Mathematics as the Paradigm for Metaphysics
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Dedicated to Prof. Cor Baayen, who retires as the scientific director of CWI, and who can therefore revive his interest in philosophy.

The ideal of mathematical exactness is strongly paradigmatic for modern science, for which mathematics practically functions as a metaphysical foundation. This strongly influenced philosophy. In our century, however, critical voices arise, even from the ranks of scientists. Reflection on the foundations of mathematics has produced a deeper insight into its nature. As a result the tendency to judge content by structure has become less prominent. Metaphysics, however, is still often rejected as only producing constructions with unjustified claims to necessity. Clear recognition of the role of the mathematical paradigm shows that this rejection is unnecessary.

MATHEMATICS AND METAPHYSICS
It is sometimes claimed as an advantage, and sometimes regretted, that modern natural science has no metaphysical foundation. The unconventional thesis might, however, be defended that mathematics has effectively functioned as the metaphysical foundation of the modern scientific tradition. The still living fundamental principle of science, from Galileo onward, is the reduction of qualitative phenomena to measurable quantities and structures. Many underlying forms of thought in which this principle has been active apart from actual mathematisation, such as the mechanistic view, determinism, and positivism, have been superseded by others such as complementarity, probabilism, and chaos-theory in science itself, and critical rationalism and even sociologism in the philosophy of science. But the idea that knowledge is scientific in the complete sense of the word only if it is expressible in mathematical structures and equations seems to be unchallenged. Seemingly extreme reactions to the mathematical perspective, such as holism, implicitly presuppose the same mathematical models as their more positivistic counterparts. This probably accounts for their apparent extremeness. Even if real mathematisation lies far behind the horizon, as it does in the cognitive sciences, it is nevertheless taken as a standard, e.g. in the form of computational models. In logic and linguistics, and even in ethics, the mathematical perspective is prevailing now. What is often called ‘formalisation’ or ‘formal methods’ by analogy to mathematical logic, is in fact the construction of mathematical models, as it originally was in mathematical logic too.²

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1) This article contains parts of the introduction and Chapter 5 from: L.E.Fleischhacker, Beyond Structure, Peter Lang, Frankfurt 1995

The paradigm of mathematical thought has also thoroughly invaded philosophy. Not only by attempts to systemize the discipline *more geometrico*, such as Spinoza's *Ethica*, but also by the ideal of demarcating a domain of pure rationality from the ambiguities and prejudices of common sense. From Descartes to Wittgenstein this ideal has exerted a strong influence on philosophy, and the consequence has been an estrangement of philosophy from its most fundamental discipline: metaphysics.

**The Problem of Mathematical Thought**

From the perspective of the philosophy of mathematical thought, the relationship of mathematical structure to observable reality has remained extremely problematic. Plato formulated the question where in the world to look for numbers and geometrical figures, and he concluded that the visible world is not the only possible mode of being. Mathematics cannot be about the world of human experience, for example, because this world resists reduction to purely mathematical structure. The reality of change especially is a hard nut to crack, as was noticed already by Aristotle. But he found a way different from Plato's for dealing with mathematical objects. He regarded them as the results of abstraction, the actualization in thought of a principle we find in the world of experience. He called this principle ὑπόθεσις: intelligible matter.

In antiquity the main philosophical problem with mathematical objectivity was to separate it from experience, without making the applicability of mathematics impossible to understand. Modern times, however, begin with the idea of the identity of mathematics and physics. Nature herself is thought to be structural, and thus accessible to mathematical investigation, not only by her external geometrical shapes - as Archimedes had already discovered - but also in her inner laws.

In philosophy this had a very strong impact. Descartes characterises the world of experience as *res extensa*, taking what in Aristotelianism had only been an outer property of material things to be their essence. The externality of nature becomes its inner principle. In the nineteenth century Hegel formulated the essence of nature as 'the Idea in the form of externality to itself'.

For philosophy this meant that the problem was no longer one of the relation of the mathematical to the physical, but of the relationship of a knowing subject, Descartes' *res cogitans*, to an objective world which is mathematical and physical at the same time. This produced strongly mathematically coloured, but never really mathematical, metaphysics.

Spinoza's attempt to construct metaphysics *more geometrico* has led to points of view which actually went beyond mathematical reasoning, but remained strongly influenced by it. Even when in modern philosophy the paradigm of geometry, or mathematics in general, is explicitly rejected, as in the case of Hegel's system, the lure of structural rigour is still present as can be seen in his rigorously systematic approach.

In the nineteenth century the identification of mathematical and physical objectivity became less and less obvious. Mathematics, liberated from its close connec-

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3) Plato, *Republic* 526a
tion to physics and technology, began to develop highly speculative theories such as complex number theory, abstract algebra, Fourier analysis, non-Euclidean geometry and projective geometry. It started looking for a foundation of its own, independently of physics, and thereby more and more overtly showed its ideal character.

In philosophy, on the eve of the twentieth century, two - apparently opposite - impulses emerge, which may eventually undermine the ideal of mathematical rigour: Husserl's phenomenology, which introduces another ideal of philosophical rigour, and Frege's mathematical logic, which objectifies mathematical reasoning.

Gödel's results teach us that, as a consequence of this objectification, the foundation of mathematics cannot be formulated explicitly as a mathematical theory. Mathematical thought as such cannot be free from intuitive presuppositions demanding investigation by a discipline other than mathematics itself.

For a 'working mathematician' this is no problem at all. She or he is perfectly happy with Hilary Putnam's 'Yes, we have no foundations'; but the philosopher experiences a change of problem-field again. Now both the subject-object relationship and the relationship of structure and reality have become problematic. The mathematical point of view appears to be based on an intuitive insight, constituting a certain perspective - which I call structural - on the world of experience.

But if that is true, mathematical structure is not necessarily the only or even the most adequate form in which scientific knowledge can be expressed. Perhaps the success of measuring-science has blinded us to metaphysical perspectives, whether or not they justify or radicalise the mathematical approach. Even philosophies which are generally considered to be anti-mathematical, such as Hegel's speculative dialectics or Heidegger's existential philosophy, when inspected more closely, appear to share certain essential presuppositions with the mathematical approach, e.g. the denial of real potentialities. In fact the ideal of 'exactitude' - the possibility of making all presuppositions explicit and developing a body of thought consistently from them - seems to be all-pervading in our culture. Wittgenstein's Philosophical Investigations, because of its anti-systematic tendency, may be regarded as an outstanding exception; but this work also shows clearly the kind of trouble that arises if one tries to leave the mathematical paradigm behind. For what other method than allusion remains, if an explicit development of ideas is forbidden? The very different ways in which

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4) The words 'subject' and 'object' are used in different senses, but the tendency always is that they are correlative in the performance of some (theoretical or practical) action - as the linguistic use suggests. The subject is the active pole, the object not necessarily passive, but the action is always directed towards it. Subjective is what belongs to the subject as such (i.e. in its function of being the active pole), objective what belongs to the object as such (i.e. in its function of being the 'aim' of the activity), which does not necessarily mean that it exists independently of the subject. Mathematical objects for instance, need not be conceived of as existing independently of mathematical thought. Subjectivity and objectivity are the properties of being subjective, respectively objective. Objectification is the act of giving objectivity to some content, either by theory - conceiving of it in the form of objectivity - or by practice - bringing about a state of affairs which may be understood as representing the said content in an objective form.
Wittgenstein's philosophy has been interpreted make this clear, for if one cannot express one's ideas explicitly, there is no limit to interpretation.

Also in structuralism, in spite of its name, a tendency to leave mathematical grounds is present. It is real structure the structuralists are after, not ideal, mathematical structure. But as long as nothing but structure is seen, it is already surreptitiously being idealised. Therefore, in structuralism there is always an essentially non-structural principle - such as power, force or spontaneity - lurking in the background. Critics of the mathematical point of view usually underestimate its power. Either it eventually turns out that they have remained within it or they adopt its abstract opposite and in this way remain indebted to it.

THE AGE OF MATHEMATISM
Dijksterhuis concludes his Mechanisation of the World Picture with the remark:

The mechanisation, which the world-picture underwent in the transition from antique to modern natural science, consisted in the introduction of a description of nature by means of the mathematical concepts of classical mechanics; it indicates the beginning of the mathematisation of natural science, which obtains its completion in twentieth century physics.

But this is only seen from the direction of the ultimate effect. In my view, the technical as well as the philosophical sources of the rise of modern science already introduced very strong tendencies towards mathematisation. It is in accordance with the natural development of technology that technical concepts are made more and more explicit. Of course this does not explain that this development took place in this particular historical period. But one thing is clear: in order to make technical concepts explicit, one must measure and calculate. Moreover, on the philosophical side, medieval Aristotelianism hardly left room for another basis to be criticised on than precisely the mathematical Platonism that arose in the Renaissance. The breakthrough of both tendencies - the technological development and mathematical Platonism - and their fruitful meeting in a particular place and time can probably be explained by fundamental changes in society. What is important here, however, is the result of the breakthrough: the firm belief that measurement and mathematical calculation, and nothing else, will lead to insight into the phenomena of nature. For Galileo the book of nature was written in mathematical signs, and for Newton mathematical space and time were absolute, whereas experienced space and time were considered to be only

5) Especially for those who - like Cusanus and the humanists, and unlike most of the modern philosophers - knew perfectly well what it was all about, and where the strength and weakness of this world-picture was to be located.


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relative. Nature came to be seen as mathematical *in itself*, and the distinction between mathematics and physics became obsolete. In the eighteenth century 'mathematics' still encompassed a whole range of disciplines, from arithmetic to machine-construction. Only in the nineteenth century did a new form of 'pure' mathematics emancipate itself from natural science and technology. But by then the mathematical style of thinking had been thoroughly spread among scientists and technicians.

**WHAT IS MATHEMATISM?**

But the prevalence of a particular style does not itself constitute mathematics, which is rather a - usually implicit - metaphysical position connected to the feeling that the 'mathematical' style is so self-evident that it does not need any foundation. In this way this style is itself taken as the foundation of science and philosophy. As a consequence, the objectivity and generality of the style have to be regarded as objectivity and generality without qualification. The object of mathematical thought can be characterised as *structure*, which is more general than what is usually understood by quantity, but is by no means identical to metaphysical universality or being. If unqualified objectivity is identified with mathematical objectivity, the fundamental nature of reality becomes structure, which is differentiated only by higher or lower degrees of complexity. This is exactly in line with the philosophy, ascribed to Pythagoras, according to which the essence of the universe is *number*. Number for the ancients was the principle of what is mathematical, and it is still often regarded as a fundamental paradigm of structure. The Pythagorean world view is a fundamental and ever recurring metaphysical perspective. In Plato's Academy, Speusippos and Xenocrates took up this line of thought and in the Renaissance it was popular with humanists such as Pico della Mirandola. Even today it is explicitly adhered to by some theoretical physicists, who doubt whether 'matter' is to be regarded as a useful concept in physics.

On the other hand we all know that structure is not something immediately given. We can see different structures in one and the same phenomenon and we can technically give different structures to our surroundings. And in pure mathematics, structure is the result of postulation or thought-construction. So structure is in a certain sense our product. It is the *structurability* of the world, which is fundamental.

So mathematics has two sides to it, expressible in two ideal-typical theses:

1. Structurability is the *essence* of everything.
2. To know something is the same as to give it structure.

This is a completely coherent metaphysical position, in which being is identified with *mathematical intelligibility*, instead of intelligibility without qualifications. But the question is, whether or not this world view is unduly *restrictive*. Does it rule

7) In this connection Kronecker's saying: 'The natural numbers are made by the Lord, the rest [of mathematics] is human work' is usually quoted.
out any other perspectives which we find particularly plausible? One could ask whether anything exists which is not - in a certain sense - structurable, and it would be difficult to find an example. On the other hand, one could ask whether in fact there exists anything the essence of which is its structurability. Perhaps one could think that the essence of space is its structurability. But once one imagines something in it, a non-structural quality is introduced, which distinguishes the space occupied by the 'something' from empty space. Trying to reduce this quality to structure again, could very well lead to an infinite regress. If indeed it is felt as absurd from the point of view of common sense to express mathematicism as an explicit philosophy - in the same way as it is felt as absurd to express scepticism\(^8\) as an explicit position - what then are the grounds for this feeling of absurdity?

Let me compare this situation with the current aporia in debates about the scope of artificial intelligence. If one mentions a human skill, not yet simulable by computer programs, the AI defender will say: if you describe it exactly and clearly (i.e. mathematically) to me, I shall find a way to simulate it, and if you cannot describe it in this way, then it is nothing at all. But then, if it is so described, it is probably not the same as it was before. What, however, is the difference? We have the feeling that, as soon as we describe this difference, a corresponding correction of the program will eliminate it.

We are so immersed in mathematicism that we simply cannot imagine a kind of exactness surpassing mathematical exactness. For how could we prove that e.g. intelligence is not reconstructible in mathematical terms, if not by using a description of mathematical reconstructibility itself, showing its restrictions. But such a description should evidently be clearer and more self-evident than any mathematical construction.

In a traditional philosophical framework metaphysics could perform this task, and that is why mathematics and metaphysics must be rivals in a mathematicistic world.

On the other hand, it seems to be precisely the development of information-technology that tends to change this situation. In this field structures are of course important, but they can no longer be considered as purely mathematical. They are not invented for the sake of clarifying the domain of the ideally structural or the inner laws of nature, but for the sake of their use in a context of human practice. In the perspective of pure mathematics they are clumsy and opportunistic. They have nothing of the proverbial mathematical elegance, their adequacy cannot be rigorously proven and their functioning cannot be completely tested.

Mathematicians as well as metaphysicians stand here awkwardly looking at something of which they claim to know the principles, but to which they cannot apply them. The two may become brothers again. But before this new brotherhood is celebrated, it is

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advisable to analyze the past period of rivalry, in which victory seemed to dwell on the mathematical side.

**Metaphysics in a mathematical style, and its fate**

Mathematical abstraction results in a certain structure, which is essentially one of the specific realizations of the structurability of a field of experience, and therefore it is contingent. Mathematical structure is grasped by - ideal or real - actualization of the potency of all things of our experience to be divided in thought into interrelated parts. This actualization essentially includes arbitrariness, and can in that sense be called a *construction*, although it may very well be a *reconstruction* of a known phenomenon. Philosophical reflection on the other hand aims at necessity, for the coherence of its objects - which I shall call *principles* - cannot depend upon tradition, convention or postulation. Any blending of mathematical and philosophical reflection bears the suggestion that there exist *necessary constructions*, which is a contradictio in adjecto. So if metaphysics is implicitly contaminated with such a blending, it is an easy prey to criticism depicting it as either absurd or obscure. A construction has definite inner relationships, definite elements and definite properties. All these are definite, because they have been *defined* to be such as they are, and this means that there is arbitrariness in them. Principles nor their relationships, on the other hand, can be understood as the result of definition, they must on the contrary be *presupposed* in definitions. They constitute the perspectives in which we can try to conceptualize or reconstruct experience. Their relationships are beyond definition, because they are constitutive for the meaning of a definition. Nevertheless, in their implicit form, these relationships are better known than explicitly defined structures. They are implicitly but effectively known to us, and attempts to express them explicitly are experienced as highly artificial. They are not axioms, nor 'necessary truths,' nor expressible in a judgement or theorem without already presupposing them. We can investigate them, but we can never use them, apply them or draw conclusions from them *outside* the perspective they constitute. Yet, if we want to investigate principles, we must somehow express the results of this investigation. This is where the difficulties begin, for how to express such results in a form which must necessarily be determined either by tradition or by construction? Philosophy seems to hesitate continuously in its form of expression between mathematics and literature.

Literature is suggestive to us on the basis of culture and tradition. It can express truth, it can make one think, and it can point towards insights into necessary connections. But it lacks liability to critical investigation of its evidence. It either convinces or does not, but in the latter case one can rarely lay one's finger on the spot.

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9) If an axiomatic theory is e.g. understood as a definition of a particular kind of structure (not of a definite structure, for all interesting theories are non-categorical), this presupposes the consistency of the theory. But it follows from Gödel's well known results that this consistency requires a *stronger* theory to be proven. The real reason why we believe in the consistency of the theory is, that we believe we already understand nature of the kind of structure it is meant to deal with. And we believe this, because we are thoroughly convinced of the applicability of the principle of structurability to a certain field of experience. Therefore, the principle of structurability is a presupposition of any mathematical theory.
Reducing philosophical prose to ‘literary text’ means depriving it of its real ambition: expressing intelligible necessity as such.

Mathematics, on the other hand, owes its intellectual force and its certainty to the systematic representation of its objects. In its various forms of representation there exists a structural relationship between the intended mathematical objects and the way they are expressed. This specifically mathematical relationship of sign and meaning is not necessary in a strict sense, but it characterizes a mathematical discipline so strongly, that within the discipline it appears as necessary. Geometry without figures and algebra without formulas is not impossible, and in some periods of the development of these disciplines purely linguistic expression was even normal, but, as Leibnitz observes, it is very hard in this way to travel a long distance without getting exhausted.10

Philosophical systematization, however, cannot aim at representing certain structures in such a rigorous way. It has to transcend its own particular structure, not into a literary expressive imagination, but into the intellectual challenge of its proper aim: establishing real insight. Such systematization has the function to prevent thought from stopping at too low a level of understanding, it provides the formulations of the problems, but it is never itself a solution. Philosophy is the encouragement of the intellect to recognize that it knows more than it thinks it knows. Participating in philosophical thought always requires that we give up some prejudice concerning what we imagine to be the definition of ‘knowing.’ This distinguishes the intellectual challenge in philosophy from its mathematical counterpart. In mathematics the challenge is directed towards the faculty of imagining of and reasoning about new and unheard-of structures. The principle of ‘knowing’ in mathematical reflection, however, always remains the same: structurability. In philosophy there is no fixed principle of knowing, only the attempt at explicitly knowing the principles guiding all of our knowledge. The ‘exactness’ of philosophical expression, therefore, is of a negative nature. Its function is to prevent a premature feeling of understanding. All beginning students of philosophy complain about this. They justly feel that philosophical language aims at making things more difficult instead of easier. Why cannot this be said in a more simple way? In a certain sense this resembles the situation in mathematics. There too things are said in a complicated form. But one feels that the reason for it is, that they are really complicated. In philosophy, however, anyone who has feeling for what it is all about, becomes convinced that understanding the complicated cannot be the ultimate aim here. Principles must be simple, and it is because of their simplicity that it is difficult to grasp them. So why cannot simple things be expressed in a simple way? The answer of course is, that simple expressions suggest to the untrained the wrong kind of simplicity of the content. In ‘ occult disciplines’ of certain religious societies this is no problem, because the expressions

are only meant for the initiated, who are supposed to understand them properly. Philosophy, being a rational discipline, must necessarily provide its own initiation. It cannot separate a cult of initiation from the expression of its actual contents. In a rational discipline one becomes initiated because one takes up the intellectual challenge, one understands what is interesting about it, whereas in initiation rites one participates not in the first place because one understands what they are good for, but because someone with authority says they must be undergone in order to understand what they were good for afterwards.

Mathematical and philosophical expression have, as we now understand, diametrically opposed criteria of adequacy. Mathematical expression is better, in the measure in which it allows us to connect mathematically the structure of our language with the structures expressed in that language. The more rigorous this connection becomes, the more our way of expression gains the character of a formalism useful for accurate proof and computation. Philosophical expression, on the other hand, is better in the measure in which it prevents the intellect from clinging to certain definite structures of knowledge and self-expression. Mathematical language should enable us to concentrate on definite structures, philosophical language should prevent such concentration with the aim of opening up our minds for the origin of our perspectives without presupposing any initiation into extraordinary realms of experience.

For this reason any attempt to develop metaphysics following the mathematical paradigm must necessarily end in the fundamental rejection of all forms of metaphysics. If both disciplines start to claim a common domain, they become rivals, and if this common domain is structurability, mathematics is in for a glorious victory. The dilemma between the mathematical and the philosophical criterion of adequacy of expression is unsolvable. There is no dialectical solution either, because to choose for dialectics is already to choose for the philosophical criterion. As soon as it is tried to understand dialectics as a formalism in the sense of mathematical logic, complete rejection of it is not far behind.

On the one hand to choose the mathematical criterion for philosophy, leads to nihilism. If, on the other hand, mathematical reflection acquires metaphysical pretensions, it cannot very long remain content to be pure mathematics. It has to incorporate some philosophical reflection, and in the measure in which it succeeds in expressing this incorporation explicitly, it disqualifies itself as mathematics. In the measure, however, in which it succeeds in satisfying the requirements of mathematical expression, it becomes philosophically irrelevant. In its naive form it becomes dogmatic because it postulates some more or less arbitrary constraining framework, which nevertheless is infected by the metaphysical claim that it expresses necessity. In reaction to this dogmatism it then becomes nihilistic, for the arbitrary character of the construction is brought to the foreground. It will be insisted then that ‘anything goes.’ In this case, ‘anything’ is not really anything of course, but any construction, and that is not what we are looking for in metaphysics. Therefore this trail leads us into nothingness.

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In fact such a development has taken place in contemporary philosophy. The period in the history of philosophy which is currently labelled ‘modern philosophy’ ends with Hegel’s famous system. The tension between systematic rigour on the one hand, and the aim of expressing the openness of the human mind and the dynamical character of thought on the other, is still reconciled here. The dogmatic and nihilistic side are still held together in the truly philosophical conception of the absolute idea. But the suggestion is very strong that the absolute idea, binding together all principles like a ‘one ring’\footnote{See J.R.R. Tolkien, The Fellowship of the Ring.} is not only meant to be completely intelligible to us, but also to justify the specific structure of Hegel’s system. Yet this suggestion is somewhat misleading, because Hegel himself never hesitated to make additions and corrections. The problem is, that in its systematic structure, there is no place for expressing this openness \textit{with respect to the structure itself}. This is a curious paradox: the system aims at the expression of the transcendental openness of the human mind, that is its ability to grasp transcendental principles by intellectual perception; yet it is not able to express this openness with respect to the philosophical method by which it is composed! The system, therefore, still has some traits of the ‘necessary constructions’ of modern philosophy. This is precisely the impossible contamination of mathematics and metaphysics which tends to discredit all modern metaphysical positions,\footnote{Heidegger’s notion of ‘Seinsvergessenheit’ can be interpreted as a philosophical expression of this confusion. It is coined to criticize ontological fixation of the opposition of subject and object. Such a fixation is a characteristic of mathematical reflection. It seems to be this mathematical element in modern metaphysics, which falls under Heidegger's criticism, and that makes it also clear why he understands modern technology as the realization of such metaphysics. The ‘Verdinglichung des Seins,’ the blurring of the ontological difference, reminds us of what is done in mathematical reflection: creating ideal entities as actualizations of a potency. This potency - structurability - is of another order than its ideal actualizations - the mathematical objects -, and it is indeed ‘forgotten’ and inexpressible in mathematical thought. In the mathematical degree of reflection mathematical being as such is indeed in a certain sense \textit{absent}, but absence cannot be written on the account of ancient and scholastic metaphysics: as a metaphysical trend it is thoroughly modern. Heidegger understood rightly that the confusion of the mathematical and the philosophical degree of reflection, which he did not interpret as a confusion but as a fate - \textit{Seinskgeschick} -, must necessarily lead to nihilism.} and which seems to be the basis of the widespread present consensus on the impossibility of philosophical systems.

But the ‘\textit{faute hypercorrecte}’ is as usual in philosophy as it is in practice. Because it has not become clear that it is the mathematical paradigm which still constituted the trap of Hegel’s system, philosophical positions opposing to German idealism or to modern philosophy in general, such as Marxism, vitalism, existentialism, positivism and structuralism, however critical they are with respect to the modern tradition, are by no means free from this same paradigm. Essentially they all switch to and fro between the dogmatic form, which suggests a necessary construction, and the voluntaristic form, which essentially expresses the abstract notion of a freedom which is only limited by the consequences of its own decisions, such as only the
mathematician really possesses in relation to the sphere of ideal structures. Those philosophies all present themselves as absolutely valid insights on the one hand, but reject any claim to knowledge of the absolute on the other. To such philosophical currents, metaphysics counts as an ideological claim to authority which hampers human freedom. The 'necessary construction' is deconstructed and shown to be only one of infinitely many possible ones. Dogmatic systematics has passed into dogmatic nihilism and the project of metaphysics as such has become suspect. 14 Only a clear recognition of the role of the mathematical paradigm in the process leading to this conclusion can still save it.

14) Th. Adorno expressed the suspicion that this anti-metaphysical trend has been a process of flight from something which could not be left behind. "The process by which metaphysics continuously ended up where it was conceived to lead away from, has reached its vanishing point" [Th. W. Adorno, Negative Dialektik, Suhrkamp, Frankfurt a/M, 1966 p.356.]