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# Strong Viscous-Inviscid Interaction and the Effects of Streamline Curvature[1]

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Asymptotic theory has led to the development of the quasi-simultaneous method for treating viscous-inviscid interaction. In the present paper we summarize this historical relation. Furthermore it is demonstrated how effects of streamline curvature can be included in this framework.

## 1. Lessons from asymptotic theory

*The triple deck* Asymptotic theory for large Reynolds numbers, in particular the triple-deck theory as developed by Stewartson and Messiter around 1970 (see e.g. Stewartson [6]), has laid the foundation for one of the more succesfull viscous-inviscid interaction methods: the quasi-simultaneous method. Triple-deck theory describes the structure of the flow field near singular points like an airfoil trailing edge or a point of separation. The message of this theory is three-fold:

- Near a singular point a smaller length scale in streamwise direction exists. This scale has to be reflected in a finer distribution of grid points in a numerical solution method.
- The importance of the various terms in the equations of motion is indicated, with result that in first approximation the classical shear-layer equations are sufficient to describe the flow in the vicinity of the singular point.

[1] The main part of the research has been performed in the period 1977-1990 while the author was employed at the National Aerospace Laboratory NLR (Amsterdam, The Netherlands). The contribution of Monique Somers to the research described in this paper is especially acknowledged.

– The interaction between the shear layer and the outer inviscid flow can be described by thin-airfoil theory. Furthermore there is no hierarchy between the viscous shear layer and the outer inviscid flow: this is called strong interaction. The lack of hierarchy has to be reflected in the iteration process for solving the flow equations.

*Quasi-simultaneous interaction* The quasi-simultaneous method (VELDMAN ET. AL. [8], [10]) has been designed based on these three messages, suggesting the choice of the mathematical model and the strategy for the numerical method. We will describe it here in terms of the viscous pressure distribution $p_e$ and the displacement thickness $\delta^*$. Let the external flow be described by $p_e = E[\delta^*]$ (where $E$ denotes e.g. a transonic full-potential equation), and let the shear layer be described by $p_e = B[\delta^*]$ (where $B$ represents the shear-layer equations). The lack of hierarchy between both flow regions suggests a simultaneous treatment of the viscous and inviscid flow equations.

However, when both flow regions are described by a differential (field) approach, a simultaneous solution of both equations leads to relatively complex software. Therefore, in the quasi-simultaneous method, a simple approximation of the inviscid-flow equations is introduced, called the interaction law, which describes the relevant part of the interaction: $p_e = I[\delta^*]$. The triple-deck theory suggests to base the interaction law on thin-airfoil theory

$$I[\delta^*] = -\frac{1}{\pi}\int_0^\infty \frac{d\delta^*}{d\xi}\frac{d\xi}{x-\xi}. \tag{1}$$

In the viscous-inviscid iterations the interaction law is solved simultaneously with the shear-layer equations; only the difference between the interaction law and the 'exact' inviscid flow is treated iteratively

$$\begin{cases} p_e^{(n+1)} - I[\delta^{*(n+1)}] &= E[\delta^{*(n)}] - I[\delta^{*(n)}] \\ p_e^{(n+1)} - B[\delta^{*(n+1)}] &= 0. \end{cases} \tag{2}$$

*Numerical analysis* The discretization of the thin-airfoil integral (1) (exclusive the minus sign) leads to a positive-definite matrix $H$. E.g. the discretization as presented in VELDMAN [9] yields the following expression on a uniform grid with mesh size $h$

$$\begin{aligned} I[\delta^*]|_i &\approx -(H\delta^*)|_i \\ &\equiv -\frac{1}{\pi}\sum_{j\neq i-1,i}\left\{\frac{1}{h}(\delta^*_{j+1}-\delta^*_j)\ln\left|\frac{i-j}{i-j-1}\right|\right\} + \frac{2}{\pi h}(\delta^*_{i+1}-2\delta^*_i+\delta^*_{i-1}). \end{aligned}$$

We recognize in the latter term the discretization of the second-order derivative $d^2\delta^*/dx^2$, which describes the local contribution from the two intervals adjacent to the $i$-th grid point. It easily follows that $H$ is symmetric and diagonally dominant with diagonal entry $4/\pi h$. The discrete form of (2), setting $E = I$ for convenience, now reads

$$p_e + H\delta^* = R_1; \quad p_e - B\delta^* = R_2,$$

where $R_1$ contains contributions to the integral (1) from the end points of the computational domain. $B$ now stands for the Jacobian of the shear-layer equations (it is lower triangular in attached flow), and in $R_2$ we can hide effects from their nonlinearity. After elimination of $p_e$ we are left with

$$(H + B)\delta^* = R_1 - R_2. \tag{3}$$

Experience has learned that the diagonal of $B$ is positive in regions of attached flow, but it vanishes in a point of separation after which it is slightly negative. This phenomenon is responsible for the break-down of the classical approach for solving the shear-layer equations. But when the interaction law is added to the formulation, the matrix $H + B$ becomes relevant. With $H$ being diagonally dominant there is some room for adding negative contributions of $B$ without the matrix becoming singular. Practice shows that a simple Gauss-Seidel procedure (i.e. a number of traditional boundary-layer sweeps) suffices to solve the system (3). When the separated flow regions grow in size this approach fails (the negative contribution from $B$ becomes too large), but in these situations the flow is unlikely to be steady and a time-derivative should have been added to the equations (making them better conditioned again). For a more detailed discussion of the numerics involved we refer to VELDMAN [9].

## 2. INCLUSION OF STREAMLINE CURVATURE

*Modelling* Especially for rear-loaded airfoils the streamlines immediately behind the trailing edge are highly curved. In such a situation the assumption of constant pressure across the shear layer is no longer justifiable. In asymptotic terms it means that higher-order effects, in regions even smaller than the triple-deck, are becoming relevant. Figure 1 shows a typical pressure profile illustrating the differences between the pressure of the real viscous flow (RVF) and the extrapolated pressure of the external (equivalent) inviscid flow (EIF). The effect of streamline curvature can be modelled by a jump between the pressure in the shear layer $p_e$ and the pressure of the equivalent inviscid flow $p_{EIF} \equiv E[\delta^*]$

$$p_e = p_{EIF} - [p] \quad \text{with} \quad [p] = \kappa \rho_e u_e^2 (\delta^* + \theta), \tag{4}$$

where $\kappa$ represents the streamline curvature, whereas $\theta$ is the momentum thickness (LOCK and WILLIAMS [4]). The extended version of (2) thus <u>could</u> become

$$\left\{ \begin{array}{rcl} p_e^{(n+1)} - I[\delta^{*(n+1)}] & = & E[\delta^{*(n)}] - [p]^{(n)} - I[\delta^{*(n)}], \\ p_e^{(n+1)} - B[\delta^{*(n+1)}] & = & 0. \end{array} \right. \tag{5}$$

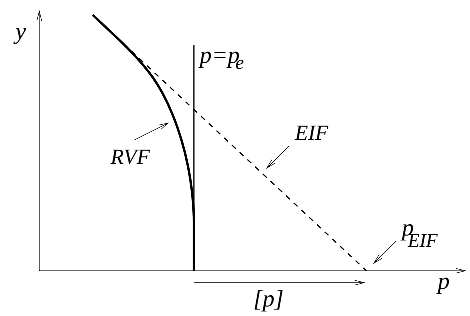


FIGURE 1. Pressure profiles across shear layer.

*Numerical analysis* The pressure jump (4) is proportional to the curvature of the streamlines $\kappa \approx d^2\delta^*/dx^2$. The fact that it implies a second-order derivative is responsible for its strong-interaction character. In numerical terms: when the pressure jump is evaluated from a previous iteration, as indicated in (5), through $\kappa$ it generates a contribution to the iterative amplification factor proportional to $h^{-2}$, which is very difficult to neutralize. This explains the numerical difficulties encountered when this effect is not treated in a simultaneous way. Extensive smoothing is then required in order to obtain convergence of the viscous-inviscid iterations (LOCK and WILLIAMS [4, page 82]).

In a simultaneous treatment of the curvature effect the term $[p]^{(n)}$ is shifted to the left-hand side in (5) and evaluated at the new iteration level $(n+1)$. Let us abbreviate (4) as $[p] = a\, d^2\delta^*/dx^2$ (with $a > 0$). Then, again after setting $E = I$, elimination of $p_e$ from (5) leads to (compare (3))

$$(H+B)\delta^* + a\frac{d^2\delta^*}{dx^2} = R_1 - R_2. \tag{6}$$

Since the entries of $H$ are of order $h^{-1}$ only, it is clear why the curvature term (being $h^{-2}$) has to be applied at the new iteration level. But there is more. The eigenvalues of $H+B$ lie in the positive half-plane, and adding a central discretization of $d^2\delta^*/dx^2$ (with a negative(!) diagonal) would completely destroy its favorable properties. Therefore the curvature term has to be discretized in a skew manner

$$\frac{d^2\delta^*}{dx^2} = \frac{\delta_i^* - 2\delta_{i-1}^* + \delta_{i-2}^*}{h^2}.$$

In this way the diagonal of $H+B$ is strengthened, although the lower triangular part receives some 'non-innocent' additions. Nevertheless, it appears that under conditions with modest amounts of separation, the system again can be solved by Gauss-Seidel sweeps without any need for additional smoothing. Also this skew differencing fits to the parabolic nature of the shear-layer equations. An at-length theoretical discussion is presented in a forthcoming publication by SOMERS and VELDMAN [5].

## 3. Results

We will demonstrate the performance of the quasi-simultaneous coupling concept on some realistic calculations of transonic flow past an RAE 2822 airfoil (flow cases 1 and 6; see Figure 2). In these airfoil-flow problems the integral (1) has been used to describe the symmetric displacement effects ('thickness problem'). Its skew-symmetric counterpart has been used to describe the effects of camber ('lift problem'); for details see Veldman et al. [10].

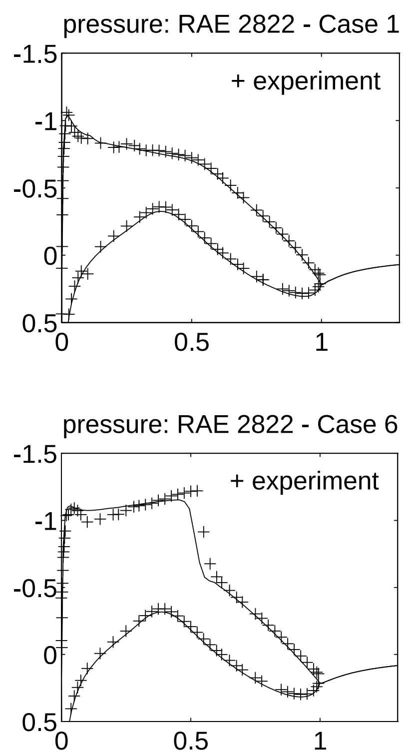


Figure 2. Pressure distributions for RAE 2822 airfoil.

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|---|---|---|---|---|---|
| Case 1: | $M = 0.676$, | $Re = 5.7$ mill., | trip = 11%, | $\alpha = 1.93$, | $C_L = 0.581$; |
| Case 6: | $M = 0.725$, | $Re = 6.5$ mill., | trip = 3%, | $\alpha = 2.44$, | $C_L = 0.743$. |

The rate of convergence of the viscous-inviscid iterations is hardly influenced by the inclusion of streamline curvature effects: a handful of quasi-simultaneous iterations suffices to obtain the aerodynamic coefficients; in 10–15 iterations machine-zero (single precision) is reached (Figure 3). The required computational effort for this kind of viscous-inviscid calculations is very modest: one minute per flow case on a personal computer. To appreciate the fast convergence even better, one has to realize that the external flow in these examples is transonic, with a significant supersonic flow region in Case 6. The interaction law (1) however is based on subsonic theory.

As expected, the inclusion of streamline curvature has a lift reducing effect, although for this particular airfoil the effect is not dramatic. More results on calculations with and without inclusion of curvature effects can be found in SOMERS and VELDMAN [5].

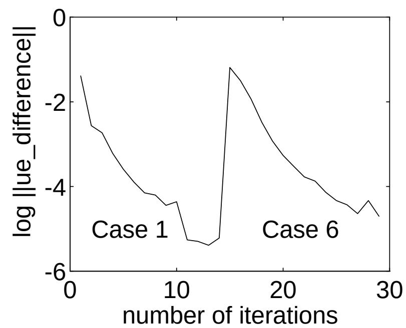


FIGURE 3. Convergence of viscous-inviscid iterations.

## 4. REMARKS

The interaction law used here is based on thin-airfoil theory, but it can be simplified even further to

$$I[\delta^*] = \frac{2h}{\pi}\frac{d^2\delta^*}{dx^2}, \tag{7}$$

after which the first equation in (2) simply reads

$$p_e^{(n+1)} - \frac{2h}{\pi}\left(\frac{d^2\delta^*}{dx^2}\right)^{(n+1)} = p_{EIF}^{(n)} - \frac{2h}{\pi}\left(\frac{d^2\delta^*}{dx^2}\right)^{(n)}. \tag{8}$$

Of course, since the description (7) contains less physics than (1) the convergence rate deteriorates (COENEN [2]). But (8) is easily implementable in existing boundary-layer codes, and it prevents the blow-up that occurs when just the pressure is prescribed.

In three dimensions the quasi-simultaneous concept is equally feasible, and in fact has been applied already (with the simplified interaction law (7)) in the MatricsV program, which simulates 3D viscous transonic flow around airplane wings (e.g. VAN DER WEES [11]).

Also the concept can be applied in a more general domain-decomposition setting (DE BOER and VELDMAN [1]). There is a close relation with the class of local coupling methods introduced by TAN [7] which make use of combinations of function values and normal derivatives in the grid points $i-1$, $i$ and $i+1$. Tan has optimized the coefficients in these formulas based on mathematical arguments. It appears that his optimum for ellipticity dominated problems is very close to the interactive coupling condition (8) which is based on physical arguments.

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