# Modelling of Conflicts and Bounds Estimation in Production Systems Thanks to Dioid Theory

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# Content of the presentation

# Next

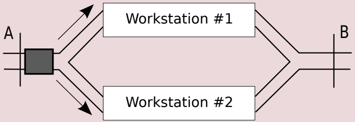
- **Context**
- Modelling
- Carrier Routing
- Resource Sharing
- Conclusion and Future Work

# Issue to Tackle

- Performance evaluation and control of production systems involving conflicts.
- Two kinds of conflict to study:
  - Routing (as in a rail road switch);
  - Resource sharing.
- Using dioid theory, which was adapted for systems without conflicts.

# First Conflict Example: Routing

## Routing between two workstations



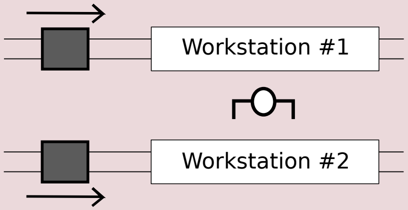

## Goal

Modelling the input/output behaviour of the global system.

- The workstations realise different operations.
- Raw material arrive at point A. Processed products are collected at point B.
- Because of the routing phenomenon, the order of the processed products can be different from the one of the incoming pieces of raw material.
- Our approach: find two behaviours, one slower and the other one faster than the one of the actual system.
  - Performance behaviour, in an approximated way.

# Second Conflict Example: Resource Sharing

## Operator working on two workstations




- Pseudo-periodic assignment policy, which is dependent of the entries.
- For a given supply, it is possible to give an exact model of the system. But we want one which is independent of the entries.
- Our approach: work on the assignment policy to find minimum and maximum waiting times before assignment of the resource.

# Next

# About Dioids

## Definition (Baccelli et al., 1992)

A dioid is a semiring $(\mathcal{D}, \oplus, \otimes)$, of which $\oplus$ law (called sum) is idempotent $(\forall a, a \oplus a = a)$.

## Canonical order in a dioid

$\forall a, b \in \mathcal{D}, a \preccurlyeq b \iff a \oplus b = b.$

## Examples: dioids $\overline{\mathbb{Z}}_{max}$ and $\overline{\mathbb{Z}}_{min}$

| $\overline{\mathbb{Z}}_{max}$ | $\overline{\mathbb{Z}}_{min}$ |
|---|---|
| $(\mathbb{Z} \cup \{+\infty, -\infty\}, \max, +)$ | $(\mathbb{Z} \cup \{+\infty, -\infty\}, \min, +)$ |
| $3 \oplus 4 = 4 \quad (\max(3,4) = 4)$ | $3 \oplus 4 = 3 \quad (\min(3,4) = 3)$ |
| $3 \otimes 4 = 7 \quad (3 + 4 = 7)$ | $3 \otimes 4 = 7 \quad (3 + 4 = 7)$ |
| $3 \preccurlyeq 4$ | $4 \preccurlyeq 3$ |

# Dioid theory

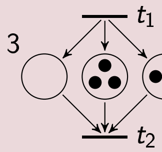
- Algebraic context useful for discrete event systems (DES's) with synchronisations and no conflicts (Cuninghame-Green, 1979; Baccelli et al., 1992).
  - Focus on delays, because of transportations, operating times and resets of workstation equipment.
- A set of intervals endowed with adequate operations can also be a dioid. This property has already been used for the study of systems with uncertain parameters (Litvinov and Sobolevskiĭ, 2001; Lhommeau, 2003).

# Graphical Representation

## Timed event graphs (TEG)

Petri net so that each place has exactly one incoming and one outgoing arc.
Delays are attached to places.
(Murata, 1989)

## A TEG example




- Possibility to handle bounded delays, using intervals.

## Specificity

The behaviour of a TEG is represented in a linear way in dioids $\overline{\mathbb{Z}}_{min}$ or $\overline{\mathbb{Z}}_{max}$, provided the focus on the counting or on the dating of events.

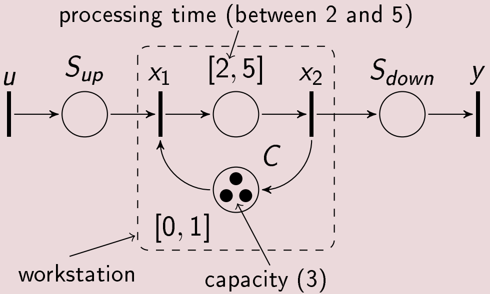
# Linearity in Dioids



- The superposition principle holds also in $\overline{\mathbb{Z}}_{min}$ and $\overline{\mathbb{Z}}_{max}$. Therefore, the outputs of a system is a convolution between its entries and its impulse response.
  - $$y(t) = \bigoplus_{i=0}^{t} H(i) \otimes u(t-i) = (H * u)(t)$$
  - The impulse response of a production system is its outputs provided an infinite stock of raw material available from the very beginning of the observation.

# TEG with Time Uncertainties

## Flexible manufacturing workstation




- 3 products can be processed at a time (the operation taking between 2 and 5 units of time depending of the product).
- After each operation, possible wait before the workstation is available (e.g. tool change).

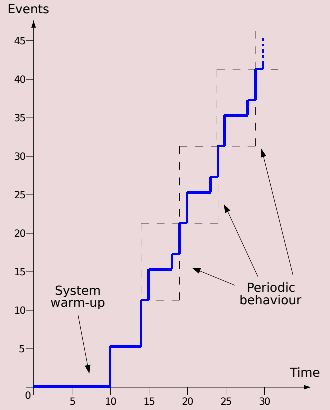
# Dioid $\overline{\mathbb{Z}}_{min}$ : Counting of Events

## Counter function

Adds up the number of events that occurred up to a given date.

- In our application cases: the total number of pallets detected by a sensor at a given date.

## Graphical representation of counter functions



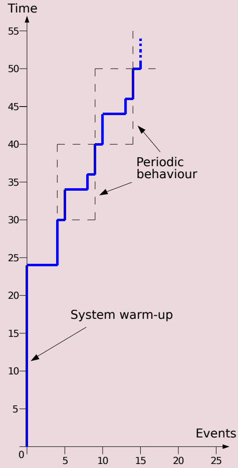
# Dioid $\overline{\mathbb{Z}}_{max}$ : Dating of Events

## Dater function

Yields the date of the occurrences of a given event.

- In our application cases: the date of each pallet detection by a sensor.

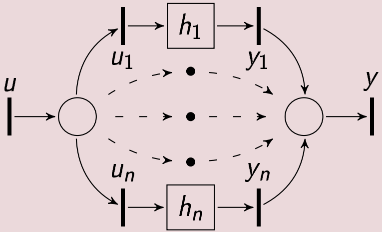
## Graphical representation of dater functions



# Next

# Conflict upon the entries

## Parallel production lines




- The entry $u$ is routed to either one of subsystems $u_i$ and output $y$ collects all outputs $y_i$. The whole system is denoted $(h_1|h_2|\cdots|h_n)$.
- Products can overlap. Hence the loss of linear input/output behaviour.
- Both lower and upper bounds of the system will be characterised, based on the routing policies.

# State of the Art of Routing in Petri Nets

- Stochastic Petri nets (Baccelli et al., 1991; Baccelli et al., 1992).
  - Models are not linear.
- Free choice Petri nets (Baccelli et al., 1996).
  - "Pseudo-linar" models.
- Usual Petri Nets (Libeaut, 1996).
  - Sets of equations and inequations that do not guarantee the unicity of solutions.
- Continuous Petri nets (Cohen et al., 1998).

## Our approach

Getting a linear model, though approximate, in $\overline{\mathbb{Z}}_{min}$ algebraic context.

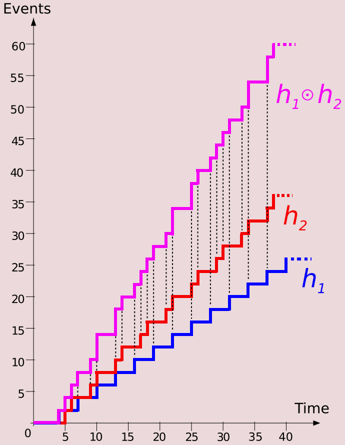
# Need for Specific Operators

## Hadamard product

Adding functions, corresponds in $\overline{\mathbb{Z}}_{min}$ to a point-to-point product, denoted $\odot$. Let $f$ and $g$ be two counter functions in $\overline{\mathbb{Z}}_{min}$:

$$\forall t, (f \odot g)(t) = f(t) \otimes g(t) = f(t) + g(t).$$

## Hadamard product example



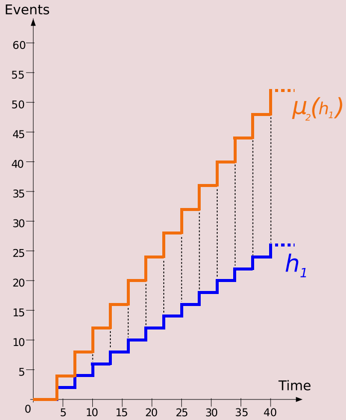
# Need for Specific Operators (cont.)

## Scaling function

Scaling function, denoted $\mu_n$, multiplies a counter function by an integer $n \in \mathbb{N}$.

- The scale of the graph is changed consequently.

## Scaling function example



# Need for Specific Operators (cont.)

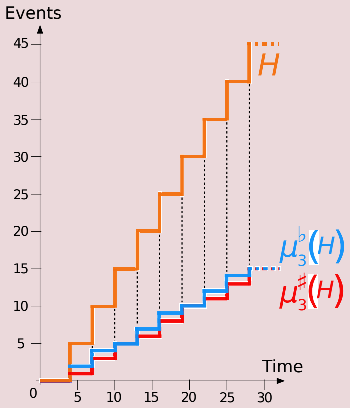
## Pseudo inverses

$\mu_m^\flat(h) \triangleq$ smallest $x$ such that $\mu_m(x) \preccurlyeq h$.

$\mu_m^\sharp(h) \triangleq$ greatest $x$ such that $\mu_m(x) \succcurlyeq h$.

- Integer division, either rounded up or down.

## Other scaling example



# Periodic Routing Between Several Sub-Systems

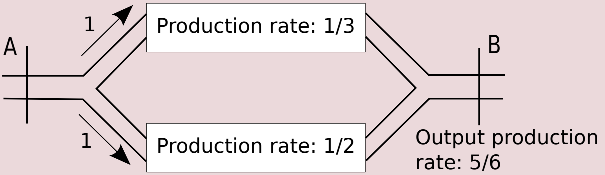
## Periodic routing function

Let system $h$ be composed of 2 sub-systems $h_1$ and $h_2$ having conflict upon the entries of raw material. $m$ incoming pieces of raw material are first routed to $h_1$, then $n$ of these pieces are routed to $h_2$, afterwards $m$ of them are routed to $h_1$ and so forth in a cyclic fashion.

- This routing function upstream $h_1$ and $h_2$ is denoted $r = m|n$.

# Best Possible Behaviour

## Routing example



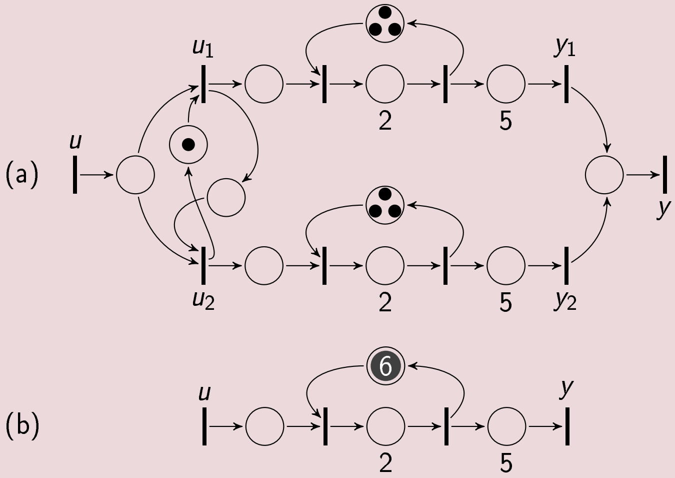

- The global system cannot provide more processed products than what can produce its different sub-systems.
- Here: $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$.
- This production rate is not dependent of the routing, when an arbitrary high quantity of raw material is available as the system starts.

# Best Possible Behaviour (cont.)

- The best possible behaviour is the sum of its internal sub-systems.
  - In dioid $\overline{\mathbb{Z}}_{min}$: the Hadamard product of their impulse responses.
- This establishes a lower bound of all possible behaviours.

# Case of Identical Sub-Systems

## Equivalence between $(h|h)_{1|1}$ (a) and $\mu_2(h)$ (b)



# Case of Identical Sub-Systems (cont.)

**Case of a system $(h|h|\cdots|h)_{1|1|\cdots|1}$**

The impulse response is exact and equal to $\mu_n(h)$.

- This is common sense: adding identical resources amounts to multiplying the production rate of one resource by the number of resources.

# Influences of the Routing in the Case of Different Sub-Systems

- Routing has a influence when sub-systems are different, because products can overlap. There are two possible configurations:
  - Balanced routing $r = 1|1|\cdots|1$ ;
  - Batch routing.
- In the two cases, the worst possible behaviour is obtained when considering that all sub-systems are equivalent to the slowest one.

**Case $(h_1|h_2|\cdots|h_n)_{1|1|\cdots|1}$**

$\mu_n(\bigoplus_{i=0}^{n} h_i)$.

**Case $(h_1|h_2)_{m|n}$**

$\mu_{m+n}(\mu_m^\flat(h_1) \oplus \mu_n^\flat(h_2))$.

# Summary (Boutin et al., 2009a; Boutin et al., 2009b)

| Kind of sub-system | Routing policy | Input/output behaviour | |
|---|---|---|---|
| $n$ identical sub-systems $(h \mid h \mid \ldots \mid h)$ | Balanced routing $1 \mid 1 \mid \ldots \mid 1$ | $\bigodot_{i=0}^{n} h = \mu_n(h)$ | |
| | | Infimum | Upper bound |
| $n$ different sub-systems $(h_1 \mid h_2 \mid \ldots \mid h_n)$ | Balanced routing $1 \mid 1 \mid \ldots \mid 1$ | $\bigodot_{i=0}^{n} h_i$ | $\mu_n(\bigoplus_{i=0}^{n} h_i)$ |
| 2 different sub-systems $(h_1 \mid h_2)$ | Batch routing $n \mid m$ | | $\mu_{m+n}(\mu^\flat_m(h_1) \oplus \mu^\flat_n(h_2))$ |

# Optimal Periodic Routing

- In the case of batch routing between 2 different sub-systems, is it possible to find optimal parameters $m$ and $n$ such that the production rate of the upper bound is the one of the global system.

**Characterisation**

When the two bounds of the interval of behaviours have the same production rate, the size of this interval is minimal and the production rate is the best.

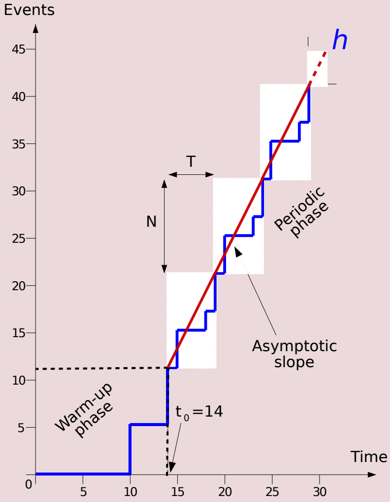
# Asymptotic Slope

## Definition

Let $h$ be a counter function such that $\forall t > t_0, h(t) = N \otimes h(t - T)$. The asymptotic slope of $h$ is denoted $\sigma(h) = \frac{N}{T}$.

- In a production engineering context, this is the actual production rate of the system.

## Graphical Representation



# Asymptotic Slope (cont.)

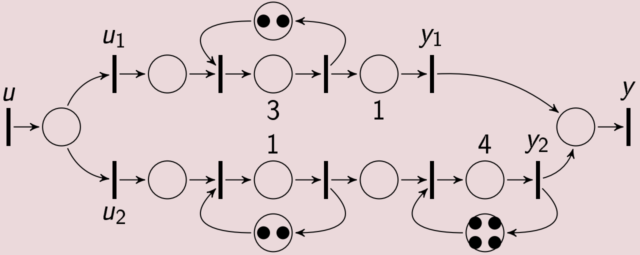


- TEG's always have a periodical behaviour, after a possible warm-up phase.
  - Routing functions between two linear parallel systems has an influence over the overall production rate of the global system.
  - Choosing $m|n$ such that $\frac{m}{n} = \frac{\sigma(h_1)}{\sigma(h_2)}$, we get

$$\sigma(h_1 \odot h_2) = \sigma\Big(\mu_{m+n}\big(\mu^{\flat}_{m}(h_1) \oplus \mu^{\flat}_{n}(h_2)\big)\Big)$$
$$= \sigma(h_1) + \sigma(h_2)$$

# Two Parallel Lines in a Flexible Shop

## Flexible shop

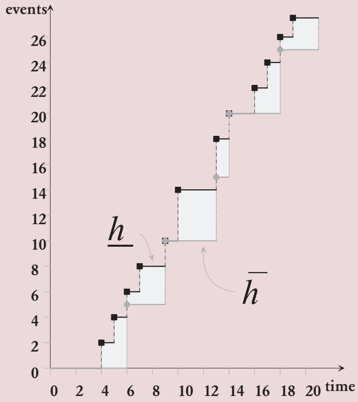



- The two production rates are $\sigma(h_1) = 2/3$ and $\sigma(h_2) = 1$. So $\frac{2/3}{1} = 2/3$, which implies $m = 2$ et $n = 3$.
  - Routing function $r = 2|3$ guarantees a global production rate of $\sigma\big((h_1|h_2)_{2|3}\big) = \sigma(h_1) + \sigma(h_2) = 5/3$.
  - This is the best possible production rate.

# Graphical Representation

- For any entry $u$, the output of system $(h_1|h_2)_{2|3}(u)$ is included in interval $[\underline{h} * u, \overline{h} * u]$.
- The white areas correspond to the uncertainties due to the routing.

## Impulse response of the two bounds of the interval



# Next

- Context
- Modelling
- Carrier Routing
- **Resource Sharing**
- Conclusion and Future Work

# Conflicts Over Resources

## First example

An operator can be working on more than one workstation at the same time.

- Problem to solve when pieces of raw material arrive on both workstations.
- An assignment policy is available, but the incoming of the products is unpredictable.

# State of the Art of Resource Sharing in Petri Nets

- Sharing only one resource (Al Saba et al., 2006a).
- Static assignment policy of the resource (Trouillet et al., 2007; Al Saba et al., 2006b).
- Set of equations and inequations (Libeaut, 1996; Corréïa et al., 2009).

## Our approach

Virtual splitting of the two production lines, duplicating the resource and limiting the production by adding uncertain delays.

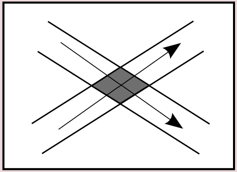
# Conflict Zones

A common section needs a predictive or in-line scheduling for parts to through.
A merging junction will be seen as a resource managed by a mutual exclusion policy.

Two possible behaviour for a pallet:

- It can go through without waiting. (best case)
- It must wait for another pallet coming from the other branch. (worst case)

**Intersection**



Context

Modelling

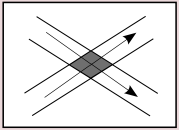
Carrier Routing

**Resource Sharing**

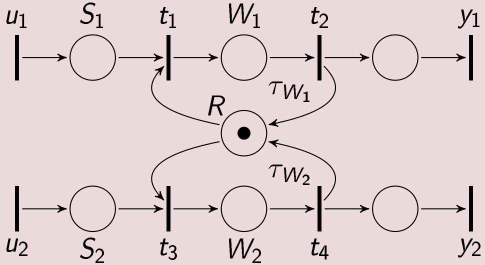
Conclusion and Future Work

# Modelling an Intersection

## Principle sketch

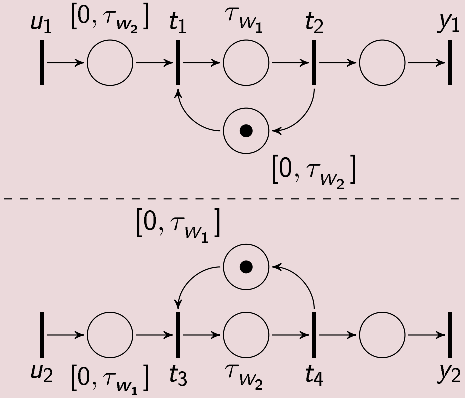


## In Petri nets



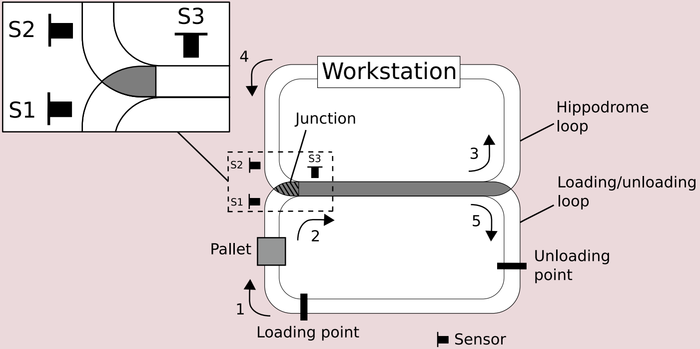
# Getting Parallel TEG's (Boutin et al., 2008a; Boutin et al., 2008b)

## Splitting



# Application Case

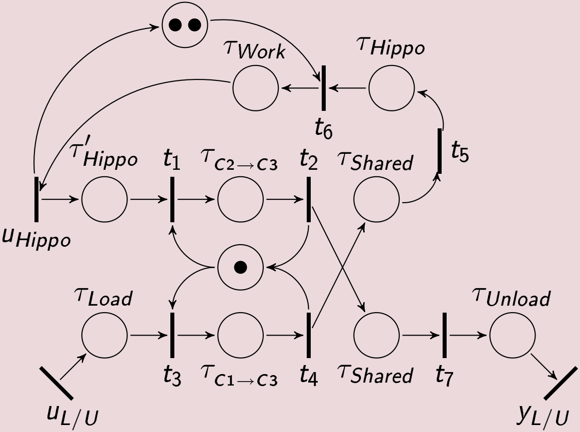
## Transfer Line with interconnected sub-systems




- Two loops sharing a common section.
- Need to rule the entry of the pallet in order to disable deadlocks. This can be done from the very loading point.

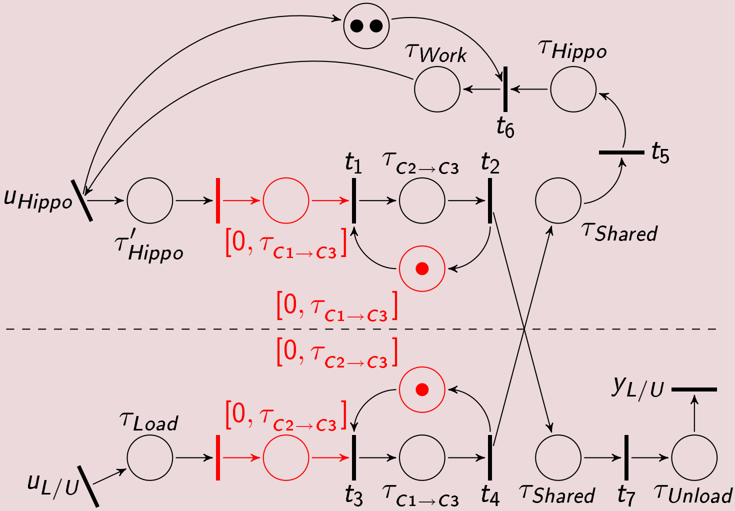
# TEG of the Transfer Line

## First Petri net model



# TEG of the Transfer Line

## TEG obtained thanks to our approach



# Next

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# Conclusion

- Approach for modelling production shops including conflicts.
- Characterisation of corresponding systems without conflicts, of which behaviours are slower or faster than the one of the studied system.
- Study of carriers routing:
  - Balanced between any number of different sub-systems.
  - With batches between 2 different sub-systems.
- Study of resource sharing with dynamic assignment policy.

# Future Work

- Study more general forms of conflicts.
- Study more complex systems, including both kinds of conflicts.
- Put this approach to the test on a real case (already done, but for exact models (Boutin et al., 2007) - without intervals).

Thanks for your attention!

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