Scanner generation for modular regular grammars

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When formal language definitions become large it may be advantageous to divide them into separate modules. Such modules can then be combined in various ways, but this requires that implementations derived from individual modules can be combined as well. In this paper we address the problem of combining regular grammars appearing in separate modules and of combining the lexical scanners generated for them.

Dedicated to J.W. de Bakker on the occasion of the 25th anniversary of his association with the Centre for Mathematics and Computer Science.

Key Words & Phrases: program generator, generation of lexical scanners, modular regular grammars, finite automata, subset construction.


1985 Mathematics Subject Classification: 68N20 [Software]: Compilers and generators.

1. INTRODUCTION
The benefits of dividing complex systems into several, smaller, modules are well-known. Apart from a reduction in complexity that can be achieved for individual modules, one also introduces the possibility of re-using a module several times. In this paper we will apply the idea of modular decomposition to the definition of formal languages (such as, e.g., programming languages and specification languages), and concentrate on lexical syntax, one of the syntactic aspects that have to be defined for a language. Typically, the lexical syntax defines comment conventions, layout symbols, and the form of identifiers, keywords, delimiters, and constants (e.g., numbers, strings) in a language. The standard method is now to use a regular grammar to specify the precise form of the various elements in the lexical syntax and to compile this regular grammar into a deterministic finite state automaton (DFA) to be used for the actual reading of program texts.

Here, we are interested in the problem of how the lexical syntax can be subdivided in separate modules and how DFAs can be obtained from various combinations of these modules. The motivation for this problem comes from two different sources:
• In a setting where definitions for more than one language are being developed, it is natural to construct a set of standard modules defining frequently used notions (e.g., identifiers, floating point numbers, or string constants). Of course, these standard notions may sometimes need adaptation depending on their use (e.g., the letters
Abbreviations:

\[
\begin{align*}
M1: \langle \text{DIGIT} \rangle & = 0 \mid 1 \mid \ldots \mid 7 \\
M2: \langle \text{DIGIT} \rangle & = 8 \mid 9 \\
M3: \langle \text{LETTER} \rangle & = a \mid b \mid \ldots \mid z
\end{align*}
\]

Rules:

\[
\begin{align*}
M4: \langle \text{INT} \rangle & = \langle \text{DIGIT} \rangle^+ \\
M5: \langle \text{REAL} \rangle & = \langle \text{INT} \rangle \ "." \ \langle \text{INT} \rangle \\
M6: \langle \text{ID} \rangle & = \langle \text{LETTER} \rangle \ (\langle \text{LETTER} \rangle \mid \langle \text{DIGIT} \rangle)^* \\
M7: \langle \text{KW} \rangle & = \text{if} \\
M8: \langle \text{KW} \rangle & = \text{end}
\end{align*}
\]

Figure 1. A modular regular grammar.

appearing in identifiers may or may not contain both lower case letters and upper case letters, integer constants may or may not contain hexa-decimal digits).

* In modular specification languages that allow user-definable syntax to be introduced in each module, the composition of modules requires, among others, the composition of lexical syntax.

The flexibility we want to achieve can best be illustrated by an example. Consider the grammar shown in Figure 1. It consists of two parts: abbreviations and rules. The abbreviations part defines named regular expression to be used in the rules part. When several regular expressions \(e_i\) are associated with one name, we associate with that name a regular expression containing all expressions \(e_i\) as alternatives. The actual regular grammar is defined in the rules part. Names appearing in rules can be completely eliminated by textual substitution. The names of rules define the token-name to be associated with a string recognized by that particular rule. Note that more than one rule may recognize the same string; in that case we associate more than one token-name with it.

In the example, we define a lexical syntax containing integer constants, real constants, identifiers and the keywords if and end. Each regular expression in the grammar is labelled with a module name. In general, several expressions may be labelled with the same name, but in this example we have the extreme case that every expression is labelled with a different name. The use of this modular regular grammar is shown in Figure 2. Given a list of selected module names, only those rules are to be used whose module name appears in the selection. For each selection of modules, the modular regular grammar thus corresponds to a (probably) different ordinary regular grammar.

An implementation of modular regular grammars should, clearly, have the following two properties:

* The time needed to construct a DFA for a given selection of modules (in the modular case) should be significantly less than the time needed to construct the automaton from scratch (in the non-modular case) using only the rules from the selected modules.
* The efficiency of the DFA generated in the modular and in the non-modular case should be comparable.
<table>
<thead>
<tr>
<th>modules</th>
<th>selection (1)</th>
<th>selection (2)</th>
<th>selection (3)</th>
<th>selection (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>M2</td>
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<td>o</td>
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<td>x</td>
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<tr>
<td>M3</td>
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<td>M4</td>
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<td>M8</td>
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<table>
<thead>
<tr>
<th>sentences</th>
<th>recognized as</th>
<th>recognized as</th>
<th>recognized as</th>
<th>recognized as</th>
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</thead>
<tbody>
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<td>&lt;INT&gt;</td>
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<td>-</td>
<td>&lt;INT&gt;</td>
</tr>
<tr>
<td>678</td>
<td>&lt;INT&gt;</td>
<td>-</td>
<td>-</td>
<td>&lt;INT&gt;</td>
</tr>
<tr>
<td>2.8</td>
<td>&lt;REAL&gt;</td>
<td>-</td>
<td>-</td>
<td>&lt;REAL&gt;</td>
</tr>
<tr>
<td>abc</td>
<td>&lt;ID&gt;</td>
<td>&lt;ID&gt;</td>
<td>&lt;ID&gt;</td>
<td>-</td>
</tr>
<tr>
<td>end</td>
<td>&lt;ID&gt;, &lt;KW&gt;</td>
<td>&lt;ID&gt;</td>
<td>&lt;ID&gt;, &lt;KW&gt;</td>
<td>&lt;KW&gt;</td>
</tr>
<tr>
<td>xy9</td>
<td>&lt;ID&gt;</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

How can modular regular grammars now be compiled into DFAs? There are two, fundamentally different, solutions to this problem:

- Compile all rules that are labelled with the same module name into a single DFA and define a composition operation on DFAs. The DFA constructed for a certain selection of modules then consists of the composition of the DFAs constructed for each individual module in the selection.

- Compile all rules into a single DFA and define a selection operation that, given a list of selected modules, extracts the sub-automaton that corresponds to that selection.

Obviously, the first solution is the most elegant one since it leads to a truly modular implementation of lexical scanners. Unfortunately, the composition operation on deterministic automata is expensive: given two DFAs $A$ and $B$ and their composition $A \cup B$, in many cases the states of $A$ and $B$ do not appear in $A \cup B$. Instead, they are combined into new states thus reflecting the interactions between the languages recognized by $A$ and $B$. As a result, the computation of $A \cup B$ requires roughly the same amount of work as the construction of a completely new automaton. This is illustrated in Figure 3, where the DFAs for the two regular expressions $a \ b$, and $(a \mid c) \ d$ are shown together with the resulting DFA for the combined language $\{a \ b, (a \mid c) \ d\}$ or, equivalently, $(a \ b) \mid ((a \mid c) \ d)$. It should be emphasized that the complexity of the composition operation is caused by our (efficiency) requirement that the result of the composition is again a deterministic automaton. Allowing a non-deterministic automaton (NFA) as result, would significantly simplify the composition operation as is shown in Figure 4.
### Figure 3. Two DFAs and their composition as DFA.

<table>
<thead>
<tr>
<th>Regular expression(s)</th>
<th>Corresponding DFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>a b</td>
<td><img src="image1" alt="DFA Diagram 1" /></td>
</tr>
<tr>
<td>(a</td>
<td>c) d</td>
</tr>
<tr>
<td>{ a b, (a</td>
<td>c) d }</td>
</tr>
</tbody>
</table>

### Figure 4. Same DFAs and their composition as NFA.

In this paper, we now concentrate on the second solution mentioned above and investigate how a selection operation can be defined on a DFA that extracts a sub-automaton corresponding to a selection of modules.

### 2. AN ALGORITHM FOR COMPILING REGULAR EXPRESSIONS

In this section we sketch an algorithm for the lazy compilation of regular expressions into deterministic finite automata. A complete description of this method can be found in [HKR87b].

### 2.1. Preliminaries

First, we introduce the notions of regular expression and labelled regular expression.
Regular expressions over a finite alphabet $\Sigma$ are composed of the symbols from that alphabet, the empty string (\(\varepsilon\)), the operators concatenation (denoted by juxtaposition), alternation (\(\mid\)), repetition (\(*\)), and parentheses. We will adopt the convention that parentheses may be omitted under the assumption that the operators in regular expressions are left associative and that * has the highest priority, concatenation has the second highest priority and \(\mid\) has the lowest priority. We will also use the undefined regular expression (\(\bot\)) denoting the empty set of strings, i.e. \(\bot\) does not recognize anything. The following identities characterize the interactions between concatenation, \(*\), and \(\bot\):

(a) \(\bot \varepsilon = \bot \varepsilon = \bot\)
(b) \(\bot * = \varepsilon\)
(c) \(r \bot = \bot r = r\)

A labelled regular expression is a regular expression in which a unique natural number \(p\) is associated with each occurrence of a symbol \(a \in \Sigma\). We say that \(a\) occurs at position \(p\) in that symbol at position \(p\) is \(a\), notation: \(a_p\). Also define \(symbol(p) = a\) for each \(a_p\).

We will use some auxiliary functions on labelled regular expressions which describe properties of the strings recognized by them:

1. The predicate nullable determines whether a regular expression can recognize the empty string.
2. The function firstpos maps a labelled regular expression to the set of positions that can match the first symbol of an input string.
3. The function lastpos maps a labelled regular expression to the set of positions that can match the last symbol of an input string.
4. The function followpos maps a position in a labelled, regular expressions \(e\) to the set of positions that can follow it, i.e., if \(p\) is a position with \(symbol(p) = a\) and \(p\) matches the symbol \(a\) in some legal input string \(\ldots ab\ldots\), then \(b\) will be matched by some position in followpos\((p, e)\).

For precise definitions of these functions we refer the reader to [HKR87b] or [BS87].

In the sequel, we will adopt the convention that a unique symbol \(\$\in \Sigma\) is used to terminate both regular expressions and input strings. A terminated, labelled, regular expression \(e\) over an alphabet \(\Sigma\), has the form \(e\$\), where \(e\) is a labelled regular expression over \(\Sigma\\{\$\}\).

An accepting sequence of positions for a labelled regular expression \(e\) can now be defined as a sequence of positions \(p_1, \ldots, p_n\) such that \(p_1 \in firstpos(e)\), \(p_n \in lastpos(e)\), and \(p_{i+1} \in followpos(p_i, e)\), \(i = 1, \ldots, n-1\). For all strings \(s \in \Sigma^*\) and for all terminated, labelled, regular expressions \(e\) over \(\Sigma\) the following holds: \(s = a_1 \ldots a_n\) with \(a_n = \$\) belongs to the set of strings denoted by \(e\) if and only if there exists an accepting sequence of positions \(p_1, \ldots, p_n\) for \(e\) such that \(a_i = symbol(p_i)\), \(i = 1, \ldots, n\). (see [YM60], Theorem 3.1).

2.2. Algorithms for the lazy construction of a DFA

Using the notions introduced in the previous section we now formulate an algorithm for the lazy construction of a deterministic finite automaton for a given set of regular expressions. The basic idea is to construct a deterministic automaton in which each state corresponds to a set of positions in the set of regular expressions. In this way, each state may represent several ways of recognizing an input string. The initial state of the automaton consists of the first positions of all the regular expressions. Transitions from the start state, as well as from any other state, are computed as follows: consider for each symbol \(a\) in the alphabet (or the end marker) the positions that can be reached when recognizing \(a\) in the input; the
set of positions that can be reached in this way form the (perhaps already existing) state to
which a transition should be made from the original state on input $a$. The set of positions
that corresponds to a state thus characterizes the progress of all possible accepting se-
quencies for input strings with a common head.

In principle, the powerset of all positions in the set of regular expressions should be con-
sidered during the construction of a DFA. The following algorithms only consider the sets
of positions that are really used during this construction. These sets are collected in the set
$\text{States}$. When a state $S$ is added to $\text{States}$, it is unexpanded and $\text{expanded}(S) = \text{false}$. A state $S \in \text{States}$ can be marked as expanded by setting $\text{expanded}(S) := \text{true}$.

The DFA that is being constructed is represented by an initial state $\text{start} \in \text{States}$ and a
transition function $\text{Trans} : \text{States} \times \Sigma \rightarrow \text{States}$.

In standard DFA construction algorithms, a complete DFA is computed for a given regu-
lar expression. In the following lazy algorithm only a Partial DFA (PDFA) is constructed
which is further extended when needed during scanning of given input strings.

First, we give the algorithms for the lazy construction of the start state and for the ex-
pansion of a state.

Algorithm $\text{L-CONSTRUCT}$
Construction of the initial part of the DFA that accepts the language described by a set of
regular expressions.

Input. A set $E$ of terminated, labelled, regular expressions over alphabet $\Sigma$.

Output. A PDFA in which only the start state has been expanded.

Method.

\begin{align*}
A.\text{start} &:= \bigcup_{e \in E} \text{firstpos}(e) \\
A.\text{States} &:= \{ \text{A.start} \} \\
A.\text{Trans} &:= \emptyset \\
\text{return}(\text{EXPAND}(E, A, A.\text{start}))
\end{align*}

Algorithm $\text{EXPAND}$
Expansion of a PDFA state.

Input. A set of terminated, labelled, regular expressions $E$, a corresponding PDFA $A$, and a
state $S$.

Output. The original PDFA expanded with all states to which $S$ has transitions, and a de-
finite of these transitions.

Method.

\begin{algorithmic}
\For{$a \in \Sigma \setminus \$$}
\State $U := \bigcup \{ p \in S \mid \text{symbol}(p) = a \} \text{followpos}(p, E)$
\If{$U \neq \emptyset$ \And $U \subseteq A.\text{States}$}
\State $A.\text{States} := A.\text{States} \cup \{ U \}$
\State $A.\text{Trans}(S, a) := U$
\EndIf
\EndFor
\State $\text{expanded}(S) := \text{true}$
\State \text{return}(A)
\end{algorithmic}
The positions $p$, $q$, $r$ and $s$ in $S$ with $\text{symbol}(p) = \text{symbol}(q) = \text{symbol}(r) = \text{symbol}(s) = a$

The subsets $\text{followpos}(p)$, $\text{followpos}(q)$, $\text{followpos}(r)$ and $\text{followpos}(s)$ of $T$

Figure 5. Positions causing a transition between states $S$ and $T$ on symbol $a$.

For later reference it is useful to emphasize that the existence of a transition between two states $S$ and $T$ on alphabet symbol $a$ may be caused by several positions in $S$ that correspond to the symbol $a$. State $T$ will contain as subsets the, possibly overlapping, sets of follow positions for each of these positions in $S$. This situation is illustrated in Figure 5.

From the definition of EXPAND it follows that a state can never correspond to an empty set of positions. For convenience, we will assume in the sequel that all automata contain an error state with the following properties:

1. The error state corresponds to the empty set of positions.
2. The error state is not an accepting state.
3. The transition function is augmented as follows:
   (a) for each state, transitions to the error state are added for all characters in $\Sigma$ for which that state has no legal transition.
   (b) for all characters in $\Sigma$, the transition function contains a transition from the error state to itself.

These additions to the generated automata are implicit and will not be shown in the diagrams.

Finally, we give the scanning algorithm associated with $L$-CONSTRUCT. It performs expansions of needed, unexpanded, states.

**Algorithm** $L$-SCAN
Simulate a given PDFA on a given input string, incrementally expanding the PDFA when necessary.
Input. A set $E$ of terminated, labelled, regular expressions, a corresponding PDFA $A$, and an input sentence $s = a_1...a_n$, with $a_n = \$$. 
Output. true or false (indicating acceptance or rejection of the input string) and a possibly extended version of $A$.

Method.

\begin{align*}
S &:= A.\text{start} \\
i &:= 1 \\
\text{while } a_i \neq \$ &\text{ do} \\
\quad &\text{if } \neg \text{ expanded}(S) \text{ then } A := \text{EXPAND}(E, A, S) \text{ fi} \\
\quad &\text{S} := A.\text{Trans}(S, a_i) \\
\quad &i := i + 1 \\
\text{od} \\
\text{return } (\text{FINAL}(S), A)
\end{align*}

The last state reached during the scanning of an input string determines whether the input string should be accepted or rejected. A state is accepting if one of its positions corresponds to the end marker $. This is defined by the following algorithm.

Algorithm FINAL
Determine whether a given state is an accepting state.

Input. A state $S$.
Output. true or false

Method.

\begin{align*}
\text{return } \exists p \in S \ [\text{symbol}(p) = \$]
\end{align*}

Note that a state may contain several positions with symbol $. This may happen when a string is recognized by more than one rule in the regular grammar.

3. AN ALGORITHM FOR COMPILING MODULAR REGULAR GRAMMARS

3.1. Modular regular expressions versus modular regular grammars
Before generalizing these lazy scanner generation techniques to the case of modular regular grammars, we first need a definition of modular regular grammars. It would be natural to describe them as (module-name, regular expression) pairs. However, it turns out that not all sub-expressions of a regular expression need to originate from the same module. This will become clear when discussing named regular expressions in Section 3.3. Therefore, we choose a method that allows more refined control over the module information and associate module names with the positions in a (terminated, labelled) regular expression and not with the regular expression as a whole. We will write $\text{module}(p)$ to denote the module name associated with position $p$ and we will write $ma_p$ to denote a position $p$ such that $\text{symbol}(p) = a$ and $\text{module}(p) = m$. We will call these regular expressions with associated module information modular regular expressions. In this section, we will only use sets of modular regular expressions. In Section 3.3, modular regular grammars will be introduced and we will show how they can be reduced to sets of modular regular expressions.
Given a modular regular expression $e$ and a list of module names $M$ we can now restrict expression $e$ to $M$ (notation: $e/M$) by replacing all $m\alpha_p$ in $e$ with $m \notin M$ by the undefined expression $\perp$. We extend the restriction operator $/$ to sets of regular expressions.

The problem we want to solve can now be formulated as follows: given a set of modular regular expressions $E$, a partially constructed automaton $A$ for these regular expressions, and a list of module names $M$, can we select a part of $A$ that precisely recognizes the language defined by $E$ restricted to $M$?

The simplest method one can imagine to restrict the language accepted by a given DFA is to use the DFA as it is, but impose restrictions on accepting states according to the current selection of modules. This method would only require some recomputations on the accepting states of the DFA.

Consider, for instance, the set of expressions $E = \{ m_{a1} m_{b2} m_{s3}, n_{c4} n_{d5} n_{s6} \}$ and the corresponding DFA $A$ shown in Figure 6 (the border lines of accepting states are shown in bold face). Choosing the set of modules $\{m\}$, $E /\{m\}$ is then equal to $\{ m_{a1} m_{b2} m_{s3}, \perp \} = \{ m_{a1} m_{b2} m_{s3} \}$ and the automaton obtained from $A$ by only retaining accepting states that are labelled with a position in the selection $\{m\}$ correctly recognizes the language defined by $E /\{m\}$ (see Figure 7).

However, on closer inspection it turns out that this simple method may be incorrect when the positions in a single modular regular expression are labelled with different module names. This is illustrated by the following counter example. Consider

$E = \{ ( m_{a1} | n_{b2} ) n_{c3} n_{s4} \}$
with corresponding DFA shown in Figure 8. When we restrict $E$ to the single module $\{n\}$ we obtain

$$E(\{n\}) = \{ \langle \bot, n, b_2 \rangle, n, c_3, n, a_4 \} = \{ n, b_2, n, c_3, n, a_4 \}$$

but the string $ac$ will (erroneously) be accepted using the simple method of restricting the accepting states of the DFA corresponding to $E$. The reason is, of course, that it is not sufficient to require that the accepting position is in the current selection of modules, as long as it can be reached using transitions that do not belong to that selection, e.g. the transition from the start state on symbol $a$. Therefore, we should remove all such invalid transitions obtaining the DFA shown in Figure 9 (invalid transitions are represented by shaded arrows).

Following this second method, we restrict the language accepted by a given DFA by eliminating all transitions in the DFA that do not correspond to the regular expressions in the current selection. This method requires the calculation of modifications to the transition table of the DFA for each new selection of modules.

### 3.2. Restricting a PDFA to a selection of modules

Given a PDFA $A$ and a list of selected modules $M$, we have to compute those parts of the transition table that are still valid in this new selection. We introduce the following notions to achieve this goal:

1. The table containing the transitions that are valid in the current selection will be called $SetTrans$, it is always a subset of the complete (but perhaps only partially computed) transition table $Trans$ of $A$.

2. With each state $S$ in $A$ we associate an attribute $specialized$, indicating whether the valid transitions leaving $S$ have already been recorded in $SetTrans$. Initially, $specialized(S) = \text{false}$ holds.

Remains the problem of formulating criteria to decide when a transition between two states $S$ and $T$ on alphabet symbol $a$ is still valid in the current selection of modules. Our goal is to restrict the automaton $A$ in such a way that it is equivalent to an automaton $A'$ that would have been constructed when using the restricted set of regular expressions right from the start. In other words a transition should be valid in the restricted automaton $A$ when it would have been constructed in $A'$ as well. Looking at the way expansion of states is de-
fined (see Section 2.2, algorithm EXPAND) we observe that the existence of a transition between $S$ and $T$ on symbol $a$ implies that

1. $S$ contains a position $p$ whose symbol is $a$;
2. $T$ contains some position $q$ in the set of follow positions of $p$.

In the restricted automaton one should impose the additional restriction that positions $p$ and $q$ are labelled with module names appearing in the current selection.

Referring to Figure 5 given in Section 2.2, we illustrate the situation in the modular case in Figure 10. Positions that are labelled with a module in the current selection are indicated by a black square. A transition between $S$ and $T$ on symbol $a$ is valid in this particular selection, since position $q$ is selected and followpos($q$) contains a selected position as well. In this case, $q$ is the only position that supports this transition! Note that states $S$ and $T$ correspond to states $S'$ and $T'$ in automaton $A'$ that contain only these selected positions.

These ideas are described more precisely in the following algorithm.

**Algorithm SPECIALIZE**

**Input.** A set $E$ of terminated labelled regular expressions, a PDFA $A$, a state $S$, and a list of modules $M$.

**Output.** A modified version of $A$ in which all valid transitions from state $S$ in the modules in $M$ have been recorded in $A$.SelTrans.

**Method.**

```
for $\forall a \in \Sigma \{\$\}$
do
if $\exists p \in S, q \in A.Trans(S,a)$
[symbol($p$) = $a$ \& module($p$) $\in M$ \& module($q$) $\in M$ \& $q$ $\in$ followpos($p,E$)]
then $A$.SelTrans($S,a$) $:= A$.Trans($S,a$) fi
specialized($S$) $:= \text{true}$
od
return($A$)
```

Remains to be described how specialization and expansion of states interact during scanning. Before actual scanning starts, all states are set to unspecialized$^1$. During scanning states are encountered that are either not yet expanded (and should be both expanded and specialized) or expanded but not yet specialized (and should be specialized). The algorithm is as follows:

**Algorithm M-SCAN**

Simulate a given PDFA for a given selection of modules on a given input string, incrementally expanding and specializing the PDFA when necessary.

**Input.** A set $E$ of terminated, labelled, regular expressions, a corresponding PDFA $A$, a list of modules $M$, and an input sentence $s = a_1...a_n$, with $a_n = \$$. 

**Output.** $\text{true}$ or $\text{false}$ (indicating acceptance or rejection of the input string) and a possibly extended version of $A$.

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$^1$Instead of setting all states to unspecialized at the beginning of M-SCAN, it would be more efficient to do this once in a separate selection operation. Subsequent applications of M-SCAN could then profit from the gradually increasing number of already specialized states.
Method.
for ∀ State ∈ A.States do specialized(State) := false od;
S := A.start
i := 1
while a_i ≠ $ do
    if ¬ expanded(S) then specialized(S) := false ; A := EXPAND(E, A, S) fi;
    if ¬ specialized(S) then A := SPECIALIZE(E, A, S, M) fi;
    S := A.SelectTrans(S, a_i)
    i := i + 1
    return (M-FINAL(S, M), A)
do

In the modular case, a state is accepting if one of its positions corresponds to the end marker $ and that position is part of the current selection. This is defined by the following algorithm.

Algorithm M-FINAL
Determine whether a given state is an accepting state in a given selection of modules.
Input. A state S and a list of modules M.
Output. true or false
Method.
return ∃ p ∈ S [symbol(p) = $ ∧ module(p) ∈ M]

The positions p, q, r and s in S with symbol(p) = symbol(q) = symbol(r) = symbol(s) = a
The subsets followpos(p), followpos(q), followpos(r) and followpos(s) of T

Figure 10. Positions causing a transition between states S and T on symbol a in the modular case.
The specialization of PDFA $A$ for selection $M$ (denoted by $A/M$) described by $SelTrans$ has three interesting properties:

- For each string in the language defined by the restricted set of regular expressions $E/M$, the accepting sequence of positions in $A/M$ is identical to the accepting sequence as it would occur in the new automaton $A'$ that is constructed independently for the restricted set of regular expressions $E/M$.

- The above mentioned automaton $A'$ does not need to occur as sub-automaton of $A/M$, since two distinct states $S$ and $T$ in $A/M$ may become effectively equivalent due to specialization (i.e., after specialization $S$ and $T$ contain the same subset of positions that are labelled with a module name in the current selection and will thus behave identically; they correspond to a single state in automaton $A'$), but they will remain distinct states in $A/M$.

- It is difficult to assess the complexity of the operations $SPECIALIZE$ and $EXPAND$. The latter constructs unions of sets (of positions) and has to perform a complex membership test to determine whether a newly constructed state already exists, while the former does no set construction at all but only performs pair-wise comparisons of set elements. Assuming that set construction is the most expensive operation, we conjecture that $SPECIALIZE$ is cheaper than $EXPAND$.

### 3.3. Modular regular grammars

In the previous section we have discussed lexical definitions that have the form of a list of modular regular expressions. Each position occurring in these expressions is labelled with both a position name and a name of a module. Now we turn our attention to the complete modular regular grammars as sketched in Section 1. Such a grammar consists of two parts: abbreviations and rules. Both abbreviations and rules contain triples of the form

$\text{module-name: } \text{token-name = regular-expression}.$

Names appearing in a regular expression should always have been defined by a previous abbreviation or rule and can always be eliminated by textual substitution. When several regular expressions $e_i$ are associated with one name, we associate with that name a regular expression containing all expressions $e_i$ as alternatives. We will now show how a modular regular grammar of this form can be reduced to a set of modular regular expressions as defined in Section 3.1. We proceed in four stages:

1. Associate module names and positions with all alphabet symbols in the modular regular grammar.

2. Replace all uses of names in regular expressions by their definition.

3. Terminate all resulting expressions in the rules section with a $\$ symbol and associate both the module name preceding the expression and a new position with that $\$ symbol. When this terminator appears in a state (= set of positions) it will uniquely identify this rule. This fact can be used to determine the token-name to be associated with the recognized input string.

4. The set of modular regular expressions obtained in step 3 is the reduced form of the original modular regular grammar.

Only step 2 is non-trivial and requires some further comments, since what will happen when a named expression that is not selected is used in a expression that is selected? Intuitively, one would like to replace the use of the named expression by $\perp$. It turns out that this can be achieved by an appropriate definition of textual substitution.
Let $e$ be the regular expression associated with some name in a modular, regular grammar $E$. Define $e' = \text{copy}(e)$ as the labelled regular expression obtained by taking a literal copy of $e$, with the exception that each symbol $a_p$ appearing in $e$ is replaced by $a_{p'}$, where $p'$ is a new, unique, position label and we define $\text{module}(p') = \text{module}(p)$. It is important to note that the module name associated with each position remains the same.

Using this definition of taking a copy of a regular expression, all named regular expressions can be removed from a grammar by replacing each occurrence of a name by a copy of its associated expression. The positions occurring in the resulting, expanded, regular expression may now be labelled with different module names (this possibility was already mentioned in Section 3.1).

We conclude this section by applying all the techniques described so far to the modular regular grammar given as example in Figure 1 (see Section 1). In Figure 11, the complete DFA corresponding to this grammar is show for the selection of modules $\{ M_1, M_3, M_4, M_5, M_6, M_7 \}$. 
in other words the modules M₂ (that includes the digits 8 and 9 in the definition of <DIGIT> and M₈ (the keyword end) are not selected. The following conventions have been used in this figure:

- Invalid transitions are (as before) indicated by shaded arrows.
- Potentially accepting states are labelled with the name of the accepting token.
- The abbreviation ≠ c stands for all letters a–z, except the letter c.

Note that all transitions labelled with 8–9 are invalid and that the state that could potentially recognize end both as a keyword and as an identifier can only recognize it as an identifier in this particular selection.

4. CONCLUDING REMARKS

In this paper we have presented the algorithms required for the generation of lexical scanners for modular regular grammars. A prototype implementation of these algorithms has been completed and indicates that the method of selecting a sub-automaton from the large automaton corresponding to all regular expressions in all modules is superior over the method of constructing a new automaton for each selection of modules. It is too early to present a quantitative analysis of the performance of this implementation.

As indicated in Section 1, the modularization of language definitions implies that all parts of such a definition have to be processed in a modular fashion. In [Rek89], a technique is sketched for the generation of parsers for modular context-free grammars. It turns out that the techniques for lazy and incremental program generation as described earlier in [HKR87a], form a good foundation for modular program generation techniques.

ACKNOWLEDGEMENTS

Jan Rekers made several comments on a draft of this paper. The technique presented here was inspired by our discussions on modular parser generation. All these methods are extensions of the lazy/incremental program generation techniques developed in cooperation with Jan Heering and Jan Rekers.

REFERENCES


