Localizing DART using the Reconstructed Residual Error

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ABSTRACT

The reconstructed residual error was recently introduced as a localized measure of the accuracy of a segmented tomogram. In this paper, we use the reconstructed residual error *during* the reconstruction, and adapt the recently proposed *Discrete Algebraic Reconstruction Technique* (DART) to only enforce discreteness in regions where this is appropriate. Simulation experiments show that the reconstructed residual error can detect non-homogeneous regions. Based on this observation, the DART algorithm is modified to allow the reconstruction of objects that are only partially discrete. The experiments show that the reconstruction quality of both homogeneous and non-homogeneous regions is improved.

1. INTRODUCTION

In discrete tomography, it is assumed that the scanned object consists of only a few different compositions. This assumption allows adding constraints to the reconstruction algorithm, which can improve the final tomogram. A straightforward way to do this, is to specify *a priori* gray levels for the reconstruction. A recently proposed reconstruction algorithm that implements such constraints, is the *Discrete Algebraic Reconstruction Technique* (DART) (Batenburg and Sijbers 2011). It has been applied successfully in practice, notably in the field of materials science (*e.g.*, Batenburg *et al.* 2009).

A major issue with adding constraints to the reconstruction algorithm, is that these are often not applicable to the whole image. The *Partially Discrete Algebraic Reconstruction Technique* (PDART) overcomes this problem for samples in which only the densest material is homogeneous (Roelandts *et al.* 2012a). However, its use is then evidently restricted to such samples.

Recently, the *reconstructed residual error* was introduced as a way to visualize the accuracy of a segmented tomogram (Roelandts *et al.* 2012b). This method exploits the projection data by comparing it with a forward projection of the final segmented image. The reconstructed residual error can be used to detect where the segmentation was not accurate. Hence, if the segmentation is based on constraints that follow from prior knowledge, it allows determining the areas of the reconstruction for which this prior knowledge was, apparently, not valid.

In this paper, we adapt the DART algorithm to exploit the reconstructed residual error *during* the reconstruction. At regular intervals, the reconstructed residual error is computed to locate pixels that do not sufficiently confirm to the constraint of homogeneity. The adapted algorithm then uses that information to no longer enforce homogeneity in those regions.

2. METHODS

The projection process in tomography can be modeled as a linear operator that is determined by the projection geometry. This leads to a system of linear equations,

$\boldsymbol{Wx} = \boldsymbol{p}$, (1)

where $\boldsymbol{p} \in \boldsymbol{R}^m$ contains the projection data and $\boldsymbol{x} \in \boldsymbol{R}^n$ corresponds to the image. The linear operator is represented by the $m \times n$ matrix $\boldsymbol{W}$, the *projection matrix*. An approximate solution $\boldsymbol{x'} \in \boldsymbol{R}^n$ of (1) can then be computed, in practice often by minimizing some norm $\|\boldsymbol{Wx} - \boldsymbol{p}\|$. This approximate solution $\boldsymbol{x'}$ can then be segmented into $\boldsymbol{s} \in \boldsymbol{R}^n$.

The reconstructed residual error is computed as follows. First, $\boldsymbol{s}$ is forward projected, yielding $\mathbf{p_s} \in \boldsymbol{R}^m$, so $\boldsymbol{p_s} = \boldsymbol{Ws}$. Second, an approximate solution of the system $\boldsymbol{We} = \boldsymbol{p} - \boldsymbol{p_s}$, where $\boldsymbol{e} \in \boldsymbol{R}^n$ is the (unknown) error image, is computed. This approximate solution $\boldsymbol{e'} \in \boldsymbol{R}^n$ is then the reconstructed residual error. We use the SIRT algorithm (Gregor and Benson 2008), both to reconstruct the residual error and as a subroutine of DART.

The original DART algorithm, illustrated as the lines with a white background in Figure 1, interleaves a continuous algebraic reconstruction method, such as SIRT, with segmentation steps. See (Batenburg and Sijbers 2011) for a detailed description and characterization of the algorithm. The segmentation is performed using simple thresholding, starting from a set of thresholds τ and a set of gray levels ρ. After each segmentation step, the boundary pixels between homogeneous regions are determined, and added to a set F, the set of *free pixels*. During the following SIRT iterations, pixels that are not in F are kept fixed. A smoothing step then compensates for possibly heavy fluctuations in the values of pixels that can be caused by only updating the boundaries. The output of the algorithm is the final segmented image.

Input:
  projection data $\boldsymbol{p}$
  thresholds τ and gray levels ρ
  error threshold c

Algorithm:
  $t := 0$
  Initialize M^0 to the empty set
  Compute an initial SIRT reconstruction $\boldsymbol{x}^0$ and an initial segmented image $\boldsymbol{s}^0$
  while stop condition is not met **do**
    Initialize the set F^t with the boundary pixels of $\boldsymbol{s}^t$
    $F^t := F^t \cup M^t$
    Compute the image $\boldsymbol{y}^t$ from $\boldsymbol{x}^t$ and $\boldsymbol{s}^t$, setting $y^t_j := x^t_j$ if $j \in F^t$ and $y^t_j := s^t_j$ otherwise
    Compute the SIRT reconstruction $\boldsymbol{x}^{t+1}$, with $\boldsymbol{y}^t$ as the start solution, keeping pixels not in F^t fixed
    $t := t + 1$
    Smooth the pixels of $\boldsymbol{x}^t$
    Compute the segmented image $\boldsymbol{s}^t$ from $\boldsymbol{x}^t$
    Reconstruct the residual error $\boldsymbol{e}^t$ of $\boldsymbol{s}^t$
    Determine M^t from $\boldsymbol{e}^t$ by including all pixels for which $e^t_j > c$
    [optional] Perform morphological operations to clean up M^t
  endwhile
  Compute the image $\boldsymbol{z}^t$ from $\boldsymbol{x}^t$ and $\boldsymbol{s}^t$, setting $z^t_j := x^t_j$ if $j \in M^t$ and $z^t_j := s^t_j$ otherwise

Output:
  $\boldsymbol{z}^t$

Figure 1: Overview of the localized DART algorithm in pseudocode. Code that was added for the localized version has a gray background.

For the localized DART algorithm, an extra set of free pixels M is added to the algorithm. After each segmentation step, the reconstructed residual error is computed, and pixels for which the error is larger than a given threshold c are added to M. This is also illustrated in Figure 1, where a gray background indicates code that was added. Setting c to infinity recovers the original DART algorithm.

3. EXPERIMENTS AND RESULTS

This Section describes simulation experiments that demonstrate the localized version of DART. All experiments were conducted on a square reconstruction grid of size 512×512 pixels. A phantom image (Figure 2a) of size 2048×2048 pixels was created. The resolution of the phantom is higher than that of the reconstruction grid, to reduce the effect of the pixelation on the reconstructions.

For the first experiment, a synthetic dataset was created using 40 equiangular parallel beam projections in the interval [0, 180]. From this synthetic dataset, a localized DART reconstruction (Figure 2b) was created. The extra set of pixels that were reconstructed continuously (Figure 2c) was determined from the reconstructed residual error. For comparison, Figure 2d shows a SIRT reconstruction of the same dataset.

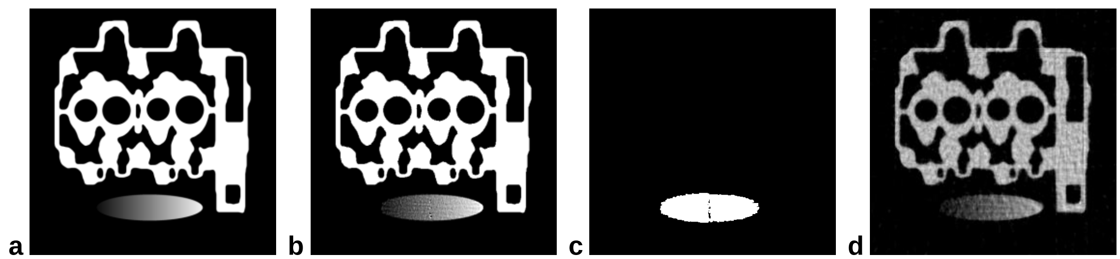

Figure 2: (a) Phantom. (b) Localized DART reconstruction. (c) Final mask. (d) SIRT reconstruction.

The mechanism behind the localized DART reconstruction is illustrated in Figure 3. Figure 3a shows the final fully segmented reconstruction $\boldsymbol{s}^t$. For each of these fully segmented images, the reconstructed residual error (Figure 3b) is computed. Figure 3b clearly indicates that the ellipse at the bottom is not segmented correctly, and applying the threshold c to Figure 3b, followed by the morphological operations of removing very small clusters of pixels and dilation with a small square, results in the final mask of Figure 2c. For a regular, *i.e.*, non-localized, DART reconstruction (Figure 3c), the reconstructed residual error (Figure 3d) is much less discriminating, and cannot be used to create a usable mask. Hence, it is advantageous to compute the reconstructed residual error *during* the execution of DART.

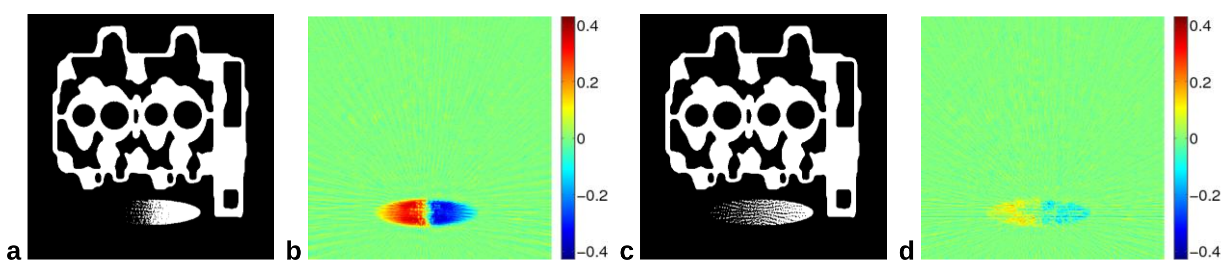

Figure 3: (a) Fully segmented image (internal to DART). (b) Reconstructed residual error of (a).
(c) Regular DART reconstruction. (d) Reconstructed residual error of (c).

For the second experiment, a synthetic dataset with only 20 parallel beam projections was created. For this experiment, the set M, denoting the non-homogeneous region, was assumed to be known a priori (Figure 4c). A localized DART reconstruction was created (Figure 4a), and a SIRT reconstruction is shown for comparison (Figure 4d). In Figure 4b, the non-homogeneous part was reconstructed again, now starting from the localized DART reconstruction. The homogeneous part was projected forward and subtracted from the original projections. This new dataset was then reconstructed, and the result was used to replace the non-homogeneous ellipse with. This improves the accuracy of the non-homogeneous ellipse.

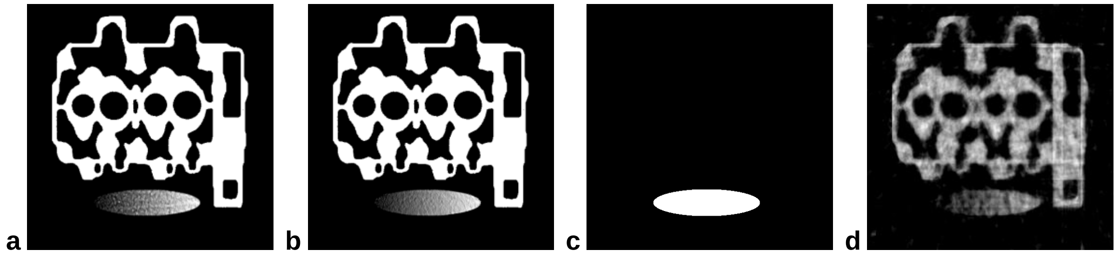

Figure 4: (a) Localized DART reconstruction. (b) Same as (a), but with the non-homogeneous part reconstructed again separately. (c) Manual mask. (d) SIRT reconstruction.

4. DISCUSSION AND CONCLUSION

A main strength of discrete tomography is that it allows creating high quality reconstructions from less data than is needed for a classical reconstruction. It is clear from Figures 2 and 4 that this strength is retained by the localized version of DART. Hence, the proposed approach effectively expands the applicability of DART to objects that contain both homogeneous and non-homogeneous regions. The reconstructed residual error allows discriminating between these regions automatically, and its interaction with the DART algorithm results in both homogeneous and non-homogeneous regions being reconstructed more accurately than with SIRT. If the non-homogeneous area is known a priori, the amount of data can be further reduced (Figure 4) without impairing the reconstruction quality.

We have presented an adapted DART algorithm that localizes constraints automatically, using the reconstructed residual error. The localized DART algorithm retains a superior reconstruction quality, as compared to classical algorithms, both for homogeneous and non-homogeneous regions.

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