Partial Partial Preference Order Orders

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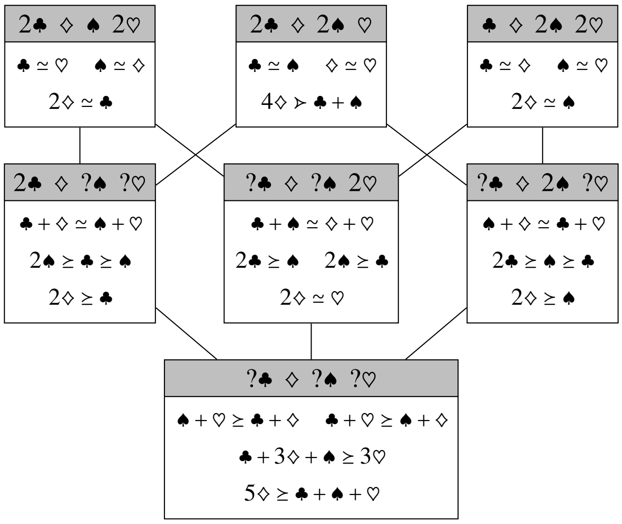
Consider the following collection of six-sided dice:

> *There are four faces, each present at least once: clubs $\clubsuit$, spades $\spadesuit$, diamonds $\diamondsuit$, and hearts $\heartsuit$. A face only becomes visible after applying a drop of white wine to its side. There are at least three black faces. There are either more hearts than diamonds or an equal number of clubs and spades. A die is fair unless it has more black than white-faced sides, then each of the latter is equally more likely to land up than each of the former.*

Because of the *exclusive disjunctions*—either/or statements—in this description, the uncertainty we must model when gambling with dice from this collection cannot be handled using a single convex credal set, set of desirable gambles, preference order, or other such uncertainty model. Arguably, also non-convex credal sets are inadequate here.

I wish to discuss the following conceptual approach for dealing with this modeling issue:

- The *possibility space* is restricted to observables only ($\clubsuit$, $\diamondsuit$, $\spadesuit$, and $\heartsuit$) and so should not involve, e.g., the die variant. (There are three such variants; see the gray boxes in the top row of the diagram.)
- We consider the *partial order* X generated by the exclusive disjunctions. (See the gray boxes and their interconnections in what is in fact a Hasse diagram.)
- We attach an *uncertainty model* to each element of X, e.g., a partial preference order, that reflects the information common to its upset in X. (In the diagram we use $\geq$ for non-strict acceptance, $\succ$ for strict preference, and $\simeq$ for indifference [1]. Also, in the expressions, the faces denote the corresponding indicator gamble.)
- We can furthermore assign an *optimality criterion* to each element of X. Maximality and maximin variants thereof are natural candidates, E-admissibility perhaps less so, due to its use of individual probability measures, which can be replaced by exclusive disjunctions.
- With any set of decision options, we can then associate the corresponding partial order of optimal options. *Choice functions* [cf. 2] may be derived as functions thereof, for example the union of optimal options for the maximal elements of X.



Acknowledgments Research financed by the *Safe Statistics* project of the Netherlands Organisation for Scientific Research (NWO). Thanks go to Jasper De Bock, Gert de Cooman, and Arthur Van Camp for stimulating discussion.

Keywords. Exclusive disjunction, partial order, uncertainty model, credal set, set of desirable gambles, preference order, optimality criterion, choice function.

References

[1] Erik Quaeghebeur, Gert de Cooman & Filip Hermans. Accept & reject statement-based uncertainty models. *International Journal of Approximate Reasoning* 57 (Feb. 2015), 69–102. DOI: `10.1016/j.ijar.2014.12.003`. arXiv: `1208.4462`.

[2] Teddy Seidenfeld, Mark J. Schervish & Joseph B. Kadane. Coherent choice functions under uncertainty. *Synthese* 172 (2010), 157–176. DOI: `10.1007/s11229-009-9470-7`.