Nuclear Engineering and Design 265 (2013) 319–329

Contents lists available at ScienceDirect

Nuclear Engineering and Design

journal homepage: www.elsevier.com/locate/nucengdes

Construction strategies and lifetime uncertainties for nuclear projects: A real option analysis

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HIGHLIGHTS

- Real options can be used to value flexibility of modular reactors.
- Value of NPPs increases with implementation of long term cost reductions.
- Levels of uncertainties affect the choice between projects.

ARTICLE INFO

Article history:
Received 26 October 2012
Received in revised form 18 July 2013
Accepted 15 August 2013

ABSTRACT

Small and medium sized reactors, SMRs (according to IAEA, 'small' are reactors with power less than 300 MWe, and 'medium' with power less than 700 MWe) are considered as an attractive option for investment in nuclear power plants. SMRs may benefit from flexibility of investment, reduced upfront expenditure, and easy integration with small sized grids. Large reactors on the other hand have been an attractive option due to economy of scale. In this paper we focus on the advantages of flexibility due to modular construction of SMRs. Using real option analysis (ROA) we help a utility determine the value of sequential modular SMRs. Numerical results under different considerations, like possibility of rare events, learning, uncertain lifetimes are reported for a single large unit and modular SMRs.

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1. Introduction

Deregulation of the electricity market has been driven by the belief in increased cost-efficiency of competitive markets, but also leads to increased uncertainties in the market. There is a need for valuation methods to make economic decisions for investment in power plants in these uncertain environments. Kessides (2010) emphasizes the use of real options analysis (ROA) to estimate the option value that arises from the flexibility to wait and choose between further investment in a power plant and other generating technologies as new information emerges about energy market conditions.

The real options approach for making investment decisions in projects with uncertainties, pioneered by Arrow and Fischer (1974), Henry (1974), Brennan and Schwartz (1985) and McDonald and Siegel (1986) became accepted in the past decade. Dixit and Pindyck (1994) and Trigeorgis (1996) comprehensively describe the real options approach for investment in projects with uncertain future cash flows. Using real options enables us to value the option to delay, expand or abandon a project with uncertainties, when such decisions are made following an optimal policy.

ROA has been applied to value real assets like mines (Brennan and Schwartz, 1985), oil leases (Paddock et al., 1988), patents and R&D (Schwartz, 2003). Pindyck (1993) uses real options to analyse the decisions to start, continue or abandon the construction of nuclear power plants in the 1980s. He considers uncertain costs of a reactor rather than expected cash flows for making the optimal decisions. Rothwell (2006) uses ROA to compute the critical electricity price at which a new advanced boiling water reactor should be ordered in Texas.

In this paper we focus on the inherent value of flexibility that arises in construction scenarios of nuclear power plants (NPPs). We use the Stochastic Grid Bundling Method (SGBM) (Jain and Oosterlee, 2013) for the valuation of the real option of investing in NPPs for different construction scenarios. The method has been validated for valuing flexibility that arises during the modular construction of nuclear reactors in Jain et al. (2013). In Section 2 we state the context of different construction strategies for nuclear

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http://dx.doi.org/10.1016/j.nucengdes.2013.08.060

power plants and its corresponding mathematical forumaltion. Section 3 deals in detail with the real option analysis of projects under different construction strategies, while Section 4 describes the effect of a stochastic life time of operation for nuclear plants. Finally, Section 5 gives some concluding remarks.

2. Context

We consider a competitive electricity market where the price of electricity follows a stochastic process. A utility needs to make a choice between different projects to meet the same generation capacity expansion. The following construction scenarios are considered:

- The utility is planning a capacity expansion of 1200 MWe and needs to make the choice between a single large reactor of 1200 MWe that benefits from the economy of scale or four modules of 300 MWe each, that benefit from flexibility, learning, and site sharing costs.
- The utility has a choice between two twin units at the same site or four individual reactors at different sites. Twin units, in order to benefit from site sharing costs, are constrained to be constructed one after the other. The latter project, although it does not benefit from site sharing costs, has the flexibility to order reactors at favourable times.

In addition to the above two scenarios, we also study how sensitive the utilities decisions would be to changes in underlying parameters like decision horizon and uncertainty in electricity prices. For this we take a case from Gollier et al. (2005), where the utility makes a choice between a single large reactor or four small-sized reactors to meet its generation capacity expansion.

We use the following notation: the total number of series units is denoted by n. Unit number i is characterized by discounted averaged cost per kWh equal to θ_i, its construction time is denoted by C_i and the lifetime of its operation by L_i. Construction and lifetime are expressed in years. It is assumed that different modules are constructed in sequence, and the construction of unit $i+1$ can be decided from any time subsequent to the start of the construction of unit i.

We assume a constant risk free interest rate, denoted by r here.

The utility needs to take the decision to start construction of the modules within a finite time horizon, denoted by T_i for the ith module. In terms of financial options, T_i represents the expiration time for the 'option to start the construction of the ith module'. Unlike financial options, it is difficult to quantify the *expiration time* for real options, and it is usually taken as the expected time of arrival of a competitor in the market, or the time before which the underlying technology becomes obsolete. In case of an electricity utility, it also represents the time before which the utility needs to set up a plant to meet the electricity demand with certain reliability.[2]

The value of different construction strategies can be affected by the uncertain life time of operation of nuclear power plants. In Section 4 we discuss the effect of stochastic life time of operation on the value of NPP.

2.1. Electricity price and revenue model

The uncertainty in our pricing model is the electricity price. Modelling electricity spot prices is difficult primarily due to factors like:

- Lack of effective storage, which implies electricity needs to be continuously generated and consumed.
- The consumption of electricity is often localized due to constraints in grid connectivity.
- The prices show other features like daily, weekly and seasonal effects, that vary from place to place.

Models for electricity spot prices have been proposed by Pilipovic (1998) and Lucia and Schwartz (2002). Barlow (2002) develops a stochastic model for electricity prices starting from a basic supply/demand model for electricity. These models focus on short term fluctuations of electricity prices which is beneficial for accurate pricing of electricity derivatives.

As decisions for setting up power plants look at a long term evolution of electricity prices, we, like Gollier et al. (2005), use the basic Geometric Brownian Motion (GBM) model as the driving electricity price process. However, it should be noted that within our modelling approach we can easily include other price processes.

Fig. 1 illustrates the profit from the sale of electricity for one such realized electricity price path. The cost of operation, θ, in the illustration is 3.5 cents/kWh and the area between the electricity path and θ gives the profit from the sale of electricity. We are interested in the expected profit, i.e. the mean profit from all possible electricity paths in the future. This expected profit (or net cash flow) is the payoff, $h_i(X_t)$, or the amount the utility receives when the real option is exercised.

The *revenue*, R_i, for the ith module, for every unit power of electricity sold through its lifetime L_i, starting construction at time t, when the electricity price is $X_t = x$, can be written as

$$R_i(X_t = x) = \mathbb{E}\left[\int_{t+C_i}^{t+C_i+L_i} e^{-ru} X_u du | X_t = x\right]. \quad (1)$$

R_i is the discounted expected gross revenue over all possible electricity price paths. The revenue starts coming in once the construction is finalized, and therefore the range for the integral starts from $t+C_i$, and lasts as long as the plant is operational, i.e. until $t+C_i+L_i$. Similarly, the *cost* of operating the ith module, K_i, through its lifetime for every unit power of electricity generated, is given by:

$$K_i = \int_{t+C_i}^{t+C_i+L_i} e^{-ru}\theta_i du. \quad (2)$$

Here θ_i, the cost of operating the reactor per kWh, is assumed to be constant. Therefore, the net discounted cash flow, for module i, is given by:

$$h_i(X_t = x) = R_i(X_t = x) - K_i. \quad (3)$$

Eqs. (1)–(3) give the expected profit from the sale of electricity through the life of the nuclear reactor. Eq. (3) is the mean profit from all possible electricity paths in the future.

2.2. Dynamic programming formulation

In order to construct all modules at the optimal time we use Bellman's principle of optimality, where the optimal decisions are made recursively moving backwards, starting from the final reactor. At the expiration time for the last module the firm does not have the option to delay an investment. Therefore, the decision to start the construction is taken at those electricity prices for which the expected NPV of the last module is greater than zero. The option value of the last module at the expiration time is then given by:

$$V_n(t_m = T_n, X_{t_m}) = \max(0, h_n(X_{t_m})). \quad (4)$$

At time t_k, $k = m-1, \ldots, 0$, the option value for the last of the series of reactors is the maximum between immediate pay-off h_n and

[2] Reliability is measured as the probability of the number of unplanned outages in a year when excess demand of electricity cannot be met by the utility.

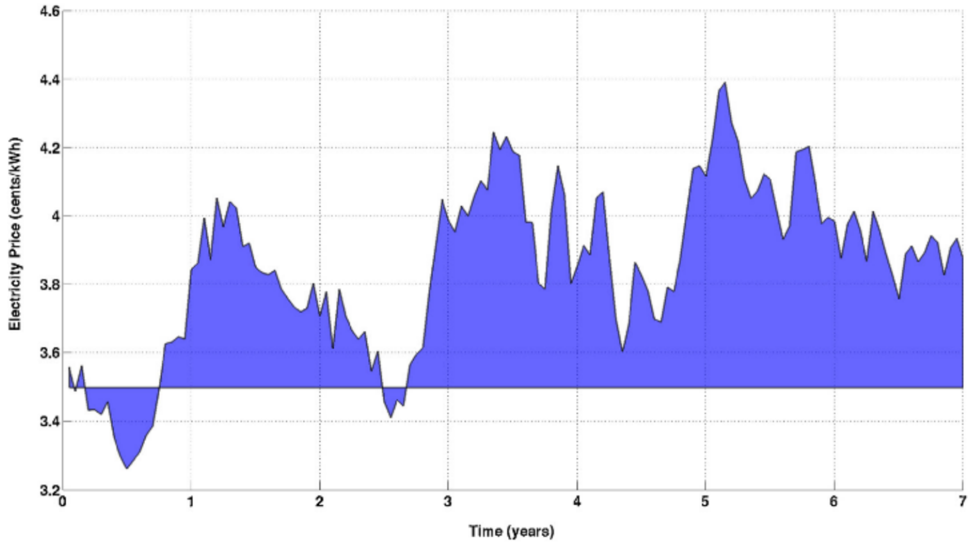

Fig. 1. The area between the electricity path (starting at 3.5 cents/kWh) and the cost of operation = 3.5 cents/kWh, gives the cash flow for the reactor.

its continuation value Q_n. The continuation value is the expected future cash flow if the decision to construct the reactor is delayed until the next time step. The reactor is constructed if at the given electricity price the net present value is greater than the expected cash flows if the reactor is constructed sometime in the future. This can be written as:

$$V_n(t_k, X_{t_k}) = \max\left(h_n(X_{t_k}), Q_n(t_k, X_{t_k})\right), \quad k = 0, \ldots, m-1. \tag{5}$$

Given the present state X_{t_k}, the continuation value, or, in other words, the discounted cash flows if the decision to start the construction is delayed for the last reactor is,

$$Q_n(t_k, X_{t_k}) = e^{-r(t_{k+1}-t_k)}\mathbb{E}\left[V_n(t_{k+1}, X_{t_{k+1}})|X_{t_k}\right]. \tag{6}$$

Once the option value at each time step for the last module is known, we move on to the modules $n-1, \ldots, 1$. At the expiration time for the i-th module, the decision to start its construction is taken when the combined NPV of the present reactor and the expected future cash flow from the optimally constructed modules $i+1, \ldots, n$ are greater than zero. Therefore, the option value for the ith module at its expiration time T_i is given by:

$$V_i(T_i, X_{T_i}) = \max\left(0, h_i(X_{T_i}) + Q_{i+1}(T_i, X_{T_i})\right), \tag{7}$$

where h_i gives the direct future cash flow from the ith module and $Q_{i+1}(T_i, X_{T_i})$ gives the expected cash flow from the optimal construction of modules $i+1, \ldots, n$, given the information X_{T_i}. The option value for the module at time step t_k, where $t_k < T_i$, is given by

$$V_i(t_k, X_{t_k}) = \max(h_i(X_{t_k}) + Q_{i+1}(t_k, X_{t_k}), Q_i(t_k, X_{t_k})), \tag{8}$$

i.e. the decision to start the construction of module i is taken if the cash flow from its immediate construction (given by $h_i(X_{t_k})$) and the expected cash flow from the modules $i+1, \ldots, n$, constructed optimally in the future (modeled by $Q_{i+1}(t_k, X_{t_k})$), is greater than the expected cash flows from the modules $i, \ldots, n$, if the decision to start its construction is delayed to the next time step (given by $Q_i(t_k, X_{t_k})$). The expected cash flow if the decision to start the construction of modules $i, \ldots, n$ is delayed to the next time step is given by:

$$Q_i(t_k, X_{t_k}) = e^{-r(t_{k+1}-t_k)}\mathbb{E}\left[V_i(t_{k+1}, X_{t_{k+1}})|X_{t_k}\right]. \tag{9}$$

The option value, $V_i(t_k, X_{t_k})$, at time t_k for constructing the module i does not only carry the information about the cash flows from module i, but also about the cash flows from the optimal construction of the modules $i+1, \ldots, n$ in the future.

For sequential modular construction the payoff for module i is given by $h_i(X_{t_k}) + Q_{i+1}(t_k, X_{t_k})$. The payoff does not only contain h_i, the direct discounted revenue from module i, but also Q_{i+1}, the value of the option to start or delay the construction of new modules, that opens up with the construction of module i.

The problem of sequential modular construction stated above can be solved using the *Stochastic Grid Bundling Method* (SGBM) (Jain and Oosterlee, 2013), which is a Monte–Carlo based simulation technique. It is chosen because:

- It can efficiently solve the multiple exercise option problem;
- SGBM can be used to compute the sensitivities of the real option value;
- The method can be easily extended to higher dimensions;
- The method is flexible on the choice of the underlying stochastic process;
- Improved confidence intervals are obtained with fewer paths when compared to the so called *Least Squares Method* (LSM) (Longstaff and Schwartz, 2001).

Appendix A describes the method details for solving the above problem using SGBM.

3. Effects of construction strategies

In this section we perform the real option analysis for investment decisions, which arise due to sequential construction of SMRs, for the different scenarios discussed earlier.

3.1. Economy of scale vs modularity

One of the measures identified to reduce capital costs of nuclear power by the NEA report (NEA, 2000) was increased plant size. The savings arising from the economy of scale when the unit size of power plants increases from 300 to 1300 MWe range have been studied by experts around the world since the early 1960s. The specific costs ($/kWe) of large nuclear power plants have been quoted within such a broad range that the derivation of scaling factors becomes difficult. In addition to savings arising from increased reactor unit size, cost reductions due to other factors such as construction of several units at the same site, effects of replication and

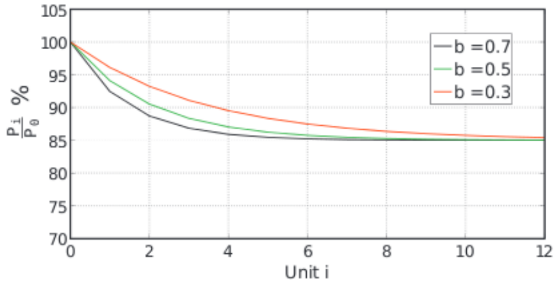

Fig. 2. Cost savings factor for different local learning rates.

series construction, and learning effects need to be incorporated in the analysis as well. In this test case we consider two projects, one with a single large reactor which benefits from the economy of scale considerations, while the other project consists of a series of four SMRs which benefit from learning and site sharing costs. Moreover, the modular units benefit from the flexibility to order the reactors at optimal times.

For many years, bigger has been better in the utility industry. The economy of scale arguments have, for some time, and in many cases, reduced the real cost of power production. The economy of scale can be expressed by the following scaling function, which relates the effect of changing the unit size to the cost of the unit,

$$\frac{TC_1}{TC_0} = \left(\frac{S_1}{S_0}\right)^{\gamma}, \tag{10}$$

where TC_0, TC_1 are the total cost for construction of two reactors with size S_0, S_1, respectively, γ is the scaling factor which is usually in the range of 0.4–0.7. It is assumed that the two reactors differ only in size, with *other details being equal*.

The effect of a learning curve and the associated cost reduction for nuclear technology has been studied in detail by Zimmerman (1982). Modular SMRs benefit from learning economies which result from the replicated supply of SMR component by suppliers and from the replicated construction and operation of SMR units by the utilities and their contractors (see Carelli et al., 2010). Boarin and Ricotti (2011) separate four effects of modular construction:

1. Learning factor. The number of similar plants constructed world-wide will lead to increased experience in construction and therefore in decreased costs;
2. Modularity factor. Modularization assumes capital cost reduction for modular plants, based on the reasonable assumption that the lower the plant size, the higher is the degree of design modularization;
3. Multiple Units factor. The multiple units saving factor shows progressive cost reduction due to fixed cost sharing among multiple units on the same site;
4. Design factor. The design factor takes into account a cost reduction by assumed possible design simplifications for smaller reactors.

We use the following equation to model the combined impact of multiple units and learning effects on cost savings, as a function of number of units on site.

$$P_i = P_0((1-a) + ae^{-bi}), \tag{11}$$

where a is the cost-savings factor, which is asymptotically achieved by an increasing number of units, and b gives the rate of on-site cost savings. Factor a would depend on the number of units constructed world wide, the amount of R&D effort put in the technology, etc. Factor b depends on the contractor, the skills of the labour involved, etc. Fig. 2 shows the price of subsequent units constructed at the same site for varying values of learning rate b. For increasing value of b, the subsequent reactor converges faster to the final cost efficiency gained by learning.

Table 1
Construction time and discounted averaged cost used for the modular units with learning and a single large unit.

	Construction time (months)	Discounted average cost (cents/kWh)	Discounted average cost ($/kWe)
Modular units			
Unit 1	36	4.71	4950
Unit 2	24	4.06	4250
Unit 3	24	3.77	3950
Unit 4	24	3.63	3800
Large reactor			
Unit 1	60	2.9	3000

We now consider four modules, each with size 300 MWe, and compare this project with a single unit of size 1200 MWe. The scaling factor for the economy of scale is taken to be $\gamma = 0.65$ (Kessides, 2012). In order to benefit from local learning, we put a constraint that the construction of a next module can begin only after one year of the start of the construction of the previous module. We take as the rate of local learning $b = 0.8$ and assume that the cost saving for large numbers of modules would approach a value of 25%. These parameter values correspond to those suggested by Mycoff et al. (2010), based on various studies in the literature, posit that the combined impact of multiple units and learning effects is a 22% reduction in specific capital costs for the SMR-based power plant. The discounted average costs of a large reactor are taken from Gollier et al. (2005) and the corresponding values for modular units are computed based on the discussion above. With these parameters, the construction costs of the modules can be summarized in Table 1.

Fig. 3 compares the option value obtained for different volatilities in the electricity price model. It is clear that with increasing uncertainties in the electricity prices the flexible modular project becomes more attractive. However, in a more certain environment a single large reactor seems to be profitable. Table 2 reports the option values for the two projects for different decision horizons for ordering the first unit. The gain due to the learning curve and the flexibility of construction, although it improves the option value for the modular units, does not seem to be sufficient to compensate for the economy of scale factor in the case of moderate uncertainty in electricity prices.

Fig. 4 illustrates the sensitivity of the real option value of the modular project with respect to parameter a, which represents the asymptotic cost reduction that will be achieved from learning, when the rate of learning (given by parameter b) is kept constant and equal to 0.8. We see that the option value of the project grows proportionally to the asymptotic cost saving due to learning.

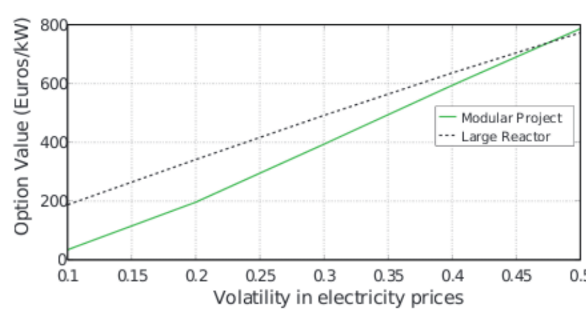

Fig. 3. The real option value for the two projects for different uncertainties in the electricity prices when the decision horizon for ordering the first unit and the large reactor is equal to 7 years.

Table 2
Option value (Euro/kW) for the modular case (4× 300 MWe) and for the large reactor (1200 MWe). The volatility for the electricity price is 20% and the initial price of electricity equals 3 cents/kWh.

Decison time (years)	Real option value (Euro/kW)
Modular case	
7	196.17
10	229.72
13	250.15
Large reactor	
7	341.48
10	364.04
13	376.98

Table 3
Construction times and discounted average costs used for the two twin units at same site and for four units at different sites.

	Construction time (months)	Discounted average cost (cents/kWh)
Two twin units		
Unit 1	60	3.5
Unit 2	48	1.67
Unit 3	48	1.81
Unit 4	48	1.60
Four independent units		
Unit 1	60	3.5
Unit 2–4	48	2.25

Fig. 5 illustrates the sensitivity of the real option value of the modular project for different rates of learning, given by parameter b, when a is kept constant and equal to 0.25. We see that for a faster rate of learning the option value of the project is higher as for large b values the cost savings are reflected rapidly in the subsequent units, while for a smaller b value the benefit is reflected only after few units have been constructed.

It can be seen from Figs. 4 and 5 that a higher *final cost saving factor* which is reflected by the value of parameter a increases the option value of the project more significantly than a higher on site learning rate as given by the parameter value b. Under our model assumptions a higher final cost saving could be achieved by long term cost reductions resulting from plant upgrades and/or increased R&D efforts to increase the real option value of the nuclear power plants.

3.2. Two twin units vs four single units

We look at the case describing the sharing of facilities by constructing multiple units at a single site. The parameter values are taken from a real case observed at EdF as described in the NEA report (NEA, 2000). The averaged costs of a unit reduce with an increasing number of units per site. We consider the case where two pairs of units per site are constructed with a case where four individual units are constructed. All units considered are of the same size, so the economy of scale does not play any role in this case.

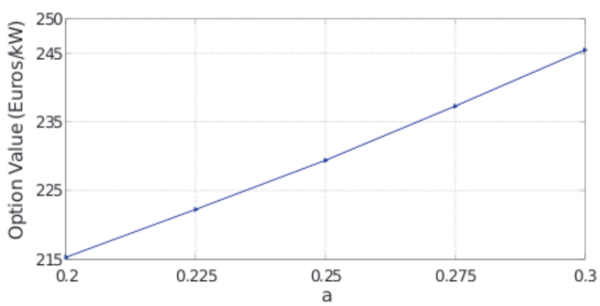

Fig. 4. The real option value for the modular project when b = 0.8, the decision horizon is 10 years and the volatility in the electricity prices is 20%, for different values of parameter a.

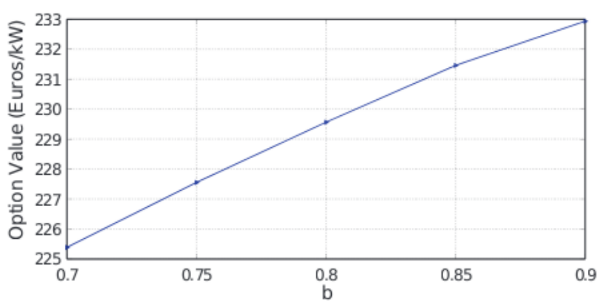

Fig. 5. The real option value for the modular project when a = 0.25, the decision horizon is 10 years and the volatility in the electricity prices is 20%, for different values of parameter b.

The aim of the test case is to compare a project with two twin units on a single site with four individual units constructed at different sites. We assume that the reactors involved are of the same size (1200 MWe each), and hence the only cost difference comes from the sharing of costs if constructed at a single site. The first reactor, in both cases, is considered to be first-of-a-kind (FOAK), and we assume the cost of generating electricity for this reactor to be 3.5 cents/kWh. The costs of the other units are summarized in Table 3. In order to achieve a cost benefit for the reactors constructed at the same site, the reactor units are constrained to be constructed in a phased manner. Therefore, the two twin reactors are constructed immediately one after the other, with the construction of the first unit starting when the electricity price crosses the corresponding critical price. The benefits of cheaper subsequent units come at the loss of flexibility to order the subsequent units at optimal electricity prices. The reasons for phased construction include considerable efficiencies and associated savings to be gained from the phased construction and rolling the various craftsmen teams from one unit to the next. In addition, by repetition of construction, there is the *craft labour learning effect* that reduces the time to perform a given task and correspondingly reduces labour cost and schedule. We take the decision horizon for ordering the first reactor to be 7 years, and assume that once a unit becomes operational it operates at its maximum capacity factor.

On the other hand, when the four units are constructed at separate sites, they do not have the cost benefits of sharing the site specific costs, neither of the productivity effects. However, when constructed individually they benefit from the flexibility to order each unit at its corresponding optimal time.

The following assumptions are made when determining the overnight costs:

- Unit 1 bears all of the extra first-of-a-kind (FOAK) costs.
- The cost of engineering specific to each site is assumed to be identical for each site.
- The cost of facilities specific to each site is assumed to be identical for each site.
- The standard cost (excluding the extra FOAK cost) of a unit includes the specific engineering and specific facilities for each unit.

If θ_0 is the standard cost (excluding the extra FOAK cost) of a sole unit on a site, the:

- Cost of the first unit is $\theta = (1 + x)\theta_0$;
- Cost of the following units is θ_0 (for 1 unit per site);
- Cost of the 2nd unit at a site with one pair is $y\,\theta_0$;

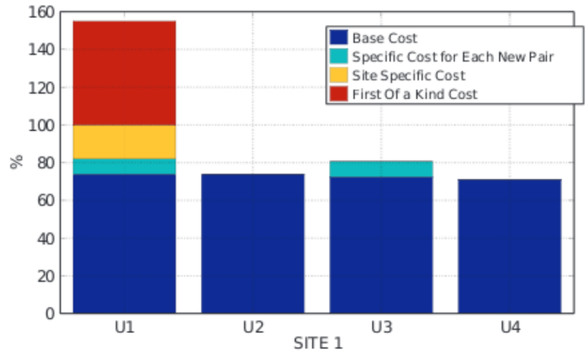

Fig. 6. Relative cost of the four units when constructed as two twin units on a single site.

- Cost of the 3rd unit at a site with two pairs is $z\,\theta_0$;
- Cost of the 4th unit at a site with two pairs is $y\,\theta_0$ (it is assumed that the cost of the 2nd unit of a pair is independent of the rank of the pair on the site).

The productivity effect depends on the rate of commitment of the units and on the owner's procurement policy. The optimum commitment rate is the one which provides good operational feedback from one construction project to the other, while serving to maintain the apprenticeship effect in the manufacturers' facilities and on the sites. The most favourable procurement policy to obtain the best prices from the suppliers consists in ordering the equipment of all the units under the same contracts NEA (2000).

A productivity effect is considered to only occur from the 3rd unit on of a series. If n is the rank of the unit in the series, and θ_n is the cost which results from taking into account the individual unit, it follows that:

$$\theta'_n = \frac{\theta_n}{(1+k)^{n-2}} \quad n \geq 2,$$

where θ'_n represents the cost of a module if there is a productivity gain involved. Using the above formulation, for the case of EdF ($x = 55\%$, $y = 74\%$, $z = 82\%$, $k = 2\%$) (NEA, 2000) in the case of the two pairs of units per site the relative costs are illustrated in Fig. 6.

In the case of two twin units the cash flow due to the sales of electricity once the units become operational is modified from Eq. (1) into

$$R_1(X_t = x) = \mathbb{E}\left[\left(\left(\int_{t+C_1}^{t+C_1+L_1} e^{-ru}X_u du + \int_{t+(C_1+C_2)}^{t+(C_1+C_2)+L_2} e^{-ru}X_u du + \cdots + \int_{t+(C_1+C_2+C_3+C_4)}^{t+(C_1+C_2+C_3+C_4)+L_4} e^{-ru}X_u du\right) | X_t = x\right]. \quad (12)$$

Eq. (12) can be read as the revenue from unit 1, when ordered at time t, starts coming in once its construction is finalized at $t + C_1$, and continues till the end of its lifetime, i.e. until time $t + C_1 + L_1$. The construction of unit 2 starts at $t + C_1$, and its revenues start flowing in from $t + C_1 + C_2$, till the end of its lifetime at $t + C_1 + C_2 + L_2$. Eq. (2) can be modified accordingly. It can be seen that only the first reactor can be ordered at an optimal time, and the construction of the ith reactor is forced immediately after the completion of the $(i - 1)$th reactor to achieve the cost reductions shown in Fig. 6.

For the four independent reactors the revenue is given by Eq. (1). As significant cost benefits can be achieved after the construction of the FOAK unit, the decision for the construction of subsequent units is made only when the construction of the first unit is done. Also the usual constraint that subsequent modules can be ordered only after the decision on starting the construction of all previous

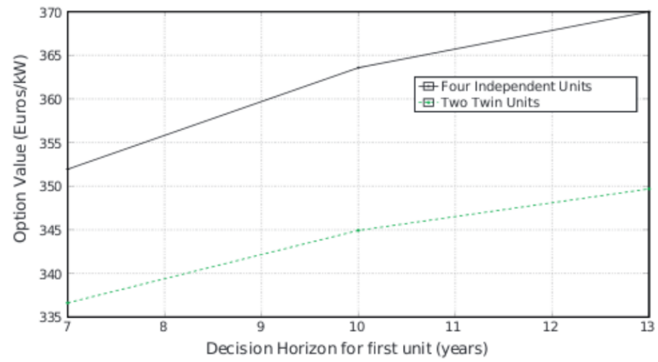

Fig. 7. Option values (Euro/kW) for the twin units, and the four independent units for different decision horizons. The option values are computed when the initial electricity price is 3 cents/kWh and the volatility in the electricity prices is 20%.

modules has been taken into account. The decision horizon for unit i is $T_i = T_{i-1}$ + *Construction Time of Unit* $(i - 1)$, where T_{i-1} is the decision horizon for the $(i - 1)$th unit.

Fig. 7 compares the option values of the two projects for different decision horizons for starting the construction of the first unit. When a firm has more time to decide when to construct the reactor, the option value of the project increases. From these results it seems that for the parameters chosen the project involving four independent units appears to be a better choice, under our model assumptions.

Fig. 8 compares the two projects for different uncertainties in the electricity prices. The project involving four independent units seems to be less profitable in a more certain environment of electricity prices. The benefit of flexibility in a project becomes more apparent with increasing uncertainties in electricity prices.

It can be seen that although building two twin reactors at a single site significantly reduces the costs of producing electricity for the units, the project loses on the value of flexibility. The four units constructed at different sites, although produce electricity at higher prices, can benefit from the opportunity to construct at more optimal market electricity prices. It should also be noted that the construction of the units in this manner may also benefit since in an event of natural disaster not all units would shut down (see Takashima and Yagi, 2009, for more details). Also it is clear that when the uncertainty in electricity price increases, it appears to be advisable to focus on flexibility, by constructing independent reactors, rather than the increased cost efficiencies, by constructing twin units at the same site. On the other hand, when the electricity price is less uncertain not much is gained by the flexibility in the

Fig. 8. Option values (Euro/kW) for the twin units, and the four independent units for different levels of uncertainties in the electricity prices. The option values are computed when the initial electricity price is 3 cents/kWh and decision horizon for the first unit is 7 years.

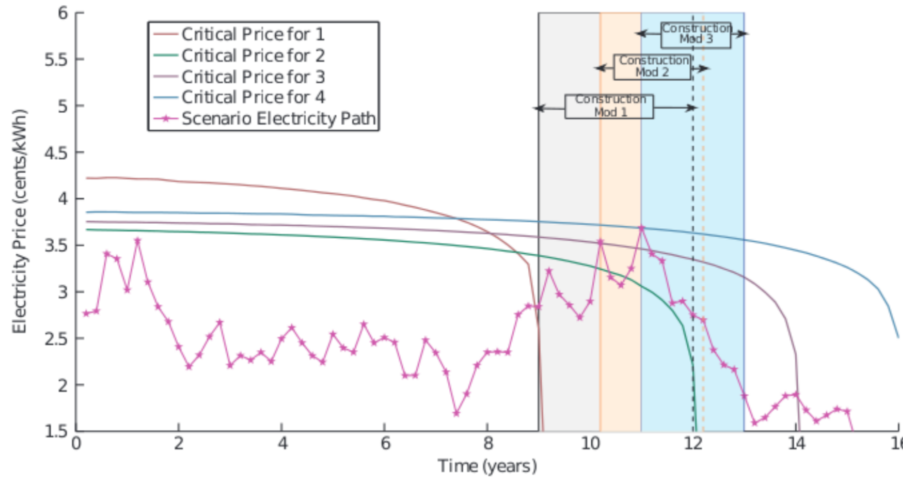

Fig. 9. Optimal investment policy for ordering sequential modular reactors, with a sample scenario path. Overlap between construction period of two reactors is possible in this case.

time to order and it appears more profitable to choose cheaper twin units in our model.

3.3. Sequential construction

We consider the theoretical case where an investor needs to decide between two projects, one involving a single large reactor of 1200 MWe and the other consisting of four modules of 300 MWe each. The construction time and costs for the two projects, given in Table 4, are taken from the reference case by Gollier Gollier et al. (2005). The discount rate is taken as 8% per annum and the predicted growth rate of electricity price is 0% here. The cost of electricity production for the first unit is relatively expensive when compared to the series units, as a large part of the fixed costs for the modular assembly, like the land rights, access by road and railway, site licensing cost, connection to the electricity grid are carried by the first unit.

We assume that a new unit can be ordered if all previous units have been *ordered* (not constructed). It is common practice to have parallel construction of different units in order to achieve cost savings, as it allows rotation of specialized labour between different units (NEA, 2000).

Fig. 9 shows a scenario path and investment policy for a modular project with the above considerations. It can be seen that in this case overlap in the construction period of different units is possible.

3.3.1. Effect of decision horizon

Fig. 10 compares the real option values of the two projects for different decision times. The decision time for the large unit is kept the same as that for the first unit in the modular project. Decisions of generation capacity expansion are based on meeting increasing electricity demands with a certain minimum reliability and therefore the decision horizon is chosen to be the same for the first module and the large reactor. In this case the modular project appears more profitable than the single large reactor.

In order to detail the results obtained, we compute the expected cashflow from different units of the modular project and the fraction of modules ordered for different decision times. Fig. 11 gives the expected cashflow from the four units for different decision

Table 4
Construction times and discounted averaged costs used for the large reactor and the modular case.

	Construction time (months)	Discounted average cost (cents/kWh)
Large reactor	60	2.9
Modular case		
Module 1	36	3.8
Module 2–4	24	2.5

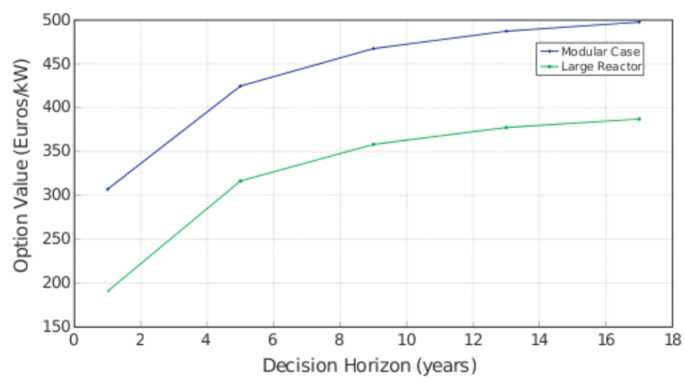

Fig. 10. Real option value for the large reactor and the modular project for different decision horizons when the initial price of electricity is 3 cents/kWh.

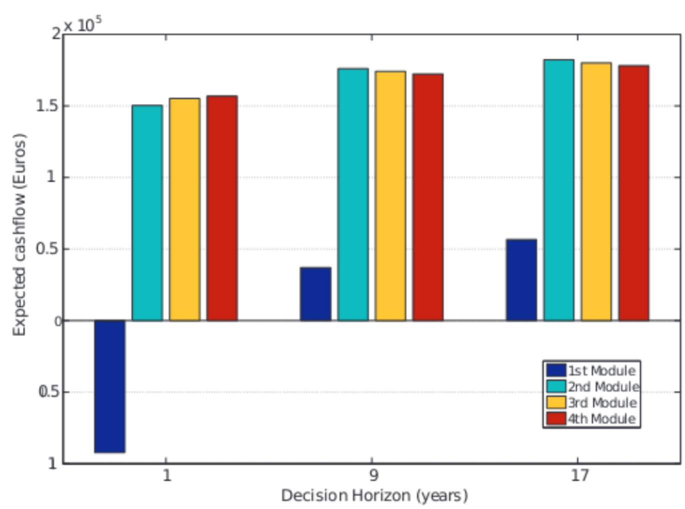

Fig. 11. Cash flow from different modules with increasing decision time. The initial price of electricity is 3 cents/kWh and volatility in the electricity prices is 20%.

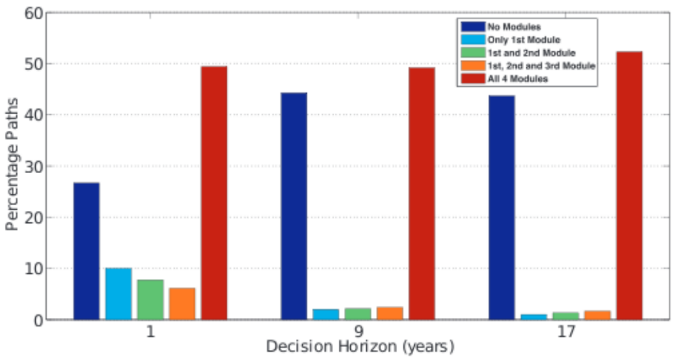

Fig. 12. Fraction of modules ordered at the end for different scenario paths with increasing decision times. The initial price of electricity is 3 cents/kWh and the volatility in the electricity prices is 20%.

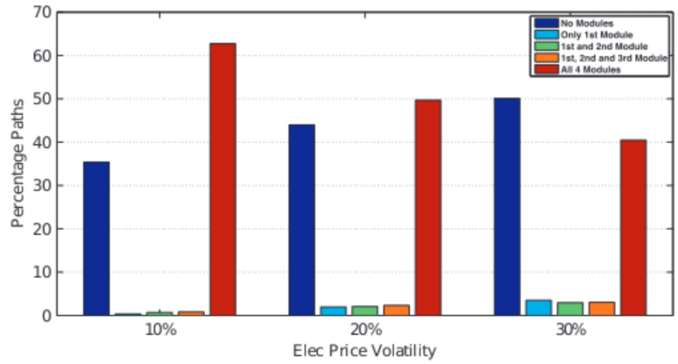

Fig. 14. Fraction of modules ordered at the end for different scenario paths with increasing volatility. The initial price of electricity is 3 cents/kWh and the decision horizon for the first module is 9 years.

times. It can be seen that the expected cashflow grows with decision time. When overlap in the construction periods of different units is allowed, the cashflow from the three similar costing units is almost the same. The reason for this is that most often the three units are ordered around the same time, and so the effect of discounting to present time is almost the same. An important reason for modular projects having higher real option value is that the effective decision horizon for the modular project is significantly longer than that of the large reactor (which is the same as that of the first unit). Another factor which adds up to the profitability of the modular project (when parallel construction is allowed) is that modular units have less construction time in our model, which allows cashflows from the sale of electricity to start before it would start from the large reactor.

Fig. 12 shows the fraction of different modules constructed by the end of the decision time for the modular project. It is clear that when the constraint of waiting for completion of a unit before ordering a new one is relaxed, that once the first unit is ordered, in most cases it results in all four units being ordered.

3.3.2. *Effect of electricity price volatilities*

Fig. 13 compares the real option values of the two projects for different volatilities in the electricity prices. We see in this case that the modular project is always more profitable than a single large reactor.

Most of the units are ordered around the same time, as can be concluded from the discussion above. However, this will not be the case when the uncertainty in the electricity price increases. Fig. 14 shows the fraction of scenario paths for which different numbers of units are ordered by the end of the decision horizon. It is clear that with increasing uncertainty more often the project ends with fewer units than were planned initially.

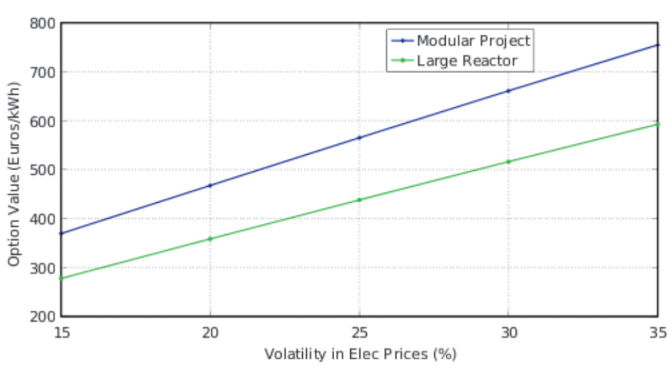

Fig. 13. Real option value for the large reactor and the modular project for different volatilities when the decision horizon is 9 years and the initial price of electricity is 3 cents/kWh.

4. Effects of uncertain life time of operation

Uncertain life times of operation should be taken into account when computing the value of investment in an NPP. A detailed analysis would not just take the uncertain life time of operation into account but also uncertain capacity factors during the operation of the reactor. Du and Parson (2010) perform a detailed analysis on the capacity factor risk in the nuclear power plants. Rothwell (2006) employs a stochastic process for varying capacity factors in his analysis. We here assume (like Gollier et al., 2005) that the nuclear reactors operate at a mean capacity factor of 90% throughout their lifetime, which is a reasonable assumption Varley (2002) for modern reactors.

In our analysis we assume uncertain life time of operation of NPPs which can be due to premature permanent shut down on one hand or due to extension of operating licenses and lifetime on the other hand.

4.1. *Effects of premature permanent shut-down*

We use the term *premature permanent shut down* for the case when an operating reactor is permanently shut down before completing its licensed operating life time. Historically, premature permanent shut down of reactors have been observed for direct reasons- like accidents or serious incidents in a reactor (e.g. Three Mile Island 2 – 1979, Chernobyl 4 – 1986, Fukushima Daiichi 1,2,3,4 – 2011), or it could be indirect by – for example shutting down of reactors due to increased safety measures, economic reasons, changing government policies, etc. (e.g. Shoreham, in US 1989).

The arrival time of such an event (elsewhere called *rare events* or *catastrophic events*) has been modelled by a Poisson process, e.g. Clark (1997) for a real options application with a single source for rare events, Schwartz (2003) uses Poisson arrival times to model catastrophic events when investing in R&D. We also model the arrival time for the cause of premature permanent shut down as a Poisson process whose arrival frequency, λ, is the expected number of such events a year.

In order to compute the frequency of premature permanent shut down we use data available from the IAEA report (2005) and a WNA report Varley (2002). In total there have been 133 reactors that have been permanently shut down after they started operating. Out of these, 11 reactors were shut down due to accidents or serious incidents, and 25 have been shut down due to political decisions or due to regulatory impediments without a clear or significant economic or technical justification. The remaining 97 reactors were

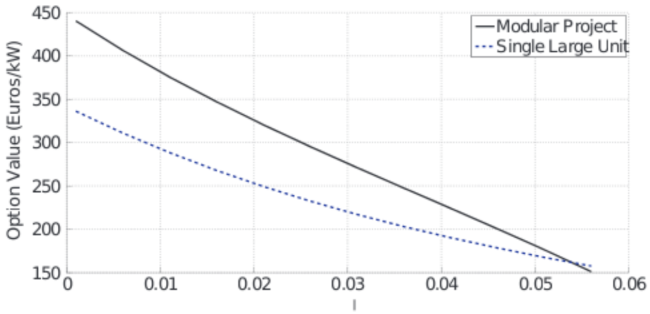

Fig. 15. The real option price for the large reactor, and the modular project for different values of λ when the volatility of the electricity price is 20% and the decision horizon for the large reactor and the first module equals 7 years.

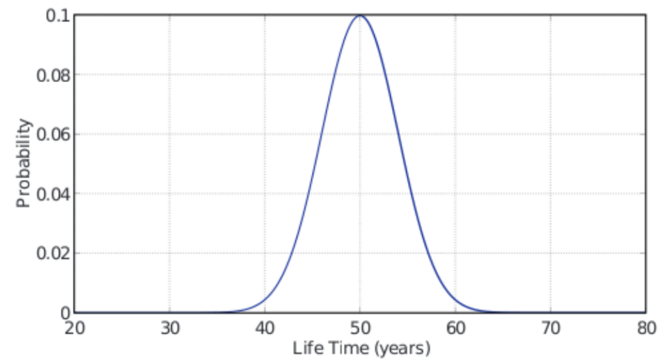

Fig. 16. The distribution for the lifetime of operation of a nuclear reactor, which was originally licensed for 40 years of operation.

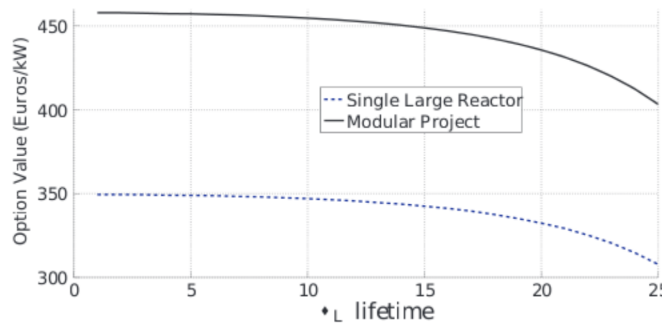

Fig. 17. Option value vs uncertainty in lifetime of operation.

shut down because they completed their designated lifetime and costs associated with a lifetime extension did not make economic sense for these reactors. The total cumulative life time of operation for the reactors in the world is approximately 14,500 years. Therefore, the number of premature permanent shut downs per reactor year is (25+11)/14,500, which is 0.0025 reactors per year. Thus, the rate of arrival of the cause for premature permanent shut down, is modelled as λ = 0.0025 events every year, from a statistical point of view.

The inclusion of catastrophic events results in an effective discount rate from r to $r+\lambda$ (see Schwartz, 2003) for more details) once the plant gets operational. Fig. 15 compares the option values for the modular project and the single large reactor, with parameter values as given in Table 4. It is clear that the value of the investment option reduces with increasing probability of catastrophic events; however, the modular project seems to be more profitable in the realistic domain of λ values.

4.2. Effects of life time extension

Most nuclear power plants originally had a nominal design lifetime of 25–40 years, but engineering assessments of many plants have established a longer operation time. In the USA over 60 reactors have been granted licence renewals which extend their operating lives from the original 40–60 years, and operators of most others are expected to apply for similar extensions. Such licence extensions at about the 30-year mark justify a significant capital expenditure for replacement of worn equipment and outdated control systems. In 2010 the German government approved a lifetime extension for the country's 17 nuclear power reactors. However, after the Fukushima accident in March 2011, Germany planned a complete phase-out by the year 2022, reverting the previous decision. It is clear therefore that the lifetime of a nuclear power plant can be uncertain.

Gen III and Gen IV reactors are mostly designed for a life time of 60 years (Kessides, 2012). However many generation II reactors are being life-extended to 50 or 60 years, and a second life-extension to 80 years may also be economic in many cases (NEI, 2010). In order to address the uncertain lifetime of operation due to the possibility of lifetime extension, we use a normal distribution with a mean reactor life of $\mu_l = 50$ years and a variance of $\sigma_l = 4$, which fits well to the discussion above, as can be seen in Fig. 16. It should be noted that such a distribution allows for negative life time, however the probability for such lifetime is almost negligible.

In the case where the electricity price follows GBM and the lifetime has a normal distribution, as described above, Eqs. (2) and (1) can be written as

$$R_i(X_t = x) = e^{-(r-\alpha)C_i}\,\frac{1-e^{-((r-\alpha)\mu_l-(\sigma_l^2(r-\alpha)^2)/2))}}{r-\alpha}\,x, \tag{13}$$

and

$$K_i = e^{-rC_i}\,\frac{1-e^{-(r\mu_l-((\sigma_l^2 r^2)/2))}}{r}\,\theta_i. \tag{14}$$

Fig. 17 compares the real option value for the modular project and the single large reactor with parameter values taken from Table 4. We plot then for various σ_l with a mean reactor life of 50 years. It can be seen that with increasing uncertainty in the lifetime the option value reduces in value, although it follows the same trend for both cases considered.

5. Conclusion

In this paper we presented a real option valuation of different construction strategies of NPPs for finite decision horizon. We analysed a few scenarios a utility might be interested in before making a choice of nuclear reactor. The conclusions drawn from the test cases *under the model assumptions in the present paper* can be summarized as follows:

1. In a finite decision horizon, sequential modular units can be ordered at more competitive electricity prices, compared to a construction of units in isolation.
2. The model shows that the real option value of nuclear power plants increases with implementation of long term cost reductions. Such cost reductions might be achieved e.g. by plant upgrades and/or increased R&D efforts.
3. When twin units are constructed at the same site, significant cost reductions can be achieved. On the other hand, in order to achieve these cost savings, the utility loses the flexibility to order units at optimal market conditions. When the electricity price uncertainty is low it appears that cost savings by a construction of twin modules at the same site would be favourable under our model assumptions, while when electricity prices are more volatile, the flexibility of choice dominates.

4. Uncertain lifetime of operations reduces the option value of both the modular and single large reactor test cases. However, with increasing frequency of premature permanent shutdown, the modular construction works out more profitable than individual large reactors. Uncertainty in life time extension affects the option value of a single large unit and modular units almost similarly in our model.
5. Specific cost of SMRs can be much higher than of a single large unit, because of the economy of scale argument. Some cost reduction is achieved by the learning effect with each new module. However, it appears that cost savings due to learning are not sufficient to make modular SMRs competitive with large units.

Appendix A.

The real option problems we are interested in have financial counterparts, i.e. the Bermudan options and multiple exercise Bermudan options (Seydel, 2012). A Bermudan option gives the holder the right, but not obligation, to exercise the option once, on a discretely spaced set of exercise dates. A multiple exercise Bermudan option, on the other hand, can be exercised multiple times before the option expires. Pricing of Bermudan options, especially for multi-dimensional processes is a challenging problem owing to its path-dependent settings.

Consider an economy in discrete time defined up to a finite time horizon T_n. The market is defined by the filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$. Let X_t, with $t = t_0, t_1, \ldots, t_m = T_n$, be an $\mathbb{R}^d$-valued discrete time Markov process describing the state of the economy, the price of the underlying assets and any other variables that affect the dynamics of the underlying. Here $\mathbb{P}$ is the risk neutral probability measure. The holder of the multiple exercise Bermudan option has n exercise opportunities, that can be exercised at $t_0, t_1, \ldots, t_m$. Let $h_i(X_t)$ represent the payoff from the ith exercise of the option at time t and underlying state X_t. The time horizon for the ith exercise opportunity is given by T_i.

We define a policy, π, as a set of stopping times $\tau_n, \ldots, \tau_1$ with $\tau_n < \ldots < \tau_1$, which takes values in $t_0, \ldots, t_m = T_n$, and τ_i determines the time where the ith remaining exercise opportunity can be used. The option value when there are n early exercise opportunities remaining is then found by solving an optimization problem, i.e. to find the optimal exercise policy, π, for which the expected payoff is maximized. This can be written as:

$$V_n(t_0, X_{t_0} = x) = \sup_{\pi} \mathbb{E}\left[\sum_{k=0}^{n} h_k(X_{\tau_k}) | X_{t_0} = x\right]. \qquad (15)$$

In simple terms, Eq. (15) states that of all possible policies for ordering the reactor in the given decision horizon, the real option value is the one which maximizes the expected future cash flows.

SGBM solves a general optimal stopping time problem using a hybrid of dynamic programming and Monte–Carlo simulation. It extends SGM (Jain and Oosterlee, 2012) to efficiently solve such problems in high dimensions. The method first determines the *optimal stopping policy* and an estimator for the option price. The optimal stopping policy for the ith module at time step t_k involves finding the critical electricity price $X^*_{t_k}$. When the market price of electricity is equal to the critical price, the value of delaying the construction of the module to the next time step is equal to the value of starting the construction immediately, i.e.,

$$Q_i(t_k, X^*_{t_k}) = h_i(X^*_{t_k}).$$

Therefore, the critical price is taken to be the largest value of X_{t_k}, for which $Q_i(t_k, X_{t_k}) > h_i(X_{t_k})$. The module is ordered if the present market price of electricity is greater than the critical price for the given time step. Once the policy for all the time steps is known, SGBM computes lower bound values, using a new set of simulated electricity paths, as the mean of the cashflows from each simulated path where the module is ordered following the policy obtained above.

SGBM for multiple exercise Bermudan options begins by generating N stochastic paths for the electricity prices, starting from initial state X_0. The electricity prices realized by these paths at time step t_k constitute the grid points at t_k. Numerically the main task involved is computing the continuation value Q_i at all time steps.

In order to obtain the continuation value for grid points at t_k, we need to determine the functional approximations of the option value at t_{k+1}. Once the option values at the grid points at t_{k+1} are known, the functional approximation is obtained using combination of *least squares regression* and bundling (Jain and Oosterlee, 2013). Therefore, to compute the continuation value for grid point $X_{t_k}(n)$, $n \in [1, \ldots, N]$, with n the path number, we only use only those paths which belong to bundle containing $X_{t_k}(n)$. Let $\mathrm{B}_n(t_k)$ be the set of indexes including n which are bundled with $X_{t_k}(n)$. The corresponding functional approximation for this bundle is obtained by regressing the option values for these paths at t_{k+1} on a set of basis functions of state variables at t_{k+1}. This functional approximation is then used to compute the continuation value at the particular grid point $X_{t_k}(n)$.

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