Perspectives in Mathematical Sociology

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A strong tradition exists in the application of mathematics in physics. More and more, mathematical methods are applied in other disciplines such as biology and the humanities. It is noticed that each field demands its own type of mathematics. The question arises whether beside mathematical physics and mathematical biology a mathematical sociology can also be discerned.

1. Introduction

Recently, in a special issue of the *Journal of Mathematical Sociology,* attention was given to the role of mathematics in the social sciences. In particular the articles by Wilson [42] and Marsden and Laumann [30] give an impression of the progress that has been made in this direction. It would be inappropriate to write an article based on excerpts from this issue. An interested reader will certainly enjoy the original articles. In the present paper perspectives in mathematical sociology are discussed from the point of view of a mathematician, working on the modeling of physical and biological phenomena, with an eagerness to explore new fields.

Between mathematics and the applied disciplines an interaction exists which uncovers new areas of research: mathematics provides us with new methods to analyse problems in the sciences and the humanities. On the other hand problems in these disciplines may require a new type of mathematics. The study of nonlinear diffusion equations was strongly motivated from biology and chemistry. The discovery of the mechanism underlying chaotic behaviour of physical and biological systems (turbulence) required new mathematical tools such as the concept of fractional dimension and methods for analyzing discrete dynamical systems. The question arises whether sociological problems may also induce a new type of mathematics. The survey paper of Marsden and Laumann [30] shows how in quantitative sociology a set of mathematical techniques are brought together forming a coherent method for analyzing social

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structures. From the sociological point of view, a new theory was developed and for mathematics a new application was found. However, the mathematical techniques are not fundamentally new, as in the above example of chaotic dynamics.

In our exposé on the perspectives of mathematical sociology we focus our attention on the problem of relating individual action to collective phenomena. We will dwell upon cases for which no model can yet be formulated because of the fact that at a microscopic level structural changes are induced by macro-variables of the interaction process. In sociological problems, the complicated cause-effect chain between an individual action and a collective phenomenon is fairly well unraveled by a theory known as *structural individualism.* Reverse effects may also be described in this way. However, structural individualism is a qualitative explanatory theory, in which an explanation of a collective phenomenon is formulated in the context of sharply defined sociological problems. It is hoped that a mathematical formalism can be found, which quantifies the principles underlying this theory. Such a development would enhance the explanatory power of structural individualism and create the possibility of application in other disciplines like biology, see also LEVINS and LEWONTIN [26] for a discussion of the problem of analyzing ecological systems.

The principle of *self-organization* in physics, biology and sociology is an example of the type of interaction process described above. In the last ten years various theories of self-organization have been formulated. However, none of them describes micro-macro processes in a way that accounts adequately for structural changes at the microlevel induced by macro variables.

In Section 2 we review present, non-statistical, applications of mathematics in sociology. Two major types of applications can be distinguished: game theoretical problems and network analysis (based on several mathematical techniques, such as graph theory, optimization, and the theory of stochastic processes). In Section 3 sociological theories with a strong mathematical content are discussed. The work of Boudon is taken as a basis from where we make our excursions. Special attention is given to the work of LINDENBERG [27] about structural individualism. Parallels between modeling in physics, biology and sociology are made in Section 4, where we deal with the phenomenon of self-organization.

In this paper we will not discuss the role of statistical methods in sociology. We refer to FOX [12] for recent developments in regression analysis of sociological problems. FREEDMAN [13] has some critical remarks on the application of statistics in the social sciences. In the same volume responses are given by S.E. Fienberg and by K.G. Jöreskog and D. Sörenbom.

2. MATHEMATICS IN SOCIOLOGY TODAY

From the applications of mathematics in sociology we will discuss game theoretical problems and network analysis of social structures. The latter has been set forth as a widely accepted direction in sociology with its own journal (Social Networks). Furthermore, we will touch upon the mathematical and sociological aspects of voting.

In *game theory* a number of players all try to realize a maximal benefit for themselves in a game, being a set of rules. Each player has resources at his disposal and different choices of actions may be taken. A player will have preferences for a possible outcome of the game because of its pay off. In order to achieve such an outcome players may collaborate. Usually, in the game, players will have a conflict of interest and they may loose or win if the benefit is below or above an expected value. For a survey of game theory in the social sciences we refer to SHUBIK [38]. A game frequently cited in sociological studies is the 'prisoners dilemma', see HOFSTADTER [22].

In *social network analysis* a group of interacting individuals are represented by a graph. The nodes are the actors and a directed arc indicates the presence of a communication channel between two actors. This can be seen as a static structure in which one may analyse maximally connected subgraphs (cliques). It is also possible to make the graph time dependent. HOLLAND and LEINHARDT [23] construct such a dynamic social network model. The graph is given by a matrix with entries having a value zero (no communication) or one (directed communication). The entries change stochasticly at each time step with the parameters of the stochastic process depending on the current structure. The evolution of the process can be investigated by Monte Carlo simulations and stable stationary solutions may be interpreted in sociological terms. The sociological distance between two actors in a network can be measured from the minimal number of existing arcs that is needed to make a connection.

Social distance is also used in a different way. For each item of a set of n, an individual will have a score on a one-dimensional scale. SCHIFFMAN et al. [36] describe a method of multi-dimensional scaling in which they construct the smallest underlying space of dimension $m \leq n$, where the individuals take such positions that their mutual distances satisfy the requirements of a metric in this space.

In the study of *voting* a wide spectrum of applications of mathematics and sociology is found. First there is the problem of proportional representation. Let political party i have a fraction f_i of the votes. How many seats should it have in a parliament with N seats? That is, find the vector n/N such that it has, in some sense, a minimal distance to f, see TE RIELE [39]. In a second type of voting problem, members of a regional council are appointed by and from the local councils participating in the regional co-operation. The delegation from each council should reflect the political composition of this council, while in the regional council the political parties should have a proportional representation from all over the region. ANTHONISSE [1] studies this problem from a point of view of optimal flow in networks, see also PELEG [34]. Less mathematical is the question of voting behaviour. One may approach it as a problem of competition, as in biological population dynamics (COLLINS and KLEINER [9]) or as a sociological problem (LYPHART [29]), see also BERELSON and LAZARSFELD [4].

3. Sociological theories open to mathematical modeling

In this section we give a survey of sociological theories with a logical structure suited for mathematical modeling.

We start this overview with the founder of modern sociology Emile Durkheim. His investigations focus on social causes for the presence of collective phenomena. In his study on suicide he relates this act to social factors like religion and economical depression and revival. From the work of Durkheim one gets the impression that in society constants exist which rise above the acting of the individual. This idea is also met in the work of Quetelet [35] on his statistical description of the physiognomy of men.

At a later stage the idea evolved to connect acts of individuals to collective phenomena. First the attention went more in the direction of the interaction between some individuals (microsociology). This is seen in the work of Simmel, who studied the influence of the size of the group on social phenomena. Coleman [8] gives a quantitative mathematical description of the effect of group size. Another way of studying the behaviour of the individual is the stimulus-response theory of Skinner. Eventually, such theories of behaviour were put aside by theories of individual action as these are also applicable in a social context. The exchange theory of Homans [24] and the choice theory are examples of this new development.

In the last twenty years a new movement in sociology came into existence: *interpretive sociology.* Starting point for theories, brought together under this name, is their method of empirical research: the investigator should project himself in the social happening taking place in its natural environment. This attitude is in conflict with the classical ideas of observation and the use of a laboratory type of setting for doing experimental research. From these theories *symbolic interactionism* is the one with elements that are also found in self-organizing biological systems. It stresses the role of the acting individuals (actors) in the building of a social interaction pattern, see Mead [32] and Blumer [5]. Important in this theory is the meaning that is given to a social act. By this process the actor constructs images of himself, his co-actors and the environment. These images, in turn, control his activities in a social context.

Returning to analytical sociology we arrive at *methodological individualism,* in which the individual is the smallest logical unit of a social interaction system. An exponent of this theory is Boudon [6] with his analysis of social mobility. In his study on the relation between school career and social background, he analyses cohorts of students. In a stochastic model transition probability coefficients are determined and their dependence on the social parameters is measured. The coefficients are determined by flows at macrolevel and chance at microlevel is a reflection of these flows. Consequently, this is not a type of micro-macro system structure as we discussed in the A.

In his book *La Logique du Social* Boudon [7] gives a systems approach to social processes. He introduces this subject with a reference to the work of Hägerstrand [17] on the spread of an innovation. Hägerstrand's study concerns a Swedish governmental grant to farmers for fencing their land. It was noticed that in the five years after the start of subsidization in 1928, the process developed from spreading centers with personal contacts between farmers playing an important role, see fig. 1 for the comparable process of the spread of an infection. Hägerstrand had a sufficiently accurate picture of the microsociological structure to construct a model of the actual process. The premises of his model are:

a. At the start one actor has accepted the innovation.
b. The actors meet two by two.
c. The degree of acceptance of the innovation has a known distribution.
d. The willingness to accept increases with the number of meetings with a positive personal influence.
e. The meeting probability of two actors depends on their mutual distance.

Although the model is not cast in formula's, as one is used to in the exact sciences, the description is sufficiently accurate to understand the mechanism and to simulate the process.

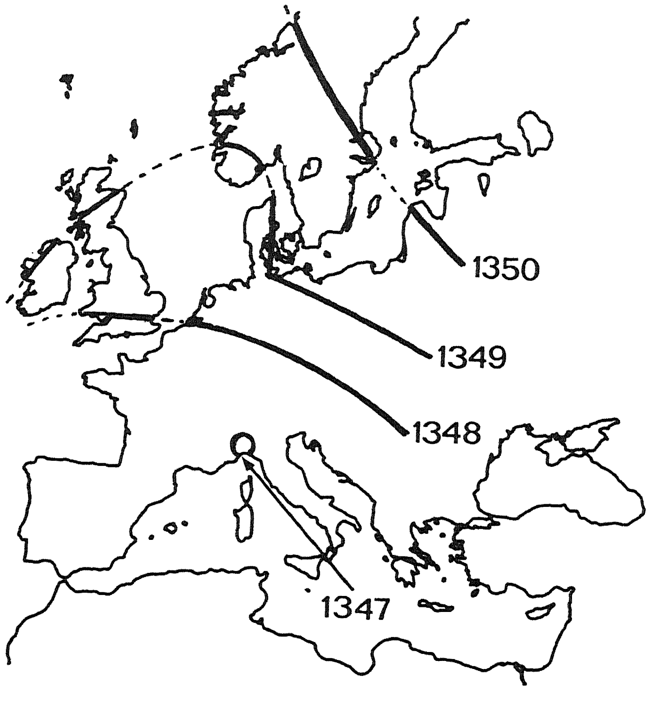

FIGURE 1. Spread of pestilence over Europe in the middle ages

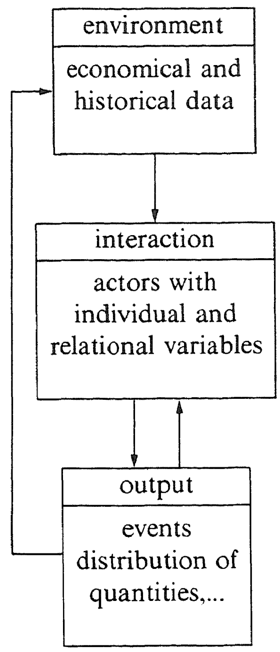

FIGURE 2. A systems approach to the interaction process; the arrows indicate the possible mutual influences of the components

In an interaction process, as described above, a number of components can be distinguished: the environment, the interaction system itself and the output of the process, see fig. 2. When all influences indicated by arrows are present we have a so-called transformation process containing different feedbacks. In fig.3 we sketch two reductions: the reproduction process and the accumulation process with one feedback. An example of the latter is the fluctuation of market prices in the process of supply and demand.

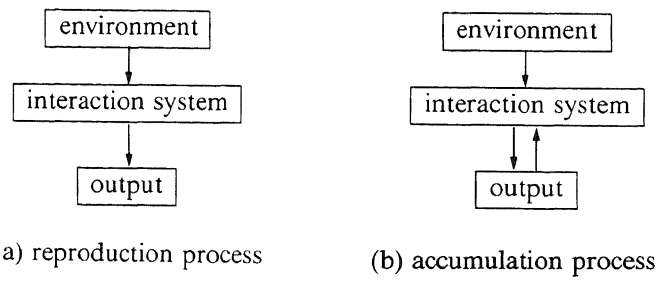

FIGURE 3. Two types of reduced transformation processes: farming can be seen as an example of (a) and a prey-predator system as an example of (b)

Last in our survey of sociological theories is *structural individualism.* This movement in Dutch sociology, which started in the seventies (WIPPLER [43]), links up with the methodological individualism of Boudon. The most important difference between the two is the quantitive approach of Boudon versus the qualitative explanatory theory of collective phenomena in the structural individualism. In a tight way individual laws (e.g. making profit or the benefit question) are coupled to individual activities by a clear bridge theory and an explanans and explanandum of a collective phenomenon are formulated. LINDENBERG [27] brought the composite explanation of a collective effect in the scheme of fig.4 (different definitions are used as Lindenberg's expressions have a meaning of their own in mathematics).

individualistic laws		transformation process
individualistic rules		collective rules
individual effect	⟶	individual effects
		collective effect

FIGURE 4. Scheme for an explanatory theory of collective effects (LINDENBERG [27])

We now come to the point of discussing the possible role of mathematics in this theory. In some problems a model can be used to describe a transformation process, such as Hägerstrand's diffusion model for the spread of an innovation. It is possible to analyse, in that case, the dynamics of the collective effect. It is remarked that then the microlevel is left completely: the dynamical system is studied in its macro-variables. In a similar way individual decisions are modeled stochasticly in Boudon's mobility theory and these decisions are reflected in the change of macro-variables (fractions of students). For those problems the models give a satisfying description. However, there are problems in which the collective effect influences the individualistic rules and makes it necessary to analyse simultaneously the micro- and macro structure. Such a type of modeling is not yet developed. Directions in research exist in which one tries to master this problem. The change of connections between individual nerve cells and the global behaviour of a neural network is an example of such a micro-macro structure. In the next section we will come back on this problem.

4. MATHEMATICAL PHYSICS, BIOLOGY AND SOCIOLOGY

The title of this section suggests the presence of a continuum of mathematical models of 'real world' phenomena. Such a continuation is found in the class of diffusion processes, as one may observe in the following five examples:

a. *Chemical reaction and diffusion.* In a medium there is a chemical reaction; the reactants diffuse in the medium and take part in the reaction process at neighbouring positions. The change of the concentration of the reactants as a function of time and position has the shape of a travelling wave. For this process a mathematical formulation exists, see ARIS [2].

b. *Pulse propagation in nerve cells.* In the membrane of a nerve cell an electro-chemical process takes place; the ions diffuse freely at both sides of the membrane. The process yields a propagating electric pulse wave along the membrane. HODGKIN and HUXLEY [21] formulated a mathematical model, which gives an excellent description of the phenomenon, although some details of the process (transport through the membrane) were not completely understood.

c. *Spread of a contagious disease.* In a biological population a contagious disease spreads out as a wave over the area. Also for this process a mathematical formulation exists, see DIEKMANN [10] (cf. figs 1 and 5).

d. *Spread of a new genotype.* In a biological population a new genotype spreads out over many generations, see for example FISHER [11].

e. *Spread of an innovation.* In a community an innovation is accepted in wider circles after periods of time, see HÄGERSTRAND [17].

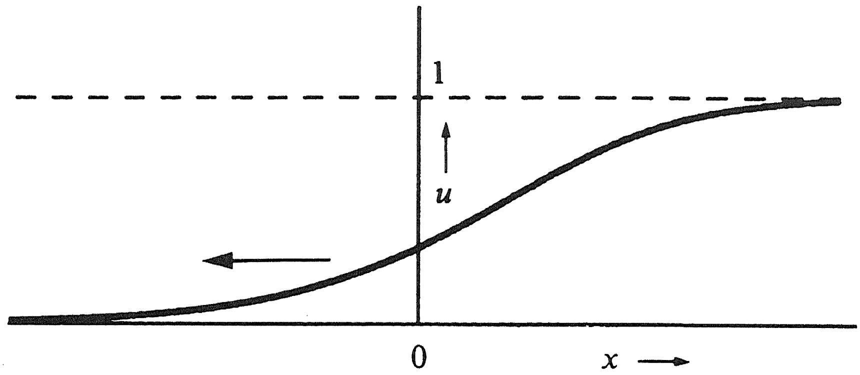

FIGURE 5. Travelling wave solution that describes the spread of a substance such as an infection over a population

Using a mathematical formulation we may construct a differential equation model that describes the underlying mechanism in the five diffusion processes. First we consider the case that within a closed box the carriers of a substance mix perfectly. Then the increase of the fraction of taken carriers is assumed to increase logisticly. That is it satisfies the differential equation

$$\frac{du}{dt} = \alpha u(1-u), \quad u(0) = \delta > 0.$$

For a 1-dimensional continuum of connected boxes, we have to take into account the effect of diffusion between neighbouring boxes. The model equation is then a partial differential equation of parabolic type

$$\frac{\partial u}{\partial t} = \alpha u(1-u) + \beta \frac{\partial^2 u}{\partial x^2}.$$

Let us assume that

$$u(x,t)=0 \quad \text{for} \quad x \to -\infty,$$

$$u(x,t)=1 \quad \text{for} \quad x \to \infty.$$

Using analytical methods one can prove that a travelling wave exists with a minimal velocity of $2\sqrt{\alpha\beta}$, see fig.5. FISHER [11] dealt with this problem in his analysis of the geographic spread of a genotype over a population.

A similar continuation of interrelated phenomena is found in the modeling of self-organization. Chemical reaction-diffusion processes may lead to stable inhomogeneous spatial concentration distributions. This, in turn, may explain the self-organized differentiation of cell functions in an embryo.

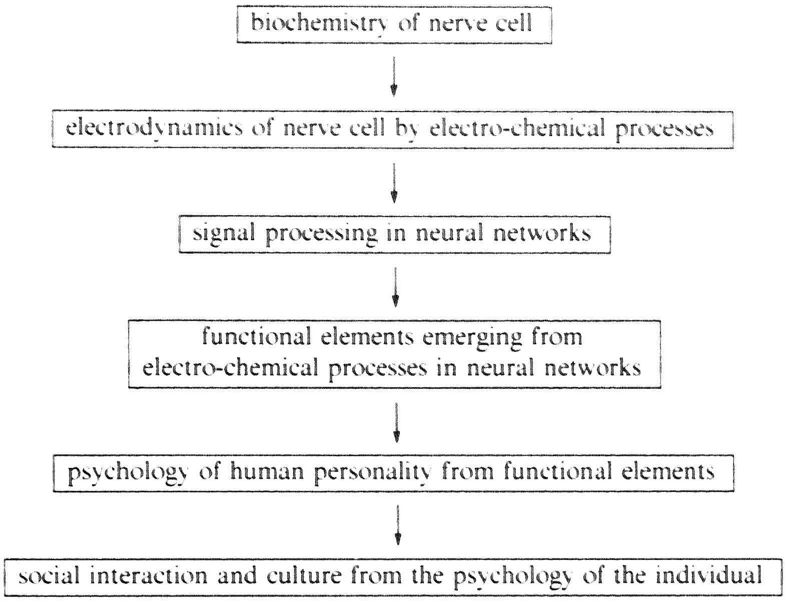

FIGURE 6. Scheme of levels in mental activity embedded in a wider framework (SCOTT [37])

Nerve cells (neurons) organize themselves. HEBB [19] introduced this element in neurodynamics through the concept of 'assembly'. An assembly is a group of neurons with mutual connections (synapses) that are strengthened if during a period of time the cells exhibit synchronous activity. As a result of this the probability, that the cells will be active at the same time in future, will increase. Of course the cells must have a certain freedom: they should not be tied up completely to the control of physiological functions. These unconstrained cells have the ability to organize their own structure within certain limitations. Consequently, this structure will differ from individual to individual; it carries thoughts and memories. The schematic representation of mental processes of fig.6 shows at which level self-organization enters in neurodynamics. Surprisingly the explanatory scheme of collective effects of Lindenberg applies quite well to mental activity, see figs. 4 and 7.

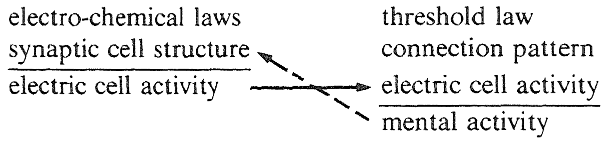

FIGURE 7. An explanatory scheme for mental activities in the manner of structural individualism

The change of the interaction structure in a network of neurons caused by the formation of cell assemblies is known as plasticity of the neuronal system. GRASMAN and JOHANNESMA [15] have indicated that social processes have properties comparable to plasticity of neural activities. The following three examples illustrate this relation, see the scheme of fig. 8 for a summary of the parallels.

a. *Hebb's cell assembles.* We already mentioned this model. In a neuronal network the activities consist of generation and conduction of electric pulses. This process yields structural changes in the network: strengthening and weakening of connections.
b. *Communication between elephant fishes (Mormyriden).* These fishes communicate through electric pulses comparable with the ones between nerve cells. Because of this activity (and environmental causes) the fish will change its position or will get in a different state of behaviour, which may make it react differently upon signals from other fishes, see HEILIGENBERG [20].
c. *A cocktail party.* At a party the most important activity is communication with one another. By this process one will change position and behaviour such that communication with certain others is advanced or prevented, e.g. by joining or leaving a group in conversation.

System	Activity	Structure
Cell assembly	electric pulses	synaptic connections
Elephant fishes	electric pulses	position, behaviour
Cocktail party	conversation	position, behaviour

FIGURE 8. Scheme of three transformation processes admitting plastic changes

The sociologist Weber described social interaction as the probability that social actions follow a certain pattern. Although it is not mentioned explicitly, the possibility exists that this probability changes in time. A pattern is culturally determined. Individuals may act differently and if they do it in the long term, the probability pattern (cultural system) will change, which indeed is a form of plasticity. In Mead's symbolic interactionism the role of the actors in the formation of the probability pattern is stressed. Not so clear in this theory is the presence of two time scales: the actual time scale for the social action of the individuals and the larger time scale with the change in interpretation that is given to social actions. Plastic changes are in the latter scale.

GROSSBERG [16] constructs a differential equation model of a neuronal network in which plastic changes are possible. Let x_i be the electric potential of the i^{th} neuronal cell and z_{ki} a measure for the number and strength of the synaptic connections from cell k to i (long-term memory trace), then the system of differential equations is of the form

$$\frac{dx_i}{dt} = f_i(x_i) + g_i(x_i)\sum_k A_{ki} z_{ki} e_k(x_k),$$

$$\frac{dz_{ki}}{dt} = -B_{ki} z_{ki} + C_{ki} h_i(x_i) d_k(x_k).$$

This network has learning properties. In simulation runs, macro-phenomena occur, that are easily perceived, but difficult to quantify.

The relation between micro- and macrolevel is of fundamental importance for the understanding of the dynamics of nonlinear systems with a large number of interacting components. In the literature dealing with the problem of self-organization we distinguish three theories which we discuss next.

The work of the Belgium group around Prigogine is based upon thermodynamic principles for *dissipative structures* in physical and chemical systems, see NICOLIS and PRIGOGINE [33]. A hypothetical chemical reaction, known as the Brusselator, plays a central role in their theory of self-organization, see fig. 9.

$$\begin{aligned} A &\rightleftarrows X, \\ B+X &\rightleftarrows Y+D, \\ 2X+Y &\rightleftarrows 3X, \\ X &\rightleftarrows E. \end{aligned}$$

FIGURE 9. The Brusselator; a hypothetical chemical reaction scheme admitting nonuniform concentration distributions over the medium

The reaction occurs in a medium where the reactants may diffuse freely. Stationary states exist for which the concentrations of the reactants have a nonuniform distribution over the medium. The nonlinearity in the third autocatalytic step of the scheme is responsible for this behaviour. The model can be used in developmental biology. The spontaneous change in the shape of the beginning embryo can be explained by this process (morphogenesis). It demonstrates the application of this principle of self-organization in biological problems.

A second theory of self-organization is the synergetics approach of HAKEN [18], who studies the occurrence of qualitative macroscopic changes caused by microscopic action principles of elementary subunits. The applications range from astrophysics over biology to sociology. In the series *Synergetics* edited by Haken, the mathematical techniques and their applications are brought together. We mention the application in sociology (WEIDLICH and HAAG [41]), in brain modeling (BASAR et al. [3]) and in various other fields (FREHLAND [14]). Haken was confronted with remarkable properties of nonlinear systems in his study on laser dynamics. Depending on the value of the parameters of a system a few state variables may change from stable to unstable. This subsystem determines the qualitative properties of the total system; the stable modes just follow (slaving principle).

Bifurcation theory is the branch of mathematical analysis that deals with the change in number, type and stability of stationary solutions of a nonlinear system as a function of the parameters. With the discovery of chaotic dynamics a new element entered the study of nonlinear systems: stable solutions may exhibit a seemingly random behaviour. LORENZ [28] was the first who noticed this phenomenon in a simple system of three coupled differential equations. This new concept in the theory of dynamical systems was rapidly admitted to existing theories of self-organization. However, this new development is drawing attention away from a competing problem. We can put this as follows: the insight in the complex dynamics of simple systems has increased dramatically in the last ten years, but on the other hand we still need to make a lot of progress in the analysis of simple dynamics of complex systems. Examples of such systems in physics, biology and sociology are easily found. We mention the well-known Ising problem dealing with the spin orientation of a ferro-magnetic object, the threshold phenomenon in epidemics and in nerve excitation and the brain as a complex system which handles questions having a yes/no outcome. In sociology we have the example of the race for the presidency between two candidates in a democratic electorial system.

Formal system theoretical aspects of self-organization are analyzed in the third approach, called *autopoiesis*. It deals with the structural coupling between systems and between a system and its environment. Autopoiesis is the realization through a closed organization of production processes such that the same organization of processes is generated through the interaction of their own products (components). Proliferation of biological cells is an illustrative example of autopoiesis. MATURANA and VARELA [31] formulate general principles which constitute a theory that is applicable in several disciplines, e.g. in the study of human organizations.

For a discussion of the principles of self-organization we refer to JANTSCH [25]. The above theories have in common that their scope ranges beyond the limitation of a discipline, as it is the case in Berthalanffy's general system theory, see VON BERTHALANFFY [40]. This can be seen as a strong point, but it also has its weakness. No authority in any of the accepted scientific disciplines can defend such a theory in all its elements spread out over the various fields of science. The switching from a metaphor in one field to another in a different field (as we did) indicates the missing of an appropriate mathematical formalism.

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