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A specification system for statistical software

V.J. de Jong
"Unfortunately statistics has got divided up. There is a U-group called Mathematical Statisticians and a Non-U group called Applied Statisticians. The effect of all the U-manship has, not surprisingly, been to produce a U-shaped distribution of talents with these two groups of people either ignoring each other or else eyeing each other distrustfully and getting further and further apart. The result is that instead of having a productive iteration between theory and practice, which history and common sense both show is the key to progress, we have theoreticians with less and less acquaintance with the real world, and we have work being done by (and advice being given by) applied people having less and less acquaintance with theoretical ideas."
ACKNOWLEDGEMENTS

A large piece of software is something that one can not develop on his own. A major influence in the choice of the design tools was exerted by Paul Klint from The Center of Mathematics and Computer Science in Amsterdam. He ‘forced’ me to use the formal specification language ASF. Though it seemed to lead me further away from my goal of implementing a specification environment for statistical software, it saved me from getting lost in the complexity of a large computer program.

Also I am indebted to Larry Menninga from Western Washington University, who spent his sabbatical leave at the University of Groningen. He introduced me in the world of computer science. Many of his solutions to problems are used in the prototype of CONDUCTOR.

It is, of course, hardly possible to know all the details of existing statistical packages. In this respect I greatly benefitted from stimulating comments of Leo van der Weele.

The prototype of CONDUCTOR was constructed with assistance of Theo Stam and Jan-Herman Veldkamp. They have implemented several modules and spent many hours finding bugs in the system.

I also would like to thank the Computer Science Department of the University of Groningen for letting me be their guest on their UNIX system. Without this system a project of the size of CONDUCTOR is hardly possible.

Finally I must thank Tom Wansbeek, Albert Verbeek and numerous other colleagues for reading preliminary drafts of this book.
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PART I

BASIC CONCEPTS AND TOOLS
1. **INTRODUCTION**

Statistical software developed in the last two decades can only be maintained by professional programmers. It goes without saying that experts in statistics lacking both the required programming skills and the knowledge of the architecture of the software are unable to maintain statistical software. Improvements in statistical techniques, therefore, first have to be explained to programmers. This takes time, costs money and causes errors. In software engineering the problem of *maintainability* is considered to be enormous. Wiener and Sincovec [1984] estimated that maintenance often accounts for more than 80% of the life cycle cost of a software product. A survey by Lientz and Swanson [1980] revealed that approximately 65% of maintenance was preventive (i.e., was the result of changes demanded by the user after the first version of the software product was finished).

An important development that may improve the maintainability of software is the use of *specification* systems. In these systems a *specification language* is available in which the design of the software can be expressed. The specification language is not just some variant of a conventional higher-level programming language. Rather it is a language which is much closer to the problem domain, enabling domain experts to add their knowledge to the system. A program written in a specification language is automatically transformed into an efficient executable program. An instruction in a specification language is equivalent to a large number of instructions in a higher-level programming language. Higher-level programming languages are often referred to as third generation languages; the specification languages evolving from these
languages are also called fourth generation languages. An example of a specification system is given in Cheng et al. [1984]. This system allows managers to create software for financial accounting.

In this book we describe a specification system for statistical software. Developing and maintaining statistical software is a combined effort of:

- **Technical statisticians**. This group of experts must implement and maintain the statistical techniques in the statistical software.
- **Data experts**. This group of experts must implement and maintain the database in the statistical software.
- **Computer scientists**. This group of experts must implement and maintain the computer science aspects, such as the numerical methods and the user interfaces, of the statistical software.

In a specification system for statistical software each of these groups must be able to contribute their knowledge in a language close to their problem domain. The main design goal of a specification system for statistical software is to orchestrate the individual contributions of each of these groups.

```
  technical statisticians
    |
    |
  computer scientists ----> SPECIFICATION SYSTEM ----> statistical software
    |
    |
  data experts
```

Of course the subdivision of experts is arbitrary. Each group of experts itself again is heterogeneous. For the construction of a specification system for statistical software, however, it is only relevant that the experts in each group are able to express their knowledge in the same language.

A crucial step in the design of a specification system for statistical software is the choice of an appropriate language for each of the groups.
For technical statisticians we created a new language: the *statistical language*. For computer scientists and data experts we chose already existing languages. For computer scientists this is a higher-level programming language and for data experts a database design language. Even though these languages have many drawbacks, in this book no effort is undertaken to improve them. For a discussion on available higher-level programming languages see for example Pratt [1984]; database systems and database design languages are discussed in for example Ullman [1985] and Date [1981].

The statistical language enables a technical statistician to add his knowledge to statistical software in such a way that it can be used by applied statisticians. In the statistical language he can specify for each statistical technique:
- the required input data,
- the resulting statistics,
- the equations describing the calculation of the parameters,
- (pre)tests,
- advice for the user of the technique.

To give concrete form to our ideas in this book a prototype of a specification system for statistical software is developed called CONDUCTOR. To simplify the construction of CONDUCTOR, the current version focusses on econometricians, a subclass of all technical statisticians (CONDUCTOR can of course be tailored for technical statisticians). The statements in the statistical language resemble the mathematical notation used in econometric textbooks, such as Maddala [1977] and Judge et al. [1980]. A technical statistician may use a subset, *matrix notation*, of the mathematical notations used to describe estimation techniques as in these books. In this notation the dimension bounds of matrices are index expressions. This is one of the differences between the statistical language and other matrix languages, like APL (see Pakin [1972]) and SAS/IML [1985].

The main goal of a specification system for statistical software is to produce efficient software. The matrix languages APL and SAS/IML can only be used for what is called rapid prototyping. This is due to the fact that programs written in these languages are interpreted and not compiled. As a result many of the optimization techniques used in compilers for higher-level programming languages can not be applied. If
the programs in matrix languages however could be compiled, the performance of the resulting software would increase dramatically.

A part of the compilation of a program is called type checking. In type checking the type restrictions on particular constructs of the language are verified. A matrix operator, for example, can be restricted to operate on two matrices with equal dimension bounds. The use of symbolic dimension bounds in matrix types complicates the type checking of the statistical language. An important part of this book tries to tackle this problem. Solving this problem removes one of the bottlenecks towards the construction of a specification system for statistical software.

The user of a statistical technique initializes the index variables that determine the dimension bounds of the matrices during the execution of the statistical technique. The index variables must be initialized in such a way that no type or dimension bound conflicts occur during the execution of the statistical technique. In software generated by a specification system for statistical software, the user is not aware of the fact that matrices are used in the statistical language. All error messages concerning matrices are therefore meaningless to a user, and should be avoided as much as possible. **Type and dimension bound restrictions** in the statistical language should therefore be checked during the compilation of a statistical program (static type checking).

A matrix with symbolic dimension bounds, as defined in the statistical language, is the equivalent of a dynamic array of reals in a higher-level programming language. Static type and dimension bound checking of dynamic arrays in a higher-level programming language, however, is impossible. A program in these languages may have infinitely many execution trees. That is, there are infinitely many paths leading through a program. To prove that the type and dimension bounds restrictions are not violated in these programs is tedious and in practice often impossible. In higher-level programming languages, and in other matrix languages one, therefore, relies on run-time checks on these restrictions.

The statistical language of our prototype CONDUCTOR has a less complicated control structure than higher-level programming languages:
1. it does not have conditional statements,
2. the only loop control statement is the for-statement,
3. a for-loop is executed at least once,
4. loop control variables may not be reassigned inside a loop.
5. the input index variables may not be reassigned.
6. index expressions are monotone non-increasing or non-decreasing.
7. dimension ranges are strictly positive.

Due to these properties a statistical program in CONDUCTOR has a unique symbolic execution tree: an unconditional symbolic expression, in terms of the input variables, can be calculated for each index variable after each statement in the program. The symbolic expression calculated for the dimension bounds of matrix types are used by the symbolic type and dimension bound checker in CONDUCTOR. If a type or dimension bound restriction can not be verified symbolically, input restrictions are generated on the input index variables in a statistical program. The input variables of a statistical technique are the variables that the user of the statistical technique must initialize in the generated statistical software. This group of variables, and the restrictions on these variables, are assumed to have meaning for an applied statistician who uses the statistical technique. Symbolic evaluation was, for example, also used by King [1976] to test higher-level programming language programs with infinitely many execution trees.

The restrictions 1 and 6 on the statistical language are rather strong and seem to demonstrate the impossibility of the creation of a useful statistical language. How these restrictions can be relaxed is one of the things we learned from construction of the prototype. Both restrictions are only imposed in order to make symbolic evaluation of the statistical language possible. We may relax restriction 1 allowing conditional statements, if we impose the restriction that index variables may not be reassigned inside a conditional statement. Under this restriction we still can make an unconditional symbolic evaluation, even though a matrix program as a whole may have infinitely many execution trees. In our prototype CONDUCTOR, however, conditional statements in the statistical language are not implemented. Restriction 6 can be relaxed by introducing an additional data type integer. Of course integers do not have to obey the restrictions on the data type index as long as they are not used in the symbolic evaluation.

Besides the tackling of problems involved in the creation of a statistical language, a major effort discussed in this book is the definition of the interaction between parts of software generated by the different groups of experts. To simplify this definition, in our
prototype CONDUCTOR, a kernel is introduced. The kernel has a processor that can execute the semantic actions that take place during the evaluation of a statistical program. For the kernel a statistical program is a sequence of these actions, also called kernel instructions. Two examples of kernel instruction are:
- an instruction that requests the user of the statistical technique to initialize an input variable,
- an instruction that calls a numerical function.

These kernel instructions and the numerical counterparts of the functions in the statistical language, must be implemented by computer scientists. CONDUCTOR provides the generated statistical software with a database interface. Data in the database is not stored as merely a collection of numbers. Additional information, such as, sample design, instrument and context, collected during the data production process (see David [1985]) can be added to the database. A data expert can write background queries, that are used by the database interface to check the consistency of the retrieved data. When a user of the generated statistical software retrieves a particular series from the database the background query is automatically invoked to do the required consistency checks. Thus detailed knowledge of the data production process is hidden from the user of the data, yet inconsistent data is labeled by the database interface. CONDUCTOR allows each group of experts to look at a statistical technique from their own level of abstraction. An expert at each of these levels might detect that the execution of a statistical technique must be interrupted because necessary conditions for execution are not satisfied. In computer science such conditions are called exceptions, and causing an interrupt is called raising an exception (see Goodenough [1975]). In systems, that must remain in continuous operation, it is important that the execution of a program is not stopped when an exception occurs. Several higher-level programming languages, like ADA, PL/I and PL/C, therefore, provide facilities for exception handlers. When an exception is raised, control of the program is passed to the exception handler. After completion program control is returned to the point where the exception occurred.

Statistical analysis is seen in CONDUCTOR as a continuous process. During statistical analysis, it often happens that a statistical technique can not calculate the required statistics, because necessary conditions are
not satisfied by the analyzed data. Such a situation is seen as the occurrence of an exception. Yet the applied statistician wants to continue the analysis and needs advice. Are alternative statistical techniques available that can tackle the problem? Or do data preprocessing techniques make analysis of his data set possible? The answers, of course, should be given by the experts. The exception handling mechanism in CONDUCTOR allows the experts to provide this information. Exceptions in CONDUCTOR are raised when:
- test results indicate that the applied statistical technique is inadequate for the analyzed data (detected by the software created by the technical statistician),
- computational problems make the calculation of a numerical function impossible (detected by the software created by the computer scientist),
- data is missing or inconsistent (detected by software created by the data expert).

When an exception occurs, the kernel of CONDUCTOR looks for an exception handler. Exception handlers describe how the statistical software must react if an exception occurs. Exception handlers can be written by either a technical statistician, a data expert or a computer scientist. For the kernel, exception handlers are independent sequences of kernel instructions. The exception handler mechanism opens the possibility for experts to react on exceptions raised by one of the other groups of experts. An exception handler implemented by a technical statistician, for example, can handle an exception that is raised by a numerical procedure implemented by a computer scientist.

To summarize, the main design goal of a specification system for statistical software is to offer experts in computer science, data collection and technical statistics the possibility to implement and maintain their own restricted contribution to efficient statistical application software. Technical statisticians are trained to think in terms of matrix notation. The widespread use of matrix languages, like APL, among these experts is not surprising. The use of these languages is, however, only suited for rapid prototyping. One can not develop and maintain efficient application software in these languages. In the developed prototype CONDUCTOR, a technical statistician can add his knowledge in a statistical language. CONDUCTOR transforms a program,
written in this language, into efficient statistical software. It combines the software written by the technical statistician with software written by computer scientists and data experts. And, it also generates the appropriate input restrictions.

An applied statistician should not notice the difference between existing statistical software and software produced in CONDUCTOR. He can apply statistical techniques to explore data gathered by data experts. Software constructed in a specification system, however, has two big advantages compared with existing software. The first advantage lies in the maintainability of the generated statistical software. New developments in the scientific areas of the different expert groups can be implemented without the need for deliberation with experts in one of the other groups. This will make modern techniques in technical statistics, data production and computer science more rapidly available for applied statisticians. The second advantage is, that software generated in a specification system for statistical software can produce more than just statistics. When during the execution of a statistical technique an exception is raised, the user can be given advice by the appropriate expert. Technical statisticians, computer scientists and data experts can implement their messages in exception handlers.

Constructing a specification system for statistical software is a large software project. To make such a project successful it has to be thoroughly specified. To specify the system yet an other language is used: a formal specification language. Such a language enables computer scientists (system developers) to give a formal definition of a software project. Yes this is complicated, this book contains a formal specification of a specification system for statistical software. The formal specification language is used to make a blueprint of CONDUCTOR. Both a UNIX and an MS-DOS version of this prototype exist.

For the specification of our prototype CONDUCTOR we use the formal specification language ASF (Bergstra, Heering and Klint [1987]). ASF is based on initial algebraic semantics for algebraic specifications with conditional equations. Modularization mechanisms in ASF, such as parameterization, imports and exports are similar or identical to the ones discussed in Klaeren [1983], Loeckx [1984] and Bergstra et al. [1985]. The formal specification of CONDUCTOR contains definitions of: the data types in the statistical language and in the user language, the
abstract syntax of these languages, the kernel, the translation of a statistical program into a kernel program, the symbolic type and dimension bound checking, and a top level view of the CONDUCTOR environment. A by-product of the CONDUCTOR project is the evaluation of usefulness of ASF.

This book is organized as follows. In the remaining chapters of part I a general introduction is given to the basic ideas behind CONDUCTOR and the formalisms used in the definition of CONDUCTOR. Chapter 2 contains a short review of existing software tools in statistics. Information hiding, as applied in CONDUCTOR, is discussed in chapter 3. Chapter 4 shows an example of an implementation of a statistical technique in CONDUCTOR. The formalisms used in the specification of CONDUCTOR, grammars and algebraic specifications, are discussed in chapter 5. In chapter 6 we discuss the general outline of the formal specification of CONDUCTOR. In part II the definition of the statistical language is given. The specification of the concrete and abstract syntax of this language is discussed in chapter 7, the symbolic type checking in chapter 8, and symbolic dimension bound checking in chapter 9. In part III the formal specification of the kernel is given. In chapter 10 the kernel and the kernel instructions are specified. The translation of the statistical programs into kernel instructions is discussed in chapter 11. The exception handler mechanism of CONDUCTOR is described in chapter 12. In part IV both the user language and the data interface are specified. The user language in chapter 13, the data interface in chapter 14. In part V, we conclude, in chapter 15, with a discussion of the prototype, and, in chapter 16, of the use of specification language ASF. In chapter 17, we discuss the current status of the CONDUCTOR project and suggest future developments.
2. EXISTING SOFTWARE TOOLS IN STATISTICS

Most empirical work in statistics is done with the use of a few leading statistical packages. These so-called general statistical packages contain the commonly used statistical techniques. Well known examples of general packages are SAS [1982], SPSS [1986] and BMDP [1985]. For statisticians with a more specialized field of interest also specialized software exists: the special purposes packages. Good examples of special purpose packages are LISREL, a package tailor-made for the estimation of parameters in models with unobservable variables (see Jöreskog and Sörbom [1981]), and TSP [1980], a package for time series analysis. If a statistician wants to use a statistical technique which is not contained in any of the statistical packages, he has to use a higher-level programming language. In the statistical community, frequently used higher-level programming languages are FORTRAN, PASCAL, PL/I and C. Also matrix oriented languages, like APL and SAS/IML, are popular among statisticians. In this chapter we will briefly discuss the flexibility and ease of use of both statistical packages and programming languages.

2.1. FLEXIBILITY AND EASE OF USE OF SOFTWARE TOOLS

Statistical packages make statistical techniques available for large groups of users. In order to reach this goal the statistical techniques are implemented as a 'black box'. The user only has to give the input data and the package returns an impressive amount of statistics.
Getting output from a statistical technique in statistical packages, as a result, requires only minimal knowledge of the statistical technique. An example is the use of the instrumental variables technique (INST) in TSP. INST is a statistical technique that can be used if the explanatory variables in a linear model are correlated with the disturbance term. In such a situation ordinary least squares estimates are inconsistent. To solve this problem a group of variables is sought that is both highly correlated with the explanatory variables, and is uncorrelated with the disturbance term. These variables are called instrument variables. INST uses these variables to produce consistent estimates of the coefficients in the linear model. In the TSP program fragment below, a consumption equation is estimated using INST. The first statement in this example gives the name of the program. The second statement in the program loads the data. The third statement gives the equation that is to be estimated. In this example the dependent variable is consumption and the explanatory variables are income and consumption price. The variables import price, export, import and a constant are used as instrument variables, as specified in the second part of the statement.

```
INST program fragment
name example "instrumental variables example";
load;
INVR consumption income consumption price
   INST import price export import constant;
stop;
end;
```

To get output from INST in TSP only modest knowledge of this statistical technique is needed. The main effort lies in accessing and manipulating the data. Other statistical techniques can be used in a similar way. Of course, for the interpretation of the output, knowledge of INST is highly recommended. Unfortunately this requirement is never enforced by statistical packages. Table 2.1 gives an overview of available techniques for the estimation of equations in standard packages as reported by
Francis [1981], Rodler [1985] and Srba [1985]. The even larger market of statistical software for microcomputers is for example discussed in Woodward, Elliot and Gray [1985] and Van Nes [1987].

The 'black box' approach also has its price from a software engineering point of view. Modifications in the software can only be made by experts with considerable programming skills, knowledge of the statistical technique and knowledge of the architecture of the package in question. And even if one has such rare skills, most commercial software producers do not make available the source programs of their statistical packages. Thus in practice the 'black box' packages are inflexible.

In recent years many features have been added to statistical packages in order to improve the flexibility. Examples of such features are (1) parameters allowing the user to choose from different options of a statistical technique, (2) macro facilities and (3) interfaces with subroutines written in higher-level programming languages. Mostly the ease of use of a statistical package (sp) decreases when these facilities are added, while of course the flexibility improves.

<table>
<thead>
<tr>
<th>high</th>
<th>high level programming language(hlp)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>blp + subroutine library</td>
</tr>
<tr>
<td></td>
<td>sp + subroutine library</td>
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<tr>
<td></td>
<td>sp + macro facilities</td>
</tr>
<tr>
<td></td>
<td>sp + options</td>
</tr>
<tr>
<td>low</td>
<td>standard packages(sp)</td>
</tr>
</tbody>
</table>

If the statistical packages do not contain the desired estimation techniques, a statistician has to use a higher-level programming language. The disadvantage of using a higher-level programming language is obvious: one has to start all over again. Not only the statistical technique has to be implemented, but also user interfaces, report facilities, documentation, etc. This is time consuming. In most cases, after a promising start, the new software engineer ends up in a labyrinth of problems. Only a few will find a reasonable way out. A result is that statisticians write programs that are used by a few friends at most.
Table 2.1 Available estimation techniques in a few leading standard packages for econometric applications.

<table>
<thead>
<tr>
<th>statistical package</th>
<th>IAS</th>
<th>TSP</th>
<th>SAS/ETS</th>
<th>TROLL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ordinary least squares</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>with AR(1)-correction</td>
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<td>x</td>
<td>-</td>
<td>x</td>
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<tr>
<td>CORC</td>
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<tr>
<td>HILU</td>
<td>x</td>
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<td>-</td>
<td>x</td>
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<tr>
<td>ML-proc</td>
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<td>x</td>
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<td>iter.</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>-</td>
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<tr>
<td>2. generalized least squares</td>
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<td>x</td>
<td>-</td>
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<td>3. non-linear least squares</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
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<tr>
<td>4. two stage least squares</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
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<td>5. instrumental variables</td>
<td>x</td>
<td>x</td>
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<td>x</td>
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<td>6. k-class estimator</td>
<td>x</td>
<td>-</td>
<td>x</td>
<td>x</td>
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<tr>
<td>7. limited information</td>
<td>x</td>
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<td>x</td>
<td>x</td>
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<tr>
<td>maximum likelihood</td>
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<tr>
<td>8. full information</td>
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<td>x</td>
<td>-</td>
<td>x</td>
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<tr>
<td>maximum likelihood</td>
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<tr>
<td>9. limited information</td>
<td>x</td>
<td>-</td>
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<td>-</td>
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<tr>
<td>instrumental variables</td>
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<tr>
<td>10. full information</td>
<td>x</td>
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<tr>
<td>instrumental variables</td>
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<tr>
<td>11. three stage least squares</td>
<td>x</td>
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<td>x</td>
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<td>12. non-linear 3-stage least squares</td>
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<td>x</td>
<td>x</td>
<td>-</td>
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<tr>
<td>13. non-linear multivariate</td>
<td>-</td>
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<td>x</td>
<td>-</td>
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<tr>
<td>regression</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

source: Rodler[1985].
Making the technique available for a large community is simply too much of an effort. One dreams of a kind statistical tool box in which previous efforts can be reused to create software for new developed statistical techniques. A specification system for statistical software is a meant to make that dream come true.

2.2. TWO INSPIRING EXAMPLES

Two software tools developed by large organizations formed an inspiring example for the construction of the statistical language in CONDUCTOR: S, developed by AT&T Bell Laboratory, and IML, an interactive matrix language developed by SAS. S is a software tool for data analysis and graphics. It emphasizes interactive analysis and graphics, ease of use, flexibility and extendibility. S is developed at AT&T Bell Laboratories and is currently in use under the UNIX operating system. An extensive treatment of S is given in Becker and Chambers [1984a], a short overview of S can be found in Becker and Chambers [1984b]. The design goal of S is stated by Becker and Chambers as: "to enable and encourage good data analysis, that is to provide users with specific facilities and a general environment that helps them quickly and conveniently to look at many displays, summaries and models for their data and to follow a kind of iterative, exploratory path that most often leads to thorough analysis". Particular interesting features of S are:

- in S the language resembles common algebraic notation, using operators and functions,
- S has an interface to user-written functions, which allows functions to be written in a higher level programming language,
- S is centered around "an executive": an interactive parser that parses and evaluates the expressions; "the executive" is an interpreter,
- S focusses on a research environment where statisticians continuously develop new techniques and thus is highly extensible,
- there is a special value NA (not available) which can be used to signify missing data,
- S allows the use of vectors and matrices with fixed dimensions,
- changes in "the executive" of S should not require changes in the
code of the user.

IML is a programming language developed by SAS. The basic data elements
of IML are matrices. IML can be seen as a successor to the programming
language APL in which the rather cryptic special symbols in APL are
replaced by a more familiar notation. IML tries to let the user think in
terms of matrix notation. One of the big advantages of IML is that it can
be used in combination with other software products of SAS such as
SAS/GRAPH, which makes IML a powerful tool for matrix oriented
scientists. Some interesting features of IML are:
- the matrices in IML are dynamic. The dimension and type of a variable
can be changed at any time in a program,
- IML contains a large set of matrix functions and operators,
- no declarations are required in IML, the attributes of a matrix are
determined when the matrix is given a value (late binding),
- IML allows data processing,
- IML provides graphic commands.

For a detailed description of SAS/IML see the SAS/IML User's Guide
[1985].

Both S and IML are excellent tools for technical statisticians to tackle
their problems. Neither tool, however, is intended to be a specification
system for statistical software. Programs written in S or IML are used by
their creators, and are not meant to be used by others. Both S and IML
are suitable for what is called rapid prototyping, and do not produce
efficient application software. A program in the statistical language of
CONDUCTOR is compiled in order to make efficient execution possible,
whereas a program in S or IML is interpreted. The difference between the
statistical language in CONDUCTOR and both S and IML is, among other
things, reflected in the fact that the matrices in CONDUCTOR's
statistical language may have symbolic dimension bounds. Whereas in S and
IML the exact dimensions at any time during the interpretation of a
program are given.
3. INFORMATION HIDING CONCEPTS

In a specification system for statistical software, where different experts cooperate, knowledge implemented by one expert must be completely transparent for experts in other scientific disciplines. This concept, called information hiding, is well-known in computer science. In this chapter two basic concepts of information hiding are discussed: the multi-layered approach and modularization. Both in the multi-layered approach and modularization, a problem is tackled at different levels of abstraction. The difference between the two concepts of information hiding is whether or not a separate language is defined at each level of abstraction. In the multi-layered approach a separate language is created for each level, whereas in modularization the solutions of problems at different levels of abstraction are expressed in the same language.

3.1. THE MULTI-LAYERED APPROACH

Consider running a statistical program on a computer. A user of a statistical program gives an instruction, using the command language of the statistical program, to calculate certain statistics. This instruction is equivalent to a large number of micro-code instructions that are executed by the hardware of the computer. In modern computers many intermediate levels exist between the hardware level of the computer and the statistical program level. The statistical program level is like the top of an iceberg. Underneath the surface are a lot of other levels. Instructions in the statistical language are interpreted by a program
written in a higher-level programming language. Statements in the higher-level programming languages are compiled into statements in lower-level languages until finally the hardware level is reached where the instructions are executed by the electric circuits of the computer. For a discussion on the multilevel architecture of computers see Tanenbaum [1976]. Another beautiful example of the multi-layered approach is in the construction of distributed database systems (see Ceri and Pelagatti [1985]).

The multi-layered approach makes it possible to make modifications at one level without influencing the other levels. Changes can be made at, for example, the assembly level without influencing the other levels. Of course, certain changes require that the interfaces between the levels also must be modified. The advantage of the multi-layered approach is that problems can be solved at the appropriate level of abstraction, in a language close to the domain language of the expert who must make the changes.
3.2. MODULARIZATION

At one level of abstraction in the multi-layered approach, a problem can be so complex that it has to be divided into subproblems that can be tackled separately. This form of information hiding is called modularization. In contrast to the multi-layered approach, all subproblems are solved using the same language.

In the academic world it is considered to be 'self-evident' that large and complex pieces of software are constructed using the principles of modular design. Using these design principles in practice, however, appears to be difficult. Parnas et al. [1985] have specified the following goals of module decomposition:

- each module's structure should be simple enough to be understood fully,
- it should be possible to change the implementation of one module without the knowledge of the implementation of other modules and without affecting the behaviour of other modules,
- only very unlikely changes should require changes in the interface of widely used modules,
- it should be possible to make a major software change as a set of independent changes to individual modules,
- a software engineer should be able to understand the responsibilities of a module without understanding the details of the internal design,
- a reader with a well-defined concern should easily be able to identify the relevant modules without studying irrelevant modules,
- the number of branches at each non-terminal module should be small enough that the designer can give convincing arguments that the submodules have no overlapping responsibilities.

For large projects the number of modules is enormous and the modular design principles are difficult to check. Therefore much effort is undertaken to improve modularization techniques and tools. Important in this respect are the development of new programming languages and formal specification languages.

Again a warning for the reader. Do not confuse formal specification languages with the statistical language. A formal specification language is meant for software engineers who develop any kind of software, not
just statistical software. Using this language a software engineer can formalize the requirements and properties of his software. In this book it is used to formalize the requirements of a specification system for statistical software. The statistical language is part of this system that has to be formalized.

3.2.1. Programming languages.
New programming languages facilitate the construction of modular software. Important languages in this respect are ADA and MODULA-2. The ADA programming language was developed at the initiative of the U.S. Department of Defense (USDoD) between 1979 and 1983. In April 1979 a language design team, headed by Jean Ichbiah of CII Honeywell-Bull won a four-way competition for the best language design. This design was thoroughly tested and revised between April 1979 and July 1982. Nevertheless many computer scientists hold critical views with respect to ADA. For a further introduction to ADA see Wiener [1983] and USDoD [1983]. MODULA-2 was introduced by Niklaus Wirth, the founder of PASCAL, in 1980 (Wirth [1983]). The MODULA-2 programming language overcomes many of the deficiencies of PASCAL. It combines PASCAL’s simplicity with much of ADA’s power. ADA and MODULA-2 provide facilities for reducing two major difficulties in large scale software design:
- poor interface between separate software components,
- interference between components because shared data (global data)
  is incorrectly modified by some program unit.
As a result the interfaces between separate modules in ADA and MODULA-2 are precisely defined.
Though ADA and MODULA-2 are excellent programming languages for large scale software development, they are not the type of language in which non-computer scientists, like statisticians, easily maintain and develop software. The languages simply contain too many features irrelevant to non-computer scientists, while other necessary features, like dynamic arrays of reals, are missing. Another problem is that, even though ADA and MODULA-2 are superior to earlier programming languages, FORTRAN still has the historic advantage that many procedures are already available in procedure libraries like IMSL and NAG.
3.2.2. Formal specification.

The new programming languages offer the possibility to implement modularized projects. An even bigger problem is how to modularize a large project. Many modularization techniques exist these days that offer a discipline to modularize problems. An example is the Jackson-design method (see Jackson [1975,1983]). Not surprisingly, no algorithm has ever been found to modularize large scale problems. The process of modularization remains and probably will always remain dependent on the creativity of the problem analyst. Therefore, it is still very important that a system analyst thoroughly defines his problem before he actually starts implementing it. This specification process, however, is also not without problems. The pitfalls in specifying a software project were listed by Meyer [1985] as "the seven sins":

- **noise**: elements in the specification do not add information,
- **silence**: aspects of the problem are not treated in the specification,
- **overspecification**: aspects of the specification do not deal with the problem but with a possible solution of the problem,
- **contradiction**: elements of the specification contradict with other elements in the specification,
- **ambiguity**: elements of the specification can be interpreted in more than one way,
- **forward reference**: elements in the specification refer to problems solved later in the specification,
- **wishful thinking**: there are elements in the specification for which no realistic solution exists.

Some of these sins can be averted if the analyst makes use of a formal specification language. The mathematical notation in such a language is better suited to give a precise description of a problem than natural languages. The language used in this book, for example, helps to remove noise, contradiction, ambiguity and forward reference in the specification of CONDUCTOR. Silence, overspecification and wishful thinking in this specification, however, remain the responsibility of the author of this book.

An important aspect of a formal specification language is, that it has facilities to describe the modularization of a problem. This offers a system analyst the possibility to describe how his problem is divided in
subproblems, before he starts implementing such modules. The formal specification language ASF has such facilities.

3.3. APPLICATIONS OF INFORMATION HIDING

Modularization and the multi-layered approach are applied almost everywhere in computer science. In this section we discuss a few of the applications that influenced the design of CONDUCTOR. These applications are: procedure libraries, the UNIX environment, and very high-level programming producing systems.

3.3.1. Procedure libraries.
A beautiful and simple application of modularization is the standard function in higher level programming languages. These functions can be linked into a program without having to be coded line by line by the programmer who wants to use these functions. For some scientific areas also special libraries are created containing non-standard functions and procedures. Good examples of such libraries are the collection of over 500 mathematical and statistical routines in the IMSL library and the NAG library. Another good example is given by the 400 functions associated with the UNIX "programmers workbench"(see Ivie [1977]).

3.3.2. The UNIX environment.
The UNIX environment contains a variety of facilities that apply information hiding. Besides function libraries, a good example is the UNIX pipe. The UNIX pipe makes whole programs building blocks of larger computational structures. This has led to the development of a literature of specialized programs. These programs structured as simple filters can be applied in many applications.

Other examples of information hiding in UNIX are the shell and the generic facilities. The shell hides the implementation details of UNIX on a particular computer from the UNIX user. Examples of generic facilities are the screen management software (cursors and termcap) and program generators (lex and yacc). The program generator yacc can be used to create a parser for a programming language. The implementation details of the parsing algorithm in yacc are hidden from the user of yacc. A discussion of the information hiding principles applied in UNIX can be
found in Kernighan [1984]. A detailed discussion of UNIX is given in Kernighan and Pike [1984].

3.3.3. Very high-level program-producing systems.
In computer science, a system that in interaction with an expert can produce software for solving the expert's problems, is called a very high-level program-producing system (VHLPPS). Such a system does not accept a variant of a conventional high-level programming language, rather it accepts a language closer to the problem domain of the expert. The system may have a great deal of information built into it, either about the domain or about how to create programs for this domain. Different types of VHLPPS are discussed in Horowitz and Musson [1984]. Examples of VHLPPS are DRACO a system developed by Neighbours and Freeman [1980, 1984], and MODEL developed by Prywes et al. [1977, 1979]). An approach taken by several researchers is to use a formal specification language as the domain language, and try to transform a program in this language into an efficient program. This approach has only been used for small prototypes and the question is if this approach will work for large-scale software projects. Examples of this approach are given in Arsac [1979] and Balzer [1981].

3.4. INFORMATION HIDING IN CONDUCTOR

CONDUCTOR is a VHLPPS: a system for the development of statistical software. CONDUCTOR has built-in knowledge to generate efficient statistical software from the statistical programs written by the statistical expert. Given a statistical program CONDUCTOR generates:
- a user interface,
- an interface with a database,
- input restrictions,
- links with numerical procedures.
Both modularization and the multi-layered approach are applied in the design of CONDUCTOR. The emphasis on the multi-layered approach is reflected by the fact that the two upper levels of abstraction in conventional statistical software, the user command level and the higher-level programming language level, are subdivided into five levels:
- a statistical level: at this level statistical techniques are implemented,
- a data expert level: at this level data experts can specify how data sets are collected,
- a user command level: the command level of the resulting software. It resembles the conventional user command level in statistical software packages. Using a command language a user can apply statistical techniques to data stored in a database,
- a kernel level: statements at the statistical level are translated into kernel instructions. The kernel level is a shell that allows computer scientists, technical statisticians and data experts to make changes on their own level without knowledge of the other levels,
- The higher-level programming level: At this level, computer science experts can add their knowledge to CONDUCTOR, and, of course, CONDUCTOR itself is implemented.

Procedures and functions in libraries are the building blocks of CONDUCTOR: the conductor kernel calls these procedures and functions. Examples are procedures to calculate the inverse of a matrix, and procedures to calculate the eigenvalues and eigenvectors of a matrix. CONDUCTOR allows technical statisticians to combine these procedures in infinitely many ways, freeing them of the burden of the tedious parameter substitution problems involved in the use of these procedures in higher-level programming languages. From the UNIX operating system CONDUCTOR 'steals' the idea of the shell (the kernel of CONDUCTOR), which hides tedious details involved in the implementation of UNIX on a particular computer from the main design of UNIX.

CONDUCTOR consists of the kernel plus the four interfaces with the surrounding levels, to wit a compiler for the statistical language, a user language interpreter, a database interface, and facilities to connect numerical procedures written in a higher-level programming language. The statistical and data expert level are built on top of a higher-level programming level. This is the general outline of a specification system in which statistical and data experts can create, debug and modify their (restricted) contribution to the software.
Creating a specification system for statistical software is a large software project. A necessary condition for such a project to be successful is that all requirements are described, and that the project is modularized. Therefore a complete formal specification of CONDUCTOR is given in the specification language ASF. ADA or MODULA-2 are very good higher-level programming languages to implement CONDUCTOR. One could benefit from the elegant way in which modularization is possible in these languages. CONDUCTOR, however, is implemented in C and runs under both the UNIX and the MS-DOS operating systems. The choice for C was mainly based on practical reasons. ADA and MODULA-2 compilers were not available at the computers used by the author. Yet, C in combination with the UNIX operating system forms an excellent software development environment, and was available on the VAX computer used by the author.
In this chapter it is demonstrated how a statistical technique can be implemented in our prototype specification system for statistical software. As an example we use a statistical technique known as the bootstrap method. After a brief introduction of this statistical technique, it is shown how a technical statistician can implement this technique, and how it can be used by an applied statistician in the resulting software. Furthermore the role of the kernel and the contributions of both the data expert and the computer scientist in this example are discussed.

4.1. THE BOOTSTRAP METHOD

The bootstrap method is a statistical method that can be used to estimate the statistical error of estimated parameters. In this example the method is used to estimate the variance of regression coefficients in a linear model. In the linear model it is assumed that the variation in the dependent variable $y$ can be explained by the variation in the independent variables $X_1, \ldots, X_k$.

Suppose $(y, X)$ is a realization of the random matrix $(y, X)$, whose rows are identical independently distributed with unknown distribution $F$; $X$ is a matrix of order $n \times k$ and contains the observations of the independent variables, and $y$ is an $(n \times 1)$-vector of the observations on the dependent variable. Define
\[ b = (X'X)^{-1}X'y \]

so that \( Xb \) is the orthogonal projection of \( y \) on the space spanned by \( X \) (a general formulation of the well-known regression coefficient in the linear model). The distribution of \( b \) is unknown, but asymptotic theory usually yields useful approximations. The bootstrap offers a valid alternative replacing mathematical analysis in the field of asymptotic theory by 'massive calculations'. Its basic idea is simple: estimate \( F \) by the empirical distribution \( \hat{F} \) of the observations \( (y, X) \) and perform a Monte Carlo study drawing samples from \( \hat{F} \).

\[
\hat{F} = \text{mass } (1/n) \text{ on each observed data point of } y.
\]

More precisely the bootstrap method proceeds as follows:
- draw with replacement a sample from the \( n \) observations of \( (y, X) \)
- calculate estimator \( b \) for the sample,
- repeat the first two steps mentioned above and calculate the average and variance of the sample estimates.

The calculated average and variance are respectively the bootstrap estimates of the expected value of the regression coefficient and the variance of this parameter. For a more detailed discussion of the bootstrap method see for example Efron and Gong [1983].

4.2. IMPLEMENTATION OF THE BOOTSTRAP METHOD

A statistical technique is implemented in CONDUCTOR in the statistical language. A program in this language defines:
- the name of the technique,
- the input variables, the variables that must be initialized by the user, and the output variables, the variables that contain the results of the statistical technique,
- the equations that specify how the required statistics are calculated,
- the tests of either the basic assumptions of the implemented technique, or the significance of the estimated parameters; when the test results are negative an exception is raised
- the exception handlers for exceptions that may occur during the execution of the statistical technique.

For the bootstrap technique we first have to decide which identifier must be used in the resulting software to call the technique. An obvious choice is the identifier 'bootstrap'. This identifier is given in the name section of a statistical program.

```
NAME bootstrap
```

Next it has to determined what are the input and output variables of the bootstrap technique. Clearly the user has to give the observations for the dependent and independent variables, Y and X. Furthermore he must indicate the number of observations n and the number of independent variables k in his application¹. In the example below, also the size of each random sample s and the number of times a sample is drawn d must be determined by the user. The output variables are the estimated expected value of the regression coefficients b and the corresponding variance (var_b).

Messages, that are used to prompt for input, may accompany the declaration of input variables. Similar, a message in an output declaration may explain the calculated result. The input and output variables are declared in the input/output section of a statistical program.

```
<table>
<thead>
<tr>
<th>INTERFACE</th>
<th>INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDEX k</td>
<td>MESSAGE: &quot;number of independents&quot;;</td>
</tr>
<tr>
<td>INDEX n</td>
<td>MESSAGE: &quot;number of observations&quot;;</td>
</tr>
<tr>
<td>INDEX s</td>
<td>MESSAGE: &quot;size of bootstrap sample&quot;;</td>
</tr>
<tr>
<td>INDEX d</td>
<td>MESSAGE: &quot;how many samples must be drawn&quot;;</td>
</tr>
</tbody>
</table>
```

¹ Of course CONDUCTOR can be made smarter by adding a function that can determine the dimensions of a matrix, thus removing the burden from the user to enter this data. In the current version of CONDUCTOR, however, this is not implemented.
The equations in the implementation section of a statistical program specify how the resulting statistics $b$ and $\text{var}_b$ are calculated. The implementation section consists of a part in which internal variables are declared and a part in which the equations are given. Here a remark on the choice of the functions in the specification of the bootstrap technique is on its place. In statistical computing it is well known (see Kennedy and Gentle [1980]) that using the inversion procedure is not the most efficient and stable approach to estimate the regression coefficients. Other procedures, like Cholesky and Householder decomposition, in many cases have better numerical properties. For the design of CONDUCTOR, however, this is irrelevant. The choice of a particular set of functions only influences which experts are able to use the statistical language. To improve the readability of the examples we have chosen the notation used in econometric textbooks. Adding Householder and Cholesky decompositions function to CONDUCTOR would require technical statisticians with more knowledge of the numerical problems in statistical computing. Surely this would improve the efficiency of the generated software, but it would reduce the group of potential users.
VARIABLES
INDEX i,j,sample:
MATRIX [1 to s, 1 to m] X_temp:
VECTOR [1 to s] y_temp:
VECTOR [1 to k] b_temp:

EQUATIONS
beta[i] = 0  i = 1,...,k:
FOR i := 1 TO d DO
{ /* draw sample */
  FOR j := 1 to s DO
  { sample := draw_random_index(i,n);
    X_temp[j,1 to m] := X[sample,1 to m];
    y_temp[j] := y[sample];
  };

  /* calculate the OLS estimator */
  b_temp := inv(X_temp'*X_temp)*(X_temp'*y_temp);
  /* update b and b_var */
  beta := ((i-1)/i)*beta + (1/i)*b_temp;
  b_var := b_var - (i-1)/i * (b_var-beta)^2
}

In test sections the technical statistician can specify tests that check the significance of the resulting statistics. A test, for example, may check if all estimated regression coefficient are significant. In the test section below this test is 'the rule of thumb'. This rule states that an estimated regression coefficient is significant if it is at least twice the size of the estimated standard deviation of the coefficient.
Note that the functions abs and sqrt and the operator <= operate element-wise on the vectors. If all the inequalities hold the result of the <= operator is the boolean value true. Though the matrix notation suggests that this is a combined test of the significance of the estimated coefficients, it consists in fact of independent tests of the individual coefficients in the vector beta.

After the execution of the implementation section of the bootstrap technique the conditional expression in the test is evaluated. If the result is true, an exception is raised and the execution of the technique is interrupted. During the execution of the bootstrap technique also exceptions can be raised in software created by other experts. Exceptions are regarded as signals that further execution of the technique is impossible or meaningless.

In the statistical program the technical statistician can write exception handlers. An exception handler determines how the software reacts in case an exception occurs. An exception handler may consist of a warning for the user of the statistical technique, as shown in the following exception handler section of a statistical program.

```
exception handler section statistical program

WHEN near_singular:
MESSAGE:
 "THE INDEPENDENT VARIABLES IN SOME BOOTSTRAP SAMPLES ARE
 STRONGLY CORRELATED (MULTICOLLINEARITY). DUE TO THIS THE
 NUMERICAL RESULTS OF THE TECHNIQUE MAY BE INACCURATE."
```

If an exception near_singular is raised, during the execution of the bootstrap technique, the message in the exception handler is displayed. Because no explicit STOP statement is added to the exception handler, the execution of the statistical technique continues after the exception handler is executed. The example shows that the exception handling mechanism makes it possible for a technical statistician to react on exceptions raised by the software created by other experts. In the example the check on near-singularity is assumed to be implemented by a computer scientist in a higher-level programming language. When a near-singular matrix is inverted the numerical results may be inaccurate. The inversion function in the bootstrap program inverts the matrix (X'•X).
The notion of near-singularity in this case is interpreted by the technical statistician as multicollinearity: the independent variables are strongly correlated. The exception handler of the technical statistician communicates this meaning to the user of the statistical technique.

In the example no STOP statement was added to the exception handler. By adding such a statement, the technical statistician determines that the execution of the statistical technique must be stopped when an exception occurs, because the exception is considered to be fatal. This decision is clearly an expert opinion; it may be different for different statistical techniques.

4.3. INPUT RESTRICTIONS

The applied statistician who uses the generated statistical software must not be confronted with cryptic errors concerning the matrix notation used by the technical statistician. CONDUCTOR, therefore, performs extensive type and dimension bound checks to reduce the number of cryptic error messages that may occur during the execution of a statistical technique. Not all the type and dimension bound restrictions can be checked during the compilation of a statistical program. The input restrictions, however, that remain to be checked during the execution of the statistical technique are restrictions on the input index variables. These variables, and restrictions on these variables, are assumed to have meaning for the user of the statistical technique. In our example the following restrictions on the index variables of the bootstrap technique are generated:

\[^2\] In the current version of CONDUCTOR all input restrictions are written as

<table>
<thead>
<tr>
<th>expression</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>expression &gt;= 0,</td>
<td>or expression &gt; 0,</td>
</tr>
</tbody>
</table>
\[ n - 1 \geq 0 \]
\[ m - 1 \geq 0 \]
\[ s - 1 \geq 0 \]
\[ n - s \geq 0 \]
\[ d - 1 \geq 0 \]

The input restrictions are checked after the user has initialized all input index variables of the statistical technique.

4.4. USING THE BOOTSTRAP TECHNIQUE

The use of the bootstrap technique is demonstrated in the following user session. In this example the user first retrieves data from a database. He indicates which sample is to be retrieved from the database and which variables he wants to retrieve. The symbol \( \gg \): is the prompt of the user language interpreter in CONDUCTOR. After retrieving the data from the database, the user calls the bootstrap technique, and the user is asked to initialize the input variables.

```
\( \gg u: \) set sample 1945 to 1985
\( \gg u: \) retrieve income, indirect_tax, credit, consumption
\( \gg u: \) bootstrap
   number of independents
   \( \gg u: 3 \)
   number of observations
   \( \gg u: 41 \)
   size of bootstrap sample
   \( \gg u: 20 \)
   how many samples must be drawn
   \( \gg u: 100 \)
   The independent variables are
   \( \gg u: \) income, indirect_tax, credit
   The dependent variable is
   \( \gg u: \) consumption
```

After the results of the bootstrap method are calculated, the requested output is displayed. If exceptions occur during the execution of the statistical technique, warnings and suggestions from experts may accompany this output. For example, it may have been detected that the matrix inversion was near-singular. The interrupt, handled by the
exception handler defined by the technical statistician in the bootstrap technique, displays the specified message in this handler.

4.5. THE DATA

In the user session, in the previous section, series were used for consumption, income, indirect tax and credit. In statistical research these series are often produced by data experts. In the Netherlands a large part of the data in statistical research is produced by the Central Bureau of Statistics. During the data production process additional information on the series is gathered. In CONDUCTOR it is assumed that this additional information is stored in a database. Unfortunately such a database is still a dream. CONDUCTOR only contains the interface with this dream, and hopefully is an extra motivation to make this dream come true.

When the series is retrieved from the database, the additional information is checked for consistency. The consistency checks must, of course, be written by the data expert. In our example a consistency check may reveal that the series consumption is based on different types of measurement in the sample period. When an inconsistent series of observations is used by a statistical technique an exception is raised. A data expert can specify an exception handler to warn the user. Note that this exception handler is not a part of a statistical program. Such an exception handler is called an external handler in CONDUCTOR.

4.6. THE KERNEL

A statistical program is executed when it is called in a user session. The executing technique may invoke exception handlers written by either the data expert, the technical statistician or the computer scientist. During the execution of a statistical technique a limited number of semantic actions are executed. These actions form the instruction set of a virtual machine called the kernel. The kernel has instructions to
- request the user to initialize the input variables,
- evaluate expressions on a stack,
- change the sequential pattern of execution,
- control the exception handling,
- display messages,
- check input restrictions,
- terminate execution.

A compiler in CONDUCTOR translates a statistical program into a sequence of kernel instructions. The bootstrap technique specified in section 4.2, for example, is translated into the following instruction sequences. Of course, only a summary of all generated instructions is shown.

```
kernel instruction sequence

USER_LOAD           number_of_independents
  :                  :
USER_LOAD           y
CHECK-GE-RESTR
LOAD_INT            1
  :                  :
JUMP                (relative address)
USER-STORE          b
USER-STORE          var_b
HALT
```

The instruction sequence generated for the exception handlers in the statistical technique, are stored separate from the main instruction sequence of the statistical technique.

4.7. THE CONTRIBUTION OF COMPUTER SCIENTISTS

The numerical procedures in the bootstrap technique, such as matrix inversion and the generation of a random number, must be implemented by computer scientists in a higher-level programming language. Furthermore, these experts, of course, have to create CONDUCTOR.
In this chapter we introduce the formalisms used in the definition of CONDUCTOR. The definitions of the statistical, the user and the kernel language, together form the definition of CONDUCTOR. The definition of programming languages has received a lot of attention in the last two decades (see McGottrick [1980]). From the informal description of the first programming language many, more formal, definitions have evolved. A complete language definition nowadays consists of:

- a set of building blocks (words), this set is called lexicon, or sometimes alphabet,
- a possibly infinite set of sequences of words, called sentences,
- a predicate on sentences indicating whether or not a sentence is an element of the language,
- the specification of the semantics of the sentences.

The first three parts of this definition are referred to as the syntax of the language. In section 1 of this chapter a short review of syntax definitions is given. For a discussions on this topic see also Aho and Ullman [1977] and Aho, Sethi and Ullman [1986].

To define the semantics of the languages in CONDUCTOR we use the formal specification language ASF. ASF is discussed in section 2 of this chapter.
5.1. SYNTAX DEFINITIONS

Among the many notations used in describing the syntax of a programming language are context-free grammars and syntax trees.

5.1.1. Context-free grammars.

A context-free grammar G is a 4-tuple \((N, T, S, P)\), where \(N\) is a finite set of non-terminals, \(T\) is a finite set of terminals, \(S\) is a start symbol, and an element of \(N\), and \(P\) is a finite set of production rules. The terminals are the symbols from which sentences are formed in the language. The set of non-terminals \(N\) are the grammatical categories of the language. The start symbol \(S\) is a special non-terminal indicating a correct sentence in the language. The production rules \(P\) specify the ways in which sentences can be constructed from \(S\). A string of terminals is a correct sentence in the grammar \(G\), if and only if it can be derived from \(S\) using the production rules.

Example 5.1.: (from Aho and Ullman [1977]).

Consider the grammar \(G\) for simple arithmetic expressions. The non-terminals are expression and operator, with expression as the start symbol.

\[
N = \{ \text{expression, operator} \}
\]

\[
S = \text{expression}
\]

The set of terminals is:

\[
T = \{ \text{id, +, -, *, /, \(\), (, )} \}
\]

The set of productions \(P\) contains the productions:

\[
\begin{align*}
\text{expression} & \rightarrow \text{expression} \text{ operator expression} \\
\text{expression} & \rightarrow ( \text{ expression } ) \\
\text{expression} & \rightarrow \text{id} \\
\text{operator} & \rightarrow + | - | \bullet | / | \text{"} \\
\end{align*}
\]
Examples of correct sentences in this language are:

\[
\text{id } \ast \text{id} - \text{id} \\
\text{id} \\
\text{id } \ast (\text{id} - (\text{id } \ast \text{id}))
\]

{end example 5.1.}

The process of checking whether a sentence can be generated by the grammar is called the parse of a sentence. In order to be able to write an efficient parser for a grammar, the grammar must satisfy certain restrictions. For most programming languages either LL(1) or LALR(1) grammars are used to describe their syntax. These grammars are subclasses of the general class of context-free grammars, but they are powerful enough to describe most syntactic constructs in these languages. Both grammar classes guarantee that no back-tracking is required in the parse.

LL(1) parsers are top-down, deterministic parsers with one symbol lookahead, and were first described by Foster [1968], and received a theoretical treatment in Knuth [1971]. LL(1) parsers are for example used in compilers for programming languages like PASCAL and MODULA-2. LALR(1) parsers are bottom-up deterministic parsers with one symbol lookahead. The LALR(1) parsers are a subclass of the more general LR parsers introduced by Knuth [1965]. Examples of languages using LALR(1) grammars are ADA and C.

For the LALR(1) grammars there exist algorithms which can automatically construct a parser. LR(1) parser-construction algorithms are due to DeReemer [1969, 1971]. An efficient parser-constructor algorithm is also given in Park, Choe and Chang [1985]. An example of a parser generator is yacc written by Johnson [1975]. In CONDUCTOR languages are defined using grammars that satisfy the LALR(1) restrictions. The parser in the prototype of CONDUCTOR is developed with the use of yacc.

5.1.2. Syntax trees.

A useful representation of the result of a parse are syntax trees. The tree representation of the parse of a sentence in the grammar is called the concrete syntax tree or parse tree. Each node in this syntax tree represents a non-terminal of the grammar, the leafs represent terminals.
Example 5.2:

The parse of the last sentence of example 5.1. can be represented by the following concrete syntax tree (the non-terminals are abbreviated):

```
expr
 / \
expr op expr
| | / \
id * ( expr )
expr op expr
| | / \
id - ( expr )
  expr op expr
| | / \
id + id
```

{ end example 5.2.}  

A condensed version of the concrete syntax tree is called the abstract syntax tree. In the abstract syntax tree superfluous information in the concrete syntax tree is removed. Superficial distinctions in form, unimportant for the translation, do not appear in the abstract syntax tree. McKeeman [1974] showed that the transformation of a concrete syntax tree into a abstract syntax tree can be described by the use of a transduction grammar.

Example 5.3:

Using the transduction grammar with the following "tree constructing rules"

```
expr --> expr op expr  =>  op
       / \  
       expr expr
   |    ( expr ) =>  expr
   | id     =>  id
   op    =>  +
      / \  
     expr expr
  |    ( expr ) =>  expr
  | id     =>  id
  op    =>  *
    / \  
   expr expr
  |    ( expr ) =>  expr
  | id     =>  id
  op    =>  *
    / \  
   expr expr
  |    ( expr ) =>  expr
  | id     =>  id
  op    =>  *
```

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the concrete syntax tree of example 5.2. reduces to the abstract syntax tree

```
    *  \\  /
   id  \\  /
  id   +  \\
 id   id
```

{end example 5.3}

5.2. ALGEBRAIC SPECIFICATION

For the description of CONDUCTOR we use the formal specification language ASF, defined in Bergstra, Heering and Klint [1987]. This description includes a description of the semantics of the various languages in CONDUCTOR. ASF is based on algebraic specification techniques as described in Klaeren [1983], Wirsing [1983], Gaudel [1984] and Loeckx [1984]. ASF extends the algebraic specification formalism based on signatures and sets of equations in several ways. It supports (1) prefix and infix operators, (2) multiple output values of functions, and (3) module expressions. In ASF it is possible to give an algebraic specification, with conditional equations, of the languages in CONDUCTOR. For a review on algebraic specifications see Meseguer and Goguen [1982] and Klaeren [1984].

The basic concepts in algebraic specification are sorts, carrier sets and signatures. One can think of a sort as an abstract data type. The elements of a carrier set represent distinct instances of a sort. Consider, for example, the sort Booleans. If we take the intuitive meaning of this sort it could be represented by the carrier set \( \{0, 1\} \) or the carrier set \( \{\text{true, false}\} \). Of course infinitely many other carrier set (representations) of Booleans can be chosen. A signature describes a set of functions. For each function it is exactly specified which sorts are expected as input and what sort is returned as output. A signature is defined more thoroughly by Meseguer and Goguen [1982]. A signature can be defined as
Let $S$ be a set of sorts. If $a$ and $b$ are both sorts in $S$. Then a family of functions is defined as

$$\{ f: a \to b \mid a, b \in S \}$$

These are typically all the function with as input of sort $a$ and output of sort $b$. If we allow more than one sort as input we get what is called an $S$-sorted signature $\Sigma$

$$\Sigma_{w,s} = \{ f: w \to s \mid w \in S^*, s \in S \}$$

A signature defines some structure of interest. For example the signature Boolean specifies booleans using a set of sort ($\text{bool}$) and a set of functions ($\text{true, false, and}$).

$$\text{Boolean} = \{ \text{(bool)}, \text{(true, false, and)} \}$$

with

- $\text{true}: \to \text{bool}$
- $\text{false}: \to \text{bool}$
- $\text{and}: \text{bool} \# \text{bool} \to \text{bool}$

By assigning a carrier set to each of the sorts we get a what is called a $\Sigma$-algebra. Many $\Sigma$-algebras may exist for the same signature, therefore an initial $\Sigma$-algebra is defined as:

A $\Sigma$-algebra $A$ is initial in a class of $\Sigma$-algebras $\mathcal{S}$ that describe the same structure if and only if there is only one $\Sigma$-homomorphism for each $\Sigma$-algebra $C$ in $\mathcal{S}$ from $A$ to $C$ (see Meseguer and Goguen [1982]).

In other words an initial $\Sigma$-algebra is the 'smallest' representation of a structure. Note that there may be more than one initial $\Sigma$-algebra in the same class. Two initial algebras in the same class $\mathcal{S}$ are abstractly the same but differ in the representation given to the elements.

From the functions in a signature $\Sigma$-terms can be formed, similar to
sentences in context-free grammars. This set of terms can be used as a carrier set. All distinct terms are a possible representation of the distinct data-items in a signature. For example, in our signature Boolean we may form the terms

\[ \text{true, false, and(true,false), and(and(true,false),false),...} \]

It is clear that many of the above terms represent the same data item. Equations are introduced to specify which terms are equal. To restrict the \( \Sigma \)-algebra \( \text{Booleans} \) to only two data items the functions in this signature must satisfy the equations

\[
\begin{align*}
\text{and(true,false)} &= \text{false} \\
\text{and(false,true)} &= \text{false} \\
\text{and(false,false)} &= \text{false} \\
\text{and(true,true)} &= \text{true}
\end{align*}
\]

To get meaningful restrictions the sets of equations should obey the following restrictions:
- applying the equational logic to deduce new equations should always yield equations that are satisfied by any algebra satisfying the equations (soundness),
- every equation, satisfied by all algebras satisfying the given equations, can be deduced using the equational logic (completeness).

Using one signature to describe a large software product would yield an enormous amount of sorts, function and equations. In ASF, therefore, a signature can be modularized. In a module expressions make one can import sorts and functions specified in another module by importing that module. Each module may contain an export clause indicating which sorts and functions can be imported by other modules. Our \( \text{Booleans} \) can be expressed in the following module

```plaintext
module Booleans
begin
export
begin
sorts BOOL
functions
  true: -> BOOL
  false: -> BOOL
  and: BOOL # BOOL -> BOOL
end
```
functions
not: BOOL -> BOOL

equations
and(true, false) = false
and(false, true) = false
and(false, false) = false
and(true, true) = true
not(true) = false
not(false) = true

end Booleans

The module Booleans is imported in the module Integers. This module can use the functions true, false and and specified in module Booleans. For example, the constant functions true and false are used in the specification of a function equal in module Integers.

module Integers
begin
export
begin
sort INT
functions
null: -> INT
increment: INT -> INT
equal: INT # INT -> BOOL
end

imports Booleans

variables
i, i1, i2 :-> INT

equations
equal(null, null) = true
equal(null, increment(i)) = false
equal(increment(i), null) = false
equal(increment(i1), increment(i2)) = equal(i1, i2)

end Integers

To make modules more generally applicable, parameterization is available in ASF. Each formal parameter is a submodule and contains one or more sorts or functions, which at a later stage have to be bound to the actual parameter. Consider the formal specification of a sequence
module Sequences
begin
  parameter Items
  begin
    sorts ITEM
    end Items
  exports
  begin
    sorts SEQ
    functions
      add-item: ITEM # SEQ -> SEQ
    end
  end Sequences

A sequence of integers can be specified by binding the parameter Items to the module Integers.

module Integer-sequences
begin
  imports Sequences
  begin
    [ renamed by
      [ SEQ   -> INT-SEQ ]
      Items bound by
      [ ITEM  -> INT ]
      to Integers
    end Integer-sequences
  end

The module Integer-sequences defines the sort INT-SEQ. All functions specified for the sort SEQ are also defined for the sort INT-SEQ. Note that not all parameters have to be bound when a module with parameters is imported. Such unbound parameters are called inherited parameters.

The overall structure of specifications is illustrated by structure diagrams. Each module is represented by a rectangular box. For example the module Booleans is represented by

\[
\text{Booleans}
\]

Modules imported by a module M are represented by structure diagrams inside the box representing M. For example the module Integers imports the module Booleans.
Parameters of the module are represented by ellipses carrying the name of the parameter. The module Sequences with parameters Items, for example, is represented by:

```
  Booleans

  Integers

  Items

  Sequences
```

The binding of formal parameters is represented by joining the formal parameter and the module to which it is bound. This leads to the following representation of the module Integer-sequences:

```
  Booleans

  Integers

  Items

  Sequences

  Integer-sequences
```
5.3 THE DEFINITION OF THE LANGUAGES IN CONDUCTOR

An important part of the definition of CONDUCTOR consists of the definition of the statistical language, the kernel language and the user language. The first part of the definition of the statistical language defines the concrete syntax. The next part is the definition of the abstract syntax. In the abstract syntax tree the 'syntactic sugar' of the concrete syntax is removed. The third part defines the type restrictions in the statistical language. A type correct statistical program is represented as a type correct abstract syntax tree plus a symbol table containing the type information of all variables in the abstract syntax tree. The fourth part is the definition of the dimension bound restrictions. A type and dimension bound correct abstract syntax tree includes a set of input restrictions. In the final part of the definition of the statistical language, the evaluation of the abstract syntax tree is defined.

```
concrete syntax tree
  ↓
abstract syntax tree
  ↓
type correct
abstract syntax tree
  + symbol table
  ↓
type and dimension bound correct
abstract syntax tree
  + symbol table
  + input restrictions
  ↓
evaluation
```

A type and dimension bound correct statistical program is represented in the kernel as a kernel program. The semantic actions of the statistical program in this representation are a sequence of kernel instructions. The set of all possible kernel instructions is called the kernel language. Of the kernel language only the abstract syntax and a description of the
evaluation are specified. The definition of the user language consists of a concrete syntax, an abstract syntax, and the evaluation process of abstract syntax trees. One of the semantic actions in the user language is the evaluation of a statistical program.

The concrete syntax of the user and the statistical language is described by a grammar. This grammar satisfies the LALR(1) restriction and can be found in appendix A. The remaining parts of the language definitions are described in the formal specification language ASF. The definition of the statistical language is given in part II of this book. The kernel language and the evaluation of statistical kernel programs can be found in part III. The data interface and the user language are given in part IV.
6. THE FORMAL SPECIFICATION OF CONDUCTOR

The definition of CONDUCTOR is a combined definition of the statistical, the user and the kernel language. The definition consists of the grammar describing the concrete syntax of these languages, as given in appendix A, and the formal specification, as given in appendix C through L. In this chapter we discuss the modules that describe CONDUCTOR at a high level of abstraction. These modules can be found in appendix L.

6.1. A TOP LEVEL VIEW OF CONDUCTOR

CONDUCTOR maintains a global state. This state is determined by:
- the statistical techniques implemented by the technical statisticians,
- the external handlers implemented by either the computer scientists or the data experts,
- the state of a user session.

All implemented statistical techniques are stored in the statistical technique table. This table is represented in the formal specification by the sort STAT-TECH-TABLE. All external handlers are stored in the external handler table, represented by the sort EXT-HANDL-TABLE. The state of a user session is specified by the sort USER-STATE. Every combination of the sorts USER-STATE, STAT-TECH-TABLE and EXT-HANDL-TABLE is a state of the CONDUCTOR. This state is represented by the sort CDT-STATE. The function state in module Conductor-states specifies the state of CONDUCTOR.
state: USER-STATE # STAT-TECH-TABLE # EXT-HAN DL-TABLE -> CDT-STATE

The details of the sorts USER-STATE, STAT-TECH-TABLE and EXT-HAN DL-TABLE are defined in the imported modules of module Conductor-states.

The state of CONDUCTOR can be modified in session. Module Conductor-sessions specifies that a session is either:
- a session of a technical statistician; in such a session a technical statistician implements a statistical technique (represented by the sort STAT-PRO),

stat-session: STAT-PRO -> SESSION

- a session in which either a data expert or a computer scientist implements an external handler (represented by the sort HANDLER),

hand-session: HANDLER -> SESSION

- a session in which the implemented statistical software is used (such a session consists of a user program represented by the sort USER-PRO).

user-session: USER-PRO -> SESSION

A session may modify the state of CONDUCTOR, as specified in the module Conductor in the function execute

execute: SESSION # CDT-STATE -> CDT-STATE

The module Conductor specifies the most abstract notion of CONDUCTOR. Details are defined in the modules imported by the module Conductor. Of course, one easily loses track in the modules. The complete specification consists of 114 modules with a total length of 4500 lines. It contains, for example, the formal specification of the abstract syntax of the statistical language, the static symbolic type checking of this language, and the kernel. To get better insight in the relation between the modules, a tree representation is presented, that gives a top-down overview of the import relations between the modules of CONDUCTOR. The tree, given below, shows the import relations between the modules, that
specify an abstract notion of CONDUCTOR.

Tree 1. The import structure of CONDUCTOR modules.

The complete tree representation of the system can be found in appendix M. In the trees the information on inherited parameters, and the import of basic modules such as the modules Booleans and Sequences is omitted. The graphical representation of the modules generated by the ASF specification checker is used as illustration throughout this book. The graphical representation of the module Conductor on page 52 illustrates that CONDUCTOR has three parameters: Current-func-types, Current-func-code and Current-database. These parameters are discussed in sections 3 and 4 of this chapter.

6.2. THE CONTRIBUTION OF THE TECHNICAL STATISTICIAN

A technical statistician can implement statistical techniques in CONDUCTOR. He does this by writing a program in the statistical language. CONDUCTOR compiles this statistical program and stores it in the
statistical technique table. A successful session of a technical statistician modifies the state of CONDUCTOR as defined by the following equation for the function `execute` in module `Conductor`:

\[
[416] \quad \text{execute}(\text{stat-session}(sp), \text{state}(ust, stt, eht))
\quad - \text{state}(ust, \\
\quad \quad \text{store-stat-tech}(sp, at:), \\
\quad \quad \quad \quad \quad \quad \text{eht})
\]

This equation specifies that, given that CONDUCTOR is in a state determined by user state `ust`, statistical technique table `sst` and external handler table `eht`, the execution of a session of a technical statistician results in a state where the statistical program is compiled and stored in the statistical technique table. The tag `[416]` is the number of the equation in appendices.

More details of the compilation and storage process are specified in the imported module `Compiler`. In this module the function `store-stat-tech` is
specified

store-stat-tech: STAT-PRO # STAT-TECH-TABLE ←→ STAT-TECH-TABLE

with equation

[403] \( \text{store-stat-tech}(sp, stt) = \text{insert}(\text{name}(sp), \text{compile}(sp), stt) \)

implying that a statistical program, \( sp \), is compiled and stored in the statistical technique table \( stt \). The module Compiler also contains the specification of the functions compile and name. These functions are discussed in chapter 11. Of course, all details involved in the compilation and storage of a statistical program, including the function insert, are specified in the imported modules of module Compiler.

6.3. THE CONTRIBUTION OF THE DATA EXPERT

A data expert has to construct the database that is connected to CONDUCTOR. This is represented, in the formal specification by the parameter Current-database of the module Conductor. This parameter is inherited from module Database-interface. Recall that an inherited parameter is a parameter of an imported module that is not bound in the importing module. The parameter Current-database in module Database-interface reads:

parameters Current-database
begin
  functions
    current-db: DATA-BASE # ID # → DATA-BASE
    data-query: DATA-BASE # ID # CONST-RANGE-SEQ → USER-DATA
    retrv-data: DATA-BASE # ID # CONST-RANGE-SEQ → SCALAR-SEQ
    bg-query: DATA-BASE # ID # CONST-RANGE-SEQ → IL-SEQ
  end Current-database

This parameter contains the functions current-db, data-query, retrv-data and bg-query. The parameters specify, respectively, the database, how
data is retrieved from the database, and how a background query checks
the consistency of the data. The sort ID in these functions specifies the
name of the series that is to be retrieved, and the sort CONST-RANGE-SEQ
describes the sample. A background query may result in a sequence of
exception identifiers: the sort ID-SEQ. A specification of all sorts in
the definition of parameter Current-database can be found in the modules
imported by the module Database-interface.

A background query may reveal that a retrieved series is inconsistent. If
such an inconsistency is detected an exception identifier is added to the
retrieved series. If this series is used in a statistical technique, the
accompanying exception is raised and the execution of the statistical
technique is interrupted.

The data expert can add external handlers to CONDUCTOR in an external
handler session. An external handler session of a data expert modifies
the state of CONDUCTOR as defined by the following equation for the
function execute in module Conductor

\[
\text{execute}(	ext{hand-session}(\text{hnd}), \text{state}(\text{ust}, \text{sst}, \text{eht})) = \text{state}(\text{ust}, \text{sst}, \text{store-ext-handler}(\text{hnd}, \text{eht}))
\]

Given that CONDUCTOR is in a state determined by user state \text{ust},
statistical technique table \text{sst} and external handler table \text{eht}, the
execution of a handler session results in a state where the external
handler is compiled and stored in the external handler table. The defini-
tions of the function \text{store-ext-handler} is given in the module Gen-ext-
handlers, a module that is imported by the module Conductor

6.4. THE CONTRIBUTION OF THE COMPUTER SCIENTIST

The contribution of the computer scientist is, to specify CONDUCTOR, and
to make efficient implementations in a higher level programming language
of all modules in the specification. This book can be seen as the
description of the contribution of the computer scientists in CONDUCTOR
as far as the formal specification of the modules concerns. Which
functions are available in the statistical language is left unspecified.
The type restrictions checks on these functions, as well as the numerical
procedures for the actual calculations, have to be constructed by the computer scientists. For example, functions in the statistical language, such as, matrix inversion, matrix multiplication and Kronecker product are not specified. The fact that functions in the statistical language are left unspecified is represented in the formal specification by unbound parameters Current-func-types and Current-func-code. These parameters are discussed in respectively chapter 8 and chapter 10. To handle such exceptions, the computer scientist may, like the data expert, add external exception handlers to CONDUCTOR in an external handler session.

6.5. USING THE IMPLEMENTED STATISTICAL SOFTWARE

A user can apply the statistical techniques implemented by the technical statisticians to the data collected by the data experts in an environment created by computer scientists. A user session modifies the state of CONDUCTOR, as defined by the following equation for the function execute in module Conductor.

\[
\text{execute(user-session}(\text{up}),\text{state}(\text{ust},\text{stt},\text{eht})) \\
= \text{state(ejec-user-pro}(\text{ust},\text{available}(\text{stt},\text{eht}),\text{up}), \\
\quad \text{stt}, \\
\quad \text{eht})
\]

This equation specifies that, given that CONDUCTOR is in a state determined by user state ust, statistical technique table stt and external handler table eht, a user session results in a state where the user program is interpreted and the user state is modified. The definitions of the function exec-user-pro is given in the module Resulting-software, a module imported by the module Conductor.
PART II

FORMAL SPECIFICATION OF THE
STATISTICAL LANGUAGE
In statistical analysis a theoretical model is postulated that must explain the observed data. If the basic assumptions of this model are not rejected by (pre)tests, the unknown parameters of the model can be estimated. Particular test results and parameter estimations may lead to modification of the postulated model, and thus to a better description of the observed data.

Statistical software makes it possible to calculate test results and parameter estimations very fast. Besides positive effects, this also introduced new perils. Trying many variants of the same theoretical model can easily lead to data peeping and chance capitalization (see Lovell [1983] and Meyer [1975]). The contribution of a technical statistician to statistical analysis, therefore, should not only be restricted the creation of tests and estimation techniques for specific theoretical
models. A technical statistician should also advise the users of his
techniques how to modify a postulated model given the calculated results.
The statistical language enables the technical statistician to add his
knowledge to statistical software. In this chapter both the concrete and
the abstract syntax of the statistical language are discussed. The
complete set of production rules of the concrete syntax of the
statistical language is given in appendix A. The formal specification of
the abstract syntax in appendix F.

7.1. ABSTRACT SETS OF STATISTICAL TECHNIQUES

For the definition of the statistical language one only needs to know
what operators, functions and data types are required to describe the
statistical techniques a technical statistician want to implement.
Knowledge of the statistical technique is not required. In this section
we define statistical techniques in terms of these building blocks. This
will lead to a few straightforward restrictions on the statistical
language.

Definitions:

1. TARGET is the set of n statistical techniques that the statistician
intends to express in the statistical language:

$$\text{TARGET} = \{\text{TEC}_1, \ldots, \text{TEC}_n\}$$

2. FUNCTIONS is a set of m functions used in the equations of the
statistical techniques in the set TARGET.

$$\text{FUNCTIONS} = \{F_1, \ldots, F_m\}$$

3. OPERATORS is the set of p operators used in the equations of the
statistical techniques in the set TARGET.

$$\text{OPERATORS} = \{\text{OP}_1, \ldots, \text{OP}_p\}$$

4. TYPES is the set of k data types used in the equations of the
statistical techniques in the set TARGET.

$$\text{TYPES} = \{T_1, \ldots, T_k\}$$
5. SST is the set of all statistical techniques that can be expressed in terms of the sets FUNCTIONS, OPERATOR and TYPES.

\[ \text{SST} = \{ \text{TEC} \mid g(\text{TYPES,FUNCTIONS,OPERATORS}) \rightarrow \text{TEC} \} \]

Note that the difference between operators and functions is not fundamental. Operators can be regarded as functions with the appropriate number of parameters. Nevertheless operators are used frequently in statistical textbooks and can not be omitted if one wants to keep close resemblance to statistical notational conventions.

**Example 7.1:**
Assume we live in 'a simple statistical world' and have the following estimation techniques at our disposal: ordinary least squares (OLS) and two stage least squares (2-SLS). These techniques give an estimation of regression coefficients in a linear (simultaneous) model:

\[ y = Y_1 \beta + X_1 \tau + u \]

Where \( y \) is a \((n \times 1)\)-vector of observations of the endogenous variable that must be explained in the equation, \( Y_1 \) is a \((n \times m)\)-matrix of the observations of the other endogenous variables in the equation, \( X_1 \) is a \((n \times p)\)-matrix of the observations of the \( p \) exogenous variables in the equation, \( u \) is a \((n \times 1)\)-matrix of random disturbances, \( \beta \) and \( \tau \) are, respectively \((m \times 1)\), \((p \times 1)\) vectors of model parameters.

To estimate the model parameters the OLS-estimator can be used, however this estimator results in inconsistent estimates of \( \beta \) and \( \tau \). The OLS-estimator combines the matrix \( X_1 \) and \( Y_1 \) in the matrix \( X \)

\[ X = (Y_1 \mid X_1) \]

and estimates \( \delta = (\beta \mid \tau)' \) as

\[ d_{olsq} = (X'X)^{-1}X'Y \]

whereas the 2-SLS estimator estimates the coefficients in two rounds can be written as (see Maddala [1977])
\[ d_{2-\text{SLS}} = \begin{bmatrix} (Y_1'Y_1 - V_1'V_1) & Y_1'X_1 \\ X_1'Y_1 & X_1'X_1 \end{bmatrix}^{-1} \begin{bmatrix} (Y_1-V_1)' \\ X_1'y \end{bmatrix} \]

where \( V_1 \) is the \((n \times m)\)-matrix from the least-squares regression of \( Y_1 \) on \( X \). The two estimation techniques are the target set of the statistical language.

\[ \text{TARGET} = \{\text{OLS, 2-SLS}\} \]

The equations that describe the two techniques in \( \text{TARGET} \) make use of the following functions, operators and data types:

\[ \text{FUNCTIONS} = \{\text{matrix inversion}\} \]

\[ \text{OPERATORS} = \{\text{matrix subtraction (\(-\)}, \text{matrix multiplication (\(*\), \text{matrix transpose ('), matrix assignment (\(\sim\))}\} \]

\[ \text{TYPES} = \{\text{scalar, matrix, submatrix, partitioned matrix}\} \]

Using the sets \( \text{FUNCTIONS}, \text{OPERATORS} \) and \( \text{TYPES} \) we can also describe a statistical technique called generalized least squares. This technique therefore belongs to the set \( \text{SST} \).

\[ \text{SST} = \{\text{OLS, 2-SLS, GLS, \ldots}\} \]

(end example 7.1.)

Given the sets \( \text{FUNCTIONS}, \text{OPERATORS} \) and \( \text{TYPES} \) we can derive the following necessary conditions for the grammar of the statistical language. Recall that the grammar describes the syntax of the language, as discussed in chapter 5. A grammar \( G \) consisted of a set of non-terminals \( N \), a set of terminal symbols \( T \), a set of production rules \( P \), and a starting symbol \( S \). \( S \) is one of the non-terminal symbols.

**Condition 1:** The identifiers of elements of the sets \( \text{FUNCTIONS}, \text{TYPES} \) and \( \text{OPERATOR} \) are in \( T \).

This condition implies that all identifiers (function identifiers, operator symbols and data type identifiers) in the description of the
statistical technique must be terminals of the statistical language.

Condition 2: The equations describing the statistical techniques in TARGET are correct sentences in the grammar.

In other words the production rules in the grammar must enable the technical statistician to form the required expressions.

7.2. THE GENERAL STRUCTURE OF A STATISTICAL PROGRAM

In the statistical language the statistician determines:
- the user interface of the statistical technique; the user interface consists of the input variables and the resulting statistics of the technique,
- the equations that describe the calculation of the statistic
- the (pre)tests that check the basic assumptions of the statistical technique and the significance of the calculated parameters,
- the exception handlers that describe how the software reacts if an exception occurs (assumptions are violated or problems occur on other levels of CONDUCTOR).
- a unique name for the statistical technique.

Each of these parts can be specified in separate sections of a statistical program. A statistical program consists of one or more sections as stated in the following production rules.

```
statistical_program --> statistical_program section | section

section --> name_section | input_output_section | implementation_section | test_section | exception_handler_section
```

The corresponding abstract syntax is specified in the module Statistical-programs.
module Statistical-programs
begin
  exports
  begin
  sorts STAT-PRO, SECTION
  functions
  abs-prog: STAT-PRO # SECTION -> STAT-PRO
  abs-prog: SECTION -> STAT-PRO
  abs-sect: ID -> SECTION
  abs-sect: IO-SEC -> SECTION
  abs-sect: IMPL-SEC -> SECTION
  abs-sect: TEST-SEC -> SECTION
  abs-sect: HANDL-SEC -> SECTION
  end
  imports Decl-abstr-synt, Impl-abstr-synt
  Test-abstr-synt, Handler-abstr-synt,
  Identifiers
end Statistical-programs

Note that this module imports several other ones, which define the abstract syntax of separate language constructs. The reader is referred to appendix F for the definition of these modules; they will not further be discussed in this chapter.

7.3. DECLARATION OF VARIABLES

Variables in the statistical language are divided in three categories: input variables, output variables and the internal variables. The input variables are the variables that must be initialized by the user of the statistical technique; the output variables will contain the results of the statistical technique that are returned to the user. The input and output variables are declared in a input/output section of a statistical program. A message string may accompany the declaration of input/output variables. For input variables this message string is used to prompt the user of the statistical technique to enter his data. For output variables this message is clarifies the calculated results. Internal variables can be declared in an implementation, a test and an exception handler section. These variables remain invisible for the user of the statistical technique.

Each declaration is a list of identifiers followed by the type of the declared variables and a message.
declaration  →  var_type id_list  message

In the current version of CONDUCTOR the data types booleans, scalars, indices and matrices are available. Submatrices, matrix elements and partitioned matrices can also defined. These matrix references, however, are regarded as functions to access particular parts of matrices.

var_type  →  'BOOL'|'INDEX'|'SCALAR' |
          vec_type | mat_type

The syntax of vector and matrix types includes the declaration of the dimension bounds.

vec_type  →  'VECTOR'  range

mat_type  →  'MATRIX' '[ ranges ]'

ranges    →  ranges ',,'  range |
          range

range     →  index_expr  'TO'  index_expr

The upper and lower bound of a dimension range are expressed in terms of index expressions. These index expressions are somewhat restricted compared to integer expressions in higher-level programming languages in order to make symbolic type and dimension bound checking possible. Introducing a data type integers next to the data type indices relaxes these restriction considerable (See chapters 8 and 9.) The abstract syntax of declarations is specified in module Decl-abstr-syntax in appendix F.

Example 7.2: Declarations in the statistical language.

Examples of correct declarations in the statistical language are:
7.4. THE IMPLEMENTATION SECTION

The implementation section of a statistical program contains the equations that describe how the parameters in the theoretical model are calculated. An implementation section consists of internal declarations of variables, and statements.

\[
\text{implementation\_section} \rightarrow \text{internal\_declarations} \begin{array}{|l|} \hline \text{\'EQUATIONS\' statements} \hline \end{array}
\]

In CONDUCTOR five types of statements are distinguished.

\[
\text{statement} \rightarrow \text{assignment} | \text{index\_assignment} | \text{message} | \text{for\_statement} | \text{compound\_statement}
\]

Conditional statements are not included in CONDUCTOR because they would make symbolic evaluation of a statistical program extremely difficult. The assignment statement, index assignment statement and compound statement are identical to similar statements defined in higher level programming languages. The message statement is a rudimentary print statement.

The for-statement in the statistical language is adapted to statistical notational convention. In statistical textbooks the following notation is found frequently

\[
X_{i,j} = 0 \quad i = 1, \ldots, m \quad j = 1, \ldots, n
\]

This is reflected in the following syntax rules

\[
\text{for\_statement} \rightarrow \text{compound\_statement} \text{ offsets}
\]
offsets  \rightarrow offsets ', offset

offset  \rightarrow \text{index\_var} '='
         \text{index\_expr} ',...,' \text{index\_expr}

Besides the 'statistical' syntax, also the 'usual' syntax of for-statements in higher-level programming languages is available in the statistical language

\text{for\_statement}  \rightarrow 'FOR'\text{index\_var} ':=' \text{index\_expr}
                        'TO' \text{index\_expr} \text{compound\_statement}

Note that, though two different forms of for-statements exist in the concrete syntax, there is only one representation of a for-statement in the abstract syntax.

\textbf{Example 7.3:} an implementation section.

In example 7.1. a matrix \(X\) is declared. In the following implementation section all diagonal elements of matrix \(X\) are added.

\begin{verbatim}
Implementation section statistical program

VARIABLES
INDEX i;
SCALAR sum;

EQUATIONS
MESSAGE:" adding all diagonal elements of matrix X"
; sum := 0
; sum := sum + X[i,i]  i = 1, ..., N

{ end example 7.3. }
\end{verbatim}

\textbf{7.5. THE TEST AND EXCEPTION HANDLER SECTION}

Exceptions are a well known phenomenon in computer science, and can occur at every level of the hardware and/or software. Exceptions are unexpected situations which make further execution of a program undesirable or sometimes impossible. Typical examples of exceptions are underflow, overflow and division by zero. These situations are called exceptions
because they were not intended to appear during the normal course of execution of the program.

In the test sections of a statistical program a technical statistician can specify when an exception must be raised. For instance, an exception can be raised if one of the assumptions underlying a statistical technique is rejected by a test: the software created by the technical statistician will signal that there is something wrong with the analyzed data set and interrupts the calculation of the statistical technique.

The syntax of a test in the statistical language is:

```
test_section    -->  'TEST' internal_declarations 'EQUATIONS' statements raises

raises         -->  raises raise |
                 raise

raise          -->  'RAISE' exception_flag 'WHEN' condition

exception_flag -->  IDENTIFIER_NAME
```

Exception handlers describe how the program must react if an exception is raised. The syntax of an exception handler in the statistical language is:

```
handler        -->  'WHEN' exception_flag ':'
                 internal_declarations 'EQUATIONS' statements stop_or_continue

stop_or_continue -->  'STOP' |
                    /* default action continue */
```

Exception handlers in CONDUCTOR can be created by the technical statistician in a statistical program. The exception handler is only executed if an exception occurs during the execution of the statistical technique. In the exception handler the technical statistician can decide if the calculation of the statistical technique must be aborted or if it may continue, when an exception occurs. A further discussion of exception handling can be found in chapter 12. The abstract syntax of test and handler sections is specified in modules Test-abstr-syntax and Handler-abstr-syntax in appendix F.
Example 7.4. Tests and exception handlers.

In chapter 4 we described the implementation of a bootstrap method to estimate the regression coefficients in a linear model. In this example we used the 'rule of thumb' to check the significance of the calculated parameters.

```
TEST
   RAISE insignificant_coefficients
   WHEN abs(beta) <= 2*sqrt(b_var)
```

In that chapter also an exception handler was given for near-singularity.

```
WHEN near_singular:
   MESSAGE:
   "THE INDEPENDENT VARIABLES IN SOME BOOTSTRAP SAMPLES ARE STRONGLY CORRELATED (MULTICOLLINEARITY). DUE TO THIS THE NUMERICAL RESULTS OF THE TECHNIQUE MAY BE INACCURATE."
```

This exception handler warns the user, and continues the calculation after the message is displayed, whereas the following handler displays the same message and then aborts the interrupted statistical technique if near-singularity occurs, because a stop instruction is added to the handler.

{end example 7.4.}
8. SYMBOLIC TYPE CHECKING OF STATISTICAL PROGRAMS

The syntax rules discussed in the previous chapter do not include any restrictions on the type of particular constructs in the language. A matrix function, for example, can be restricted to have only a square matrix as its argument. A type checker verifies that a construct in a language has the type expected by its context. In the early higher-level programming languages type checking was not considered to be a serious problem (see Sheridan [1959]). One of the first languages that allowed the type of language constructs to be evaluated systematically was Algol68. In type checking a distinction is made between static and dynamic type checking. If type checking can be done before the program is executed, it is called static, whereas it is called dynamic, if it is done during the execution of a program. Programming languages that involve matrices, like APL (see Iverson [1962]), rely on dynamic type checking.

Type checking refers to a large class of problems. In the statistical language we particularly are interested in type checking of assignment statements that include matrix variables. Type checking of other constructs in the statistical language is done in exactly the same way as in higher-level programming languages. We concentrate on the restrictions on the dimension bounds of matrix types in assignment statements. The related problem of dimension bound checking in matrix element references is discussed in chapter 9.

Allowing symbolic dimension bounds in the statistical language makes checking the dimension bound restrictions complicated. The dimension bounds of matrices in the statistical language are given in terms of
index expressions. A variable representing the number of observations, for example, will often specify the dimension bounds of matrices in a statistical technique. The actual dimension bounds of matrices are determined by the values of the input variables as given by the user of the statistical technique. This user, however, has no knowledge of the implementation details of the statistical technique, such as, constraints imposed by the dimensions of matrices used in the implementation of the statistical technique. He, therefore, should never be confronted with error messages concerning violations of such constraints. To ensure that no errors occur, type and dimension restrictions concerning (matrix) operators and (matrix) functions in the statistical language are checked extensively. Example 8.1 shows some type restrictions that are checked by the type checker.

Example 8.1: Type checking of a statistical program.

Consider the following statistical program.

<table>
<thead>
<tr>
<th>statistical program</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAME</td>
</tr>
<tr>
<td>INTERFACE</td>
</tr>
<tr>
<td>INPUT</td>
</tr>
<tr>
<td>INDEX</td>
</tr>
<tr>
<td>MATRIX [1 to n, 1 to t]</td>
</tr>
<tr>
<td>OUTPUT</td>
</tr>
<tr>
<td>MATRIX [1 to t, 1 to n]</td>
</tr>
<tr>
<td>EQUATIONS</td>
</tr>
</tbody>
</table>

The type checker checks if in case of the transpose (') operator that:
- both argument and result are two-dimensional matrices
- the first dimension of the result is equal to the second dimension of the argument
- the second dimension of the result is equal to the first dimension of the argument

And in case of plus (+) and the assign (:=) operator that:
- both arguments are of the same symbolic type

{end example 8.1}
The type checking of the statistical language is based on symbolic evaluation. The symbolic evaluation of a matrix type requires the symbolic evaluation and comparison of the dimension ranges and involved index expressions. The symbolic comparison of index expressions is based on a simple mechanism, that multiplies out an index expression, and combines identical terms and factors. This mechanism is, for example, also applied in the symbolic manipulation languages REDUCE [1973] and MACSYMA [1977].

In this chapter, first the symbolic comparison of index expressions is discussed in section 8.1. In section 8.2 the algebraic specification of this comparison is given and in section 8.3 the symbolic comparison of dimension ranges and matrix types. A symbol table containing the type information of variables declared in a statistical program is specified in section 8.4, and in this section it is shown how declarations in the statistical language are stored in this table. The type information in the symbol table is used in the symbolic type checking of assignment statements in section 8.5. The specification of symbolic type checking of function calls is discussed in section 8.6. A statistical program that satisfies all symbolic type restrictions is called a type correct statistical program. The specification of such a program is discussed in section 8.7.

8.1. THE INDEX EXPRESSIONS

The index expressions in the statistical language have a restricted syntax compared to integer expressions in higher-level programming language. The index variables, however, are only intended to be used in the declaration of dimension bounds of matrices, and in matrix element and submatrix references. An index expression may consist of a combination of index variables, index constants and the operators plus, minus and multiplication as stated in the following syntax rules.

\[
\begin{align*}
\text{index_expr} & \quad \rightarrow \ '( index_expr ')' \\
& \quad \rightarrow \ index_expr \ '+' \ index_expr \\
& \quad \rightarrow \ index_expr \ '-' \ index_expr \\
& \quad \rightarrow \ index_expr \ '*' \ index_expr \\
& \quad \rightarrow \ INDEX\_VARIABLE \\
& \quad \rightarrow \ INDEX\_VALUE
\end{align*}
\]
Note that the syntax rules are ambiguous. This ambiguity is easily solved by giving precedence rules for the operators in the parser-generator yacc (see Johnson [1975]). The multiply operator (*) has the highest priority of the operators, and is right associative, the plus (+) and minus (-) operators are left associative.

During a parse of an index expression its abstract syntax tree is constructed. The abstract syntax tree of the index expression n*t-1 is, for instance:

```
  -
 / \  
*   1
/ \  
n t
```

During type checking the need arises to determine whether two index expressions are equivalent\(^1\), i.e. will have the same value when evaluated for any possible combination of values for the variables occurring in them. Unfortunately, equivalent index expressions may have different abstract syntax trees. For example, the equivalent expressions

\[ a \cdot (b \cdot c) \quad \text{and} \quad (a \cdot b) \cdot c \]

correspond to

```
  *
/ \  
*   *
/ \  
a   b
/ \  
c   a
```

Another example is given by the expressions

\[ a \cdot (b+c) \quad \text{and} \quad a \cdot b + a \cdot c \]

which are clearly equivalent (due to distributivity) but have non-equivalent abstract syntax trees.

\(^1\). We ignore the problems due to underflow and overflow of integers.
To make symbolic comparison of the abstract syntax trees possible they are "normalized". That is, all symbolic equivalent index expressions are represented by the same abstract syntax tree. In CONDUCTOR index expression are normalized using the algorithm 8.1. In the description of this algorithm we use the following definitions:

factor: an index expression consisting of either an index variable or an index constant.
term: a factor or an index expression in which factors are multiplied.

Algorithm 8.1:

This algorithm multiplies out an index expression, and combines similar terms and factors. The algorithm proceed as follows:

1. It distributes multiplication over additions and subtractions.

\[
(i_{e1} \cdot i_{e2}) \cdot i_{e3} \Rightarrow i_{e1} \cdot i_{e3} + i_{e2} \cdot i_{e3}
\]

\[
(i_{e1} - i_{e2}) \cdot i_{e3} \Rightarrow i_{e1} \cdot i_{e3} - i_{e2} \cdot i_{e3}
\]

where \( i_{e1}, i_{e2} \) and \( i_{e3} \) are index expressions.

2. It eliminates the influence of parentheses within \( \cdot \) or \( - \) operators and within the \( \cdot \) operator\(^2\). ( \( +, - \): left associative, \( \cdot \): right associative).

\(^2\) Applying this rule \( i_{e2} \) and \( i_{e3} \) change position in order to keep the rules consistent with the implementation of the algorithm where the trees are rebuilt in the following way:

```
  / \       =>       /\
 i_{e1} -  \        -/ \i_{e2}
  / \      / \   / \\
 i_{e2} i_{e3} i_{e1} i_{e3}
```
\[ ie1 \cdot (ie2 \cdot ie3) \Rightarrow ie1 \cdot ie3 \cdot ie2 \]
\[ ie1 \cdot (ie2 - ie3) \Rightarrow ie1 \cdot ie3 + ie2 \]
\[ ie1 + (ie2 - ie3) \Rightarrow ie1 - ie3 + ie2 \]
\[ ie1 - (ie2 + ie3) \Rightarrow ie1 - ie3 - ie2 \]
\[ ie1 \cdot ie2 \cdot ie3 \Rightarrow ie1 \cdot (ie2 \cdot ie3) \]

where \( ie1, ie2 \) and \( ie3 \) are index expressions.

3. It sorts factors and multiplies constant factors within terms,
\[ \text{factor1} \cdot \text{factor2} \Rightarrow \text{factor2} \cdot \text{factor1} \]

when \( \text{factor2} \prec \text{factor1} \), and where \( \prec \) is a lexicographical order on the factors (constants \( \prec \) variables) and
\[ c1 \cdot c2 \cdot \text{term} \Rightarrow c3 \cdot \text{term} \quad (c3 = c1 \cdot c2) \]

where \( c1, c2 \) and \( c3 \) are constant factors.

4. It sorts terms and adds/subtracts identical terms.
\[ \text{term1} \cdot \text{term2} \Rightarrow \text{term2} + \text{term1} \]

when \( \text{term2} \prec \text{term1} \), where \( \prec \) is a lexicographical order on the terms (constants \( \prec \) variables) and
\[ c1 \cdot \text{term} + c2 \cdot \text{term} \Rightarrow c3 \cdot \text{term} \quad (c3 = c1 \cdot c2) \]

where \( c1, c2 \) and \( c3 \) are constant factors.

5. It adjusts for negative signs in constant factors in terms.
\[ -(c) \cdot \text{term} \Rightarrow c \cdot \text{term} \]
\[ *(-c) \cdot \text{term} \Rightarrow -c \cdot \text{term} \]

**Example 8.2:** Normalization of the abstract syntax tree of an index expression.

Consider the following index expression
\[(a+3) \times (a-6)\]

The abstract syntax tree of this index expression is

```
          *
         / \-  
        /   /  /
       a  3  a  6
```

Applying the algorithm gives

Step 1: distribution of \(\ast\) over \(+/\): 

\[
(a+3) \times (a-6) \Rightarrow (a \ast (a-6)) + (3 \ast (a-6)) \\
\Rightarrow ((a-6) \ast a) + ((a-6) \ast 3) \\
\Rightarrow (a \ast a - 6 \ast a) + (a \ast 3 - 6 \ast 3)
\]

resulting in the tree

```
          +
         /  \-  
        /    /  /
       a  a  6  a  a  3  6  3
```

Step 2: remove influence of parentheses:

\[
(a \ast a - 6 \ast a) + (a \ast 3 - 6 \ast 3) \Rightarrow ((a \ast a - 6 \ast a) - 6 \ast 3 + a \ast 3)
\]

resulting in the tree

```
          +
         /  -  
        /    /  
       *  a  3
      /    /
     a  a  6  3
```

75
Step 3: sort factors and multiply constant factors within terms:

\[ a^3 - 6a - 6a^3 + a^3 \Rightarrow a^3 - 6a - 18 + 3a \]

resulting in the tree

```
+  
\-  
/ \  
*  
\18 / \ 
-  \ 
*  
\a / \ 
a \a \6 \a
```

Step 4: Sort terms and add/subtract identical terms:

\[ a^3 - 6a - 18 + 3a \Rightarrow a^3 - 6a + 3a - 18 \]

\[ \Rightarrow a^3 + (-3)a - 18 \]

resulting in the tree

```
-  
/  
\18 
/ \ 
*  
/ \ 
\a / \ 
a \a \-3 \a
```

Step 5: Adjust for negative signs:

\[ a^3 + (-3)a - 18 \Rightarrow a^3 - 3a - 18 \]

resulting in the tree:

```
-  
\-  
/ \ 
\18 
/ \ 
*  
/ \ 
\a / \ 
a \a \3 \a
```

Equivalent index expressions are always represented in CONDUCTOR by the same abstract syntax tree. An example of an equivalent index expression is \( a^3 - 3(a-6) \).

{end example 8.2.}
8.2. ALGEBRAIC SPECIFICATION OF INDEX EXPRESSIONS

The abstract syntax trees of index expressions are specified in module
Ind-expr-abstr-syntaz.

module Ind-expr-abstr-syntaz
begin
exports
begin
sorts IND-EXPR
functions
  abs-plus: IND-EXPR # IND-EXPR -> IND-EXPR
  abs-minus: IND-EXPR # IND-EXPR -> IND-EXPR
  abs-mul: IND-EXPR # IND-EXPR -> IND-EXPR
  abs-const: INDEX -> IND-EXPR
  abs-ind: ID -> IND-EXPR
  null: -> IND-EXPR
  eq-result: IND-EXPR # IND-EXPR -> BOOL
end
imports Indices, Identifiers, Booleans
end Ind-expr-abstr-syntaz

Equivalence classes of index expressions are defined by equations in
module Ind-expr-abstr-syntaz. Each step in the normalization algorithm
requires that the functions in the module Ind-expr-abstr-syntaz satisfy
a certain equations.

Step 1. Distributivity of the multiply operator over the add and
minus operator.

[59]  eq-result(abs-mul(e1,abs-plus(e2,e3)),
         abs-plus(abs-mul(e1,e2),abs-mul(e1,e3)))
         = true

and

[60]  eq-result(abs-mul(e1,abs-minus(e2,e3)),
         abs-minus(abs-mul(e1,e2),abs-mul(e1,e3)))
         = true

Step 2. Associativity of the plus, minus and multiply operator.

[61]  eq-result(abs-plus(e1,abs-plus(e2,e3)),
         abs-plus(abs-plus(e1,e2),e3))
         = true

and
\[62\] \text{eq-result(\text{abs-minus}(e_1, \text{abs-minus}(e_2, e_3)),} \\
\text{\quad \text{abs-minus}(\text{abs-minus}(e_1, e_2), e_3))} \\
\quad = \text{true}
\]

and

\[63\] \text{eq-result(\text{abs-mul}(e_1, \text{abs-mul}(e_2, e_3)),} \\
\text{\quad \text{abs-mul}(\text{abs-mul}(e_1, e_2), e_3))} \\
\quad = \text{true}
\]

The other steps in the algorithm require that:
- the multiplication of indices can be applied to the constant factors in one term,
- the multiply operator is associative and commutative,
- the plus and minus operator are associative and commutative, and the arithmetic rules of indices must be applicable to add/subtract terms,
- subtracting a negative term is equal to adding the negation of that term.

A complete specification of these requirements is given in module \text{Ind-expr-absic-syntax} in appendix E. The equations in this module define the equivalence relation on index expressions.

### 8.3. Symbolic Equivalence of Dimension Ranges and Data Types

A dimension range consists of a pair of index expressions. This pair describes the lower and upper bound of the range. The symbolic equivalence of dimension ranges is specified by straightforward application of the symbolic equivalence relation on index expressions.

\[
\text{range: INDEX-EXPR \# INDEX-EXPR -\rightarrow RANGE}
\]
\[
\text{eq-range: RANGE \# RANGE -\rightarrow BOOL}
\]

with equation

\[82\] \text{eq-range(range(e_1, e_2), range(e_3, e_4))} \\
\quad = \text{eq-result(\text{abs-minus}(e_2, e_1), \text{abs-minus}(e_4, e_3))}
\]

In words, two ranges are symbolic equivalent if the difference between the upper bound and lower bound of the two ranges are symbolic equivalent.

A matrix in the statistical language may have more than one dimension. A sequence of dimension ranges is specified in the module \text{Range-sequences}. 78
This module is imported in the module `tech-types`, where the type
descriptions of the variables in the statistical language are specified.

```plaintext
module Technique-types
begin
  exports
  begin
    sorts TECH-TYPE
    functions
    tech-type: SIMPLE-TYPE -> TECH-TYPE
    matrix-type: RANGE-SEQ -> TECH-TYPE
    eq-type: TECH-TYPE # TECH-TYPE -> BOOL
  end
end
```

```plaintext
imports Range-sequences, Booleans
```

end Technique-types

The type of a variable in the statistical language, is either a simple
type (boolean, index or scalar) or a matrix type. Symbolic equivalence of
type descriptions is specified in the function `eq-type`. Two matrix type
descriptions are symbolic equivalent when the dimension ranges (rs1 and
rs2) are symbolic equivalent.

\[ eq-type(matrix-type(rs1), matrix-type(rs2)) = eq-ranges(rs1, rs2) \]

A complete specification of the function `eq-type` is found in appendix E.

**Example 8.4:** Comparison of matrix-types.

Let the `INPUT` section of a statistical program contain the following
variable declarations:

```
<table>
<thead>
<tr>
<th>declarations stat. program</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDEX</td>
</tr>
<tr>
<td>k,n,m,p;</td>
</tr>
<tr>
<td>VECTORS [n*p TO n+k] X;</td>
</tr>
<tr>
<td>VECTORS [m*p TO m+k] Y;</td>
</tr>
</tbody>
</table>
```

To simplify the example we assume that in the formal specification the
index variables `k, n, m` and `p` are represented by the same identifier.
Note that in general the identifiers in the specification are different
from the identifiers in the program.
$k,n,m,p :\Rightarrow ID$

Using the functions in the formal specification the type description of the variable $X$ reads

\[
   t1 = \text{matrix-type}(
      \text{add-item(range(abs-plus(abs-ind(n),abs-ind(p)),}
      \quad \text{abs-plus(abs-ind(n),abs-ind(k))},
      \quad \text{null-range}))
\]

and the type description of variable $Y$ reads

\[
   t2 = \text{matrix-type}(
      \text{add-item(range(abs-plus(abs-ind(m),abs-ind(p)),}
      \quad \text{abs-plus(abs-ind(m),abs-ind(k))},
      \quad \text{null-range}))
\]

The two type descriptions $t1$ and $t2$ are equivalent as the following derivation shows:

\[
\text{eq-type}(t1,t2) =
\text{eq-ranges(add-item(range(abs-plus(abs-ind(n),abs-ind(p)),}
      \quad \text{abs-plus(abs-ind(n),abs-ind(k))},
      \quad \text{null-range),}
      \quad \text{add-item(range(abs-plus(abs-ind(m),abs-ind(p)),}
      \quad \text{abs-plus(abs-ind(m),abs-ind(k))},
      \quad \text{null-range}))}
\]

\[
\text{and(}
   \text{eq-range(range(abs-plus(abs-ind(n),abs-ind(p)),}
   \quad \text{abs-plus(abs-ind(n),abs-ind(k))},
   \quad \text{range(abs-plus(abs-ind(m),abs-ind(p)),}
   \quad \text{abs-plus(abs-ind(m),abs-ind(k))},
   \quad \text{eq-ranges(null-range,null-range))}
\]

\[
= \text{eq-range(range(abs-plus(abs-ind(n),abs-ind(p)),}
   \quad \text{abs-plus(abs-ind(n),abs-ind(k))},
   \quad \text{range(abs-plus(abs-ind(m),abs-ind(p)),}
   \quad \text{abs-plus(abs-ind(m),abs-ind(k))})
\]

\[
\text{eq-result(abs-minus(abs-plus(abs-ind(n),abs-ind(p))),}
   \quad \text{abs-plus(abs-ind(n),abs-ind(k)))},
\quad \text{abs-minus(abs-plus(abs-ind(m),abs-ind(p))),}
\quad \text{abs-plus(abs-ind(m),abs-ind(k)))}
\]

\[
= \text{eq-result(abs-minus(abs-ind(p),abs-ind(k))),}
\quad \text{abs-minus(abs-ind(p),abs-ind(k))})
\]

\[
\text{true}
\]

{ end example 8.4. }
8.4. MANAGING TYPE INFORMATION DURING TYPE CHECKING

In order to manage the type information obtained during type checking we introduce a symbol table that relates variables in a statistical program with their type. This table is called the technique symbol table. For each variable the data type description (the sort TECH-TYPE), the visibility (the sort USER-VIEW), the value (the sort TECH-DATA) and a message for the user (the sort STRING) is stored. The information for each variable in the symbol table can be located by the variable identifier (the sort ID).

![Diagram of the technique symbol table]

The variables in the declaration sections of a statistical program are stored in the technique symbol table. The module Store-declarations imports the abstract syntax of declarations in the statistical language from the module Decl-abstr-syntax, and imports the technique symbol tables from the module Tech-symtabs. The function store-id specifies that the information in a variable declaration, the identifier (id), the type description (tt), the user view (uw) and the message (mess), is inserted
in the technique symbol table (ts). A variable is not assigned a value when it is declared. This is specified by the constant function uninitialised.

\[118\] \text{store-id}(ts, td, uw, \text{mass}) \\
= \text{insert}(id, \\
\text{ts-info}(tt, uw, \text{uninitialized}, \text{mass}), \\
ts)

8.5. TYPE CHECKING OF ASSIGNMENT STATEMENTS

Symbolic type checks are performed for all assignment statements in a statistical program. The other statements do not, or only indirectly, impose type restriction. The function type-check-stmt,

\[
\text{type-check-stmt: TECH-SYMTAB \# STATEMENT} \rightarrow \text{BOOL}
\]

with equation

\[156\] \text{type-check-stmt}(\text{abs-assign}(\text{var}, \text{expr}), \text{tst}) \\
= \text{eq-type}(\text{expr-type}(\text{expr}, \text{tst})), \text{var-type}(\text{var}, \text{tst})

specifies the type restrictions on statements. The type of the variable (\text{var}) in the left-hand-side of the assignment statement is determined by the function \text{var-type}(\text{var}, \text{tst}). This function retrieves the type of the variable from the technique symbol table \text{tst}. The type of the expression on the right-hand-side of an assignment statement is specified in the function \text{expr-type} in module Stat-check-expressions

\[
\text{expr-type: EXPR \# TECHN-SYMTAB} \rightarrow \text{TECH-TYPE}
\]

If, for example, an expression only consists of a scalar (sca) its resulting type is scalar type, as expressed in the equation

\[150\] \text{expr-type}(\text{abs-expr}(\text{t-data(sca)}), \text{tst}) = \text{tech-type}(\text{scalar-type})

where \text{tst} is a technique symbol table. The specification of the type of other expressions can be found in module Stat-check-expressions in appendix H.
8.6. TYPE CHECKING OF FUNCTION CALLS

Type checking of functions calls in the statistical language is organized around the notion of a type list. The type list contains the types of the function arguments. The types in the type list are used to check the type restrictions in the function table. The latter captures the type information for each function in the statistical language. Type restrictions on functions describe the required types of the result and the argument(s) of a function. The symbolic type checker in CONDUCTOR checks two sorts of type restrictions:

(1) skeleton restrictions
(2) dimension restrictions

To check the first class of restrictions the function eq-skelet is specified in the module Tech-types.

\[ eq\text{-skelet}: TECH\text{-TYPE} \# TECH\text{-TYPE} \rightarrow BOOL \]

In this function the equivalence of matrix-types is less restrictive than in the function eq-type. The function eq-skelet only checks if the number of dimensions of the matrix-types, with range-sequences r1 and r2, are equal,

\[ eq\text{-skelet}(matrix\text{-type}(r1),matrix\text{-type}(r2)) = eq(n\text{-of-dims}(r1),n\text{-of-dims}(r2)) \]

whereas the function eq-type checks if the dimension ranges are equivalent.

A skeleton restriction consists of two indices, indicating which arguments in the type list must satisfy the skeleton restrictions, or an index and a type description where the indicated element in the type list must be equal to the given type.

\[ restr: INDEX \# INDEX \rightarrow SKELET\text{-RESTR} \]
\[ restr: INDEX \# TYPE \rightarrow SKELET\text{-RESTR} \]

A check of a skeleton restriction is specified in function check

\[ check: SKELET\text{-RESTR} \# TYPE\text{-LIST} \rightarrow BOOL \]

with equations

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[124] \texttt{check(restr(ind1,ind2),tl)}
\hspace{1cm} = \texttt{eq-skeleton(\texttt{argument(tl,ind1)},\texttt{argument(tl,ind2)})}

[125] \texttt{check(restr(ind,t),tl)}
\hspace{1cm} = \texttt{eq-skeleton(\texttt{argument(tl,ind)},t)}

A dimension restriction imposes an equality restriction or a constant restriction on particular dimension ranges of arguments. A dimension range of an argument is indicated in the type list by a constant range. The first element of the constant range specifies the argument number, the second index the dimension number. An equality restriction on two dimension ranges consists of two constant ranges; a constant restriction on a dimension range consist of a constant range and a dimension range.

\begin{itemize}
    \item \texttt{dim-restr: CONST-RANGE \# CONST-RANGE} -> \texttt{DIM-RESTR}
    \item \texttt{dim-restr: CONST-RANGE \# RANGE} -> \texttt{DIM-RESTR}
\end{itemize}

The check of dimension restrictions

\begin{itemize}
    \item \texttt{check: DIM-RESTR \# TYPE-LIST} -> \texttt{BOOL}
\end{itemize}

checks if the restriction is satisfied for a particular type list. If the dimension restriction consists of two constant ranges \texttt{cr1} and \texttt{cr2}, these constant ranges are used to retrieve the appropriate dimension ranges from the type list \texttt{tl} and these dimension ranges are compared.

[132] \texttt{check(dim-restr(cr1,cr2),tl)}
\hspace{1cm} = \texttt{eq-range(arg-range(cr1,tl),arg-range(cr2,tl))}

The type of the result of a function is specified to be either a simple type or a matrix type. In the latter case the dimension ranges are retrieved from the type list.

\begin{itemize}
    \item \texttt{res-type: TYPE} -> \texttt{RESULT-TYPE}
    \item \texttt{res-type: CONST-RANGE-SEQ} -> \texttt{RESULT-TYPE}
\end{itemize}

How the result is defined in term of the argument types in the type list is specified in module \texttt{Result-types} in appendix H. Example 8.5 demonstrates a set of symbolic type restrictions that can be checked by the type checker in \texttt{CONDUCTOR}.

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Example 8.5: Type restrictions on functions.

Assume we impose the following restrictions on a function:
1. Skeleton restrictions on arguments:
   - the first argument must be a scalar.
   - the second argument must be a two dimensional matrix.
2. Dimension restrictions on arguments:
   - the first dimension of the second argument must be equal to the
     second dimension of the second argument.
3. The type of the result of the function:
   - the result is a one dimensional matrix.
   - the dimension range of the result is equal to the first dimension
     range of the second argument.

In terms of the functions in the formal specification the skeleton
restrictions read\(^3\)

\[
\begin{align*}
\text{restr}(1, \text{tech-type}(\text{scalar-type})) \\
\text{restr}(2, \text{matrix}(\text{add-item}(r1, \text{add-item}(r2, \text{null-range}))))
\end{align*}
\]

where \(r1\) and \(r2\) are ranges. The symbolic dimension restrictions read

\[
\begin{align*}
\text{dim-restr}(\text{c-range}(1,2), \text{c-range}(2,2))
\end{align*}
\]

The result is specified as

\[
\text{res-type}(\text{add-item}(\text{cr-range}(2,1), \text{null-cr-range}))
\]

The type restrictions on functions are sequences of skeleton restrictions
and dimension restriction. A function \texttt{check} specifies the checking of all
skeleton restrictions.

\texttt{check: SKELET-RESTR-SEQ \# TYPE-LIST -> Bocl}

with equations

\[
\]
[126] \text{check}(\text{add-item}(sr,\text{null-seq}),tl) = \text{check}(sr,tl)

and

[127] \begin{align*}
\text{check}(\text{add-item}(sr,srs),tl) \\
&= \text{and}(\text{check}(sr,tl),\text{check}(srs,tl))
\end{align*}

where \textit{sr} is a skeleton restriction and \textit{srs} is a sequence of skeleton restrictions. In a similar way, module \textit{Dim-restrictions} specifies the check of dimension restrictions (the sort \textit{DIM-RESTR-SEQ}) in appendix H. All type information concerning functions is combined in the sort \textit{FUNC-TYPE-INFO} in module \textit{Function-types}

\begin{align*}
\text{f-info: INDEX} & \# \text{SKELET-RESTR-SEQ} \# \\
\text{DIM-RESTR-SEQ} & \# \text{TECH-TYPE} & \rightarrow & \text{FUNC-TYPE-INFO}
\end{align*}

The type information is stored in the function type table. The type of a function call in the statistical language, with function identifier \textit{f-id} and argument list \textit{argl}, can now be specified as:

[153] \begin{align*}
\text{expr-type}(\text{abs-f-call}(f-id,\text{argl}),tst) \\
&= \text{result-f}(f-id,\text{arg-types}(\text{argl},\text{null},tst))
\end{align*}

when \text{check-f-restr}(f-id,\text{arg-types}(\text{argl},\text{null},tst)) = \text{true}

The function \text{check-f-restr} defines the checking of both the skeleton and dimension restrictions; the function \text{result-f} returns the type of the result of the function call in the statistical language. The type of the arguments is evaluated by the function \text{arg-types}, using information in the symbol table \textit{ts}.

The above mechanism only describes equality and constant restrictions on dimension ranges of matrix types. More complicated restriction on these dimension ranges can be created using the range expressions specified in module \textit{Ranges}. Range expressions will not always yield correct ranges. These problems, caused by the lack of order on the equivalence classes of index expressions, are discussed in chapter 9.
8.7. A TYPE CORRECT STATISTICAL PROGRAM

In chapter 7 we specified a syntactically correct statistical program. Type checking of a statistical program, using the mechanisms discussed in this chapter, gives the next step in the definition of a correct statistical program: a type correct statistical program. The symbolic type checking of a statistical program is specified in module Static-type-checking.

The parameter Current-func-type-restrs is left unbound in this module. This specifies that the design of CONDUCTOR is independent of the functions defined in the statistical language.
9. SYMBOLIC DIMENSION BOUND CHECKS AND THE GENERATION OF INPUT RESTRICTIONS

In a correct statistical program, matrix element and submatrix references may never refer to elements outside the declared dimensions of a matrix. Matrix element reference can be compared with references to elements of a dynamic array in a higher-level programming language program. In practice it is difficult to check that such references are correct for all possible values of the input variables of such a program. A higher-level programming language program may have infinitely many execution trees, i.e. there are infinitely many ways to evaluate a program written in these languages. In practice, therefore, one often has to rely on testing instead of proving the correctness of a program. A way to test a program with infinitely many execution trees is called symbolic evaluation (see King [1976]). For each variable, after each statement, a symbolic denotation of the value is given and for each path in the program a conditional expression is calculated, indicating under which condition the path is taken. King [1976] suggests giving a symbolic value to the input variables and shows that, for a simple programming language, EFFIGY, symbolic evaluation is possible. The mechanism suggested by King, is used in CONDUCTOR to generate input restrictions on the values of the input variables of a statistical program. The symbolic evaluation of a statistical program is facilitated by the fact that the structure of the statistical language in CONDUCTOR is less complicated than the structure of higher-level programming languages. Important differences in comparison with higher-level programming languages are:
- the language does not have conditional statements,
- the only loop control statement is the for-statement,
- a for-loop is at least executed once,
- loop control variables may not be redefined inside a loop,
- the input index variables in the statistical program may not be
  redefined inside a statistical program,
- index expressions are a subset of general integer expressions.

Due to these restrictions a statistical program has a unique symbolic
execution tree, i.e. a symbolic range can be calculated for each index
variable after each statement in the program. This symbolic range
describes the upper and lower bound of the index variable in terms of the
input index variables of the statistical technique.

The symbolic evaluation of a statistical program consists of the
following steps:
- symbolic ranges are assigned to input index variables,
- symbolic ranges are assigned to index variables in for-statements and
  index assignment statements.
- for each index expression a symbolic range is calculated using the
  symbolic ranges of the index variables in the expression,
- the symbolic ranges of index expressions in a matrix element reference
  are compared with the symbolic dimension ranges of the referenced
  matrix; if these symbolic ranges are not equivalent, input
  restrictions on the values of the input index variables are generated.

Consider the following statistical program

```
statistical program

INTERFACE
INPUT
  INDEX n,m,p;
  VECTOR [n*m TO p*m] X

VARIABLES
  INDEX i

EQUATIONS
  X := 0;
  X[i+m] := 1/m
  i := n,...,p
```

In this example, first the symbolic ranges [n TO n], [m TO m] and [p TO p],
are assigned to the input variables n, m and p. Then a symbolic range
[n TO p] is assigned to the index variable i. And finally a symbolic
range \([n+m \text{ TO } p+m]\) is calculated for the expression \(i+m\), by adding the bounds of the symbolic ranges of \(i\) and \(m\). This range is equivalent to the symbolic dimension range of the vector \(X\), showing that the element reference \(X[i+m]\) is correct for all possible values of the input variables.

The symbolic ranges for an index expression are calculated by applying the operator in the index expression to the bounds of the symbolic ranges of the operands. The example above, of course, obscures the problems that may occur in less trivial index expressions. Symbolic ranges can only be calculated with the range calculation mechanism for monotone increasing or decreasing index expressions. And as an additional restriction all symbolic ranges in the symbolic calculations must be strictly positive.

9.1. AN ORDERING ON INDEX EXPRESSIONS

In the symbolic dimension check mechanism, it has to be determined whether the symbolic range of an index expression is a subrange of the declared symbolic range. For example, the vector \(X\), declared in the statistical program in the beginning of this chapter, has dimension bounds \([n + m \text{ TO } p + m]\). For the index expression \(i+m\), in the vector element reference \(X[i+m]\), we calculated the same range. And we had to determine whether the restriction

\[
n+m \leq n+m \leq n+p \leq n+p
\]

holds for all possible values of the variables in the expressions. Thereto, we have to define an ordering on the index expressions. This ordering is based on the ordering of integers. If two expressions always yield the same integer value after evaluation, for all possible values of the variables in these expressions, they are equivalent. If one index expression always yields a greater/smaller integer than another index expression, the order is greater/smaller. The order of two expressions is said to be undecided if the comparison is undecided. The order on index expressions is specified in module Ind-expr-order.
module Ind-expr-order
begin
exports
begin
functions
  order: IND-EXPR # IND-EXPR -> RELATION
end

imports Order, Ind-expr-abstr-syntax
end Ind-expr-order

The function order specifies the order relation of two index expressions, \( iel \) and \( ie2 \), based on the order relation on the integers. If the difference between two index expression is a constant \( c \), it can be decided if the order relation between the two expressions is equal, greater or less, by comparing the constant \( c \) with 0. If \( c \) is equal to 0 the boolean function \( eq(c,0) \) returns the value true, if \( c \) is greater than 0, the boolean function \( ge(c,0) \) returns the value true.

\[ \text{order}(iel,ie2) = \begin{cases} eq(c,0), & \text{equal}, \\ ge(c,0), & \text{greater,less} \end{cases} \]

when eq-result(abs-minus(iel1,ie1),abs-const(c)) = true

If the difference between two index expressions is not a constant, the expressions are not comparable, and the order is said to be undecided.

\[ \text{order}(iel,ie2) = \text{undecided} \]

when eq-result(abs-minus(iel2,ie1),abs-const(c)) = false

If the difference between two index expressions consist of the sum of quadratic terms, also the order relation can be determined. If all quadratic terms are positive, the order relation is greater and if all quadratic terms are negative, the order relation is smaller. If the resulting difference is a mixture of positive and negative quadratic terms, the order is again undecided. In the formal specification of the function order in appendix I, the order relation based on quadratic terms is not incorporated, it can however easily be added.
9.2. CALCULATION OF A SYMBOLIC RANGE FOR AN INDEX EXPRESSION

For each index expression in a statistical program a symbolic range is calculated during the symbolic evaluation. A symbolic range for an index expression is calculated by applying the operator in the index expression to the bounds of the symbolic ranges calculated for the operands.

The range operator $\cdot_r$, $-r$, and $\ast_r$ are defined as follows:

$$[n \text{ to } m] \cdot_r [p \text{ to } q] = [(n \cdot p) \text{ to } (m \cdot q)]$$
$$[n \text{ to } m] -_r [p \text{ to } q] = [(n \cdot q) \text{ to } (m \cdot p)]$$
$$[n \text{ to } m] \ast_r [p \text{ to } q] = [(n \cdot p) \text{ to } (m \cdot q)]$$

An index expression that only consists of a variable (x) or constant (c) the function range results in a range with the variable or constant as lower and upper bound. The calculation of symbolic ranges is specified in the function `calc-range` in module `Range-calculations`. 

---

**Diagram:**

- **Ind-expr-abstr-syntax**
  - **Items**
  - **Sequences**
  - **Ind-expr-sequences**
  - **Booleans**
  - **Monotone-restrictions**

- **Ranges**
  - **Items**
  - **Sequences**
  - **Range-sequences**

- **Ranges**
  - **Identifiers**
    - **Entries**
    - **Addresses**
    - **Tables**
    - **Range-tables**

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For instance, the equation [205] specifies the calculation of a range for the plus operator in an index expressions.

\[205\] \text{calc-range}(\text{abs-plus}(iel, ie2), rt) \\
= \text{range-plus}(\text{calc-range}(iel, rt), \\
\text{calc-range}(ie2, rt)) \\
\text{when } \text{is-monotone}(\text{abs-plus}(iel, ie2)) = \text{true}

Where \(iel\) and \(ie2\) are index expression and \(rt\) is a range table. This equation states that if we want to calculate the symbolic range of an expression that is the sum of two expressions, we first calculate the symbolic ranges for the two sub-expressions \((iel\) and \(ie2\)) and then add these two symbolic ranges. The restriction that the index expressions must be monotone increasing or decreasing is discussed in the next section of this chapter. The addition of two symbolic ranges is specified in module Ranges in equation [??]. The addition of two ranges, \(\text{range}(iel, ie2)\) and \(\text{range}(ie3, ie4)\) results in a new range where the lower and upper bounds are added.

\[??\] \text{range-plus}(\text{range}(iel, ie2), \text{range}(ie3, ie4)) \\
= \text{range}(\text{abs-plus}(iel, ie3), \text{abs-plus}(ie2, ie4))

Similar equations are given for multiplication and subtraction.
Ranges for index expressions can only be calculated if the intermediate ranges in the calculations are strictly positive. If this can not be verified, input restrictions are generated as specified in module Range-calc-restr. For example, the restriction that a range retrieved from the range table \((id’s)\) must always be positive is specified in function restr-calc-range.

\text{restr-calc-range}: \text{IND-EXPR} \# \text{RANGE-TABLE} \#
\text{TECH-SYMTAB} \# \text{INP-RESTR-SEQ} \rightarrow \text{INP-RESTR-SEQ}

\[218\] \text{restr-calc-range}(\text{abs-ind}(id), rt, ts, rs) \\
= \text{conc}(\text{pos-range-restr}(id^\prime rt, ts), rs)

The function \text{conc} concatenates the generated restrictions and the already existing input restrictions \(rs\).
9.3. RESTRICTIONS ON INDEX EXPRESSIONS

A symbolic range for an index expression is calculated by applying the operator in the index expression to the bounds of the symbolic ranges of the operands. This mechanism can only be applied to positive monotone increasing or decreasing index expressions. Furthermore all ranges in the calculations should be positive.

Example 9.1: A not monotone increasing index expression.

Consider the following simple index expression

\[ p(i) = i*i - 4*i + 4 \]

This expression is neither monotone increasing nor monotone decreasing.

\[ \frac{\delta}{\delta i} p(i) > 0 \quad i > 2 , \]

\[ \frac{\delta}{\delta i} p(i) < 0 \quad i < 2 . \]

Assume the variable \( i \) is assigned the symbolic range \([n \text{ to } p]\). The symbolic range calculation mechanism will first assign a range to the variables and constants in the expression, then it will use these ranges to assign ranges to the terms \( i*i, 4*i \) and \( 4 \), etc.

\[
\begin{align*}
[n \text{ to } p] & \quad [4 \text{ to } 4*p] & \quad [4 \text{ to } 4]\[4] \\
[n \text{ to } p-4*p] & \quad [p-4*p \text{ to } p-4*n] & \\
[n \text{ to } p-4*p+4] & \quad [p-4*p+4 \text{ to } p-4*n+4]
\end{align*}
\]

The calculated range is obviously wrong. Substitution of \( n = 1 \) and \( p = 10 \) leads to a calculated range of -35 to 100, while the actual range of this quadratic function is 0 to 64.

{end example 9.1.}

For monotone increasing expression it can easily be shown that the range
calculation mechanism is correct. A normalized index expressions (see algorithm 8.1 in chapter 8) is a polynomial in the integer variables \( x_1, \ldots, x_n \). Calculating the upper bound of a normalized index expression is equal to finding the maximum of the polynomial for the variables \( x_1, \ldots, x_n \). For the variable \( x_i \) this reads

\[
\max_{x_1, \ldots, x_n} \sum_{i=1}^{n} c_i (x_i)^i f_i(x_2, \ldots, x_n)
\]

where \( c_i \) is the constant factor in a term of the polynomial, and \( f_i \) a function of the other variables in each term. The function \( f_i \) is a product of powers of the variables \( x_2, \ldots, x_n \). All variables in \( f_i \) are positive

\[ f_i(x_2, \ldots, x_n) \geq 0 \]

The variable \( x_i \) is also restricted to a positive range, with maximum \( \min(x_i) \) and minimum \( \max(x_i) \)

\[ 0 < \min(x_i) \leq x_i \leq \max(x_i) \]

If also \( c_i \geq 0 \), the partial derivative of the polynomial with respect to \( x_i \) is positive for all values of \( x_1, \ldots, x_n \), and we may simply substitute the maximum of \( x_i \) for every occurrence of \( x_i \) to get the maximum of the polynomial with respect to \( x_i \).

\[
\max_{x_2, \ldots, x_n} \sum_{i=1}^{n} c_i (\max(x_i))^i f_i(x_2, \ldots, x_n)
\]

It can also be shown that, for strictly negative terms, where \( c_i \leq 0 \), one may substitute the minimum of \( x_i \) to get the maximum of the polynomial. This shows that we can use the symbolic evaluation mechanism under the restriction that a given polynomial \( p \) can be written as the sum of two polynomials \( p_1 \) and \( p_2 \)

\[ p(x_1, \ldots, x_n, y_1, \ldots, y_t) = p_1(x_1, \ldots, x_n) + p_2(y_1, \ldots, y_t) \]

where the function \( p_1 \) only contains the positive terms of polynomial \( p \)
and \( p_2 \) the negative terms. The sets of variables \( x_1, \ldots, x_n \) and \( y_1, \ldots, y_k \) must be disjoint. The check whether an index expression is monotone increasing or decreasing, amounts to a check whether the set of variables in negative terms and the set of variables in positive terms of a normalized index expression is disjoint. The monotonicity restriction is specified by the function \texttt{is-monotone} in module \texttt{Monotone-restrictions}.

\begin{verbatim}
module Monotone-restrictions
begin
  exports
    begin
      functions
        is-monotone: IND-EXPR -> BOOL
        var-list: BOOL # BOOL # IND-EXPR # ID-SEQ -> ID-SEQ
      end

    imports Ind-expr-abstr-syntax, Id-sequences,
        Boolean { renamed by [true -> pos, false -> neg] }
end Monotone-restrictions
\end{verbatim}

The function \texttt{is-monotone} makes two lists of identifiers. One list contains the variables in the positive terms, the other the variables in the negative terms. The first boolean flag in this function indicates if a variable must be added to the positive list or to the negative list. The second flag indicates the sign of the term that is examined. To increase the readability of the module the boolean functions \texttt{true} and \texttt{false} are renamed. If the two lists are disjoint the expression is monotone increasing or decreasing \(^1\).

\texttt{[204] is-monotone(ie) = disjoint(var-list(pos,pos,ie,null),
    var-list(pos,neg,ie,null))}

A complete specification of the function \texttt{var-list} in is found in module \texttt{Monotone-restrictions}.

All symbolic ranges in the symbolic evaluation must be positive. In general this restriction is not satisfied. Consider the input declarations

\[^1\] Here we pay the price for not having specified a normalized index expression. The \texttt{is-monotone} function is only correct if applied to a normalized index expression. For other representations it may lead to a wrong result.
The range \([n \text{ to } p]\) is positive if \(n \geq 1\) and \(p \geq n\). CONDUCTOR generates during the compilation of a statistical program a sequence of these input restrictions.

The positive restriction of symbolic ranges is specified in the module \textit{Range-restrictions} in equation [193] for function \textit{pos-range-restr}.

\begin{verbatim}
[193] pos-range-restr(range(iel,ie2),ts)
    = add-imp-restr(iel,abs-const(:nd(1)),ts,
                     add-imp-restr(ie2,iel,ts,no-restrictions))
\end{verbatim}

The function \textit{add-imp-restr} adds an input restriction (\textit{imp-restr (iel,ie2,ts)}) to the input restriction sequence \textit{irs}, when the order relation of the two index expressions in the range is undecided.

\begin{verbatim}
[192] add-imp-restr(iel,ie2,ts,irs)
    = if(eq(order(iel,ie2),undecided)),
        add-item(imp-restr(abs-minus(iel,ie2),ts),irs),
        if(eq(order(iel,ie2),less),
            error-imp-restr-seq,
            irs)
\end{verbatim}
9.4. INPUT RESTRICTIONS

All input restrictions in CONDUCTOR are given as a positive restriction on an index expression.

index expression \geq 0

The variables in the index expression in a restriction may only be input variables. CONDUCTOR checks whether all variables are input variables by examining the technique symbol table. The specification of input restrictions reads

\texttt{inp-restr: \ IND-EXPR \ # \ TECH-SYMTAB \ --> \ INF-RESTR}

with equation

\[ \text{inp-restr}(ie,ts) = \text{pos-restr}(ie) \]

\text{when are-input-vars}(ie,ts) = \text{true}

The function \texttt{are-input-vars} specifies that all the variables in the expression \((ie)\) must be declared in the input section of a statistical program. This function uses information stored in the technique symbol table \((ts)\) and is specified in module \texttt{Are-input-vars}. The function \texttt{pos-restr}(ie) specifies that the restriction \(ie \geq 0\) must hold.

9.5 ASSIGNMENT OF A SYMBOLIC RANGE TO AN INDEX VARIABLE

During the symbolic evaluation of a matrix language program symbolic ranges are assigned to index variables in:
- an INPUT declaration; an index variable \(n\) declared in an input section is assigned the range \([n \ to \ n]\),
- an index assignment statement; a non-input index variable \(p\) in the assignment \(p = ie\) is assigned the range calculated for the index expression \(ie\),
- a for-statement; a loop control variable \(i\) in the for loop
for i := iel to ie2 do statements

is assigned the range determined by the lower bound of the range calculated for index expression iel and the upper bound calculated for the index expression ie2.

To ensure that the for-statement is at least executed once, it must be checked if the upper bound of the range calculated for index expression iel is smaller or equal to the lower bound of the range calculated for index expression ie2. If this restriction is undecided input restrictions are generated for the input index variables of the matrix language program in which the for-statement occurs.

These symbolic ranges assigned to index variables are stored in the symbolic range table, specified by the sort RANGE-TABLE in the module Range-tables; the assignment is specified in module Range-assignments.

The abstract syntax of a for-statement is given in the module Statements-abstr-syntax in the function abs-for.

\[
\text{abs-for: INDEX-VAR \# INDEX-EXPR}
\]
\[
\text{# INDEX-EXPR \# STATEMENTS \rightarrow STATEMENT}
\]

The symbolic range calculated for the range consisting of the two index expressions (iel and ie2), is stored in the range table (rt) for the control index variable (id) in the for-loop.

\[
[229] \text{assign-range(abs-for(i, iel, ie2, stmts), rt)}
\]
\[
= \text{assign-ranges}
\]
\[
\text{stmts,}
\]
\[
\text{insert(id,}
\]
\[
\text{range-calc-range(range(iel, ie2),}
\]
\[
\text{rt))}
\]

Similar equations are specified for the range assignments in index assignment statements and INPUT index declarations in module Range-assignments.
9.6. MATRIX ELEMENT REFERENCES

The restriction that a symbolic range \((\text{range}(t_{e1}, t_{e2}))\) is a subrange of another symbolic range \((\text{range}(t_{e3}, t_{e4}))\) is specified in module Range-restrictions in function

\[
\text{range-within-range: } \text{RANGE} \# \text{RANGE} \# \text{TECH-SYMTAB} \\
\rightarrow \text{INP-RESTR-SEQ}
\]

with equation
[104] \text{range-within-range} (\text{range}(i_{e1},i_{e2}),\text{range}(i_{e3},i_{e4}),\text{ts})
= \text{add-imp-restr}(i_{e1},i_{e3},\text{ts},
\text{add-imp-restr}(i_{e2},i_{e1},\text{ts},
\text{add-imp-restr}(i_{e4},i_{e2},\text{ts},\text{no-restrictions}))))

Equation [104] specifies that, if the subrange restriction cannot be
verified, a sequence of input restrictions, \(i_{e3} \leq i_{e1} \leq i_{e2} \leq i_{e4}\), is
generated.

The generation of input restrictions for variables is specified in module
\text{Gen-restr-variables} in function \text{gen-restr-var}.

\text{gen-restr-var: INF-RESTR-SEQ # RANGE-TABLE #}
\text{TECH-SYMTAB # VARIABLE}
\rightarrow INF-RESTR-SEQ

A matrix element reference \text{abs-var}(id,\text{inds}) consists of an identifier \(id\)
and an index expression sequence \(\text{inds}\). For each of these index
expressions a symbolic range is evaluated, and the static type checker
checks if these symbolic ranges are subranges of the declared dimension
ranges of the matrix. If the static type checker cannot determine if
this restriction holds, input restriction are generated. Note that the
generated input restrictions consist of:
- input restrictions that guarantee that all symbolic ranges in the
  symbolic evaluation of an index expression \(ie\) are positive; this was
  specified in the function \text{restr-calc-ranges},
- input restrictions that guarantee that the calculated range lies
  within the referenced range; this was specified in function \text{range-
  with-in-range}.

[254] \text{gen-restr-var}(\text{irs},\text{rt},\text{ts},\text{abs-var}(id,\text{inds}))
= \text{conc} (\text{restr-calc-ranges}(id,\text{rt},\text{ts},\text{irs}),
\text{range-within-range} (\text{calc-ranges}(\text{inds},\text{rt}),\text{rs},\text{ts}))

\text{when eq-type(type(id,ts),matrix-type(rns)) = true}

9.7. A TYPE AND DIMENSION BOUND CORRECT STATISTICAL PROGRAM

In module \text{Input-restr-generator} it is specified how all input
restrictions are generated during the symbolic evaluation of a
statistical program.
The module Input-restr-generator imports modules that specify how input restrictions are generated for sections of the statistical program. These modules in turn import modules that specify how restrictions are generated for for-statements and declarations, etc. A complete specification of the input generation mechanism can be found in appendix I. A type correct statistical program in combination with the generated input restrictions is called a type and dimension bound correct statistical program.
PART III

FORMAL SPECIFICATION

OF THE KERNEL
The kernel is a simple virtual machine, with a processor that can execute kernel instructions. The instruction set of the kernel (the kernel language) shows close resemblance with a simple language, called postfix notation, used for intermediate code generation in compilers for higher-level programming languages. A kernel (language) program is a postfix representation of a statistical program. Postfix notation is, for example, used as intermediate code in most SNOBOL compilers (see Griswold [1972]).

The kernel instructions can be regarded as the semantical actions of the statistical language. There is, for example, an instruction that can access and execute numerical functions written by computer scientists, and an instruction that can prompt the user of a statistical technique for input. A statistical techniques, implemented by the technical statistician, is compiled into a kernel program and stored in the statistical technique table. This table contains the collection of available statistical techniques in the generated software. If in a user session a statistical technique is executed, the user language interpreter instructs the kernel to retrieve the technique from the statistical technique table, and to execute the kernel instruction sequence of the technique.

Besides the processor, important parts of the kernel are the data memory, the data stack and the external handler tables. The data memory of the kernel contains the values of the variables in the user session, and the value of the variables of an executing statistical technique. Expressions in the kernel language are evaluated on the data stack. Stacks are, for
example, also used in the design of the VAX-11 assembly language (see Peeters [1985]). The external handlers, written by either a computer scientist or a data expert, are stored in the external handler table. In this chapter we will give an informal description of the various parts of the kernel and the kernel language. A complete formal specification of the kernel can be found in appendix K.

10.1. MEMORY ORGANIZATION

The data memory of the kernel consists of, on the one hand, the symbol table of the user session and, on the other hand, the technique symbol table of an executing statistical program. In these tables the values of the variables in, respectively, a user session, and an executing statistical technique are stored. Note that these tables also contain type information for the static type checking. The use of the technique symbol table in type checking has already been discussed in section 8.4.

In the memory of the kernel, data transfer is possible between the user symbol table and the technique symbol table. This makes it possible for a user to initialize variables in a statistical technique. In the formal specification of the memory, in module Memory, this is represented by the functions user-store and user-load. Data transfer is also possible between the technique symbol table and the data stack. Recall that the
expressions in the kernel language are evaluated on the data stack. In the formal specification, transfer between the data stack and the technique symbol table is represented by the functions get-data and store-data.

10.2. THE INSTRUCTION SET

In the following subsections we give a short description of each of the kernel instructions. A formal specification of the effect of the instructions on the state of the kernel is discussed in section 10.5.

10.2.1. User interface instructions.

The USER-LOAD and USER-STORE instructions form the user interface of an implemented statistical technique. The USER-LOAD instruction prompts the user to initialize an input variable. If no message is specified in the declaration of that variable in the statistical program, its identifier is displayed followed by a question mark and an assignment symbol:

    variable_id ?=

A user can either enter a constant value or a variable declared earlier in his session. The user answer, of course, must be of the correct type. The value assigned to an input variable of a statistical program is

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stored in the technique symbol table. The address of the variable in this
table is a parameter of the USER-LOAD command. Matrices can also be
initialized using series of observation. The observations of the series
will form the columns of the matrix (see chapter 13).

Example 10.3: loading a matrix on the data stack.

Two index variables n and m and a matrix X are declared in the
INPUT/OUTPUT section of a statistical program.

```
input/output section of statistical program

INPUT
INDEX n, m;
MATRIX [1 TO n, 1 TO m] X
MESSAGE: "Enter a matrix identifier:"
```

At the kernel level this is represented as three USER-LOAD instructions.

```
kernel instructions

USER-LOAD n
USER-LOAD m
USER-LOAD X
```

These three instructions are the first to be executed when the
statistical technique is called by a user. Because no messages are
specified in the declaration of the variables n and m, the identifiers
are used in the prompt in combination with the prompt of the user command
language (the string 'u:'). After the user has entered the values for n
and m, the run-time type of the matrix X is calculated, and the user is
prompted with the specified message to enter a matrix identifier. In this
example a matrix with dimension ranges [1 TO 2, 1 TO 3] is expected.
The matrix usermat should be declared and initialized in a user session
before the statistical technique is called (see chapter 14). A copy of
the matrix usermat is stored in the technique symbol table entry of X.
{ end example 10.3 }

The USER-OUTPUT instruction transfers the calculated output variable back
to the user. The output variable is copied from the technique symbol
table to the user symbol table, and the output variable is displayed. If
a message is specified in the declaration of the output variable, this
message will accompany the output.

10.2.2. Data transfer between the symbol tables and the data
stack.

A LOAD instruction pushes a copy of the value of a variable from the
technique symbol table on the data stack. A STORE instruction pops the
top element of the data stack, and stores it in the technique symbol
table. In both instructions the address of the variable in the technique
symbol table is a parameter. Separate load and store instructions are
defined for submatrices and matrix elements. These instructions have as
an additional parameter the number of dimensions of the matrix. This
parameter tells the processor how many element or subrange references are
on the data stack.

10.2.3. Function calls.

Functions operate on the data stack of the kernel. When a function is
called, the results of evaluating its arguments, have already been pushed
on the data stack. The FUNCION-CALL instruction calls the numerical
function, and replaces the argument values on the stack by the result. The address of the numerical function in the function code table is a parameter of this instruction.

Example 10.4. Execution of a FUNCTION-CALL instruction.

Assume that in the statistical language there exists a predefined function \( f_4 \) with 3 arguments of type index and a result also of type index.

\[
f_4(\text{arg1}, \text{arg2}, \text{arg3}) = \min(\text{arg1} + \text{arg2}, \text{arg3})
\]

When the function \( f_4 \) is called, during the execution of a program, the data stack contains the values of the arguments. After execution of the function the arguments are popped from, and the result is pushed on the stack.

\[
\begin{array}{c}
\text{arg3} \rightarrow 9 \quad \text{<-- TOP} \\
\text{arg2} \rightarrow 7 \\
\text{arg1} \rightarrow 1 \\
\ldots
\end{array}
\]

\[
\begin{array}{c}
\text{result} \rightarrow 8 \quad \text{<-- TOP} \\
\ldots
\end{array}
\]

{ end example 10.4 }

10.2.4. The JUMP instruction.

To alter the sequential execution of instruction sequences both conditional and unconditional jump instructions exist. The conditional jump instructions, JUMP_TRUE and JUMP_FALSE, inspect the top of the data stack. If the top of the data stack contains a boolean with value true, respectively false, the address of the next instruction to be executed is obtained by adding the relative address specified in the jump instruction to the absolute address of the current instruction. Otherwise, the next instruction in the instruction sequence is executed. The unconditional JUMP instruction always jumps to the calculated address.
10.2.5. Instructions for exception handling.

Exception handlers in CONDUCTOR are represented at the kernel level as sequences of kernel instructions. The RAISE instruction transfers control from the instruction sequence of the statistical program to the instruction sequence of an exception handler. The address of the instruction after the raise instruction is saved. The exception handler is identified by its name, which is an argument of the RAISE instruction. This identifier is the address of an exception handler instruction sequence in an exception handler table. An UNRAISE instruction transfers control back to the interrupted instruction sequence of the statistical program. No nested exception handling is allowed in CONDUCTOR: if an exception handler invokes yet another exception the execution of the statistical technique is stopped.

10.2.6. The DISPLAY instruction.

The DISPLAY instruction prints a message. The only argument in this instruction is a message string.

10.2.7. The input restriction check.

The index input variables of a statistical technique must obey the generated input restrictions. This check is represented at the kernel level by the CHECK-RESTR instruction. This instruction evaluates the index expression in the input restrictions, and checks if they obey the restriction. If the input restrictions are not satisfied the user can restart or abort the technique. The input restrictions are not a parameter of the CHECK-RESTR instruction, but are part of the state definition of the kernel.

10.2.8. Evaluating index expressions.

The evaluation of index expression is represented at the kernel level by the EVAL-EXPR instruction. This instruction evaluates the index expression that is a parameter of this instruction, using the values of the variables in the technique symbol table.
10.2.9. A halt instruction.

The halted instruction terminates the execution of the kernel instruction sequence.

10.2.10. Algebraic specification of instructions.

The complete instruction set of the kernel is specified in module Instructions.

module Instructions
begin
  exports
  begin
    sorts INSTR
    functions:
      -- user interface --
      user-load: ID -> INSTR
      user-store: ID -> INSTR

      -- load instruction --
      load: TECH-DATA -> INSTR
      load: ID -> INSTR
      load-elem: ID # INDEX -> INSTR
      load-subm: ID # INDEX -> INSTR

      -- store instruction --
      store: ID -> INSTR
      store-elem: ID # INDEX -> INSTR
      store-subm: ID # INDEX -> INSTR

      -- index expression instructions --
      check-restr: INF-RESTR-SEQ -> INSTR
      eval-expr: INDEX-EXPR -> INSTR

      -- other instruction --
      jump: INDEX -> INSTR
      jump-true: INDEX -> INSTR
      jump-false: INDEX -> INSTR
      incal: ID -> INSTR
      f-call: ID # INDEX -> INSTR
      display: STRING -> INSTR
      raise: ID -> INSTR
      unraise: -> INSTR
      halt: -> INSTR
  end

  imports Technique-data, Identifiers, Ind-expr-abstr-syntax, Strings
end Instructions

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A sequence of instructions is specified by binding the sort \textit{ITEM} in the parameter \textit{Items} of the module \textit{Sequences} to the sort \textit{INSTR} in the module \textit{Instructions}. The sort \textit{INSTR-SEQ} is the formal abstraction of an instruction sequence at the kernel level. An instruction address in the kernel is specified as an instruction sequence combined with an integer indicating the position in the sequence. Given the address a function \textit{fetch} can retrieve an instruction from a sequence of instructions.

\[ \text{fetch}(\text{addr}(\text{is}, i)) = \text{item-no}(i, \text{is}) \]

Where \text{is} is an instruction sequence and \text{i} an index.

\section*{10.3. EXCEPTION HANDLER TABLES}

The kernel has tables to store exception handlers. The exception handler tables for exception handlers specified by a technical statistician are specified in module \textit{Tech-handler-tables}. Exception handlers created by a technical statistician are stored as sequences of kernel instructions, and each exception handler is uniquely determined by its name.

Two exception handler tables are defined in the kernel: a table with handlers specified by the technical statistician, and a table with external handlers specified by either the data expert or the computer scientist. The RAISE-instruction searches the tables to find a handler with a given name. If a handler with that name is found in the handler table of the technical statistician this handler is returned, otherwise the external handler table is searched. If no handler is found in either of these tables, a sequence containing only a halt instruction is returned, causing the statistical program that is calling the exception handler to terminate (a detailed discussion can be found in chapter 12).
10.4. THE DATA STACK

Expressions in the statistical language are evaluated on the data stack. A stack is a sequence of data. Items can be popped from the stack or pushed on the stack. This is specified in the module Data-stacks.

10.5. THE PROCESSOR

The state of the kernel is determined by (1) the data memory, (2) the data stack, (3) the current instruction, (4) the address of the next instruction, (5) the information needed to restart an interrupted instruction sequence, (6) the exception handler tables, (7) the input restrictions and (8) the user display, as specified in the function state in module Kernel-states.
state: MEMORY #
DATA-STACK #

INSTR #   -- current instruction --
INSTR-ADDR #  -- address next instruction --

RESET-INFO #
HANDLER-TABLES #

INP-RESTR-SEQ #
STRING      -- display --

--> STATE

The execution of a kernel instruction modifies the state of the kernel.

execute: STATE --> STATE

The working of the individual instructions is specified in the equations
for the function execute in module Processor. Consider, for instance, the
JUMP-TRUE instruction. This instruction checks the top element of the
data stack. If the flag \( b \) on top of the data stack \( ds \) is true, the
relative address \( int 1 \) is added to the current address \( addr(is,int2) \),
and the instruction at this address is executed next. The flag is popped
from the data stack. The other information remains unchanged.

\[ \text{execute}(state(m,}
\quad \text{push}(t-data(b),ds),
\quad \text{jump-true}(int1),
\quad addr(is,int2),
\quad rt,ht,irs,dis)
\] 
\[ \quad = \text{state}(m,}
\quad ds,
\quad \text{if}(b,
\quad \quad \text{fetch(addr(is,add(int1,int2))),}
\quad \quad \text{fetch(addr(is,int2))),}
\quad \quad \text{if}(b,
\quad \quad \quad \text{next(addr(is,add(int1,int2))),}
\quad \quad \quad \text{next(addr(is,int2))),}
\quad \quad \quad rt,ht,irs,dis)
\]

The specification of other kernel instructions is given in module
Processor in Appendix K.
10.6. KERNEL PROGRAMS

A statistical technique is reduced at the kernel level to the following abstract notions:
- the name of the technique,
- a sequence of instructions,
- a sequence of input restrictions,
- an exception handler table,
- a technique symbol table.

Such a representation of a statistical technique at the kernel level is called a kernel program. A kernel program is formalized in the module Kernel-programs.

```
module Kernel-programs
begin
  export
    begin
      sorts KERNEL-PRO
      functions
        kern-pro: INSTR-SEQ #
        INP-RESTR-SEQ #
        TECH-SYMTAB #
        TECH-HANDLER-TABLE -> KERNEL-PRO
    end

  imports Tech-symtabs, Instruction-sequences,
         Tech-handler-tables, Input-restr-sequences
end
```

The collection of all kernel programs is stored in the statistical technique table. The kernel can execute the kernel programs in this table, as specified in the functions run-technique in module Kernel.

```
run-technique: USER-SYMTAB # KERNEL-PRO #
                  EXT-HANDLER-TABLE -> USER-SYMTAB
```

The kernel retrieves the statistical technique from the table. The symbol table (ts) and exception handler table (eh) of the technique are entered in the kernel, and the processor starts executing the first instruction in the instruction sequence (ts) of the statistical technique.
run-tech(us,kern-pro(is,irs,ts,tvt),eht)
   = result(
     execute(
       state(memory(ts,us),empty-stack,
       first(is), addr(is,1),
       no-reset-info, handlers(tvt,eht),
       irs,string(blank))))

The result statistics, as calculated by a statistical technique, are
stored in the user symbol table (us). Execution of a statistical
technique, therefore, can be regarded as the modification of a user
symbol table.

The code of actual functions in the statistical language is left
unspecified. The function code table in the kernel is not instantiated.
The parameter Current-func-code emphasizes that the design of CONDUCTOR
is independent of these functions.
A statistical program in CONDUCTOR is translated into a kernel program. This translation process is divided in five steps: lexical analysis, parsing, type checking, the generation of input restrictions, and code generation.

```
lexical analysis
  ↓
parsing
  ↓
type checking
  ↓
input restriction generation
  ↓
code generation
```

The formal specification of the translation process starts with the abstract syntax of the statistical language. The lexical analysis and parsing is left unspecified. A complete formal specification of these processes, for a simple programming language, can be found in Bergstra et al. [1986]. Type checking and the generation of input restrictions were already discussed in the chapters 8 and 9. The formal specification of the code generation is discussed in this chapter.

A sequence of kernel instructions is a postfix representation of a statistical program. The generation of kernel instructions, in CONDUCTOR, can be compared with syntax directed intermediate code generation for
higher-level programming languages. The intermediate code in compilers for these languages hide the details of a particular target machine (see Davidson and Fraser [1984] and Tanenbaum et al. [1983]). The only difference is, that most of these compilers use three-address statements as intermediate language, and CONDUCTOR uses postfix notation. The code generation for PASCAL, for example, is discussed in Wirth [1971] and Ammann [1977].

11.1. GENERAL STRUCTURE OF THE COMPILER

A compiler is characterized by three languages: the source language, the target language and the implementation language (see McKeeman [1974]). The source language of the specified compiler is the abstract syntax of the statistical language, and the target language the kernel language. By specifying of the compiler in ASF, we, by definition, leave the implementation language undetermined.

The kernel language has been formalized in the previous chapter (see section 10.6). The compiler constructs each of the parts of a kernel program for a particular statistical program sp, as specified in the function compile in module Compilers.

\[ 402 \] compile(sp) = kern-pro( gen-instr-seq(sp),
                    gen-restr-pro(sp, type-check-pro(sp)),
                    type-check-pro(sp),
                    gen-handlers(sp))

All the functions on the right hand side of this equation are specified in the imported modules of module Compilers. The function gen-restr-pro was discussed in section 9.8 and the function type-check-pro in section 8.7. In this chapter we describe the functions gen-instr-seq, that specifies the generation of kernel instructions, and gen-handlers that specifies the generation of an exception handler table.
11.2. GENERATION OF INSTRUCTION SEQUENCES

An instruction sequence in a kernel program consists of five parts: a sequence of input instructions, an instruction that checks the input restrictions, a sequence of calculation instructions, a sequence of output instructions and a halt instruction.

<table>
<thead>
<tr>
<th>input seq</th>
<th>check restr.</th>
<th>calc. seq.</th>
<th>output seq.</th>
<th>halt</th>
</tr>
</thead>
</table>

sequential execution

The generation of the instruction sequence is specified in the module Gen-instructions. The function gen-ing-instr, in this module, specifies how input instructions are generated. The function check-restr specifies that an instruction to check the input restrictions is added to this sequence. The calculation instructions are added as specified by the function gen-calc-instr, and the output instructions by the function gen-out-instr. At the end of each instruction sequence a halt instruction is place. Note that in the formal specification the inner-most function call is evaluated first.
11.2.1. Generation of input and output instructions.

For each variable in the input/output section of a statistical program a
user-load/user-store instruction is generated. The function gen-code-id,
in module Gen-code-declarations, specifies that a user-load instruction
is added to the sequence (is) for each input variable (id)

[363] gen-code-id(is, id, load)
    = add-item(user-load(id), is)

and a user-store instruction is added for each output variable

[363] gen-code-id(is, id, store)
    = add-item(user-store(id), is)

The complete specification of the generation of user-load/user-store
instructions is given in module Gen-code-declarations.
Note that the functions user-load and user-store are only instructions to
load and store user variables. The actual loading and storing is done
when the kernel executes these instructions.

11.2.2. Generation of instructions for the calculation of the statistics.

The function gen-code-stm in module Gen-code-statements specifies how
kernel instructions are generated for each type of statement in the
statistical language. The generation of instructions for a for-statement
with control variable id, index expressions te1 and te2 and loop
statements stmts, is specified as follows

[351] gen-code-stm(heb-for(id,te1,te2,stmts))
    = incr2-cod(id, stmts,
            gen-code-stms(
            check-up-limit(id,te2,stmts,
                int-control-var(id,te1,te1),stmts)))

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This is represented by the well-known flow chart of a for-statement.

```
initialize
control
variable
↓
increment
control
variable

contr. var. > upper limit -> false -> statements

true
continue
```

The instruction sequence for the initialization of a control variable is specified in function `init-control-var`. This sequence contains an instruction to evaluate the index expression (iels) and an instruction to store the result in the control variable `id`.

```
[352] init-control-var(id,iels,is)
- add-item(store(id),
  add-item(eval-expr(iels),is))
```

After these instructions, the instructions must be generated that check if the control variable is larger than the upper limit of the for loop. A conditional jump will force the kernel to jump to the statement following the for-statement instructions, in case the control variable is larger than the upper limit. Also the size of the jump and the incrementing of the control variable is specified. These equations can be found in module `Gen-code-statement`, where also the specifications are given for the generation of instructions for the other types of statements in the statistical language.
11.2.3. Generation of exception raise instructions.

A test consists of internal declarations, statements and exception raises. The function \texttt{gen-instr-test-sec} in module \texttt{Gen-code-tests-section} specifies that first instructions are generated for the statements (\texttt{stmts}) and then for the for exception raises (\texttt{rs}).

\begin{verbatim}
[365] gen-code-test-sec(abs-test-sec(decl,stmts,rs),is)
    = gen-code-raises(rs,
       gen-code-stmtsn(stmts,is))
\end{verbatim}

Recall that the instruction sequence of a statistical technique is interrupted if the test indicates that the data violates the assumptions of the statistical technique.

An exception raise statement consists of a boolean expression and an identifier. First the instructions for the evaluation of expression \texttt{expr} are generated, then a \texttt{jump-false} and a \texttt{raise} instruction are added to the instruction sequence.

\begin{verbatim}
[368] gen-code-raise(is,abs-raise(expr,id)
    = add-item(raise(id),
       add-item(jump-false(increm(1)),
       gen-code-expr(is,expr)))
\end{verbatim}

If the boolean expression in the raise statement is true the exception will be raised. Otherwise the kernel will continue with the instruction following the raise instruction. The \texttt{jump-false} instruction will force the kernel to skip the \texttt{raise} instruction.

11.3. GENERATION OF A HANDLER TABLE

Instruction sequences generated for an exception handler section are not added to the instruction sequence of the statistical program, but are stored in an exception handler table. The statements (\texttt{stmts}) in the handler are translated and the resulting instruction sequence is inserted in the handler table (\texttt{ht}), at the given address (\texttt{id}).

\begin{verbatim}
[371] gen-code-handl(ht,abs-handl(id,decl,stmts))
    = insert(id,
       gen-code-stmtsn(multi-instr-seq,stmts),
       ht)
\end{verbatim}
This function is specified in module Gen-cois-handler-section.

11.4. OPTIMISING KERNEL INSTRUCTIONS

The problem of optimising instruction sequences was not discussed in this chapter. For an efficient implementation of CONDUCTOR, however, the instruction sequences must be optimised. In the current prototype, the only optimization technique that is applied is common subexpressions elimination. This improves the execution time of the statistical techniques considerably. Recall that kernel instructions are instructions for a virtual machine. These instructions may involve time consuming functions, such as, matrix inversion. The common subexpression elimination is specified completely independent of the specification in this book, because it is considered to be an implementation problem, instead of a fundamental part of the specification of CONDUCTOR. Also other optimization techniques, such as, constant folding or removal of loop invariant computations, will probably increase the performance of the implemented statistical techniques.
12. EXCEPTION HANDLING

Technical statisticians, data experts and computer scientists each look at a statistical technique from their own level of abstraction. Software implemented by one of these experts might detect that the execution of a statistical technique must be interrupted, because necessary conditions for execution are not satisfied. In computer science such conditions are called exceptions and the interrupt is called the raising of an exception. Especially in systems, that must remain in continuous operation, it is important that the execution of a program is not stopped when an exception occurs. Before a program can continue with the 'normal' operations, it must deal with the unexpected situation, without completely terminating execution.

The response to an exception condition is called the handling of an exception (see Goodenough [1975] and Wiener[1983]). Two basic approaches in exception handling can be distinguished:
- when an exception occurs, normal program flow is interrupted and control is passed to the exception handler; after completion of the exception handling, control is returned to the point at which the exception occurred,
- when an exception occurs, this makes normal flow of the program impossible, and the program is terminated.

In the first approach it is possible to make a repair action and thereafter continue operation. Examples of programming languages with exception handlers that allow repair actions are ADA, PL/1 and PL/C.

The process of statistical analysis in CONDUCTOR is seen as a continuous process. In statistical analysis, an applied statistician uses
statistical techniques to analyze his data set. These statistical techniques can only be applied under certain conditions. Therefore, during statistical analysis, it often happens that a statistical technique can not calculate the required statistics. Such situations can be seen as the occurrence of an exception. Yet, the applied statistician wants to continue the analysis and needs advice. Are there any alternative statistical techniques that can tackle the problem? Or are there data preprocessing techniques that do make analysis of his data set possible? The answers, of course, can only be given by the experts. The exception handling mechanism allows the experts to provide this information. The problem of given proper advice when an exception occurs, can be compared with the problem of giving understandable error messages, when an error occurs in procedures, that are hidden for the user of the software, in higher-level programming languages. For a discussion on this topic see Efe [1987].

In CONDUCTOR an exception is raised when:

- test results at the statistical level indicate that a basic assumption of a statistical technique is violated,
- inconsistencies are detected in the series used in a statistical technique,
- numerical problems occur in function calls.

In this chapter we will discuss exceptions in tests and in numerical functions. Exceptions raised due to inconsistencies in the data are discussed in chapter 13.

12.1. RAISING OF EXCEPTIONS

To explain the exception handling mechanism in CONDUCTOR we start at the kernel level. The kernel instruction `raise(h-id)` forces an interrupt of the instruction sequence of the statistical technique. Where `h-id` is the exception identifier. If an exception handler, with the given identifier, is found in the exception handler tables, the next instruction executed by the kernel is the first instruction in this exception handler.

Two groups of exception handlers are distinguished in CONDUCTOR:

- **the technique handlers**: the exception handlers specified by the technical statisticians inside a statistical technique,
EXECUTION OF A RAISE INSTRUCTION

A. State before execution.

```
instruction sequence statistical technique

............ raise xl ...
```

```
kernel

```

```
technique handlers

<table>
<thead>
<tr>
<th>x0</th>
<th>instr. seq. handler 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>instr. seq. handler 2</td>
</tr>
</tbody>
</table>

```

```
external handlers

<table>
<thead>
<tr>
<th>y0</th>
<th>instr. seq. handler 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y1</td>
<td>instr. seq. handler 2</td>
</tr>
</tbody>
</table>

```

B. State after execution.

```
instruction sequence statistical technique

............ raise xl .................
```

```
kernel

```

```
technique handlers

<table>
<thead>
<tr>
<th>x0</th>
<th>instr. seq. handler 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>instr. seq. handler 2</td>
</tr>
</tbody>
</table>

```

```
external handlers

<table>
<thead>
<tr>
<th>y0</th>
<th>instr. seq. handler 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y1</td>
<td>instr. seq. handler 2</td>
</tr>
</tbody>
</table>

```

129
- the external handlers: exception handlers specified outside a statistical technique by either a technical statistician, a data expert or a computer scientist.

The technique handlers have the highest priority. If an exception is raised the technique handler table is searched first. If no handler is found in this table the external handler table is searched. If no handler is found in either of these tables the execution of the statistical technique is aborted.

After the exception handler is executed the kernel returns to the interrupted instruction sequence of the statistical technique. To make this return possible the address of the next instruction to be executed in the statistical technique is saved, when the technique is interrupted. If an external exception handler is taking over control, also the symbol table of interrupted statistical technique is saved.

The exception handling mechanism is specified in the function handle in module Exception-handling. For example, it is specified that if the processor is in a state 1.

<table>
<thead>
<tr>
<th>state description of the kernel 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>memory</td>
</tr>
<tr>
<td>data stack</td>
</tr>
<tr>
<td>current instruction</td>
</tr>
<tr>
<td>address next instruction</td>
</tr>
<tr>
<td>reset information</td>
</tr>
<tr>
<td>handler tables</td>
</tr>
<tr>
<td>input restrictions</td>
</tr>
<tr>
<td>display</td>
</tr>
</tbody>
</table>

and if the handler is found in the exception handler table of the technical statistician tht, execution of the raise instruction will force the kernel to execute the first instruction of the exception handler first(h-id^tht), set the address of the next instruction to the address of the second instruction in the exception handler addr(h-id^tht,1) and save the next instruction of the interrupted instruction
sequence \texttt{reset-info}(\texttt{ia}). The resulting state of the kernel is state 2.

<table>
<thead>
<tr>
<th>state description of the kernel 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>memory</td>
</tr>
<tr>
<td>data stack</td>
</tr>
<tr>
<td>current instruction</td>
</tr>
<tr>
<td>address next instruction</td>
</tr>
<tr>
<td>reset information</td>
</tr>
<tr>
<td>handler tables</td>
</tr>
<tr>
<td>input restrictions</td>
</tr>
<tr>
<td>display</td>
</tr>
</tbody>
</table>

On the other hand, if the handler is found in the external handler table \texttt{eht}, the kernel starts executing the first instruction in the external handler table \texttt{first(han-instr-s(h-id'eht))}, sets the address of the next instruction to the second instruction in this instruction sequence \texttt{addr(han-instr-s(h-id'eht),1)} and saves both the return address and the symbol table of the interrupted statistical technique \texttt{reset-info}(\texttt{ia,ts}). The symbol table of the external handler replaces the symbol table of the statistical technique in the memory of the kernel \texttt{memory(han-symtab(h-id'eht),us)}. This results in state 3.

<table>
<thead>
<tr>
<th>state description of the kernel 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>memory</td>
</tr>
<tr>
<td>data stack</td>
</tr>
<tr>
<td>current instruction</td>
</tr>
<tr>
<td>address next instruction</td>
</tr>
<tr>
<td>reset information</td>
</tr>
<tr>
<td>handler tables</td>
</tr>
<tr>
<td>input restrictions</td>
</tr>
<tr>
<td>display</td>
</tr>
</tbody>
</table>

If the exception handler identifier is found in neither of the tables the
execution is stopped. The formal specification of the exception handling in the module **Exception-handling** reads

```
[305] handle(state(memory(ts,us),ds,raise(h-id),ia,
no-reset-info,handlers(tht,eht),irs,dis)
  = if(found(h-id,tht),

  -- handler found in the technique symbol table --

  state(memory(ts,us),ds,
  first(h-id',tht),addr(h-id',tht,1),
  reset-info(ia),handlers(tht,eht),
  irs,dis),
  if(found(h-id,eht),

  -- handler found in the technique symbol table --

  state(memory(han-symtab(h-id',eht),us),ds,
  first(han-instr-a(h-id',eht),
  addr(han-instr-seq(h-id',eht),1),
  reset-info(ia),handlers(tht,eht),
  irs,dis),

  -- handler found in the technique symbol table --

  stop))
```

For a complete specification of the function **handle** see appendix J, where also the unraise instruction that returns control to the interrupted instruction sequence is specified.

12.1.1. Raising exceptions in the statistical language.

At the statistical level a technical statistician may specify tests. The syntax of these tests was already discussed in chapter 7. For example, a statistician may test the significance of an estimated parameter, using "the rule of thumb", as discussed in chapter 4.

```
<table>
<thead>
<tr>
<th>TEST section statistical program</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST</td>
</tr>
<tr>
<td>WHEN  abs(beta) &lt;= 2*sqrt(b_var)</td>
</tr>
<tr>
<td>RAISE insignificant_coefficients</td>
</tr>
</tbody>
</table>
```

A test at the statistical level is translated into a raise instruction at
the kernel level. For the statistical language this translation process is specified in module Gen-code-test in the function gen-code-raise.

\[368\] \text{gen-code-raise}(\text{is}, \text{abs-raise}(\text{id}, \text{expr}))
= \text{add-item}(\text{raise}(\text{id}),
\text{add-item}(\text{jump-false}(1),
\text{gen-code}(\text{is}, \text{expr})))

Equation [368] specifies that first the code for the boolean expression (expr) is generated, then a jump-false instruction is added, and finally a raise instruction. The raise instruction will only be executed if the given condition is true.

12.1.2. Raising exceptions in function calls.

The kernel calls functions implemented on the higher level programming. Function calls are specified in the kernel instruction \textit{f-call}. If an exception occurs during the execution of a function an exception is raised. The function \textit{except-raised} in the formal specification signals if an exception is detected in a function call.

\textit{except-raised}: \text{ID} \# \text{DATA-STACK} \rightarrow \text{BOOL}

The function \textit{exception-f} returns the exception identifier

\textit{exception-f}: \text{ID} \# \text{DATA-STACK} \rightarrow \text{ID}

The possible states of the processor after the execution of a function call is executed are given in equation [325].

[325] \text{execute}(\text{state}(\text{m}, \text{ds}, \text{f-call}(\text{f-id}, \text{t}), \text{ta}, \text{r'}, \text{ht}, \text{trs}, \text{dis}))
= \begin{align*}
\text{execute}(
\text{if}(\text{except-raised}(\text{f-id}, \text{ds}),
\text{-- raise the exception --}
\text{state}(\text{m},
\text{push}(\text{execute}(\text{f-id}, \text{ds}), \text{pop}(\text{ds}, \text{t})),
\text{raise}(\text{exception}(\text{f-id}, \text{ds}), \text{ta},
\text{r'}, \text{ht}, \text{trs}, \text{dis})),
\end{align*}
-- execute function and continue with --
-- the next instruction --

state(m,
  push(execute(f-id,ds),pop(ds,i)),
  fetch(ta),next(ta',
  ri,ht,irs,dis))

After a successful completion of the f-call instruction the data stack is
modified. The arguments are popped from the stack and the result is
pushed on the stack. If an error is detected an exception is raised using
the identifier returned by f-excep.

12.2 HANDLING A RAISED EXCEPTION

If an exception is raised, the kernel will look for an exception handler.
This can be an exception handler created by either the technical
statistician or one of the other experts. In this section we will discuss
how technique handlers and external handlers are created.

12.2.1. Exception handlers in the statistical language.

The technical statistician can create exception handlers in a statistical
program. The syntax of exception handlers was already discussed in
chapter 7. An exception handler consists of an exception identifier and
the statements that must be executed if the exception occurs. The
statements in an exception handler may only consist of a message
statement, that explains to the user of the statistical technique what
caused the exception. A call of a matrix inversion, for example, raises
an exception if the matrix is near singular. This exception can be
handled by an exception handler written by a statistical expert. The
statistical expert is often able to give a higher level interpretation of
an exception. If, for instance, the inversion routine is called in the
ordinary least squares estimation of regression coefficients, the higher
level interpretation of near-singularity is multicollinearity. The
technical statistician can communicate this interpretation in his
exception handler.
The statistician may also specify that the execution of the statistical technique is stopped after the exception handler is executed by adding a stop instruction at the end of his exception handler.

The fact that an exception can be handled that is raised at an other level of abstraction introduces complications. Sometimes, a technical statistician will use the same function several times in a statistical technique. Consider the following simple statistical program

```
exception handler section statistical program

WHEN
  near_singular:

  MESSAGE:
  "THE INDEPENDENT VARIABLES IN SOME BOOTSTRAP SAMPLES ARE
  STRONGLY CORRELATED (MULTICOLLINEARITY). DUE TO THIS THE
  NUMERICAL RESULTS OF THE TECHNIQUE MAY BE INACCURATE."

```

If an exception is raised by the function inverse it can not be determined if it was caused by the first or the second call of the function. In an exception handler, therefore, a technical statistician can specify different handlers for different calls.

```
statistical program

INPUT
    index n,m;
    matrix [1 to n, 1 to n] X;
    matrix [1 to m, 1 to m] Y

EQUATIONS
    X = inv(X) ; Y = inv(Y);

exception handler section

WHEN near-singular

  CALL(inv,1):
  {
    MESSAGE:"The X matrix is near singular"
    STOP
  }
```
CALL(inv, 2):
{
    MESSAGE:" The Y matrix is near singular";
}

In the formal specification of CONDUCTOR, only exception handlers without
the call mechanism are specified. An exception handler consists of an
identifier, local declarations and statements as given in function abs-
handler in module Handler-abstr-syntax in appendix F.

abs-handler: ID # LOC-DECLS # STATEMENTS -> HANDLER

At the kernel level an exception handler is an kernel instruction
sequence. The statements in an exception handler are translated and
stored in the technique handler table as specified in function gen-code-
hdnl in module Gen-code-handler-section.

gen-code-hdnl: TECH-HANDLER-TABLE # HANDLER ->
TECH-HANDLER-TABLE

with equation [371].

[371] gen-code-hdnl(ht, abs-handler(id, id, stmts))
    = insert(id,
          gen-code-stmts(null-instr-seq, stmts),
             ht)

12.2.2. External handlers.

Data experts and computer scientists can also write exception handlers in
CONDUCTOR. The only differences with the exception handlers written by
the technical statistician are:
- it is not possible for these experts to distinguish between different
  function calls,
- variables inside a statistical technique are not allowed.
The external handlers are stored in the external handler table. External
handlers are created in external sessions. At the kernel level external
handlers consists of an instruction sequence and a symbol table.
In module `Gen-ext-handlers` in appendix L it is shown how an exception handler section in an external session is translated into an external handler.

\[
\text{gen-ext-handler: HANDLER} \rightarrow \text{EXTERNAL-HANDLER}
\]

with equation

\[
[404] \text{gen-ext-handler(abs-handler(id,ld,stmts))} \quad
\begin{align*}
\text{ext-handl}( & \text{gen-code-stmts(null-instr-seq,stmts),} \\
& \text{typcheck(abs-handler(id,ld,stmts),empty-mem)}) \\
\end{align*}
\]

\[
\text{when eq-inp-restr-seq(} \\
\text{gen-restr-handler(handler(id,ld,stmts),} \\
\text{no-restrictions),} \\
\text{no-restrictions) = true}
\]

The `when` clause specifies that the external handler may not generate input restrictions for the statistical technique.

12.2.3. All handlers.

The technique handler table and the external handler table are combined in the sort `HANDLER-TABLES` in module `Handler-tables` in appendix J. Together they form the collection of all exception handlers available during the execution of a statistical technique.
PART IV

FORMAL SPECIFICATION

OF THE

USER LANGUAGE AND DATA INTERFACE
13. AN INTERFACE BETWEEN DATA AND STATISTICAL TECHNIQUES

An applied statistician often relies on data collected by specialized agencies. David [1985] has pointed out that additional information is available at these agencies that could lead to more intelligent use of the data. One could think of information such as the sample design, the context in which the observations are made, and the type of measurement instruments used to collect the data. An applied statistician only sees data as published in official publications of the specialized agencies. To make things even more complicated, he analyses his data set using statistical techniques developed by a technical statistician, who looks at the data from yet another perspective. For a technical statistician a data set is a rectangular matrix and a set of assumptions on the type and distribution of the data.

In this chapter the interface between the different views on data is discussed. The main design goals of this data interface are:
- to transfer knowledge available in the data production process in an understandable way to users of data,
- to offer technical statisticians and data experts the possibility to suggest appropriate statistical techniques if inconsistencies are detected in data.

13.1. THE DATA PRODUCER'S VIEW ON DATA

Producing statistical data is a complicated and time consuming job. Often it is impossible to make exact measurements. Therefore, sample techniques
are used to make estimates of the required information. The resulting figures are of course conditional on the sample design chosen. Besides the sample design, also the measurement instrument used to collect the data and the context play an important role. David [1985] sees a set of observations $X$ as conditional on the design $D$, the instrument $S$ and the context $C$.

$$\{X \mid D, S, C\}$$

Comparing two observations measured at different moments in time $T$ and $T'$, one compares

$$\{X_t \mid D_t, S_t, C_t\}$$

and

$$\{X_{t'} \mid D_{t'}, S_{t'}, C_{t'}\}$$

Of course, one hopes for the best that $D$, $S$ and $C$ do not change over the sample period, but using estimation periods of several years this seems hardly realistic.

To get an indication of the quality of the data, a user must have access to all information gathered in the data production process. Also evaluation functions combining $X$ and external information, calculated to give an indication of the quality of the data, such as bias, mean square error and reliability, etc., should be part of the documentation for any data set.

13.2. THE USER'S VIEW ON DATA

An applied statistician, with a little common sense, will not try to collect large data sets on his own. By the time he is finished with the data production, the question he initially wanted to answer will surely be out of date or even no longer existent. An applied statistician can spend only limited time worrying about all the details involved in the gathering of data. The view of the user of data, therefore, is restricted
to the resulting data of a data production process.

13.3. THE TECHNICAL STATISTICIAN'S VIEW ON DATA

Statistical techniques are developed to make estimates of parameters in theoretical models which must explain the observed data. Griliches [1984], for example, classifies the data along several different dimensions:
- objective versus subjective: for example, prices versus expectations about them,
- type and periodicity: for example, time series versus cross-section; monthly, quarterly or annual data; nominal, ordinal, interval or cardinal data,
- level of aggregation: for example, data on individuals, firms, districts or states,
- quality: for example, the reliability or validity of data,
- level of fabrication: for example, primary or secondary data.

For each of the different categories of data, special statistical techniques are developed. An excellent overview of the econometric techniques that are developed for different dimensions and types of economic data is given in Judge et al. [1980]. If a statistical technique is used, it is assumed that data set has certain properties. For example, the technique might expect outliers, missing data, ordinal data or cardinal data. If the data set has other properties the statistical inference with the use of that particular technique may be incorrect. The technical statistician assumes that the user of his statistical technique is aware of this fact. And in traditional statistical software indeed this responsibility lies completely in the hands of the user. For the technical statistician the properties of the data are not a problem but a premiss.
13.4. THE INTERFACE BETWEEN THE DATA PRODUCER'S VIEW AND THE USER'S VIEW

Information and documentation collected during the data production process can be stored in a relational database system. Though other database models exist, the relational database model has the advantage of clarity and ease of use (see Ullman [1980] and Kroenke [1983]). The software produced in CONDUCTOR, can, on request of the user, retrieve data from this database. When data is retrieved, the additional information on the retrieved data is checked by a background implemented by the data expert. If an inconsistency is detected by the background query, this inconsistency is signalled by adding an exception flag to the retrieved series.

Example 13.1:

A large organization produces data on consumption, production and export in the Netherlands. Besides the figures published by the organization,

<table>
<thead>
<tr>
<th>Name</th>
<th>Sample</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption</td>
<td>1982</td>
<td>1312.4</td>
</tr>
<tr>
<td>production</td>
<td>1980</td>
<td>400.7</td>
</tr>
<tr>
<td>consumption</td>
<td>1980</td>
<td>2000.4</td>
</tr>
<tr>
<td>export</td>
<td>1979</td>
<td>487.9</td>
</tr>
<tr>
<td>production</td>
<td>1981</td>
<td>500.5</td>
</tr>
<tr>
<td>consumption</td>
<td>1981</td>
<td>900.8</td>
</tr>
<tr>
<td>consumption</td>
<td>1983</td>
<td>1200.4</td>
</tr>
<tr>
<td>production</td>
<td>1982</td>
<td>700.3</td>
</tr>
</tbody>
</table>
also additional information is available about the sample design used to estimate the figures. This information is stored in three tables (called relational schemes in relational database theory). The first table contains the estimated figures.

Table 2. Design.

<table>
<thead>
<tr>
<th>Name</th>
<th>Sample</th>
<th>Design number</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption</td>
<td>1982</td>
<td>DS2</td>
</tr>
<tr>
<td>production</td>
<td>1982</td>
<td>DS2</td>
</tr>
<tr>
<td>consumption</td>
<td>1980</td>
<td>DS1</td>
</tr>
<tr>
<td>export</td>
<td>1979</td>
<td>DS1</td>
</tr>
<tr>
<td>consumption</td>
<td>1981</td>
<td>DS1</td>
</tr>
<tr>
<td>consumption</td>
<td>1983</td>
<td>DS2</td>
</tr>
<tr>
<td>production</td>
<td>1981</td>
<td>DS2</td>
</tr>
<tr>
<td>production</td>
<td>1982</td>
<td>DS2</td>
</tr>
</tbody>
</table>

Table 3. Design description.

<table>
<thead>
<tr>
<th>Design number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS1</td>
<td>............</td>
</tr>
<tr>
<td>DS2</td>
<td>............</td>
</tr>
</tbody>
</table>

In the following queries two basic operations are applied: selection and projection. The first operation simply means that all rows of the table are selected that satisfy a given condition. Projection stands for extracting given columns from a table. In the queries projection is indicated by the operator \( \pi \), and selection by the operator \( \sigma \). The published data on consumption in the period 1980 and 1982 is retrieved by the query

\[
\text{Rawdata}(\sigma_{\text{name}=\text{consumption}, \text{1980}<=\text{Sample}<=\text{1982}})(\text{Observations})
\]

resulting in the table
The background query that checks if the same design is used over the sample period.

\[
\text{\texttt{Design\_name}(\alpha\texttt{Sample}=1980\leq\texttt{Sample}\leq1982(\texttt{Documentation}))}
\]

returns table

<table>
<thead>
<tr>
<th>Design_name</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS1</td>
</tr>
<tr>
<td>DS2</td>
</tr>
</tbody>
</table>

The resulting relation contains more than one row and an exception flag "design-unequal" is added to the series, indicating that the retrieved sample is based on different designs DS1 and DS2. The user of software produced by CONDUCTOR will not see much of these queries. In the resulting software he simply sets the sample and requests retrieval of the series consumption.

```
user session
-u:  set sample 1980 to 1982
-u:  retrieve consumption
```

The database design and the background queries are completely hidden for the user.

{ end example 13.1. }

In the formal specification of CONDUCTOR the user data is represented by the sort \texttt{SERIES}, specified in module \texttt{Series}. The observations are represented by the sort \texttt{SCALAR-SEQ}, the sample description is represented by the sort \texttt{CONST-RANGE-SEQ}, and the exception identifiers by the sort \texttt{ID-SEQ}.

\[
\texttt{ser: SCALAR-SEQ \# CONST-RANGE-SEQ \# ID-SEQ -> SERIES}
\]
A well-formed series $ss$ satisfies the restriction that the number of samples described in the constant range sequence $crs$ is equal to the number of observations in the scalar sequence $scs$.

[19] $ser(ss, scs, ids) = wf-ser(scs, crs, ids)$

$\text{when } eq(n-of-items(scs), length(crs)) = true$

and

[20] $ser(ss, scs, ids) = error-ser$

$\text{when } eq(n-of-items(scs), length(crs)) = false$

The retrieval process of CONDUCTOR is specified in module Database-interface in the function query.

\[
data-query: DATA-BASE \# ID \# CONST-RANGE-SEQ \\
\rightarrow USER-DATA
\]

which invokes the functions

\[
bg-query: DATA-BASE \# ID \# CONST-RANGE-SEQ \\
\rightarrow ID-SEQ
\]

\[
retrv-data: DATA-BASE \# ID \# CONST-RANGE-SEQ \\
\rightarrow SCALAR-SEQ
\]

A query of a database $db$ for a series with identifier $id$ over the sample given by constant range sequence $crs$ will invoke a background query to check the consistency of the data.

[416] $data-query(db, id, crs) = u-data( \\
\text{series(retrieve-data(db, id, crs),} \\
\text{crs,} \\
\text{bg-query(db, id, crs))))$

The data retrieval process is not further specified in CONDUCTOR. It is assumed that a database, with the appropriate queries, can be constructed.
13.5. THE INTERFACE BETWEEN THE USER’S VIEW AND THE TECHNICAL STATISTICIAN’S VIEW ON DATA

The series retrieved from the database can be used in statistical techniques, that is implemented by a technical statistician. The observations in the series will form the columns of the matrix.

Example 13.2:

Assume that the user has retrieved the consumption and production series from the database in example 1, and he wants to use these series in a statistical technique. The user interface of the statistical technique is specified by the technical statistician in the INPUT section of a statistical program specifying the technique

```
input/output section statistical program

INTERFACE
INPUT
  INDEX n  MESSAGE: 'give the number of observations'
  ; INDEX m  MESSAGE: 'give the number of independent variables'
  ; MATRIX [1 TO n, 1 TO m] X  MESSAGE: 'list the independent variables'
```

When the user calls the statistical technique n, m and X must be initialized. The user is prompted to enter values for these variables.

```
user session

give the number of observations
\$u: 3

give the number of independent variables
\$u: 2

list the independent variables
\$u: consumption, production
```

At the kernel level the prompting of user information and the loading of
the matrix \( X \) is seen as the execution of a \texttt{USER_LOAD} instruction. The input of the variables \( n, m \) and \( X \) at the kernel level is represented by the instruction sequence

\[
\text{USER_LOAD n} \quad \text{USER_LOAD m} \quad \text{CHECK_RESTR} \quad \text{USER_LOAD X} \quad \ldots
\]

\text{sequential execution}

To calculate the required output of a statistical technique other instructions will follow the \texttt{USER_LOAD} instructions. The result of the \texttt{USER_LOAD} instructions in the above example is a \((3 \times 2)\) matrix with the observations on consumption and production in the columns.

\[
X = \begin{bmatrix}
2000.4 & 400.7 \\
900.8 & 500.5 \\
1312.4 & 700.3
\end{bmatrix}
\]

{ end example 13.2 }

A matrix in \texttt{CONDUCTOR} is specified in module \texttt{Matrices} in the function \texttt{mat}.

\[
\text{mat: SCALAR-SEQ \# CONST-RANGE-SEQ} \rightarrow \text{MATRIX}
\]

A matrix consists of a scalar sequence containing the values of the matrix elements and a constant range sequence, describing the dimensions. A matrix is well-formed if the product of the length of the dimension ranges \texttt{crs} is equal to the number of items in the scalar sequence \texttt{scs}.

\[16\] \text{mat(scs,crs) = wf-mat(scs,crs)}

\[\text{when eq(n-of-items(scs),product(crs)) = true}\]

A complete specification of all function and sorts involved can be found in appendix D. The \texttt{USER_LOAD} command has a special facility to transfer sequences of series into a matrix as specified in module \texttt{Series-matrix-interfaces} in function \texttt{data-matrix}.

\[
\text{data-matrix: ID-SEQ \# USER-SYMTAB} \rightarrow \text{MATRIX}
\]

The details of this operation are left unspecified. When a series, with
an exception flag attached to it, is loaded in a matrix an exception is raised. The software will look for an exception handler for this particular exception. This behaviour is specified in equations for the user-load command in module Processor in appendix J.

\[ \text{execute}(state(m,ds,user-load(id),ta,rt,ht,irs,dis)} \]
\[ \quad - \text{execute(} \]
\[ \quad \quad \text{if(user-load-excep(id,m),} \]
\[ \quad \quad \quad \text{-- raise the exception --} \]
\[ \quad \quad \quad \text{state(user-insert(id,m),ds,} \]
\[ \quad \quad \quad \quad \text{raise(load-excep-id(id,m),ia,} \]
\[ \quad \quad \quad \quad \quad \text{rt,ht,irs,dis),} \]
\[ \quad \quad \quad \text{-- else load the variable and continue --} \]
\[ \quad \quad \quad \text{-- with the next instruction --} \]
\[ \quad \quad \text{state(user-insert(id,m),ds,} \]
\[ \quad \quad \quad \text{fetch(ia),next(ia,} \]
\[ \quad \quad \quad \quad \text{rt,ht,irs,dis))} \]

Example 13.3:

In example 13.2 we loaded two series in a matrix $X$. The consumption series, however, was produced using different sample designs. This inconsistency was detected by the background query and a exception flag "design_unequal" was added to the retrieved series. When the series is loaded in the matrix, the exception flag "design_unequal" is raised. The data expert may have written an external exception handler for this exception.

```
external exception handler of data expert

WHEN: design_unequal

MESSAGE:
"You use a series which is based on different sample designs. Be careful with the interpretation of the results based on these series"
```

After this warning the execution of the interrupted statistical technique continues. For a particular statistical technique a statistician may overrule an external exception handler by writing a exception handler in the specification of the statistical technique.
When the exception "design_unequal" occurs the statistical technique is stopped and the user is advised to use the more appropriate technique XXX.

{ end example 13.3.}
In CONDUCTOR, data experts, technical statisticians and computer scientist each can add their knowledge to the software in a language close to their problem domain. To demonstrate the use of software created in such an environment, a user command language is developed. To keep the prototype of CONDUCTOR simple, it has not been attempted to create a revolutionary new user interface. The user command language is needed only to demonstrate the fact that the software implemented by the different experts can be used by an applied statistician. In the prototype of CONDUCTOR the user language consists of commands to
- declare and initialize input variables,
- retrieve series from a database,
- call a statistical technique.

Of course, many facilities should be added in order to compete with the interfaces of modern statistical packages. One could think of help facilities, multiple screens, graphical facilities etc. Adding these facilities however is merely a matter of hard work and is not considered to be within the scope of this book.

After giving an informal description of the user language in the first three section of this chapter, we will discuss its formal specification.

14.1 Initializing Variables

The input variables of a statistical technique must be initialized by the user of the statistical technique. In the user language, therefore, an applied statistician must be able to declare and initialize variables of
the types scalar, index, boolean, vector, matrix or matrix. In the user
session given below, a scalar x and a matrix m are declared. In contrast
to the statistical language the dimension bounds of matrices in a user
session are fixed. The variables are initialized in a user session using
the set command. The values of x and m can be displayed using the display
command.

```plaintext
>u: scalar x
>u: matrix [1 to 2, 1 to 2] m
>u: set x 12.4
>u: set m

Enter the values of m row by row:
>u: 1.0 2.0
>u: 3.0 4.0
>u: format 10.2
>u: display x

    x = 12.40
>u: display m

    m =
        1.00 2.00
        3.00 4.00
```

In the prototype of CONDUCTOR the user may also change the format of the
reals using the format command. The format instruction in the example
above changes the output format of a real to a total of 10 positions and
2 position after the decimal point.

### 14.2 INITIALIZING SERIES

In statistical software, series of observations play an important role.
At the user level a series can be initialized in two ways. It can
initialized in a way similar to other variables, or it can be retrieved
from a database. To initialize a series the user must set the current
sample in his session. After this must declare the identifiers of the
series. In the example session below a series consumption is declared
over the sample 1952 to 1958 and 1962 to 1964.

```
user session

> u: set format 1952 to 1958, 1962 to 1964
> u: series consumption
> u: set consumption

Enter the observations of the series consumption

> u: 1.2 2.4 5.6 5.9 3.5
> u: 1.2 3.4 1.9 3.4 0.2
> u: display consumption

The observations of consumption are

<table>
<thead>
<tr>
<th>Year</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952</td>
<td>1.20</td>
</tr>
<tr>
<td>1953</td>
<td>2.40</td>
</tr>
<tr>
<td>1954</td>
<td>5.60</td>
</tr>
<tr>
<td>1955</td>
<td>5.90</td>
</tr>
<tr>
<td>1956</td>
<td>3.50</td>
</tr>
<tr>
<td>1957</td>
<td>1.20</td>
</tr>
<tr>
<td>1958</td>
<td>3.40</td>
</tr>
<tr>
<td>1962</td>
<td>1.90</td>
</tr>
<tr>
<td>1963</td>
<td>3.40</td>
</tr>
<tr>
<td>1964</td>
<td>0.20</td>
</tr>
</tbody>
</table>

> u: set sample 1957 to 1960, 1963 to 1965
> u: display consumption

The observations of consumption are

<table>
<thead>
<tr>
<th>Year</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>1.20</td>
</tr>
<tr>
<td>1958</td>
<td>3.40</td>
</tr>
<tr>
<td>1959</td>
<td>NA</td>
</tr>
<tr>
<td>1960</td>
<td>NA</td>
</tr>
<tr>
<td>1963</td>
<td>3.40</td>
</tr>
<tr>
<td>1964</td>
<td>0.20</td>
</tr>
<tr>
<td>1965</td>
<td>NA</td>
</tr>
</tbody>
</table>
```

The values of this series are initialized using the `set` command. The resulting series `consumption` is displayed using the `display` command. The `display` command always prints the series for the current sample. If the current sample contains missing values the symbol NA is printed. The other way to initialize a series is by retrieving it from a database, as discussed in chapter 13.
14.3 EXECUTING A STATISTICAL TECHNIQUE

After having initialized variables a user can call a statistical technique specified by a technical statistician. The user only has to enter the name of a statistical technique. For example if a technique called bootstrap_olsq is specified by the technical statistician the user can apply this technique to his data set. The statistical technique will prompt the user to initialize the input variables. Then it will calculate the specified statistics and return the results.

```
user session
>u: bootstrap
number of independents
>u: 3
number of observations
>u: 10
size of bootstrap sample
>u: 5
how many samples must be drawn
>u: 100
The independent variables are
>u: income, indirect_tax, credit
The dependent variable is
>u: consumption

The expected value of the regression coefficients is:
  0.78
  0.45
  0.06

The variance of these estimates are:
  0.23
  0.12
  0.02
```

14.4. FORMAL SPECIFICATION OF THE USER LANGUAGE

In the formal specification of CONDUCTOR the display and format commands are omitted. The abstract syntax of the other user language
commands can be found in module Use-programs in appendix L in the functions.

user-decl: USER-TYPE ID -> USER-COMMAND
set: ID # USER-DATA -> USER-COMMAND
set-sample: CONST-RANGE-SEQ -> USER-COMMAND
retrieve: ID -> USER-COMMAND
call-tech: ID -> USER-COMMAND

The semantics of these instructions is defined in module Resulting-software. In this module it is specified how a user command modifies the state of a user program given the set of implemented statistical techniques and external handlers.

The state of a program is defined by the user symbol table, represented by the sort USER-SYMTAB, and the current sample, represented by the sort CONST-RANGE-SEQ. Recall that the user symbol table contains both the type description and the value of each variable declared in a user session.

user-state: USER-SYMTAB # CONST-RANGE-SEQ -> USER-STATE

The semantics of the set sample instruction is defined in function exec-user-com in module Resulting-software. If the user state is determined by the user symbol table us and the sample crs1, the execution of a set-sample instruction results in a user state where the sample description crs1 is replaced by crs2.

\[ 410 \] exec-user-com(user-state(us,crs1),tt,set-sample(crs2)) = user-state(us,crs2)

The specification of the semantics of the other user commands can be found in appendix L.
PART V
IMPLEMENTATION AND EVALUATION
OF CONDUCTOR
15. IMPLEMENTATION OF CONDUCTOR

CONDUCTOR is implemented in C and runs under both the UNIX and the MS-DOS operating systems. Given the formal specification of CONDUCTOR, still many practical problems had to be solved to get a program performing the specified tasks. In this chapter we will discuss some of these problems to wit the implementation of the formal modules, the hashing mechanism for associating information with identifiers, storage problems, and error recovery.

15.1. IMPLEMENTATION OF FORMAL MODULES

An implementation of a formal module consists of

- the internal definition file; this file contains the definitions of the hidden data types; such data types are hidden from the other modules in the system,
- the external definition file; this file contains the definitions of the data types that are visible for the other modules,
- the C-code file; this file contains the C-implementation of the functions.

The data types specified in the internal definition files and the implementation of the functions in the C-code files can be changed without influencing other modules. The implementation of the module Tables, for example, consists of the files tables.def containing the internal definitions, tables.h containing the external definitions, and tables.c containing the C code.
The modules that describe the separate passes of the compilation and/or interpretation of the languages are not implemented using the method described above. The parser for the languages in CONDUCTOR is implemented using the parser generator yacc (see Johnson [1975]). Using yacc, all the separate passes in the formal specification of the statistical language (type checking, input restriction generation and code generation) are implemented in one pass. The formal specification in these modules describes at which point actions must be added to the syntax rules in yacc.

Example 15.1: Implementation of the assignment statement in the statistical language.

The concrete syntax of an assignment statement in the statistical language reads

\[ \text{assignment} \rightarrow \text{variable} \; ':=' \; \text{expression} \]

the abstract syntax is specified in the function abs-assgn.

\[ \text{abs-assgn:} \quad \text{VARIABLE} \; \# \; \text{EXPR} \rightarrow \text{STATEMENT} \]

The type checking of an assignment statement is specified in module Stat-check-statements in equation

\[ \text{[156]} \quad \text{type-check-stmt}(\text{abs-assgn(var,expr),tst)} \]
\[ \quad = \text{eq-type(var-type(var,expr),} \]
\[ \quad \text{expr-type(expr,tst))} \]

where \( \text{tst} \) is the technique symbol table. The equation specifies that for each assignment statement it must be checked that the type of the left-hand side variable is equal to the type of the expression.

The generation of input restrictions for a assignment statement is specified in module Gen-restr-statements in appendix I and the code generation in module Gen-code-statements in appendix K.

The formal specifications in these modules are reflected in the following rules for yacc
assignment: variable BECOMES expression
{
    -- code to check the type restrictions & --
    -- add store instruction for left-hand --
    -- side variable --
}

No actions are added for the generation of input restrictions, because the only input restrictions for assignment statements are the input restrictions generated by the variable and the expression in the assignment statement. The actions which generate input restrictions for variables and expressions, of course, accompany the syntax rules for variables and expressions.
{ end example 15.1.}

15.2. FINDING INFORMATION THROUGH IDENTIFIERS

In most tables in the formal specification we used identifiers to locate an entry in a table. Information on, for example, exception handlers, variables and functions is located through an identifier. A simple hashing scheme, called 'open hashing' as defined in Aho, Sethi and Ullman [1986], is used to search identifiers. Such a scheme consists of:

- a hash table: a fixed array of m records (hash entries) pointers.
- hash entries organized into m separate linked lists called buckets (some buckets may be empty).

A hashing function can calculate in which bucket the hash entry of an identifier can be found. For each identifier the character string and the type of the identifier is stored in a hash entry. An identifier can be the name of a statistical technique, a keyword, a function name, the name of a variable or the name of an exception. Additional information for each of these types of identifiers is stored in separate tables. The hash entries contain pointers to the table entries for each of these identifier types and vice versa.

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15.3. STORAGE PROBLEMS

Making abundant copies of large matrices during the execution of a statistical technique will rapidly create storage problems. Sharing storage instead of making complete duplicates of matrices, reduces the storage problem considerably. A matrix is implemented as a record containing a pointer to the data, and a pointer to the dimension descriptions. The data and the dimension descriptions can be shared by more than one matrix. The pointers in the record of several matrices may point to the same address. Each matrix record, therefore, contains a flag indicating if it is the original record or a copy. To avoid dangling pointers one of course must be sure that the original matrix is not removed as long as copies exist. In C the definition reads

```c
typedef struct matrix
{
  bool: original;
  float: *data;
  pointer: dims;
} 
```

When an input matrix at the statistical level is initialized by a user, or a matrix is pushed on the data stack, a copy is made of the matrix involved. In the first case the original matrix is stored in the user symbol table and a copy is placed in the technique symbol table. In the second case the original matrix is stored in the technique symbol table and a copy is pushed on the data stack. In both case the copy is removed
before the original matrix can be removed. A complete copy of the matrix, including a full copy of the data and dimension descriptions, is made when a matrix at the data stack is stored in the technique symbol table, or when a matrix operator or function creates a new matrix.

Example 15.1: storage of matrices.

A statistical program

```
NAME example
INTERFACE
INPUT
   INDEX     n;
   MATRIX [1 TO n, 1 TO n] X
OUTPUT
   MATRIX [1 TO n, 1 TO n] Y
EQUATIONS
   Y := X * X
```

is represented at the kernel level as

```
USER-LOAD  n
USER-LOAD  X
CHECKRESTR
LOAD_ID    X
LOAD_ID    X
OPCALL     + 2
STORE_ID   Y
USER-STORE Y
HALT
```

In a user session a user can declare and initialize a matrix MAT and call the technique example.
The matrix MAT is stored in a record of type matrix. This record contains a boolean flag, indicating that it is an original matrix, and pointers to the data and the dimension description. The values of the pointers, such as AEA0 and AEFO are addresses on an imaginary computer.

```
record matrix MAT

  true          AEA0: 1.0 2.0 3.0 4.0
  AEA0          AEFO: 1 to 2 1 to 2
```

When the user runs the statistical technique 'example' he must initialize the input variables. The initialization is represented in the kernel by the USERLOAD-instruction. The USERLOAD-instruction stores a copy of the original matrix MAT in the entry of the matrix X in the technique symbol table. The pointers in the record of X point to the same data and dimension description as the pointer in the record of matrix MAT. A flag indicates that the matrix is a copy.
record matrix X

false
AEAO
AEFO

After the input restrictions of the statistical technique are checked, two LOAD_ID instructions push copies of the matrix X on the data stack. Now three copies of the matrix MAT exist, the data and the dimension description however are stored only ones. The OPFALL instruction uses the two copies to create the sum of the matrix. This instruction creates a new matrix containing the resulting sum of $X \times X$.

record matrix $X + X$

true
AEBO
AFOO

AEBO: 2.0 4.0 6.0 8.0
AFOO: 1 to 2 1 to 2

The storage used for the records of the two copies of the X matrix on the data stack is freed. The STORE_ID instruction stores the resulting matrix in the entry of Y in the technique symbol table.

{ end example 15.1. }

Besides avoiding copying the data of matrices, the storage problems can further be reduced by making use of the structure of the matrices. One can distinguish:

- A normal matrix. Every element of the matrix may be different and must be stored separately,
- A sparse matrix. The matrix contains many zero elements and only the position of an element in combination with the value are stored,
- An identity matrix. All elements on the main diagonal are one. Elements not on the main diagonal are zero,
- A symmetric matrix. Element[i,j] is equal to element[j,i],

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- A triangular matrix. All elements below or above the main diagonal are zero,
- A constant matrix. All elements of the matrix are equal,
- A large matrix. The matrix is so large that it is not stored in central memory,
- A binary matrix. The elements of the matrix are either 0 or 1.

On the one hand, special storage types will complicate the implementation of the matrix functions. For every type of matrix, separate functions must be written. On the other hand, it will increase the speed of the matrix functions dramatically. In CONDUCTOR, special storage types for matrices are not implemented.

15.4. ERROR RECOVERY

CONDUCTOR is an interactive system. It is inevitable that users make mistakes during interactive sessions. Therefore the system should offer the user the possibility to undo the effect of erroneous commands. Important in this respect is that the system should give proper error messages, making it possible for a user to find errors during a session. Once the error is located the user must be able to undo the effects of the command and restart the system as efficient as possible.

A model for error recovery is given in Archer et al. [1984]. An interactive session S is viewed as a sequence of commands

\[ c_1 ; \ldots ; c_n \]

S

Assume that after j commands an error is detected. This leads to the situation that j commands are already executed. This set of executed commands is called E. The remaining commands are in P the set of pending commands

\[ c_1 ; \ldots ; c_{j-1} ; c_j ; \ldots ; c_n \]

E \hspace{1cm} P

Modification of S will lead to S', a modified session. This session can be separated into a set U, a prefix of S, which remained unchanged and a
set \( M \) containing the modified commands.

\[
\begin{array}{c}
c_1 : \ldots : c_{i-1} : c'_1 : \ldots : c'_s \\
\hline
U \quad M
\end{array}
\]

In error recovery clearly the most complicated situation is the restart of the system with command \( c'_i \) if \( i < j \). The problem is in this case how to return to the state prior to \( c_i \). The simplest solution is a complete restart. In Archer et al. [1984] it is suggested that the introduction of checkpoints ('saved intermediate states') is profitable. If in our model we had saved the state after command \( c_{i-1} \) was executed we could restart the system with command \( c'_1 \). Saving all states after each command however is space and time consuming, therefore, only a few checkpoints should be used in a system.

In CONDUCTOR error recovery is possible. When an error occurs, CONDUCTOR displays the last commands entered by the user and indicates the position where the error occurred. Also the possible options to modify the commands are given. In an external handler session and a statistical session four options are available

1. Stop executing CONDUCTOR and return to the operating system.
2. Edit the commands with the use of an editor.
3. Replace the text that caused the errors.
4. Insert text before the text that caused the errors.

In a user session also the last command can be aborted. Editing commands in a user session, however, is not possible. Options 3 and 4 are only available if the error is still in the input buffer of the system. After the correction is made the system will start from the latest saved checkpoint.

The error recovery is build into the parser generated by the yacc parser generator. A check point in yacc amounts to an error flag on the parse stack. Checkpoint are placed at the beginning of sessions, the beginning of expressions in the statistical language, and at the beginning of user commands. These checkpoints allow the user to recover faster from minor errors.
Example 15.3:

If a technical statistician makes an error in a expression CONDUCTOR will give the following error message:

```
NAME example
INTERFACE
INPUT
  VECTOR [1 TO 3] X
  VECTOR [3 TO 1] Y
EQUATIONS
  X := X > Y
  X := <<X + Y>>
```

line 5: illegal type of arguments in function or operator call

0 Exit to system
1 Edit with PRIVE
2 Replace text between <<_ and _>>
3 Insert text before <<_

If the user chooses option 2 or 3 and repairs the error, for example by replacing Y by the transpose Y', only the expression is recompiled. If however the error was due to improper declaration of Y the program is recompiled.

{ end example 15.3.}
16. THE USE OF FORMAL SPECIFICATIONS IN CONDUCTOR

In this book a formal specification of a large software project is discussed. Of course, the specification presented is a product that has gone through many stages. In this chapter we will discuss our experience in using the specification formalism ASF, and demonstrate how it helped to modularize and define CONDUCTOR. We also will discuss a major deficiency of ASF: the impossibility to hide error cases in specifications. And finally we will discuss the problem of keeping the formal specification consistent with the prototype.

16.1. USING ASF IN THE DESIGN OF CONDUCTOR

In a large software project one can not just start writing procedures in a higher-level programming language, and hope that, at the end, the software performs the ideas one had in mind. The problem is that half way through the project it might be discovered that previously written procedures should be modified, and that the efforts in writing the procedures were a waste of time. As in all large projects, one has to carefully plan before one starts working. In the CONDUCTOR project this plan is described in ASF. This formalism provides the means to give a rigourous specification of CONDUCTOR, and one can abstract from many of the problems one has to tackle in a higher-level programming language. A few of these problems were discussed in the previous chapter. The modules in the formal specification can be seen as an abstract descriptions of the higher-level programming language implementation of the modules
(hlpl-modules). Of course, this abstraction is only useful if the formal specification of a module is considerable smaller than the hlpl-module. The hlpl-modules in the CONDUCTOR project were, on average, a factor 10 larger than their abstract counterparts.

During the design of CONDUCTOR ASF was used as a bookkeeping mechanism. Modules were specified at a certain stage in the design and later on, functions and sorts were added, or the specifications of existing functions and sorts were modified. Thus we knew all functions and sorts of a module before we implemented it. For instance, the module Ranges, given in appendix E, is described in 30 lines in the formal specification. The implementation of this module in the programming language C (Range.c) consists of 340 lines of code, including comments. In the module Range.c functions had to be added for storage management and I/O of ranges. In the first version of module Ranges the functions range-plus, range-min and range-mul did not exist. Later on, when we designed the static type checking and the generation of input restriction, these functions turned out to be needed. Thus, the formal module Ranges provided an excellent abstract model that could be used during the implementation of the hlpl-module Range.c.

16.2. MODULARIZATION

In chapter 3 of this book we listed the goals of modularization as given by Parnas [1985]. We will discuss how ASF contributed to each of these goals.

16.2.1. Small modules.

The first goal of modularization, as given by Parnas [1985], is that each module should be small so that it can be understood fully. The formal specification language ASF does not really force such modularization. One could think of the horrible solution to specify the CONDUCTOR in just one module with hundreds of sorts and functions. By restricting the number of newly exported sorts and functions in a module - the average number of newly defined sorts in the specification of CONDUCTOR is near 1 and the
newly defined functions near 5 - one obtains ASF specifications consisting of simple modules.

16.2.2. Implementation changes.

One must be able to change the implementation of a module without affecting other modules. Again ASF is, albeit indirectly, of help in this respect. The formal specification contains a complete specification of the interfaces of all modules involved. If the modules are indeed implemented using only the knowledge of the interfaces of imported modules, the implementation of modules can be changed without influencing other modules.

In our implementation (see 15.1), the data types defined in internal definition files and the implementation of the functions in the C-code files can be changed without influencing other modules. Changes in the parameter lists or the result of functions in the C-code files and changes in the external definition files do influence other modules because they are part of the interface of a module.

16.2.3. Unlikely changes.

Only unlikely changes should require changes in the interface of the modules. The benefit of ASF in this respect is that the use of formal specification languages forces the system analyst to define the system thoroughly before he implements it. Evidently this does not guarantee that later on in the development process some aspects of the problem change or new wishes occur. If, however, simple modules are specified as discussed in 16.1.1, a change in the interface of a module will only add new functions to the interface. In using ASF to specify CONDUCTOR, we found that the specification of basic sorts such as the data types (appendix D), the type descriptions (appendix E), the abstract syntax of the statistical language (appendix F) hardly changed during the specification process. More delicate parts like the symbolic evaluation of index variables (appendix I) and the exception handling mechanism (appendix J) were changed several times before they got their final shape. This indicates, that if we had started the implementation of these modules too early, we would have been forced to make changes in these
modules. By using ASF as a design language, the need for such changes is reduced considerably.

16.2.4. Major changes.

Major changes in the software should only result in independent changes in the individual modules. A major change in the software leads to a major change in the specification of the software. In the formal specification the following changes can be distinguished:
- adding new modules,
- changing the equations that specify sorts and functions in the module,
- modifying the exported sorts and functions of the module.

If a major change only consists of the addition of modules or the changes of equations, this change can be regarded as a complete independent modification of modules. Of course the use of ASF does not guarantee that no modifications of module interfaces are necessary. In the development process of CONDUCTOR these changes turned out to be inevitable. Most modifications, however, were either changes in the parameter lists of functions or the addition of new functions.

16.2.5. The responsibility of a module.

A software engineer should be able to understand the responsibility of a module without understanding the details of its internal design. If a software engineer is capable of reading the formal specification language ASF, the formal specification of a module gives him an exact description of its responsibility. For example, the responsibility of hlpl-modules, such as, Range.c, Type.c, Table.c and Seq.c can be understood be studying the formal specification as given in the formal modules Ranges, Types, Tables and Sequences.

16.2.6. Relevant modules.

A reader with a well-defined question is able to find the relevant modules in the formal specification. The formal specification introduces a strict hierarchy in the modules. A module can be understood by studying its formal specification and the specification of its imported modules.
For example, the module Compilers can be understood by studying besides this module, module Statistical-programs (the source language), module Kernel-programs (the target language), module Input-restr-generator (the input restriction generator), module Static-type-checking (the static type checking), module Gen-instructions (the generation of kernel instructions) and module Gen-handlers (the generation of exception handlers).

*Figure 16.1. The Compiler*

```
Compilers

<table>
<thead>
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</tr>
</thead>
</table>
```

16.2.7. The number of branches.

The submodules imported by a module should not have overlapping responsibilities. In order to avoid this, the number of submodules should be small. In ASF this goal is reached by keeping the number of imported modules small. The average number of imported modules in the specification of CONDUCTOR is between 2 and 3. ASF does not force the number of branches to be small; this remains the responsibility of the system analyst.

16.3 FORMAL DESCRIPTION

We also used the formal specification technique to avoid "the seven sins" as discussed in Meyer [1985] and to get an unambiguous definition of CONDUCTOR.

16.3.1. Noise.

Noise is the problem that the specification contains elements that do not add information. Deliberately some noise is left in the specification of
module Scalars. This module contains the exported functions add, sub, mul and div. These functions do not add any information, because they are not imported by any other module in the specification, nor are they used in the equations of the module Scalars. So neither in the definition of scalars nor in the use of scalars these functions are needed. We suggest to add a simple bookkeeping mechanism to the implementation of ASF to list all unused exported functions.

16.3.2. Silence.

It is even more difficult to detect whether some aspects of the problem are not specified. Examples of silence in the specification are the database, and the inconsistencies and redundancy in the input restrictions. The database is, in the specification, represented by the sort DATABASE and a function current-db. How a database is constructed and how additional data is stored and interrogated is left unspecified. The problem with input restrictions is, that CONDUCTOR may generate restrictions that are contradicting, such as,

\[ n - 1 \geq 0 \quad \text{and} \quad - n - 1 \geq 0 \]

or redundant, such as,

\[ n - 7 \geq 0 \quad \text{and} \quad n - 3 \geq 0 \]

In the first case, the contradicting restrictions, there is no feasible set of input variables, and in the second case the second restriction need not be tested.

The level of abstraction chosen to describe CONDUCTOR, makes these problems implementation problems. Nonetheless, a complete specification of these problems would help to get a good implementation. The same is true for other implementation problems like the optimization of the instruction sequences in the kernel, or the error recovery used in the prototype.
16.3.3. Overspecification.

Where does the specification of a problem end and where does the implementation begin? This is probably one of the most difficult problems for a system analyst. Defining the problem is a matter of finding a good level of abstraction to describe the problem. For example, at the highest level of abstraction CONDUCTOR is defined by the module Conductor and its imported modules.

*figure 16.2: The overall CONDUCTOR system.*

```
Conductor
  Conductor-sessions Resulting-software Compilers Gen-ext-handlers Conductor-states
```

Limiting the specification to 6 modules, it would only slightly contribute to the definition of CONDUCTOR. One must admit that also the level of abstraction chosen in the formal specification, presented in the appendices of this book, is arbitrary. In this specification the problems involved in optimising the kernel instructions, the hashing mechanism, the construction of the database, etc., are omitted. On the one hand, by defining these problems to be implementation problems this can be defended, on the other hand if one needs to implement code optimization or hashing mechanisms one could surely benefit from a thorough specification of these problems. A natural solution is to make an independent specification of these subproblems, emphasizing that these are implementation problems for the "main" specification.

16.3.4. Contradiction.

ASF does not offer the possibility to check the consistency of the specification. The equations may contain contradictions in terms of the logic of the specified sort, for example,

```
eq-result(abs-minus(e1,e2),abs-minus(e2,e1)) = true
```
is obviously wrong. It specifies that the \texttt{minus} operator in index expressions is symmetric. Using the equations in module \texttt{Ind-expr-abstract-syntax} it can be derived from this equation that an index value is equal to its opposite

\[
\text{eq-result}(\text{abs-const}(c), \text{abs-const}(\text{neg}(c))) = \text{true}
\]

which is of course not what we had in mind. Even worse, if we added two equations to module \texttt{Ind-expr-abstract-syntax}

\[
\begin{align*}
\text{eq-result}(\text{abs-minus}(e1,e2), \text{abs-minus}(e2,e1)) & = \text{true} \\
\text{eq-result}(\text{abs-minus}(e1,e2), \text{abs-minus}(e2,e1)) & = \text{false}
\end{align*}
\]

the contradiction in these equations is not detected by \texttt{ASF}. We actually specified that the constant functions \texttt{true} and \texttt{false} are identical. The problem is that, by giving meaningful names to functions, the system analyst assumes that the specified mathematical constructs have the suggested semantics. The examples above make it clear that \texttt{ASF} does not guarantee that the suggested semantics of a specification is without contradictions.

The examples also make it clear that modifications of the semantics of sorts, outside the module where they are declared should be avoided. These modifications make it, in a large specification, almost impossible to verify if the suggested and mathematical semantics coincide. A book-keeping mechanism that warns the user of \texttt{ASF} if the semantics of a sorts is redefined outside the module where it is declared should be of great help to a system analyst.

16.3.5. Ambiguity.

From the mathematical point of view, a specification always has a meaning. This meaning is certainly not the intended one if the specification contains contradictions. The discussion in 16.3.4. showed that in practice contradictions still exist in the suggested semantics, and thus we still have to face an ambiguous specification. This implies that we have to face ambiguities due to this difference between mathematical and intended meaning.
16.3.6. Forward reference.

In the formal specification language ASF forward reference is impossible. A module can only import functions and sorts of previously defined modules.

16.3.7. Wishful thinking

Is wishful thinking impossible if one uses ASF? Of course not. By remaining silent over the exact specification of a particular subproblem a delicate problem may remain unspecified and the system analyst may hope that someone might solve the problem. In CONDUCTOR an example of wishful thinking is the database. The database is left completely unspecified and the author of this book simply hopes and believes that a database in combination with background queries can be implemented.

16.4. SPECIFICATION OF ERRORS

At many points in the formal specification we had to use conditional equations. That is, we had to impose conditional restrictions on the sorts used as arguments in formal functions. An example is the specification of a constant range in module Constant-ranges. A constant range consists of two indices under the restriction that the first argument is equal to or less than the second argument. Legal examples of constant ranges are

\[
[1 \text{ to } 5] \quad \text{or} \quad [27 \text{ to } 217]
\]

In the formal specification a range is represented by the sort CONST-RANGE through the function c-range.

\[
c-range: \quad \text{INDEX \# INDEX} \rightarrow \text{CONST-RANGE}
\]

To specify a well-formed constant range the functions \(\text{wf-c-range}\) and \(\text{error-c-range}\) are introduced

\[
\text{wf-c-range:} \quad \text{INDEX \# INDEX} \rightarrow \text{CONST-RANGE}
\]

\[
\text{error-c-range:} \quad \rightarrow \text{CONST-RANGE}
\]
These functions make it possible to specify a well-formed constant range consisting of two indices \textit{ind1} and \textit{ind2} as given in equation [1].

[1] \( c\text{-range}(\text{ind1}, \text{ind2}) = \text{wf-c-range}(\text{ind1}, \text{ind2}) \)

\hspace{1cm} \text{when ge(\text{ind2}, \text{ind1}) = true}

A constant range is specified to be illegal when the greater equal condition \( \text{ge}(\text{ind2}, \text{ind1}) \) does not hold.

[2] \( c\text{-range}(\text{ind1}, \text{ind2}) = \text{error-c-range} \)

\hspace{1cm} \text{when ge(\text{ind2}, \text{ind1}) = false}

In this specification we reduced all combinations of indices to a set of correct constant ranges and one error case. The other functions in module \textit{Constant-ranges}, such as \textit{length} and \textit{eq-range}, were all specified in terms of well-formed constant ranges (the function \textit{wf-c-range}). For example the equation [3] specifies the length of a constant range.

[3] \( \text{length}(\text{wf-c-range}(\text{ind1}, \text{ind2})) = \text{add}(1, \text{sub}(\text{ind2}, \text{ind1})) \)

The error case is left unspecified. Taking into account all error cases would considerably increase the length of the specification. For example, all functions specified for the sort \textit{INDEX} would need extra equations for the constant function \textit{error-index}. Probably the best solution is to incorporate the error propagation in \textit{ASF} in such a way that the error cases due to conditional equations can only be seen in the module where such errors occur. At all points, in the specification of \textit{CONDUCTOR}, where we used conditional equations, we made a comment that the error case is left unspecified. If a conditionally specified sort is imported by an other module one may assume that the well-formed sort is imported. Thus we avoided specifying lengthy error cases.

16.5. KEEPING THE PROTOTYPE CONSISTENT WITH THE SPECIFICATION

It is important to develop a prototype to check the suggested semantics of the specification. Of course this introduces the problem of keeping
the specification and the prototype consistent. One of the problems in this respect is that the exported sorts and functions should indeed form the interface of the implemented modules of the prototype. The implemented prototype is by-hand kept consistent with the specification. This approach naturally introduces errors. For example, function names in the formal specification may differ from the names of their counterparts in the implementation. The only way to avoid this would be, that a specification is automatically translated into an implementation. This approach is taken by Arsac [1979] and Balzer [1981]. An other way to check the consistency of specification is the use of term-rewriting systems, as implemented by Hendriks [1988]. In such a system the equations in an ASF specification are used as rewriting rules. Given a term, the generated prototype for a specification tries to find the smallest possible representation, using the rewriting rules. This approach up till now only works for small examples. To automatically produce an implementation from a specification of the size of the specification of CONDUCTOR still seems to be a dream. A small step in making this dream come true is the following. In many specifications one will need modules for structures such as booleans, sequences, and tables. There seems to be no need to specify these modules over and over again, and an efficient implementation of these modules can be made and stored in a module library. These modules can be used as building blocks for other specifications, that are also using these structures. And the efficient implementations of these modules should be linked automatically in the implementation of these specifications.

16.6. EQUALITY OF CONSTANT FUNCTIONS

At many places in the specifications we defined a function eq for equality of specified sorts. If constant functions were defined for these sorts, we had to specify that all these functions were unequal. This was boring work, and the result can almost be called noise. If we wanted constant functions to be equal we would have specified only one constant function. Therefore it seems worthwhile to incorporate an equal function in ASF, in order to avoid lengthy specifications of the inequality of constant functions. For instance, the specification of equality of three
terms $t_1$, $t_2$ and $t_3$ in the current version of ASF reads

$$eq: \text{SORT \# \text{SORT} -> BOOLEAN}$$

with equation

$$eq(t_1,t_2) = \text{true}$$
$$eq(t_1,t_3) = \text{false}$$
$$eq(t_2,t_3) = \text{false}$$

In a new version this should be replace by one equation.

$$t_1 = t_2$$

in combination with a built in function $eq$, that is specified to return the value $\text{true}$ if two terms are equivalent in terms of the equational logic in a module, and otherwise returns the value $\text{false}$.

16.7. ADVICE FOR USERS OF ASF

The use of ASF by now means solves all problems of system analysts. If, however, the system analyst obeys a few simple rules, ASF opens the possibility to thoroughly define and modularize his problem before he starts implementing it. These rules are:
- keep the number of sorts specified in each module small,
- keep the number of imported modules small.

Furthermore, it is important to develop a prototype to check the suggested semantics of a specification. Of course, this introduces the problem of keeping the specification and the prototype consistent.

Also, realize that a specification should be an abstraction of a problem. There is no such thing as the one and only correct abstraction. Important is to keep the specification readable by abstracting from subproblems (implementation problems). A useful approach is to
- make the specification of subproblems completely independent of the specification of the 'main' problem.

Finally, finding better ways to deal with error cases in the specifications, and incorporating a function for equality of terms, will surely improve the specification formalism ASF used in this book.
CONDUCTOR is not a commercial product. The current prototype, that runs under both the operating systems UNIX and MS-DOS, can compile and run the statistical techniques discussed in this book. It still has to be tested on a larger set of examples. These tests probably will reveal that the efficiency of some of the implemented modules can be improved. Furthermore, a collection of statistical functions, comparable to the functions in languages like S and IML, should be added in order to make it possible to implement large sets of statistical techniques. Also the user language has to be improved if one wants to make CONDUCTOR competitive with modern statistical software packages. Important in this respect are virtual screens and pull-down menus. The virtual screens and the pull-down menus were first introduced in SMALLTALK (see Goldberg [1983-1984]), and have strongly influenced the user interface of modern operating systems and software packages. Though adding such features is merely a matter of time, it is necessary if one wants to compete with existing statistical software packages.

Besides these improvements, other more fundamental concepts can be added to CONDUCTOR. One can think of:

- Making it possible to create statistical expert systems in CONDUCTOR. An exception, in software generated by CONDUCTOR, signals when the execution of a statistical technique must be interrupted. In the current version of CONDUCTOR, the generated software reacts on a single exception. For example, the software generated in CONDUCTOR may
detect that the data used in a statistical technique is constructed using different measurement instruments. The execution of the statistical technique is interrupted, and the software looks for an appropriate exception handler.

If the exception handling mechanism is given a 'memory', a combination of exceptions can be examined, using inference rules given by the technical statistician. Thus, different combinations of exceptions may invoke different exception handlers. The created statistical expert system, would be able to give a user advice in more complicated cases in statistical analyses. For a discussion on the slow emergence of expert systems in statistics see Gale et al. [1986].

- Adding a syntax directed editor. A statistical technique written in the statistical language shows close resemblance with the notation used in statistical textbooks. However, partitioned matrices can not be entered the way they are written in these books. The reason is that the input of a statistical technique is line oriented rather than screen oriented. With the use of syntax directed editors that know the lay out of a partitioned matrix, partitioned matrices can be entered the way they are written in textbooks. A second advantage of syntax directed editors is that the abundant typing of keywords can be avoided. Syntax directed editors are for example used in the programming language B¹ (see Geurts et al. [1985] and Meertens [1981]).

- Generating code for a parallel processor. In a statistical language the techniques are described in a matrix language. Some of the matrix functions can be executed much faster by parallel than by sequential processors. Parallel algorithms for matrix functions are discussed in O'Leary and Stewart [1985]. Instead of generating instructions for the kernel, as in the prototype of CONDUCTOR, code could be generated for a parallel processor. This would, of course, dramatically improve the performance of the resulting software.

Besides these 'dreams', also other problems remain open. The most

¹ The current version of this language is called ABC.
important 'open ends' in the CONDUCTOR project are:

- **Contradicting input restrictions.** For each statistical technique, a sequence of input restrictions on the value of the input index variables is generated. These input restrictions may be contradicting. For example, the input restrictions

\[ n - 1 \geq 0 \quad \text{and} \quad n - 1 \geq 0 \]

both must hold. In this case no feasible input exists. This simple case is detected by the prototype, however more complex cases should be considered.

- **CONDUCTOR will detect many exceptions during statistical analysis.** Probably there will occur combinations of exceptions for which statistics does not offer any solution. Statistical theory focusses on single exceptions to a general model. In applied statistics many exceptions occur simultaneously. For some of the combinations the statistical theory has no answers. We hope that the statistical software generated by CONDUCTOR will stimulate statisticians to find answers for such questions.

The current version of CONDUCTOR is already a useful piece of software. It can generate efficient software for small statistical applications. By adding new features to it we also hope to improve it in the near future.
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APPENDIX A: THE CONCRETE SYNTAX

This appendix contains the concrete syntax of the statistical language. The user language, the language that can be used to create external exception handlers. In the syntax rules the following notational conventions are employed: (1) All words written in lowercase letters are non-terminals. (2) All words and symbols between single quotes (" and ") are terminals. (3) All words written in upper-case letters not between quotes are terminals.

A. THE OVERALL CONDUCTOR SYSTEM

    session  --> session sub_session |
               sub_session

    sub_session  --> 'STATISTICIAN' statistical_program |
                   'USER'  user_session |
                   'EXTERNAL' ext_handler_program

B. THE STATISTICAL LANGUAGE

1. General Structure.

    statistical_program  --> statistical_program_section |
                            section

    section  --> name_section |
               input_output_section |
               implementation_section |
               test_section |
               exception_handler_section

2. Name section.

    name_section  --> 'NAME' IDENTIFIER

3. Import/export section.

    input_output_section  --> INTERFACE input_declaration |
                             output_declaration

    input_declaration  --> 'INPUT' declarations

    output_declaration  --> 'OUTPUT' declarations

4. Implementation section.

    implementation_section  --> local_declarations |
                              'EQUATIONS' statements

5. Test section.

    test_section  --> 'TEST' local_declarations |
                    'EQUATIONS' statements |
                    raises |
                    raise |
                    'RAISE' exception_decl |
                    'WHEN' expression |
                    exception_decl  --> IDENTIFIER |
                                        EXCEPTION_FLAG

6. Handler section.

    exception_handler_section  --> 'HANDLERS' handlers |
                                   handler

    handler  --> handlers handler |
               handler

    handler  --> 'WHEN' 'EXCEPTION_FLAG' |
                local_declarations |
                'EQUATIONS' statements |
                stop_or_continue

    stop_or_continue  --> STOP |
                        /* default continue */

7. Messages.

    message  --> 'MESSAGE' '"' STRING |
                /* empty */
8. Expression.

expression → expression OP term |
  term => (' expression ') | function_call |
  factor ⇒ PREFIX_UNARY_OP term |
factor → factor POSTFIX_UNARY_OP |
  variable ⇒ SCALAR_VALUE | INDEX_VALUE |
variable → VARIABLE_NAME | VARIABLE_NAME offset |
offset → ['subranges '] | |
  [' sat-indices '] |
subranges → subranges ',' subrange |
  subrange |
subrange → index_expr 'TO' index_expr |
  sat-indices |
  sat-index |
sat-index → index_expr |


function_call → FUNCTION_NAME '(' argument_list ')' |
argument_list → argument_list ',' expression |
  expression |

10. Declarations.

local_declaration → 'VARIABLES' declarations |
  /* empty */ |
declarations → declarations ';' declaration |
  declaration |
declaration → var_type id_list message |
id_list → id_list ',' IDENTIFIER |
  IDENTIFIER |
var_type → 'INDEX' | 'SCALAR' | 'BOOL' | |
  sat_type | vec-type |
sat_type → 'MATRIX' '[' ranges ']' |

vec_type → 'VECTOR' range |
ranges → ranges ',' range |
range ⇒ index_expr 'TO' index_expr |
index_expr → (' index_expr ') | |
  index_expr ',' index_expr |
  index_expr '=' index_expr |
  index_expr '-' index_expr |
  INDEX_VARIABLE |
  INDEX_VALUE |

C. USER LANGUAGE

user_session → user_session user_command |
user_command → 'RETRIEVE' user_variables |
  'DISPLAY' user_variables |
  'DELETE' user_variables |
  'INITIALIZE' user_variables |
  'FORMAT' INDEX_VALUE INDEX_VALUE |
  'SEP' user_variables |
  'SAMPLE' const_ranges |
  'TECHNIQUE_ID' |
  user_declaration |
user_variable → user_variable ',' user_variable |
  user_variable |
user_variable → MATRIX_ID |
  SCALAR_ID |
  INDEX_ID |
  SERIES_ID |
  BOOL_ID |
user_declaration → user_type id_list |
user_type → 'BOOL' |
  'INDEX' |
  'SCALAR' |
  'VECTOR' |
  'MATRIX' |
  'SINESS' |
const_ranges → const_ranges ',' const_range |
  const_range |
const_range → INDEX_VALUE 'TO' INDEX_VALUE |

D. EXTERNAL HANDLERS

ext_handler_program → exception_handler_section |
A many-sorted signature is a set of declarations of sorts and functions over these sorts. A signature defines a language of strongly typed terms (expressions). A basic ASF module consists of a signature, a set of variable declarations and a set of positive conditional equations in the language defined by the signature and the variable declarations. ASF modules may be parameterized. Parameter binding and importing modules in other ones, are the two ways in which modules can be combined in ASF. ASF specifications are sequences of modules. A module can be normalized in the context of a specification to which it belongs by eliminating all imports and binding as many parameters as possible. Normalization is a textual operation. The semantics of a module is the initial algebra of its normal form, provided the latter does not have any remaining unbound parameters.

1. Syntax of ASF

In this section we give a context-free grammar for ASF. The following notational abbreviations are used in this definition:
- { x } denotes an optional occurrence of x.
- (x) and {x} denote, respectively, one or zero and one or more occurrences of x.
- (x t) and {x t} denote, respectively, zero or more and one or more occurrences of x separated by terminal symbol t.

ASF has the following grammar:

```
(module) ::= (module) (module-ident)
    "begin"
    { (parameters) ]
    { (exports) ]
    { (imports) ]
    { (functions) ]
    (end) (module-ident) .

(module-ident) ::= (ident).

(parameters) ::= "parameters" (parameter) ".

(parameter) ::= (parameter-ident)
    "begin"
    { (functions) ]
    (end) (parameter-ident) .

(export-ident) ::= (ident).

(export) ::= (exports) "begin"
    { (functions) ]
    (end) .

(imports) ::= "imports" (module-expression) "."

(module-expression) ::= (module-ident) [ ("(" (modifier) "") ]
    { (renamed) [ (bound) ] [ (bound) [ (renamed) ]
    "renamed" (by) (renamings) .
```

2 Lexical Syntax

Layout or comment may separate the following lexical notions of ASF: (ident) (operator) and (implies). Layout has no significance other than separating consecutive lexical tokens that would otherwise not be distinguished. Layout may never occur embedded in a lexical token. In cases of ambiguity, the longest lexical token is preferred. The lexical conventions of ASF are summarized below.

- Layout characters are space, horizontal tabulation, carriage return, line feed and form feed.
- Comments follow a layout character and begin with two hyphens and end with either and end of line (i.e., carriage return or line feed) or another pair of hyphens.

APPENDIX C. ASF SPECIFICATION OF BASIC STRUCTURES

In this appendix the the simple data types, sequences and tables, are specified. It is assumed that each sort and function in the modules corresponds to its "natural" meaning. A full specification of the modules, including equations, can be found in Bergstra et al. [1995].

1. SIMPLE DATA TYPES.

The simple data types are booleans, indices and scalars.

1.1. BOOLEANS.

1.1.a. Global description.

In the module Boolean the usual boolean operators are specified, such as, and, or, not, etc. Also the sort BOOL can be used in an "if-then-else" construct.

1.1.b. Specification.

module Boolean
begin
exports
begin
  sorts BOOL
  functions
  true : -> BOOL
  false: -> BOOL
  and : BOOL # BOOL -> BOOL
  or : BOOL # BOOL -> BOOL
  not : BOOL -> BOOL
  eq : BOOL # BOOL -> BOOL
  if : BOOL # * # -> *
end
end Boolean

1.2. SCALARS.

1.2.a. Global description

The module Scalars specifies reals. For scalars, the functions add (add), subtract (sub), multiply (mul), divide (div) and equal (eq) are specified. Even though all these functions are on the export list of the module Scalars, only the function eq is imported by other modules in the formal specification.

1.2.b Specification.

module Scalars
begin
exports
begin
  sorts SCALAR
  functions
  add: SCALAR # SCALAR -> SCALAR
  sub: SCALAR # SCALAR -> SCALAR
  mul: SCALAR # SCALAR -> SCALAR
  div: SCALAR # SCALAR -> SCALAR
  eq: SCALAR # SCALAR -> BOOL
end
end Boolean, Scalars

1.3. INDICES.

1.3.a. Global description.

Indices are specified in module Integers. The constant functions 0 and 1 are specified to be indices, and also every index that can be constructed using the functions increment (increment), round (round), add (add), subtract(sub), multiply(mul) and negate(neg). Furthermore, the boolean relations less than or equal to (le) and equal to (eq) are specified for indices.

1.3.b. Specification.

module Indices
begin
exports
begin
  sorts INDEX
  functions
  0: -> INDEX
  1: -> INDEX
  increment: INDEX -> INDEX
  round: SCALAR -> INDEX
  add: INDEX # INDEX -> INDEX
  sub: INDEX # INDEX -> INDEX
  mul: INDEX # INDEX -> INDEX
  neg: INDEX -> INDEX
  ge: INDEX # INDEX -> BOOL
  le: INDEX # INDEX -> BOOL
  eq: INDEX # INDEX -> BOOL
end
end Boolean, Scalars, Indices
2. SEQUENCES.

2.a. Global description.

Sequences are linear lists of items; they are parameterized with the type of the item. The following functions are specified for sequences:

- **add-item**: add an item to a sequence
- **del-item**: delete an item from a sequence
- **conc**: concatenate two sequences
- **eq-seq**: check the equality of two sequences
- **disjunct**: test if two sequences are disjunct
- **n-of-items**: count the number of items in a sequence
- **first**: return the first item in a sequence
- **last**: return the last item(s) in a sequence
- **item-no**: return the specified item in a sequence

2.b. Specification.

module Sequences
begin
  parameters Items
  begin
    sorts ITEM
    functions
    eq-item: ITEM # ITEM -> BOOL
  end
  Items
  exports
  begin
    sorts SEQ
    functions
    null: -> SEQ
    add-item: ITEM # SEQ -> SEQ
    del-item: SEQ -> SEQ
    conc: SEQ # SEQ -> SEQ
    eq-seq: SEQ # SEQ -> BOOL
    disjunct: SEQ # SEQ -> BOOL
    n-of-items: SEQ -> INDEX
    first: SEQ -> ITEM
    last: SEQ -> ITEM
    item-no: SEQ # INDEX -> SEQ
  end
  end
end

Imports Booleans, Indices

3. TABLES.

3.a. General description.

In several parts of the formal specification we need tables to store information. Therefore, a general module Tables is specified with parameters Entries and Addresses. In this module the following functions are specified:

- **null-table**: an empty table
- **insert**: insert an entry at a specified address in the table
- **lookup**: lookup an entry at a specified address and check if entry is found
- **delete**: delete an entry at the specified address in the table
- **eq-tab**: equality of a table
- **entry**: return an entry if found with lookup


module Tables
begin
  parameters
  begin
    parameters Items
    begin
      sorts ENTRY
      functions
      eq-entry: ENTRY # ENTRY -> BOOL
    end
    Entries,
  Addresses
  begin
    sorts ADDRESS
    functions
    eq-addr: ADDRESS # ADDRESS -> BOOL
  end
  Addresses
  exports
  begin
    sorts TABLE
    functions
    null-table: ADDRESS # ENTRY # TABLE -> TABLE
    insert: ADDRESS # ENTRY # TABLE -> TABLE
    lookup: TABLE # TABLE -> BOOL
    found: ADDRESS # TABLE -> TABLE
    delete: ADDRESS # TABLE -> TABLE
    eq-tab: TABLE # TABLE -> BOOL
    entry: ADDRESS # TABLE -> ENTRY
  end
  end
end

Imports Booleans

end Tables

4. IDENTIFIERS AND STRINGS.


Strings are used in CONNECTOR to communicate messages. Strings are a sequence of characters. Identifiers are a restricted set of strings.
APPENDIX D. ASF SPECIFICATION OF THE STRUCTURED DATA TYPES

In this appendix the formal specification is given of the structured data types, to wit matrices and series. Also the data types of variables in the user language, and the data types of variables in a statistical program are defined.

1. STRUCTURED DATA TYPES.

1.1. MATRICES.

1.1.a. Global description.

A matrix in CONDUCTOR is a sequence of scalars, containing the values of the individual matrix elements, combined with a sequence of constant ranges describing the dimensions of the matrix. A matrix XI:

\[
x_{11} \ x_{12} \\
\ x_{21} \ x_{22}
\]

is thought of as a sequence of scalars \(x_{11}, x_{12}, x_{21}, x_{22}\) and the dimension description \((1\ to\ 2, 1\ to\ 2)\). In a well-formed matrix the product of the length the dimensions must be equal to the length of the sequence of reals. In the module Matrices the functions for matrix element and submatrix reference are not further specified.

1.1.b. Specification.

module Scalar-sequences
begin
imports Sequences

\{renamed by
\[\begin{align*}
&\{ SEQ \to SCALAR-SEQ, \\
&\text{null} \to \text{null-seq}, \\
&\text{eq-seq} \to \text{eq-id-seq}\}
\end{align*}\]

Items bound by

\[\text{eq-item} \to \text{SCALAR, eq}\]
to Scalars

\}
end Scalar-sequences

module Id-sequences
begin
imports Sequences

\{renamed by
\[\begin{align*}
&\{ SEQ \to ID-SEQ, \\
&\text{null} \to \text{null-id-seq, eq-seq} \to \text{eq-id-seq}\}
\end{align*}\]

Items bound by

\[\text{eq-item} \to \text{eq-id}\]
to Ids

\}
end Id-sequences

module Index-sequences
begin
imports Sequences

\{renamed by
\[\begin{align*}
&\{ SEQ \to INDEX-SEQ, \\
&\text{null} \to \text{null-ind-seq}, \\
&\text{eq-seq} \to \text{eq-ind-seq}\}
\end{align*}\]

Items bound by

\[\text{eq-item} \to \text{INDEX, eq}\]
to Indices

\}
end Index-sequences
module Constant-ranges
begin
exports
begin
sorts CONST-RANGE
functions
  o-range: INDEX # INDEX --> CONST-RANGE
  length: CONST-RANGE --> INDEX
  eq: CONST-RANGE # CONST-RANGE --> BOOL
  eq-length: CONST-RANGE # CONST-RANGE --> BOOL
end
imports Booleans, Indices
functions
  -- functions used in specification of well-formedness --
  -- constant ranges
  -- error range
  length: INDEX # INDEX --> CONST-RANGE
  error-c-range: INDEX # INDEX --> CONST-RANGE
variables
cr, cr1, cr2 :--> INDEX
end
module Matrices
begin
exports
begin
  sorts MATRIX
  functions
    mat: SCALAR-SEQ # CONST-RANGE-SEQ --> MATRIX
    eq: MATRIX # MATRIX --> BOOL
    elements: INDEX-SEQ # MATRIX --> INDEX
    submat: CONST-RANGE-SEQ # MATRIX --> MATRIX
end
imports Scalar-sequences, Constant-range-sequences, Booleans, Index-sequences, Indices
functions
   -- functions used to specify well-formed matrices --
   uf-mat: SCALAR-SEQ # CONST-RANGE-SEQ -> MATRIX
   error-mat: -> MATRIX
variables
   i : INDEX
   t : SCALAR-SEQ
   ss, ssl, ssl2 : SCALAR-SEQ
   cr : CONST-RANGE
   crs, cros, crs2 : CONST-RANGE-SEQ
   m : MATRIX
equations
   [16] mat(ss, crs) = uf-mat(t, ss)
       when eq(n-of-item(ss), product(crs)) = true
   [17] mat(t, crs) = error-mat
       when eq(n-of-item(ss), product(crs)) = false
   [18] eq(uf-mat(s, ssl), uf-mat(s, ssl2)) = and(eq-sca-seq(ssl, ssl2), eq-cr-seq(crs, crs2))
       -- the error case is left unspecified --
end Matrices

1.2 SERIES.
1.2a. Global description.
A (time) series of observations consists of a sequence of scalars (the observations in the series) and a sequence of sample indicators. For example, a series of observation on consumption over the period 1945-1949 and 1960-1962 is represented as a scalar sequence containing the 8 observations in this period
12.4 13.9 14.8 13.7 15.9 20.9 22.6 21.8
and a constant ranges sequences, containing the sample description
(1945 to 1949, 1960 to 1962)
In a well-formed series the sum of the length the samples is equal to the number of observations (the length of the sequence of reals).

1.2b. Specification.
module Series
begin
exports
begin
   sorts SERIES
   functions
      ser: SCALAR-SEQ # CONST-RANGE-SEQ # ID-SEQ -> SERIES
      eq-series: SERIES # SERIES -> BOOL
   end
   imports Const-range-sequences, Scalar-sequences, ID-sequences, Booleans
functions
   -- functions used to specify well-formed series --
   uf-ser: SCALAR-SEQ # CONST-RANGE-SEQ # ID-SEQ -> SERIES
   error-ser: -> SERIES
variables
   s : SERIES
   crs, crs2 : CONST-RANGE-SEQ
   ida, ida1, ida2 : ID-SEQ
   ssc, ssl, ssl2 : SCALAR-SEQ
   crs, crs2 : CONST-RANGE-SEQ
   ida, ida1, ida2 : ID-SEQ
   sscs, ssls, ssl2s : SCALAR-SEQ
   crs, crs2 : CONST-RANGE-SEQ
   ida, ida1, ida2 : ID-SEQ
   equations
      [19] ser(ssc, crs, ida) = uf-ser(ssc, crs, ida)
          when eq(n-of-item(ssc), length(crs)) = true
      [20] ser(ssc, crs, ida) = error-ser
          when eq(n-of-item(ssc), length(crs)) = false
      [21] eq-series(uf-ser(ssc, crs, ida), uf-ser(ssc, crs, ida)) =
          and(eq-sca-seq(ssc, ssl), eq-cr-seq(crs, crs2),
               ida, ida1, ida2)
          -- the error case is left unspecified --
end Series

2. DATA TYPES.
2.a. Global description.
Variables in a statistical technique can be of type boolean, index, scalar or matrix. Variables declared in a user session can, besides these data types, also be of the type series. The module technique-data given the specification of the data types in a statistical technique: the module User-data specifies data types in a user session. Besides the data types, also data type descriptions are specified. The simple data types boolean, index and scalar are described by a constant function as specified in module Simple-types. The type description of variables
declared in a user session are specified in module User-types. A user type description is either a simple type, a matrix type or a series type description. The last two type descriptions contain a constant range sequence, describing respectively the dimensions or the sample of the data type. Type descriptions for variables in the statistical language are discussed in appendix E.

2.b. Specification.

module Technique-data begin
exports
begin
sorts TECH-DATA
functions
uninitialized: Bool

begin

end

end

(module User-data
begin
exports
begin
sorts USER-DATA
functions
u-data: SERIES (-> USER-DATA)

begin

end

end

(module Technique-data, Series, Booleans
begin
variables

begin

end

end

(module Simple-types
begin
exports
begin
sorts SIMPLE-TYPE
functions
bool-type: (-> SIMPLE-TYPE)

begin

end

end

end
APPENDIX E. ASF SPECIFICATION OF INDEX EXPRESSIONS, SYMBOLIC RANGES AND
TYPE DESCRIPTIONS

1. INDEX EXPRESSIONS

1.1.a. Global description.

In index expressions the operators plus (abs-plus), minus (abs-minus) and
multiply (abs-mul) are specified. An index expression consists of
identifiers (abs-ind) or index constants (abs-const). A function
eq-result specifies the equivalence of two symbolic index expression. An
empty abstract syntax tree of an index expression is specified by the
constant function nil.

1.1.b. Specification.

module Ind-expr-abstr-syntx
begin
exports
begin
sorts IND-EXPR
functions
abs-plus: IND-EXPR × IND-EXPR → IND-EXPR
abs-minus: IND-EXPR × IND-EXPR → IND-EXPR
abs-mul: IND-EXPR × IND-EXPR → IND-EXPR
abs-const: INDEX → IND-EXPR
abs-ind: ID → IND-EXPR
nil: IND-EXPR → BOOL
eq-result: IND-EXPR × IND-EXPR → BOOL
end
imports Indices, Identifiers, Booleans

variables
e,e1,e2,e3,e4 :→ IND-EXPR
n,c1,c2 :→ INDEX
id1,id2 :→ ID

end

[55] eq-result(e,e) = true
[56] eq-result(e1,e2) = eq-result(e2,e1)

-- symmetry operators --

[57] eq-result(abs-plus(e1,e2), abs-plus(e2,e1)) = true
[58] eq-result(abs-mul(e1,e2), abs-mul(e2,e1)) = true

-- distributivity operators-

[59] eq-result(abs-mul(e1,abs-plus(e2,e3)), abs-plus(abs-mul(e1,e2),abs-mul(e1,e3)))
   = true

variables
st,tt,tt2 :→ SIMPLE-TYPE

end

module User-types
begin
exports
begin
sorts USER-TYPE
functions
user-type: SIMPLE-TYPE → USER-TYPE
series-type: CONST-RANGE-SEQ → USER-TYPE
matrix-type: CONST-RANGE-SEQ → USER-TYPE

end

exports Simple-types, Const-range-sequences, Booleans

variables
or1,or11,or22 :→ CONST-RANGE-SEQ
st,tt,tt2 :→ SIMPLE-TYPE
u1,u2,u22 :→ USER-TYPE

end

end

---

[43] eq(st,t1) = true
[44] eq(st,t2) = eq(tt1,tt2)

---
[60] eq-result(abs-mul(e1, abs-minus(e2, e3)), abs-minus(abs-mul(e1, e2), abs-mul(e1, e3)))
   = true

-- associativity operators --

[61] eq-result(abs-plus(e1, abs-plus(e2, e3)), abs-plus(abs-plus(e1, e2), e3))
   = true

[62] eq-result(abs-minus(e1, abs-minus(e2, e3)), abs-minus(abs-minus(e1, e2), e3))
   = true

[63] eq-result(abs-mul(e1, abs-mul(e2, e3)), abs-mul(abs-mul(e1, e2), e3))
   = true

-- arithmetic rules indices --

[64] eq-result(abs-plus(abs-const(c1), abs-const(c2)), abs-const(add(c1, c2)))
   = true

[65] eq-result(abs-minus(abs-const(c1), abs-const(c2)), abs-const(sub(c1, c2)))
   = true

[66] eq-result(abs-mul(abs-const(c1), abs-const(c2)), abs-const(mul(c1, c2)))
   = true

[67] eq-result(abs-minus(e1, abs-mul(abs-const(c2), e2)), abs-plus(e1, abs-mul(abs-const(reg(c1), e2))))
   = true

-- null element --

[68] eq-result(abs-mul(abs-const(0), e), abs-const(0))
   = true

[69] eq-result(abs-minus(e, abs-const(0)), abs-const(0))
   = true

-- unity --

[70] eq-result(abs-mul(abs-const(1), e), e) = true

-- empty tree --

[71] eq-result(abs-const(0), nil) = true

-- equivalence --

[72] eq-result(abs-plus(e1, e2), abs-plus(e3, e4))
   = and (eq-result(e1, e3), eq-result(e2, e4))

[73] eq-result(abs-minus(e1, e2), abs-minus(e3, e4))
   = and (eq-result(e1, e3), eq-result(e2, e4))

[74] eq-result(abs-mul(e1, e2), abs-mul(e3, e4))
   = and (eq-result(e1, e3), eq-result(e2, e4))

[75] eq-result(abs-nd(id1), abs-nd(id2))
   = eq-id(id1, id2)

[76] eq-result(abs-const(c1), abs-const(c2))
   = eq(c1, c2)

end ind-expr-absr-syntax

module ind-expr-sequences
begin
exports Sequences

(renamed by)

<table>
<thead>
<tr>
<th>SEQ</th>
<th>IND-EXPR-SEQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>m-of-items</td>
<td>n-of-dims</td>
</tr>
<tr>
<td>null</td>
<td>mul-nd-expr- seq</td>
</tr>
<tr>
<td>item-no</td>
<td>dim-no</td>
</tr>
<tr>
<td>eq-seq</td>
<td>eq-dims</td>
</tr>
<tr>
<td>Items bound by</td>
<td>IND-EXPR, eq-item</td>
</tr>
<tr>
<td>to ind-expr-sequences</td>
<td></td>
</tr>
</tbody>
</table>

end ind-expr-sequences

2. SYMBOLIC RANGES.

2.a. Global description.

Two index expressions form a symbolic range as specified in module Range. Equality of symbolic ranges is specified using the equivalence of index expressions. The restrictions that specify well-formed symbolic ranges are not given in this module but in the modules in appendix I. The functions range-plus, range-min, and range-mul specify the range operators.

2.b. Specification.

module Range
begin
exports begin
sorts RANGE functions
range: IND-EXPR # IND-EXPR -> RANGE
range-plus: RANGE # RANGE -> RANGE
range-min: RANGE # RANGE -> RANGE
range-mul: RANGE # RANGE -> RANGE
end

lower: RANGE -> IND-EXPR
upper: RANGE -> IND-EXPR
eq-range: RANGE # RANGE -> BOOL

imports Booleans, Ind-expr-abstract-syntax
variables
t1, t2, t3, t4, t5 -> IND-EXPR
equations

[77] range-plus(range(t1, t2), range(t3, t4))
   = range(abs-plus(t1, t3), abs-plus(t2, t4))

[78] range-nil(range(t1, t2), range(t3, t4))
   = range(abs-nil(t1, t2), abs-nil(t3, t4))

[79] range-mul(range(t1, t2), range(t3, t4))
   = range(abs-mul(t1, t2), abs-mul(t3, t4))

[80] lower(range(t1, t2)) = t1
[81] upper(range(t1, t2)) = t2
[82] eq-range(range(t1, t2), range(t3, t4))
   = eq-result(abs-nil(t1, t2), abs-nil(t3, t4))

end Ranges

module Range-sequences
begin
imports Sequences
(renamed by
   [SEQ] -> RANGE-SEQ,
   null -> null-range,
   n-of-items -> n-of-ranges,
   item-no -> item-no,
   eq-seq -> eq-seq]

Items bound by
   [ITEM] -> RANGE,
   eq-item -> eq-range]
to Ranges
end Range-sequences

3. TYPE DESCRIPTIONS IN THE STATISTICAL LANGUAGE.

3.a. Global description.

Variables in the statistical language can be of type index, scalar, boolean or matrix. The type system can determine the visibility of a declared variable (input, output or internal). The type descriptions are specified in module Tech-types, the visibility in module User-visibility. Two matrix type descriptions are equal if their symbolic dimension ranges are equal, as specified in the function eq-type. The function eq-skeleton only checks if two matrix types have the same number of dimensions.


module Technique-types
begin
imports
  Booleans, Simple-types, Range-sequences
exports
  begin
    sorts TECH-TYPE
    functions
      tech-type: SIMPLE-TYPE -> TECH-TYPE
      matrix-type: RANGE-SEQ -> TECH-TYPE
      dim-no: INDEX # TECH-TYPE -> RANGE
      n-of-dims: TECH-TYPE -> INDEX
      2-dim-matrix: TECH-TYPE -> BOOL
      eq-type: TECH-TYPE # TECH-TYPE -> BOOL
      eq-skeleton: TECH-TYPE # TECH-TYPE -> BOOL
    end

end

variables
  tt, t1, t2, t3, t4, t5 -> TECH-TYPE
  rs, rz, rz1, rz2 -> RANGE-SEQ
  st, s1, s2 -> SIMPLE-TYPE
  t

equations

[83] dim-no(i, matrix-type(rs)) = dim-no(rs, i)
[84] n-of-dims(matrix-type(rs)) = n-of-ranges(rs)
[85] 2-dim-matrix(matrix-type(rs))
    = eq(n-of-ranges(rs), 1)
-- the error cases of the above functions
-- referring to simple types are left unspecified
[86] eq-type(tt, tt) = true
[87] eq-type(tt, t1) = eq-type(tt, rz)
[88] eq-type(tech-type(s1, tech-type(s2)), eq(s1, s2))
[89] eq-type(tech-type(s1, matrix-type(s2))) = false
[90] eq-type(matrix-type(s1), matrix-type(s2)) = eq-ranges(s1, s2)
-- skeleton restrictions
[91] eq-skeleton(tt, tt) = true
[92] eq-skeleton(tt, t1) = eq-skeleton(t1, t1)
[93] eq-skeleton(tech-type(s1), tt) = eq-type(tech-type(s1), tt)
[94] eq-skeleton(matrix-type(rs), matrix-type(rz))
    = eq(n-of-ranges(rs), n-of-ranges(rz))

end Technique-types
module User-visibility
begin
exports
begin
sorts USER-VIEW
functions
input: -> USER-VIEW
output: -> USER-VIEW
internal: -> USER-VIEW
end
end User-visibility

APPENDIX F. ASF SPECIFICATION OF THE ABSTRACT SYNTAX OF THE
STATISTICAL LANGUAGE

The abstract syntax of the statistical language is divided over several
modules:
- a group of modules for the basic elements in the language:
  - variables, expressions, and statements.
- a group of modules for the abstract syntax of the program
  sections.
- a module for the abstract syntax for a complete program in
  the statistical language.

1. BASIC ELEMENTS.

1.1. VARIABLES

1.1.a. Global description.

A variable in the statistical language is either of type boolean, scalar,
index or a matrix. A matrix variable identifier can be followed by a
range sequence or a index expression sequence, forming respectively a
submatrix or matrix element reference.

1.1.b. Specification.

module Variable-abstr-syntax
begin
exports
begin
sorts VARIABLE
functions
abs-var: ID -> VARIABLE
abs-exp: ID # IND-EXPR-SEQ -> VARIABLE
abs-rs: ID # RANGE-SEQ -> VARIABLE
end
imports Ind-expr-sequences, Range-sequences, Identifiers, Indices
end Variable-abstr-syntax

1.2. EXPRESSIONS.

1.2.a. Global description.

An expression in the abstract syntax is either a variable, a constant,
an index expression or a function call. The arguments of a function call are
a list of expressions.

1.2.b. Specification

module Expr-abstr-syntax
begin
exports
begin
sorts EXPR, ARG-LIST
functions
abs-exp: VARIABLE -> EXPR
abs-exp: TECH-DATA -> EXPR
abs-exp: IND-EXPR -> EXPR
abs-f-call: ID # ARG-LIST -> EXPR
abs-arg-l: ARG-LIST # EXPR -> ARG-LIST
abs-arg-l: EXPR -> ARG-LIST

end

Imports Technique-data, Variable-abstr-syntax, Ind-exp-abstr-syntax, Identifiers

end Expr-abstr-syntax

1.3. STATEMENTS.

1.3.a. Global description.

Statements are used in the implementation, test and exception handler section. A statement is either an assignment statement, a For, a compound or an index assignment statement.

1.3.b. Specification.

module Statements-abstr-syntax
begin
    exports
    begin
        sorts STATEMENTS, STATEMENT
        functions
        abs-statements : STATEMENTS # STATEMENT -> STATEMENTS
        abs-statements : STATEMENT -> STATEMENTS
        abs-assgm : VARIABLE # EXPR -> STATEMENT
        abs-messstmt : STRING -> STATEMENT
        abs-for : ID # IND-EXPR # IND-EXPR # STATEMENTS
        abs-compound : STATEMENTS -> STATEMENT
        abs-ind-assgm : ID # IND-EXPR -> STATEMENT
        end
    Imports Variable-abstr-syntax, Expr-abstr-syntax, Strings, Ind-exp-abstr-syntax
end Statements-abstr-syntax

2. PROGRAM SECTIONS IN THE STATISTICAL LANGUAGE.

2.1. DECLARATIONS.

2.1.a. Global description.

Variables in a statistical program must be declared in the input/output section of the program, or in the internal declarations in the other sections. An input/output section consists of two lists of declarations as specified in function abs-to-sec. The first list contains the input declarations, the second set the output declarations. A declaration consists of a sequence of identifiers (the names of the declared variables), the type description of the declared variables and a message string.

module Decl-abstr-syntax
begin
    exports
    begin
        sorts ID-SEC, INTERN-DECLS, DECLS, DECL
        functions
        abs-to-sec: DECLS # DECLS -> ID-SEC
        abs-interm-decl: DECLS -> INTERN-DECLS
        abs-empty-decls: -> DECLS
        abs-decs: DECLS # DECL -> DECLS
        abs-decl: ID-SEQ # TECH-TYPE # STRING -> DECL
        end
    Imports Strings, Id-sequences,
    User-visibility, Technique-types
end Decl-abstr-syntax

end Impl-abstr-syntax
2.3. THE TEST SECTION.

2.3.a. Global description.

In a test section the technical statistician can describe a test and indicate under which conditions the test should interrupt the calculation process (raise an exception). A raise statement consists of an exception identifier and a boolean expression as specified in function abs.Raise.

2.3.b. Specification.

\[\text{module Test-abstract-syntax} \begin{align*}
\text{begin} & \\
\text{exports} & \\
\text{begin} & \\
\text{sorts} & \text{TEST-SEC, RAISES, RAISE} \\
\text{functions} & \\
\text{abs-test-sec: INTERN-DECLS \# STATEMENTS \# RAISES \# TEST-SEC} \\
\text{abs-raise: RAISE \# RAISE \# \# RAISE} \\
\text{abs-raise : ID \# EXPB \# RAISE} \\
\text{end} & \\
\text{imports} & \text{Statements-abstract-syntax, Expr-abstract-syntax,} \\
& \text{Identifiers, Decl-abstract-syntax} \\
end \text{Test-abstract-syntax}
\]

2.4. THE EXCEPTION HANDLER SECTION.

2.4.a. Global description.

An exception handler consists of: the exception identifier, the internal declarations, and the statements. The statements in an exception handler are only executed when the exception occurs.

2.4.b. Specification.

\[\text{module Handler-abstract-syntax} \begin{align*}
\text{begin} & \\
\text{exports} & \\
\text{begin} & \\
\text{sorts} & \text{HANDL-SEC, HANDLER} \\
\text{functions} & \\
\text{abs-handl-sec: HANDL-SEC \# HANDLER \# HANDL-SEC} \\
\text{abs-handl-sec: HANDLER \# HANDL-SEC} \\
\text{abs-handler: ID \# INTERN-DECLS \# STATEMENTS \# HANDLER} \\
\text{end} & \\
\text{imports} & \text{Identifiers, Decl-abstract-syntax,} \\
& \text{Statements-abstract-syntax} \\
end \text{Handler-abstract-syntax}
\]
APPENDIX G. ASF SPECIFICATION OF THE SYMBOL TABLES

In this appendix the symbol tables are specified. First a symbol table is specified for each statistical technique (read statistical programs). This table contains the technique description, the value and some additional information for each variable that is declared in a statistical technique. Also a symbol table is defined for the user of a statistical technique. This table contains the technique description and the value of each variable declared in a user session. In this appendix it is also specified how declaration sections in a statistical program are stored in a technique symbol table. The storage of declarations in the user languages is specified in appendix N.

1. THE SYMBOL TABLES.

1.a. Global description.

The value, the type, the message string and the visibility of the variables in a statistical program are stored in the technique symbol table. In the user symbol table the value and the type of variables declared in a user session are stored.

All information for a variable is stored in one table. Instead, separate tables could have been defined for each separate piece of information in these tables. In order to keep the number of tables low, we have chosen for the combined approach, in which both information needed for type checking and information needed for the evaluation of programs is stored in the same table.

1.b. Specification.

module Techn-symb-tables
begin
  exports
  begin
    sorts TECH-INFO
    functions
    ts-info: TECH-TYPE # USER-VIEW # TECH-DATA # STRING -> TECH-INFO
    type: TECH-INFO -> TECH-TYPE
    data: TECH-INFO -> TECH-DATA
    view: TECH-INFO -> USER-VIEW
    string: TECH-INFO -> STRING
    eq: TECH-INFO # TECH-INFO -> BOOL
  end

  imports Technique-data, Technique-types, User-visibility, Strings

  variables
  i, i, id, id

  equations
  [100] data(ts-info(t, w, td, str)) = td
  [101] type(ts-info(t, w, td, str)) = t
  [102] view(ts-info(t, w, td, str)) = w
  [103] string(ts-info(t, w, td, str)) = str
  [104] eq(ts-info(t, w, td, str), ts-info(t2, w2, td2, str2))
      = (eq-type(i, i2),
        and(eq-id(id, id2),
          eq-str(str, str2)))

  end Techn-symb-tables
end

module User-symtab-info
begin
  exports
  begin
    sorts USER-INFO
    functions
    ws-info: USER-TYPE # USER-DATA -> USER-INFO
    data: USER-INFO -> USER-DATA
    type: USER-INFO -> USER-TYPE
    eq: USER-INFO # USER-INFO -> BOOL
  end
end User-symtab-info

imports User-types, User-data, Booleans

variables
  ut, ut1, ut2 :-> USER-TYPE
  wd, wd1, wd2 :-> USER-DATA

equations
  [106] data(ws-info(ws, wd)) = wd
  [107] type(ws-info(ws, wd1)) = ut
  [108] eq(ws-info(ws, ut2), ws-info(ws, ut2)) = eq(ws-info(ws, ut2), eq(ws, wd2))
end User-symtab-info

module User-symtabs
begin
  exports
  begin
    functions
    -- this function stores types in the symbol table --
    -- during the evaluation of a user program. --
    -- insert-type: ID # USER-TYPE # USER-SYMTAB -> USER-SYMTAB
    -- this function stores values in the symbol table --
    -- during the evaluation of a user program. --
    -- insert-data: ID # USER-DATA # USER-SYMTAB -> USER-SYMTAB
  end
end User-symtabs

imports Tables
{ renamed by
  [ TABLE -> USER-SYMTAB,
    null-table -> empty-u-symtab]

Entries bound by
[ ENTRY -> USER-INFO,
  eq-entry -> eq]
to User-symtab-info
Addresses bound by
[ ADDRESS -> ID,
  eq-address -> eq-id ]
to identifiers
}

variables
  td
  ws :-> ID
  us :-> USER-SYMTAB

[109] insert-type: ID, ut, insert(ID, ws-info(ws, ut), us)
    = insert(ID, ws-info(ws, ut), us)
[110] insert-data(ID, us, insert(ID, ws-info(ws, us), us))
    = insert(ID, ws-info(ws, us), us)
end User-symtabs

2.1. STORING DECLARATIONS IN THE TECHNIQUE SYMBOL TABLE.

2.1.a. Global description.

For each variable declared in a statistical program an entry is made in
the technique symbol table. This entry contains the type information, the
visibility and the message string in the declaration. The value of the
variable is uninitialized. The constant function uninitialized is
specified in module Technique-data.

2.2.b. Specification.

module Store-declarations
begin
  exports
  begin
    functions
      store-loc-set: TECH-SYMTAB # ID-LOC -> TECH-SYMTAB
      store-loc-set: TECH-SYMTAB # ID-LOC -> TECH-SYMTAB
      store-decls: TECH-SYMTAB # DECLS # USER-VIEW -> TECH-SYMTAB
      store-decl: TECH-SYMTAB # DECL # USER-VIEW -> TECH-SYMTAB
      store-id: TECH-SYMTAB # ID # TECH-TYPE # USER-VIEW # STRING
                  -> TECH-SYMTAB
  end
end Store-declarations

imports Technique-symtabs, Decl-abstr-symtabs

variables
  te
  decls, imp-decls, out-decls
  dec1
  id1
  td
  tt
  us
  mess
    :-> TECH-SYMTAB
    :-> DECL
    :-> ID-SEQ
    :-> ID
    :-> TEC-TYPE
    :-> USER-VIEW
    :-> STRING
equations

[111] store-to-sect(ts, abs-to-sect(mp-decls, out-decls))
    = store-decls(store-decls(ts, mp-decls, input), out-decls, output)

[112] store-intern-decls(ts, abs-intern-decl(decls))
    = store-decls(ts, decls, internal)

[113] store-decls(ts, abs-empty-decls, uw)
    = ts

[114] store-decls(ts, abs-decls(decls, decl1), uw)
    = store-decls(store-decls(ts, decls, uw), decl1, uw)

[115] store-decls(ts, abs-decls(decl1, decl1), uw)
    = store-decl(ts, decl1, uw)

[116] store-decl(ts, abs-decl(add-item(id, id), tt, mess), uw)
    = store-id(store-decl(ts, abs-decl(id, tt, mess), uw),
                   id, tt, ui, mess)

[117] store-decl(ts, abs-decl(add-item(id, null-id-seq),
                       tt, mess), uw)
    = store-id(ts, id, tt, uw, mess)

[118] store-id(ts, id, tt, uw, mess)
    = insert:id(id, ts, info(tt, uw, uninitialized, mess),
                   ts)

end Store-declarations

APPENDIX H. ASF SPECIFICATION OF THE STATIC SYMBOLIC TYPE CHECKER

This appendix contains the specification of the static symbolic type checking mechanism of CONDUCTOR. It contains modules to specify:
- the dimension and skeleton restrictions on function arguments,
- the type of the result of a function,
- the type of variables and expressions,
- symbolic type checking of a statistical program.

1. SYMBOLIC TYPE CHECKING OF FUNCTION CALLS.

1.4. Global description.

The type restrictions of the function arguments are expressed in terms of skeleton and dimension restrictions. Two skeletons of a type description are equal if the type descriptions have the same simple type or, in case of a matrix type description, the number of dimensions is equal. Consider a function sum (A, B) that adds two matrices, A and B. A skeleton restriction on the function sum is the restriction that both arguments must be matrices with an equal number of dimensions. The range restrictions on the function sum are the restrictions that the ranges of the arguments must be equal.

A type list is used to store the type descriptions of the arguments of a function call. A skeleton restriction consists of either two indices, indicating the position of two arguments in the type list that must have equal skeletons, or an index and a type indicating the skeleton of a referenced argument must be equal to the skeleton of a given type. For the function sum the skeleton restriction consists of the constant range (1, 2), indicating that the skeletons of the first and second element are equal. The range restrictions are specified in a similar way, but instead of an index a constant range is used. The first element of this constant range gives the argument number, the second element the dimension number. For the function sum the range restrictions consists of two constant range sequences

\\[(1,1), (1,2), (2,1), (2,2)\\]

The result of a function in the statistical language is either a simple type or a matrix type. The ranges describing the dimension of a matrix can be retrieved from the type list, using a constant range as with range restrictions. For the function sum, the type of the result is given by the constant range sequence (1,1), (1,2) stating that the resulting type is a matrix type and that the dimensions are the first and second dimension of the first argument.

The restrictions on the arguments and the type of the result of a function are stored in the function type table. The type restrictions of the functions actually defined in the statistical language are left unspecified. The parameter Current-value-types in module Func-type-table emphasises that the design of CONDUCTOR is independent of the functions actually defined in the statistical language.
i.e. Specification.

module Type-lists
begin
  exports
    begin
      functions
        arg-range: CONST-RANGE \ TYPE-LIST \ RANGE
      end
    imports Constant-ranges, Sequences
      renamed by [seq] \ TYPE-LIST
      (seq-no \ argument)
      items bound by [item] \ TECH-TYPE
      eq-item \ eq-type
    to Technique-types
  }
  variables
    ind1, ind2 \ INDEX
    tl \ TYPE-LIST
  equations
    [119] arg-range(c-range(ind1, ind2), tl) = dim-no(ind3, argument(tl, ind1))
end Type-lists

module Skeleton-restart-sequences
begin
  exports
    begin
      functions
        check: SKELETON-RESTR-SEQ \ TYPE-LIST \ BOOL
      end
    imports Sequences
      renamed by [seq] \ SKELETON-RESTR-SEQ
      eq-seq \ eq-skeleton-restr
      items bound by [item] \ SKELETON-RESTR
      eq-item \ eq-skeleton-restr
    to Skeleton-restart-sequences
  }
  variables
    sr : \ skeleton-restart
    srs \ SKELETON-RESTR-SEQ
    tl \ TYPE-LIST
  equations
    [126] check(add-item(sr, null), tl) = check(sr, tl)
    [127] check(add-item(sr, srs), tl) = and(check(sr, tl), check(srs, tl))
end Skeleton-restart-sequences

module Dim-restrictions
begin
  exports
    begin
      functions
        check: DIM-RESTR \ TYPE-LIST \ BOOL
      end
    imports CONST-RANGE, Technique-types, Type-lists, Booleans
      renamed by [seq] \ CONST-RANGE
      eq-seq \ eq-CONST-RANGE
      dim-restr \ CONST-RANGE
      dim-restr \ RANGE
      eq-restr \ DIM-RESTR
      items bound by [item] \ DIM-RESTR
      eq-item \ eq-dim-restr
    to Dim-restrictions
    }
  variables
    x1, x2, x3 \ \ DIM-RESTR
    ind, ind1, ind2, ind3, ind4 \ INDEX
    tl, t1, t2 \ TYPE-LIST
  equations
    [120] eq-restr(x1, x2) = eq-restr(x2, x1)
    [121] eq-restr(restr(ind1, ind2), restr(ind3, ind4)) = and(eq(ind1, ind2), eq(ind3, ind4))
imports Constant-ranges, Ranges, Type-lists

variables
dr, dr1, dr2 ::= DIM-REST
cr, cr1, cr2, cr3, cr4 ::= CONST-RANGE
tl ::= TYPE-LIST
r, r1, r2 ::= RANGE

equations
[128] eq-restr(dr1, dr2) = eq-restr(dr2, dr1)
[129] eq-restr(dim-restr(cr1, cr2), dim-restr(cr3, cr4)) = and(eq(cr1, cr3), eq(cr2, cr4))
[130] eq-restr(dim-restr(cr1, cr2), dim-restr(cr3, r2)) = and(eq(cr1, cr2), eq-range(r1, r2))
[131] eq-restr(dim-restr(cr1, cr2), dim-restr(cr3, r)) = false
[132] check(dim-restr(cr1, cr2), tl) = eq-range(arg-range(cr1, tl), arg-range(cr2, tl))
[133] check(dim-restr(cr1, r), tl) = eq-range(arg-range(cr1, tl), arg-range(cr1, r))

end Dim-restrictions

module Dim-restr-sequences
begin
exports
begin
functions
check DIM-REST-SEQ # TYPE-LIST -> BOOL
end
imports Sequences

[replaced by]
SEQ
null
no-dim-restr
eq-seq

end Dim-restrictions

variables
dr ::> DIM-REST
drseq ::> DIM-REST-SEQ
tl ::> TYPE-LIST

end Dim-restr-sequences

module Result-types
begin
exports
begin
sorts RESULT-TYPE

functions
res-type: TECH-TYPE -> RESULT-TYPE
res-type: CONST-RANGE-SEQ -> RESULT-TYPE
result: RESULT-TYPE # TYPE-LIST -> TECH-TYPE
end

imports Technique-types, Const-range-sequences,
Type-lists

functions
res ranges: CONST-RANGE-SEQ # TYPE-LIST -> RANGE-SEQ

end Result-types
2. The Data Type of the Basic Elements of the Statistical Language.

2.a. Global description.

In this section the type of variables, expressions and statements is specified. The type description of a variable is found in the technique symbol table. A matrix element reference is specified to be of type scalar; a submatrix of type matrix. The number of dimensions in matrix references must be equal to the number of dimensions in the matrix declaration.

2.b. Specification.

module Stat-check-variables
begin
exports
begin
functions
var-type: VARIABLE # TECH-TYPETAB -> TECH-TYPE
end
end
end

variables
id  --> ID
rt  --> TYPE-LIST

equations
[142] get-n-arg(id) = n-arg(id)
[143] check-f-restr(id,rt) = check(f:info(id,rt,rt),rt)
[144] result-f(id,rt) = result(id,rt,rt)

end

end
end
module Stat-check-expressions
begin
exports
begin
functions
expr-type: EXPR # TECH-SYNTAX -> TECH-TYPE
arg-types: ARG-LIST # TYPE-LIST & TECH-SYNTAX -> TYPE-LIST
end

imports
Expr-abstr-syntax, Type-lists,
Func-type-tables, Stat-check-variables

variables
expr
  ::= EXPR
  ::= TYPE-LIST
stmt
  ::= TECH-SYNTAX
var
  ::= VARIABLE
argc
  ::= SCALAR
ind
  ::= INDEX
b
  ::= BOOL
f-id
  ::= ID
argl
  ::= ARG-LIST

equations
  -- simple expressions --
[149] expr-type(aba-expr(var),stmt) = var-type(var,stmt)
[150] expr-type(aba-expr(f-data(var)),stmt) = tech-type(scalar-type)
[151] expr-type(aba-expr(t-data(ind)),stmt) = tech-type(scalar-type)
[152] expr-type(aba-expr(f-data(b)),stmt) = tech-type(boolean-type)

  -- function calls --
[153] expr-type(aba-f-call(f-id,argl),stmt) = result-f(f-id, arg-type(argl, null, stmt))

  when check-f-restr(f-id, arg-type(argl, null, stmt)) = true

  -- the error case in the above equation is not specified --
[154] arg-type(arg-arg-l(argl, exp),stmt)
  = add-item(expr-type(expr, stmt), arg-type(argl, stmt, stmt))
[155] arg-type(arg-arg-l(expr),stmt)
  = add-item(expr-type(expr, stmt), stmt)
end

Stat-check-expressions

module Stat-check-statements
begin
exports
begin
functions
  type-check-stmt: STATEMENTS # TECH-SYNTAX -> BOOL
end

imports
Statements-abstr-syntax, Stat-check-expressions

variables
var
  ::= VARIABLE
expr
  ::= EXPR
stmt
  ::= TECH-SYNTAX
id
  ::= ID
l1, l2
  ::= IND-EXPR
s
  ::= STRING
stmts
  ::= STATEMENTS
stmt
  ::= STATEMENT

equations
[156] type-check-stmt(aba-assign(var, exp), stmt)
  = eq-type(var-type(var, stmt), expr-type(expr, stmt))
[157] type-check-stmt(aba-for(id, l1, l2, stmts), stmt)
  = type-check-stmt(stmts, stmt)
[158] type-check-stmt(aba-mosstmt(m), stmt)
  = true
[159] type-check-stmt(aba-uncount(stmts), stmt)
  = type-check-stmt(stmts, stmt)
[160] type-check-stmt(aba-ind-assign(id, l1), stmt)
  = true
end

Stat-check-statements

3. STATIC TYPE CHECKING OF SECTIONS OF A STATISTICAL PROGRAM.


The static type checking of the sections gives an updated technique
symbol table. The internal declarations in each section are stored in the
technique symbol table, and the type restrictions in the statements are
checked.

3.1.1. Specification

module Stat-check-impl
begin
exports
begin
functions
  type-check-impl->: IMPL-SEC # TECH-SYNTAX -> TECH-SYNTAX
end

imports
Stat-check-statements, Store-declarations,
Impl-abstr-syntax
variables
dcl  ::= INTERN-DECLS
stmts ::= STATEMENTS
tst  ::= TECH-SYNTAX

equations
[161] type-check-impl-s(abse-impl-sec(dcl,stmts),tst)
   = store-interm-decls(tst,dcl)
   when type-check-stmts(stmts,store-interm-decls(tst,dcl)) = true

-- the error case in the above equation is not specified --
end Stat-check-impl

module Stat-check-tests
begin
exports begin
  functions
    type-check-test-s: TEST-SEC # TECH-SYNTAX -> TECH-SYNTAX end
end
imports Stat-check-statements, Store-declarations, Test-abstr-syntax

variables
dcl  ::= INTERN-DECLS
stmts ::= STATEMENTS
rexs ::= RAISES
tst  ::= TECH-SYNTAX

equations
[162] type-check-test-s(abse-test-sec(dcl,stmts,rexs),tst)
   = store-interm-decls(tst,dcl)
   when type-check-stmts(stmts,store-interm-decls(tst,dcl)) = true

-- the error case in the above equation is not specified --
end Stat-check-tests

module Stat-check-handlers
begin
exports begin
  functions
    type-check-handl-s: HANDL-SEC # TECH-SYNTAX -> TECH-SYNTAX
    type-check-handler: HANDLER # TECH-SYNTAX -> TECH-SYNTAX
end
imports Stat-check-statements, Store-declarations, Handler-abstr-syntax

variables
  h-sec ::= HANDL-SEC
dcl  ::= INTERN-DECLS
stmts ::= STATEMENTS
rexs ::= RAISES
tst  ::= TECH-SYNTAX

id  ::= ID

equations
[163] type-check-handl-s(abse-handl-sec(h-sec,dcl),tst)
   = type-check-handl-s(abse-handl-sec(h-sec),
     type-check-handl-s(h-sec,tst))
[164] type-check-handl-s(abse-handl-sec(dcl),tst)
   = type-check-handl-s(dcl,tst)
[165] type-check-handler(abse-handler(id,dcl,stmts),tst)
   = store-interm-decls(tst,dcl)
   when type-check-stmts(stmts,store-interm-decls(tst,dcl)) = true

-- the error case in the above equation is not specified --
end Stat-check-handlers

module Static-type-checking
begin
exports begin
  functions
    type-check-set: SECTION # TECH-SYNTAX -> TECH-SYNTAX
    type-check-pro: STAT-PRO -> TECH-SYNTAX
end
imports Statistical-programs, Store-declarations, Stat-check-handlers, Stat-check-tests, Stat-check-impl

variables
  id ::= ID
tst ::= TECH-SYNTAX
  pro ::= STAT-PRO

impl-sec ::= IMPL-SEC
test-sec ::= TEST-SEC
handl-sec ::= HANDL-SEC

end
APPENDIX I. ASF SPECIFICATION OF THE GENERATION OF THE INPUT RESTRICTIONS

This appendix contains the formal specification of the input restriction generation mechanism in CONDUCTOR. Input restrictions are generated in order to guarantee that no type or dimension bound conflicts will occur during the execution of a statistical technique. Modules are given to specify: (1) input restrictions, (2) the calculation of a symbolic range for an index expression, (3) the assignment of a symbolic range to index variables, (4) the generation of input restrictions for variables, expressions and statements, (5) the generation of input restrictions for sections of a statistical program, and (6) the generation of input restrictions for a statistical program.

1. INPUT RESTRICTIONS.

1.a. Global description.

An input restriction is defined as a restriction that an index expression is greater or equal zero. The index expressions in the input restrictions may only consist of input variables and constants. This is specified in module Input-var-restrictions.

In module Ind-expr-order the order relation on index expressions is specified. If an index expression always yields a greater (equal or smaller) value than another index expression, for all possible values of the variables in the expressions, the order relation is greater (equal or smaller). For pairs of index expressions where this problem is undecidable, the relation is undecidable.

In module Input-restr-sequence it is specified how an input restriction is added to a sequence of these restrictions, if the order relation of two index expressions is undecided. In module Range-restrictions the following restrictions on ranges are defined:

- positive restrictions, the upper bound of a dimension range must be larger than the lower bound, and
- subrange restriction, the upper bound of the subrange must be smaller than the upper bound of the range, and the lower bound greater than the lower bound of the range.

1.b. Specification.

module Input-var-restrictions begin
exports
begin
functions
are-input-vars: IND-EXPR # TECH-SYMTAB --> BOOL
end

(imports Ind-expr-abstract-syntax, Techn-syntabs, User-visibility

variables
\{i.e, i, e\} --> IND-EXPR
ts --> TECH-SYMTAB
c --> INCSI
id --> ID
equations

[174] are-input-vars(abs-plus(ta1,ta2),ta)  
    = and_are-input-vars(ta1,ta2,are-input-vars(ta1,ta))

[175] are-input-vars(abs-minus(ta1,ta2),ta)  
    = and_are-input-vars(ta1,ta2,are-input-vars(ta1,ta))

[176] are-input-vars(abs-mul(ta1,ta2),ta)  
    = and_are-input-vars(ta1,ta2,are-input-vars(ta1,ta))

[177] are-input-vars(abs-ind(ta),ta)  
    = (if(eq(typeof(ta),true),input,true,false))

[178] are-input-vars(abs-const(ta),ta)  
    = true

end Input-var-restrictions

module Input-restrictions
begin
  exports
  begin
    sorts INF-RESTR
    functions
    pos-restr: INF-EXPR  
               --> INF-RESTR
    (op-restr: INF-EXPR # TECH-SYNTAX  
                --> INF-RESTR
    eq-op-restr: INF-RESTR # INF-RESTR  
                        --> BOOL
  end

imports Ind-expr-abstr-syntax, Input-var-restrictions

functions

  error-op-restr:  
                  --> INF-RESTR

variables

  ta1, ta2  
            :--> INF-EXPR

  ts  
            :--> TECH-SYNTAX

end Input-restrictions

module Order
begin
  exports
  begin
    sorts ORDER
    functions
    greater:  
              --> ORDER
    less:  
              --> ORDER
    equal:  
              --> ORDER
    undecided:  
                           --> ORDER
    eq:  
        ORDER # ORDER  
                  --> BOOL
  end

imports Booleans

variables

0,0,0  
        :--> ORDER

end Order

module Ind-expr-order
begin
  exports
  begin
    functions
    order: INF-EXPR # INF-EXPR  
             --> ORDER
  end

imports Order, Ind-expr-abstr-syntax

variables

  ta1, ta2  
            :--> INF-EXPR

  c  
            :--> INDEX

end Ind-expr-order

module Ind-expr-order
begin
  exports
  begin
    functions
    order: INF-EXPR # INF-EXPR  
             --> ORDER
  end

end Ind-expr-order
module Input-restr-sequences
begin
exports
begin
functions
add-inp-restr: IND-EXPR # IND-EXPR #
  TECH-SYMTAB # INP-RESTR-SEQ -> INP-RESTR-SEQ
end
imports Ind-expr-order, Techn-symtabs,
Sequences
(functioned by
[ SEQ  -> INP-RESTR-SEQ,
  eq-seq  -> eq-inp-restr-seq,
  null  -> no-restrictions ]
items bound by
[ ITEM  -> INP-RESTR,
  eq-item  -> eq-inp-restr ]
to Input-restr-sequences
functions
error-inp-restr-seq:  -> INP-RESTR-SEQ
variables
ts  ::=  TECH-SYMTAB
  r,
end Imports

variables
rs  ::=  TECH-SYMTAB
  r,
end Imports

2. RANGE CALCULATION FOR INDEX EXPRESSIONS.

2. Global description.

For each index variable a range is stored in the range table.
These ranges can be used to calculate symbolic ranges for index
expressions. Ranges can only be calculated for symbolic index expressions
if the index expressions are monotone increasing or decreasing, as
specified in module NonMon-restrictions. The specification of monotony
amounts to the construction of two lists of variables. One list contains
the variables in the negative terms of the expression, the other the
variables in the positive terms. If these two list are disjoint the index
expression is monotone. The calculation of ranges for index expressions
leads to input restrictions as specified in module Range-calc-restr.
These input restrictions assure that all ranges involved in the calculations are positive.

2.2. Specification.
module Monotone-restrictions
begin
exports
begin
functions
(line-monotone: IND-EXPR --> BOOL
var-list: BOOL # BOOL # IND-EXPR # ID-SEQ --> ID-SEQ
end
imports Ind-expr-abstract-syntax, Id-sequences,
Booleans
[renamed by [true --> pos, false --> neg]
variables
i, i1, i2
id
agm1, agm2

equations
[199] var-list(agm1, agm2, abs-plus(i1, i2), id)
   = var-list(agm1, agm2, i1,
              var-list(agm1, agm2, i2, id))
[200] var-list(agm1, agm2, abs-mul(i1, i2), id)
   = var-list(agm1, agm2, i1,
              var-list(agm1, agm2, i2, id))
[201] var-list(agm1, agm2, abs-minus(i1, i2), id)
   = var-list(agm1, agm2, i1,
              var-list(agm1, agm2, i2, id))
[202] var-list(agm1, agm2, abs-ind(i), id)
   = if(eq(agm1, agm2), add-item(id, id, id), id)
[203] var-list(agm1, agm2, abs-const(0), (id))
   = id
   when ge(0, i) = true
   -- the error case is left unspecified --
   -- monotone restriction --
[204] line-monotone(i)
   = disjoint(var-list(pos, pos, i, null-id-seq),
              var-list(neg, pos, i, null-id-seq))
and Monotone-restrictions
module Range-tables
begin
imports Tables
{ renamed by [TABLE --> RANGE-TABLE]
Entries bound by
[ENTRY --> RANGE,
   eq-entry --> eq-range]
to Ranges
Addresses bound by
[ADDRESS --> ID,
   eq-addr --> eq-id]
to Identifiers
}
end
Range-tables
module Range-calculations
begin
exports
functions
(calc-range: IND-EXPR # RANGE-TABLE --> RANGE
calc-ranges: IND-EXPR-SEQ # RANGE-TABLE --> RANGE-SEQ
range-calc-range: RANGE # RANGE-TABLE --> RANGE
range-calc-ranges: RANGE-SEQ # RANGE-TABLE --> RANGE-SEQ
end
imports Ind-expr-sequences, Range-sequences,
Range-tables, Monotone-restrictions
variables
i, i1, i2
id
agm1, agm2

equations
[205] calc-range(abs-plus(i1, i2), id)
   = range-plus(calc-range(i1, id),
                calc-range(i2, id))
   when line-monotone(abs-plus(i1, i2)) = true
   -- the error case is left unspecified --
[206] calc-range(abs-minus(i1, i2), id)
   = range-min(calc-range(i1, id),
                calc-range(i2, id))
   when line-monotone(abs-minus(i1, i2)) = true
   -- the error case is left unspecified --
[207] calc-range(abc-mul(la,la,rt)
   = range-mul(calc-range(la,rt),
   calc-range(la,rt))
when (a-monotone(abc-mul(la,la)) = true
   -- the error case is left unspecified --
[208] calc-range(abc-ind(id,rt) = id'rt
[209] calc-range(abc-const(c,rt)
   = range(calc-range(abc-const(c),
   abc-const(c)))
-- calculations of ranges for a sequence of index
expressions --
[210] calc-range(add-item(le,le,rt)
   = add-item(calc-range(le,rt),
   calc-range(le,le))
[211] calc-range(add-item(a,mul-ind-expr-seq,rt))
   = add-item(calc-range(a,rt),
   mul-range)
-- calculation of a range of a range --
[212] range-calc-range(range(la,la,rt)
   = range(calc-range(la,la),
   range-lower(calc-range(la,la)),
   range-upper(calc-range(la,la)))
when (a-monotone(range-plus(la,la)) = true
   -- the error case is left unspecified --
-- calculations of ranges for a sequence of ranges --
[213] range-calc-range(add-item(r,rt,rt)
   = add-item(range-calc-range(r,rt),
   range-calc-range(rt))
[214] range-calc-range(add-item(r,mul-range,rt)
   = add-item(range-calc-range(r,rt),
   mul-range)
end Range-calc-range

module Range-calc-restr
begin
exports
begin
Functions
 restr-calc-range: IND-EXPR # RANGE-TABLE #
 TECH-SYNTAB # ALPHA-EXPR-SEQ => INP-RESTR-SEQ
 restr-calc-ranges: IND-EXPR-SEQ # RANGE-TABLE #
 TECH-SYNTAB # ALPHA-EXPR-SEQ => INP-RESTR-SEQ
 restr-range-calc-range: RANGE # RANGE-TABLE #
 TECH-SYNTAB # ALPHA-EXPR-SEQ => INP-RESTR-SEQ
end

imports Ind-expr-sequences, Input-restr-sequences,
Range-tables, Range-calculations, Range-restrictions

variables
le,le,le
   =: IND-EXPR
le
   =: IND-EXPR-SEQ
id
   =: ID
k
   =: INDEX
tr
   =: INP-RESTR-SEQ
rt
   =: RANGE-TABLE
r
   =: RANGE
rs
   =: RANGE-SEQ
ts
   =: TECH-SYNTAB

equations
[215] restr-calc-range(range-plus(abc-plaintext(la,la),rt,rs,rs)
   = restr-calc-range(la,rt,rs,rs)
   restr-calc-range(la,rt,rs,rs)
[216] restr-calc-range(range-mul(la,la,rt,rs,rs)
   = range(lower(calc-range(la,la,rt)),
   upper(calc-range(la,la,rt)))
when (a-monotone(range-plus(la,la,rt)) = true
   -- the error case is left unspecified --
-- calculations of restrictions for a sequence of
expressions --
[217] restr-calc-range(add-item(la,rt,rs)
   = restr-calc-range(la,rt,rs)
   restr-calc-range(la,rt,rs)
[218] restr-calc-range(add-item(id,rt,rs)
   = restr-calc-range(id,rt,rs)
[219] restr-calc-range(add-item(c,rt,rs)
   = restr-calc-range(c,rt,rs)
   restr-calc-range(c,rt,rs)
-- calculation of a range of a range --
[220] restr-calc-range(add-item(a,mul-ind-expr-seq,rt,rs)
   = restr-calc-range(a,rt,rs,rs)
   restr-calc-range(a,rt,rs,rs)
[221] restr-calc-range(add-item(a,mul-ind-expr-seq,rt,rs)
   = restr-calc-range(a,rt,rs,rs)
   restr-calc-range(a,rt,rs,rs)
-- calculation of a range of a range --
[222] restr-calc-range(range-plus(la,la,rt,rs,rs)
   = restr-calc-range(la,rt,rs,rs)
   restr-calc-range(la,rt,rs,rs)}
3. RANGE ASSIGNMENTS FOR INDEX VARIABLES.


Symbolic ranges are assigned to index variables in index assignment statements for statements and index input declarations. The specification is given in module Range-assignments. The range assignments are stored in the range table specified in module Range-tables.

3.2. Specification.

module Range-assignments
begin
exports
begin
functions
assign-range: STATEMENTS -> RANGE-TABLE
assign-range: STATEMENT -> RANGE-TABLE
assign-range: ID -> RANGE-TABLE
assign-range: ID-SEQ -> RANGE-TABLE
end

imports Statements-abstr-syntax, Range-tables,
Range-calculations, Id-sequences

variables
stmts
stmt -> STATEMENTS
var -> VARIABLE
expr -> EXP
str -> STRING
id id, id1, id2 -> IND-EXP
rt -> RANGE-TABLE

[225] assign-range(abc-stmt(stmts, stmt), rt)
  = assign-range(stmt, assign-range(stmts, stmt))

[226] assign-range(abc-stmt(stmts, stmt), rt)
  = assign-range(stmt, assign-range(stmts, stmt))

-- assignment for individual statements --

[227] assign-range(abc-assign(var, expr), rt) = rt

end

4. GENERATION OF INPUT RESTRICTIONS FOR THE BASIC ELEMENTS OF THE STATISTICAL PROGRAM.


In this section it is shown how input restrictions are generated for variables, expressions and statements in the statistical language. The input restrictions generated for matrix element references and consist of two groups of restrictions:
- the restrictions generated during the calculation of symbolic ranges for index assignments used as matrix elements in automatic references;
- the dimension bound restrictions, checked with the use of the calculated symbolic ranges.

4.2. Specification.

module Gen-restr-variables
begin
exports
begin
functions
gen-restr-arr: INV-RESTR-SEQ -> RANGE-TABLE
Tech-syntab -> INV-RESTR-SEQ
end

imports Variable-abstr-syntax, Input-restr-sequences,
Range-tables, Tech-syntabs, Range-restrictions,
Range-calculations, Range-cal-restr

variables
arrs, id -> INV-RESTR-SEQ
id -> ID
inds, index, index -> IND-EXP
ranges, range1, range2 -> RANGE-TABLE
tech -> TECH-SYNTAB
equations
-- simple variables --
[231] gen-restr-var((rs, rt, ts, abs-var((id)))) = (rs
-- matrix element --
[234] gen-restr-var((rs, rt, ts, abs-var((id, inda)) = conc restr-range-col-range((inda, rt, ts, rs),
range-within-range(col-range((inda, rt, rs),
rngs, ts)))
when eq-type(type(id'ts), matrix-type(rngs)) = true
-- submatrix --
[235] gen-restr-var((rs, rt, ts, abs-var((id, rngs)))
= cons restr-range-col-range((rngs, rt, ts, rs),
range-within-range(col-range((rngs, rt, ts),
rngs2, ts)))
when eq-type(type(id'ts), matrix-type(rngs2)) = true
end Gen-restr-variables

end Gen-restr-expressions

module Gen-restr-expressions
begin
exports
begin
functions
[238] gen-restr-expr((rs, rt, ts, abs-expr(f-id, expr)))
= gen-restr-argl((rs, rt, ts, expr))
[239] gen-restr-argl((rs, rt, ts, abs-argl((argl, expr))
= gen-restr-expr((rs, rt, ts, argl, rt, ts, expr))
[240] gen-restr-argl((rs, rt, ts, abs-argl((expr)))
= gen-restr-expr((rs, rt, ts, expr))
end Gen-restr-variables

end Input-restr-info

module Gen-restr-statements
begin
exports
begin
functions
[241] gen-restr-stms((Expr-stms, expr-assignments))
= gen-stmt((Expr-stms, expr-assignments))
end

end Gen-restr-expressions

module Gen-restr-expressions
begin
exports
begin
functions
[242] gen-restr-expr((rs, rt, ts, abs-expr))
= gen-restr-var((rs, rt, ts, abs-var))
[243] gen-restr-expr((rs, rt, ts, abs-expr))
= (rs
end

end Gen-restr-statements

end Input-restr-info

import Range-tables, Input-restr-sequences

end Input-restr-info
5.b. Specification.

module Gen-restr-declarations begin
  exports begin
    imports Input-restr-info, Decl-abstr-syntax, Range-restrictions, Range-assignments
    variables
      ts        --> TECH-SYNTAX
      lstr      --> GEN-RESTR-INFO
      id        --> RANGE-TABLE
      iri        --> TECH-SYNTAX
      decls,inp-decls,out-decls --> DECL
      idl       --> ID-SEQ
      range     --> RANGE-SEQ
      mass       --> STRING
      at         --> SIMPLE-TYPE
      uv         --> USER-VIEW

    functions
      gen-restr-io-sect (iri.ts, abs-io-sect(inp-decls,out-decls))
        = gen-restr-decls (iri.ts,iri.decls.input, ts,out-decls.output)
      gen-restr-int-decls (iri.ts,abs-intern-decl (decls))
        = gen-restr-decls (iri.ts,decls,decls.output)
      gen-restr-decls (iri, decs, decls, uv)
        = gen-restr-decls (iri, decls, decls, uv)

  end
end

end

5. GENERATION OF INPUT RESTRICTIONS FOR SECTIONS OF A STATISTICAL PROGRAM.
5.a. Global description.

In this section it is shown how restrictions are generated for the sections in a statistical program.
[253] gen-restr-decl(inp-res-info(irs,rt),ts,
    abs-decl(id1,matrix-type(rms),mess),uv)
  = inp-res-info(conc(irs.pos-range-restr(rms,ts)),
                  rt)

[254] gen-restr-decl(inp-res-info(irs,rt),ts,
    abs-decl(id1,tech-type(index-type),mess),
    input)
  = inp-res-info(irs.assign-rms(id1,rt))

end Gen-restr-declarations

module Gen-restr-impl
begin
exports
begin
Functions
gen-restr-impl-s: IMPL-SEC # TECH-SYNTAX #
                IMP-RESTR-INFO -> IMP-RESTR-INFO
end

imports Gen-restr-statements, Gen-restr-declarations,
      impl-abstr-syntax

variables
decl   ::= INTERN-DECLS
stmts  ::= STATEMENTS
iri    ::= IMP-RESTR-INFO
ts     ::= TECH-SYNTAX

equations
[255] gen-restr-impl-s(abs-impl-sec(stmts),ts,iri)
  = gen-restr-stats(
    gen-restr-int-decls(iri,ts,decl),ts,stmts)

end Gen-restr-impl

module Gen-restr-tests
begin
exports
begin
Functions
gen-restr-test-s: TEST-SEC # TECH-SYNTAX #
                IMP-RESTR-INFO -> IMP-RESTR-INFO
end

imports Gen-restr-statements, Gen-restr-declarations,
       Test-abstr-syntax

variables
decl   ::= INTERN-DECLS
stmts  ::= STATEMENTS
rss    ::= RAISES

iri    ::= IMP-RESTR-INFO
ts     ::= TECH-SYNTAX

equations
[256] gen-restr-test-s(abs-test-sec(stmts),ts,iri)
  = gen-restr-state(
    gen-restr-int-decls(iri,ts,decl),ts,stmts)
end Gen-restr-tests

module Gen-restr-handlers
begin
exports
begin
Functions
gen-restr-handl-s: HANDL-SEC # TECH-SYNTAX #
                IMP-RESTR-INFO -> IMP-RESTR-INFO

  gen-restr-handler: HANDLER # TECH-SYNTAX #
                IMP-RESTR-INFO -> IMP-RESTR-INFO
end

imports Gen-restr-statements, Gen-restr-declarations,
       Handler-abstr-syntax

variables
h-sec   ::= HANDL-SEC
hndl   ::= HANDLER
decl   ::= INTERN-DECLS
stmts  ::= STATEMENTS
iri    ::= IMP-RESTR-INFO
id     ::= ID
ts     ::= TECH-SYNTAX

equations
[257] gen-restr-handl-s(abs-handl-sec(h-sec,hndl),ts,iri)
  = gen-restr-handl-s(abs-handl-sec(hndl,ts),
    gen-restr-handl-s(h-sec,ts,iri))

[258] gen-restr-handl-s(abs-handl-sec(hndl),ts,iri)
  = gen-restr-handler(hndl,ts,iri)

[259] gen-restr-handler(abs-handler(id,stmts),ts,iri)
  = gen-restr-stats(
    gen-restr-int-decls(iri,ts,decl),ts,stmts)
end Gen-restr-handlers
6. THE GENERATION OF INPUT RESTRICTIONS FOR A STATISTICAL PROGRAM.


In this section it is specified how restrictions are generated for a statistical program.

6.2. Specification.

module Input-restr-generator
begin
  exports
    begin
      functions
        gen-restr-set: SECTION # TECH-SYNTAB #
                           INP-RESTR-INFO # INP-RESTR-SEQ -> INP-RESTR-INFO
        gen-restr-pro: STAT-PRO # TECH-SYNTAB
                           -> INP-RESTR-INFO
      end

  imports Statistical-programs, Gen-restr-declarations,
        Gen-restr-handlers, Gen-restr-tests, Gen-restr-impl

  variables
    id   ::= ID
    if   ::= INP-RESTR-INFO
    to-sec ::= ID-SEC
    impl-sec ::= INP-SEC
    test-sec ::= TEST-SEC
    handl-sec ::= HANDL-SEC
    sect ::= SECTION
    pro ::= STAT-PRO
    ts ::= TECH-SYNTAB

  equations
    [260] gen-restr-set(abs-name(id),ts,tri)
            = tri
    [261] gen-restr-set(abs-sec(to-sec),ts,tri)
            = gen-restr-to-sec(tri,ts,to-sec)
    [262] gen-restr-set(abs-impl-sec,ts,tri)
            = gen-restr-impl-sec(to-impl-sec,tri)
    [263] gen-restr-set(abs-sect(to-sect),ts,tri)
            = gen-restr-sect(to-sect,ts,tri)
    [264] gen-restr-set(abs-handl-sec,ts,tri)
            = gen-restr-handl-sec(to-handl-sec,ts,tri)
    [265] gen-restr-pro(abs-prog(pro,sect),ts)
            = gen-restr-act(sect,ts,gen-restr-pro(pro,ts))
    [266] gen-restr-pro(abs-prog(sect),ts)
            = gen-restr-set(sect,ts,gen-restr-pro(pro,ts))
            (inp-restr-info/no-restrictions,mult-table))
  end
end Input-restr-generator

APPENDIX J. ASF SPECIFICATION OF THE KERNEL

The kernel is a virtual machine that can execute the semantic actions that take place during the execution of a statistical technique. A statistical technique is represented at the kernel level by a kernel program. The specification of the kernel consists of the specification of (1) the evaluation of index expressions, data types and input restrictions, (2) the data memory, (3) the instructions set, (4) the data stack, (5) exception handler tables, (6) the function code table, (7) a kernel program, (8) the processor: execution of the instructions, (9) the kernel.

1. EVALUATION OF INDEX EXPRESSIONS, TYPES AND RESTRICTIONS.

1.1. Global description.

The values of the variables in an index expression are stored in the technique symbol table. The kernel uses these values to evaluate the index expression. The evaluation of the two index expressions in this range, and results in a constant range.

The evaluation of a matrix type description in the statistical language requires the evaluation of the symbolic ranges describing the dimension ranges. The evaluation of a matrix type description results in a matrix description with fixed dimensions, as used at the user level.

The evaluation of an input restrictions results in the value true or false.

1.2. Specification.

module Evaluate-inf-exp
begin
  exports
    begin
      functions
        evaluate: IND-EXPR # TECH-SYNTAB -> INDEX
      end

  imports Ind-expr-abstract-synt, Techn-symbols, Indices

  variables
    (exprexpr1,exprexpr2) ::= IND-EXPR
    ts ::= TECH-SYNTAB
    id ::= ID
    [267] evaluate(abs-mult(exprexpr1,exprexpr2),ts)
        = mul(evaluate(exprexpr1,ts),evaluate(exprexpr2,ts))
    [268] evaluate(abs-plus(exprexpr1,exprexpr2),ts)
        = add(evaluate(exprexpr1,ts),evaluate(exprexpr2,ts))

  end
end Evaluate-inf-exp
evaluate(abe-minus( (exper1,expr2),ts) + sub(evaluate(expr1,ts),evaluate(expr2,ts))

[270] evaluate(abe-const(i),ts) - i

[271] evaluate(abe-ind(id), (mateq(id, ts-infin(st,um,r-data(i),str),ts)) - i

-- the error case is not specified --

end Evaluate-ind-exp

module Evaluate-ranges
begin
exports
begin
functions
evaluate: RANG, # TECH-SYMTAB --> CONST-RANGE
evaluate: RANG-SEQ, # TECH-SYMTAB --> CONST-RANGE-SEQ
end

imports Range-sequences, Const-range-sequences,
Evaluate-ind-exp

variables
expr1,expr2 :-> IND-EXPR
i :-> TECH-SYMTAB
r :-> RANG
rs :-> RANGE-SEQ

equations
[272] evaluate(rangel( (exper1,expr2),ts) + c-range(evaluate(expr1,ts),evaluate(expr2,ts))
[273] evaluate(add-item(r,rs),ts) + add-item(evaluate(r,ts),evaluate(rs,ts))
[274] evaluate(add-item(r,mt-range),ts) + add-item(evaluate(r,ts),mt-range)

end Evaluate-ranges

module Evaluate-types
begin
exports
begin
functions
evaluate: TECH-TYPE # TECH-SYMTAB --> USER-TYPE
end

imports User-types, Technique-types,
Evaluate-ranges

variables
st :-> SIMPL-TYPE
rangs :-> RANGE-SEQ
ts :-> TECH-SYMTAB

equations
[275] evaluate(tech-type(st),ts) = user-type(st)
[276] evaluate(matrix-type(rangs),ts) = matrix-type(evaluate(rangs,ts))

end Evaluate-types

module Evaluate-restrictions
begin
exports
begin
functions
check: INP-RESTR # TECH-SYMTAB --> BOOL
check: INP-RESTR-SEQ # TECH-SYMTAB --> BOOL
end

imports Evaluate-ind-exp, Input-restr-sequences

variables
ter :-> IND-EXPR
te :-> TECH-SYMTAB
ir :-> INP-RESTR
ires :-> INP-RESTR-SEQ

equations
[277] check(inp-restr(te,ts),ts) = ge(evaluate(te,ts),0)
[278] check(no-restrictions,ts) = true
[279] check(add-item(ir,ires),ts) = and(check(te,ts),check(ires,ts))

end Evaluate-restrictions

2. THE DATA MEMORY OF THE KERNEL.

2.a. Global description.

The memory of the kernel consists of two symbol tables: a technique and a user symbol table. The kernel can transfer data between the technique symbol table and a data stack. How data is retrieved from the technique symbol table is specified in the functions get-data, get-elem (for matrix elements) and get-sum (for submatrices). The data on the data stack are stored in the technique symbol table in specified by the functions store-data, store-elem (for matrix elements) and store-sum (for submatrices).

The input variables must be initialized by the user of the statistical technique. At the kernel level this is represented by a transfer from in the user symbol table to the technique symbol table. A special case is the situation where series form the columns of a matrix. This is specified in module Series-matrix-interface. No equations are given in this module. The resulting statistics calculated by a statistical
technique are stored in the user symbol table, as specified in module
User-to.

2.b. Specification.

module Memory
begin
  exports
  begin
    sorts MEMORY
    functions
    memory: TECH-SYMB * USER-SYMB --> MEMORY
    get-data: ID * MEMORY --> TECH-DATA
    get-elem: ID * INDEX-SEQ * MEMORY --> TECH-DATA
    get-subseq: ID * CONST-RANGE-SEQ * MEMORY --> TECH-DATA
    store-data: ID * TECH-DATA * MEMORY --> MEMORY
    store-elem: ID * TECH-DATA * INDEX-SEQ * MEMORY --> MEMORY
    store-subseq: ID * TECH-DATA * CONST-RANGE-SEQ * MEMORY --> MEMORY
  end
end Memory

module Series-matrix-interface
begin
  exports
  begin
    functions
    data-matrix: ID-SEQ * USER-SYMB --> MATRIX
    load-mat-epoch: ID-SEQ * USER-SYMB --> BOOL
    mat-excep-id: ID-SEQ * USER-SYMB --> ID
  end
end Series-matrix-interface

module User-to
begin
  exports
  begin
    functions
    user-insert: ID * MEMORY --> MEMORY
    user-load-epoch: ID * MEMORY --> BOOL
    load-excep-id: ID * MEMORY --> ID
    store-result: ID * MEMORY --> MEMORY
  end
end User-to

variables
id --> ID
t --> TECH-SYMB
us --> USER-SYMB
t --> INDEX-SEQ
td --> TECH-DATA
drs --> CONST-RANGE-SEQ

equations
-- data retrieval --
[260] get-data(id, memory(ts, us)) = data(id, ts)
[261] get-elem(id, s, memory(ts, us)) = element(s, data(id, ts))
[262] get-subseq(id, s, memory(ts, us)) = subseq(s, data(id, ts))

-- data storage --
[263] store-data(id, td, memory(ts, us)) = memory(insert-data(id, td, ts, us))

end Memory

module Series-matrix-interface
begin
  exports
  begin
    functions
    data-matrix: ID-SEQ * USER-SYMB --> MATRIX
    load-mat-epoch: ID-SEQ * USER-SYMB --> BOOL
    mat-excep-id: ID-SEQ * USER-SYMB --> ID
  end
end Series-matrix-interface

module User-to
begin
  exports
  begin
    functions
    user-insert: ID * MEMORY --> MEMORY
    user-load-epoch: ID * MEMORY --> BOOL
    load-excep-id: ID * MEMORY --> ID
    store-result: ID * MEMORY --> MEMORY
  end
end User-to

variables
id --> ID
us --> USER-SYMB
t --> TECH-SYMB
rng1, rng2 --> RANGE

equations
-- initialising input variables --
[264] user-insert(id, memory(ts, us))
  = memory(insert-data(id, us-data(user-input-scalar),
    ts),
    us)

  when eq-type(type:id, tech-type(scalar-type)) = true
3. THE KERNEL INSTRUCTIONS.


The kernel has instructions to:
- transfer data between the user and statistical level,
- pop data from and push data on the data stack,
- evaluate index expressions and check index expression conditions,
- call functions that operate on the data stack,
- alter the sequential control of kernel,
- control the exception handling,
- display a message,
- stop the execution of the kernel.

The abstract syntax of these instructions is specified in module Instructions. The semantics of the instructions can be found in module Processor in section 8 of this appendix. An address of an instruction is specified as an instruction with an index indicating the position in the sequence.

3.2. Specification.

module Instructions
begin
exports
begin
sorts INSTR
functions
-- interface with user level --
user-load: ID -> INSTR
user-store: ID -> INSTR

-- load instructions --
load: TECH-DATA -> INSTR
load: ID -> INSTR
load-elem: ID # INDEX -> INSTR
load-subm: ID # INDEX -> INSTR

-- store instructions --
store: ID -> INSTR
store-elem: ID # INDEX -> INSTR
store-subm: ID # INDEX -> INSTR

-- index-expression instructions --
equal-exp: IND-EXPR -> INSTR
check-gte-expr: IND-EXPR -> INSTR

-- other instructions --
jump: INDEX -> INSTR
jump-true: INDEX -> INSTR
end
end

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jump-false: INDEX -> INSTR
increment: ID -> INSTR
fetch: ID # INDEX -> INSTR
display: STRING -> INSTR
raise: ID -> INSTR
halt: -> INSTR
eq: INSTR # INSTR -> BOOL

4. THE DATA STACK


The kernel evaluates an expression on a data stack. The functions index-
and range-s are special functions that can retrieve, respectively, a
sequence of indices (matrix element reference) or a sequence of constant
ranges (submatrix reference) from the data stack.

4.2. Specification.

module Data-stacks
begin
exports
begin
functions
index-s: DATA-STACK -> INDEX-SEQ
range-s: DATA-STACK -> CONST-RANGE-SEQ
end
imports Index-sequences, Const-range-sequences,

Sequences

{ renamed by

[ SEQ
  => DATA-STACK,
  add-item => push,
  del-item => pop,
  last => top,
  null => empty-stack

Items bound by

[ ITEM
  => TECH-DATA,
  eq-item => eq ]

to technique-data
}

variables
i,i1,i2 -> INDEX
i a -> INDEX-SEQ
a s -> CONST-RANGE-SEQ
a r -> DATA-STACK

equations

[258] index-s(push(t-data(i),r)) = add-item(i,index-a(r))
[257] index-s(empty-stack) = null-ind-seq
[256] range-s(push(t-data(i),push(t-data(i),r))) = add-item(s-range(i),range-a(r))
[255] range-s(empty-stack) = null-cr-seq

end Data-stacks

end Instruction-sequences
5. HANDLER TABLES.

5.a. Global description.

Instruction sequences of exception handlers declared in a statistical program are stored in a technique handler table. External exception handlers consist of a technique symbol table and instructions sequence. The external handlers are stored in the external handler table. Both the handlers stored in both the technique and the external handler table are available during the execution of a statistical technique.

5.b. Specification.

module Tech-handler-tables
begin
  imports
    Tables
      [ renamed by
        [ TABLE          -> TECH-HANDLER-TABLE,
        null-table     -> empty-tech-hand-tab ]
      Entries bound by
        [ ENTRY          -> INSTR-SEQ,
        eq-entry       -> eq-instr-SEQ ]
      to Instruction-sequences
        Addresses bound by
          [ ADDRESS       -> ID,
          eq-addr         -> eq-id ]
      to Identifiers
    ]
end Tech-handler-tables

module External-handlers
begin
  exports
    begin
      sorts EXT-HANDL
      functions
        ext-handl: INSTR-SEQ # TECH-SYMTAB  -> EXT-HANDL
        han-syntab: EXT-HANDL                -> TECH-SYMTAB
        han-instr-s: EXT-HANDL                -> INSTR-SEQ
        eq-ext-handl: EXT-HANDL # EXT-HANDL   -> BOOL
    end
  imports Tech-syntabs, Instruction-sequences
end External-handlers

module Ext-handler-tables
begin
  imports
    [ renamed by
      [ TABLE          -> TECH-HANDLER-TABLE,
      null-table     -> empty-ext-hand-tab ]
    ]
end Ext-handler-tables

end Handler-tables

6. FUNCTION CODE.


The functions in the statistical language are left unspecified. Only an abstract notion of such a function is defined. Given the current values on the data stack a function returns a value. Also a function may return an exception identifier. The fact that the function table still has to be initialized is emphasized by a parameter Current-function-code in the module Func-code-table.


module Function-code
begin
  exports
    begin
      sorts FUNC-CODE
      functions
        execute: FUNC-CODE # DATA-STACK  -> TECH-DATA
        except-f: FUNC-CODE # DATA-STACK  -> ID
        except-raised: FUNC-CODE # DATA-STACK  -> BOOL
        eq-f-code:  FUNC-CODE # FUNC-CODE  -> BOOL
    end
  imports Data-stacks , Identifiers
end Function-code
module Func-code-tables
begin
  parameters Current-func-code
  begin
    Functions
    int-funcode-tab: -> FUNC-CODE-TABLE
  end Current-func-code
end

module Kernel-programs
begin
  exports
  begin
    sorts KERNEL-PRO
    kern-pro: INSTR-SEQ #
    INF-REST-SEQ #
    TECH-SYNTAX #
    TECH-HANLER-TABLE -> KERNEL-PRO
    eq-pro: KERNEL-PRO # KERNEL-PRO -> BOOL
  end
  imports Tables,
    Instruction-sequences,
    Tech-handler-tables, Input-restr-sequences
end

module Stat-technique-tables
begin
  imports Tables
  begin
    [ renamed by ]
    TABLE
      Entries bound by
        [ ENTRY
          eq-entry
        to Function-code
          Addresses bound by
            [ ADDRESS
              eq-add
            to identifiers
        ]
  variables
  id :: ID
  ds :: DATA-STACK
  equations
  [500] execute-f(id,ds) = execute(id'int-funcode-tab,ds)
  [501] exception-f(id,ds) = execp-f(id'int-funcode-tab,ds)
  [502] execp-raised(id,ds) = execp-raised(id'int-funcode-tab,ds)
end Func-code-tables

7. KERNEL PROGRAMS.

7.a. Global description.

A statistical program is represented at the kernel level by a kernel program. A kernel program consists of an instruction sequence an input restriction sequence, a technique symbol table and a technique handler table. The collection of all available statistical techniques at the kernel level is stored in the statistical technique table.


module Kernel-programs
begin
  exports
  begin
  sorts KERNEL-PRO
    kern-pro: INSTR-SEQ #
    INF-REST-SEQ #
    TECH-SYNTAX #
    TECH-HANLER-TABLE -> KERNEL-PRO
    eq-pro: KERNEL-PRO # KERNEL-PRO -> BOOL
  end
  imports Tables,
    Instruction-sequences,
    Tech-handler-tables, Input-restr-sequences
end

module Stat-technique-tables
begin
  imports Tables
  begin
    [ renamed by ]
    TABLE
      Entries bound by
        [ ENTRY
          eq-entry
        to Kernel-programs
          Addresses bound by
            [ ADDRESS
              eq-add
            to identifiers
        ]
  variables
  id :: ID
  ds :: DATA-STACK
  equations
  [500] execute-f(id,ds) = execute(id'int-funcode-tab,ds)
  [501] exception-f(id,ds) = execp-f(id'int-funcode-tab,ds)
  [502] execp-raised(id,ds) = execp-raised(id'int-funcode-tab,ds)
end Func-code-tables

8. EXECUTION.


The kernel executes sequences of instructions until a halt instruction is reached. Each instruction modifies the state of the kernel. The instructions for exception handling are treated in a separate module. The state of the kernel is determined by:
- the data memory,
- the data stack,
- the current instruction,
- the address of the next instruction,
- the reset information for an unraise instruction,
- the exception handler tables,
- the input restriction sequence,
- a user display.
8.3. Specification

module Kernel-states
begin
exports
begin
sorts STATE, RESET-INFO
 functions
reset-info: INSTR-ADDR # TECH-SYNTAX --> RESET-INFO
reset-info: INSTR-ADDR --> RESET-INFO
reset-info:

state: MEMORY #
 DATA-STACK #
 INSTR #
 INSTR-ADDR #
 -- current instruction --
 -- address next instruction --

RESET-INFO #
 HANDLER-TABLES #

INV-REST-SEQ #
 STRING -- display --
 --> STATE

stop:
 result: STATE --> STATE
 --> USER-SYNTAX
end

imports Func-code-tables, Memory, Data-stacks,
 HANDLER-TABLES, Instructions, Instruction-sequences,
 Input-rest-sequences, Strings

variables
m : MEMORY
us --> USER-SYNTAX
ts --> TECH-SYNTAX
da --> DATA-STACK
ta --> INSTR-ADDR
rt --> RESET-INFO
hta --> HANDLER-TABLES
inv --> INV-REST-SEQ
str --> STRING

functions

[505] handle(state(memory(ta,wa),da,raise(h-d),ta,
no-reset-info,handlers(th,t,eht,tes,dis))

[506] handle(state(memory(ts,wa),da,raise(h-d),ta,
reset-info(ta,handlers(th,t,eht,tes,dis)...)...
[309] handle(state(m,da,load(h-id),1al,reset-info(a0,2al),h1s,irs,dis))
  = stop
[310] reset(state(m,da,uname,1al,reset-info(a2),h1s,irs,d1s))
  = state(m,da,fetch(a2),next(a2),
           no-reset-info,h1s,irs,d1s)
[311] reset(state(memory(t2,1al),da,load(h-id),1al,reset-info(a2,a0),h1s,irs,d1s))
  = state(memory(t2,1al),da,fetch(a2),next(a2),
           no-reset-info,h1s,irs,d1s)
end Exception-handling

module Processor
begin
exports
begin
functions
execute: STATE -> STATE
end
imports Kernel-states, Evaluate-restrictions,
Evaluate-indent-expression, Exception-handling, User-IO
variables
m -> MEMORY
s -> STATE
a -> USER-STATE
da -> DATA-STACK
irs -> INPUT-RESTRICTION-SEQ
ri -> REST-INFO
id -> ID
ia -> INDEX-EXPRESS
f -> INT-EXPRESS
ia -> INDEX-ADDR
ia -> INDEX-SEQ
id -> ID
ht -> HANDLER-TABLES
ia -> ID
is -> ID
b -> BOOL
dis, mes -> STRING

equations
-- user interface instructions --
[312] execute(state(m,da,load(id),ia,ri,ht,irs,dis))
  = execute(if(load-except(id),da,
               state(user-insert(id,m),da,
               load-except(id,m),ia,ri,ht,irs,dis),
               state(user-insert(id,m),da,
               fetch(a),next(a),
               ri,ht,irs,dis)))
[315] execute(state(m,da,user-store(id),ia,ri,ht,irs,dis))
  = execute(state(store-result(id,m),da,
               fetch(a),next(a),
               ri,ht,irs,dis))
  = load data stack instructions --
[314] execute(state(m,da,load(id),ia,ri,ht,irs,dis))
  = execute(state(m,push(get-data(id,m),da),
               fetch(a),next(a),
               ri,ht,irs,dis))
[315] execute(state(m,da,load(id),ia,ri,ht,irs,dis))
  = execute(state(m,push(get-data(id,m),da),
               fetch(a),next(a),
               ri,ht,irs,dis))
[316] execute(state(m,da,load-data(id,m),ia,ri,ht,irs,dis))
  = execute(state(m,
               push(get-data(id,m),
               range-s(top(id,m),m),
               pop(id,m),
               fetch(a),next(a),
               ri,ht,irs,dis))
[317] execute(state(m,da,store-addr(id,m),ia,ri,ht,irs,dis))
  = execute(state(m,
               push(m,
               range-s(top(id,m),m),
               range-s(pop(id,m),m),
               pop(id,m),
               fetch(a),next(a),
               ri,ht,irs,dis))
  = store from data stack instructions --
[318] execute(state(m,da,store-data(id,da),ia,ri,ht,irs,dis))
  = execute(state(store-data(id,da),m),
               pop(id,m),
               fetch(a),next(a),
               ri,ht,irs,dis))
[319] execute(state(m,da,store-addr(id,da),ia,ri,ht,irs,dis))
  = execute(state(store-addr(id,da),m),
               index-s(top(pop(id,m),1,m),m),
               pop(id,m),
               fetch(a),next(a),
               ri,ht,irs,dis))
[320] execute(state(m,da,store-addr(id,m),ia,ri,ht,irs,dis))
  = execute(state(store-addr(id,m),m),
               range-s(pop(id,m),1,m),
               pop(id,m),
               fetch(a),next(a),
               ri,ht,irs,dis))
  = index expression instructions --
[321] execute(state(memory{ts,du}),ds,eval-equip{e},ia, 
  ri,ht,tra,dis))  
  = execute(state{m}, 
  push(t-data{evaluate{e,ts}},ds), 
  fetch(ia),nest(ia), 
  ri,ht,tra,dis))

[322] execute(state(memory{ts,du}),ds,check-e-restr{a, 
  ri,ht,tra,dis}))  
  = (if(check(tra,ts), 
  execute(state(memory{ts,du}),ds, 
  fetch(ia),nest(ia), 
  ri,ht,tra,dis)), 
  stop)

  -- other instructions --

[323] execute(state{m},ds,jump{int1},addr{a,int2},ri,ht, 
  tra,dis))  
  = execute(state{m},ds, 
  fetch(addr{a,addr{int1,int2}}), 
  nest(addr{a,addr{int1,int2}}), 
  ri,ht,tra,dis))

[324] execute(state{m},push(t-data{b},ds), 
  jump-true{int1},addr{ts,int2}, 
  ri,ht,tra,dis))  
  = execute(state{m},ds, 
  if(b, 
  fetch(addr{a,addr{int1,int2}}), 
  fetch(addr{ts,int2})), 
  if(b, 
  nest(addr{a,addr{int1,int2}}), 
  nest(addr{ts,int2})), 
  ri,ht,tra,dis))

  -- the error case (no boolean on the stack) --
  -- is left unspecified

[325] execute(state{m},push(t-data{b},ds), 
  jump-false{int1},addr{a,int2}, 
  ri,ht,tra,dis))  
  = execute(state{m},ds, 
  if(not(b), 
  fetch(addr{a,addr{int1,int2}}), 
  fetch(addr{ts,int2})), 
  if(not(b), 
  nest(addr{a,addr{int1,int2}}), 
  nest(addr{ts,int2})), 
  ri,ht,tra,dis))

  -- the error case (no boolean on the stack) --
  -- is left unspecified

[326] execute(state{m,ds,foall{f-id,i}},a,ri,ht,tra,dis,))  
  = execute(if(foall{f-id,ds}), 
  state{m, 
  push(execute-f{f-id,ds}), 
  pop(ds,)), 
  raise(exception-f{f-id,ds}),i, 
  ri,ht,tra,dis))

[327] execute(state{m,ds,raise{h-id}},ia,ri,ht,tra,dis))  
  = execute( 
  handle( 
  state{m,ds,raise{h-id}},i,ri,ht,tra,dis))

[328] execute(state{m,ds,unraise{a,ri,ht,tra,dis}}, 
  = execute( 
  reset(state{m,ds}, 
  unraise{a,ri,ht,tra,dis})))

[329] execute(state{m,ds,display{mes}},ia,ri,ht,tra,dis))  
  = execute(state{m,ds,fetch{a}},nest{ia}, 
  ri,ht,tra,dis))

[330] execute(stop)  = stop

end Processor

9. KERNEL


The kernel can execute the statistical techniques stored in the 
statistical technique table. The execution of a statistical technique 
may involve exception handlers. The first instruction to be executed is the 
first instruction in the instruction sequence of the statistical technique.
The results of the calculations after the execution of the statistical 
technique are stored in the user symbol table.


module Kernel
begin
  exports
  begin
    functions
      run-tech: USER-SYNTAX # KERNEL-PROC # 
      EXT-HANDLER-TABLE # USER-SYNTAX
      run-tech: ID # USER-SYNTAX # STAT-TECH-TABLE # 
      EXT-HANDLER-TABLE -> USER-SYNTAX
  end
end
APPENDIX K. ASF SPECIFICATION OF THE GENERATION OF KERNEL INSTRUCTIONS

A statistical program is represented at the kernel level as a kernel program. The generation of a kernel instruction sequence for a statistical program and a technique handler constructed is specified in this appendix. It contains:

- modules that describe the generation of kernel instructions for variables, expressions and statements,
- modules that describe the generation of kernel instructions for statistical program sections; the instruction sequence generated for an exception handler section is stored in the technique symbol table,
- a module that describes the generation of a kernel instruction sequence and a technique handler table for an entire statistical program.

1. GENERATION OF KERNEL INSTRUCTIONS FOR THE BASIC ELEMENTS OF THE STATISTICAL LANGUAGE.

1.a. Global description.

In this section the generation of kernel instructions for variables, expressions and statements is specified. The assignment statements are written in postfix notation, making it possible for the kernel to evaluate the statements on the data stack. The message statement generates a display instruction. The instructions for a compound statement consists of the instructions generated for the individual statements in the compound statement. For a for-statement instruction are generated that initialize the control variable, check the restriction that the initial variable is smaller than the upper limit, execute the statements inside the loop and increment the control variable. Also the size of the involved jumps is specified.

1.b. Specification.

module Gen-code-variables
begin
exports
begin
Functions
  gen-code: BOOL # INSTR-SEQ # VARIABLE -> INSTR-SEQ
  gen-code: INSTR-SEQ # END-EXPR-SEQ -> INSTR-SEQ
  gen-code: INSTR-SEQ # RANGE-SEQ -> INSTR-SEQ
end
end

imports Variable-abstract-syntax, Instruction-sequences,

variables:
  ・ IX :-> INSTR-SEQ
  ・ ID :-> ID
  ・ IND :-> END-EXPR-SEQ
  ・ SUBS :-> RANGE-SEQ
  ・ IE, IE1, IE2 :-> END-EXPR
  ・ FLAG :-> BOOL
equations
  -- simple variables --
  [333] gen-code(flag, is, abs-var(id))
  + if (eq(flag, load)
       add-item(load(id), is)
       add-item(store(id), is))

  -- matrix element references --
  [334] gen-code(flag, is, abs-var(id, inds))
  + if (eq(flag, load)
       add-item(load-elem(id, n-of-item(inds)),
       gen-code(is, inds))
       add-item(store-elem(id, n-of-item(inds)),
       gen-code(is, inds)))

  -- submatrices --
  [335] gen-code(flag, is, abs-var(id, subre))
  + if (eq(flag, load)
       add-item(load-subre(id, n-of-range(subre)),
       gen-code(is, subre))
       add-item(store-subre(id, n-of-range(subre)),
       gen-code(is, subre)))

  -- index expr sequences --
  [336] gen-code(is, add-item(is, inds))
  + add-item(eval-expr(is), gen-code(is, inds))

  -- multi-index expr-seq --
  [337] gen-code(is, add-item(is, multi-index-seq))
  + add-item(eval-expr(is), is)

  -- note that eval-expr(is) is only the instruction --
  -- that is must be evaluated, the actual evaluation --
  -- is defined in the kernel. --

  -- range sequences --
  [338] gen-code(is, add-item(range(is, is) subre))
  + add-item(eval-expr(is), is)
  + add-item(eval-expr(is), gen-code(is, is))

  [339] gen-code(is, add-item(range(is, is) multi-range))
  + add-item(eval-expr(is), is)
  + add-item(eval-expr(is), gen-code(is, is))
end Gen-code-variables

module Gen-code-exports begin
  exports begin
    Functions
    gen-code-expr: INSTR-SEQ # EXPR -- INSTR-SEQ
equations
  -- function for the for-statement --
end Gen-code-variables

end Gen-code-exports

end
variables

\begin{align*}
\text{te} & \quad \Rightarrow \text{INSTR-SEQ} \\
\text{stmts} & \quad \Rightarrow \text{STATEMENTS} \\
\text{stmt} & \quad \Rightarrow \text{STATEMENT} \\
\text{var} & \quad \Rightarrow \text{VARIABLE} \\
\text{exp} & \quad \Rightarrow \text{EXPR} \\
\text{m} & \quad \Rightarrow \text{STRING} \\
\text{id} & \quad \Rightarrow \text{ID} \\
(e_1, e_2) & \quad \Rightarrow \text{IND-EXPR}
\end{align*}

equations

\begin{align*}
[345] \text{gen-codestmts}(e, \text{abs-in-stmt}(\text{stmts}, \text{stmt})) & = \text{gen-codestmt}(\text{gen-codestmt}(s, \text{stmts}), \text{stmt}) \\
[346] \text{gen-codestmt}(s, \text{abs-in-stmt}(\text{stmts}, \text{stmt})) & = \text{gen-codestmt}(s, \text{stmt}) \\
\quad \text{-- assignment statement} \\
[347] \text{gen-codestmt}(s, \text{abs-asgn}(\text{var}, \text{exp})) & = \text{gen-codestmt}(s, \text{load}, \text{gen-codeexpr}(s, \text{exp}), \text{var}) \\
\quad \text{-- index assignment statements} \\
[348] \text{gen-codestmt}(s, \text{abs-ind-asgn}(\text{id}, \text{exp})) & = \text{add-item}(\text{store}(\text{id}), \text{add-item}(\text{eval-exp}(s, \text{exp}), \text{id})) \\
\quad \text{-- message statement} \\
[349] \text{gen-codestmt}(s, \text{abs-message}(\text{m})) & = \text{add-item}(\text{display}(s, \text{m})) \\
\quad \text{-- compound statement} \\
[350] \text{gen-codestmt}(s, \text{abs-compound}(\text{stmts})) & = \text{gen-codestmt}(s, \text{stmt}) \\
\quad \text{-- for statement} \\
[351] \text{gen-codestmt}(s, \text{abs-for}(\text{id}, \text{exp}, \text{stmts})) & = \text{gen-codestmt}(s, \text{stmts}) \\
\quad \text{-- increment code, if statements,}
\quad \text{gen-codestmt(}
\quad \text{check-up-limit}(\text{id}, \text{exp}, \text{stmts}),
\quad \text{int-control-var}(\text{id}, \text{exp}, \text{stmts}),
\quad \text{stmts}))
\quad \text{-- interrupt control var (id, exp, stmt)}
\quad \text{add-item}(\text{store}(\text{id}),
\quad \text{add-item}(\text{eval-exp}(s, \text{exp}), \text{id})))
\quad \text{check-up-limit}(\text{id}, \text{exp}, \text{stmts}, \text{e})
\quad \text{add-item}(\text{jump-true}(\text{id}, \text{stmts}),
\quad \text{add-item}(\text{call}(\text{greater-int}, \text{increment})))
\quad \text{add-item}(\text{load}(\text{id}),
\quad \text{add-item}(\text{eval-exp}(s, \text{exp}), \text{id}))
\end{align*}

end Gen-code-statements

2. GENERATION OF KERNEL INSTRUCTIONS FOR SECTIONS OF A STATISTICAL PROGRAM

2.1. Global description.

Each section of a statistical program generates specific instruction sequences. These instruction sequences are added to instruction sequences generated in previous sections. An input/output section only generates user-load and user-store instructions. The exception handler section generates instruction sequences that are stored in a technique handler table.

2.2. Specification.

module Gen-code-declarations

begin

exports

begin

functions

\begin{align*}
\text{gen-code-to-sect} & : \text{INSTR-SEQ} \# \text{ID-SEQ} \# \text{BOOL} \rightarrow \text{INSTR-SEQ} \\
\text{gen-code-decls} & : \text{INSTR-SEQ} \# \text{DECLS} \# \text{BOOL} \rightarrow \text{INSTR-SEQ} \\
\text{gen-code-decl} & : \text{INSTR-SEQ} \# \text{DECL} \# \text{BOOL} \rightarrow \text{INSTR-SEQ} \\
\text{gen-code-id} & : \text{INSTR-SEQ} \# \text{ID} \# \text{BOOL} \rightarrow \text{INSTR-SEQ}
\end{align*}

end

imports Instruction-sequences, Decl-abstr-syntx

variables

\begin{align*}
\text{id} & \quad \Rightarrow \text{INSTR-SEQ} \\
\text{imp-decls,out-decls} & \quad \Rightarrow \text{DECLS} \\
\text{decls} & \quad \Rightarrow \text{DECLS} \\
\text{decl} & \quad \Rightarrow \text{DECL} \\
\text{id} & \quad \Rightarrow \text{ID-SEQ} \\
\text{id} & \quad \Rightarrow \text{ID} \\
\text{ss} & \quad \Rightarrow \text{TECH-TYPE}
\end{align*}
module Gen-code-test-section
begin
exports
begin
functions
gen-code-test-sec: INSTR-SEQ # TEST-SEC --> INSTR-SEQ
gen-code-raises: INSTR-SEQ # RAISES --> INSTR-SEQ
gen-code-raise: INSTR-SEQ # RAISE --> INSTR-SEQ
end
imports Gen-code-statements, Instruction-sequences, Test-abstract-syntax
variables
t
end Gen-code-test-section

module Gen-code-impl-section
begin
exports
begin
functions
gen-code-impl-sec: INSTR-SEQ # IMPL-SEC --> INSTR-SEQ
end
imports Gen-code-statements, Instruction-sequences, Impl-abstract-syntax
variables
t
end Gen-code-impl-section

module Gen-code-handl-section
begin
exports
begin
functions
gen-code-handl-sec: TECH-HANDLER # HANDL-SEC --> TECH-HANDLER

gen-code-handl: TECH-HANDLER # HANDLER --> TECH-HANDLER
end
imports Gen-code-statements, Tech-handler-tables, Handler-abstract-syntax
variables
handl-s

end Gen-code-handl-section
is ::= INSTR-SEQ
tt ::= INST-EXCEPT
id ::= ID
decl ::= INTERM-DECLS
sta ::= STATEMENTS

equations
[369] gen-code-handl-seq(ht, aba-handl-seq(handl-s, handl1))
   = gen-code-handl(handl-seq(ht, handl-s), handl1)
[370] gen-code-handl-seq(ht, aba-handl-seq(handl1))
   = gen-code-handl(ht, handl1)
[371] gen-code-handl(ht, aba-handl(id, decl, sta))
   = insert(id, gen-code-sta(mul-instr-seq, sta), 
            gen-code-handl(id, decl, sta))
end Gen-code-handl-section

3. THE GENERATION OF KERNEL INSTRUCTION FOR A STATISTICAL PROGRAM

3.a. Global description.

An instruction sequence in a kernel program consists of a sequence of
input instructions, an instruction to check the input restrictions, a
sequence that describes the calculations of the statistics, a sequence of
output instructions, and a halt instruction. How input and output
instruction sequence are generated for a statistical program is specified
in module Gen-to-instructions. The generation of calculation instructions
is specified in module Gen-kernel-instructions. The instruction sequence
are linked in module Gen-instructions. The technique handler table of
statistical technique contains the instruction sequences of the exception
handlers in the statistical technique, as specified in module Gen-
handlers.


module Gen-to-instructions
begin
exports
begin
functions
  gen-inp-instr: INSTR-SEQ # STAT-PRO -- INSTR-SEQ
  gen-inp-instr: INSTR-SEQ # SECTION -- INSTR-SEQ
  gen-out-instr: INSTR-SEQ # STAT-PRO -- INSTR-SEQ
  gen-out-instr: INSTR-SEQ # SECTION -- INSTR-SEQ
end
end

variables
ap ::= STAT-PRO
is ::= INST-SEQ
end

3.c. Input instructions

module Gen-kernel-instructions
begin
exports
begin
functions
  gen-calc-instr: INSTR-SEQ # STAT-PRO -- INSTR-SEQ
  gen-calc-instr: INSTR-SEQ # SECTION -- INSTR-SEQ
end
end

variables
ap ::= STAT-PRO
is ::= INST-SEQ
end
to-s  ::= IO-SEC
impl-s ::= IMPL-SEC
test-s  ::= TEST-SEC
handl-s ::= HANDL-SEC
id     ::= ID

equations
[356] gen-calc-instr(s, abs-prog(ap, act)) = gen-calc-instr(gen-calc-instr(s, ap), act)
[357] gen-calc-instr(s, abs-prog(s)) = gen-calc-instr(s, act)
[358] gen-calc-instr(s, abs-sect(impl-s)) = gen-code-impl-sec(s, impl-s)
[359] gen-calc-instr(s, abs-sect(test-s)) = gen-code-test-sec(s, test-s)
[360] gen-calc-instr(s, abs-sect(handl-s)) = is
[361] gen-calc-instr(s, abs-sect(id)) = is
[362] gen-calc-instr(s, abs-name(id)) = is

end Gen-calc-instructions

module Gen-instructions
begin
exports
functions
  gen-check-instr: INSTR-SEQ # STAT-PRO -> INSTR-SEQ
  gen-instr-seq: STAT-PRO -> INSTR-SEQ
end

imports Instruction-sequences, Gen-calc-instructions,
      Gen-lo-instructions

variables
sp  ::= STAT-PRO
ht  ::= INSTR-SEQ

equations
[400] gen-check-instr(s, ap) = add-item(check-ga-restr, s)
[401] gen-instr-seq(sp) = add-item(ht, gen-check-instr(
  gen-instr(sp) -> gen-check-instr(
    gen-instr-null(sp), sp), sp))

end Gen-instructions

module Gen-handlers
begin
exports
begin
functions
  gen-handlers: STAT-PRO -> TECH-HANDLER-TABLE
  gen-handlers: TECH-HANDLER-TABLE # SECTION -> TECH-HANDLER-TABLE
end

imports Gen-code-handl-section, Statistical-programs,
      Tech-handler-tables

variables
sp  ::= STAT-PRO
ht  ::= TECH-HANDLER-TABLE
act ::= SECTION
io-s ::= IO-SEC
impl-s ::= IMPL-SEC
test-s ::= TEST-SEC
handl-s ::= HANDL-SEC
id   ::= ID

equations
[395] gen-handlers(abc-prog(ap, act)) = gen-handlers(gen-handlers(abc-prog(ap, act)))
[396] gen-handlers(abc-prog(ht, act)) = gen-code-impl-sec(ht, handl-s)
[397] gen-handlers(ht, abc-sect(impl-s)) = ht
[398] gen-handlers(ht, abc-sect(test-s)) = ht
[399] gen-handlers(ht, abc-sect(handl-s)) = ht
[400] gen-handlers(ht, abc-sect(id)) = ht

end Gen-handlers
APPENDIX L. ASF SPECIFICATION OF THE OVERALL CONDUCTOR SYSTEM

In this appendix the overall CONDUCTOR system is specified.

In the CONDUCTOR system:
- a technical statistician can implement statistical techniques; these techniques are stored in the statistical technique table,
- a data expert or computer scientist can implement external exception handlers,
- an applied statistician can use the implemented statistical techniques,
- a data expert can initialize the database system,
- a computer scientist can implement the functions that are available in the statistical language.

In this appendix we specify the compiler that generates kernel programs for statistical techniques (module Compiler), and the generation of external handlers (module Gen-ext-handlers). The statistical technique table and the external handlers table together form the collection of available techniques for the applied statistician (module Implemented-techniques). An applied statistician may call the implemented techniques in a simple user language. The definition of the user language, in section 2 of this appendix, includes the definition of a database interface. Finally, in section 3 the overall CONDUCTOR system is specified.

1. COMPILERS.

1.a. Global description.

A statistical program is compiled into a kernel program as specified in module Compiler. External handlers are compiled as specified in module Gen-ext-handlers.

1.b. Specification.

module Compiler
begin
exports
begin
functions
compile: STAT-PRO -> KERNEL-PRO
store-stat-techn: STAT-PRO * STAT-TECH-TABLE -> STAT-TECH-TABLE
end
end

module Gen-ext-handlers
begin
exports
begin
functions
gen-ext-handler: HANDLER -> EXT-HANDLER
store-ext-handler: HANDLER & EXT-HANDLER-TABLE -> EXT-HANDLER-TABLE
end
end

variables
id :: ID
decl :: INTERN-DECLS
stmt :: STATMENTS
to :: FROM-STRUCT
ext :: EXT-HANDLER-TABLE
hmd :: HANDLER

begin

end
2. THE USER LANGUAGE.

2.a. Global description.

The user language allows the user to declare variables, initialize variables, retrieve series from a database, and call statistical techniques. The user can also determine the current data sample.

2.b. Specification.

module User-programs
begin
exports
begin
sorts USER-PRO, USER-COMMAND
functions
user-pro: USER-COMMAND # USER-COMMAND -> USER-COMMAND
user-pro: USER-COMMAND -> USER-COMMAND
user-decl: USER-TYPE # ID -> USER-COMMAND
set: ID # USER-DATA -> USER-COMMAND
set-sample: CONST-RANGE-SEQ -> USER-COMMAND
retrieve: ID -> USER-COMMAND
call-tech: ID -> USER-COMMAND
end
imports Identifiers, User-types, User-data,
Const-range-sequences
end User-programs

3. THE RESULTING SOFTWARE.

3.a. Global description.

The implemented statistical techniques and exception handlers together form the implemented techniques. The statistical techniques can be called by the user in the resulting software. The resulting software also has a database interface. The user may retrieve data from the database, when data is retrieved also a background-query is started to check the consistency of the retrieved data.


module Implemented-techniques
begin
exports
begin
sorts IMPL-TECH
functions
available: STAT-TECHN-TECH # EXT-HANDLER-TECH -> IMPL-TECH
end
imports Stat-technique-tables, Ext-handler-tables
end Implemented-techniques

module Database-interface
begin
parameters Current-database
begin
sorts DATA-BASE
functions
current-db: -> DATA-BASE
data-query: DATA-BASE # ID # CONST-RANGE-SEQ -> USER-DATA
retro-data: DATA-BASE # ID # CONST-RANGE-SEQ -> SCALAR-SEQ
bg-query: DATA-BASE # ID # CONST-RANGE-SEQ -> ID-SEQ
end Current-database
exports
begin
sorts DATA-BASE
functions
query: ID # CONST-RANGE-SEQ -> USER-INFO
end
imports Identifiers, Const-range-sequences,
User-agetab-info, Series, Id-sequences
variables
db ::= DATA-BASE
id ::= ID
crs ::= CONST-RANGE-SEQ
end Database-interface
module Conductor-states
begin
exports
begin
sorts CDT-STATE
functions
state: USER-STATE # STAT-TECH-TABLE # EXT-HANDLER-TABLE -> CDT-STATE
end
imports Resulting-software, Stat-technique-tables, Ext-handler-tables, Conductor-states
begin
exports
begin
functions
execute: SESSIONS # CDT-STATE -> CDT-STATE
execute: SESSION # CDT-STATE -> CDT-STATE
end
imports Conductor-sessions, Compiler, Gen-ext-handlers, Resulting-software
begin
variables
as :-> SESSIONS
s :-> SESSION
cs :-> CDT-STATE
ws :-> USER-STATE
at :-> STAT-TECH-TABLE
wht :-> EXT-HANDLER-TABLE
sp :-> STAT-PRO
up :-> USER-PRO
hd :-> HANDLER
end
exports
begin
functions
obseessions: SESSIONS # SESSION -> SESSIONS
obseessions: SESSION -> SESSION
user-session: USER-PRO -> SESSION
state-session: STAT-PRO -> SESSION
hand-session: HANDLER -> SESSION
end
imports User-programs, Statistical-programs, Handler-abstract-syntax
end Conductor-sessions

module Conductor-sessions
begin
exports
begin
sorts SESSIONS, SESSION
functions
obseessions: SESSIONS # SESSION -> SESSIONS
obseessions: SESSION -> SESSION
user-session: USER-PRO -> SESSION
state-session: STAT-PRO -> SESSION
hand-session: HANDLER -> SESSION
end
imports User-programs, Statistical-programs, Handler-abstract-syntax
end Conductor-sessions

4. THE ENTIRE CONDUCTOR SYSTEM
4.a Global description.

In the CONDUCTOR specification environment a technical statistician can implement statistical techniques, whose outputs can implement exception handlers and a user can apply the implemented statistical techniques and exception handlers.
APPENDIX M: TREE REPRESENTATION OF CONDUCTOR

Tree L1. The overall CONDUCTOR system.

```
Tree L1. The overall CONDUCTOR system.

Conductor
  ├── Conductor-sessions
  │    ├── Resulting-software
  │    │    └── Compiler
  │    └── Gen-ext-handlers
  └── Conductor-states

  ├── Stats-programs
  │    └── Kernel
  │        └── Const-range-sequences
  │            └── Implemented-User-techniques states
  └── Handler-section

  ├── User-symbols
  │    └── Database-interface

  └── Tree L4. The Database Interface

        Database-interface
          ├── Id-sequences
          │    └── User-info
          │        └── User-data
          │            └── User-type
          └── Tree L5. The user-programs

                      User-programs
                        └── Identifiers
                            ├── User-types
                            │    └── User-data
                            │        └── Const-range-sequences

                  ├── User-programs
                  │    └── Identifiers
                  │        ├── User-types
                  │        └── User-data
                  └── Const-range-sequences
```

Tree L2. The Compiler.

```
Tree L2. The Compiler.

Compiler
  ├── Stats-programs
  │    └── Kernel
  │        └── Input-restr-type
  │            └── Static-gen-in-struct-handlers


                     Gen-ext-handlers
                       └── Handler-external-symbols
                           └── External-handlers
                               └── Stat-check-handlers
                                   └── Gen-code-statements
                                       └── Gen-restr-handlers
```


```

Gen-ext-handlers
  ├── Handler-external-symbols
  │    └── External-handlers
  │        └── Stat-check-handlers
  │            └── Gen-code-statements
  │                └── Gen-restr-handlers
```

Tree L4. The Database Interface

```
Tree L4. The Database Interface

        Database-interface
          ├── Id-sequences
          │    └── User-info
          │        └── User-data
          │            └── User-type
          └── Tree L5. The user-programs

                      User-programs
                        └── Identifiers
                            ├── User-types
                            │    └── User-data
                            │        └── Const-range-sequences

                  ├── User-programs
                  │    └── Identifiers
                  │        ├── User-types
                  │        └── User-data
                  └── Const-range-sequences
```

Tree L5. The user-programs.

```
Tree L5. The user-programs.

User-programs
  ├── Identifiers
  │    └── User-types
  │        └── User-data
  │            └── Const-range-sequences
```
Tree K1. The generation of kernel instructions.

Gen-instructions

- Gen-io-instructions
- Statistical-declarations
- Gen-code-declarations
- Gen-code-statements
- Decl-abstr-syntax

- Gen-code-instructions
- Statistical-programs
- Gen-code-test-section
- Gen-code-impl-section
- Decl-abstr-syntax
- Test-abstr-syntax
- Gen-code-statements
- Tepl-abstr-syntax

Instruction-sequences

Tree K2. The generation of an exception handler in statistical technique.

Gen-handlers

- Statistical-programs
- Gen-code-excep-section
- Handler-abstr-syntax
- Gen-code-statements
- Tech-handler-tables

Tree K3. The generation of kernel instructions for statements.

Gen-code-statements

- Gen-code-expressions
- Statements-abstr-syntax
- Gen-code-variables
- Expr-abstr-syntax

- Inst-sequence
- Variable-abstr-syntax
Tree 2. The Kernel.

Kernel

Processor Stat-technique-tables Ext-handler-tables

Kernel-programs External-handlers

Input-restr-sequences Tech-handler-tables Techn-symbols

Instr-sequences User-symbols

Tree 2. The Processor.

Processor

Exception-handling Reaff-evaluator User-input

Kernel-states Evaluate-ind-expr

Input-restr-sequences Handler-tables Func-code-tables

Tech-handler-tables Ext-handler-tables Func-de

External-handlers Data Stacks

Memory

Instruction-sequences Tech-symbols User-symbols

Instructions Strings

Identifiers Ind-expr-abstract Syntax Technique-data
Tree 14. Range restrictions.

Range-restrictions
  | Range-sequences
  |   | Input-restr-sequences
  |   |   | Input-restrictions
  |   |   | Ind-expr-order
  |   |   | Order
  |   |   | Ind-expr
  |   |   | abstr-syntax
  |   | Tech-symbols
  |   | User-visibility

Trees appendix H.

Tree H1. Static type checking of a statistical program.

Static-type-checking
  | Statistical programs
  |   | Stat-check-
  |   | handlers
  |   | Stat-check-
  |   | templ
  |   | Stat-check-
  |   | tests
  |   | Handler-
  |   | abstr-syntax
  |   | Test-
  |   | abstr-syntax
  | Store-
  |   | declarations
  |   | Stat-check-
  |   | statements
  |   | abstr-syntax
Tree OA. Evaluation of symbolic types, ranges and index expressions.

```
Evaluate-type
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Technique-types</td>
</tr>
<tr>
<td>Evaluate-ranges</td>
</tr>
<tr>
<td>User-types</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Range-sequence</td>
</tr>
<tr>
<td>Evaluate-expr</td>
</tr>
<tr>
<td>Const-ranges</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ind-expr-abstr-syntax</td>
</tr>
<tr>
<td>Techn-syntabs</td>
</tr>
<tr>
<td>Indices</td>
</tr>
</tbody>
</table>
```

Tree F1. Statistical program.

```
Statistical-programs
<table>
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<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Test-abstr-syntax</td>
</tr>
<tr>
<td>Impl-abstr-syntax</td>
</tr>
<tr>
<td>Handle-abstr-syntax</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Statements</td>
</tr>
<tr>
<td>Abstr-syntax</td>
</tr>
<tr>
<td>Decl-abstr-syntax</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expr-abstr-syntax</td>
</tr>
<tr>
<td>Technique-types</td>
</tr>
<tr>
<td>User-visibility</td>
</tr>
<tr>
<td>Id-sequences</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Strings</td>
</tr>
<tr>
<td>Identifiers</td>
</tr>
</tbody>
</table>
```
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