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# QML: A PARACONSISTENT DEFAULT LOGIC

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*Abstract*
In {5} Pequeno and Buchsbaum presented a new default logic, the so-called Inconsistent Default Logic (IDL), which is based on Da Costa's paraconsistent logic {3}, in which the *Ex Falso* rule $\phi \wedge \neg\phi \Rightarrow \psi$ does not hold. They argue that IDL is more appropriate to model conflicts between default rules than Reiter's original default logic. In this paper we present the Question Marked Logic (QML), which is a further development of IDL. We show that QML inherits the good behaviour of IDL on representing conflicts, and that in addition it can be used to model specificity between default rules in a very intuitive way. We also show that QML can be viewed as a generalisation of the meta-level architecture BMS for non-monotonic reasoning as presented in {8, 9}

## 1. *Introduction*

In {5} Pequeno and Buchsbaum argue that in default reasoning a distinction should be made between 'hard' and 'soft' inconsistencies. If $\phi$ and $\neg\phi$ are conflicting factual information data, then clearly $\phi \wedge \neg\phi$ is a hard contradiction. But if we have that $\phi$ is given as factual information, whereas $\neg\phi$ is a default assumption, then $\phi \wedge \neg\phi$ is a soft contradiction. Also if both $\phi$ and $\neg\phi$ are conflicting default assumptions, then $\phi \wedge \neg\phi$ is a soft contradiction. Soft contradictions should be treated differently from hard ones. One of the most important differences is that the *Ex Falso* rule might be applicable for strong contradictions, but not for soft ones. The *Ex Falso* rule says that we can derive any arbitrary formula $\psi$ from the contradiction $\phi \wedge \neg\phi$, i.e.

$$\phi \wedge \neg\phi \Rightarrow \psi$$

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In classical logics this *Ex Falso* rule always holds. However, for modelling default reasoning it is better to have a logic in which *Ex Falso* does not hold for the case where either $\phi$ or $\neg\phi$ in the conjunction $\phi \wedge \neg\phi$ is a default assumption. Hence, to model default reasoning one would like to have a logic in which default assumptions are syntactically distinguishable from factual information, and in which the *Ex Falso* rule only applies to hard contradictions that contain only factual statements.

In {5} Pequeno and Buchsbaum introduced a new type of default logic, called the *Inconsistent Default Logic* (IDL), which is based on a paraconsistent logic instead of the classical logic that is used in Reiter's original default logic (see {7}). Paraconsistent logics are logics that lack the *Ex Falso* principle. Hence, in these logics it is not the case that from a contradiction $\phi \wedge \neg\phi$ follows any arbitrary statement $\psi$, i.e. $\phi \wedge \neg\phi \not\Rightarrow \psi$ The two best-known paraconsistent formalisms are Da Costa's system {3} and Belnap's four-valued logic {1}. Although the underlying semantics are quite different, these differences are not relevant in this paper. For a detailed comparison of different paraconsistent systems the reader is referred to {6}. For the use of paraconsistent formalisms in knowledge representation see {2}.

In IDL paraconsistency is selectively applied. The language of IDL consists of two types of formulas; formulas with a question mark ($\phi$?) and without it ($\phi$). The basic idea is that the question marked formulas denote default assumptions which are weaker statements than the factual statements without question marks. The paraconsistency in IDL is selective in the sense that the *Ex Falso* principle is only valid for ?-free formulae, but not for question marked formulae, i.e. $\phi? \wedge \neg\phi? \not\Rightarrow \psi$. Hence, a contradiction between default assumptions does not lead to an explosion of derivable formulas, whereas a contradiction between factual statements does lead to such an explosion. Pequeno and Buchsbaum show in their article that this difference can be used to solve the problem with anomalous extensions in Reiter's default logic that was pointed out by Morris in {4}.

The logic IDL can also be viewed as a meta-level architecture for non-monotonic reasoning that has two levels: an object-level and a meta-level. At the object-level we reason about factual statements without question marks, and at the meta-level we reason about default assumptions with question marks. The reasoning at the object-level is classical, and the reasoning at the meta-level is paraconsistent. Default rules in IDL have a question marked formula as conclusion, and question mark-free formulas as conditions of these rules. Given some factual information at the object-level the default rule asserts a default assumption at the meta-level.

The logic we introduce in this paper, the so-called *Question Marked*

*Logic* (QML) can be viewed as a generalisation of IDL. Instead of only one paraconsistent meta-level as is the case in IDL, the logic QML contains a hierarchy of meta-levels. In addition to formulae of the form $\phi ?$ with one question mark QML also has formulae with arbitrary finite numbers of question marks, i.e. $\phi ?$, $\phi ??$, $\phi ???$ etc. We use the notation $\phi^i$ to indicate that $\phi$ is followed by $i$ question marks. The basic idea is that if $i < j$, then the statement $\phi^i$ is stronger than $\phi^j$ in the sense that in QML $\phi^i$ implies $\phi^j$ but not vice versa. Analogous to IDL the *Ex Falso* rule does not hold for levels $i$ with $i$ larger than 0, i.e. $\phi^i \wedge \neg\phi^i \not\Rightarrow \psi^i$ for $i > 0$.[1]

## 2. *The Idea behind QML*

The central notion of the *Question Marked Logic* (QML) is the distinction of levels at which statements are made. A problem that may occur in ordinary default logic is the possibility of applying multiple default rules to infer a statement about a certain proposition. This may especially occur in taxonomic hierarchies, where some properties of the individuals may be cancelled and reintroduced. An example of this is the hierarchy of animals, birds and penguins, where an animal cannot fly by default, a bird can fly and a penguin cannot.

In a situation as described above we would like the results of more specific default rules to cancel or block the results of less specific default rules. In order to do this we need to make a distinction among statements and rules. This distinction is made by putting every statement at a certain level. These levels establish a hierarchy among the propositions. The inference of a statement is blocked by a contradicting statement at a lower level, i.e. $\neg\alpha^4$ blocks the inference of $\alpha^5$. If we infer something using a default rule the conclusion is at a level higher than the antecedent and the justifications. At the lowest level nothing is derived from a default rule.

At the same time we would like to retain an element of paraconsistency like in IDL. This means that contradictory statements inferred at the same level are allowed to coexist peacefully. So our system should be constructed in a way that it tolerates some inconsistencies, but that others are not allowed.

The Question Marked Logic contains default rules of the form

[1] In QML expressions of the form $\phi ?^i \wedge \psi ?^j$ with $i \neq j$ are not well-defined, and hence they have no direct consequences.

$$\frac{\alpha^i:\beta^i:\gamma^i}{\beta^{i+1}}$$

where $\alpha^i$ is the antecedent of the rule, $\beta^{i+1}$ the conclusion, and $\beta^i$ and $\gamma^i$ can be compared with the justification in Reiter's default rules. The conclusion $\beta^{i+1}$ can be derived with this rule if $\alpha^i$ is true, $\neg\beta^i$ is not true, and neither $\neg\gamma^i$ nor $\neg\gamma^{i+1}$ is true. The usefulness of this subtle difference between $\gamma^i$ and $\beta^i$ will be explained later.

The application of a default rule brings the conclusion at one level higher than the antecedent and the justifications. The question mark hierarchy plays two roles with regard to default rules. The first is that the application of a default rule to the conclusion of another default rule brings the second conclusion at a higher question mark level to indicate its default nature. Secondly, we can now establish a hierarchy among the default rules, such that the application of a more specific rule prevents the use of a more general rule.

For a better understanding of what is and is not possible we discuss some examples. The diagrams show the derivability of both $\alpha^i$ and $\neg\alpha^i$. The fact that a statement is inferred is indicated by a '+' sign. If the inference of a proposition is blocked, a '-' sign is placed in its column. A $u$ indicates that nothing has been inferred about a statement.

The first example is the standard Tweety example. All penguins are birds, penguins cannot fly by default and birds can fly by default. Because being a bird ($B$) is a more general concept than being a penguin ($P$), the default rule about birds flying ($F^2$) is placed at a higher level than the default rule about the non-flying ($\neg F^1$) of penguins. These two defaults can be expressed in QML as follows:

$$\frac{P^1:\neg F^1:}{\neg F^2}$$

$$\frac{B^2:F^2:}{F^3}$$

Suppose now that we have inferred $P^1$. With the default rule that penguins do not fly we infer $\neg F^2$. Now that we inferred that Tweety does not fly at level 2, we do not want any contradicting conclusion to be inferred from more general rules, i.e. at higher levels. The diagram pictures the resulting inference of propositions.

| Question Mark Level $i$ | $F^i$ | $\neg F^i$ |
|---|---|---|
| $n$ | + | − |
| ⋮ | ⋮ | ⋮ |
| 3 | + | − |
| 2 | + | $u$ |
| 1 | $u$ | $u$ |
| 0 | $u$ | $u$ |

What happens if we derive a contradiction at a certain level? An example of this is the Nixon diamond. Quakers are pacifists by default, republicans are not pacifists by default. Nixon is both a quaker and a republican. Is he a pacifist or isn't he? These two defaults can be represented in QML as follows:

$$\frac{Q^1 : P^1 :}{P^2}$$

$$\frac{R^1 : \neg P^1 :}{\neg P^2}$$

Suppose at level 1 we have $Q^1$ and $R^1$. The default rule about the pacifism of Quakers and the default rule about the non-pacifism of Republicans lead to contradicting conclusions, because no rule is more specific than the other. This means that at level 2 both $P^2$ and $\neg P^2$ are inferred. Since there is no *Ex Falso* rule at levels $i$ greater than 0, these conflicting conclusions do no lead to an inferential explosion, and we will see that both conclusions can be contained in one extension. Because truth at a certain level is propagated upwards to higher levels, both conclusions are also inferred at the levels above 2.

| Question Mark Level $i$ | $P^i$ | $\neg P^i$ |
|---|---|---|
| $n$ | + | + |
| ⋮ | ⋮ | ⋮ |
| 3 | + | + |
| 2 | + | + |
| 1 | $u$ | $u$ |
| 0 | $u$ | $u$ |

## 3. *The Logic $B^n$*

The base logic for QML is $B^n$, which is a multi-level logic[2] .The logic $B^n$ contains an arbitrary, finite number of levels. Basically, the logic $B^n$ is simply a collection of the logics $\mathcal{L}^0, \mathcal{L}^1 \ldots \mathcal{L}^n$. We will first define the language of $B^n$. After that we will give the semantics of $B^n$.

The alphabet of $B^n$ is formed of the following:

1. A set of propositional constants $\mathcal{A}$
2. A truth constant *True*
3. The connectives $\neg$, $\vee$, $\wedge$ and $\rightarrow$
4. Punctuation marks '(' and ')'
5. An indexed indication of the level '$i$', with $0 \leq i \leq n$

To define the set of well-formed formulae in $B^n$, we first define an intermediate set $\Phi$. $\Phi$ is the smallest set satisfying the following conditions:

1. All propositional constants are members of $\Phi$
2. If $\phi$, $\psi \in \Phi$, than $\neg\phi$, $\phi \vee \psi$ and $\phi \wedge \psi \in \Phi$

Now that we have defined the set $\Phi$, we can define the set of well-formed formulae. We define a sequence of languages $L^0 \ldots L^n$, where $L^i$ is the language for the logic $\mathcal{L}^i$ at level $i$. The language $L^i$ is defined as follows:

1. If $\phi \in \Phi$, then $\phi^i \in L^i$
2. If $\phi$, $\psi \in L^i$, then $\neg\phi$, $\neg\psi$, $(\phi \vee \psi)$ and $(\phi \wedge \psi) \in L^i$

Implication $\phi^i \rightarrow \psi^i$ is defined as $\neg\phi^i \rightarrow \psi^i$. A formula $\phi$ is a well-formed formula of $B^n$ if and only if $\phi$ is a well-formed formula of one

[2] The logic $B^n$ is a 4-valued logic, whereas IDL is based on an adapted version of Da Costa's $C_\omega$ paraconsistent logic.

of the logics $\mathcal{L}^i$ that $B^n$ consists of. This definition in two stages of the set of well-formed formulae in $B^n$ is to prevent the nesting of question marks and mixing up the languages of the different levels. We wish to exclude such formulae like for example

$$(\alpha^5 \wedge \beta^5)^4$$

$$\gamma^4 \vee \beta^2$$

Examples of formulae that are well-formed are

$$\alpha^2 \vee \neg\beta^2$$

$$(\alpha \vee \neg\beta)^3$$

The logic $B^n$ consists of a finite number of logics $\mathcal{L}^0 \ldots \mathcal{L}^n$. Each of these logics has its own language $L^0 \ldots L^n$. The semantics for $B^n$ is defined in terms of the semantics of the levels. This semantics is a Belnap four-valued semantics for the logics $\mathcal{L}^1$ to $\mathcal{L}^n$. $\mathcal{L}^0$ is the only level that has a classical semantics. Let $WFF^i$ denote the set of well-formed $L^i$ formulae.

*Definition* 1. *A four-valued model $M^i$ for the logic $\mathcal{L}^i$, $1 \leq i \leq n$ is a function from the set of well-formed $L^i$-formulae to the fourtruth values {u, f, t, o}. In other words a model $M^i$ is an $\mathcal{L}^i$-model if*

(i) $i = 0 : M^i : WFF^i \rightarrow \{f, t\}$
$i \geq 1 : M^i : WFF^i \rightarrow \{u, f, t, o\}$
(ii) For all formulae in $WFF^i$ the mapping agrees with the truth tables for $B^n$ (see Figure 1)

The following notation is used for well-formed $L^i$ -formulae

$$M^i \models_i \phi^+ \Leftrightarrow M^i(\phi) = t \text{ or } M^i(\phi) = o$$

and

$$M^i \models_i \phi^- \Leftrightarrow M^i(\phi) = f \text{ or } M^i(\phi) = o$$

The '+' and '-' signs are not part of the syntax of the formula $\phi$, but are semantical symbols related to '$\models_i$' If both $M^i \not\models_i \phi^+$ and $M^i \not\models_i \phi^-$, then $\phi$ is undefined i.e. $M^i(\phi) = u$. If, on the other hand, we have both

$M^i \models_i \phi^+$ and $M^i \models_i \phi^-$ then $M^i(\phi) = o$.

The truthtables for $B^n$ are the truthtables for Belnap's logic. These truthtables are a superset of the classical truth tables. Therefore we can use them for all levels, including the 0 level. The following truthtables are schemata with $i$ ranging over 0, 1, ..., $n$.

| $\phi^i \wedge \psi^i$ | $u$ | $f$ | $t$ | $o$ |
|---|---|---|---|---|
| $u$ | $u$ | $f$ | $u$ | $f$ |
| $f$ | $f$ | $f$ | $f$ | $f$ |
| $t$ | $u$ | $f$ | $t$ | $o$ |
| $o$ | $f$ | $f$ | $o$ | $o$ |

| $\phi^i \vee \psi^i$ | $u$ | $f$ | $t$ | $o$ |
|---|---|---|---|---|
| $u$ | $u$ | $u$ | $t$ | $t$ |
| $f$ | $u$ | $f$ | $t$ | $o$ |
| $t$ | $t$ | $t$ | $t$ | $t$ |
| $o$ | $t$ | $o$ | $t$ | $o$ |

| $\neg\phi^i$ | |
|---|---|
| $u$ | $u$ |
| $f$ | $t$ |
| $t$ | $f$ |
| $o$ | $o$ |

Figure 1: $B^n$ Truthtables

*Definition* 2. A $L^m$*-formula* $\phi$ *with* $0 \leq m \leq n$ *is a* $\mathcal{L}^m$*-semantical consequence of a set of* $L^m$*-formulae* $\Sigma$ *written* $\Sigma \models_m \phi$, *if for all* $\mathcal{L}^m$*-models* $M^m$ *holds*

$$\text{if } M^m \models_m \psi^+, \text{ for all } \psi \in \Sigma, \text{ then } M^m \models_m \phi^+$$

The semantic consequence relation of the compound logic $B^n$ is composed of the semantic consequence relations of the composing logics $\mathcal{L}^0 \ldots \mathcal{L}^n$.

*Definition* 3. *A* $B^n$ *formula* $\phi$ *is a* $B^n$ *semantic consequence of a set of* $B^n$ *formula* $\Sigma$, *written* $\Sigma \models_{B^n} \phi$, *iff there exists an m with* $0 \leq m \leq n$, *such that* $\phi$ *is a* $\mathcal{L}^m$*-semantic consequence of a subset of* $L^m$*-formulae, say* $\Sigma^m$, *of* $\Sigma$, *i.e.*

$$\Sigma \models_{B^n} \phi \Leftrightarrow \textit{There is an m such that } \Sigma^m \models_m \phi,$$

*with* $0 \leq m \leq n$ *and* $\Sigma^m \subseteq \Sigma$.

The closure in $B^n$ is defined as follows:

*Definition 4. Let* $Th_{B^n}(S)$ *denote the set of all formulae that are* $B^n$ *consequences of S, i.e.*

$$Th_{B^n}(S) = \{\phi \mid S \models_{B^n} \phi\}$$

If it is clear from the context, we usually omit the subscript $B^n$ from $Th_{B^n}$.

In this paper we only give the semantic definition of $B^n$. For a proof system for each of the four-valued logics $\mathcal{L}^1, \ldots, \mathcal{L}^n$ the reader is referred to {10}[3].

4. *Properties of* QML *as a Default Logic*

Transitions between the different logics $\mathcal{L}_i$ and $\mathcal{L}_j$ in $B^n$ are strictly via default rules. We will define a QML default theory, a QML default rule and an extension of a QML theory.

*Definition* 5 *A* QML *default is of the form*

$$\frac{\alpha^i\colon \beta^i;\ \gamma^i}{\beta^{i+1}}$$

*The formulae $\alpha^i$, $\beta^i$, $\beta^{i+1}$ and $\gamma^i$ are well-formed formulae of $B^n$ with $0 \leq i \leq n$. The formula $\alpha^i$ is the antecedent, $\beta^i$ the default condition, $\gamma^i$ the proviso, and $\beta^{i+1}$ the conclusion.*

IDL defaults are QML default rules in which $i = 0$.

*Definition* 6. *A QML theory is a pair $\langle W, D\rangle$, where W is a set of $L^0$ formulae and D a finite set of* QML *default rules.*

QML default rules can be viewed as a multiple meta-level generalisation of the idea behind the so-called heuristic rules in the meta-level architecture BMS {8, 9}. For example the default rule

$$\frac{\alpha^0\colon \beta^0;\ \gamma^0}{\beta^1}$$

can be roughly compared to the heuristic rule $T(\alpha) \wedge \neg T(\neg\gamma) \rightarrow PA(\beta)$. This heuristic rule states that the default assumption $PA(\beta)$ can be de-

[3] See in particular the proof system $\vdash_K$ in Section 2.4 of {10}. The proof system $\vdash_K$ is given for the three-valued Strong Kleene Logic, but Urquhart makes the observation that if one removes the rule $\phi, \neg\phi \vdash_K \psi$ from this system, then the resulting system is a proof system for Belnap's four-valued logic.

rived at the meta-level (= level 1) if at the object-level (= level 0) $\alpha$ is true and $\gamma$ is not false (i.e. $\gamma$ has either the semantic value $u$ or $t$).

Another feature of QML is *upward reflection*. It means that everything that is true at a level $m$ must be true at higher levels, because a higher level represents a weaker notion of truth. If the strong version of a proposition is true, its weaker versions are certainly also true. This rule is a generalisation of the upward reflection in the meta-level architecture BMS for default reasoning described in {8, 9}. The upward reflection is implemented by the following rule schema:

$$\frac{\phi^i::}{\phi^{i+1}}, \text{ for all propositional symbols } \phi \text{ and } 0 \leq i < n$$

In this schema the default condition and the proviso are always true. The rule schema is included in $D$ for every QML theory. In the sequel we will not mention the schema in $D$, but we will assume that it is contained in every $D$. If $S_i$ is a set of $L^i$-formulae, then we write $Upw(S_i)$ to denote the set of $L^{i+1}$-formulas that are generated by the upward reflection schema, i.e.

$$Upw(S_i) = \{\phi^{i+1} | \phi^i \in S_i\}$$

An extension of a QML theory is defined as follows, following Reiter {7}.

*Definition 7. (Extension) Let* $\Delta = \langle W, D \rangle$ *be a QML default theory. For any set of formulae S from the logic* $B^n$ *let* $\Gamma(S)$ *be the smallest set satisfying the following three properties*

(1) $W \subseteq \Gamma(S)$

(2) $Th_{B^n}(\Gamma(S)) = \Gamma(S)$

(3) *For each default* $\dfrac{\alpha^i:\ \beta^i:\ \gamma^i}{\beta^{i+1}} \in D$, *if* $\alpha^i \in \Gamma(S)$, $\neg\beta^i \notin S$ *and* $\neg\gamma^i,\ \neg\gamma^{i+1} \notin S$ *then* $B^{i+1} \in \Gamma(S)$, *where* $0 \leq i < n$

*A set Q is an extension of* $\Delta$, *iff* $\Gamma(Q) = Q$, *i.e. Q is a fixed point of* $\Gamma$.

Note that $Q$ extensions do not always exist. This is a well-known problem in Reiter's default logic that also occurs in QML. The famous example of a Reiter default theory that has no extension is the default with the empty prerequisite $\left(\frac{:A}{\neg A}\right)$. The analogon of this in QML is, for example, the default $\left(\frac{::A^0}{\neg A^1}\right)$, i.e. the proviso violates the conclusion. This default does not have a $Q$ extension in QML either.

Upward reflection has the effect that $\neg\gamma^{i+1} \notin S$ implies $\neg\gamma^i \notin S$. Hence, a QML default rule might as well be formulated as

$$\frac{\alpha^i \colon \beta^i;\ \gamma^{i+1}}{\beta^{i+1}}$$

We prefer the current format in order to preserve the property that IDL defaults are QML defaults with $i = 0$.

The resulting extensions are different from the corresponding reiter extensions. The intuitive explanation for this is that in addition to internal consistency we take into account whether we can decide between the application of rules when defining a QML extension. In some situations different Reiter extensions are combined in a single QML extension. An example of this is the Nixon-diamond, as formulated earlier in Section 2.

$$\frac{Q^1 \colon P^1 \colon}{P^2}$$

$$\frac{R^1 \colon \neg P^1 \colon}{\neg P^2}$$

If we have $W = \{Q^0,\ R^0\}$, then the contradicting defaults about Nixon's pacifism lead to different extensions in Reiter default logic, but are incorporated into one extension in QML, namely $Th(\{Q^0,\ R^0,\ P^2,\ \neg P^2\})$. The intuition behind this is that we cannot decide which rule takes priority.

In other situations counter-intuitive Reiter extensions are eliminated in the corresponding QML situation. An example of this can be found in taxonomic hierarchies where all properties are modelled by default rules. An example is the following Tweety example:

$$W = \{P,\ P \rightarrow B\}$$

$$D = \left\{ \frac{B{:}F}{F},\ \frac{P{:}\neg F}{\neg F} \right\}$$

which has two extensions in Reiter default logic:

$$E_1 = Th(\{P,\ B,\ \neg F\})$$

$$E_2 = Th(\{P,\ B,\ F\})$$

Whereas the corresponding specificity based QML theory

$$W = \{P^0,\ P^0 \rightarrow B^0\}$$

$$D = \left\{ \frac{P^0{:}\neg F^0{:}}{\neg F^1},\ \frac{B^1{:}F^1{:}}{F^2} \right\}$$

has only one extension

$$Q = Th(\{P^0,\ B^0,\ \neg F^1\})$$

Note that, if we took $S = \{P^0,\ B^0,\ F^1\}$, then $\Gamma(S) = Th(\{P^0,\ B^0\}) \neq S$, due to the fact that $\Gamma(S)$ is the smallest set that satisfies the requirements (1)-(3) from Definition 7. The next application of $\Gamma$ gives us the extension of this QML theory, i.e. $\Gamma(\Gamma(S)) = Q$. This QML theory also illustrates how specificity can be implemented in QML. Given the implication $P^0 \rightarrow B^0$ in $W$, the first rule is more specific than the last one. Hence the last rule should be at a higher level than the first one.

Note that we cannot obtain this result if we only allow IDL defaults, i.e. default rules in which $i = 0$. If we replace $D$ for example by

$$D' = \left\{ \frac{P^0{:}\neg F^0{:}}{\neg F^1},\ \frac{B^0{:}F^0{:}}{F^1} \right\}$$

then

$$Q' = Th(\{P^0,\ B^0,\ \neg F^1,\ F^1\})$$

In this extension we derive the conflicting conclusion $\neg F^1 \wedge F^1$, and not just the preferred conclusion $\neg F^1$ as we have in $Q$.

We can solve these problems in IDL by changing the default rules from $D'$ as follows:

$$\frac{P^0 : \neg F^0 :}{\neg F^1} \tag{1}$$

$$\frac{B^0 : F^0 : F^0}{F^1} \tag{2}$$

The formula $F^0$ in the proviso of Rule 2 prevents the application of this rule, if Rule 1 is applied. This yields the desired extension $Q = Th(\{P^0,\ B^0, \neg F^1\})$. Suppose, however, that we have a third level of specificity about animals ($A$), that generally do not fly, e.g. we add the rule

$$\frac{A^0 : \neg F^0 :}{\neg F^1} \tag{3}$$

then this rule will be preferred over Rule 2, if we have $W' = \{A^0,\ B^0\}$. Hence, we get $Q' = Th(\{A^0,\ B^0, \neg F^1\})$, which is counter-intuitive because Rule 2 is more specific than Rule 3. The solution is to replace Rule 3 by the following rule about animals:

$$\frac{A^0 : \neg F^0 : \neg F^0}{\neg F^1} \tag{4}$$

In this case, Rules 2 and 4 will block each other given $W'$, which results in multiple extensions:

$$Q'_1 = Th(\{A^0,\ B^0, \neg F^1\})$$

$$Q'_2 = Th(\{A^0,\ B^0, F^1\})$$

Again this is counter-intuitive. Since Rule 2 is more specific than 4, the only intuitive extension is $Q'_2$. The best way to solve this problem is to explicitly add the more specific cases as exceptions to the justifications of the rule about animals. Hence, even Rule 4 has to be replaced by the following more complicated rule:

$$\frac{A^0 : \neg F^0 \wedge \neg P^0 \wedge \neg B^0 : \neg F^0}{\neg F^1} \tag{5}$$

The corresponding specificity-based QML default theory would be as follows:

$$D'' = \left\{ \frac{P^0 : \neg F^0 :}{\neg F^1}, \frac{B^1 : F^1 :}{F^2}, \frac{A^2 : \neg F^2 :}{\neg F^3} \right\}$$

Given $W'$, this would only yield the intuitive extension $Q'' = Th(\{A^0, B^0, F^2\})$. Hence, these examples show that QML is more appropriate than IDL to represent specificity-based reasoning.

QML inherits from IDL the nice property that it solves the anomalous extension problem as described by Morris {4}. An example is the following theory about being a bird ($B$), being an animal ($A$), being able to fly ($F$) and having wings ($W$).

$$W = \{W \rightarrow F,\ B \rightarrow A,\ B\}$$

$$D = \left\{ \frac{A : \neg F \wedge \neg W}{\neg F}, \frac{B : W}{W} \right\}$$

The extensions of this theory in Reiter default logic are

$$E_1 = Th(\{B,\ A,\ W,\ F\})$$

and

$$E_2 = Th(\{B,\ A,\ \neg F,\ \neg W\})$$

Extension $E_2$ is an anomalous extension, because it contains the strange conclusion that Tweety is wingless because he cannot fly. In QML this theory would be modelled as follows:

$$W = \{W^0 \rightarrow F^0,\ B^0 \rightarrow A^0,\ B^0\}$$

$$D = \left\{ \frac{A^1 : \neg F^1 : \neg W^1}{\neg F^2}, \frac{B^0 : W^0 :}{W^1} \right\}$$

The only extension of this theory is

$$Q = Th(\{B^0,\ W^1,\ F^1,\ A^0\})$$

As we see the anomalous extension is blocked by the presence of a hierarchy among the default rules. Moreover, in this example the role of the proviso is essential.

Situations with multiple extensions still exist in QML. The pattern for this is the presence of a combination of default rules whose conclusion is the contradiction of the proviso of another default rule. An example of this is the following pair of default rules:

$$\frac{A^i:\ B^i;\ \neg C^i}{B^{i+1}}$$

$$\frac{A^i:\ C^i;\ \neg B^i}{C^{i+1}}$$

In combination with a theory $W = \{A^0\}$, this leads to the extensions:

$$Q_1 = Th(\{A^0,\ B^1\})$$

and

$$Q_2 = Th(\{A^0,\ C^1\})$$

This pattern is the only one that leads to multiple extensions. The intuitive motivation for these multiple extensions is that we cannot decide which rule takes priority, but that we are sure that the two rules are mutually exclusive. In other cases the difference in levels can take care of blocking the application of a conflicting default.

## 5. *An Alternative Extension Definition*

The following theorem gives us an alternative definition of an extension:

*Theorem* 1 *Let T be a* QML *theory* $\langle W,\ D\rangle$ *with n the number of levels of default rules in D, then Q is an extension of T iff there are* $Q_0,\ Q_1,\ \ldots,\ Q_n$ *such that:*

(1) $Q = Q_0 \cup Q_1 \cup \ldots \cup Q_n$

(2) $Q_0 = Th_0(W)$

(3) *For* $0 \leq i < n$

$$Q_{i+1} = Th_{i+1}(Cons(Q_i))$$

*where* $(Cons(Q_i)) =$

$$\left\{ \beta^{i+1} \left| \begin{array}{l} \dfrac{\alpha^i : \beta^i : \gamma^i}{\beta^{i+1}} \in D \text{ and } \alpha^i \in Q^i, \\ \neg\beta^i \notin Q_i,\ \neg\gamma^i \notin Q_i,\ \neg\gamma^{i+1} \notin Q_{i+1} \end{array} \right. \right\}$$

*where* $Th_i$ *is the deductive closure of a set of formulae in* $B^n$ *at level i.*

*Proof.* We denote $\bigcup_{i=0}^{n} Q_i$ with $\Omega$. We must prove that $\Omega = \Gamma(Q)$. We do this by proving that $\Gamma(Q) \subseteq \Omega$ and $\Omega \subseteq \Gamma(Q)$. Remark that each $Q_i$ is the set of those elements of $Q$ that are statements at level $i$.

To show that $\Omega \subseteq \Gamma(Q)$, assume the contrary. Then there is a $k$ such that $Q_{k-1} \subseteq \Gamma(Q)$ and $Q_k \not\subseteq \Gamma(Q)$. This implies that there are $\alpha$, $\beta$, and $\gamma$ such that $\beta^k \notin \Gamma(Q)$ and

$$\frac{\alpha^{k-1} : \beta^{k-1} : \gamma^{k-1}}{\beta^k} \in D$$

$\alpha^{k-1} \in Q_{k-1}$, $\neg\beta^{k-1} \notin Q_{k-1}$, $\neg\gamma^{k-1} \notin Q_{k-1}$ and $\neg\gamma^k \notin Q_k$. $Q_{k-1} \subseteq \Gamma(Q)$ implies that $\alpha^{k-1} \in \Gamma(Q)$, and $\neg\beta^{k-1} \notin Q_{k-1}$ implies that $\neg\beta^{k-1} \notin Q$, and $\neg\gamma^{k-1} \notin Q_{k-1}$ and $\neg\gamma^k \notin Q_k$ implies $\neg\gamma^{k-1}$, $\neg\gamma^k \notin Q$. However, we have $\beta^k \notin \Gamma(Q)$. This contradicts the third property of the $\Gamma$-operator as defined in Definition 7.

To show that $\Gamma(Q) \subseteq \Omega$, we have to show that $\Omega$ satisfies the conditions for $\Gamma(Q)$ The first two conditions are trivial. Since $Q_0 = Th_0(W)$, $W \subseteq \Omega$. $\Omega$ is deductively closed, because every level $i$ is deductively closed in $Q_i$. Finally the conditions to apply a default rule are analogous with the only difference being the substitution of $\Omega$ for $\Gamma(Q)$. Since $\Gamma(Q)$ is defined as the smallest set satisfying its conditions, $\Gamma(Q) \subseteq \Omega$.

## 6. *Establishing the Specificity Hierarchy*

As we mentioned earlier a conclusion from a general rule should be at a higher level than a conclusion resulting from a specific rule. This way the specific rule will block the application of the general rule. To achieve this the ordering of the default rules should reflect the subset relation of the predicates that occur in the antecedents of these rules. For example, the rule that birds fly by default is more specific than the rule that animals do not fly by default, because being a bird implies being an animal. The idea is captured by the following constraint which every QML theory should satisfy:

*Definition 8. (Specificity constraint) Given a QML theory $T = \langle W, D \rangle$. The specificity constraint on T is, that if $W \models_{B^n} \phi^0 \rightarrow \psi^0$ then for every pair of default rules*

$$\frac{\phi^k\colon \chi^k;\ \upsilon^k}{\chi^{k+1}},\ \frac{\psi^m\colon \omega^m;\ \mu^m}{\omega^{m+1}} \in D$$

*we have $m > k$.*

## 7. *Conclusion*

We have presented the Question Marked Logic and showed that in some cases it yields more intuitive results than Reiter's original default logic and Pequeno and Buchsbaum's improvement of it, IDL. In particular, QML is more appropriate for modelling specificity among default rules than IDL. We also showed that it can be viewed as a generalisation of the meta-level architecture BMS for non-monotonic reasoning. Some possible applications may be found in legal practice and in reasoning about object systems. For a more elaborate description of QML and its development the reader is referred to {11}.

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