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Indagationes Mathematicae 26 (2015) 715–722
indagationes mathematicae
www.elsevier.com/locate/indag

Johannes Gualtherus van der Corput (4 September 1890–13 September 1975)☆

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Johannes Gualtherus van der Corput (Jan for his intimates) was born in Rotterdam, where his father Gualtherus Johannes had a grocery in colonial products. While his father was easygoing, Jan's mother Anna Maria Blomjous took life seriously. The family numbered four sons and a daughter. After Jan's father died in middle age, his mother had to watch expenses. Jan, although not the eldest, helped with the education of the younger children.

☆ Revision of 1975 obituary (Korevaar, 1975) in Dutch.

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http://dx.doi.org/10.1016/j.indag.2015.08.008

There were prominent Van der Corput's in the sixteenth and seventeenth century and there were several indications of mathematical ability. In 1566 Johannes de Corput made a beautiful city plan of Duisburg (while he later helped defend Steenwijk against the Spaniards). A certain Anna Maria van der Corput was the mother of the statesmen Johannes and Cornelis de Witt (who were also known for their mathematics). It is known that Jan's mother had mathematical insight; his oldest brother studied at Delft and became a secondary-school teacher of mathematics.

After primary school Jan attended the Erasmiaans gymnasium; he excelled at both Rotterdam schools. His family wanted him to study medicine and he himself considered history (in which he always maintained a lively interest) and the Dutch language. In the end the recommendation of his secondary-school teacher, R.H. van Dorsten, was decisive: Jan should study mathematics, pure mathematics.

Van der Corput studied at Leiden, 1908–14, where J.C. Kluyver lectured on analysis. Although he was later at times critical about the level of Kluyver's courses and research, the latter's interest in number theory must have been stimulating. Not only Van der Corput would later write a Ph.D. thesis under Kluyver's direction, but also H.D. Kloosterman, the other prominent Dutch number-theorist of the period 1920–70.

However, in 1914, at the beginning of the first world war and the precautionary mobilization of the Dutch army, Van der Corput was called up for military service. As an army captain he became interested in the only sport he ever practised: horseback riding. A job as secondary-school teacher of mathematics (Leeuwarden 1917–19) and work on a Ph.D. thesis made a leave before the end of 'mobilization' possible.

The difficult subject of the thesis, 'On lattice points in the plane' (in Dutch), 1919, [4] and cf. [5], was a fortunate choice. The problem is to determine the number of points with integral coordinates in certain planar domains. Important examples are: a circular disc with (large) radius $\sqrt{t}$ about the origin ('circle problem'), and the domain bounded by a hyperbola $xy = t$ and its asymptotes ('divisor problem'). The number of lattice points is equal to a principal term (essentially the area of the domain), plus a remainder term. The main problem is to find a sharp estimate for this remainder. For the circle problem Gauss already had a remainder estimate of order $t^{1/2}$. At the beginning of the twentieth century W. Sierpinski proved that the exponent $1/2$ could be reduced to $1/3$. Van der Corput came to this problem through a number of articles by E. Landau in Göttingen, who (along with G.H. Hardy and J.E. Littlewood in England) was a leading specialist in analytic number theory. With this choice Van der Corput left the more elementary area, related to Kluyver's work, in which he had published before. There was useful contact with Landau during the thesis work, which would ultimately lead Van der Corput to sharper and more extensive results than those of the master.

After his doctorate, Van der Corput taught mathematics at a secondary school in Utrecht for a year. He spent the following summer with Landau. As an immediate result there were three articles [36,5,6], the first jointly with his host. In the third paper Van der Corput introduced a new method for number-theoretic estimates, the method of exponential sums (which would afterwards lead to a breakthrough in lattice-point problems). In 1921 there now were five different methods which all gave the same estimate for the remainder term (the exponent $1/3$ in the circle and divisor problems). No wonder that one believed to have found the correct order of the remainder term! Landau spoke of a "wunderschöne Harmonie".

However, Van der Corput's new method soon led to essentially sharper results, and with those, again according to Landau, it brought "das Chaos". The big breakthrough came in the divisor problem, where Van der Corput could replace the exponent $1/3$ by $33/100$ [7]. Shortly thereafter, he announced similar results for other problems, among them the circle problem [8]. Here it is appropriate to remark that older work of Hardy [25] and Landau [35] had given the lower bound $1/4$ for the exponent, and it is widely believed that this bound is sharp.

From 1920 till 1922 Van der Corput worked at the University of Utrecht as an assistant to A. Denjoy. Next came a brief professorship at the University of Fribourg in Switzerland. In 1923 he became professor at the University of Groningen where he would remain until 1946.

For about ten years he occupied himself mainly with further refinement of the method of exponential sums, see for instance [9], and with the development of applications. Some of his first (and best) students took part in this work. Thus L.W. Nieland [40,41] could reduce the exponent $1/3$ in the circle problem to $27/82 \approx 0.3293$. (Tragically, Nieland lost his life during the second world war as a result of a German reprisal action at the village of Putten.) Jointly with J.F. Koksma, Van der Corput applied his new method to the Riemann zeta function in the critical strip [22]. The method was also applied to systems of diophantine inequalities, yielding two long papers in Acta Mathematica [10,11] (the first with an application to the theory of uniform distribution modulo 1), as well as Koksma's Ph.D. thesis [31]. The problem here is to obtain integer solutions to inequalities, such as arise when one wants to find close rational approximations to irrational numbers.

In 1975 I wrote in an obituary [33] of Van der Corput in Dutch: "His work on exponential sums belongs to the most advanced and sophisticated work ever done in the field of analytic number theory. It says something about his skill and ingenuity that there has been only marginal progress on the subject of lattice point problems since 1930. In fact, the best known result on the circle problem today seems to be that of Wen-lin Yin [46], who obtained the exponent $12/37 \approx 0.3243$". Now, forty years later, one should say that there has been slow but steady progress since Van der Corput's work. One may mention Kolesnik [32] with exponent $35/108 + \epsilon \approx 0.3241$, Iwaniec and Mozzochi [29] with exponent $7/22 + \epsilon \approx 0.3182$, and Huxley [28], who in 2003 obtained the exponent $131/416 + \epsilon \approx 0.3150$.

In the 1930's Van der Corput's research became broader. The refined analytic methods were now also applied to problems outside the theory of numbers. This may have been an effect of the courses he had to teach and likely also had to do with the need to find topics for some fifteen Ph.D. candidates at the University of Groningen. In the field of analysis there is his first systematic work on the asymptotic evaluation of general types of integrals. He started with publications on the method of stationary phase [12,13]. Next he combined the known methods into the theory of so-called decisive or critical points, which has important applications in the natural sciences. This was a first step in the direction of his subsequent more and more general theories of asymptotics. Already in 1933, the Ph.D. thesis of his student C.S. Meijer [38] (who later became quite well-known) gave approximations to many important special functions. There were a few more theses in this area, while other students occupied themselves with differential equations, convex sets, number theory (of course) and problems between analysis and number theory. Some of the best students worked in the latter area: besides J.F. Koksma [31], also J. Popken [43] and B. Meulenbeld [39]. There were also a few students from abroad (not so common then as nowadays): C. Pisot and J. Teghem.

Around this time, the Russian mathematician I.M. Vinogradov developed a powerful new technique for the treatment of exponential sums. Van der Corput quickly mastered the complicated new method, and applied it to diophantine equations and to the famous conjecture of Goldbach. On the basis of scant numerical data the latter had asserted in 1742 that every even positive integer can be written as the sum of two prime numbers. Vinogradov proved a related result, namely, that every sufficiently large odd integer can be written as the sum of three primes. Shortly thereafter, Van der Corput proved that even positive integers which do not satisfy Goldbach's hypothesis must be extremely rare [14]. In subsequent papers he studied and counted general representations of integers by linear combinations of prime powers, see [15] and related papers in the Proceedings of the (Koninklijke) Nederlandse Akademie van Wetenschappen [(Royal) Netherlands Academy of Sciences]. Because it is now an active subject, it may be of interest to indicate Van der Corput's result for the special case $2k = p_1 - p_2$. Let $\pi_{2k}(x)$ denote the number of prime pairs $(p, p + 2k)$ up to large x. Then for 'almost all' differences $2k \leq x/\log x$, Hardy and Littlewood's conjectured asymptotic formula [26] gives a good approximation to $\pi_{2k}(x)$. For a precise statement see also Mathematical Reviews on Lavrik [37]. When the number of mathematics articles in (K)NAW Proceedings became quite large, it was decided to collect these articles also in a separate journal, Indagationes Mathematicae, for greater visibility. The first volume (1939) contained nine articles by Van der Corput (two jointly with Pisot).

In the late thirties and the years 1940–45 of the German occupation of the Netherlands there was a big change in Van der Corput's orientation. Exaggerating a little, one might say that before the war he had lived only for mathematics, whereas in this period he became much more socially engaged. Still a bachelor he met a remarkable woman, Jeannette Cornelia Houwink (1898–1989). She had obtained a law doctorate at Groningen and married fellow-student H.W.O. ten Cate in 1920, worked at the tax department for a while, had three children and became a writer.[1] After her divorce Van der Corput married her in 1942, on August 31, birthday of the Dutch queen in exile. She would strongly support him in his activities (and share his frugality when it concerned themselves). He played an active role in the university opposition to the German occupants, for which he regularly traveled to Amsterdam. In their home, he and his wife provided shelter for persons hiding from the enemy. They also assisted Jewish mathematicians from abroad. At the beginning of 1945 Van der Corput and two persons hiding at his house were arrested, but thanks to the help of some unknown person, he was released after three anxious weeks.

He had for some time been involved in planning on the future of the universities. At the end of [16] he gave his view on the role of mathematicians: they should be prepared to assist others, but not neglect fundamental research. In close contact with his Groningen colleague G. van der Leeuw (professor of theology), plans were drawn up for fundamental changes in university education; it should become available to students from all levels of society. At meetings in Groningen and Amsterdam one also discussed the post-war organization of university mathematics, including the possibility of establishing a national mathematical center. Van der Corput made a detailed inventory of mathematics in the Netherlands in the period 1910–1945; see [18]. (As he mentioned, there had been an excellent earlier survey by Kloosterman [30].)

When the war ended in 1945, higher education in the Netherlands was in deplorable condition, also as a result of economy measures in the pre-war depression years. The mathematics staffs at most universities had always been small. Now there were problems with regard to the traditional subjects (many positions had become vacant), while newer areas were not represented at all. The situation demanded coordination and reorganization; cf. Alberts, van der Blij and Nuis [2], Alberts [1]. Van der Leeuw was appointed Minister of Education, Arts and Sciences, and Van der Corput was the natural person for a leading role with regard to mathematics. He became the chairman of a powerful coordination committee. Members were the mathematicians D. van Dantzig (Technische Hogeschool [Technical University] Delft), J.F. Koksma (Free University Amsterdam) and J.A. Schouten (in early retirement from Delft), the Leiden physicist H.A. Kramers and the Utrecht astronomer M.J.G. Minnaert. The committee's charge was to advise on (1) the

[1] Some of the information on J.C. Houwink is due to her grandson J.Th.M. Houwink ten Cate.

appointment of new mathematics faculty at the universities, and (2) the restructuring of the mathematics curriculum at the universities. There were to be greater opportunities for research and there should be provisions for the training of secondary-school teachers of mathematics. Because of the lag in development of more applied areas and the expected expense of catching up, there was a trend towards centralization. Strongly driven by the chairman, the committee was thinking of one comprehensive mathematical center for research and development at Utrecht or Amsterdam. When the City of Amsterdam offered strong support, it was decided that the center would be there. As a result Van der Corput (and with him, Van Dantzig) took up a professorship at the Municipal University of Amsterdam. His inaugural lecture 'Het Mathematisch Centrum' [The Mathematical Center] [17] is still worth reading today. In 1946 he became the first director of the new Mathematical Center in Amsterdam. [Reflecting the growing importance of computer science, the name would later be changed to 'Centrum voor Wiskunde en Informatica' (CWI).]

In his Amsterdam period (1946–53) Van der Corput's main activity was the shaping of this Center. Here a computing department was set up and a start was made with the development of computers in the Netherlands. The young Delft doctor of engineering A. van Wijngaarden (thesis [45]) was appointed to the staff, and sent to England to prepare for being head of computing. He would later become well-known for his contributions to computer science (think of Algol 68). Van Dantzig began a vigorous development of the rather neglected area of mathematical statistics and stochastics, and put his stamp on the subject ('parameter-free statistics'). A department of mathematical physics came under the leadership of B.L. van der Waerden (who had returned from Germany at the end of the war, but would not stay long). There was a department of pure mathematics under the direction of Van der Corput and Koksma. Among other things, this department had to support the other departments where necessary. In the early years, Schouten was also active at the Center, but not attached to a particular department. Under the direction of Koksma a library was started which could use Indagationes Mathematicae for exchanges with mathematical journals from abroad; the library gradually developed into the best mathematical library of the Netherlands.

Van der Corput considered it as part of the Center's mission to provide advanced mathematics courses and lectures for persons around the country who had been unable to obtain a university education. Many Dutch mathematicians participated in this program sponsored by the Center, which also included summer courses for secondary-school teachers of mathematics. The Center furthermore provided positions for promising young Dutch mathematicians ('postdocs avant la lettre') who were likely to obtain a university appointment after a few years. Lively contacts developed with mathematicians from abroad. Paul Erdös lectured here in 1948 on his "elementary" proof of the prime number theorem (which he had found jointly with A. Selberg); Van der Corput prepared an early Center publication of the famous proof [19].

This may be the place for a personal note. In September 1947, my Leiden study-friend Dr. Fred van der Blij and I had the good fortune to become (the first) junior staff members at the Mathematical Center. In theory our principal duty was to do research, but at the not-yet-well-organized Center, a large variety of odd jobs came our way; cf. the interview [42]. We learned a lot of mathematics and profited from the almost daily contact with Van der Corput. In the context of his 'mission' we helped organize a program of lectures and courses around the country, and ourselves taught evening courses in Amsterdam. Organizing and using newly arriving literature, we started a series of monthly Saturday morning lectures on current topics, under the title 'Actualiteiten'. The literature also provided the subject for my Ph.D. thesis, which I wrote at the Center; topics for other publications arose from Erdös's visit. My years at the Center (1947–49) were the beginning of a cordial relation with Mr. and Mrs. Van der Corput that developed further during our years in the U.S.

In his Center period Van der Corput published on a variety of subjects. He developed a constructive version of the so-called principal theorem of algebra, which says that every algebraic equation of degree n, with complex coefficients, has precisely n roots. He wrote on symmetric functions and various topics of number theory; also on subjects that he had not considered before. In that category several articles (some jointly with J.H.B. Kemperman) on the density of the set $A + B$ of all sums $a + b$ with $a \in A$, $b \in B$ (the $\alpha + \beta$ hypothesis). He directed a number of Ph.D. theses, one on a didactical subject, the others, among them that of his assistant H.J.A. Duparc [23], on algebraic or number theoretic subjects.

Van der Corput's main scientific interest, however, had become the study of asymptotic expansions, a subject that had already claimed his attention in the thirties. It was his plan to produce a complete survey of all existing asymptotic methods and to generalize those methods as much as possible. Such a new extended theory might be of great value for applications. At the Center a study group on asymptotics met weekly under his direction (cf. N.M. Temme [44]). In his Rouse Ball Lecture (Cambridge 1948), see [20], he solicited contributions from the international mathematical community. Also in connection with the large project he was happy to accept a visiting position at Stanford University (1950–52). Here he was in close contact with many prominent mathematical analysts and old acquaintances who had immigrated from Europe: S. Bergmann, K. Löwner, G. Pólya, M. Schiffer, G. Szegö.

On his return to the Netherlands he found the situation in Amsterdam somewhat disappointing. The relations at American universities were less formal (and a visiting professor was of course especially free); at Stanford and elsewhere there was much interest in his work. The Mathematical Center had become an established institute which was firmly set on its course and did not require day-to-day attention. An earlier plan to develop the Center into an institution of greater national or international scope came to nought. This fact contributed to Van der Corput's decision to accept a permanent position at the University of California. There he would have much time for his own research, and the atmosphere promised to be more congenial for his project in asymptotics.

From 1954 until 1966 his main residence was Berkeley, where he was a regular faculty member until 1958. After that there were visiting appointments at the Mathematics Research Center in Madison (Wisconsin) and in Rome. All this time he devoted himself mainly to asymptotics, on which he published very extensively, mostly in Indagationes Mathematicae (which coincided with the KNAW Proceedings Series A for a number of years). A notable exception is his long article [21] on Euler's sum formula. Most of his eight Ph.D. students in California wrote their thesis on asymptotics. From 1966 on Van der Corput lived alternately in Antwerp and in Amsterdam, where he still taught at the Free University for a year.

Looking back one is struck by Van der Corput's great enthusiasm, idealism and energy. He was capable of providing leadership when he felt that it was needed. That the CWI (formerly Mathematical Center) now plays such an important role in Dutch mathematics is to a large extent his merit. However, he primarily remained a man of science, witness his many publications. He liked to work with students and knew how to speak and write about mathematics for an educated public. When he got hold of a good problem he would repeatedly return to it, always looking for a sharper result and greater generality. The latter sometimes made his work difficult to read, which he would readily admit when it was pointed out. The same perfectionism may have kept him from completing the planned comprehensive work on asymptotics. Nevertheless he certainly ranks among the foremost mathematicians of the Netherlands in the twentieth century. His work on exponential sums in particular is highly valued today; cf. Huxley's book [27]. Hilfssätze 1 and 2 in his article [6] became known and were generalized as "the van der Corput lemma".

Recognition of Van der Corput's scientific merits had come relatively early: in 1929 he became a member of the Netherlands Academy of Sciences, three years later also of the Royal Belgian Academy. He was an invited speaker at the International Congress of Mathematicians at Oslo in 1936. His professorships at Stanford and Berkeley also meant recognition. Many of his pupils became professors themselves; Koksma and Popken also became Academy members. In 1952 Van der Corput received an honorary doctorate from the University of Bordeaux and in 1966 he was similarly honored by the Technical University at Delft. There is a brief biography by N.G. de Bruijn [3] in Acta Arithmetica, of which Van der Corput was an editor from the beginning in 1936. A complete list of his publications can be found in [34] and [24]; the latter publication also lists his Ph.D. students and the titles of their theses.

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