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THE ADDITION FORMULA FOR LITTLE q-LEGENDRE POLYNOMIALS AND THE $SU(2)$ QUANTUM GROUP*

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Abstract. From the interpretation of little q-Jacobi polynomials as matrix elements of the irreducible unitary representations of the SU(2) quantum group an addition formula is derived for the little q-Legendre polynomials. It involves an expansion in terms of Wall polynomials. A product formula for little q-Legendre polynomials follows by q-integration.

Key words. quantum groups, SU(2), little q-Jacobi polynomials, little q-Legendre polynomials, Wall polynomials, addition formula, product formula

AMS(MOS) subject classifications. 33A65, 33A75, 22E70

1. Introduction. Quantum groups were recently introduced by Drinfeld [5] and Woronowicz [17]. Interesting examples are provided by deformation into a noncommutative Hopf algebra of some suitable commutative Hopf algebra of functions on a specific group. The most elementary nontrivial example comes from deformation of the algebra of polynomials on $SU(2)$ (cf. Woronowicz [18]). We denote the resulting quantum group by $SU_\mu(2)$.

It has been proved by Vaksman and Soibelman [15], Masuda et al. [11], [12], and Koornwinder [10] that the matrix elements of the irreducible unitary representations of the quantum group $SU_\mu(2)$ can be expressed in terms of the little q-Jacobi polynomials. In the present paper we use this interpretation to derive an addition formula for the little q-Legendre polynomials, i.e., for the q-analogues of the Legendre polynomials within the class of little q-Jacobi polynomials. The derivation of this formula is straightforward and analogous to a proof of the addition formula for Legendre polynomials using irreducible representations of $SU(2)$ (cf. Vilenkin [16, Chap. 3]). However, the resulting formula involves noncommuting variables. It is less easy to rewrite it equivalently as a formula involving only commuting variables. We can do this by using an infinite-dimensional irreducible $*$-representation of the Hopf algebra considered as a $*$-algebra. The result (Theorem 4.1) gives an expansion in terms of Wall polynomials (little q-Jacobi polynomials with the second parameter equal to zero).

Our addition formula somewhat resembles an addition formula for (continuous) q-ultraspherical polynomials derived by Rahman and Verma [14], but for that formula a (quantum) group-theoretic interpretation is not yet known. It would have been hard to find our formula without guidance from the quantum group. Indeed, that it is possible to obtain such a formula demonstrates the power and depth of the quantum group-theoretic interpretation of special functions. It turns out to be highly nontrivial to prove this formula analytically or to show that its limit case for $q \uparrow 1$ is the addition formula for Legendre polynomials. The analytic proof has been done by Rahman [13] and the limit result is proved by Van Assche and Koornwinder [2].

The contents of this paper are as follows. In §§ 2 and 3 the preliminaries about q-hypergeometric orthogonal polynomials and quantum groups, respectively, are presented. The derivation of the addition formula is given in § 4. Finally, a product formula for little q-Legendre polynomials is derived from the addition formula in § 5.

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2. Some q-hypergeometric orthogonal polynomials. Let $1 \neq q \in \mathbb{C}$. We use the familiar definitions and notation for q-shifted factorials and q-hypergeometric functions (cf. [6, Chap. 1], [10, § 2]). The *little q-Jacobi polynomials*

$$p_n(x; a, b \mid q) := {}_2\phi_1\left(\begin{matrix} q^{-n}, abq^{n+1} \\ aq \end{matrix}; q, qx\right) \tag{2.1}$$

occur as part of a classification by Hahn [7] of orthogonal polynomials satisfying q-difference equations. Their detailed properties as orthogonal polynomials are given by Andrews and Askey [1]. For $a = b = 1$ we will call these polynomials *little q-Legendre polynomials.*

The special little q-Jacobi polynomials obtained by putting $b := 0$ in (2.1) are known as *Wall polynomials*; cf. Chihara [3, § 5, Case I], [4, Chap. 6, § 11]:

$$p_n(x; a, 0 \mid q) = {}_2\phi_1(q^{-n}, 0; aq; q, qx) \tag{2.2}$$

(Chihara uses another notation). We will put $a := q^\alpha$. These polynomials can be viewed as one of the many q-analogues of the Laguerre polynomials in view of the limit formula

$$\lim_{q \uparrow 1} p_n((1-q)x; q^\alpha, 0 \mid q) = L_n^\alpha(x)/L_n^\alpha(0).$$

By specialization of the orthogonality relations for the little q-Jacobi polynomials we obtain the orthogonality relations for the Wall polynomials:

$$\frac{(q^{\alpha+1}; q)_\infty}{(1-q)(q; q)_\infty} \int_0^1 p_n(t; q^\alpha, 0 \mid q) p_m(t; q^\alpha, 0 \mid q) t^\alpha (qt; q)_\infty \, d_q t = \delta_{n,m} \frac{q^{n(\alpha+1)}(q; q)_n}{(q^{\alpha+1}; q)_n}, \tag{2.3}$$

where the q-integral is defined by

$$\int_0^1 f(t) \, d_q t := \sum_{k=0}^\infty f(q^k)(q^k - q^{k+1}),$$

and where we suppose that $0 < q < 1$ and $\alpha > -1$. From [3, Forms. (5.1), (5.2)] together with (2.2) we obtain the three-term recurrence relation

$$\begin{aligned} xp_n(x; a, 0 \mid q) &= -q^n(1 - aq^{n+1}) p_{n+1}(x; a, 0 \mid q) \\ &\quad + q^n(1 + a - aq^n - aq^{n+1}) p_n(x; a, 0 \mid q) \\ &\quad - q^n(a - aq^n) p_{n-1}(x; a, 0 \mid q), \\ p_{-1}(x; a, 0 \mid q) &= 0, \qquad p_0(x; a, 0 \mid q) = 1. \end{aligned} \tag{2.4}$$

Put

$$\begin{aligned} P_n(q^k; q^\alpha \mid q) := {} & \left(\frac{(q^{\alpha+1}; q)_\infty (q^{k+1}; q)_\infty (q^{\alpha+1}; q)_n}{(q; q)_\infty (q; q)_n}\right)^{1/2} \\ & \cdot (-1)^n q^{(k-n)(\alpha+1)/2} p_n(q^k; q^\alpha, 0 \mid q). \end{aligned} \tag{2.5}$$

Then (2.3) can be rewritten as

$$\sum_{k=0}^\infty P_n(q^k; q^\alpha \mid q) P_m(q^k; q^\alpha \mid q) = \delta_{n,m}, \qquad n, m = 0, 1, 2, \cdots. \tag{2.6}$$

Since the orthogonality measure in (2.3) has compact support, the orthonormal system

$$\{P_n(q^k; q^\alpha \mid q)\}_{k=0,1,2,\cdots}, \qquad n = 0, 1, 2, \cdots,$$

is complete in the Hilbert space l^2. Hence, we have also the dual orthogonality relations

$$\sum_{n=0}^\infty P_n(q^k; q^\alpha \mid q) P_n(q^l; q^\alpha \mid q) = \delta_{k,l}, \qquad k, l = 0, 1, 2, \cdots. \tag{2.7}$$

We conclude this section with an expression of Wall polynomials in terms of a ${}_3\phi_2$:

$$p_n(x; q^\alpha, 0 \mid q) = \frac{(-1)^n q^{n(n+2\alpha+1)/2} x^n}{(q^{\alpha+1}; q)_n} \, {}_3\phi_2\left(\begin{matrix} q^{-n}, q^{-n-\alpha}, x^{-1} \\ 0, 0 \end{matrix}; q, q\right). \tag{2.8}$$

This follows by putting $b := 0$ in

$${}_2\phi_1\left(\begin{matrix} q^{-n}, abq^{n+1} \\ aq \end{matrix}; q, qx\right) = (q^{-n+1}x; q)_n \, {}_2\phi_2\left(\begin{matrix} q^{-n}, q^{-n}b^{-1} \\ qa, q^{-n+1}x \end{matrix}; q, q^{n+2}abx\right)$$
$$= \frac{(qb; q)_n q^{n(n-1)/2}(-aqx)^n}{(qa; q)_n} \, {}_3\phi_2\left(\begin{matrix} q^{-n}, q^{-n}a^{-1}, x^{-1} \\ qb, 0 \end{matrix}; q, q\right).$$

Here we have used a transformation formula for ${}_2\phi_1$ (cf. [6, Chap. 1]) in the first equality and reversion of summation order in the second equality (see also [8]).

3. The quantum group $SU_\mu(2)$. In the rest of this paper we fix $0 \neq \mu \in (-1, 1)$. Let $\mathscr{A}$ be the unital $*$-algebra generated by the two elements α and γ satisfying the relations

$$\alpha\gamma = \mu\gamma\alpha, \quad \alpha\gamma^* = \mu\gamma^*\alpha, \quad \gamma\gamma^* = \gamma^*\gamma,$$
$$\alpha^*\alpha + \gamma\gamma^* = I, \qquad \alpha\alpha^* + \mu^2\gamma\gamma^* = I.$$

Let $\Phi: \mathscr{A} \to \mathscr{A} \otimes \mathscr{A}$ be the unital $*$-homomorphism such that

$$\Phi(\alpha) = \alpha \otimes \alpha - \mu\gamma^* \otimes \gamma, \qquad \Phi(\gamma) = \gamma \otimes \alpha + \alpha^* \otimes \gamma.$$

Then Φ acts as a comultiplication and $\mathscr{A}$ thus becomes a Hopf algebra with involution which we say to be *associated with the compact matrix quantum group $SU_\mu(2)$*. In the limit for $\mu \uparrow 1$, $\mathscr{A}$ becomes the algebra of polynomials in the matrix elements of the natural representation of $SU(2)$, and the comultiplication is then induced by the group structure of $SU(2)$.

It is possible to embed the $*$-algebra $\mathscr{A}$ as a dense $*$-subalgebra of a C^*-algebra by a universal construction. The C^*-algebra approach is emphasized in particular by Woronowicz [17], [18]. In this paper we could work with the C^*-algebra, but only elements of the dense $*$-subalgebra $\mathscr{A}$ will be needed, so we will use this latter algebra. Other references for $SU_\mu(2)$ besides [17] and [18], are [5], [15], [11], and [12].

The irreducible unitary co-representations of $\mathscr{A}$ (which are called irreducible unitary representations of $SU_\mu(2)$ in [18] and [10]) have been completely classified [18], [9], [15], [11], [12], [10]. Up to equivalence, there is one such co-representation for each finite dimension. We will denote the co-representation of dimension $2l+1$ by $t^{l,\mu}$ ($l = 0, \frac{1}{2}, 1, \cdots$), and its matrix elements with respect to a suitable orthonormal basis corresponding to the quantum subgroup $U(1)$ by $t^{l,\mu}_{n,m}$ ($n, m = -l, -l+1, \cdots, l$). Then the co-representation property of $t^{l,\mu}$ is expressed by

$$\Phi(t^{l,\mu}_{n,m}) = \sum_{k=-l}^{l} t^{l,\mu}_{n,k} \otimes t^{l,\mu}_{k,m}. \tag{3.1}$$

The $t^{l,\mu}_{n,m}$ have been computed explicitly in terms of little q-Jacobi polynomials (cf. [15], [11], [12], [10]). Here we will only need the cases that $l = 0, 1, 2, \cdots$ and m or $n = 0$. Put

$$p^\mu_{l,k}(x) := \begin{bmatrix} l \\ k \end{bmatrix}^{1/2}_{\mu^2} \begin{bmatrix} l+k \\ k \end{bmatrix}^{1/2}_{\mu^2} \mu^{-k(l-k)} p_{l-k}(x; \mu^{2k}, \mu^{2k} \mid \mu^2), \tag{3.2}$$

$$p^\mu_l(x) := p^\mu_{l,0}(x) = p_l(x; 1, 1 \mid \mu^2). \tag{3.3}$$

Here the notation (2.1) for the little q-Jacobi polynomials is used and (3.3) gives a little q-Legendre polynomial. Then (cf. [10, Thm. 5.3]), for $k=0,1,\cdots,l$,

$$
\begin{aligned}
t^{l,\mu}_{k,0} &= (\alpha^*)^k p^{\mu}_{l,k}(\gamma\gamma^*)\gamma^k,\\
t^{l,\mu}_{0,k} &= (\alpha^*)^k p^{\mu}_{l,k}(\gamma\gamma^*)(-\mu\gamma^*)^k,\\
t^{l,\mu}_{-k,0} &= (-\mu\gamma^*)^k p^{\mu}_{l,k}(\gamma\gamma^*)\alpha^k,\\
t^{l,\mu}_{0,-k} &= \gamma^k p^{\mu}_{l,k}(\gamma\gamma^*)\alpha^k.
\end{aligned}
\tag{3.4}
$$

Hence

$$
t^{l,\mu}_{0,0} = p^{\mu}_{l}(\gamma\gamma^*). \tag{3.5}
$$

The irreducible $*$-representations of the $*$-algebra $\mathscr{A}$ on a Hilbert space are classified in [15, Thm. 3.2]. There is a family of one-dimensional and a family of infinite-dimensional representations, both parametrized by the unit circle. We pick one of these infinite-dimensional representations: Let $\mathscr{H}$ be a Hilbert space with orthonormal basis $e_0, e_1, \cdots$. Put $e_{-1}, e_{-2}, \cdots := 0$. We define a $*$-representation of τ of $\mathscr{A}$ on $\mathscr{H}$ by specifying the action of the generators of $\mathscr{A}$:

$$
\begin{aligned}
\tau(\alpha)e_n &:= (1-\mu^{2n})^{1/2}e_{n-1},\\
\tau(\alpha^*)e_n &:= (1-\mu^{2n+2})^{1/2}e_{n+1},\\
\tau(\gamma)e_n &:= \mu^n e_n,\\
\tau(\gamma^*)e_n &:= \mu^n e_n.
\end{aligned}
\tag{3.6}
$$

4. Proof of the addition formula. Let $l=0,1,2,\cdots$. A special case of (3.1) is

$$
\Phi(t^{l,\mu}_{0,0}) = \sum_{k=-l}^{l} t^{l,\mu}_{0,k} \otimes t^{l,\mu}_{k,0}.
$$

Hence, by (3.4) and (3.5),

$$
\begin{aligned}
p^{\mu}_{l}(\Phi(\gamma\gamma^*)) &= p^{\mu}_{l}(\gamma\gamma^*)\otimes p^{\mu}_{l}(\gamma\gamma^*)\\
&\quad + \sum_{k=1}^{l} (\alpha^*)^k p^{\mu}_{l,k}(\gamma\gamma^*)(-\mu\gamma^*)^k \otimes (\alpha^*)^k p^{\mu}_{l,k}(\gamma\gamma^*)\gamma^k\\
&\quad + \sum_{k=1}^{l} \gamma^k p^{\mu}_{l,k}(\gamma\gamma^*)\alpha^k \otimes (-\mu\gamma^*)^k p^{\mu}_{l,k}(\gamma\gamma^*)\alpha^k.
\end{aligned}
\tag{4.1}
$$

This formula might already be called an addition formula for little q-Legendre polynomials p^{μ}_{l}. It involves noncommuting variables. In the limit, for $\mu\uparrow 1$, the variables commute and (4.1) becomes the classical addition formula for Legendre polynomials. In three steps we will rewrite (4.1) into a formula involving commuting variables: First, we represent (4.1) as an operator identity on the Hilbert space $\mathscr{H}\otimes\mathscr{H}$ by using the representation $\tau\otimes\tau$ (cf. (3.6)). Second, we let these operators act on the standard basis of $\mathscr{H}\otimes\mathscr{H}$. Thus we obtain a family of vector identities in $\mathscr{H}\otimes\mathscr{H}$. Third, we take inner products with respect to another suitable orthonormal basis of $\mathscr{H}\otimes\mathscr{H}$. This will yield a family of scalar identities.

Apply $\tau\otimes\tau$ to both sides of (4.1) and let both sides of the resulting operator equality act on $e_{x+y}\otimes e_y$. Then

$$p_l^\mu((\tau\otimes\tau)\Phi(\gamma\gamma^*))\, e_{x+y}\otimes e_y = p_l^\mu(\mu^{2x+2y})p_l^\mu(\mu^{2y})\, e_{x+y}\otimes e_y$$
$$+\sum_{k=1}^{l}(-1)^k\mu^{k(x+2y+1)}(\mu^{2(x+y+1)};\mu^2)_k^{1/2}(\mu^{2(y+1)};\mu^2)_k^{1/2}$$
$$\cdot\, p_{l,k}^\mu(\mu^{2x+2y})p_{l,k}^\mu(\mu^{2y})\, e_{x+y+k}\otimes e_{y+k} \tag{4.2}$$
$$+\sum_{k=1}^{l}(-1)^k\mu^{k(x+2y-2k+1)}(\mu^{2(x+y)};\mu^{-2})_k^{1/2}(\mu^{2y};\mu^{-2})_k^{1/2}$$
$$\cdot\, p_{l,k}^\mu(\mu^{2x+2y-2k})p_{l,k}^\mu(\mu^{2y-2k})\, e_{x+y-k}\otimes e_{y-k}.$$

(Remember the convention that $e_n=0$ for $n<0$.)

In order to say more about the left-hand side of (4.2) we consider the action of

$$\Phi(\gamma\gamma^*)=(\gamma\otimes\alpha+\alpha^*\otimes\gamma)(\gamma^*\otimes\alpha^*+\alpha\otimes\gamma^*)$$

on $e_{x+y}\otimes e_y$. We obtain

$$(\tau\otimes\tau)(\Phi(\gamma\gamma^*))\, e_{x+y}\otimes e_y$$
$$=\mu^{x+2y+1}(1-\mu^{2x+2y+2})^{1/2}(1-\mu^{2y+2})^{1/2}\, e_{x+y+1}\otimes e_{y+1}$$
$$+(\mu^{2x+2y}+\mu^{2y}-\mu^{2x+4y}-\mu^{2x+4y+2})\, e_{x+y}\otimes e_y$$
$$+\mu^{x+2y-1}(1-\mu^{2x+2y})^{1/2}(1-\mu^{2y})^{1/2}\, e_{x+y-1}\otimes e_{y-1}.$$

Hence, if

$$f:=\sum_{y=0}^{\infty} c_y\, e_{x+y}\otimes e_y$$

belongs to $\mathcal{H}\otimes\mathcal{H}$, then

$$(\tau\otimes\tau)(\Phi(\gamma\gamma^*))f=\sum_{y=0}^{\infty}[c_{y-1}\mu^{x+2y-1}(1-\mu^{2x+2y})^{1/2}(1-\mu^{2y})^{1/2}$$
$$+c_y(\mu^{2x+2y}+\mu^{2y}-\mu^{2x+4y}-\mu^{2x+4y+2}) \tag{4.3}$$
$$+c_{y+1}\mu^{x+2y+1}(1-\mu^{2x+2y+2})^{1/2}(1-\mu^{2y+2})^{1/2}]\, e_{x+y}\otimes e_y.$$

Now choose

$$c_y:=P_y(\mu^{2z};\mu^{2x}\,|\,\mu^2),$$

where P_y is defined in terms of Wall polynomials by (2.5). Then $f\in\mathcal{H}\otimes\mathcal{H}$ and, by (2.4), the expression in square brackets on the right-hand side of (4.3) is equal to $\mu^{2z}c_y$. Define

$$f_z^x:=\sum_{y=0}^{\infty}P_y(\mu^{2z};\mu^{2x}\,|\,\mu^2)\, e_{x+y}\otimes e_y. \tag{4.4}$$

Then, by the orthogonality relations (2.7), the vectors $\{f_z^x\}_{z=0,1,2,\cdots}$ form an orthonormal basis of

$$\bigoplus_{y=0}^{\infty}\mathbb{C}\, e_{x+y}\otimes e_y. \tag{4.5}$$

We also have

$$(\tau\otimes\tau)(\Phi(\gamma\gamma^*))f_z^x=\mu^{2z}f_z^x. \tag{4.6}$$

Now take the inner product of both sides of (4.2) with respect to f_z^x and apply (4.4), (4.6), and the self-adjointness of $\Phi(\gamma\gamma^*)$ acting on $\mathscr{H}\otimes\mathscr{H}$. Assume also the convention that $P_n := 0$ and $p_n := 0$ for $n < 0$. Then we obtain

$$\begin{aligned} p_l^\mu(\mu^{2z})P_y(\mu^{2z};\mu^{2x}\mid\mu^2) &= p_l^\mu(\mu^{2x+2y})p_l^\mu(\mu^{2y})P_y(\mu^{2z};\mu^{2x}\mid\mu^2) \\ &\quad+\sum_{k=1}^{l}(-1)^k\mu^{k(x+2y+1)}(\mu^{2(x+y+1)};\mu^2)_k^{1/2}(\mu^{2(y+1)};\mu^2)_k^{1/2} \\ &\quad\cdot p_{l,k}^\mu(\mu^{2x+2y})p_{l,k}^\mu(\mu^{2y})P_{y+k}(\mu^{2z};\mu^{2x}\mid\mu^2) \\ &\quad+\sum_{k=1}^{l}(-1)^k\mu^{k(x+2y-2k+1)}(\mu^{2(x+y)};\mu^{-2})_k^{1/2}(\mu^{2y};\mu^{-2})_k^{1/2} \\ &\quad\cdot p_{l,k}^\mu(\mu^{2x+2y-2k})p_{l,k}^\mu(\mu^{2y-2k})P_{y-k}(\mu^{2z};\mu^{2x}\mid\mu^2). \end{aligned} \tag{4.7}$$

Finally substitute (3.2), (3.3), and (2.5) in (4.7) and replace μ^2 by q. Then we obtain the following theorem.

THEOREM 4.1 (addition formula for little q-Legendre polynomials). *For $x, y, z = 0, 1, 2, \cdots$ we have*

$$\begin{aligned} &p_l(q^z;1,1\mid q)p_y(q^z;q^x,0\mid q) \\ &\quad= p_l(q^{x+y};1,1\mid q)p_l(q^y;1,1\mid q)p_y(q^z;q^x,0\mid q) \\ &\qquad+\sum_{k=1}^{l}\frac{(q;q)_{x+y+k}(q;q)_{l+k}q^{k(y-l+k)}}{(q;q)_{x+y}(q;q)_{l-k}(q;q)_k^2} \\ &\qquad\cdot p_{l-k}(q^{x+y};q^k,q^k\mid q)p_{l-k}(q^y;q^k,q^k\mid q)p_{y+k}(q^z;q^x,0\mid q) \\ &\qquad+\sum_{k=1}^{l}\frac{(q;q)_y(q;q)_{l+k}q^{k(x+y-l+1)}}{(q;q)_{y-k}(q;q)_{l-k}(q;q)_k^2} \\ &\qquad\cdot p_{l-k}(q^{x+y-k};q^k,q^k\mid q)p_{l-k}(q^{y-k};q^k,q^k\mid q)p_{y-k}(q^z;q^x,0\mid q). \end{aligned} \tag{4.8}$$

Remark 4.1. We could have derived the final result (4.8) from (4.1) as well by using one of the other members of the series of infinite-dimensional irreducible $*$-representations of $\mathscr{A}$ as given by [15, Thm. 3.2]. The same result would also have been obtained by use of the faithful $*$-representation of $\mathscr{A}$ given in [18, Thm. 1.2]. Furthermore, it can be shown that formula (4.8) taken for all $x, y, z = 0, 1, 2, \cdots$ and with $q = \mu^2$ is equivalent to (4.1).

Remark 4.2. It is possible to give a more conceptual interpretation of the occurrence of Wall polynomials in the addition formula. Namely, it can be shown that Wall polynomials have an interpretation as Clebsch-Gordan coefficients for the decomposition of the $*$-representation $(\tau\otimes\tau)\circ\Phi$ of $\mathscr{A}$ as a direct integral of irreducible $*$-representations of $\mathscr{A}$.

5. The product formula for little q-Legendre polynomials. If we multiply both sides of (4.7) with $P_y(\mu^{2z};\mu^{2x}\mid\mu^2)$ and sum over z, then by (2.6) we obtain the product formula

$$p_l^\mu(\mu^{2x+2y})p_l^\mu(\mu^{2y}) = \sum_{z=0}^{\infty} p_l^\mu(\mu^{2x})(P_y(\mu^{2z};\mu^{2x}\mid\mu^2))^2.$$

After substituting (3.3), (2.5), and (2.8) and after replacing x by $x-y$ and μ^2 by q, we get the desired product formula.

THEOREM 5.1. *For $x, y = 0, 1, 2, \cdots$ we have*

$$p_l(q^x; 1, 1 \mid q) p_l(q^y; 1, 1 \mid q) = (1-q) \sum_{z=0}^{\infty} p_l(q^z; 1, 1 \mid q) K(q^x, q^y, q^z \mid q) q^z$$

with

$$K(q^x, q^y, q^z \mid q) := \frac{(q^{x+1}; q)_x (q^{y+1}; q)_x (q^{z+1}; q)_x q^{xy+xz+yz}}{(q; q)_x^2 (1-q)} \cdot \left\{ {}_3\phi_2\left(\begin{matrix} q^{-x}, q^{-y}, q^{-z} \\ 0, 0 \end{matrix} ; q, q \right) \right\}^2.$$

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