Extra back-off flow control in wireless mesh networks

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***Abstract*—CSMA is the predominant distributed access protocol for wireless mesh networks. Originally designed for single-hop settings, in multi-hop networks CSMA can exhibit severe performance problems in terms of stability and end-to-end throughput. To ensure a smoother flow of packets, we examine a new scheme referred to as *extra back-off (EB) flow control*. In this scheme a node remains silent for a certain extra back-off time (imposed on top of the usual back-off time that is part of CSMA) after it has transmitted a packet, so as to give both the downstream and upstream neighbors the opportunity to transmit. EB flow control entails only a small modification to CSMA, preserving its distributed character, yet considerably improving the network performance.**

I. Introduction

Emerging wireless mesh networks will provide the main artery of the *Internet of Things*, offering connectivity to massive numbers of nodes, such as environmental sensors, control devices in vehicles, and radio tags in logistics and supply chains [1], [14]. In contrast to today's cellular architectures, these mesh networks typically lack any centralized control entity for allocating resources and explicitly coordinating transmissions. Instead, these networks vitally rely on the individual nodes to operate autonomously and to efficiently share the medium in a distributed fashion. This requires the nodes to schedule their individual transmissions and decide on the use of shared resources based on knowledge that is locally available or only involves limited exchange of information.

A popular mechanism for distributed medium access control is provided by the so-called Carrier-Sense Multiple-Access (CSMA) protocol, various incarnations of which are implemented in IEEE 802.11 networks. In the CSMA protocol each node attempts to access the medium after a certain back-off time, but nodes that sense the medium busy will postpone their attempt until the medium is sensed idle.

Although widely deployed in IEEE 802.11 networks, it is common knowledge that CSMA can exhibit severe performance problems in multi-hop scenarios [2], [5], [7]. Originally devised for single-hop communications where all nodes mutually interfere, CSMA is not well-suited to settings where the interference among nodes is asymmetric in nature, especially when these nodes act as relays in transferring a flow of packets along the end-to-end path between a source and destination. In particular, nodes in the 'middle' of a network tend to experience more interference than the nodes on the 'edge' of the network, and therefore are at a disadvantage in competing for medium access. This issue, known as the node-in-the-middle problem, can cause extreme unfairness and starvation effects, manifesting itself in poor throughput, severe congestion, buffer overflow, and packet loss.

In order to remedy the above-described performance issues of CSMA and ensure a smoother flow of packets in multi-hop settings, we examine a scheme referred to as *extra back-off (EB) flow control* [8], [9]. In EB a node will remain silent for a certain extra back-off time after it has transmitted a packet, so as to give both the downstream and upstream nodes the opportunity to transmit. Hence, we impose this extra back-off on top of the usual back-off that is already part of the CSMA protocol. Indeed, EB flow control only requires local information, involves no message passing, and is fully backward compatible with the IEEE 802.11 standard.

To examine the performance benefits of EB flow control, we will use various Markov models, extending the baseline model developed in [4] for the case of ordinary back-offs. We focus on linear multi-hop topologies consisting of N nodes, in which packets are transferred from the source node 1 to some receiver, via relay nodes $2, \ldots, N$. In order to avoid collisions, we further assume that CSMA prevents two neighboring nodes from being active simultaneously.

The basic model for medium contention is similar to that in [3], [6], [17], [18]. It is worth observing that these studies assume saturated buffer conditions, where all nodes always have packets pending for transmission. The throughput characteristics in such scenarios provide useful first-order estimates of the system performance. In reality however, buffer contents fluctuate as packets are accumulated and flushed over time, giving rise to queueing dynamics. In particular, the buffers may empty from time to time, and nodes may refrain from competition for the medium during these periods. It is further worth drawing a distinction with the work in [10], [11], [13], [16], which also addresses throughput utility optimization and stability issues in CSMA networks. These studies however focus on adaptation of the nominal back-off parameters rather than incorporation of extra back-off periods, and do not capture the queueing dynamics in as much detail.

In the present paper, we also assume the source node to be saturated, but explicitly account for the queueing dynamics at the other nodes, which is especially important since these act as relays in transferring the flow of packets. Unfortunately, the queueing dynamics severely complicate the mathematical analysis. In particular, the models entail high-dimensional stochastic processes with infinite state spaces, which generally do not admit closed-form expressions for the stationary distribution or yield to standard numerical techniques or brute-force simulation, even under Markovian assumptions. In fact, just the existence of a stationary distribution (positive recurrence of the Markov process) is usually difficult to establish. In view of the complexity, we will consider a regime where the ordinary back-off periods are asymptotically small, and focus on relatively short path lengths. As it turns out, the results for systems with as few as three nodes already provide remarkably accurate estimates of the end-to-end throughputs for larger path lengths.

We will demonstrate that EB flow control smoothes the flow of packets and improves the end-to-end throughput and fairness. In particular, we establish that EB flow control primarily increases the throughput of node 2, and hardly impacts the throughput of the other downstream nodes. As it turns out, the connection between nodes 2 and 3 is indeed the bottleneck link, and with EB flow control enabled, this link receives a larger share of the medium, because the source node is throttled. We will show that when the extra back-off time is sufficiently large, the buffer content of node 2 stabilizes for a certain type of EB flow control. In fact, it will be shown that this form of EB flow control stabilizes the entire network when the mean back-off duration exceeds some critical value. We further find that this critical value only marginally depends on the size of the system. For the 3-hop system we derive exact expressions for the throughput and the critical value of the mean back-off duration. For the 4-hop system we provide numerical results and for systems with 5 or more hops we present simulation experiments.

The remainder of the paper is organized as follows. Section II provides a detailed description of the model and the back-off scheme. Section III presents the main results for exponential back-off times. Section IV discusses extensions to larger networks and larger blocking distances, as well as some open problems.

II. Model description

We consider multi-hop networks of N consecutive nodes on a line. Node 1 (the source) is saturated, meaning that it always has a packet ready to transmit. These packets need to be sent to an external node (the destination), while being relayed through nodes $2, \dots, N$. A transmitting node always blocks its direct neighbors from transmitting. In wireless networks interference can occur when two nodes that are too close to each other transmit at the same time. Nodes i and j can only transmit simultaneously if $|\,i-j\,| > 1$.

Upon completion of a transmission, a node is forced into back-off, during which it is not allowed to be active. When a node completes a transmission and starts a back-off period, several neighbors may become unblocked simultaneously. Any race conditions that arise in such situations are resolved uniformly at random.

Note that the present model differs from that in [5], [6], [17], [18], in that nodes only enter back-off once after each transmission, and do not re-enter the back-off mode when they find themselves blocked at the end of a back-off period. Alternatively, the back-offs in the present model may be interpreted as 'extra' back-offs, with the duration of the 'regular' back-offs vanishing to zero.

We will consider two different back-off schemes:

Definition 1 (EB scheme (i)*):* All nodes have independent back-off periods after the transmission of each packet.

EB scheme (i) will also be referred to as the *basic* EB scheme.

Definition 2 (EB scheme (ii)*):* All nodes have independent back-off periods after the transmission of each packet, with the additional rule that the back-off period of a node is terminated when a new packet arrives to that node.

EB scheme (ii) will also be referred to as *truncated* EB scheme. We assume that the nodes have large buffers (which is usually the case). In fact, we assume infinite buffers, so that the network is lossless. The assumption of infinite buffers (and no losses) gives rise to a potential stability problem, because the buffer content may grow without bound. A striking feature of the back-off mechanism is that it can have a positive effect on stability. In particular, the status of a node can change from unstable to stable when the mean back-off time becomes larger than a certain critical value. Here, we say that a node i is *unstable* when its buffer content grows without bound and for buffer size X_i it holds that $\mathbb{P}[X_i = 0] \to 0$ as time progresses for any proper initial value, and we call a node *stable* otherwise.

III. Main Results

A. General results for both back-off schemes

Let us start with the simple observation that each node i goes through a cyclic pattern, consisting of a transmission period T_i, a back-off period, and a possible additional inactivity period in which it is neither transmitting nor in back-off. We distinguish between back-off times B_i and V_i, where B_i is the 'intended' length of a back-off period, while V_i is the actual amount of time that node i spends in back-off between two successive transmissions. For scheme (i) it obviously holds that $V_i = B_i$, but this distinction is necessary for the truncated extra back-off scheme (ii). Also, since node 1 is the first node of the chain, its back-off cannot be truncated, so $B_1 = V_1$ for both schemes. Let W_i be the remaining time that node i is 'waiting' between two successive transmissions.

Lemma 3: For both back-off schemes, the throughput of node i may be expressed as

$$\theta_i = \frac{1}{\mathbb{E}[T_i] + \mathbb{E}[V_i] + \mathbb{E}[W_i]}. \tag{1}$$

Proof: Denote $C_i = T_i + V_i + W_i$ and let $N_i(t)$ represent the number of transmitted packets by node i after t units of time, then

$$\theta_i = \lim_{t\to\infty} \frac{1}{t} N_i(t) = \frac{1}{\mathbb{E}[C_i]}. \tag{2}$$

Here we assume the existence of the limits

$$\mathbb{E}[V_i] = \lim_{n\to\infty} \frac{\sum_{k=1}^n V_{ik}}{n},$$
$$\mathbb{E}[W_i] = \lim_{n\to\infty} \frac{\sum_{k=1}^n W_{ik}}{n},$$
$$\theta_i = \lim_{n\to\infty} \frac{n}{\sum_{k=1}^n C_{ik}} = \lim_{t\to\infty} \frac{N_i(t)}{t},$$

where V_{ik} denotes the k-th back-off time of node i and likewise for W_{ik} and C_{ik}. ■

In order for Lemma 3 to be useful, we need to determine $\mathbb{E}[V_i]$ and $\mathbb{E}[W_i]$ which depend on the node and the specific back-off scheme under consideration.

Proposition 4: For both EB schemes, with $N \geq 2$, and B_1 exponentially distributed,

$$\mathbb{E}[W_1] = \frac{\theta_2}{\theta_1}\mathbb{E}[\max\{T_2 - B_1, 0\}]. \tag{3}$$

Proof: Let σ_1 be the long-term fraction of time that node 1 is inactive, but not in back-off. Then $\sigma_1 = \theta_1\mathbb{E}[W_1]$. Recall that node 1 is saturated and always has packets to transmit, so it can only be inactive when it is in back-off or when node 2 is transmitting. Therefore, $\sigma_1 = \theta_2\mathbb{E}[U_2]$, where U_2 denotes the amount of inactive time of node 1 caused by an arbitrary transmission of node 2 (and not by back-off of node 1 itself), i.e., the amount of time that node 1 is inactive but not in back-off during an arbitrary transmission of node 2. Thus, $\mathbb{E}[W_1] = \frac{\theta_2}{\theta_1}\mathbb{E}[U_2]$. The fact that node 1 is saturated, also implies that a transmission of node 2 can only start during a back-off period of node 1. The residual back-off period at that moment is exponentially distributed due to the memoryless property. Hence, it follows that U_2 may be represented as $\max\{T_2 - B_1, 0\}$, yielding Equation (3). ■

The following corollary extends the results of Proposition 4.

Corollary 5: Under the assumptions of Proposition 4, with $\mathbb{E}[B_1] = \eta$, and assuming additionally that T_2 is exponentially distributed with $\mathbb{E}[T_2] = 1$,

$$\mathbb{E}[\max\{T_2 - B_1, 0\}] = \frac{1}{1+\eta}, \tag{4}$$

so that, when also T_1 has a distribution with unit mean, with $\theta_i = \theta_i(\eta)$,

$$\theta_1(\eta) = \frac{1}{1 + \eta + \frac{\theta_2(\eta)}{\theta_1(\eta)}\frac{1}{1+\eta}}. \tag{5}$$

The fact that $\theta_1 \geq \theta_2$ then yields that either

$$\theta_1(\eta) = \theta_2(\eta) = \tau(\eta) = \frac{1}{1 + \eta + \frac{1}{1+\eta}} \tag{6}$$

when node 2 is stable, or

$$\theta_1(\eta) > \tau(\eta) > \theta_2(\eta). \tag{7}$$

B. Throughput for EB scheme (i)

Proposition 6: For EB scheme (i), with $N \geq 3$, under the assumptions of Corollary 5, node 2 is unstable, i.e., $\theta_1(\eta) > \tau(\eta) > \theta_2(\eta)$.

Proof: Let $\eta > 0$. To prove the saturation of node 2 we will use contradiction. Assuming $\theta_1(\eta) = \theta_2(\eta)$, Corollary 5 implies

$$\theta_1(\eta) = \theta_2(\eta) = \tau(\eta) = \frac{1}{1 + \eta + \frac{1}{1+\eta}},$$

so that $\mathbb{E}[W_2] = \frac{1}{1+\eta}$. However, we can also calculate $\mathbb{E}[W_2]$ in a different way. We define r_1 (r_2) to be the expected number of transmissions of node 1 in between two successive transmissions of node 2, starting when node 2 is in back-off (not in back-off). When node 1 starts a transmission during a back-off time of node 2, we have again that the expected waiting time contributed by the transmission of node 1 equals

$$\mathbb{E}[Y_1] = \mathbb{E}[\max\{T_1 - B_2, 0\}] = \frac{1}{1+\eta}.$$

In case node 1 starts a transmission when node 2 is not in back-off, it will contribute $\mathbb{E}[Y_2] = \mathbb{E}[T_1]$ to the expected waiting time. However, the waiting time of node 2 can also be increased by node 3, for example when it is active while node 1 is in back-off. We call this extra waiting time Y_3. Since $r_1 + r_2 = \theta_1(\eta)/\theta_2(\eta) = 1$ and $r_2 > 0$, we now have

$$\mathbb{E}[W_2] = r_1\mathbb{E}[Y_1] + r_2\mathbb{E}[Y_2] + \mathbb{E}[Y_3]$$
$$\geq (1 - r_2)\frac{1}{1+\eta} + r_2 = \frac{1}{1+\eta} + r_2\frac{\eta}{1+\eta} > \frac{1}{1+\eta}.$$

This leads to a contradiction, and thus we can conclude that $\theta_1(\eta) > \tau(\eta) > \theta_2(\eta)$ according to Corollary 5. ■

For scheme (ii), the stability condition of node 2 is harder to establish, which will be discussed in Section III-C.

Proposition 7 (throughput last node): For EB scheme (i), with $N \geq 3$, under the assumptions of Corollary 5, and additionally assuming that T_{N-1} is exponentially distributed with unit mean and B_N is exponentially distributed with mean η, node N is stable, i.e., $\theta_{N-1}(\eta) = \theta_N(\eta)$.

Proof: First suppose that node N were unstable, so it always has packets to transmit. By the same arguments as in the proof of Proposition 4, it follows that

$$\mathbb{E}[W_N] \leq \frac{\theta_{N-1}(\eta)}{\theta_N(\eta)}\frac{1}{1+\eta}.$$

Combining (1) with the fact that $\theta_2(\eta) \geq \theta_{N-1}(\eta) \geq \theta_N(\eta)$ and $\theta_2(\eta) < \tau(\eta)$ according to Proposition 6 yields

$$1 = \theta_N(\eta)(\mathbb{E}[T_N] + \mathbb{E}[V_N] + \mathbb{E}[W_N])$$
$$\leq \theta_N(\eta)(1 + \eta + \frac{\theta_{N-1}(\eta)}{\theta_N(\eta)}\frac{1}{1+\eta})$$
$$\leq \theta_{N-1}(\eta)(1+\eta) + \theta_{N-1}(\eta)\frac{1}{1+\eta}$$
$$\leq \theta_2(\eta)(1 + \eta + \frac{1}{1+\eta}) < 1,$$

which gives a contradiction. ■

We will give an exact analysis for the model with $N = 3$, and T_i and V_i (for $i = 1, 2, 3$) both exponentially distributed, with unit mean and mean η, respectively. Since we already know that nodes 1 and 2 are saturated, we only have to keep track of the buffer content of node 3. In Appendix B we show that this gives rise to a Markov process on a strip, also referred to as a Quasi-Birth-Death (QBD) process. Determining the steady-state distribution, and hence the throughput, requires the solution of a non-linear matrix equation that can only be obtained numerically. Hence, the throughput for back-off scheme (i) can be evaluated only numerically (albeit up to an arbitrary level of precision). Therefore, we shall introduce an approximate model, which will lead to a closed-form solution for the throughput, by imposing a minor modification to EB scheme (i).

Definition 8 (Modified EB scheme (i)*):* All nodes have independent back-off periods after the transmission of each packet, apart from the last node that has no back-off.

Note that for the present setting, with $N = 3$ and exponential T_i and V_i, Propositions 4 and 6 also hold for this modified scheme. Even Proposition 7 continues to hold, but here we need some further reasoning. Namely, every packet transmitted by node 2 will immediately be forwarded by node 3, since this node directly grabs the channel when node 2 goes into back-off and therefore node 3 is stable. Since node 3 is the last transmitting node and is stable in both cases, it does not matter much whether or not it is forced into back-off after a transmission. We will indeed see that this modification has very little effect on the throughput. Hence, at first sight, there seems to be little difference between the original and the modified back-off scheme. The crucial difference though is that the modified scheme allows for an exact closed-form solution for the throughput.

Theorem 9: For $N = 3$ and modified EB scheme (i) with T_i exponentially distributed with unit mean for $i = 1, 2, 3$, and B_1, B_2 exponentially distributed with mean η,

$$\mathbb{E}[W_1] = \frac{1}{1+\eta+\frac{1}{1+\eta}}, \quad \mathbb{E}[W_2] = \frac{2}{1+\eta}, \tag{8}$$

so that the throughput functions are given as

$$\theta_1(\eta) = \frac{2+2\eta+\eta^2}{3+5\eta+3\eta^2+\eta^3}, \tag{9}$$

$$\theta_2(\eta) = \theta_3(\eta) = \frac{1+2\eta+\eta^2}{3+5\eta+3\eta^2+\eta^3}. \tag{10}$$

Proof: To determine $\mathbb{E}[W_2]$, we write it as $\mathbb{E}[W_2] = \mathbb{E}[W_{21}] + \mathbb{E}[W_{23}]$, where W_{23} represents the waiting time caused by an ongoing transmission by node 3 and $\mathbb{E}[W_{21}]$ the leftover waiting time caused by a transmission of node 1, while node 3 is silent. We find $\mathbb{E}[W_{23}] = \mathbb{E}[\max\{T_3 - B_2, 0\}] = \frac{1}{1+\eta}$. Let H_0 be the time that node 2 finishes its transmission and H_1 the first moment after H_0 in which node 2 has finished its back-off time and node 3 has finished its transmission. Then we have $\mathbb{E}[W_{21}] = \mathbb{P}[\text{node 1 is transmitting at } H_1]\,\mathbb{E}[T_1]$, with $\mathbb{E}[T_1] = 1$. Because node 1 is always in back-off at the start of a transmission of node 2, we find

$$\mathbb{P}\left[\text{node 1 is in back-off at } H_0^+\right] = \mathbb{P}[B_1 > T_2] = \frac{\eta}{1+\eta}.$$

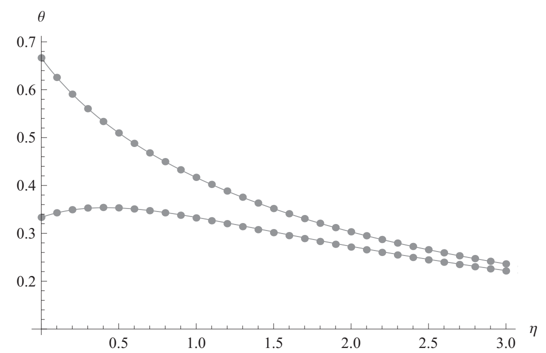

Figure 1: $\theta_1(\eta)$ and $\theta_2(\eta) = \theta_3(\eta)$ for back-off scheme (i) (dots) and its modified modified counterpart (solid line).

Since this is also the equilibrium probability distribution of node 1 being in back-off in case it could not be blocked by node 2 (which is the case between H_0 and H_1), the system retains the equilibrium distribution during this period, and thus

$$\begin{aligned} &\mathbb{P}[\text{node 1 is transmitting at } H_1] \\ &= \mathbb{P}\left[\text{node 1 is transmitting at } H_0^+\right] \\ &= 1 - \mathbb{P}\left[\text{node 1 is in back-off at } H_0^+\right] = \frac{1}{1+\eta}, \end{aligned} \tag{11}$$

which yields $\mathbb{E}[W_2] = \frac{2}{1+\eta}$. Applying Lemma 3 and Equation (5) yields $\mathbb{E}[W_1]$. ∎

An alternative, more constructive proof of Theorem 9 is given in Appendix F.

Let us now compare the schemes numerically. In Figure 1 we see the throughput of nodes 1 and 2 for both scheme (i) and the modified scheme (i). The results for scheme (i) are calculated from the numerical solution to the QBD process, and the results for the modified scheme (i) follow from Theorem 9. The difference in throughput between both schemes is negligible, and hence the modified scheme and its exact solution provides extremely sharp approximations for scheme (i). From Theorem 9 we see, among other things, that

$$\theta_1(\eta) - \theta_2(\eta) = O(\eta^{-3}),$$

which indicates that the difference in throughput between nodes 1 and 2 diminishes rapidly with η. This is confirmed in Figure 1. Moreover, we can obtain from the expression for θ_2 in Theorem 9 the following result:

Corollary 10: For $N = 3$, EB scheme (i), and T_i is exponentially distributed with unit mean for $i = 1, 2, 3$, node 2 will be saturated (not stable) for all values of η and attains the maximal throughput $\sqrt{2}/4$ for $\eta = \sqrt{2} - 1$.

C. Throughput for EB scheme (ii)

Proposition 11: For EB scheme (ii), the last node is always stable.

Proof: Whenever node N receives a packet, node $N-1$ goes into back-off, giving node N the opportunity to directly forward the packet, so that $\theta_N = \theta_{N-1}$. Therefore node N can only have 0 or 1 packet(s) in its buffer at an arbitrary moment in time. After its transmission, node N goes into back-off, but it does not have a packet to transmit anyway, and when it does receive one, its back-off will always be terminated. The system is thus insensitive to the back-off of node N. ∎

For the throughput calculations of EB scheme (ii), we will also focus on the case of $N = 3$, with both T_i and B_i exponentially distributed with unit mean and mean η, respectively, for $i = 1, 2, 3$. For all nodes except node 1, the back-off period can be terminated before expiration.

We now find that EB scheme (ii) makes node 2 stable when the mean back-off time η increases beyond a certain critical value. This observation can be of great practical significance, especially if a similar phenomenon turns out to hold when applying EB scheme (ii) to more general mesh networks.

Theorem 12: For $N = 3$, EB scheme (ii), and T_i and B_i exponentially distributed with unit mean and mean η, respectively, for $i = 1, 2, 3$, node 2 is stable and hence $\theta_1(\eta) = \theta_2(\eta) = \theta_3(\theta)$, if and only if $\eta > \sqrt{5} - 1$, and then

$$\theta_i(\eta) = \tau(\eta) = \frac{1}{1 + \eta + \frac{1}{1+\eta}}, \quad i = 1, 2, 3. \tag{12}$$

We will now take different approaches for the systems in which node 2 is saturated and in which node 2 is stable. When node 2 is saturated, the throughput of all nodes can be determined explicitly:

Theorem 13: For $N = 3$, EB scheme (ii), and T_i exponentially distributed with unit mean for $i = 1, 2, 3$, and $\eta \leq \sqrt{5} - 1$, the throughputs are given by

$$\theta_1(\eta) = \frac{8 + 4\eta + \eta^2}{12 + 14\eta + 5\eta^2 + \eta^3}, \tag{13}$$

$$\theta_2(\eta) = \theta_3(\eta) = \frac{4 + 6\eta + 2\eta^2}{12 + 14\eta + 5\eta^2 + \eta^3}. \tag{14}$$

Moreover, $\theta_1(\eta) \geq \theta_2(\eta) = \theta_3(\eta)$ with equality if and only if $\eta = \sqrt{5} - 1$.

Theorem 13 is proved in Appendix E. The proof relies on the fact that the buffer contents can be modeled as a Markov process on a finite state space that has a closed-form solution for the stationary distribution. Observe that $\theta_1(\eta) \to 2/3$ and $\theta_2(\eta) \to 1/3$ as $\eta \downarrow 0$. The maximum throughput $\theta_2 \approx 0.37513$ of node 2 is attained for $\eta \approx 0.93328$.

If node 2 is stable we can model the system as a QBD process, and numerical results up to an arbitrary level of precision can be obtained. This is explained in Appendix A. The resulting throughputs are plotted in Figure 2.

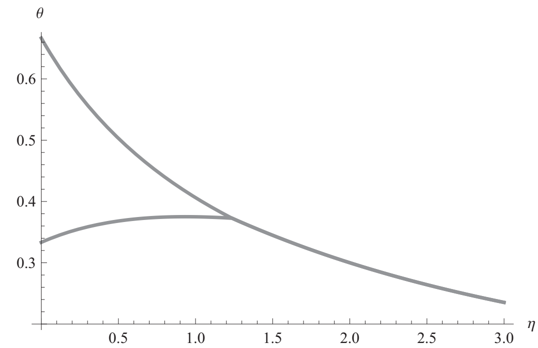

Figure 2: $\theta_1(\eta)$ and $\theta_2(\eta) = \theta_3(\eta)$ for EB scheme (ii) with $N = 3$.

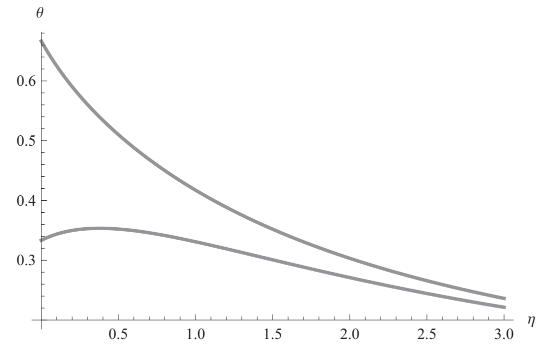

Figure 3: $\theta_1(\eta)$ and $\theta_2(\eta) = \theta_3(\eta) = \theta_4(\eta)$ for modified EB scheme (i) with $N = 4$.

IV. Discussion

Most results obtained so far were for the 3-hop network. We shall now demonstrate that networks with four or more hops behave quite similar, and that the insights obtained through analytic results for the 3-hop network to a large extent carry over to larger networks. The 4-hop network still allows for an explicit analysis via the theory of QBD processes as explained in Appendix F. The results for networks with 5 or more hops were obtained by means of simulation.

A. Robustness of throughputs for scheme (i)

As shown in Appendix F, the 4-hop network with the modified EB scheme (i) leads to the ordering of throughputs $\theta_1(\eta) > \theta_2(\eta) = \theta_3(\eta) = \theta_4(\eta)$; see Figure 3. We compare these results to the 3-hop network for which we obtained explicit results in Theorem 9. Numerical calculations show that the maximum relative difference in throughput between 3 and 4 hops is less than one percent, showing the minor effect of the addition of node 4. For $N \geq 5$ simulations have been performed for EB scheme (i), and the throughputs match almost perfectly with $N = 3$ and $N = 4$. This shows the robustness of EB scheme (i): the throughput is only minimally affected by the number of nodes N, for all $\eta > 0$. Hence, although the mathematical model becomes intractable for $N \geq 5$, the behavior is almost identical to the case $N = 3$.

B. Critical values in scheme (ii)

One of our main findings is that the 3-hop model with EB scheme (ii) can be completely stabilized when η increases beyond the critical value $\sqrt{5}-1$. The question now is whether this EB scheme can also stabilize N-hop models with $N \geq 4$.

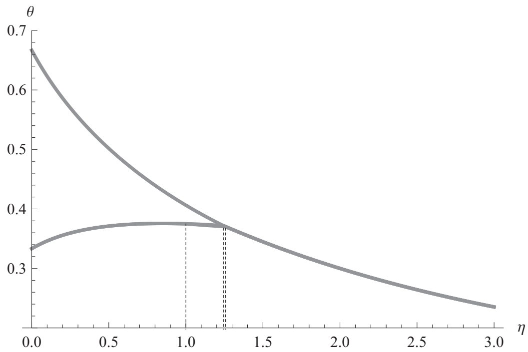

Figure 4: $\theta_1(\eta)$, $\theta_2(\eta)$ and $\theta_3(\eta) = \theta_4(\eta)$ for $N = 4$ and EB scheme (ii) for $0 \leq \eta \leq 3$.

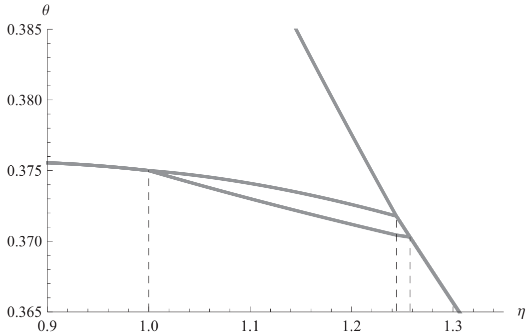

Figure 5: $\theta_1(\eta)$, $\theta_2(\eta)$ and $\theta_3(\eta) = \theta_4(\eta)$ for $N = 4$ and EB scheme (ii) for $0.9 \leq \eta \leq 1.35$

The case $N = 4$ leads to rather remarkable results, as can be seen in Figures 4 and 5.

First note that node 1 is always saturated and node 4 is always stable. It turns out that there are four possible scenarios: (1) node 2 is saturated and node 3 is stable, (2) nodes 2 and 3 are both saturated, (3) node 2 is stable (so that $\theta_1(\eta) = \theta_2(\eta) = \tau(\eta)$) and node 3 is saturated, and (4) nodes 2 and 3 are both stable. All these scenarios occur as η increases from 0 to infinity. Let $\eta_{i \to j}$ denote the value of η at which we switch from scenario (i) to scenario (j), and hence $0 < \eta_{1 \to 2} < \eta_{2 \to 3} < \eta_{3 \to 4}$. The (numerical) values for these switching points are $\eta_{1 \to 2} = 1$, $\eta_{2 \to 3} \approx 1.24415$, and $\eta_{3 \to 4} \approx 1.25763$. Hence, $\eta_{3 \to 4}$ is the critical value beyond which the whole system is stable. The main difference between $N = 3$ and $N = 4$ is thus the surprising phenomenon that node 3 is saturated for $\eta_{1 \to 2} < \eta < \eta_{3 \to 4}$.

For larger networks with $5 \leq N \leq 10$ extensive simulations have led to estimates of the critical values η_c (after which the whole network is stable) in Table I. Note that also for $N \geq 5$ stability implies that the throughput of all nodes equals $\tau(\eta)$. From $\theta_1(\eta) = \theta_2(\eta)$ it follows that both $\theta_1(\eta)$ and $\theta_2(\eta)$ are equal to $\tau(\eta)$. All other nodes are stable and hence their throughput equals $\tau(\eta)$ as well.

The critical values seem to settle around $\eta \approx 1.28$, which again suggests that nodes $5, 6, ...$ have only little influence on the mechanics of the various EB schemes. We note that $\tau(1.28) \approx 0.3678$, which still exceeds the maximum throughput of modified EB scheme (i) for $N = 3$, as found in Corollary 10. This leads us to believe that also for larger networks EB scheme (ii) can achieve stability while still maintaining a higher throughput than EB scheme (i).

N	3	4	5	6	7	8	9	10
η_c	$\sqrt{5} - 1$	1.26	1.28	1.28	1.28	1.28	1.28	1.28

Table I: Critical values η_c for systems with EB scheme (ii) and $N = 3, ..., 10$.

V. Acknowledgements

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Appendix

A. Introduction to QBD processes

Quasi-Birth-Death (QBD) processes are Markov processes on the two-dimensional lattice, whose transitions are skipfree to the left and to the right, with no restrictions upward or downward. The invariant distributions of QBD processes, under appropriate conditions, are well known to have a matrix-geometric form. More precisely, the stationary probability vector has a geometric solution in terms of a so-called rate matrix $\mathbf{R}$. We present in this section a short introduction to QBD processes and then model EB scheme (i) as a QBD process in Section B, and the EB scheme (ii) in Section C.

Consider a continuous-time Markov process $\{X(t), t \in \mathbb{R}_+\}$ on the two-dimensional state space $\{(i, j) : i \in \mathbb{Z}_+, j \in \{1, \ldots, M\}\}$, which is partitioned as $\bigcup_{i=0}^{\infty} l(i)$, where

$$l(i) = \{(i, 1), (i, 2), \ldots, (i, M)\}$$

and $\mathbb{Z}_+$, $\mathbb{R}_+$ denote the nonnegative integer and real numbers. In state (i, j) the first coordinate i is called the *level* whereas j denotes the *phase*, with the set $l(i)$ referred to as *level* i. Each level has a finite number of states, M.

This Markov process is called a QBD process when its one-step transitions from each state are restricted to states in the same level or in the two adjacent levels, and a *homogeneous* QBD process when these transition rates are additionally level-independent for levels $l(i)$ with $i > 0$.

Let $\boldsymbol{\pi}$ denote the stationary probability vector of this homogeneous QBD process. We construct $\boldsymbol{\pi}$ by concatenating subvectors $\boldsymbol{\pi}_i$, $i \in \mathbb{Z}_+$, where $\boldsymbol{\pi}_i$ has M components corresponding to the states of $l(i)$. This shows that vector $\boldsymbol{\pi}$ is of infinite size. We will assume throughout that the QBD process is irreducible and ergodic. Hence, we assume the stationary probability vector exists and therefore is uniquely determined as the solution of

$$\boldsymbol{\pi}_0 \mathbf{B} + \boldsymbol{\pi}_1 \mathbf{A}_2 = \mathbf{0}, \quad (15)$$

$$\boldsymbol{\pi}_{i-1} \mathbf{A}_0 + \boldsymbol{\pi}_i \mathbf{A}_1 + \boldsymbol{\pi}_{i+1} \mathbf{A}_2 = \mathbf{0}, \quad i \geq 1. \quad (16)$$

where matrices $\mathbf{A}_0, \mathbf{A}_2$ are nonnegative and matrices $\mathbf{A}_1, \mathbf{B}$ have nonnegative off-diagonal elements and strictly negative diagonals. Matrix $\mathbf{A}_0$ represents the transition rates from a level $i-1$ to i, while $\mathbf{A}_1$ represents transitions within the same level and $\mathbf{A}_2$ shows transitions from level i to level $i-1$. Matrix $\mathbf{B}$ serves as the rates within level $l(0)$. In our study the matrices all have dimension $M \times M$.

The infinite sized generator $\mathbf{Q}$ of the Markov process now takes the block tridiagonal form

$$\mathbf{Q} = \begin{pmatrix} \mathbf{B} & \mathbf{A}_0 & & & \\ \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & & \\ & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & \\ & & \ddots & \ddots & \ddots \end{pmatrix}, \tag{17}$$

and thus (15), (16), and the fact that the sum of all stationary probabilities must equal 1 reduce to

$$\boldsymbol{\pi}\mathbf{Q} = \mathbf{0}, \quad \boldsymbol{\pi}\mathbf{e}^T = 1, \tag{18}$$

where $\mathbf{e}$ denotes a row vector of appropriate dimension containing all ones. The matrix-geometric solution of the stationary probability vector $\boldsymbol{\pi}$ partitioned into $\boldsymbol{\pi}_i$, $i \geq 0$, is given by the following theorem (see [15]).

Theorem 14: Consider a continuous-time QBD process with generator $\mathbf{Q}$ in the form of (17). Suppose that the QBD process is irreducible and ergodic. Then its stationary distribution $\boldsymbol{\pi}$ is given by

$$\boldsymbol{\pi}_i = \boldsymbol{\pi}_0 \mathbf{R}^i, \quad i \in \mathbb{N}, \tag{19}$$

where $\mathbf{R}$ is the minimal nonnegative solution of the nonlinear matrix equation

$$\mathbf{A}_0 + \mathbf{R}\mathbf{A}_1 + \mathbf{R}^2\mathbf{A}_2 = \mathbf{0} \tag{20}$$

with spectral radius $\mathrm{sp}(\mathbf{R}) < 1$. Furthermore, the stationary probability vector $\boldsymbol{\pi}_0$ exists and is uniquely determined by solving the boundary condition

$$\boldsymbol{\pi}_0\mathbf{B} + \boldsymbol{\pi}_1\mathbf{A}_2 = \boldsymbol{\pi}_0(\mathbf{B} + \mathbf{R}\mathbf{A}_2) = \mathbf{0} \tag{21}$$

and the normalization condition

$$\sum_{i=0}^{\infty} \boldsymbol{\pi}_i \mathbf{e} = \boldsymbol{\pi}_0(\mathbf{I} - \mathbf{R})^{-1}\mathbf{e}^T = 1, \tag{22}$$

where $\mathbf{I}$ denotes the identity matrix with dimension $M \times M$.

From Theorem 14 we know that the stationary distribution is determined once $\mathbf{R}$ is obtained. Several iterative algorithms exist for numerically solving (20); an overview of such algorithms is provided in [12].

The QBD process driven by $\mathbf{Q}$ is ergodic if and only if it satisfies the mean drift condition (see [15])

$$\boldsymbol{\omega}\mathbf{A}_0\mathbf{e}^T < \boldsymbol{\omega}\mathbf{A}_2\mathbf{e}^T, \tag{23}$$

where $\boldsymbol{\omega}$ is the equilibrium distribution of the generator $\mathbf{A}_0 + \mathbf{A}_1 + \mathbf{A}_2$ and $\mathbf{e}$ the unit vector. When (23) is satisfied, the stationary distribution of the QBD process exists.

B. EB scheme (i) as a QBD process

The meaning of level and phase in this specific model must first be defined. We shall work under the assumption that node 2 is saturated. Since node 1 is saturated as well, the only buffer content that we need to keep track of is that of node 3. That is why the level $l(x_3)$ represents the state of the system with x_3 packets at node 3. The phases that form the level will now be described by all states with x_3 packets at node 3.

We denote each state by $S_1S_2S_3$, where S_i denotes the state of node i. Since nodes 1 and 2 are saturated, they can only be transmitting ($S_i = \mathrm{T}$), in back-off (B), or blocked by a neighbor (while not in back-off) (X). Node 3 can be in one of these states, or be empty (E). The phase is described as

$$l(0) = \{\mathrm{XTE}, \mathrm{XTB}, \mathrm{BTE}, \mathrm{BTB}, \mathrm{BBB}, \mathrm{BBE}, \mathrm{TBB}, \mathrm{EEE}, \mathrm{TBE}, \mathrm{TXB}, \mathrm{TXE}\} \tag{24}$$

and (for $x_3 \geq 1$)

$$l(x_3) = \{\mathrm{XTX}, \mathrm{XTB}, \mathrm{BTX}, \mathrm{BTB}, \mathrm{BBB}, \mathrm{BBT}, \mathrm{TBB}, \mathrm{BXT}, \mathrm{TBT}, \mathrm{TXB}, \mathrm{TXT}\}. \tag{25}$$

Let $(\mathbf{X})_{i,j}$ denote the element (i,j) of a matrix $\mathbf{X}$. Let $\beta = 1/\eta$. The matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{B}$ are then specified by

$$(\mathbf{A}_0)_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \{(1,9),(2,7),(3,6),(4,5)\}, \\ 0 & \text{otherwise.} \end{cases} \tag{26}$$

$$(\mathbf{A}_2)_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \{(6,5),(8,4),(9,7),(11,10)\}, \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

$$\mathbf{A}_1 = \begin{pmatrix} \Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta & \Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta & 0 & \Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & \beta & \Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & \Delta & \beta & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta & 0 & \beta & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \Delta & 0 & \beta & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta & 0 & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta & 0 & \beta \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \Delta & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta \end{pmatrix}, \tag{28}$$

and $\mathbf{B} = \mathbf{A}_1 + \mathbf{C}$, with $(\mathbf{C})_{6,3} = \beta$, $(\mathbf{C})_{6,8} = -\beta$, $(\mathbf{C})_{8,11} = -\beta + 1$, and $(\mathbf{C})_{11,3} = 1$, and all other entries 0. Here Δ is shorthand notation for the element that makes all elements in the corresponding row in the matrix $\mathbf{Q}$ in (17) add up to zero (note that Δ is row-dependent and is different in $\mathbf{A}_1$ and $\mathbf{B}$).

C. EB scheme (ii) as a QBD process

In the case of truncated back-offs, the exponential back-off times are terminated by the arrival of a new packet. This results in the back-off of node 3 becoming completely irrelevant. Again node 3 can only have 0 or 1 packet(s), since it will start transmitting right after node 2 finishes a transmission. After its transmission, node 3 will go into back-off, and will become active again only after node 2 has transmitted a new packet. It will always become active immediately, regardless of whether it was in back-off or not. Hence, whenever it is in back-off, it does not have a packet to transmit anyway.

We model the system as a QBD process, where the level $l(x_2)$ denotes all states for which the buffer content of node 2 equals x_2. The phase description is now given by

$$\begin{aligned} l(0) &= \{\mathrm{BEE}, \mathrm{BBE}, \mathrm{TBE}, \mathrm{EEE}, \mathrm{BBT}, \mathrm{TEE}, \\ &\quad \mathrm{TBT}, \mathrm{BET}, \mathrm{TET}\}, \\ l(x_2) &= \{\mathrm{BTE}, \mathrm{BBE}, \mathrm{TBE}, \mathrm{XTE}, \mathrm{BBT}, \mathrm{TXE}, \\ &\quad \mathrm{TBT}, \mathrm{BXT}, \mathrm{TXT}\}, \quad x_2 \geq 1, \end{aligned}$$

where the transition matrices can be shown to satisfy

$$(\mathbf{A}_0)_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \{(3,2),(6,1),(7,5),(9,8)\}, \\ 0 & \text{otherwise.} \end{cases} \tag{29}$$

$$(\mathbf{A}_2)_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \{(1,5),(4,7)\}, \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

$$\mathbf{A}_1 = \begin{pmatrix} \Delta & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ \beta & \Delta & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta & 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta & 0 & \beta & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \Delta & 0 & \beta \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta & \beta \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta \end{pmatrix}, \tag{31}$$

and $\mathbf{B} = \mathbf{A}_1 + \mathbf{C}$, with $(\mathbf{C})_{1,4} = -\beta$, $(\mathbf{C})_{1,6} = \beta$, and $(\mathbf{C})_{4,1} = 1$.

D. Proof of Theorem 12

Theorem 12 will be proved using the mean drift condition (23) with the transition matrices as in (29)-(31). Standard matrix calculations show that

$$\begin{aligned} \boldsymbol{\omega} = C^{-1}\{&2\beta + 4\beta^2, 1, \frac{2\beta + 5\beta^2 + 4\beta^3}{2 + 3\beta + \beta^2}, 2\beta^2 + 4\beta^3, 2\beta, \\ &\frac{2\beta^2 + 11\beta^3 + 14\beta^4 + 4\beta^5}{2 + 3\beta + \beta^2}, \frac{4\beta^2 + 4\beta^3}{2 + \beta}, 4\beta^2, \\ &\frac{6\beta^3 + 4\beta^4}{2 + \beta}\}, \end{aligned} \tag{32}$$

where $C = 1 + 5\beta + 14\beta^2 + 12\beta^3$. Using $\eta = 1/\beta$ yields

$$\boldsymbol{\omega}\mathbf{A}_0\mathbf{e}^T = \frac{8 + 4\eta + \eta^2}{12 + 14\eta + 5\eta^2 + \eta^3}, \tag{33}$$

$$\boldsymbol{\omega}\mathbf{A}_2\mathbf{e}^T = \frac{4 + 6\eta + 2\eta^2}{12 + 14\eta + 5\eta^2 + \eta^3}. \tag{34}$$

We can conclude that this system is stable if and only if

$$\frac{8 + 4\eta + \eta^2}{12 + 14\eta + 5\eta^2 + \eta^3} < \frac{4 + 6\eta + 2\eta^2}{12 + 14\eta + 5\eta^2 + \eta^3}.$$

Since the system is only well-defined for $\eta > 0$, we have $8 + 4\eta + \eta^2 < 4 + 6\eta + 2\eta^2$, with the only valid interval being $\eta > \sqrt{5} - 1$. This means that node 2 is saturated if and only if $0 < \eta \leq \sqrt{5} - 1$.

E. Proof of Theorem 13

In case $0 < \eta \leq \sqrt{5} - 1$, node 2 is saturated, and the QBD process process describing the system with EB scheme (ii) can be replaced by a more tractable Markov process. Since node 3 can have only 0 or 1 packet(s) and nodes 1 and 2 are saturated, the state space is finite, and given by

$$S = \{\mathrm{BTE}, \mathrm{BBE}, \mathrm{TBE}, \mathrm{XTE}, \mathrm{BBT}, \mathrm{TXE}, \mathrm{TBT}, \mathrm{BXT}, \mathrm{TXT}\}.$$

The transition matrix for this Markov process is given by $\mathbf{A}_0 + \mathbf{A}_1 + \mathbf{A}_2$, with the matrices as in (29)-(31). We have already calculated the equilibrium distribution to determine the mean drift condition of the QBD process in the proof of Theorem 12. The stationary distribution is equal to $\boldsymbol{\omega}$ as was found in (32). Adding the stationary probabilities of the states in which $S_i = T$ yields (13) and (14).

F. Alternate proof of Theorem 9

We will first derive the expression for $\mathbb{E}[W_2]$ in (8). Since node 3 never goes into back-off, its throughput can be written as

$$\begin{aligned} \theta_3 &= \frac{1}{\mathbb{E}[T_3] + \mathbb{E}[V_3] + \mathbb{E}[W_3]} \\ &= \frac{1}{\mathbb{E}[T_3] + \mathbb{E}[S] + \mathbb{E}[T_2]} = \frac{1}{2 + \mathbb{E}[S]}, \end{aligned} \tag{35}$$

where S represents the time node 3 is empty and not blocked by node 2. Note that $V_3 + W_3 = S + T_2$, since after a transmission node 3 first has to wait until node 2 starts another transmission and then has to wait during this transmission time. When node 3 finishes its transmission, it can find the system in four different states: (1) node 1 is in back-off, node 2 is in back-off, (2) node 1 is not in back-off, node 2 is in back-off, (3) node 1 is in back-off, node 2 is not in back-off, (4) node 1 is not in back-off, node 2 is not in back-off.

For every possible state i, we have a different S_i. Simple calculations give

$$\mathbb{E}[S_1] = \tfrac{1}{2}\eta + \tfrac{1}{2}\mathbb{E}[S_2], \quad \mathbb{E}[S_2] = 1 + \tfrac{\eta}{1+\eta}\mathbb{E}[S_1],$$

and hence $\mathbb{E}[S_1] = \eta + \frac{1}{2+\eta}$ and $\mathbb{E}[S_2] = \eta + \frac{2}{2+\eta}$. Furthermore, $\mathbb{E}[S_3] = 0$ and $\mathbb{E}[S_4] = 1$. Let P_i denote the probability that, when node 3 finishes its transmission, the system is in state i. We note that conditioned on the moment that node 3 comes out of its transmission, the event of node 1 being in back-off and the event of node 2 being in back-off are independent. This is because node 1 does not influence the length of the back-off time of node 2, which started at the moment node 3 started its transmission.

This makes it sufficient to find $P_1 + P_2$ and $P_2 + P_4$ separately. The fact that $P_1 + P_2$ is equal to

$$\begin{aligned} &\mathbb{P}[\text{Node 3 finds node 2 in back-off after its transmission}] \\ &= \mathbb{P}[T_3 < V_2] = \frac{\eta}{1+\eta} \end{aligned} \tag{36}$$

implies that $P_3 + P_4 = \frac{1}{1+\eta}$. To find $P_2 + P_4$ we condition on the moment that node 2 finished its last transmission. With probability q node 1 was not in back-off, and with probability $1 - q$ it was. We also condition on the number of transmissions node 1 has started during the transmission of node 1. Now $P_2 + P_4 =$ equals

$$\begin{aligned}
&\mathbb{P}\left[\text{Node 3 finds node 1 active after its transmission}\right] \\
=&q\mathbb{P}\left[T_3 < T_1\right] \sum_{n=0}^{\infty} \mathbb{P}\left[T_3 > T_1 + V_1\right]^n \\
&+ (1-q)\mathbb{P}\left[V_1 < T_3 < V_1 + T_1\right] \sum_{n=0}^{\infty} \mathbb{P}\left[T_3 > V_1 + T_1\right]^n \\
=&q\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{2}\frac{1}{1+\eta}\right)^n + (1-q)\frac{1}{2}\frac{1}{1+\eta}\sum_{n=0}^{\infty}\left(\frac{1}{2}\frac{1}{1+\eta}\right)^n \\
=&q\frac{1}{2}\frac{2+2\eta}{1+2\eta} + (1-q)\frac{1}{2+2\eta}\cdot\frac{2+2\eta}{1+2\eta} = \frac{1+q\eta}{1+2\eta}.
\end{aligned}$$

The probability q is easy to determine, since it is the probability that node 1 is blocked by node 2 when node 2 finishes a transmission. We note that $\mathbb{E}\left[W_1\right]$ equals

$$\begin{aligned}
&\mathbb{P}\left[\text{Node 1 has to wait for node 2 after a back-off time}\right]\cdot\mathbb{E}\left[T_2\right] \\
&= \frac{\theta_2}{\theta_1} q \mathbb{E}\left[T_2\right].
\end{aligned}$$

Since $\mathbb{E}\left[T_2\right] = 1$, we have $\mathbb{E}\left[W_1\right] = \frac{\theta_2}{\theta_1} q$. Using (3) it follows that $q = \frac{\theta_1}{\theta_2}\mathbb{E}\left[W_1\right] = \frac{1}{1+\eta}$, so that $P_2 + P_4 = \frac{1}{1+\eta}$ and $P_1 + P_3 = \frac{\eta}{1+\eta}$. Thus we conclude that

$$P_1 = (P_1 + P_2)(P_1 + P_3) = \frac{\eta^2}{(1+\eta)^2}, \tag{37}$$

$$P_2 = P_3 = \frac{\eta}{(1+\eta)^2}; \quad P_4 = \frac{1}{(1+\eta)^2}. \tag{38}$$

From this we find

$$\begin{aligned}
\mathbb{E}\left[S\right] =& P_1\mathbb{E}\left[S_1\right] + P_2\mathbb{E}\left[S_2\right] + P_3\mathbb{E}\left[S_3\right] + P_4\mathbb{E}\left[S_4\right] \\
=&\frac{1}{(1+\eta)^2}\left(\eta^2\left(\eta + \frac{1}{2+\eta}\right) + \eta\left(\eta + \frac{2}{2+\eta}\right) + 1\right) \\
=&\frac{1}{(1+\eta)^2}\left(1 + \frac{2\eta+\eta^2}{2+\eta} + \eta^2 + \eta^3\right) = \frac{1+\eta^2}{1+\eta}.
\end{aligned}$$

Using (7), it follows that

$$\mathbb{E}\left[W_2\right] = \mathbb{E}\left[S\right] + 1 - \eta = \frac{1+\eta^2}{1+\eta} + \frac{1-\eta^2}{1+\eta} = \frac{2}{1+\eta}.$$

Using (1) and (8), (10) directly follows. From this and (5), we find (9) to hold and (8) follows.

G. EB Scheme (i)

For the modification of scheme (i), in which the last node is not equipped with the back-off mechanism, we can conduct a QBD analysis for the case of $N = 4$. We have seen that taking node 1 saturated results in node 2 becoming saturated as well (Theorem 6). Since node 4 cannot have more than 1 packet, the only node of which the buffer size needs to be tracked is node 3. With x_3 denoting the the buffer size of node 3, we find the following 21 states that are required to describe the network as a Markov process:

$$\begin{aligned}
l(x_3) = \{&\text{TXTE}, \text{TXBT}, \text{TXBE}, \text{TXXT}, \text{TBTE}, \text{TBBT}, \\
&\text{TBBE}, \text{TBXT}, \text{XTXT}, \text{XTXE}, \text{XTBT}, \text{XTBE}, \\
&\text{BTXT}, \text{BTXE}, \text{BTBT}, \text{BTBE}, \text{BBTE}, \text{BBBT}, \\
&\text{BBBE}, \text{BBXT}, \text{BXTE}\}, \qquad \text{for } x_3 > 0,
\end{aligned} \tag{39}$$

$$\begin{aligned}
l(0) = \{&\text{TXEE}, \text{TXBT}, \text{TXBE}, \text{TXET}, \text{TBEE}, \text{TBBT}, \\
&\text{TBBE}, \text{TBET}, \text{XTET}, \text{XTEE}, \text{XTBT}, \text{XTBE}, \\
&\text{BTET}, \text{BTEE}, \text{BTBT}, \text{BTBE}, \text{BBEE}, \text{BBBT}, \\
&\text{BBBE}, \text{BBET}, \text{EEEE}\}.
\end{aligned} \tag{40}$$

The matrices $\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$, and $\mathbf{B}$ are constructed in the same way as for $N = 3$, but we refrain from presenting them here. The large number of states makes it infeasible for algebraic computer programs to invert these matrices, since they include variables. This makes it also impossible to give an explicit formula for the mean drift condition (23). However, we can check the mean drift condition numerically. Figure 6 shows $f(\eta) = \boldsymbol{\omega}(\eta)\left(\mathbf{A}_2 - \mathbf{A}_0\right)\mathbf{e}^T$, which is decreasing but positive for $0 < \eta \leq 10$, and we expect the mean drift condition to be satisfied for all values of η.

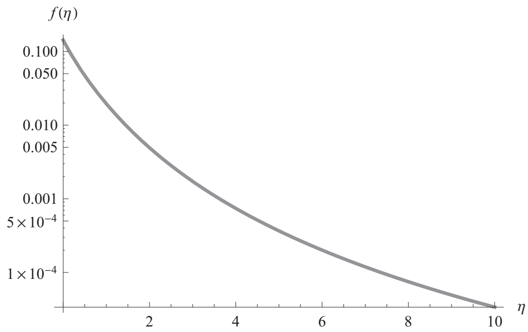

Figure 6: $f(\eta) = \boldsymbol{\omega}(\eta)\left(\mathbf{A}_2 - \mathbf{A}_0\right)\mathbf{e}^T$ being positive implies stability of node 3.

H. EB scheme (ii)

For the EB scheme (ii) it is not as evident which nodes are saturated and which are stable. We know that node 1 is saturated and that node 4 can have at most 1 packet. This leaves four possible scenarios: (1) node 2 is saturated and node 3 is stable, (2) nodes 2 and 3 are both saturated, (3) node 2 is stable (so that $\theta_1(\eta) = \theta_2(\eta) = \tau(\eta)$) and node 3 is saturated, and (4) nodes 2 and 3 are both stable. It will turn out that all these scenarios occur as η increases from 0 to infinity. Let $\eta_{i\to j}$ denote the value of η at which we switch from scenario (i) to scenario (j), and it will turn out that $0 < \eta_{1\to2} < \eta_{2\to3} < \eta_{3\to4}$.

The levels of states are the same as in (39). If the level represents x_3, then for $x_3 = 0$ we have again (40). If the level represents x_2 (the buffer size of node 2), then the zero-level

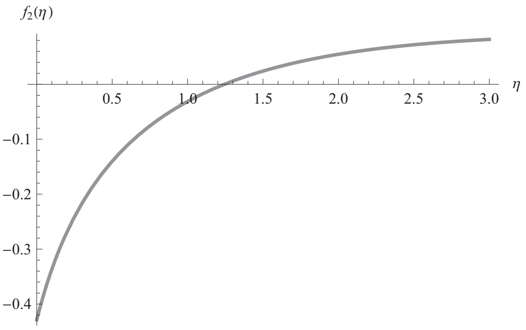

Figure 7: $f_2(\eta) = \boldsymbol{\omega}(\eta)\left(\mathbf{A}_2^2 - \mathbf{A}_0^2\right)\mathbf{e}^T$ is positive from $\eta_{2\to3} \approx 1.24415$ onwards.

is given as follows:

$$l(0) = \{\text{TETE}, \text{TEBT}, \text{TEBE}, \text{TEXT}, \text{TBTE}, \text{TBBT}, \text{TBBE}, \text{TBXT}, \text{EEEE}, \text{EEEE}, \text{EEEE}, \text{EEEE}, \text{BEXT}, \text{EEEE}, \text{BEBT}, \text{BEBE}, \text{BBTE}, \text{BBBT}, \text{BBBE}, \text{BBXT}, \text{BETE}\}. \quad (41)$$

Since the theory of QBD processes only allows for one infinite dimension (level), we will evaluate the mean drift condition for both scenarios (2) and (3). These scenarios require different matrices $\mathbf{A}_0^i, \mathbf{A}_1^i, \mathbf{A}_2^i, \mathbf{B}^i$, $i = 2, 3$, but the sum of the three matrices $\mathbf{A} = \mathbf{A}_0^i + \mathbf{A}_1^i + \mathbf{A}_2^i$ must be independent of i. Again because of the infeasibility of algebraically inverting 21×21 parameterized matrices, we will calculate this numerically. We define $f_i(\eta) = \boldsymbol{\omega}(\eta)\left(\mathbf{A}_2^i - \mathbf{A}_0^i\right)\mathbf{e}^T$, for $i = 2, 3$.

We assume continuity in throughput functions and see by looking at the case of $\eta \downarrow 0$ that the process starts with node 2 becoming saturated, since when $\eta \downarrow 0$ it does not matter whether back-off times are truncated or not and therefore the starting values are the same as for scheme (i). That is why we may assume that in the interval $(0, \eta_{1\to2}]$ node 2 is saturated and node 3 is not and thus we have scenario (1). We consider $f_2(\eta)$ to see in what interval node 3 remains stable in case node 2 is taken saturated, and it turns out $\eta_{1\to2} = 1$. So for $0 < \eta \leq 1$ we have scenario (1), and the process switches to scenario (2) at $\eta_{1\to2} = 1$. This part of the graph can be seen in Figures 4 and 5.

For scenario (2) the Markov process can be embedded on a finite state space (with 21 states). In this case, with both nodes 2 and 3 saturated, $\boldsymbol{\omega}(\eta)$ is the equilibrium distribution and we have $\theta_i(\eta) = \boldsymbol{\omega}(\eta)\mathbf{A}_0^i$ for $i = 2, 3$. Because of continuity this part of the graph lasts as long as both $\theta_1(\eta) > \theta_2(\eta)$ and $\theta_2(\eta) > \theta_3(\eta)$. The smallest $\eta > \eta_{1\to2}$ for which one of these inequalities is violated is $\eta_{2\to3}$. Numerically solving gives $\eta_{2\to3} \approx 1.24415$ with $\theta_1(\eta_{2\to3}) = \theta_2(\eta_{2\to3})$, which indicates (rate) stability at that point. Whenever this equality holds, both throughput functions have to equal $\tau(\eta)$. Since $\theta_2(\eta_{2\to3}) > \theta_3(\eta_{2\to3})$ still holds, we arrive in scenario (3) and thus look at $f_2(\eta)$ for $\eta > \eta_{2\to3}$, as depicted in Figure 7. This figure shows that from $\eta_{2\to3}$ onwards node 2 will remain stable, as $f_2(\eta) > 0$ for $\eta > \eta_{2\to3}$, conditioned on saturation of node 3. Whenever $\theta_2(\eta) > \theta_3(\eta)$ this saturation is evident. However, in this scenario the throughput of node 3 dominates the throughput of node 2 from $\eta_{3\to4} \approx 1.25763$ onwards. This implies stability of all nodes from $\eta_{3\to4}$ onwards and thus we arrive in scenario (4), where $\theta_1(\eta) = \theta_2(\eta) = \theta_3(\eta) = \theta_4(\eta) = \tau(\eta)$. Other possible scenarios for $\eta > \eta_{3\to4}$ can be discarded after evaluating the QBD processes (scenario (2) and (3)), or the finite state Markov process (scenario (1)), from which we will see that the trivial requirement

$$\theta_1(\eta) \geq \theta_2(\eta) \geq \theta_3(\eta) \geq \theta_4(\eta) \quad (42)$$

is violated. As it turns out, this reasoning can also be used to verify the situation for $\eta < \eta_{3\to4}$, since for the first three scenarios other possibilities will also violate either the mean drift condition or (42).

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