

## Everything about Kolmogorov was Unusual... <sup>1</sup>

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Andrei Nikolaevich Kolmogorov was one of those rare men who gave you the feeling of witnessing something unusual, something great, of genius, the feeling of witnessing a miracle.

Everything about Kolmogorov was unusual—his whole destiny, his pioneering discoveries in many branches of mathematics, and not just mathematics—meteorology, hydrodynamics, history, linguistics, pedagogy,... Exceptional was the range of his life-long interests—music, architecture, poetry, travelling,... Exceptional, too, his erudition; he seemed to have opinions and knowledge about wellnigh everything.

His slight stammering was out of the ordinary, and so was his energetic pace.

Unusual feelings were aroused by contact with Kolmogorov, even by the simplest talk with him. All the time, you felt the powerful activity that was going on in your partner's brain. He had an exceptionally wide circle of pupils. The letters Kolmogorov wrote to his friends and pupils were extraordinary, real models of epistolary art, full of unexpected ideas and turns of thought.

His childhood was unusual—he grew up without his mother, who died giving birth to him. Being 5-6 years old, he announced his first mathematical discovery, that  $1 = 1^2$ ,  $1 + 3 = 2^2$ ,  $1 + 3 + 5 = 3^2$ ,... He used to confront his peers with various mathematical and logical problems. For instance, in how many ways can one fix a button with four holes? (They were exhibited in the family wall newspaper *Spring Swallows*, which was edited by Andrei Kolmogorov's aunts who took care of his education and founded a small school for him and his peers.) As to the standard problem about the meeting point of two travellers starting from points  $A$  and  $B$  with different velocities, he used to recall that it was 'logically uninteresting' to him.

His classmates at grammar school pointed out that Kolmogorov was a concentrated boy, always thinking about something, but who did not refuse his part in some pranks in the senior classes. To the merriment of his classmates and the annoyance of his young physics teacher, he invented a 'Perpetuum Mobile'. He came up to the blackboard and described his usually very cleverly conceived 'apparatus', the flaws of which were not easy to detect. His teacher would try to raise objections, but her arguments were easily refuted...

Kolmogorov the schoolboy had a wide range of interests. He seriously occupied himself with biology. Later, he wrote that 'the first great impression of the force

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and significance of a scientific investigation I got from K.A. Timiryazev's book *Botany*'. When he was fourteen years old, he studied higher mathematics from Brockhaus' and Efron's encyclopedia, reconstructing the proofs by himself. He got carried away by chess, frequented chess clubs for a while, but quickly lost interest. He took an interest in history and sociology. In those years, he dreamt about a just society, and he wrote a utopian constitution for his 'island state'. He dreamt about devoting himself to forestry.

At grammar school, he decided to take his final examinations as an external. Armed with knowledge, he came to school, and declared that he was ready for the examination. The teachers told him to come back after lunch. He went for a stroll, then came back, and had lunch. And then the teachers' reunion awarded him the final certificate without any examination. He recalled that this was one of the greatest disappointments in his life.

When he went to study at Moscow University, he started to work almost at once (in order to supplement his modest scholarship) as a teacher at an ordinary school, where he taught mathematics and physics, and took an active part in the pupil's lives—he was the secretary of the school committee, something of which he remained very proud all his life, more than of the highest awards.

When he was a graduate student at university (he was a student of N.N. Luzin) he had to take 14 examinations in various mathematical disciplines. But it was possible to replace the examinations with an original result in the corresponding subject. Andrei Nikolaevich didn't take a single exam, but wrote 14 papers with new results on various subjects instead. 'One of the results', he recalled, 'turned out to be false, but I noticed only when the paper had already been accepted'.

Kolmogorov had extraordinary things to say about scientific creativity in general and about his own in particular. He said that he never was able to concentrate fully on one mathematical problem for more than two weeks. He said that any single discovery could be expounded in a four-page paper in the *Doklady*,—because 'the human mind is not able to create anything very complex at one go'.

Andrei Nikolaevich maintained a lively interest in the subject of his work only as long as it was unclear along what lines the problem was to be solved. As soon as the situation clarified, he tried to get rid of the task of working out the proofs, and began to look for someone to whom the whole subject could be handed over.

According to Kolmogorov, three stages can be distinguished in the development of every scientific subject. First—the pioneering stage, the break-through into a new field, the brilliant and unexpected discovery, often upsetting established views. Then, the technical stage ensues—laborious and time-consuming. At this stage the theory is overgrown with details, becomes unwieldy and less accessible, but covers a growing number of applications. Finally, at the third stage, a new, more general outlook on the problem and its relation to other, possibly distant problems appears, which makes possible a breakthrough into new fields of investigation.

It is characteristic of Kolmogorov's mathematical works that he was a pioneer and first discoverer in many fields, that he sometimes solved problems that had been standing for 200 years. Andrei Nikolaevich tried to avoid the technical work associated with the generalization of the constructed theory. At the third stage,

however, when it is necessary to reflect on the obtained results and to discern new directions, at the stage of developing fundamental generalizing theories, Andrei Nikolaevich's achievements are remarkable. An example of such an unexpected breakthrough is furnished by Kolmogorov's topological works, published as four small notes in the *Comptes Rendus* (1935).

In 1953, at Kolmogorov's 50th birthday, P.S. Aleksandrov said: 'I have known Andrei Nikolaevich longer than anybody else—from 1923. At that time, he worked under A.K. Vlasov's supervision on projective geometry, and attended the course of P.S. Uryson. On an autumn day he came to me and brought his work on descriptive set theory with him'. P.S. Aleksandrov went on to point out that both in that classical work by Kolmogorov, and in his work on diverging Fourier series, the following characteristics of Kolmogorov the mathematician were manifest: 1) the unusual simplicity and generality of the ideas; 2) the striking force of his technique.

A.Ya. Khinchin, who spoke after Aleksandrov, began as follows: 'Andrei Nikolaevich and I entered the theory of probability simultaneously, and by the same door. Neither of us intended to work on probability theory, but we noticed that probability theory and the metric theory of sets and functions had much in common. Andrei Nikolaevich noticed the unsatisfactory state of the theory of probability. Kolmogorov's book *Basic concepts of probability theory* did for the theory of probability what D. Hilbert's book did for geometry at the beginning of the century'.

A.Ya. Khinchin further points out that Kolmogorov combined a gift for extremely abstract investigations with a feeling for applied problems. 'The most important, and most enchanting side of Andrei Nikolaevich as a mathematician', Khinchin said, 'is the wealth of his ideas. A dissertation can arise from every phrase that he utters about a work. Many works in which Andrei Nikolaevich isn't mentioned as an author are strongly influenced by him'. 'I wrote', Khinchin continued, 'the book *Asymptotic laws of probability theory*, where I.G. Petrovsky's methods are expounded. But Petrovsky and I of course know very well that the real inspiration for all this came from Andrei Nikolaevich'.

In his speech, which followed Khinchin's, I.M. Gelfand mentioned among other things that 'the fact that mathematics is still felt to be a single science is due in large part to Kolmogorov'.

In his closing speech, P.S. Aleksandrov said: 'A.N. Kolmogorov belongs to those mathematicians, whose every work in every field causes a complete reappraisal of established values. It is difficult to find another mathematician in the past few decennia, not just of his breadth, but who also had such a strong influence on mathematical taste and the development of mathematics... Hardy considered him a specialist on trigonometric series, von Kármán considered him a specialist on mechanics, Gödel once said that the secret of human giftedness lies in a prolonged adolescence. Among other things, adolescence is characterized by the capability to feel excitement. The excitement in his attitude to mathematics is at the basis of Kolmogorov's talent... the excitement that transpires in Andrei Nikolaevich's supreme achievements as well as in his articles for the Great Soviet Encyclopedia and in his elaboration of the graduate program. That is one side. The other side is his self-effacing hard work'.

Andrei Nikolaevich's numerous students more than once remarked that it was an incomparable happiness to be his pupil. But they all always added that it also imposed a great responsibility. One felt a constant desire to give one's best in order to be able, at the next meeting with Andrei Nikolaevich, to report progress in the solution of the problem posed by him. One of my friends, when asked about his attitude to his teacher, answered, 'a panic respect'.

Another diverting anecdote that relates to this: once, the spouse of a famous professor of mathematics at Moscow University, who was reknowned for her high intelligence, got very excited upon learning that Kolmogorov was coming for dinner that night. She began to tidy up feverishly, personally prepared several dishes, and wouldn't trust the maid to do anything. Seeing her in such a state, her maid asked: 'my god, who is coming for dinner that you should make such a fuss?'. 'I don't know how to explain', was the answer. 'Just imagine that the *Czar* is coming to visit you tonight!'

Andrei Nikolaevich himself enjoyed good humor. He told the following tale about Hadamard. (I know this anecdote from V.I. Arnold.)

Hadamard was a passionate collector of ferns. When he came to Moscow, Andrei Nikolaevich and Pavel Sergeevich Aleksandrov took him for a boat trip. Suddenly Hadamard spotted something on the bank, and demanded that they pull in at once. He climbed to the prow of the little boat, and when they approached the bank, he got so excited that he fell into the water. It turned out that he had spotted a fern of a rare species that he had been looking for years. Hadamard was perfectly happy. But shortly afterwards, he was to be received by the president of the Academy of Sciences. Consequently, they had to put Aleksandrov's suit on him. But it showed very much (Hadamard was much taller). At the reception, everybody asked Hadamard: 'professor, what happened to you? It is rumored that you fell into the water'. Upon which Hadamard playfully retorted: 'and what makes you suppose there are no other types of adventures a French professor of mathematics could have?'

As is well known, in 1963 Kolmogorov obtained the international Bolzano Prize. The prize *For Peace and Humanism* was awarded in the same year to pope John XXIII. After the ceremony, Kolmogorov had a conversation (in French) with the pope, on the subject of the 'Universal Urge for Peace on Earth'.

Kolmogorov was a great connoisseur of the history of the Vatican and the Roman popes. From him I learned that John XXIII was the 261st pope, and that the current pope John Paul II is the 264th. He told me about his observation that fat popes alternated with slim ones during the 19th century. That one of these was a mountaineer, well-known for his photographs of mountain landscapes. That pope John Paul II (on a visit to the Vatican, when he still was the Polish cardinal Wojtyla), who was a ski enthusiast, inquired about the percentage of the Roman cardinals that were skiers. And when it turned out to be zero, he said that 40% of the Polish cardinals were skiers. When asked how this was possible, as there were only two cardinals in Poland, Wojtyla jokingly answered that in Poland, cardinal Wysinski counts as 60%.

Another episode. Once, on the beach of the Black Sea, my ten-year old son told a young woman that she was lying. Andrei Nikolaevich, hearing this, turned towards him and said, "You shouldn't have said this to a representative of the

beautiful sex. You should have said—‘It appears to me that you slightly departed from the truth’ ”.

Everything was unusual about Kolmogorov, who was one of those incomparable geniuses who enrich life by the mere fact of their existence.