Nonlinear Multigrid applied to a
1D Steady Transistor Problem

P.M. de Zeeuw

CWI, Centre for Mathematics and Computer Science
P.O. Box 4079, 1009 AB Amsterdam, The Netherlands

1. Introduction
In this paper we restrict ourselves on purpose to one space dimension as a preparatory study for the case or more space dimensions. We study a particular example problem which has been put forward by Dr. W.H.A. Schilders, Philips, The Netherlands. This problem models a transistor and turns out to be a lot harder to solve than the forward or reversed biased diode problem. We apply nonlinear multigrid and encounter a serious difficulty due to the nonlinearity of the problem. The difficulty is analysed and this provides insight equally well for the case of more space dimensions. Some modifications are proposed which significantly increase the robustness of the nonlinear multigrid method and which look promising also for the more-dimensional case.

2. The problem
The behaviour of a steady semiconductor device can be described by the following set of equations (cf. e.g. [5]):

\[-\nabla J_\psi = \frac{n_g}{\alpha} (\exp(\alpha(\phi_n - \psi)) - \exp(\alpha(\psi - \phi_p))) + qD, \tag{2.1a}\]
\[-\nabla J_n = +qR, \tag{2.1b}\]
\[-\nabla J_p = -qR, \tag{2.1c}\]

where \(J_\psi\) is defined by

\[J_\psi = e\nabla \psi, \tag{2.2a}\]

and

\[J_n = \frac{n}{\alpha} \exp(\alpha(\psi - \phi_n)) \nabla (\alpha \phi_n), \tag{2.2b}\]
\[J_p = \frac{n}{\alpha} \exp(\alpha(\phi_p - \psi)) \nabla (\alpha \phi_p), \tag{2.2c}\]

with

\[\frac{n_i \alpha}{\alpha}, \frac{n_i \alpha}{\alpha} = \frac{n_i \alpha}{\alpha}.
\]

Substitution of (2.2) into (2.1) results in a system of three nonlinear partial differential equations for the variables

\[\psi, \phi_n\text{ and }\phi_p,\]

In (2.1) \(\psi\) represents the electrostatic potential, \(\phi_n\) and \(\phi_p\) are the hole and electron quasi-Fermi potentials. On the one hand, by choosing these variables, the problem is strongly nonlinear, on the other hand the values assumed by \((\psi, \phi_n, \phi_p)\) are in a moderate range. The quasi-Fermi potentials satisfy the well-known relations:

\[n = n_i \exp(\alpha(\psi - \phi_n)), \tag{2.3a}\]
\[ p = n_e \exp(\alpha(\phi_p - \Psi)) \]  
(2.3b)

where \( p \) and \( n \) describe the concentration of holes and electrons respectively. Equations (2.1b) and (2.1c) are called the continuity equations; \( \mathbf{J}_n \) is the electron current density, \( \mathbf{J}_p \) is the hole current density, \( R \) is the recombination-generation rate, a function of \( n \) and \( p \). The doping profile \( D \) is a function of the space variable \( x \). The quantities \( \varepsilon, q, \alpha, \mu_n, \mu_p \) represent the permittivity, the elementary charge, the inverse of the thermal voltage and the electron and hole mobility respectively. In this paper we consider the case of only one space dimension and assume \( \varepsilon, \alpha, \mu_n \) and \( \mu_p \) to be constant.

2.1 A particular 1D model problem.

We will focus our attention to a particular (hard) 1D model problem which has been supplied by Dr. W.H.A. Schilders (cf.[6,7]). Here the problem constants are

\[ \varepsilon = 1.035918 \times 10^{-12}, \quad q = 1.602176 \times 10^{-19}, \quad \mu_n = \mu_p = 500, \quad n_i = 1.22 \times 10^{-10}, \]

\[ \alpha = q/\kappa T, \quad \kappa = 1.38054 \times 10^{-23}, \quad T = 300. \]  
(2.4)

The function \( R \) is given by

\[ R = \frac{pn - n_i^2}{\tau(p + n + 2n_i)}, \quad \tau = 10^{-6}. \]

In figure 2 a graph of the doping function \( D(x) \) is shown after the transformation \( D \rightarrow \text{sign}(D) \times 10 \log(1 + |D|) \).

![Graph of doping profile](image)

**Figure 1.** The doping profile.

The problem is defined on the domain \( \Omega = [0, \frac{4}{10}] \). We have three contacts to our semiconductor device (the 1D-model of a transistor):

the emitter \( E \) at \( x = 0 \),

- 54 -
the basis B at $x = 1.9 \times 10^{-4}$,
the collector C at $x = 8 \times 10^{-4}$.

Boundary conditions at the emitter are:
$p - n + D = 0$ (i.e. vanishing space charge),
$\phi_n = V_E$,
$J_p = 0$.

Boundary conditions at the basis:
$\phi_p = V_B = 0$.

Boundary conditions at the collector:
$p - n + D = 0$,
$\phi_n = V_C$,
$\phi_p = V_C$.

For fifteen different cases, each characterized by a pair of voltages $(V_E, V_C)$, the solution is required.

<table>
<thead>
<tr>
<th>case</th>
<th>$V_E$</th>
<th>$V_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>1</td>
<td>0.</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>0.</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>0.</td>
<td>1.</td>
</tr>
<tr>
<td>6</td>
<td>-0.2</td>
<td>1.</td>
</tr>
<tr>
<td>7</td>
<td>-0.4</td>
<td>1.</td>
</tr>
<tr>
<td>8</td>
<td>-0.6</td>
<td>1.</td>
</tr>
<tr>
<td>9</td>
<td>-0.7</td>
<td>1.</td>
</tr>
<tr>
<td>10</td>
<td>-0.8</td>
<td>1.</td>
</tr>
<tr>
<td>11</td>
<td>-0.85</td>
<td>1.</td>
</tr>
<tr>
<td>12</td>
<td>-0.9</td>
<td>1.</td>
</tr>
<tr>
<td>13</td>
<td>-0.95</td>
<td>1.</td>
</tr>
<tr>
<td>14</td>
<td>-1.</td>
<td>1.</td>
</tr>
</tbody>
</table>

Table 2.1. Subsequent voltages at the emitter and collector for which a solution is required.

3. The discretization
The interval $\Omega = (0, 8 \times 10^{-4})$ is split up into disjoint boxes $B_j = (x_{j-1}, x_j), j = 1 (1) N$. A point $x_j$ is called a wall, a point $x_{j-1/2} = (x_{j-1} + x_j)/2$ is called a center. Another set of subintervals $\{D_j\}$ is defined by
$D_0 = (x_0, x_{1/2})$,
$D_j = (x_{j-1/2}, x_{j+1/2})$,
$D_N = (x_{N-1/2}, x_N)$.

This set is called the set of dual boxes. The following conditions should be satisfied:
(i) $x_{j-1} < x_j$,
(ii) $x_0 = E, x_N = C$,
(iii) $\exists j^* \text{ with } 0 < j^* < N$ such that $x_{j^*} = B$, i.e. the basis B is at the partition-wall between two boxes.
By using a finite volume technique we arrive at a cell-centered version of the well-known Scharfetter-Gummel scheme (cf. e.g. [4,7]). Firstly we apply the Gauss divergence theorem in one dimension to the equations (2.1) on the domains $B_j$. Secondly we use the assumption that $J_{\psi j}, J_{n j},$ and $J_{p j}$ are piecewise constant on the dual set $\{D_j\}$ (cf. e.g. [1]); therefore we introduce the notation

$J_{\psi j}, J_{n j},$ and $J_{p j}$.

At the basis B special measures are taken, for full details cf.[7]. Further we introduce the variables

$(\psi_j, \phi_{n_j}, \phi_{p_j})^T, j = 1 \ldots N$,

which are associated with the centers of the boxes $B_j$. In this way we arrive at the following set of discrete equations:

\begin{align}
-J_{\psi j} + J_{\psi j-1} - S_j &= F_j, j = 1 \ldots N, \\
-J_{n j} + J_{n j-1} - R_j &= 0, j = 1 \ldots N, \\
-J_{p j} + J_{p j-1} + R_j &= 0, j = 1 \ldots N, j \neq j^*, j \neq j^* + 1.
\end{align}

(3.1a)

(3.1b)

(3.1c)

The piecewise constant functions are derived by (cf.[4])

\begin{align}
J_{\psi j} &= e^{\psi_{j+1/2} - \psi_j} \\
J_{n j} &= \bar{\mu}_n \frac{\exp(-\alpha \phi_{n,j+1}) - \exp(-\alpha \phi_{n,j})}{\exp(-\alpha \psi_{1/2}) - \exp(-\alpha \psi_j)} \\
J_{p j} &= \bar{\mu}_p \frac{\exp(\alpha \phi_{p,j+1}) - \exp(\alpha \phi_{p,j})}{\exp(\alpha \psi_{1/2}) - \exp(\alpha \psi_j)}
\end{align}

(3.2a)

(3.2b)

(3.2c)

The other functions are defined by:

\begin{align}
F_j &= q \int_{B_j} D \, d\Omega, \\
S_j &= n_q \int_{B_j} \left( \exp(\alpha (\phi_p - \psi)) - \exp(\alpha (\psi - \phi_n)) \right) d\Omega, \\
R_j &= q \int_{B_j} R \, d\Omega
\end{align}

(3.3a)

(3.3b)

(3.3c)

where $\approx$ denotes approximation by midpoint quadrature. For full details on the discretization of the boundary conditions cf.[7]. In this way we have now discretized the equations (2.1) into a set of $3N$ nonlinear equations (3.1). We can write (3.1) in symbolic form as

$M_h(q_h) = f_h,$

(3.4)

where $M_h$ denotes the nonlinear difference operator corresponding with the lefthandside of (3.1) and where $f_h$ corresponds with the righthandside of (3.1) and $q_h$ with the variables $(\psi_j, \phi_{n_j}, \phi_{p_j})^T$.

3.1. Properties of the Jacobian

In this subsection we study how the Jacobian of $M_h$ depends on the discrete solution. We confine ourselves to the dependency on $\phi_p$, results for $\phi_n$ can be derived analogously.

First we assume the recombination term to be zero. We freeze the solution components $\psi$ and $\phi_n$ and consider the $\phi_p$-stencil, at box $B_j$, defined by the triplet
\[ [s(t_p(j-1), s(t_p(j), s(t_p(j+1))] \]

with

\[ s(t_p(j,k)) = \frac{\partial(-J_{p,j} + J_{p,j-1})}{\partial \phi_p(j+k)}, \quad k = -1, 0, 1. \]  

(3.5a)

(3.5b)

Because of

\[ s(t_p(j-1)) < 0, \quad s(t_p(j)) > 0, \quad s(t_p(j+1)) < 0 \]

(3.6)

the stencil corresponds with an \( l \)-matrix.

In ref.[7] the following observation is made upon this stencil:

\[ s(t_p(j,0)) = -(s(t_p(j-1) + s(t_p(j+1))) + \alpha(-J_{p,j} + J_{p,j-1}). \]

(3.7)

It follows from (3.7) that at the discrete solution, i.e. when

\[ -J_{p,j} + J_{p,j-1} = 0 \]

is satisfied, the following equality holds:

\[ s(t_p(j,0)) = -(s(t_p(j-1) + s(t_p(j+1))) \]

so then the \( l \)-matrix possesses also weak diagonal dominance. However, in the middle of some iterative process to determine the solution, we may well have negative residuals so that the equality (3.7) implies the loss of diagonal dominance. Therefore ill-conditioning and numerical difficulties can be expected. (If the recombination term is not neglected then \( s(t_p(j,0) \) is enlarged with some positive value.) This is one explanation for the difficulties encountered when applying the Newton method.

4. The Newton Method and Expedients

Applying the Newton method directly to (3.4) involves large storage requirements and the solution of large linear systems (it would extremely so in two or more space dimensions). Therefore it should be applied only for relatively coarse grids. In fact we will use it as a coarsest grid solver for multigrid methods which will be treated in the next section. We use some additional tools to enhance the global convergence of the Newton method:

1) correction transformation cf.[5],
2) continuation (with boundary voltages as parameter) cf. e.g. [6],
3) smoothing of the Newton-iterates by means of Collective Symmetric Gauss Seidel relaxation (CSGS) cf.[4].

5. The Multigrid Method

Advanced ways of solving a set of nonlinear equations are the Full Approximation Scheme (FAS), cf.[2] and the Nonlinear Multigrid Method (NMGM), cf.[3]. Both multigrid methods are very similar although the NMGM is more general. Recently, in the field of semiconductor equations research on multigrid methods has been initiated. If well applied, a multigrid method can be optimal in the sense that the rate of convergence is independent of the meshsize. An important advantage of the FAS/NMGM-method is that no large linear systems need to be stored and solved. The subsequent stages of an usual FAS-method, applied to (3.4), are:

1) apply \( p \) nonlinear relaxation sweeps; thus we get an approximation \( q_h \) of the solution which has a smooth residual \( d_h \equiv f_h - M_h(q_h) \).

2a) transfer \( d_h \) from \( \Omega_h \) to a coarser grid \( \Omega_H \) by means of the restriction operator \( R_H \), and choose a representation \( q^{old}_h \) on the coarse grid of \( q_h \) on the fine grid,
2b) solve (approximately) on $\Omega_H$ the equation
\[ M_H(q_H^{\text{new}}) = M_H(q_H^{\text{old}}) + R_H d_h, \] (5.1)

2c) prolongate the correction, computed on $\Omega_H$, onto $\Omega_h$ and add to $q_h$:
\[ q_h^{\text{new}} = q_h + (P_h q_H^{\text{new}} - P_h q_H^{\text{old}}), \] (5.2)

3) apply $q$ nonlinear relaxation sweeps.

Stage 2 is called the coarse grid correction (CGC). Stage 2b may be obtained by applying a number of $\sigma$ FAS-cycles on the coarser grid. In this way a recursive procedure is obtained in which a sequence of increasingly coarser grids is used. In this paper we use $p = q = \sigma = 1$ throughout. As relaxation we use CSGS. Let the coarsest grid $\Omega_H$, a discretization of $\Omega$, be given by the set of boxes $\{B_H\}$. From $\Omega_H$ we construct the next finer grid $\Omega_h$ by division of each box $B_H_j$ into two disjoint boxes $B_{h,2j-1}$ and $B_{h,2j}$. By repetition we obtain thus a sequence of increasingly finer grids. For the numerical experiments in this paper we assume in addition that $B_{h,2j-1}$ and $B_{h,2j}$ have equal size.

The restriction operator $R_H$ for right hand side functions is defined by adding two adjacent values:
\[ (R_H q_h)_{j} = f_{h,2j-1} + f_{h,2j} \] (5.3)

As a prolongation operator $P_h$ which transfers a solution from a coarse grid to the next finer one we use a prolongation introduced by Hemker cf.[4] which is based on the assumption of smoothness of fluxes and which satisfies the Galerkin condition.

Various possibilities exist for choosing the approximation $q_H^{\text{old}}$, e.g. the application of fullweighting to $q_h$. Another possibility is to take $q_H^{\text{old}}$ equal to $q_H^{\text{new}}$ obtained from the previous CGC. In practice, the solution efficiency of nonlinear problems was never shown to be much influenced by either choice of $q_H^{\text{old}}$, in our case however it is (see Section 6).

6. Adaptation of the coarse grid correction
Hemker successfully applied boxcentered multigrid FAS iteration to the forward and the reversed biased diode problem (cf.[4]). However, application of the same algorithm to the transistorproblem in this paper gives rise to a complication in the CGC due to drastically varying problem coefficients. This complication and possible remedies are the topics of this section.

6.1 Improper solution transfer
The first attempt of applying multigrid to our specific problem was done by employing FAS, embedded within the Full Multigrid Method (FUGM), with only two grids. The coarse grid problem within the CGC of FAS, was to be solved up to machine-accuracy by means of the Newton method with expedients (see Section 4). For several cases of the testproblem it turned out that the twogrid-algorithm gets stuck precisely at this stage. This is remarkable because the Newton method (with expedients) on his own is successful even for very coarse grids. Apparently the right hand side of the equation (5.1) in stage 2b of the CGC, may be outside a proper range of $M_H$. Whenever such a difficulty
occurred one or more of the three solution components depicted a steep gradient and indeed, in a way, this is causing the trouble.

For an explanation, again consider the $\phi_{p}$-stencil (see section 3.1). Let the operator $\Delta_{j}$ denote the variation of a grid function over the centers of two adjacent boxes $B_{j}$ and $B_{j-1}$. In [7] it is shown that if both $|\Delta_{j-1}\psi|$ and $|\Delta_{j}\psi|$ are sufficiently small then

$$st_{p}(j,0) = \alpha \hat{\mu}_{p} \exp(\alpha(\phi_{p,j-1}\psi)) \cdot (\frac{1}{\Delta_{j-1}x} + \frac{1}{\Delta_{j}x}),$$

and if both $|\Delta_{j-1}\psi|$ and $|\Delta_{j}\psi|$ are sufficiently large then

$$st_{p}(j,0) = \alpha \hat{\mu}_{p} \exp(\alpha(\phi_{p,j-1}\psi)) \cdot \frac{\Delta_{j-1}\psi}{\Delta_{j-1}x} \text{ if } \Delta_{j-1}\psi \geq 0,$$

$$\cdot \frac{\Delta_{j}\psi}{\Delta_{j}x} \text{ if } \Delta_{j}\psi \leq 0.$$

These approximations show that the stencil is extremely sensitive to the difference $(\phi_{p,j-1}\psi)$. Hence the $\phi_{p}$-stencil on the coarse grid is extremely sensitive to how $\phi_{p,j}$ and $\psi_{j}$ on the coarse grid are determined from their counterparts on the fine grid. Because of a steep gradient it may well occur that the problem coefficients, i.e. the entries of the Jacobian of $M_{h}$, show a quite different order of magnitude on two adjacent boxes. Therefore, on the coarse grid, the problem coefficients do heavily depend on the particular coarse grid representation $q_{H}$ of $q_{h}$. When this particular coarse grid representation $q_{H}$ generates small problem coefficients, the righthand side of equation (5.1) may easily become out of the range of $M_{H}$ (note that $R_{H}d_{p}$ does not depend on the particular choice of $q_{H}$). We conclude that at locations where such a phenomenon occurs we cannot expect to be able to construct a coarse grid operator $M_{H}$ which is a fair representation of the fine grid operator $M_{h}$. For a more detailed discussion cf.[7].

6.2 Possible remedies
A radical remedy to meet the above sketched difficulty is to prevent the variation $\Delta_{j}(\alpha(\phi_{p}\cdot\psi))$ over two adjacent boxes from getting large, i.e. to introduce local refinement of the mesh by equidistributing the variation. However, we want to be able to apply coarse grids in our multigrid method. Therefore we resort to another remedy.

Let $L$ and $R$ be the centers of two adjacent boxes on the fine grid $\Omega_{h}$ which together constitute a box with center $M$ on the coarse grid $\Omega_{H}$ (see figure 2).

![Nested boxes](image)

**Figure 2.** Nested boxes.
The centers of the $\phi_p$-stencils at $L$, $R$ and $M$ are the coefficients $a^h_L$, $a^h_R$ and $a^H_M$ respectively. Let $d_h(L)$ and $d_h(R)$ be the residuals at $L$, $R$ (e.g. for the third equation only). At $M$ the difference between $q_H^{new}$ and $q_H^{old}$ has the order of magnitude $d_h(M)/d^H_M$ with
\[
d_H(M) = d_h(L) + d_h(R),
\]
see equation (5.1). Because of a steep gradient in the solution it may occur that
\[
d^H_M << \max \{a^h_L,a^h_R\},
\]
which implies that the coarse grid correction prolonged to the fine grid becomes far too large and the solution $q_h$ gets spoiled. A way to prevent this situation is to multiply the restricted residual $d_H(M)$ with
\[
\theta^H_M = \frac{d^H_M}{\max \{a^h_L,a^h_R\}}, \quad 0 < \theta^H_M \leq 1.
\]
(6.2)
For a smooth part of the solution this fraction will be near to 1, for a rapidly varying part of the solution it will be near to 0 so that the old solution $q_h$ will be preserved (see (5.2)).

Note that $\theta^H_M$ is a different number for each box on the coarse grid. Note also that for increasingly fine grids all numbers $\theta^H_M$ become 1. With this local suppression of the restricted residual our FAS-algorithm has been modified. It has been applied at all coarser grids of the nonlinear multigridalgorithm for the continuity-equations. A technical but noteworthy detail is that in stead of $\theta^H_M$, the number
\[
\theta^H_M' = \min \{2\theta^H_M , 1\},
\]
(6.3)
has been used as suppression-factor in the numerical experiments.
For more details cf.[7]. Numerical results are shown in the next section.

7. Numerical results
In this section we investigate the performance of our nonlinear multigridalgorithm. We focus our attention on the effects of local suppression of the restricted residual and the particular choice of the coarse grid solution $q_H^{old}$. In table 7.1 the performance of the nonlinear multigrid algorithm is measured by the average number of sweeps necessary to obtain a reduction factor $10^{-1}$ of the maximumnorm of the residual. In the heading of the table the following abbreviation are used:

case : this refers to table 2.1,
no $\theta$ : no suppression-operator for the restricted residual has been applied,
$\theta$ yes : the suppression-operator for the restricted residual has been applied,
$q_H^{old}$ etc. : defines how $q_H^{old}$ in (5.1) has been chosen.
In the event of divergence we write *div.

\[
q_H^{old} : q_H^{new}
\]

of

previous CGC

\[
q_H^{old} : \text{by}
\]

fullweighting on \(q_h\)

<table>
<thead>
<tr>
<th>case</th>
<th>no (\theta)</th>
<th>(\theta) yes</th>
<th>no (\theta)</th>
<th>(\theta) yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.66</td>
<td>0.66</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>1</td>
<td>0.88</td>
<td>0.73</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>*div</td>
<td>0.85</td>
<td>*div</td>
<td>1.15</td>
</tr>
<tr>
<td>3</td>
<td>*div</td>
<td>1.03</td>
<td>*div</td>
<td>*div</td>
</tr>
<tr>
<td>4</td>
<td>*div</td>
<td>1.06</td>
<td>*div</td>
<td>*div</td>
</tr>
<tr>
<td>5</td>
<td>*div</td>
<td>0.89</td>
<td>*div</td>
<td>*div</td>
</tr>
<tr>
<td>6</td>
<td>*div</td>
<td>0.89</td>
<td>*div</td>
<td>*div</td>
</tr>
<tr>
<td>7</td>
<td>*div</td>
<td>0.89</td>
<td>*div</td>
<td>1.38</td>
</tr>
<tr>
<td>8</td>
<td>0.89</td>
<td>0.89</td>
<td>1.40</td>
<td>1.37</td>
</tr>
<tr>
<td>9</td>
<td>0.87</td>
<td>0.90</td>
<td>1.54</td>
<td>1.37</td>
</tr>
<tr>
<td>10</td>
<td>0.88</td>
<td>0.82</td>
<td>2.59</td>
<td>2.52</td>
</tr>
<tr>
<td>11</td>
<td>1.25</td>
<td>1.25</td>
<td>1.22</td>
<td>1.22</td>
</tr>
<tr>
<td>12</td>
<td>2.01</td>
<td>2.01</td>
<td>1.75</td>
<td>1.74</td>
</tr>
<tr>
<td>13</td>
<td>2.19</td>
<td>2.19</td>
<td>4.66</td>
<td>4.66</td>
</tr>
<tr>
<td>14</td>
<td>1.76</td>
<td>1.77</td>
<td>2.44</td>
<td>2.43</td>
</tr>
</tbody>
</table>

**Table 7.1.** Performance of FAS, average number of sweeps necessary to obtain a reduction factor \(10^{-1}\) of the maximum norm of the residual; 3 grids: \(N = 16, 32, 64\) respectively.

Table 7.1 shows that the use of the \(\theta\)-operator is not sufficient on its own to guarantee convergence, apparently the full weighting approximation of the fine grid solution on the coarse grid may be a poor one (case 3 - 6). The use of the \(\theta\)-operator combined with a proper choice of the coarse grid solution gives convergence for all cases. The use of the \(\theta\)-operator does not slow down convergence in the cases where it is not needed.

For two typical cases, case = 4 and case = 12, we investigate the grid-dependence of the multigrid convergence in detail. In figure 3 we show the 10-logarithm of the scaled residual norms after subsequent FAS-sweeps, starting from the result by fullmultigrid with one FAS-sweep at each level. The coarsest grid always contains 16 boxes, for the finest grid we take \(32, 64, 128\) and \(256\) boxes respectively. Thus we apply multigrid with 2, 3, 4 and 5 grids respectively. The local suppression of the restricted residuals (\(\theta\)-operator) has been applied at all coarser grids.
a. case = 4.

b. case=12.

FIGURE 3. Multigrid convergence histories; the coarsest grid numbers 16 boxes.

We observe that the multigrid convergence becomes grid-independent for increasingly finer grids.

8. Conclusions
At first, when applying the nonlinear multigrid to our 1D-transistorproblem we encounter several cases with severe divergence. The difficulty is caused by discrepancies among the discretizations of the nonlinear differential operator on the subsequent grids, due to the rapidly varying problem coefficients. This difficulty is met by adaptation of the coarse grid correction. A proper choice of the coarse grid solution is of importance too, the full weighting approximation is not satisfactory. With the improvements as proposed we obtain fast multigrid convergence with a convergence rate that is independent of the meshsize.

References
Wolfgang Joppich (Hrsg.)

Device Simulation
2. Workshop in der GMD

März 1990

GESellschaft fÜr matematik
und datenverarbeitung MBH