle$ is G-Tychonov. Using a technique which modifies an arbitrary $f \in C^*(X)$ into an element of $UC^*\langle X, \pi \rangle$, it can be shown that if G is

locally compact, then the G-space $\langle X, \pi \rangle$ is G-Tychonov iff X is a Tychonov space. Cf. [11].

(3) Let G be locally compact; if $\langle X, \pi \rangle$ is a G-space with X a normal Hausdorff space, and if F is a closed invariant subset of X, then for every $f \in UC^*\langle F, \pi|_{G \times F} \rangle$ there is $f' \in UC^*\langle X, \pi \rangle$ such that $f'|_F = f$ (Tietze's theorem for G-spaces; invariantness of F need not be required, but then "$UC^*\langle F, \pi|_{G \times F} \rangle$" is meaningless, and f must be chosen in $C^*(F)$ such that condition (*) is fulfilled with the additional requirement "and $tx \in F$"). This result has not yet been published earlier.

2. PSEUDOCOMPACTNESS

In this subsection, G will always be assumed to be locally compact, and all G-spaces are assumed to be Tychonov (so that Fact 1 and Fact 2 of Section 1 can be used). We want to find a workable definition of G-pseudocompactness. Generalizing various characterizations of pseudocompactness according to the principles, mentioned in Section 1, we obtain the following alternatives: A G-space $\langle X, \pi \rangle$ is said to have property

$(G - P_1)$, whenever every π-uniformly continuous function is bounded;

$(G - P_2)$, whenever each bounded π-uniformly continuous function on X assumes its maximum and minimum values;

$(G - P_3)$, whenever every countable infinite sequence $\{B_n\}_{n \in \mathbb{N}}$ of mutually disjoint non-empty open subsets of X with the property

$$(**) \qquad \exists U \in \mathscr{V}_e \ \forall n \in \mathbb{N} \ \exists x_n \in B_n \mid Ux_n \subseteq B_n$$

has a clusterpoint (i.e. is not locally finite);

$(G - P_4)$, whenever the orbit space X/G is pseudocompact.

The following implications between these properties are universally valid and rather easy to prove (see [14], Proposition 2.5):

$$(G - P_2) \Rightarrow (G - P_3) \Rightarrow (G - P_1) \Rightarrow (G - P_4).$$

Indeed, $(G - P_1) \Rightarrow (G - P_4)$ is almost obvious; for $(G - P_3) \Rightarrow (G - P_1)$, consider $f \in UC\langle X, \pi \rangle$ and the sets $B_n := \{x \in X \mid n \leqslant f(x) < n + 1\}$, and for $(G - P_2) \Rightarrow (G - P_3)$, one needs a result from [10], about a method of transforming a member of $C^*(X)$ into a member of $UC^*\langle X, \pi \rangle$; see also [14], Proposition 2.4.

In general, one has

$$(G - P_4) \not\Rightarrow (G - P_1) \quad \text{and} \quad (G - P_1) \not\Rightarrow (G - P_3).$$

For an example of a G-space having $(G - P_1)$ but not $(G - P_3)$, see [14]. An example of a G-space with pseudocompact (even compact, or even a one-element) orbit space which does not have $(G - P_1)$ is obtained from $\langle G, \omega \rangle$ with G any locally compact group on which not all right-uniformly continuous functions are bounded (e.g. $G = \mathbf{R}$); cf. Example (d) in Section 1.

Concerning the implication $(G - P_3) \Rightarrow (G - P_2)$, the following is true (and this also gives the relationship with ordinary pseudocompactness):

Proposition 1. *The following properties are equivalent for a G-space $\langle X, \pi \rangle$:*

(i) $\langle X, \pi \rangle$ *satisfies* $(G - P_2)$;

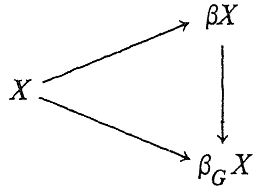
(ii) $\langle X, \pi \rangle$ *satisfies* $(G - P_3)$;

(iii) X *is pseudocompact.*

Proof. The implication (iii) $\Rightarrow$ (i) is obvious from a known characterization of pseudocompactness. So it remains to show that (ii) $\Rightarrow$ (iii). So assume (ii) holds, and let $\{W_n\}_{n \in \mathbf{N}}$ be an infinite sequence of non-empty open subsets of X, mutually disjoint. Let U be a compact symmetric neighbourhood of e in G and let $x_n \in W_n$ for every $n \in \mathbf{N}$. Since $\langle X, \pi \rangle$ is assumed to satisfy condition $(G - P_3)$, no sequence $\{W'_n\}_{n \in \mathbf{N}}$ with W'_k an open neighbourhood of Ux_k for every $k \in \mathbf{N}$ can be locally finite (if there would be such a sequence which is locally finite, then there would also be such a sequence which is *disjoint* and locally finite; for the straightforward proof of this, see [12], 2.2 (4°)). In particular the sequence $\{UW_n\}_{n \in \mathbf{N}}$ is not locally finite: there exists a point x_0 in X such that every neighbourhood V of x_0 intersects infinitely many of the sets UW_n. Let V be a neighbourhood of Ux_0. Since the action of G on X is continuous as a mapping of $G \times X$ into X and U is compact, there exists a neighbourhood V' of x_0 such that $UV' \subseteq V$. For infinitely many values of $n \in \mathrm{N}$ we now have $V' \cap UW_n \neq \neq \phi$, hence $UV' \cap W_n \neq \phi$ (for $U^{-1} = U$), and, consequently, $V \cap W_n \neq \phi$. If the sequence $\{W_n\}_{n \in \mathrm{N}}$ were locally finite, then the compact set Ux_0 would have a neighbourhood, intersecting only finitely many of the sets W_n. Thus, the sequence $\{W_n\}_{n \in \mathrm{N}}$ is not locally finite. This shows that X is pseudocompact. ∎

In [14], a Tychonov G-space $\langle X, \pi \rangle$ was called *G-pseudocompact* whenever it has property $(\mathrm{G} - \mathrm{P}_3)$. Because of the usefulness of this property (see e.g. the proof of the main results in [14]) we still propose to stick to this terminology. So Proposition 1 says, that if G is locally compact, then G-pseudocompactness if equivalent to ordinary pseudocompactness; it also states that condition $(\mathrm{G} - \mathrm{P}_2)$ is a characterization of G-pseudocompactness. It is an inelegant circumstance, that condition $(\mathrm{G} - \mathrm{P}_1)$ is *not* a good characterization of G-pseudocompactness. I have not yet investigated the problem, under which additional conditions one has $(\mathrm{G} - \mathrm{P}_1) \Rightarrow (\mathrm{G} - \mathrm{P}_3)$.

In [2], another notion of G-pseudocompactness was introduced: there a (Tychonov) G-space $\langle X, \pi \rangle$ is called G-pseudocompact whenever the equality $\beta_G X = \beta X$ holds (in [2], only compact groups are considered, but this definition can be given for arbitrary G). A little explanation is in order: recall from the Facts 1 and 2 from the Introduction that a (Tychonov) G-space $\langle X, \pi \rangle$ may be considered as an invariant subspace of its universal G-compactification $\beta_G \langle X, \pi \rangle$. For convenience, we shall denote the underlying compact Hausdorff space of $\beta_G \langle X, \pi \rangle$ by $\beta_G X$. We say that $\beta_G X = \beta X$ whenever we have a commutative diagram



where the vertical arrow is a homeomorphism (here $X \to \beta_G X$ and $X \to \beta X$ denote the canonical inclusions). The following result is stated without proof in [2]; in [14] it is proved for the case that G in a k-group.

Proposition 2. *Let $\langle X, \pi \rangle$ be a G-space. If X is pseudocompact, then $\beta_G X = \beta X$.* ∎

The converse is not true: if G is discrete, then $\beta_G X = \beta X$ for *every* G-space $\langle X, \pi \rangle$. Similarly, if G is arbitrary and $\langle X, \pi \rangle$ is a G-space with X arbitrary and π a trivial action, then $\beta_G X = \beta X$. So an additional condition under which the equality of $\beta_G X$ and βX would imply pseudocompactness of X must involve non-triviality of the action as well as non-discreteness of G.

The following is a result in this direction. If $\langle X, \pi \rangle$ is a G-space then for every $x \in X$, $G_x := \{t \in G \mid tx = x\}$ is called the *isotropy subgroup* of x in G. Since we are assuming that X has a Hausdorff topology, G_x is a closed subgroup of G for every $x \in X$. Let $X_0 := \{x \in X \mid G_x$ is open in $G\}$. Then we have:

Proposition 3. *Let $\langle X, \pi \rangle$ be a G-space such that $\beta_G X = \beta X$. Then either the set X_0 has non-empty interior, or X is* lw (G)*-pseudocompact.*

(Recall, that if α is a cardinal number, then a space is called α-*pseudocompact* whenever every locally finite family of mutually disjoint, non-empty open subsets has cardinality less than α. In the proposition above, lw (G) stands for the *local weight* (or: *local character*) of G.)

Proof. Suppose the contrary: there exists a dense set of points in X, each having a non-open isotropy group, and X is not lw (G)-pseudocompact. Then there exists a locally finite, disjoint family $\mathscr{W}$ of non-empty open subsets of X with cardinality lw (G). Let $\mathscr{B}$ be a local basis at e having cardinality lw (G), and let $U \mapsto W_U$ be an injective mapping from $\mathscr{B}$ into $\mathscr{W}$. For every $U \in \mathscr{B}$ there exists a point x_U in W_U with non-open isotropy group, i.e. the isotropy group of x_U has empty interior. So there exists $t_U \in U$ such that $t_U x_U \in W_U$ and $t_U x_U \neq x_U$. Let f_U be a continuous function from X into the interval $[0; 1]$ such that $f_U(x_U) = 1$ and $f(x) = 0$ for $x \in \{t_U x_U\} \cup (X \setminus W_U)$. As $\{W_U\}_{U \in \mathscr{B}}$ is locally finite, the function $f := \sum_{U \in \mathscr{B}} f_U$ is well-defined, bounded and continuous. Now the assumption that $\beta_G X = \beta X$ implies (in fact, is equivalent to) the equality $UC^*\langle X, \pi\rangle = C^*(X)$, hence $f \in UC^*\langle X, \pi\rangle$. Consequently, there exists $V \in \mathscr{B}$ such that

$$|f(tx) - f(x)| < \frac{1}{2} \quad \text{for all } x \in X \text{ and } t \in V.$$

Taking $x := x_V$ and $t := t_V$ in this inequality, we arrive at a contradiction. ∎

Remark. The result above is in some sense a modification of Proposition 3.4 of [4]. One of the difficulties which prevent a generalization of that result to the present context is, that the mappings $\pi_x : t \mapsto tx : G \to X$ for $x \in X$ are, in general, not open.

Note, that if G is discrete, or if the action of G on X is trivial, then $X_0 = X$. Moreover, if G is connected, then every open subgroup of G is all of G, so that in that case X_0 consists just of all invariant points. We shall say, *that* $\langle X, \pi\rangle$ *has almost no open isotropy groups* whenever X_0 has empty interior. The following result is an easy consequence of Proposition 2 and 3.

Corollary 1. *Suppose that* G *is metrizable and that* $\langle X, \pi\rangle$ *is a G-space with almost no open isotropy groups. Then* $\beta_G X = \beta X$ *iff* X *is pseudocompact.* ∎

Corollary 2. *Let* $\langle X, \pi\rangle$ *be a G-space with almost no open isotropy groups, and let* X *be a separable metric space. Then* $\beta_G X = \beta X$ *iff* X *is compact.*

Proof. We may assume, that G acts effectively on X; otherwise, pass to the corresponding effective action of G/G_0, where $G_0 := = \bigcap \{G_x \mid x \in X\}$, and observe, that G/G_0 is locally compact (beacuse G is), and that for every $x \in X$ the isotropy subgroup in G is open iff the corresponding isotropy subgroup in G/G_0 is open. It follows, that G may assumed to be metrizable (see [9], 1.1.23). Now our Corollary follows from Corollary 1 and the fact that in metric spaces pseudocompactness is equivalent to compactness. ∎

Remark 1. Proposition 1 answers two open questions from [14] and makes a few other ones unimportant. However, Problem 5.3 of [14] remains open.

Remark 2. Also open is the general question for necessary and sufficient conditions for the equality $\beta_G X = \beta X$ (compare with Proposition 3 and its Corollaries). In this context, see also Theorem 6 in [2].

Remark 3. Using Proposition 1, the main result of [14] can be extended to the case of infinite products. Details will be published elsewhere.

Remark 4. Apart from the problems, mentioned in the text, there is yet one important question: what of the preceding theory remains valid if G is not assumed to be locally compact; in which direction should definitions be adapted in order to get a theory at all?

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