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FORMAL METHODS IN THE STUDY OF LANGUAGE

PART 1

edited by

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PREFACE

From March 25-28, 1980, the Third Amsterdam Colloquium on 'Formal Methods in the Study of Language' was held. This book presents its proceedings.

The Amsterdam colloquia are held bi-annually, with the aim to bring together people from different fields but with a common interest: the formal study of language. The third colloquium was jointly organized by the Centrale Interfaculteit of the University of Amsterdam and the Mathematisch Centrum in Amsterdam.

In these Proceedings all colloquium papers are presented, with two exceptions. The paper read by David Dowty will appear in G. Pullum & P. Jakobson (eds), 'On the nature of Syntactic Representation'. The paper by Jan Landsbergen reproduced here is not the one presented at the colloquium.

For technical reasons only, it was necessary to divide this book into two volumes. There is no division by subject over the volumes, the papers are arranged in alphabetical order.

The Dutch Ministry of Education and Sciences provided financial support for the colloquium, which is gratefully acknowledged here. Further we would like to thank Mrs. S.J. Kuipers for her assistance in organizing the colloquium, Fred Landman for his help in reading the proofs, and the Mathematisch Centrum for the opportunity to publish these papers in their series Mathematical Centre Tracts.

Amsterdam, January 1981.

Jeroen Groenendijk
Theo Janssen
Martin Stokhof

SEMANTICS AND SYNTAX OF NOMINALIZATIONS

by

Renate Bartsch

0. INTRODUCTION

This paper will only deal with nominalizations that are gerunds, although the semantical framework that will be proposed is useful for derivative noun phrases. Verbal gerunds and nominal gerunds will be contrasted with *that*-clauses. WASOW & ROEPER (1972) treat verbal gerunds (Verb-*ing* with adverbials and/or direct and indirect object noun phrases) as being derived from an underlying sentence that is embedded in a matrix clause that determines the control of the deleted subject. This control property was their main argument for treating verbal gerunds as transformationally derived from embedded clauses, while treating nominal gerunds lexically. WASOW & ROEPER (1972) were aware of some counterarguments with respect to the control properties of verbal gerunds which they tried to "explain away"; but THOMPSON (1973) showed that the control properties can not be explained syntactically, rather they depend on the semantics of the context, especially the verb of the matrix sentence. CRESSWELL (1973) treats verbal gerunds semantically as *that*-clauses. The nominal gerunds are treated like predicates over individuals: the nominalization operator is semantically vacuous, it maps a predicate of type <e,t> onto itself. This is criticized by ULLMER-EHRICH (1977) because it neglects the difference in sorts: "eat potatoes" is true of individuals, while "eating potatoes" (in sentences like *Eating potatoes is fun*) is true of acts. The mixed form in, for example, *I hate John's eating Piggy* is treated as a nominal gerund by CRESSWELL (1973), but as a verbal gerund by WASOW & ROEPER (1972). Cresswell does not capture that it has a *that*-clause paraphrase. This, according to ULLMER-EHRICH 1977, has a good effect: There are *that*-clause constructions that do not permit the verbal gerund as a paraphrase (compare examples 5b,c with 5a and 6b,c with 6a, below). Because the mixed gerunds and the verbal gerunds have a different distribution from *that*-clauses, they should not be treated alike semantically. That also holds for the verbal gerund: *John eating Piggy* is not possible as object of *believe,* though the *that*-clause is. On the other hand, certain paraphrase relationships have to be explained. But this explanation does not require that one assumes the same logical structures as underlying representations of their meanings.

The nominalizations exemplified by the sentences under (1) - (9) below show a different distribution with respect to predications.

1. a. John eats Piggy. It/This is disgusting.
   b. John eats Piggy. It/This looks disgusting.
   c. John eats Piggy. It/This was to be expected.
   d. John eats Piggy. It/This takes place at 3 o'clock.
   e. John eats Piggy. It/This lasts three hours.

2. a. That John eats Piggy is disgusting.
   b. John's eating Piggy is disgusting.
   c. John's eating of Piggy is disgusting.
   d. *John eating Piggy is disgusting.
   e. John, eating Piggy, is disgusting. [pres. participle]

3. a. *That John eats Piggy looks disgusting.
   b. ?John's eating Piggy looks disgusting.
   c. John's eating of Piggy looks disgusting.
   d. *John eating Piggy looks disgusting.
   e. John, eating Piggy, looks disgusting. [pres. participle]

4. a. I am surprised that John eats Piggy.
   b. I am surprised by John's eating Piggy.
   c. I am surprised by John's eating of Piggy.
   d. I am surprised by John eating Piggy.
   e. I am surprised by John, eating Piggy. [pres. participle]

5. a. I expect that John eats Piggy (will eat Piggy).
   b. ?I expect John's eating Piggy.
   c. *I expect John's eating of Piggy.
   d. I expect John eating Piggy.
   e. I expect John (to be here).
   f. I expect John's eating Piggy to take place.
   g. *I expect John eating Piggy to take place.

6. a. I believe that John eats Piggy.
   b. *I believe John's eating Piggy.
   c. *I believe John's eating of Piggy.
   d. *I believe John eating Piggy.

7. a. *That John eats Piggy/is eating Piggy lasts for an hour.
   b. John's eating Piggy lasts for an hour.
   c. John's eating of Piggy lasts for an hour.
   d. *John eating Piggy lasts for an hour.

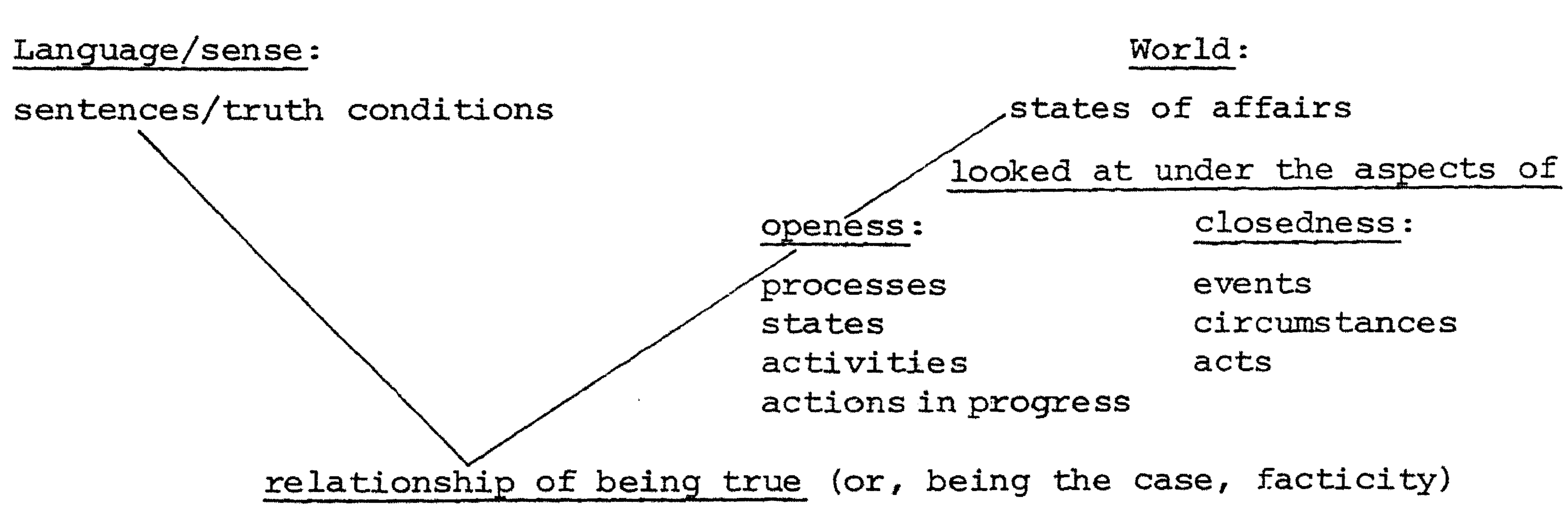
8. a. *That John eats Piggy takes place at four o'clock.
   b. John's eating Piggy takes place at four o'clock.
   c. John's eating of Piggy will take place at four o'clock.
   d. *John eating Piggy will take place at four o'clock.

9. a. *That John eats Piggy will take place in the garden.
   b. John's eating Piggy will take place in the garden.
   c. John's eating of Piggy will take place in the garden.
   d. *John eating Piggy will take place in the garden.

This difference in distribution can be explained by taking into account the semantics of the types of nominalization. This will be done first in a preliminary orientation, and later I will try to incorporate this in a formal treatment of the semantics in a model.

1. THE FUNCTION OF NOMINALIZATIONS

A rough sketch of the relation between language and world may serve as a point of departure.



With respect to this rough picture, at least the following topics can be subject of predications.

(1) Facticity can be topic in at least three ways:

(a) whether it holds or not

(b) with regard to expectation

(c) with regard to attitudes.

(2) States of affairs in the world can be looked at under the aspect of being closed or completed (result is achieved), and then be topic under several points of view (predications under external points of view):

(a) causal relations between events/circumstances

(b) temporal and local relations

(c) motivational relations, means-ends relations

(d) attitudes towards events/circumstances.

(3) States of affairs in the world can be looked at under the aspect of being open, for example, being in progress, and then be topic under several points of view (predications under internal points of view):

(a) internal characterizations of processes and states, and especially of the performance of acts (i.e. of the activity or action in progress)

(b) temporal and local relations viewed from within processes and states.

The different nominal forms contain indicators for the different points of view under which the topics for the predications are chosen.

(1) If the topic is facticity under one of the three points of view mentioned above, the nominalization expressing this topic is a *that*-clause. This is the case with examples 2a, 4a, 5a, 6a; and it is the reason why 3a, 7a, 8a, 9a are semantically unacceptable. Facticity is not something that can be looked at, or that can take place or last.[1]

(2) If the topic is a closed state of affairs the nominal form expressing this topic is a lexical deverbative noun or a verbal gerund. The simple or complex verb that is the basis for the nominalization operation can have incorporated in its meaning the aspect of achievement of result or closedness. This is the case if an action verb is accompanied by an object phrase referring to a specific object.[2] In the examples (b) we have a nominal corresponding to the complex verb *eat Piggy*. The nominal refers to the specific event of John's eating Piggy. (5b) is questionable because expectation of an event seems to be more an attitude towards facticity and thus

(a) is preferred. Since (6b) has a predicate expression that clearly refers to facticity, it is deviant.

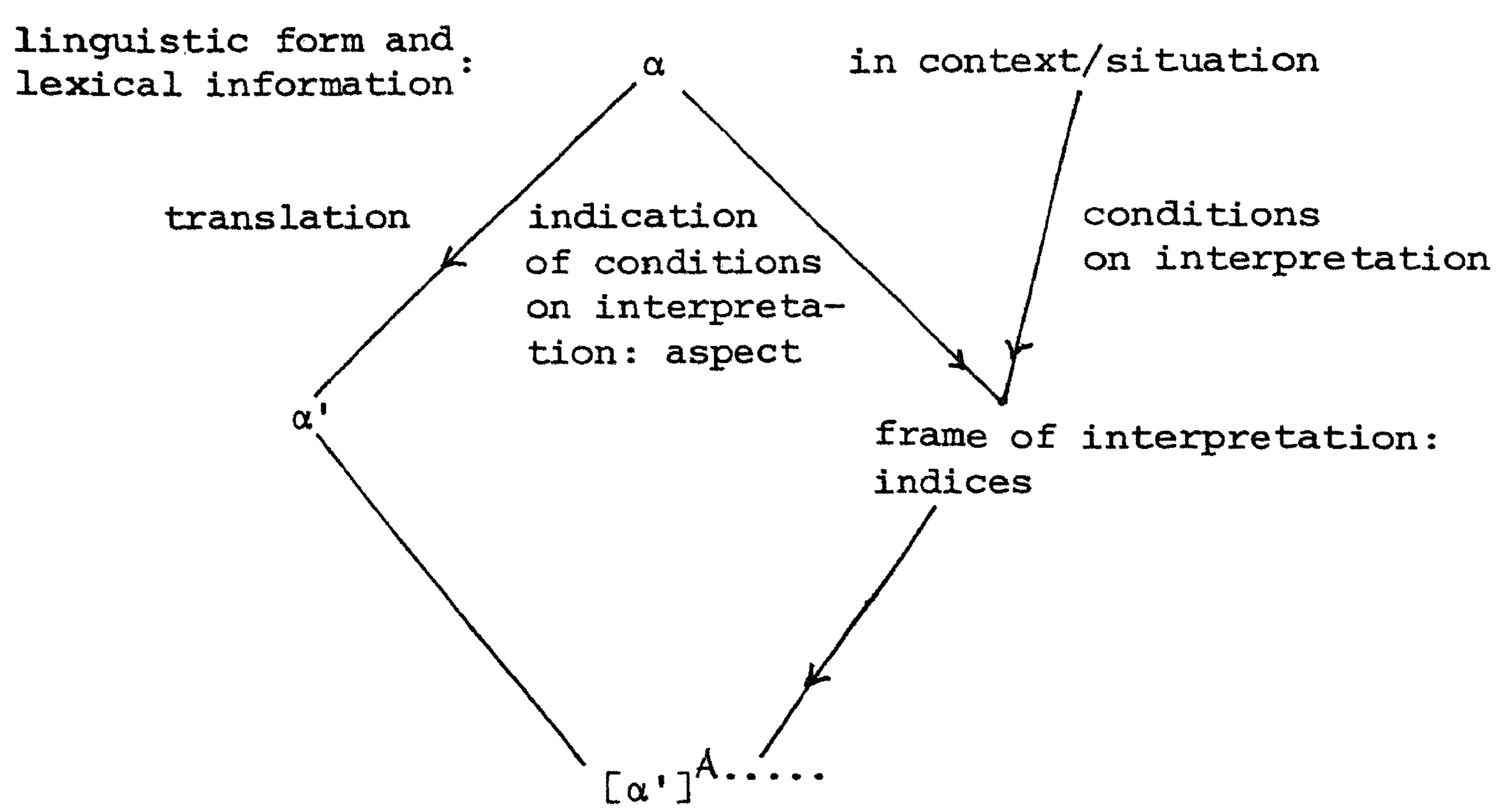
(3) If the topic is an open state of affairs and is looked at under internal points of view, then the nominal form expressing this topic is the *ing* nominalization of the lexical verb. Instead of an object phrase, there can be a prepositional attribute expressing goal directedness. In (2c), (3c), (4c), (7c), (8c), (9c) it is the performance or process of eating that is predicated about. (6c) is deviant because "believe" cannot be about processes or states, but only about facticity. (5c) is deviant because expectation requires the external point of view and thus looks at the state of affairs from the outside as an event. Thus (5b) is acceptable, though a bit strange without a temporal or local indication. It would be better as *John's eating Piggy can be expected any minute*.

The deictic pronoun *it/this* can be interpreted in the examples of (1) in all three manners: as referring to facts/facticity, to closed states of affairs (events, circumstances, acts), or to open states of affairs (processes, states, activities, actions in progress).

Notice, that the term "open state of affairs" and "closed state of affairs" can refer to the same state of affairs in reality; the difference only lies in the point of view (internal or external), that is, the aspect under which the state of affairs is perceived as topic of predication. There need not be a difference in the world involved, rather a different way of being looked at. In interpreting a nominal, we refer to a state of affairs in the world and, at the same time, interpret the form of the nominal as indicating the aspect under which we are considering that state of affairs.[3] If *s* (situation) refers to a state of affairs, we have:

$$(s, \text{open}), (s, \text{closed}).$$

I will construct a model in which reference to states of affairs (events, processes, etc.) and the manner of reference (open, closed) can be represented. The latter will be treated as a condition on interpretation, and thus appear in the frame of interpretation (i.e. in the indices). In the logical language, into which natural language will be translated in the spirit of Montague's PTQ, states of affairs will be represented, but not the aspectual properties. These only appear in the model (see schema below).



ULLMER-EHRICH 1977 has shown for German that the aspectual properties of a nominalization depend on two things: on the syntactic-morphological kind of nominalization and on the internal aspect of the lexical content of the verb phrase under the nominalization operation. If we apply this to English, we can expect a different aspect indication for *ing* nominalizations, depending on the internal aspect of the content of the lexical verb or complex verb phrase. If there is an object that influences the internal aspect of the verb phrase, the nominalization of the "Verb + object-NP"-phrase will have a different aspect than the nominalization of the lexical verb alone. The aspect of a verb phrase is influenced by the definiteness or indefiniteness of plural object phrases or mass noun phrases, by the use of accusative or partitive objects, and possibly other factors.

If a verb is an activity verb, then its gerund nominalization denotes an activity, except if the predication requires the begin and/or endpoint of the activity to be included in the topic. In that case the nominal refers to an act (cf. 8c, 9c). If a verb is an action verb (activity + result or goal directedness), then its nominalization (verbal gerund) denotes an act (result or end achieved), except if the predication requires the end or result not to be included. Then the nominalization of the complex verb phrase denotes an action, i.e. it is neutral with respect to the achievement of the result or end, that means, it is open (cf. 3b, 7b).

2. ATTEMPTS TOWARDS A SEMANTICS OF EVENTS

REICHENBACH (1947), in the context of a logical analysis of adverbial constructions, puts forward two proposals for dealing with events.

1. Specific properties:

Annette dances: $d(x_1) =_{df} \exists f\ f(x_1) \cdot \delta(f)$.

That means, Annette has a specific property and that has the properties of dancing. δ is of a higher type than d and f.

Annette dances beautifully: $\exists f\ f(x_1) \cdot \delta(f) \cdot \beta(f)$.

That means, the specific property has the properties of dancing as well as those of beauty.

2. Event predicates:

$[f(x_1)]^*$ is a "situational fact function" or "event predicate".

$(\iota v)[f(x_1)]^*(v)$ is an event description that refers to the event indicated by sentence $f(x_1)$.

"Event splitting" means that a sentence can be split up into an event predicate and a proper name for an event:

$$f(x_1) \doteq [f(x_1)]^*(v_1).$$

The main problem with this approach is that not all sentences "indicate" events, and logically equivalent sentences do not describe the same events (cf. KIM 1969 and DAVIDSON 1967). Further, Reichenbach does not distinguish facts and events. DAVIDSON (1967) incorporates event variables into sentences at a place in their predicates; not the sentences refer to events, rather a place in their predicates. Every predicate has an event place, filled by the variable *e*.

Brutus stabs Caesar: ∃e Stabs(Brutus, Caesar, e)
or: ∃e (Brutus stabs Caesar, e).

The predicate *Capsizing of the canoe by Doris yesterday* can apply to several events that happened yesterday:

λe(Doris capsized the canoe,e & e yesterday).

From this, the definite term *Doris' capsizing the canoe yesterday* can be formed by use of the iota-operator, and likewise an indefinite term as in

Some capsizing ... was a disaster: $\exists e(\ldots \& e \text{ was a disaster})$.

This purely extensional treatment gives rise to the question, what the notion of identity is for events. DAVIDSON (1969) points out that the identity of their space-time regions is not sufficient, and he proposes, that events are identical if and only if they have the same causes and effects. In this formulation a conception of events as intensional entities seems to be disguised, since having the same causes and effects, if all possible causes and effects are included, can only mean that they have the same properties.

It has, on the other hand, been proposed that events should be eliminated by reduction to properties of moments of time, e.g. in MONTAGUE 1960, or to properties of space-time regions, e.g. in KIM 1975. These treatments take into account that events are intensional entities that cannot be identified with time intervals or space-time regions. In both proposals the relationship between generic and individual events remains problematic.

Montague's reduction is:

'The event P occurs at moment $t' =_{red.}$ t' possesses the property P'.

The generic event of the sun's rising is the property of being a moment of time at which the sun rises:

$^\wedge\lambda t\ Rises(s,t)$, *or*, $^\wedge t\ Sun\ rises(t)$.

The generic event can further be specified by time t_1:

$^\wedge\lambda t(Rises(s,t)\ \&\ t=t_1)$.

For the individual event at t_1, Montague uses the notion of specific property: the specific property of t_1 has the property Sunrising. With T as the operator "the exactly one" (iota-operator), the individual event is expressed by: $\mathit{T}P\ (Sunrising(P)\ \&\ P(t_1))$. Thus we get by reduction that if there is an individual sunrising at t, then the sun rises at t:

$\forall P \forall t(Sunrising(P)\ \&\ P(t) \rightarrow Sun\ rises(t))$.

From this formula, we see that specific properties are propositional functions, and if t is incorporated, just propositions. This mixes up the difference between *that*-clauses and nominals. The second shortcoming is that space has to be included. This is done in KIM 1975.

KIM (1975) does not give a formalization, though he writes that '$[x,P,t]$' refers to the individual event of x's exemplifying property P at t. For x we can take a space-time region or an object. A generic event is a property of space-time regions, represented by a complex of properties $[A,B,C]$ realizable at one single occasion. P is a variable over generic events. We should define *P* as a propositional function, since, for example, the event of John meeting Mary can take place at different places and times. Thus we have, by stating Kim's proposal in logical notation: $[{}^{\wedge}John\ meets\ Mary, s_2, t_2]$ for the individual event of John's meeting Mary taking place at (s_2, t_2), and ${}^{\wedge}\lambda s,t\ John\ meets\ Mary,s,t]$ for the generic event of John meeting Mary. Notice, that the relationship between the generic event and the corresponding individual events is not parallel with that between the generic lion and individual lions: If the property ${}^{\wedge}\lambda x\ lion(x)$ holds of an individual, then it is a lion, while, if the property ${}^{\wedge}\lambda s,t[John\ meets\ Mary,s,t]$ holds of a space-time region, this region is not an event of John meeting Mary. The problem is that the generic event is a property of space-time regions without the individual space-time regions being individual events; rather individual events occur at (in or on) them. We also have to consider the intensions of space-time regions if we want this relation to hold. The individual event that exemplifies the propositional function ${}^{\wedge}John\ meets\ Mary$ is not simply the space-time region, but the "conceptualized" space-time region, that means, the space-time region together with its characterizing concept. This is expressed in Kim's notation '$[P,s,t]$'. This idea will be worked out formally in the following part.

3. THE REGION MODEL

States of affairs (events, processes, etc.) extend over regions in space and time (short: space-time regions). The universal (space-time) region U consists of a space coordinate S and a time coordinate T. *S* is the set of all space-regions, and *T* is the set of all time-intervals. Both, S and T, are taken as a continuum, into which special space-and-time-structures can be embedded. The idea of a continuum can be represented by taking S to be $\mathbb{R}^3$, T to be $\mathbb{R}$, and U to be $\mathbb{R}^4$, with $\mathbb{R}$ as the set of real numbers. $\mathbb{R}^4$ with its natural topology may serve as the mathematical region model in this paper. But to start out with this model right away would mean to base natural language interpretation on a fairly abstract concept, namely the notion of "point", which is an abstract construct that can be based on the less abstract notion of "region". Therefore, I will, in accordance with recent developments in Tense-Logic (KAMP 1979, 1980, VAN BENTHEM 1980), start out with "region" as the basic concept and the "part-whole"-relationship as the basic relation between regions. A region is built up from a space- and a time-component.

My aim is not, to eliminate the idea of absolute space and time and to think of space and time merely as structures between objects and events (Leibnizian position), but rather, in the Newtonian or Kantian way, of space and time as a background or receptacle (be it absolute, or "Form der Anschauung") into which different structures can be embedded, according to the different kinds of objects and events between which by different means different relationships can be established. Thus, different from Newton and Kant, space is not presupposed as being Euclidean.

The parts of U form the set $\mathcal{R}$ of space-time regions; W is the set of possible worlds; 2 is the set of truth values; $\subset$ is the "part-whole"-relationship; and F is the interpretation function for the basic constants of the language to be interpreted in the model. Our model can be represented by $\langle U,\subset,W,F,2\rangle$, or by $\langle \mathcal{R},\subset,W,F,2\rangle$, with $\mathcal{R} = \{R \mid R \subset SXT\}$, that is, for every $R \in \mathcal{R}$ the first projection $\pi_1(R) \in \mathcal{S}$, and the second projection $\pi_2(R) \in \mathcal{T}$.

The following notions can be defined:
Two regions *overlap* iff they have a common part:

$$\text{overlap}\ (R_1,R_2) =_{\text{def.}} \exists R_3(R_3 \subset R_1\ \&\ R_3 \subset R_2).$$

Two regions *merge* iff they are united to form a new region that extends over both:

$$\text{merge}(R_1,R_2) =_{\text{def.}} \exists R_3(R_1 \subset R_3\ \&\ R_2 \subset R_3\ \&\ \forall R(R \subset R_3 \rightarrow \text{overlap}(R_1,R) \lor \lor\ \text{overlap}(R_2,R))).$$

The overlap of two regions is their largest common part; the merger of two regions is the smallest region of which they both are parts.
A *path* is the merger of a series of regions $R_1,\ldots,R_i,\ R_{i+1},\ldots,R_n$, such that for each i, $1 \le i \le n$, R_i overlaps with R_{i+1}.
A region R is *connected* iff for every two parts R' and R" of R there exists a path that overlaps with R' and R".

For the universal region the property of *continuity* is required which is defined by the following properties of the relations between regions:
1. The "part-whole"-relationship between regions is transitive:
$R_1 \subset R_2\ \&\ R_2 \subset R_3 \rightarrow R_1 \subset R_3$. See Figure 1.
2. Every region contains other regions as parts of itself.

3. Every region is part of other regions. See Figure 2.
4. Given any two regions R_1 and R_2, there are connected regions which contain both of them as parts. See Figure 3.
5. For every two regions R_1 and R_2 with $R_1 \subset R_2$, there exists a complementary region of R_1 in R_2; that is an R_3 which does not overlap with R_1, and the merger of which with R_1 is R_2. (That means, R_1 and R_3 form together a junction that makes up R_2.) That means, the set of regions is closed with respect to relative complementation. See Figure 4.

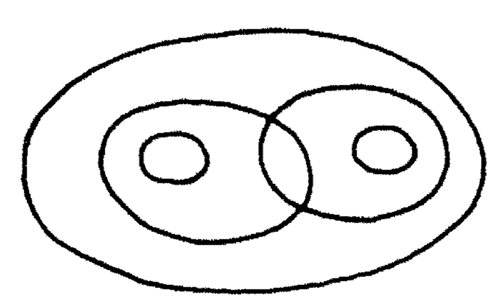


Figure 1

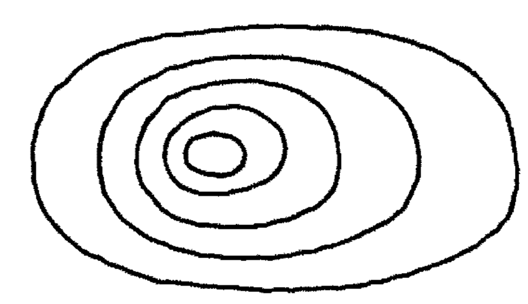


Figure 2

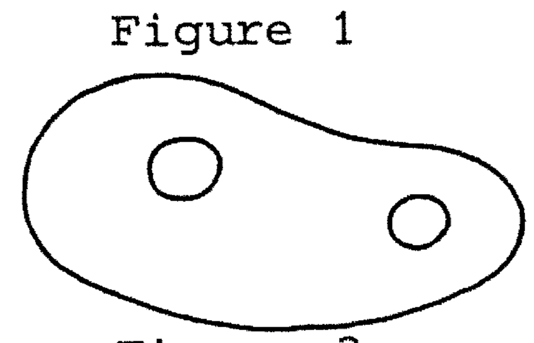


Figure 3

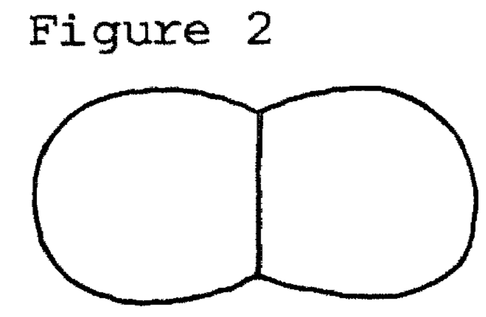


Figure 4

An interesting subset of $\mathcal{R}$ is the set of connected regions. These are the regions occupied by events on individuals.
An *event* is a pair consisting of a region concept (i.e. a function from indices to regions) and a region.
Then we take events as having connected space-time regions as their "extensions" and we can define corresponding notions about events (and other states of affairs), following WHITEHEAD 1919:
Event e extends over event f iff the region of f is part of the region of e.
Two *events overlap* or *merge* iff their respective regions overlap or merge.

In KAMP 1979 and 1980 time structure is defined by means of an event structure in which next to the relation "overlap" between events the relative notion "precede" is used as a basic notion between events. Likewise, we can say that space-time regions precede or follow each other with respect to time. With respect to space, the notion "precede" is relative to the dimensions left-right, below-above and front-back, which can be established in different ways, according as to which regions of orientation (paths, individuals, paths between individuals) they are fixed. The model would have to be worked out with respect to space structures, if we were to explicate the semantics of space adverbials and demonstratives.

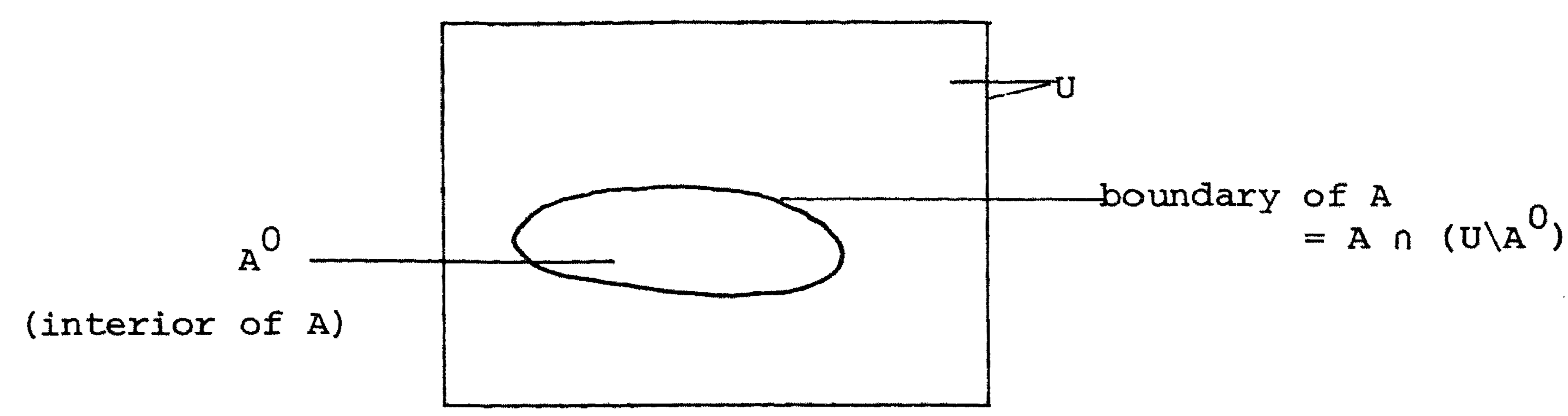
To be able to talk about boundaries of regions we need to introduce the notion of "point" of U. This has been done by WHITEHEAD 1919 in a manner that is generally used in completion of mathematical structures, for example in the construction of the real numbers out of the rational numbers with their usual topology. Instead of families of Cauchy filters used in the completion of uniform spaces, Whitehead uses "families of abstractive sets of events", which are families of descending filters of events. The same method has been employed by KAMP 1979 and 1980 and by VAN BENTHEM 1980 to construct instances of time out of events or intervals as basic notions.

A *descending filter of regions* is a set of regions, F, such that
(1) for all $R' \in F$ and $R'' \in F$, $R' \subset R''$ or $R'' \subset R'$;
(2) there is no region which is a common part of every member of F.
Two descending filters are "equal", i.e. approach the same limit, iff for every element in one filter there is an element in the other filter which is part of it.
A *point* is a family of "equal" descending filters of regions.

Because of the property of continuity, I conclude that the universal region can be conceived of as the set of all points defined by families of "equal" descending region filters. This means that there are no pointlike holes in U. This can be proved by providing a method by which for any assumed pointlike hole a descending filter can be constructed which converges to it and thus defines this assumed pointlike hole as a point belonging to U:
Assume A to be a pointlike hole in U. If A is not situated in subregion U_1 of U, then A is situated in the complement region U_1' with respect to U. In the next step, U_1' can also be subdivided into two regions U_2 and U_2', in one of which A is situated, let us say in U_2. In the next step this U_2 can be subdivided again into two parts in one of which A is situated, and so on. The set of subregions containing A which we get following this method form a descending filter that converges to A and thus defines it as a point of U. Thus the assumed pointlike hole is not a hole in U but a point of U.

A *point* is *element of a region* iff the region is element of a descending filter belonging to the family of descending filters that defines this point.

With the help of the notion of "point" we can define boundaries of events. Now we are able to distinguish between open states of affairs (processes, states, activities) and closed states of affairs (events, occasions, acts). Closed states of affairs include their boundary, open states of affairs do not. The universal region and its complement, the empty region are taken as being both, open and closed. All other regions are either open or closed.



A neighbourhood of a point x is a region A such that there is an open region A' with $x \in A' \subset A$.

If the state of affairs e has as its extension region A, i.e. the statement p characterizing the state of affairs e is true at A, the open state of affairs (i.e. the process, state or activity) can be defined as that part of e that has as its extension the interior of A; while the closed state of affairs e (event, occasion, act) has as its extension the interior united with the boundary of A. At all points of the interior we have a neighbourhood region at which p is true. Points belonging to the boundary of A have no neighbourhood region that is part of the extension of the event characterized by the statement p, i.e. p is not true for the whole stretch of any neighbourhood region. But every neighbourhood region contains a part at which p is true. With respect to the begin and end point of the associated time interval (the time projection of A) this means:

A is *open with respect to time* $=_{\text{def.}}$ For all R, $R \subset A$, there are R', $R'' \subset A$ such that if $R' \supset R$ and $R'' \supset R$ and $R \cap B_A = \emptyset$ and $R \cap C_A = \emptyset$, then $R' \cap B_A = \emptyset$ and $R'' \cap C_A = \emptyset$; with B_A and C_A as the set of boundary points projected on the begin and end point of the time interval, respectively.

A is *closed with respect to time* $=_{\text{def.}}$ A overlaps with the set of boundary points that are projected on the begin and end point of the associated time interval, respectively.

The set of points that are projected on the end point of the associated time interval can be understood as the points where the result or end of an action, process, or state is achieved, which together with the action, process, or state form the act, event or occasion. This means that a closed state of affairs overlaps with its result or end. We can look at states of affairs e as "open" or "closed"; that means, we look at them as taking place at the open region (interior) or the closed region (interior ∪ boundary).

The closure of a process, state, action or activity includes begin and end. To look at a state of affairs as "closed" means to perceive it as a whole or unit from the outside, as a "point", so to speak, located within an external structure. The set of event features that are realized at the end of the time interval is "the achievement of result".

We use the following technical terms:

(1) activity: open (ex. John was writing)

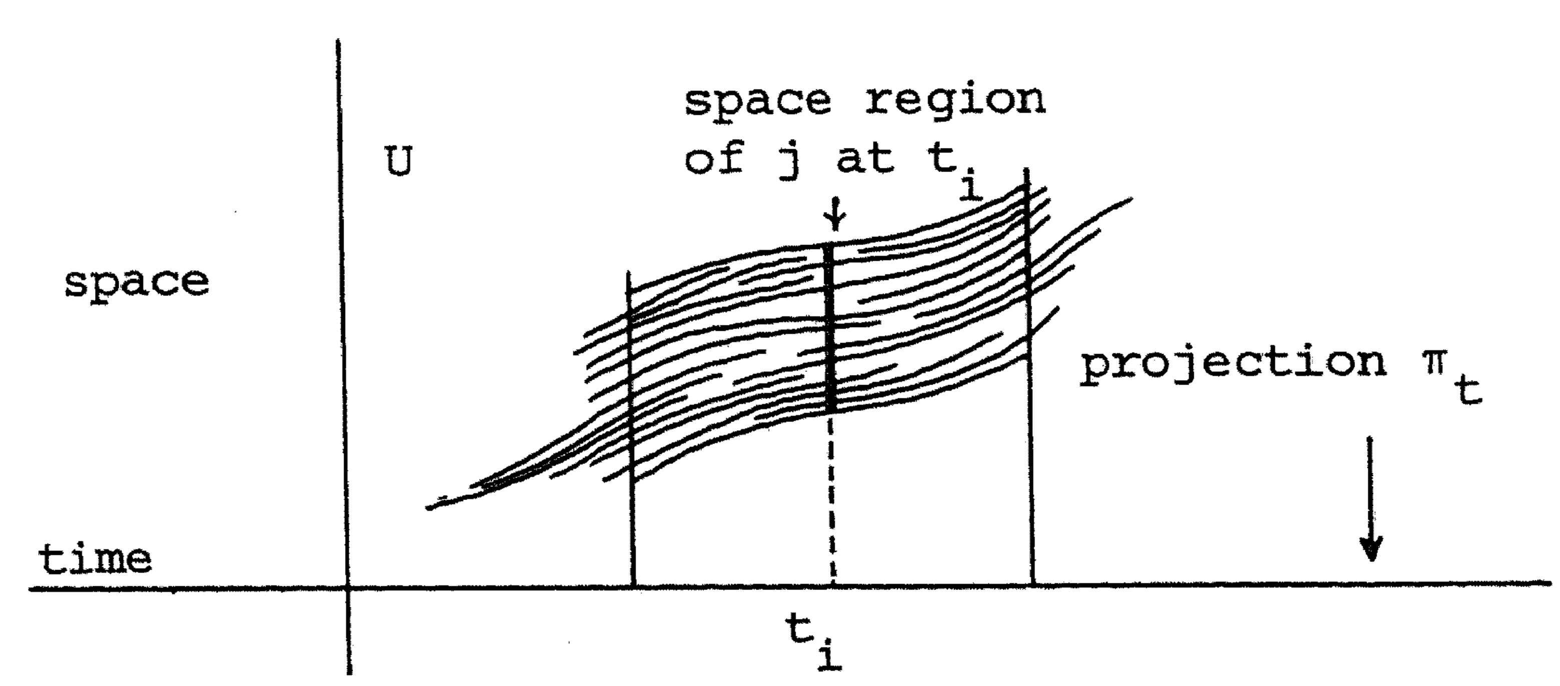
(2) action = goal or result directed activity: open (ex. John was writing a book)

(3) act = activity or action with closure or achievement of result respectively: closed (ex. John wrote a book)

(4) process, state: open

(5) event, occasion: closed.

In our model, *individual concepts* can be treated as intensions of connected regions. Viewed under the aspect of time, an *individual* can be conceived of as a string extending over a time interval (its life time). The properties the individual has for some time intervals within his life time can be conceived of as threads in the string. More exactly, the realizations of properties by an individual during an interval are states, processes, events (states of affairs), and these are the threads of which the string consists.



These threads are shorter or longer, depending on how long the individual has the property. A thread is the intersection of the set of states of affairs that are realizations of the property, with the individual. There might be some threads that run through the whole life of the individual, and there might be threads that are already there before the individual comes into existence (e.g. realizations of cell properties before the birth of an animal) and there might be threads that are still continued after death or destruction of the individual (the remains of the individual). The space time region of the individual is determined by the realization of the defining or essential properties of its kind. These can be called "the core" of the individual. During his life time different sets of threads that overlap form the characterizing properties of the individual, by which it is recognized. They form the "characterizing core" of the individual, which makes up the individual concept. In the model, events and objects are of the same logical type, but there are differences in sorts which can be expressed in the language by meaning postulates. Objects, for example, have a certain Gestalt (substance and form) that they carry through their space-time region and that varies only gradually. It belongs to the main properties that serve for recognizing the object at different times and places. The boundary between characterizing core and other property realizations is vague, and might be different for different speakers, when they refer to objects. The core of an event, like the storm Alia, can simply just be the realization of one property, e.g. "storm". The identity of Alia consists in the connectedness of the realization of this property, from the beginning to the end of the storm.

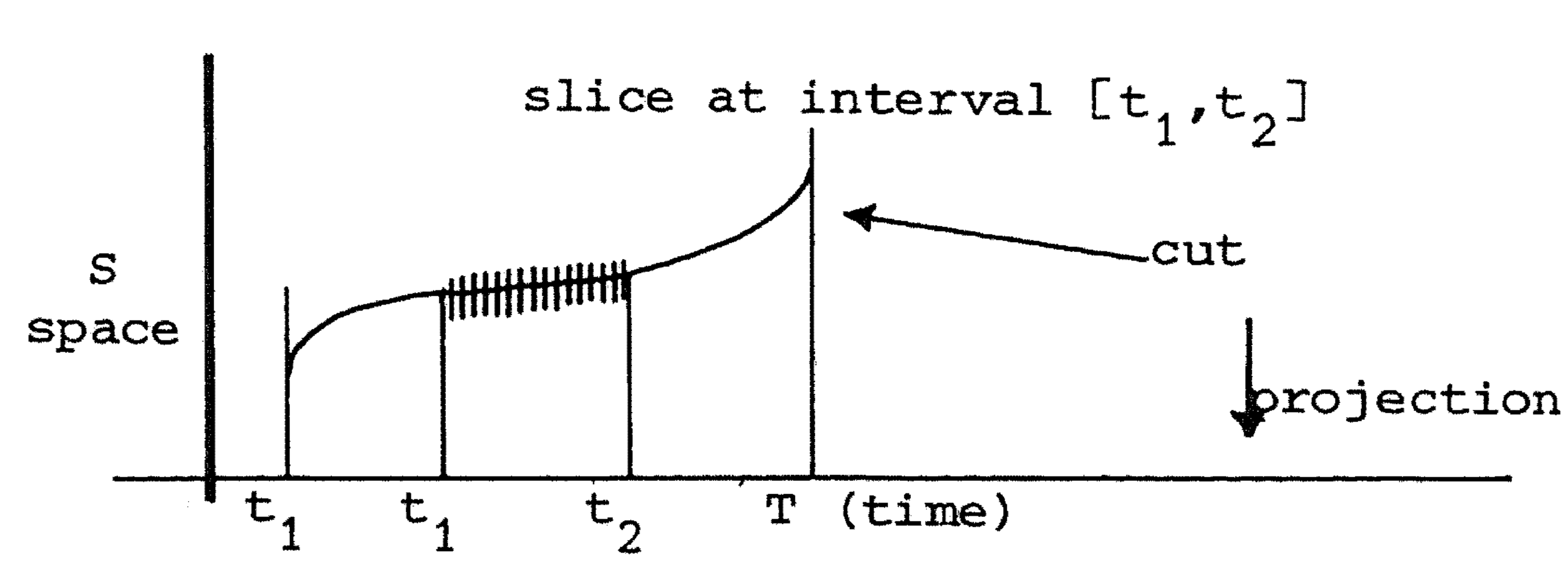
With respect to a point of time we can have an instant slice of an individual, and with respect to intervals of time we have interval slices of the individual. They can be conceived of as sets of instant slices that form a junction. The projection π_t from space-time regions onto intervals of the time axis (defined pointwise) is continuous in the sense that if $\pi_t(R)$ is open, then R is open. Extensions (regions) of individuals can then be represented as continuous cuts over connected intervals of time. The cut is a function from the points of the time interval to the instant slices of the individual.

The function j is a *continuous cut* in U with respect to projection π_t iff:

(1) j preserves time openess and connectedness

(2) $\pi_t \circ j = id_T$.

The slices corresponding to a junction of intervals form a junction of slices.



4. INTENSIONAL LOGIC FOR EVENTS AND OTHER STATES OF AFFAIRS

For the sake of brevity, I omit everything that is similar in other intensional logics, especially the logic of Montague's PTQ. We have as a new type the type of states of affairs (events, etc.), pairs (<s,e>,e). There are:

I. Individual constants of type (<s,e>,e), where e is the type of regions, and s is the type of indices, such that <s,e> is the type of region concepts. These constants denote individuals, events, situations, (etc.). As constants we use *a, b, j, ..., s,* and as variables *x, y, z.*

II. Individual constants of type e: v_1, r_3, etc.; and variables: *r, v.* Expressions of this type denote space-time regions. We have derived expressions of this type: If *a* is of type (<s,e>,e) then $r_a = \pi_2(a)$; that is the expression of the region the individual denoted by *a* occupies. π_2 is the projection of the second place.

III. Constants of type <s,e>, i.e. expressions denoting individual concepts or region intensions: c_1, c_2; and variables *c*. We have derived expressions of this type: If a is of type (<s,e>,e), then $c_a = \pi_1(a)$; that is the expression of the individual concept denoted by *a*. π_1 is the first projection. We can write $a = (c_a, r_a)$.

Further, there are basic and derived predicates of type <(<s,e>,e),t>, <e,t>, and <<s,e>,t>, and expressions of higher types for predicate modifiers, transitive verbs, etc.

<u>EXAMPLE.</u> *Alia* (a storm) : *a*

The interpretation function F maps *a* on F(*a*) which is the individual concept. This is a function from world-region-pairs on regions. (Instead of the index "region", we also can consider the index "time" or "space".) Let w be a world and u a region. The interpretation in our model is:

$$[c_a]^{A,w,u} = F(a)$$
$$[r_a]^{A,w,u} = F(a)(w,u)$$
$$[a]^{A,w,u} = (F(a), F(a)(w,u)).$$

These are slices of Alia at certain regions u. The whole individual Alia we get by interpreting with respect to the universal region: $[a]^{A,w,U} = (F(a), F(a)(w,U))$. For this we write shorter: $[a]^{A,w}$. We can say: Proper names, like *Alia*, refer not just to regions, but rather to conceptualized regions. The same is true of a derived event expression, like *John's meeting Mary*, that does not just refer to the space-time region where they meet, but to this region as conceptualized in this way, namely as a meeting between John and Mary. If a state of affairs s is a situation of John meeting Mary, then the sentence *John meets Mary* is true at the extension (region) of this situation. I shall arrange the translation of gerunds and, accordingly their interpretation, in a way that the following relationships hold for event-(state of affairs) describing statements[4]:

$$\Box\forall s\ (\textit{John meeting Mary}'(s) \leftrightarrow \textit{John Meets Mary at } r'_s)$$
$$\Box\forall s\ (\textit{Sunrising}'(s) \leftrightarrow \textit{The sun rises at } r'_s)$$

With p^* as the event predicate (nominalization) characterized in statement p, and with $p(r)$ for p at r, we get:

$$\Box\forall s\ (p^*(s) \leftrightarrow p(r_s)).$$

The valuation for expressions $p(r)$ is the following:

$$[p(r_1)]^{A,g,u,w} = 1 \quad \text{iff} \quad [p]^{A,g,u_1,w} = 1, \text{ with } u_1 = [r_1]^{A,g,U,w}$$
$$[p(r)]^{A,g,u,w} = 1 \quad \text{iff} \quad [p]^{A,g,g(r),w} = 1.$$

That means, if a non-deictic region expression is included in a sentence the interpretation is independent of the region index, i.e. it remains constant with respect to that index. Further, existence statements come out true for all slices of an individual's life-region, for example:

$$[\textit{Alia exists}']^{A,g,u,w} = 1 \quad \text{for all } u \subseteq [r_a]^{A,g,w}.$$

There are two types of predicates, extensional ones and intensional ones:

(1) *Alia extended over Florida*
(2) *Alia lasted three hours*
have both extensional predicates.

π_s is the projection of a region on space; π_t is the projection on time. Then sentence (1) is true iff $\pi_s(F(a)(w,u)) \supseteq F(\textit{Florida})(w,u)$. Sentence (2) is true iff $M(\pi_t(F(a)(w,u))) = 3$ hours, with M as a measure function established on the time coordinate.

Generally, if δ is an extensional predicate <(<s,e>,e),t>, then we have the meaning postulate:

$$\exists M\Box(\delta(x) \leftrightarrow M(\pi_2(x))).$$

If we define $\delta_* = \lambda v \delta(\pi_1(x), v)$, then, for M we can take δ_*. δ_* is the region predicate corresponding to the event predicate.

(3) *Alia is a storm* (compare meaning postulate for *sunrising*).

$$\Box[storm'(a) \leftrightarrow storm'_*(\pi_2(a))];$$

we can read $storm'_*$ as "carries storm", "realizes storm", or simply "it storms". This means:

$$[storm'(a)]^{A,u,w} = F(storm')^{u,w}(F(a), F(a)(u,w)) = 1 \quad \text{iff}$$
$$F(storm'_*)^{u,w}(F(a)(u,w)) = 1.$$

Intensional predicates range over "conceptualized regions" or "intended objects", like *heavy* in *Alia is heavy*, meaning 'Alia as a storm is heavy', or *good* in *John as a teacher is good*. If we just have *John is good*, the object John will be intended by the speaker as a teacher, as a father, etc., or just as a human being, if the context gives no further specification. We conceive of the individual concept $F(a)$ or $F(j)$ as a set of characterizing properties and a history of the individual, which mainly consists in its realizing properties. The function SEL selects from an individual concept in a certain context and situation the characterizing property that is relevant for the predication of the intensional predicate. In our examples the selected properties from the respective individual concepts $F(a)$ and $F(j)$ are identical with $F(storm')$ and $F(teacher')$, respectively.

$$[Heavy'(a)]^{A,u,w} = 1 \quad \text{iff} \quad [Heavy'_{Ad}({}^\wedge storm')(a)]^{A,u,w} = 1;$$

the complex predicate is treated extensionally, exactly like *storm'*.

Generally we have the following meaning postulate: If δ_{Ad} is the adnominal corresponding to the predicate δ, and γ is the characterizing property that is contextually relevant (= SEL(context, $F(\alpha)$)), and α is a proper name or (in)definite description, then:

$$\Box[\delta(\alpha) \leftrightarrow \delta_{Ad}({}^\wedge\gamma)(\alpha)].$$

This is also applicable to *John as a teacher is good*, or to *This sunset is beautiful*. The interpretation of the intensional predicates *good* and *beautiful* with respect to John and the sunset amounts to interpreting *John is a good teacher* and *This is a beautiful sunset*, respectively.

5. TRANSLATION INTO INTENSIONAL LOGIC AND INTERPRETATION

The operation of gerund nominalization can apply to zero-place verbs (V^0), e.g. *John meets Mary, it rains* etc., or to one-place verbs (V^1), e.g. *dance, eat Piggy, give the book to Mary*.

<u>SYNTACTIC RULE</u>. If α is a verb (V^0 or V^1), then Nom(α) is a noun.

<u>MORPHOLOGICAL RULE</u>. If α is a lexical verb, then Nom(α) = α-*ing*; if α is a complex (derived) verb, then Nom(α) = (X)β_α-*ing* (Y), with β_α as the basic verb, and X, Y adverbials or complements.

<u>TRANSLATION RULE</u>. If α is V^0, then Nom(α) translates into $\lambda s\alpha'(\pi_2(s))$ if α is V^1, then Nom(α) translates into $\lambda s(\exists x\alpha'(x))(\pi_2(s))$.[5]

Thus, the activity verbs *run* and *eat* translate as $\lambda x\ run'(x)$, and $\lambda x\ eat'(x)$, respectively. For their nominalizations we get $\lambda s\ running'(s) := \lambda s(\exists x\ run'(x))(\pi_2(s))$, and $\lambda s\ eating'(s) := \lambda s(\exists x\ eat'(x))(\pi_2(s))$. For the respective action verbs *run a mile* and *eat Piggy*, the translations are $\lambda x\exists y(run'(x,y)\ \&\ 1\ mile'(y))$ and $\lambda x\ eat'(x,p)$; accordingly we have for *running a mile'* and *eating Piggy'*: $\lambda s\exists x\exists y(run'(x,y)\ \&\ 1\ mile'(y))(\pi_2(s))$ and $\lambda s\exists x\ eat'(x,p)(\pi_2(s))$. For every two-place action verb, the corresponding one-place verb is an activity verb. Thus in *John eats* the one-place verb *eat* is an activity verb, and in *John eats Piggy* the two-place verb *eat* is an action verb. The first translates as $\lambda x\ eat'(x)$, and the second as $\lambda xy\ eat'(x,y)$ (or: $\lambda y\lambda x\ eat'(x,y)$ in Montague's PTQ).

The above translations are adequate for sentences like *Running a mile will be rewarded* or *Eating Piggy will be criticized*, but they seem not to be good for the cases with subject control, like *I hate running* or *I hate eating Piggy*. The most efficient way, as far as syntax and semantics is concerned, is to handle these cases as participles, parallel with the infinitives *I hate to run* and *I hate to eat Piggy*.[6] Another possibility would be to transfer the problem to pragmatics, in the following manner: If I hate situations in which somebody runs or somebody eats Piggy, I also hate it, if I myself run, or I myself eat Piggy. Thus, the literal meaning implies the meaning with subject control. In German, *Ich hasse Krach machen* ('I hate making noise') might mean that I hate it if somebody makes noise, while *Ich hasse es, Krach zu machen* clearly has subject control.[7] The same is true for *Ich hasse Schnarchen* ('I hate snoring'), which does not have subject control, versus *Ich hasse es, zu schnarchen* ('I hate to snore'), which has subject control.

The following tentative rules serve to calculate the aspectual character of a complex verb and of gerunds. A verb can be combined with a noun phrase or adverbial that adds to its character the feature IGD "indefinite goal directedness" (object-NP in genitivus partitivus or accusative object-NP with the feature [-count], i.e. indefinite mass nouns and not-quantified indefinite plural NPs, or local adverbials consisting of a local preposition and such a noun phrase, or frequency adverbials. The result is a verb or noun with the aspect "activity" or "iterative act" (cf. rule (1)). Directional adverbials and [+count] accusative object-NPs add to the character of the verb the feature "definite goal directedness" (DGD); this makes from an activity an action (cf. rule (2)). Rules (3) - (5) describe the categorial and aspectual effect of the gerund operation.

<u>Rules of aspectual character</u>:

(1) $$\left[\begin{Bmatrix}\text{Verb}\\ \text{Noun}\end{Bmatrix}, \text{activity}\right] + \text{IGD} = \left[\begin{Bmatrix}\text{Verb}\\ \text{Noun}\end{Bmatrix}, \begin{Bmatrix}\text{activity}\\ \text{iterative act}\end{Bmatrix}\right]$$

(2) $$\left[\begin{Bmatrix}\text{Verb}\\ \text{Noun}\end{Bmatrix}, \text{activity}\right] + \text{DGD} = \left[\begin{Bmatrix}\text{Verb}\\ \text{Noun}\end{Bmatrix}, \text{action}\right]$$

(3) $$\text{Gerund ([Verb, activity])} = \left[\text{Noun}, \begin{Bmatrix}\text{activity}\\ \text{act}\end{Bmatrix}\right]$$

(4) $$\text{Gerund ([Verb, action])} = \left[\text{Noun}, \begin{Bmatrix}\text{act}\\ \text{? action}\end{Bmatrix}\right]$$

(5) Gerund ([Verb, iterative]) = [Noun, iterative].

"Activity" and "action" mean interpretation with respect to an open region, "act" means interpretation with respect to a closed region. Rules for the characters "process" and "state" (open) and "event" and "occasion" (closed) have to be formulated accordingly.

Rule (4) explains the slight deviancy of (3b) *John's eating Piggy looks disgusting,* since the predicate "looks disgusting" requires process/activity/action interpretation (i.e. "open"), while the gerund according to rules (2) and (4) requires act/event interpretation (i.e. "closed") in the first place.

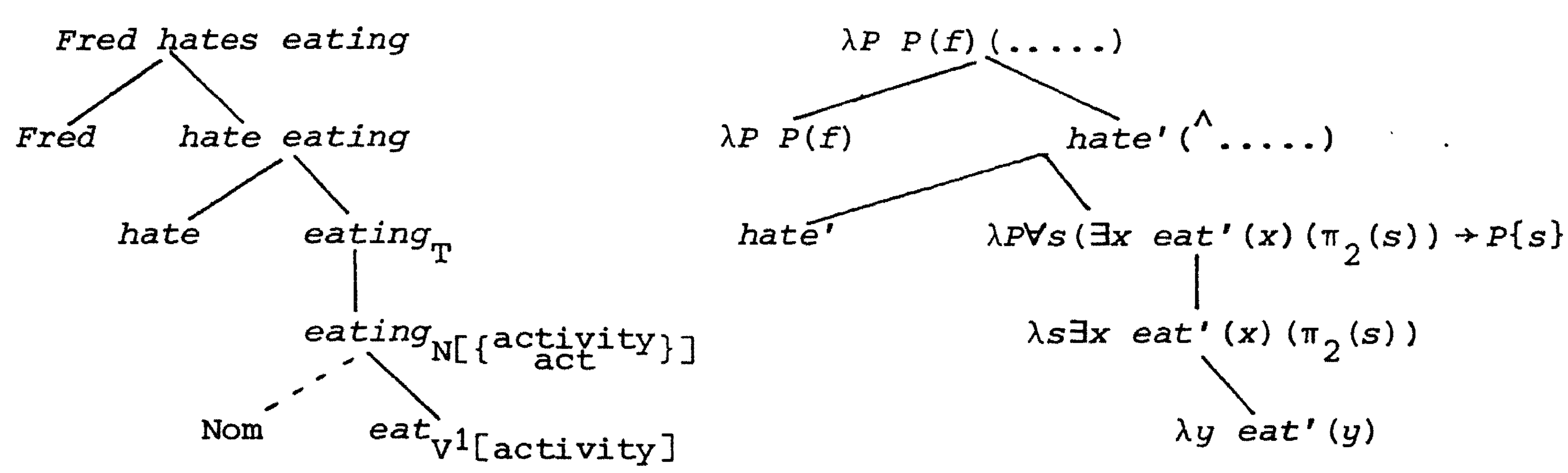
The gerund operation was treated as a syntactic nominalization operation that makes common nouns (N) from verbs. To use gerunds in sentences requires to have term formation rules. In the examples below, except in Example 6, we need rules to form terms with a general, a particular, or a generic reading.

<u>SYNTACTIC RULE SCHEMA</u>. If α is a noun of the subcategory "gerund" then $F_i(\alpha)$ is a term (T), with $i \in \{\text{general, particular, generic}\}$, and $F_i(\alpha) = \alpha$. That means that the term operation is morphologically zero.

<u>TRANSLATION RULES</u>. $F_{general}(\alpha)$ translates into $\lambda P \forall s(\alpha'(s) \rightarrow P\{s\})$, $F_{particular}(\alpha)$ translates into $\lambda P \exists s(\alpha'(s) \ \& \ P\{s\})$, $F_{generic}(\alpha)$ translates into $^\wedge\alpha'$, with α' being the translation of α.

<u>EXAMPLES</u>.

1.



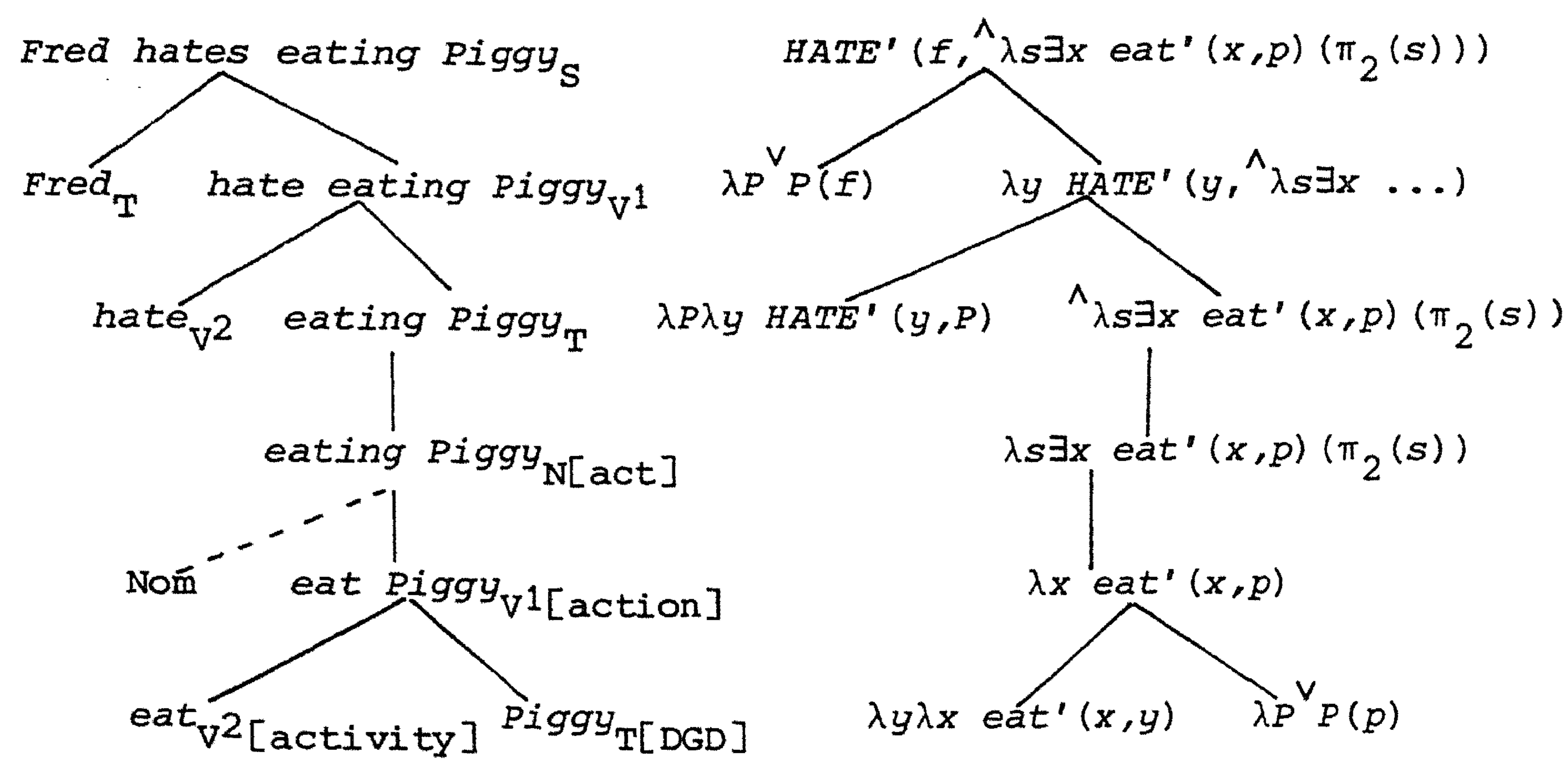
After the possible reductions, we get:

$$\forall s(\exists x \ eat'(x)(\pi_2(s) \rightarrow hate'(f,s)).$$

We also could provide for a generic reading, next to the general reading derived above, with other types: $HATE'(f,{}^\wedge\lambda s \exists x \ eat'(x)(\pi_2(s)))$. Generic readings seem preferable in cases like *Eating is fun, Black is beautiful,* etc.

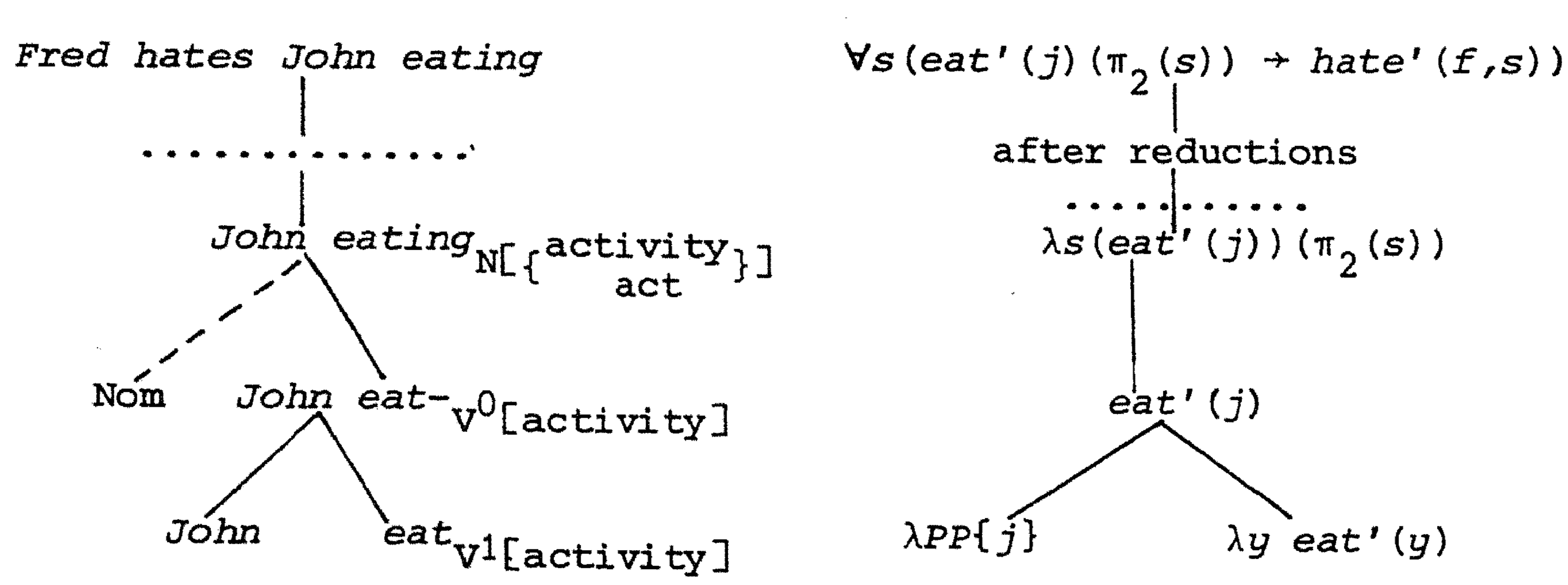
2. *Fred hates eating Piggy, Fred criticizes eating Piggy.*

These sentences will be analyzed without subject control, in the light of what was said above.



This is the translation that gives the generic reading of *eating Piggy*. Parallel with the above, we also can provide for a general reading. Interpreting *eating Piggy* with the character act we have to take the closure of the assignment for *s*: $\pi_2(g(s))$, i.e. the whole act, with "Piggy eaten", is considered. This is not represented in the language of intensional logic.

3.



4. *Fred hates John eating Piggy*

This runs like Example 2, except that *John eat- Piggy*$_{V0}$ gets nominalized. In this case, we get:

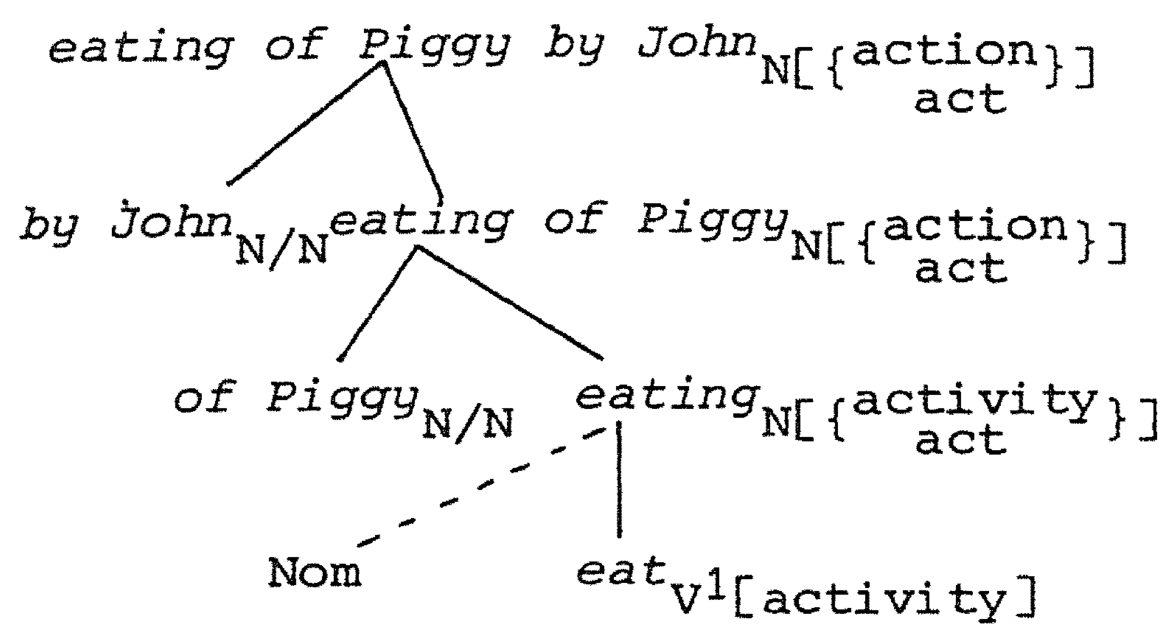
$$\forall s(eat'(j,p)(\pi_2(s) \rightarrow hate'(f,s)).$$

We also could construct a generic reading, like

$$HATE'(f,{}^{\wedge}\lambda s\ eat'(j,p)(\pi_2(s)).$$

In the generic reading we refer to the generic event (compare: generic lion, etc.), while in the general reading, we refer to every individual event of a certain kind. The aspectual character is "act".

5.



The prepositions *of* and *by* are treated as two-place relation-expressions: $\lambda sx\ of'(s,x)$, $\lambda sx\ by'(s,x)$. Without treating adnominals in this context, let us assume the following translations:

of Piggy': $\lambda Q\lambda s(of'(s,p)\ \&\ Q(s))$; *by John'*: $\lambda Q\lambda s(by'(s,j)\ \&\ Q(s))$.

Then we get:

eating of Piggy$'_N$: $\lambda s(of'(s,p)\ \&\ eating'(s))$

eating of Piggy by John$'_N$: $\lambda s(of'(s,p)\ \&\ eating'(s)\ \&\ by'(s,j))$.

By substituting the translation for which *eating'* stands, we get:

$\lambda s(of'(s,p)\ \&\ (\exists x\ eat'(x)(\pi_2(s)\ \&\ by'(s,j)))$.

The interpretation of *of'* and *by'* is very context-specific. In this case *by'* represents "actor of", and *of'* represents "infected object of". From the noun *eating of Piggy by John* a term can be formed in the usual way, either by determiners (quantifiers) or the morphological zero-determiner, translatable by the general or existential quantifier.

6. *John's eating*$_T$ (and likewise:
John's eating Piggy, John's eating of Piggy).

The determiner *John's* can have a general or a particular reading.

John's (particular): $\lambda P\lambda Q(\exists s(P(s)\ \&\ of'(s,j)\ \&\ Q(s)))$

John's (general): $\lambda P\lambda Q(\forall s(P(s)\ \&\ of'(s,j) \rightarrow Q(s)))$

The translation of the term is then straightforward, for example the particular reading:

$$\lambda Q \exists s(eating'(s) \ \& \ by'(s,j) \ \& \ Q(s))$$

John's eating is a mess will then translate into:

$$\exists s(eating'(s) \ \& \ by'(s,j) \ \& \ a\ mess'(s)).$$

Finally, let us compare the readings of the gerund constructions with the corresponding *that*-clause constructions:
Fred hates John eating Piggy (A) translates in its generic reading into

$$HATE'(f, {}^{\wedge}\lambda s\ eat'(j,p)(\pi_2(s))).$$

The corresponding general reading is expressed by

$$\forall s(eat'(j,p)(\pi_2(s)) \rightarrow hate'(f,s));$$

that is, for all occasions at which John eats Piggy it is true that Fred hates these occasions in this respect. The difference with *Fred hates John's eating Piggy* (B) is, that *John's eating Piggy* can have a general or a particular reading, but no generic reading. The general reading of (B) amounts to the same as the general reading of the above sentence (A); the particular reading of (B) implies that there is an occasion at which John eats Piggy:

$$\exists s(\exists x\ eat'(x,p)(\pi_2(s)) \ \& \ by'(s,j) \ \& \ hate'(f,s)).$$

Under the assumption of some appropriate meaning postulate for the use of *by'* in the context of an activity or action verb, this will be equivalent with:

$$\exists s(eat'(j,p)(\pi_2\ (s)) \ \& \ hate'(f,s)).$$

The situation described by this expression has some practical relationship to the one expressed by *Fred hates it that John eats Piggy* (C): If there is an occasion at which *John eats Piggy* is true and Fred hates that occasion in this respect, then it is likely that he also hates the fact that John eats Piggy. On the other hand, if Fred hates the fact that John eats Piggy, he, certainly, will also hate the occasion at which John eats Piggy, as far as this property of this occasion is concerned. But notice, there is no formal connection: The corresponding *that*-clause is no paraphrase of the nominal in any logical or linguistic sense. But there is a relation via the corresponding general reading and the generic reading, and the "nearly equivalence" of the generic reading with the *that*-clause reading:

Generic reading of *Fred hates John eating Piggy:*

$$HATE'(f, {}^{\wedge}\lambda s\ eat'(j,p)(\pi_2(s))),$$

with the interpretation of the gerund as:

$$[{}^{\wedge}\lambda s\ eat'(j,p)(\pi_2(s))]^{A,w,u,g} = \lambda uw[[eat'(j,p)]^{A,w,u,g} \& u{=}\pi_2(g(s))].$$

The restriction is $u = g(\pi_2(s)) = \pi_2(g(s))$.
That means, the interpretation of $[eat'(j,p)]$ is restricted to $g(\pi_2(s))$. Against this, the interpretation of the *that*-clause (C) is not restricted in this way:

$$Hate'(f, {}^{\wedge}eat'(j,p)); \quad [{}^{\wedge}eat'(j,p)]^{A,w,u,g} = \lambda uw[eat'(j,p)]^{A,u,w,g}.$$

Here, we have no restriction on the index u. Facts are not in space and time, i.e. not restricted to situations, while events are. The basic difference between facts and events is thus expressed in semantics, even with respect to the generic event reading.

The difference between "open" and "closed" is not represented in intensional logic. It could be done if we would mark region variables and constants as open or closed. Here, it is treated as an extra interpretation device that says with respect to what kind of regions the expression has to be interpreted. This still has to be worked out formally. Some predicates, e.g. *looks disgusting,* require an interpretation at an open interval, others, e.g. *is disgusting*, permit both, with a slight difference in interpretation: The activity of eating (that is an eating, accidentally of Piggy) can be disgusting, or the act, including the result, of the action, that is the activity in its result directedness, can be disgusting.

The deviancy of the examples with so-called verbal gerunds can be explained semantically in case of (6d) of the examples in the beginning of this paper, since the gerund does not admit a "facticity"-reading, which the predicate "believe" requires. But the deviancy of (2d), (3d), (8d) and (9d) can not be explained semantically, rather by a kind of surface constraint: The subject of the verbal gerund phrase can not function as object of the matrix verb in these examples, as it should according to its accusative case. From this it follows that the verbal gerund can not function as a subject term with respect to the matrix verb.

The model presented here can be elaborated further with respect to time structure and space structure such that the aspects "progressive" and "perfect" get included, as well as time and place adverbials and demonstratives. It, furthermore, presents a natural possibility to interpret tensed noun phrases.

FOOTNOTES

1. For the difference between facts and events see PATZIG 1970, VENDLER 1967, BARTSCH 1976, ULLMER-EHRICH 1977.
2. The internal aspect of verb phrases and the influence of their internal aspect on the aspectual properties of the corresponding nominals has been investigated for German by ULLMER-EHRICH 1977.
3. In previous writings (e.g. BARTSCH 1976), I used different sorts of variables to refer to "closed" states of affairs (events, etc.), and to "open" states of affairs (processes, etc.). Although I repeatedly stressed that the difference is only one of aspect, the use of different variables and constants suggested that different entities would be referred to.
4. A characterization of those statements that are event describing statements has been attempted by KIM 1969; excluded are identity statements, among others.
5. Instead of treating nominalizations syncategorematically, we could also deal with them categorematically with the respective translations $\lambda \mathcal{P} \lambda s \mathcal{P}\{\pi_2(s)\}$ for Nom_0, and $\lambda P \lambda s (\exists x\ P(x))\{\pi_2(s)\}$ for Nom_1, with $\mathcal{P}$ as variable for propositions and P as variable for properties of individuals.
6. By providing for a subject control translation for *hate* (like in BARTSCH 1978) and translating participles by $\lambda x\ \exists s\ run'(x)(\pi_2(s))$ and $\lambda x\ \exists s\ eat'(x,p)(\pi_2(s))$, we get the translation $Hate'(f, {}^\wedge\exists s\ run'(f)(\pi_2(s)))$ and $Hate'(f, {}^\wedge\exists s\ eat'(f,p)(\pi_2(s)))$, which amounts to the *that*-clause reading. The so-called verbal gerunds, then would have to be treated as "accusative with participle" constructions parallel with the "accusativus cum infinitivo".
7. I have to leave it to the judgement of native speakers of English, whether they also get a non-subject control reading for *I hate making noise* or *I hate snoring*. If that would be the case, we should not treat

the subject control of hate+α-*ing* in syntax.

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WHY IS SEMANTICS WHAT?

by

Johan van Benthem

ABSTRACT

"Formal Semantics" has become an enterprise in which both philosophers and linguists are active. This paper aims at conceptual clarification as to exactly what is being achieved in this way. For this purpose, some systematic questions are asked about this field of study, under the following three headings: *what* is it, *how* is it practiced, and *why*? It is claimed that such searching questions are neglected by many semanticists - more "conceptual phantasy" is pleaded for. This plea is backed up by several examples of logico-semantical research which could lead to a less incidental cooperation between the various kinds of participants in the field.

1. INTRODUCTION

Many of us would agree that someone doing pure syntax, be he a formalist logician or a diehard transformationalist linguist, is groping around in a self-imposed dark. For, he need only realize that his language refers to a reality outside- and the helping hand of semantics will open a window through which the fresh air of real life rushes in. Thus, Alfred Tarski achieved Total Enlightenment for extensional logic, Saul Kripke for intensional logic, and- to complete the trinity of church fathers, Richard Montague opened a crack in the wall for natural language.

Now, like amorous feelings, philosophical convictions periodically become in need of heart-searching re-examination. To mention a personal misgiving, from my own research into possible world semantics I emerged with the conviction that the best way to view and study Kripke's truth definition is as being a syntactic translation from a modal language into classical theories of a binary relation.[1] And I could quote similar misgivings on the part of other people.[2] To add to the confusion, not even all misgivings point towards the same conclusion. What about, e.g., the recent tendency in Sneed semantics for empirical theories to suppress all reference to the language in which these are formulated, concentrating exclusively upon classes of models?[3]

The uneasiness caused by such thoughts may be summed up in a variant of Pilate's well-known question: "What is semantics?" Certainly, as a starting point for further reflection such a question is much too vague. But, in this paper, more specific ones will be extracted from it, which are not without consequences for our research. If only philosophical background attitudes were at stake, then one need not bother: after all, in our enlightened discipline "muss jeder nach seiner Façon selig werden".

The merest look at the abstracts submitted for this Colloquium will reveal a caleidoscope of the most diverse interests and activities. If formal semantics is to become more than a mere aggregate of (at best) parallel linguistical and logical research lines, integrative schemes are needed, which manage to make it clear exactly what is meant by the "cooperative enterprise" aimed at in our Classics.[4] Ideally, such a procedure will help us to see more clearly what linguists, logicians and philosophers can learn from each other- if only by way of inspiration. Hopefully, this paper will contribute towards that end by means of its concrete examples, but also by its (rather more numerous) tentative suggestions. At worst I will have charted the full extent of my present perplexity.

2. WHAT IS SEMANTICS?

The subject of semantics could be defined ostensively by citing paradigmatic cases like DAVIDSON & HARMAN 1972. Nevertheless, more systematic definitions are around, like "the theory of meaning of natural language". Now, it would be tedious to repeat the well-known (and well-taken) criticisms of such a phrase- Quinean or otherwise. I take it to be obvious that the crucial terms "meaning" and "natural language" are notoriously problematic. For one thing, there are various types of "meaning" one would wish to associate with language: *psychological* (which mental pictures are evoked by a sentence?), *ontological* (which structures are models for it?), *discursive* (which discourse commitments are embodied in it?), etcetera. For another, the idea of "natural language", as an all-encompassing medium of discourse, may be attractive- but it is *very* boundless. A more workable type of subject for semantical study would be provided by "fields of speech"; rather like the "fields of argument" in TOULMIN 1958.

Now, these are noble- and, therefore, rather unexciting- philosophical views. Does anything follow from them? Here are two concrete examples. When treating modalities, many semanticists will follow Montague in taking over Kripke's referential ("ontological") possible worlds analysis, rather than Frege's pragmatic ("discursive") remarks.[5] More and more people are coming back from this right now- but the point here is rather that such methodological choices are always in need of *justification*. Why would a referential analysis be necessarily preferable?[6] As for the second relativization made above, one immediate implication is that many contemporary philosophers of science are to be counted as doing semantics: of mathematical or physical fields of speech. Such hospitality forces the "ordinary" semanticist (linguist or logician) to think about new problems. E.g., the philosopher's problem of "theoretical terms" and their functioning in a context of "observational" discourse[7] might well become a common concern. One would like to see reactions to the semantical views of Frank Ramsey according to whom theoretical predicates do not possess a pre-given denotation in our models, but rather serve as instructions to create such denotations- surely a realistic position concerning many non-scientific fields of speech as well.

Apart from these particular consequences, the above point of view has more general implications. Many people (the present author included) used to picture the discipline of semantics as a gigantic *coral reef* with ever growing islands which would one day be united to form a continent- viz. a complete semantical description of language. In the above, however, this picture has changed into that of an exotic *archipelago* surrounded by a great nourishing ocean.[8] There is no need to drain that ocean (polder by polder?), and hence generalization and combination of "partial" theories is not a goal in itself.[9]

In the remainder of this paper attention will be restricted to semantics of the denotational type usually encountered in contemporary research. Typically, this brand of semantical analysis for some field of speech requires the following ingredients: a grammar generating the ideal sentences (or texts) of the field ("language"), a description of the kind of "situation" referred to by that field ("reality"), and a systematic connection between the first and the second ingredient ("interpretation"). This semiotic triad suffices for a catalogue of most logical formal languages. Starting from the paradigm case of predicate logic (with the Tarski truth definition in set-theoretic structures), one arrives at ever more complex languages by varying one or more ingredients. It is also a convenient focus for the considerations to come in the following sections.

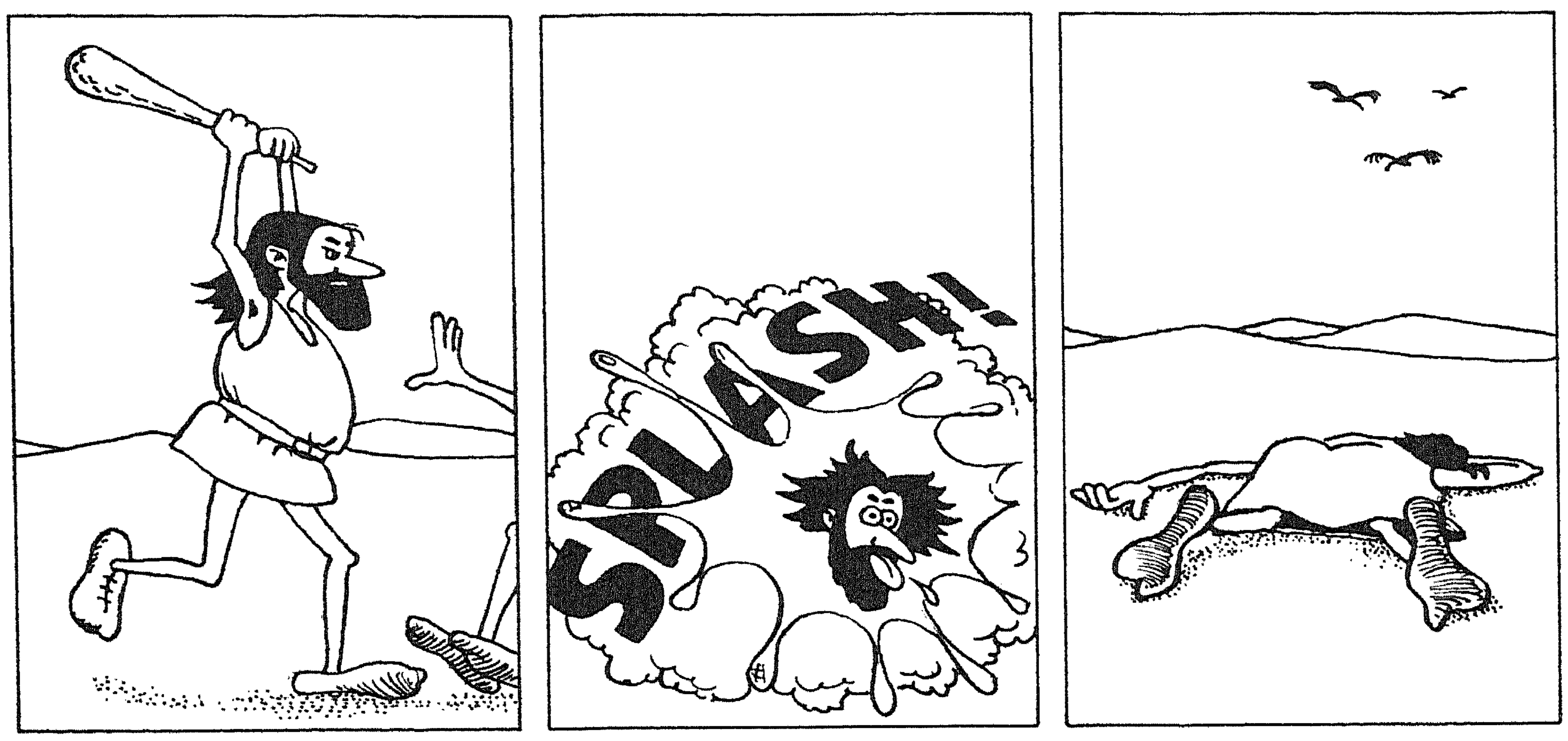
3. THREE QUESTIONS AS TO "HOW"

Before embarking upon semantical adventures, it seems reasonable to think about the choices that will have to be made. Postponing discussion of the deeper philosophical decisions until the next section, we will consider some sample issues of a more methodological nature first. No doubt, careful consideration of such matters has preceded the genesis of all serious papers on formal semantics- but they deserve explicit consideration all the same.

To begin with, here is a question concerning "language". What is it one is going to interpret- sentences, texts, yet other textual units? Evidently, the curious fact that logical semantics has been able, by and large, to get away with interpreting single sentences seems of doubtful significance;[10] the more so since the logical working languages used to formalize actual mathematical fields of speech produce texts rather than isolated sentences.[11] And even if it be true that all texts are in principle reducible to single sentences, the technical reductions involved may well deprive the grammar of efficiency and naturalness.

The standard answer is that a sentence semantics will probably generalize uneventfully to a texts semantics. Still, it may be of interest to point out that the latter move may become unavoidable; by means of a simple example. Suppose one wants to explain comparatives like "taller than" as composites in terms of an adjective "tall". By Frege's Principle of Compositionality, one looks for a corresponding semantical operation- say as follows. A "natural" measure M_{tall} is introduced on the universe of discourse such that the "tall" individuals get suitably high M_{tall}-values, and "taller than" becomes the obvious numerical notion.[12] Now, it is easy to see that this measure is to remain constant throughout whole portions of text: otherwise the inference from "x is taller than y" and "y is taller than z" to "x is taller than z" would not be valid. Consequently, on this analysis, the comparative would have to be interpreted at the text level.

The second question is concerned with the kind of "reality" our field of speech is supposed to describe. What is it that is vital to our semantical theory- the Real World, parts of it, or their conceptual *representations*? Clearly, nobody wishes to deny that a sentence like "Cain kills Abel" refers to some event outside language. One imagines a desolate plain, an ominous light in the sky, an Enrico Morricone tune, and there they are: one man standing, the other lying still. How does reference come about? Well, *of course*:



Cain kills Abel

But, even granting the difficulty in explaining the reference of "kills" in this connection, notice how little of this picture is actually used in our semantical *theory*. There would be individuals c,a in some suitable domain of discourse D such that the ordered couple <c,a> belongs to the extension of "kill" in D. Thus, formally, the above explanation amounts to the poetic phrase <I(Cain), I(Able)> ∈ I(kill). Reality may be at the back of semantical *practice*, but only its representations go into semantical *theory*.

One would expect such a matter to be of vital and lively interest to semanticists. Which type of representation is best suited to the needs of one's selected field of speech? Evidently, set-theoretic structures like above need not always be the obvious choice. After all, even in their homeland of mathematics, a "categorial" revolt is under way, in which classes of set-theoretic structures are replaced by so-called categories consisting of "objects" connected by "morphisms". In this perspective, objects (and morphisms) are no longer determined by their set-theoretic inner structure, but their "functional interaction". Notably, morphisms exemplify a primitive notion of "function" which is no longer to be identified with its extensional input/output-behaviour.[13] Maybe the chronic problem that logically equivalent propositions threaten to become mutually substitutable in intensional contexts like belief sentences will vanish without leaving a trace when "functions" from indices to truth values are taken in this new sense. This is a mere suggestion, of course: the point is just that various alternative representations could be tried.

Another example of current semantical indifference is provided by set-theoretic modelling itself. Philosophers of science have at least worried about the prolixity of set-theoretic structures, trying to reduce their number to the "natural" ones.[14] E.g., given certain domains of individuals one wants to allow no more than one interpretation of the language over them, not pointless variants. There may even be a case for admitting only one single structure for the language, whose substructures form the "situations" referred to by pieces of text. This is an important move, since the whole traditional account of logical consequence will change.[15] Thus, both semantics and logic profit from such a change in perspective, in terms of new research problems.

But, even more traditionally, there are obvious choices. E.g., many people accept Montague's "full" intensional models (with their non-axiomatizable logic) without giving a thought to the possible greater semantical adequacy of "general models" in Henkin's sense[16] - in which not all functions from individuals to truth values are automatically accepted as predicates. And in fact, there are various reasons to take the relevant second-order quantifiers to range over just a restricted set of functions, say the "definable" ones. An observation like this takes the bite out of spectacular claims like the one in HINTIKKA 1974 to the effect that the semantics of English quantifiers "seems to be powerful beyond the wildest dreams of linguists and philosophers of language". Such statements are based upon, amongst others, the non-axiomatizability of valid inference in the branching-quantifier language. But, the latter logical result is *relative* to the underlying semantics: if general models are allowed, then the resulting theory of inference becomes axiomatizable- (without become uninteresting though).[17]

In brief, what is being pleaded for here is a little semantical phantasy. Life becomes much more interesting with semanticists proposing all types of structures- and logicians happily proving or disproving representation theorems.

The third and final question of this section concerns the "interpretation" linking "language" to "reality". Technically, one constructs truth definitions or valuations- whose formal implementation is seldom in doubt. But, what is supposed to be established by such connections? The traditional answer is in terms of "truth conditions": given a structure and a sentence, the truth definition tells us what it means for the sentence to be true in that structure. This account is not to be confused with that of a "truth criterion", enabling one to actually find out if the sentence is true in the structure. The latter requirement is much more demanding- and, some would add[18], the only interesting one. Anyhow, what about a third perspective of "truth instructions" telling one how to construct an ever growing model for a given sentence? Now that the truth condition approach has come to be rather well-understood, the time seems ripe for a fruitful interaction of these different points of view- connecting truth *definitions* with interpretation *procedures*.

In section 5, some relevant logical research problems will be given.

4. THREE REASONS WHY

4.1. General considerations

A certain lack of interest in the conceptual aspects of semantical representation was noticed in the preceding section. Hence arises the danger of a division of labour whereby the logicians think (or rather, dream) up new technical gadgets- while the linguist does the occasional window-shopping. Instead of such passive consumption, one would prefer a closer cooperation in which common research projects give rise to common developments. But, if there is to be any such "rapprochement", clarity of purpose is a neccessary pre-requisite. Which specific *goals* of semantics does one find in the literature? I have found no systematic account in any of my favourite authors: only certain salient recurring topics.

For a start, one often encounters *semantic regulation of syntax* as a goal in itself. Leaving this interesting subject to people expert in both of its components, I pass on to another prominent theme, expressed in the dictum that "implications are part of the linguistic data" (Barbara Partee). Accordingly, one searches for a semantic explanation of sentences which will make their *inferential behaviour* perspicuous: which consequences follow, which ones do not?[19,20]

Which are the data against which one can measure the predictions of a semantical theory? Presumably, there are some raw intuitions as to which inferences in the original language are valid and which are not. But, then, the role of semantical theory becomes similar to that in ordinary logic. There, one has purely syntactic (e.g., axiomatic) approaches to the systematic description of valid inference; giving a catalogue of valid argument semantics. But, as for its *subject matter*, one would do well not to use this predicate. E.g., the fact that one can sometimes give semantical representations using so-called "exact" languages like that of set theory is not by itself of great importance- unless, again, the aim is to *reform* natural language. In fact, the art of formal semantics consists in careful selection of the *minimal* formal apparatus needed to elucidate this or that point of meaning. Over-elaborate set-theoretic machinery that would have caused Cantor nightmares need not be more "exact" than a simple well-chosen ad-hoc notation.

4.2. Specific examples

It is instructive to review the three paradigms of logical semantics from the above points of view.

It all began with Tarski's truth definition of 1933, in which predicate-logical formulas are systematically interpreted in structures presented set-theoretically. More specifically, Tarski showed how to describe the semantics of first-order languages in set-theoretic terms, such that the relation TRUE($\underline{A}$,$\mathcal{D}$) - where $\underline{A}$ is a set-theoretic name for A, and $\mathcal{D}$ is an appropriate structure- admits of a precise definition. This definition produces all equivalences of the form, e.g.,

(+) $\quad$ TRUE($\underline{\forall x \exists y Rxy}$, <D,S>) if and only if $\forall x \in D\ \exists y \in D\ \langle x,y\rangle \in S$;

like it should, according to the "Convention (T)".
What this means is that the notion (rather: relation) of truth for well-organized object-languages admits of an explicit definition in a meta-language that is rich enough: e.g., that of set-theory. Thus, semantical notions turned out to possess precise formulations- and hence proved worthy of meta-mathematical research; witness the resulting logical discipline of Model Theory (cf. CHANG & KEISLER 1973). What this does not mean is that predicate-logical sentences were "explained". If anything, the formula $\forall x \exists y Rxy$ needs *less* explanation than its set-theoretic counterpart in (+). Predicate-logical inference was affected by the new perspective, however. Beth's semantic tableaus could be regarded as a succesful syntactic theory of deduction owing its existence to "semantic regulation".

The age of darkness for intensional logic lasted until 1959, when Kripke first presented his possible worlds semantics. The relevant truth definition could be formulated in the above set-theoretical form: e.g.,

$$\text{TRUE}(\Box\Diamond p, \langle W,w_0,R,V\rangle) \text{ if and only if } \forall w(\langle w_0,w\rangle \in R \rightarrow \exists v(\langle w,v\rangle \in R \wedge v \in V(p)))$$

forms plus rules of derivation- which will, hopefully, produce all intuitively "valid" cases, while leaving out the "invalid" ones. The main drawback of such methods is that negative results ("this inference is not derivable") are usually hard to establish. This is where semantics comes in, enabling one to account for intuitions of non-validity by simply producing suitable "counter-examples". On the other hand, how can it account for the positive intuitions of validity? Some reflection shows that the semantics presupposes, not a mysterious blue sky of Immediate Insight, but a meta-language plus theory of deduction. And it is the latter which supports the required "positive" predictions.

Two remarks are in order here. First, there is no *necessary* connection between a semantic representation language which is perspicuous with respect to the determination of denotation and one which illuminates the theory of deduction.[21] So, the *inferential* aim of semantics is not a priori tied up with the *referential* one. Second, interesting logical questions arise- of which the following is an example.

Suppose there to be a notion $A \vdash B$ of "valid" inference at the level of the original language: either explicit, though complicated (otherwise, there would be no need for further explanation at all), or extrapolated from paradigmatic (non-)examples. Semantics provides a corresponding notion of consequence, in the sense that "all models of A are models of B". Now, the latter statement is established by means of some theory of deduction; say the Zermelo Fraenkel axioms, in case of a set-theoretic representation. Have you ever wondered whether, e.g., the following held true: "the inference $A \vdash B$ is provable in predicate logic if and only if the corresponding semantical statement is provable in ZF"? And what about non-provability? Is $A \vdash B$ non-provable if and only if the corresponding semantical statement is refutable in ZF?[22]

These considerations place inferential semantics in a more general context of, again, philosophy of science. The semantical representation language appears as a kind of "theoretization" of the original language, intended to stream-line its theory of deduction: a common enough phenomenon in science.[23]

"Semantic regulation of syntax" and "theory of inference" are respectable goals for semantics. Nevertheless, has not its main task been brushed aside too lightly at the very outset? Is not the main purpose of a semantical theory to *explain* the original language, rather than to *organize* it? Let us see.

Some text books convey the impression that semantics "explains" the validity of certain principles of *inference*. A popular example are "proofs" in terms of Tarski semantics for predicate-logical principles, like $\exists x \forall y Rxy \vdash \forall y \exists x Rxy$. Closer analysis will invariably reveal that the semantical justification is based upon, at least, the very logic being justified - and usually much more besides.[24] Now, Michael Dummett urges us[25] to distinguish between *triviality* and *circularity* in this respect (what an enviable choice!): good semantic explanations are circular, but not trivial. Most sane people (e.g., our subsidizers) would consider such a phrase the final verdict upon our activities. But, when interpreted in the right frame of mind, Dummett's point appears to be that semantical explanation can only be organization of already existing implicit knowledge.

Even so, surely semantics provides explanations of *words*; or- leaving *lexical* semantics aside as relatively problematic, explanations of the *syncategorematic* words? Is not there a stock of "compositional" words- the grease of our linguistic machinery- which will be explained through obeying Frege's Principle? Well, here is an example- adapted from CRESSWELL 1973:

"More men run than walk" is true if and only if there exist more men who run than men who walk. Now, this is disappointing. Having been exposed for a long time to equivalences like " 'Snow is white' is true if and only if snow is white", one is prepared to swallow the fact that lexical items like "men", "run" and "walk" are not explained- but, at the very least, *some* words should be. All right, then, here is a *real* explanation for "more":

"More men run than walk" is true if and only if there exists an injection from the set of walking men into the set of running men, but not conversely. Granted the set-theoretic concept of "injection", this is a perfectly legitimate explanation. Notice, however, that one may very well have got much more than was bargained for. E.g., this explanation commits one to holding that there are "more" words in the Dutch language than grains of sand at Scheveningen beach; or that there are no "more" instants of time in all Eternity than there are in my typing the dot at the end of this sentence.

What has just been offered, then, is not a harmless "explanation" but a full-fledged *explication* in the sense of those positivist philosophers who practice rational reform of our language. More generally, what has been illustrated in the preceding paragraph is the background dilemma of all formal semantics: providing either mere Formal Dress for informal content, or- at the other extreme- engaging in Rational Reconstruction.[26] Which middle course is steered by the semantical Argonauts?[27]

A short digression is relevant here. A metaphor underlying many formal semantical explanations is that one can sail between the above Symplegades by laying bare the "formal substratum" of syncategorematical words; which can be isolated from their full meaning. In accordance with the Fregean procedure, this substratum may be localized in the formal semantical operation corresponding to the syntactic introduction of the relevant word. Now, consider for instance the phrase "each other" as it applies to predicates to form sentences like

"The monkeys scratched each other".

Can a semantical operation be found on the individual predicate "scratch" so as to yield the collective predicate "scratch each other"? Proposals have been make like the following:

(1) $\lambda A.\ \forall x(Ax \rightarrow \exists y(Ay\ \&\ x \neq y\ \&\ scratch(x,y)))$

or

(2) $\lambda A.\ \forall x(Ax \rightarrow \forall y(Ay\ \&\ x \neq y \rightarrow scratch(x,y)))$.

Nothing quite succeeds, as may be seen by drawing pictures of "scratching patterns" that one would like to call "scratching each other". Here is a good example of our problem. We have a syncategorematic phrase about which we seem to know enough to expect more than a "token explanation". On the other hand, no exact semantic operation is available. There is the usual panacea, or course, of "finitely many distinct, but all crystal clear *readings*" – but it sounds unconvincing. We rather want to say something like the following:

- there are formal "upper and lower bounds" to the meaning range: (2) implies "The A's scratch each other" implies (1).
- the scratching activity in A is to exhibit a certain "regularity"; in some sense to be made formally precise.

Thus, the requirements of Frege's principle for syncategorematic constructions may sometimes have to be weakened to postulating *some* corresponding semantical operation, satisfying certain "meaning postulates" in the form of set-theoretic conditions on its image.

The final remarks of this section are devoted to a term which has occurred at several places in the above, viz. "exactness". It is often claimed that formal semantics promises to become an exact science. In the light of the above, there are several ways in which one could subscribe to such a view- all of them having to do with the explicit *procedures* and *aims* of searching for a "duality" between syntactic notions concerning the sentences and "structural" notions concerning their models. Of course, most of the technical theory one finds in the standard text books has no direct semantic application.[28] But it forms a useful fund of background notions and results which might be explored more systematically. Here are some examples of potential semantic interest. The first of these will, hopefully, transmit something of the atmosphere of duality results.

(i) Let the "regularity" idea in the "each other"-example of section 4 be explained as follows. Each monkey is to behave *exactly* like any other, in that there exists a scratching-automorphism of the whole set of monkeys interchanging the two.[29] E.g., the four monkeys in figs. 1,2 form such a homogeneous pattern, whereas those in fig. 3 do not:

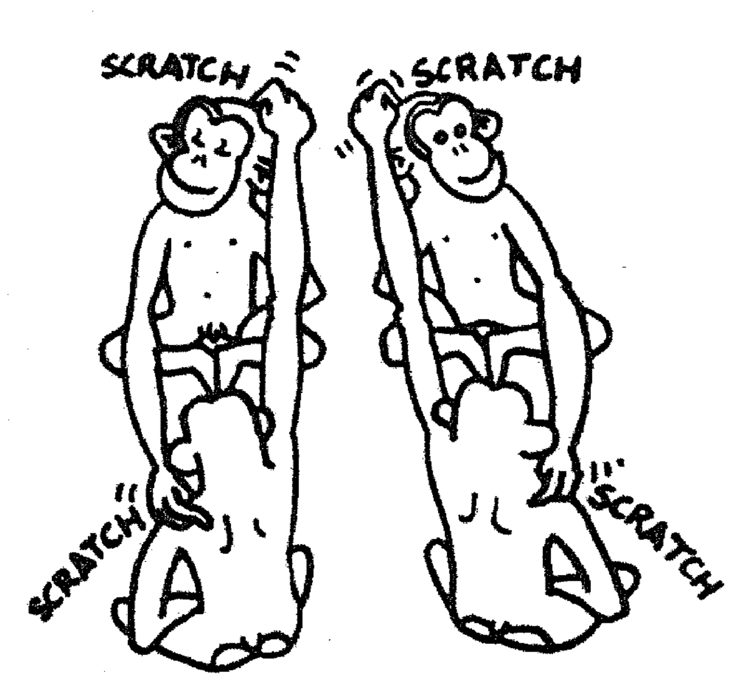


Figure 1

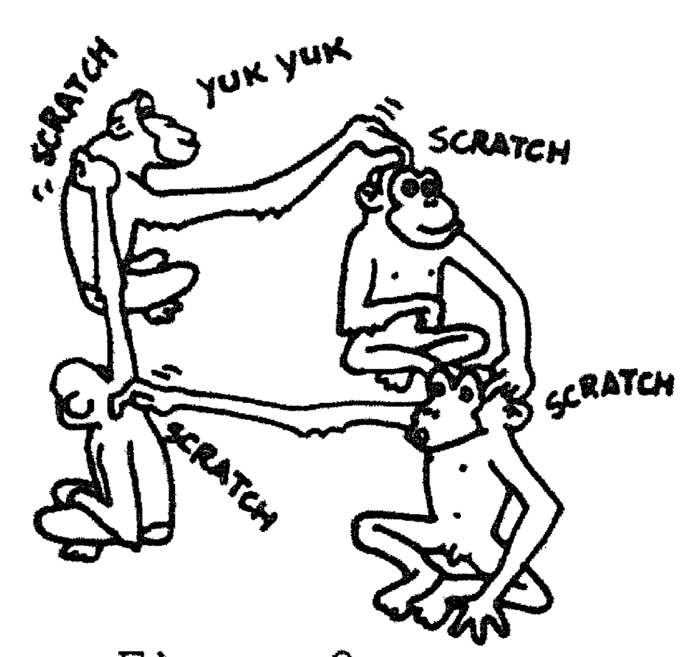


Figure 2

Figure 3

Now, a model theorist might like to know if the possession of a homogeneous model (a "structural" property) has a syntactic counterpart. And indeed it has:

– a set of sentences possesses a homogeneous model *in the above sense* if and only if it is consistent with the totality of all "all-or-nothing" principles of the form $\forall x A(x) \vee \forall x \neg A(x)$ expressible in its language.[30]

(ii) This homogeneity example also served to point out how one may sometimes have to be content with general "structural" conditions on the class of models for a sentence A, without actually being able to give an explicit definition for it. Nevertheless, one is curious which combinations of global conditions will be strong enough to force such a class to become explicitly definable after all. E.g., with respect to the language of predicate logic, "characterization theorems" to this effect exist- be it in terms of the

(When all was said and done, however, an ordinary predicate-logical formulation turned out to be all that was required:

$$\text{TRUE}(\forall x(Rx_0x \rightarrow \exists y(Rxy \wedge Py)), \langle W, w_0, R, V\rangle).)$$

After decades of laborious syntactic calculation, the feeling of liberation was immense. At last one was able to "see" what was *really* going on: in terms of the behaviour of the alternative relation R connecting the possible worlds. Many old questions became solvable without further ado, many interesting new ones arose. Modal logic has become a flourishing discipline with model-theoretic and algebraic connections. All the rhetorical artillery of the preceding sentences is of little relevance, however, to the simple question which explanatory advance was effected by Kripke's ideas. The answer is that it was the first (and, up to date, the only) succesful and clear referential account of the meaning of the modalities □, ◇ . (That it is possibly not the right *kind* of account does not detract from this merit.)

Finally Montague's PTQ-semantics for a certain fragment of natural language displays openly the two-step procedure implicit in Kripke's truth definition. One translates into an intensional typed language having a standard set-theoretic interpretation. The virtues (and vices) of all this are left to more expert chroniclers. Just one point will be of some importance for the sequel. It is sometimes remarked that the intermediate translational stage is, in principle, dispensable with. This is very true. It is equally true- and, in certain respects, much more enlightening- that such intermediate stages can always be introduced whenever convenient: the very presence of well-defined truth conditions guarantees their availability.

5. THREE PERSPECTIVES FOR FURTHER RESEARCH

The previous considerations have shown the desirability of combined logico-linguistical research projects in the area of semantics. No detailed proposals have been made, however. In this section, a more modest course is taken: some new directions are pointed out for *logical* research.

For a start, the traditional perspective of ordinary Model Theory has still got much to recommend it. There, one makes a study from two points of view:

- from a class of sentences to its class of models; and
- from a class of structures to its theory,

admittedly rather esotheric notion of an "ultraproduct". But, one can search for others.

(iii) Model Theory has studied more than just the Tarski truth definition. E.g., there is the *forcing* concept of "A is *verified* in M by the finite amount of evidence p".[31] Semanticists might find uses for the intriguing "generic" models M for which a growing sequence $p_1, p_2, \ldots$ exists of finite amounts of evidence concerning M such that M makes precisely those sentences true (in Tarski's sense) which are already verified by some p_i in the sequence. May not we be allowed to hope that The World is a generic structure?[32] On the assumption that it is, truth conditions and truth criteria become much closer than one would have thought possible.

The above questions may be interesting, but one seldom finds such "advanced" theory in papers on formal semantics- where isolated truth definitions are more of a rule. Now, if the interests lie neither in the conceptual background of the semantic representation (cf. Section 3), nor in the logical model theory based upon it, the following perspective has got much to recommend it: if only as a challenge to make it clear what more it is one is doing. Could not it be that all that is really made use of by most semanticists is the *syntactic translation* implicit in any truth definition?

When viewed in this light, semantics becomes an activity of translation: different ones for different purposes. Predicate logic has an excellent simple theory of deduction: translate your language into it and borrow. Set theory has got an exceedingly simple underlying ontology: translate your language into it and benefit from that as well. Category theory has such a nice way of looking at functions intensionally: so see how you can profit from it, by translating. Etcetera.

This view has several virtues in addition to its austerity. Amongst others, it stresses the plurality of the semantic enterprise- while deflating pompous claims that referential semantics is superior to "mere paraphrase".[33] But, can any solid research be based upon it? Again, three examples will be adduced.

(i) David Dowty mentioned the extreme freedom one has in set-theory when explaining the reference of temporal words, and asked for suitable *restrictions*. One obvious way to implement this is by considering a sequence of tense-logical languages of ascending complexity, and study the behaviour of their definable operations: as possible targets for translating into.

(ii) Indeed, one would like to see more of a logical theory of translation. Given two formal languages, when will there be an effective mapping from one onto the other satisfying Frege's Principle of Compositionality, while preserving valid inferences? Could criteria be developed guaranteeing this?
(iii) The previous question may be connected with special theories. In ordinary Model Theory, one asks when a theory T has got a model- and the answer is forthcoming from the Completeness Theorem: if and only if T is consistent. But, now, focusing attention upon possible translations, what is wanted is rather a so-called "inner model" for T in some other theory T'.[34] In other words, a translation is asked for from the language of T to that of T' mapping T-theorems onto T'-theorems. When do such translations exist? This topic is receiving more and more attention from logicians.[35] To get into the spirit of the thing, the reader might wish to prove-or-disprove the following syntactic version of the well-known Compactness Theorem[36]: "T is interpretable in T' if and only if each *finite* subtheory of T is".[37]

One general reason for carrying out such a "syntactization" of current model theory is that syntactic versions of old results often carry more constructive information than their semantic counterparts. This is one more notable virtue of the syntactic translational perspective.

The third and final perspective is obtained by setting out from ordinary Model Theory in a quite different direction. Up to now, the idea has been that a truth definition correlates sentences as un-interpreted syntactic objects with the class of situations in which they are true. But, in certain respects this is not a realistic picture. More often, a partial interpretation exists already from certain parts of the language to one (or several) situations, and the question is to find suitable "completions". Two examples may make such an idea of a "step-wise interpretation" clearer.
(i) As is well-known, Bernard Bolzano's original definition of logical consequence is not quite the same as the current one.[38] E.g., nowadays an inference like "x is taller than y, y is taller than z; therefore: x is taller than z" would not be called valid by the logic teacher- to the great dismay of generations of students. For, the *schema* "xRy,yRz; therefore: xRz" is not valid, as may be seen by substituting, say, the relation "mother of". Bolzano, however, would have called the first inference valid "with respect to the constituent 'taller'". For, no matter how the other constituents are interpreted, truth of the premises will entail truth of the conclusion.

The appropriate model-theoretic implementation of this notion would seem to presuppose *one* structure in which "constant" constituents have their interpretation given in advance, whereas that of the other constituents may vary. Then, the situations described by fully interpreted sentences could be taken to be substructures of this single "mother structure", with suitable additional denotations for the "running" constituents. The resulting notion of a structure-relative partial logical consequence remains to be studied for most obvious structures.

Another semantical source for partial notions is the following.
(ii) The kind of semantical activity going on when one is reading a text seems to lie half-way between "interpretation from scratch" and "perfect knowledge". One takes the text to refer to a certain situation ("structure"), say the city of Groningen, and "enriches" this structure with denotations for new names or concepts as these are introduced in the text. (Say, it is a novel about a boy "Hans" and a girl "Grietje" afflicted by a mysterious disease called "philosophily".) At once, an interesting question arises: under what conditions on the text can we be sure that the given structure admits of a suitable expansion so as to make the text "completely" true?

Formally, one may ask how familiar model-theoretic results fare in this new setting. Consistency used to be a syntactic guarantee for the possession of a model "from scratch". Likewise, will consistency of a theory T (in a language L_1+L_2, say) taken together with the full L_1-theory of some L_1-structure D suffice for the existence of an expansion of D to some L_1+L_2-structure D^+ which is a model for T? Fortunately, the answer is negative[39]; and interesting complications arise, awaiting further study. Similarly, the analogon to the Compactness Theorem will fail in this setting. (Cf. its failure in the above syntactic perspective.) As it happens, finite "embeddability" of T into D is no guarantee for "total embeddability".[40]

These topics, no matter how sketchily presented, will have shown that there is a non-trivial logical inspiration to be derived from semantics. Such stimuli are necessary to a logic which wants to be more than a handmaiden of mathematics.

6. EPILOGUE

In many areas of research, the following perpetuum mobile produces publications. First, some authors make certain abstractions to get their theories under way. Next, other authors, pretending not to know why these were made, attack them for "neglect of relevant aspects". The intention of this paper has not been to join the latter chorus, even where semantical narrow-mindedness is criticized. Its constructive aim has been to promote greater clarity as to relevant aspects; thus paving the way for further unifying research. Moreover, some pertinent suggestions have been made to this effect. Making suggestions is not, of course, the same as doing honest work oneself. Be that as it may, the heart-searching of the preceding sections may yield its fruits in the end.

FOOTNOTES

1. Cf. VAN BENTHEM 1980a.
2. E.g., in the semantics of certain programming languages, it turns out that relations between languages are all that matters: the machine language, the user language, the mathematical language of problem representation, etc. (This point is due to Bert de Brock.)
3. Cf. BALZER & SNEED 1977/8. In the spirit of the previous metaphor, such philosophers have taken a vow of *silence*.
4. Cf. the editorial introduction to DAVIDSON & HARMAN 1972.
5. Cf. section 4 of FREGE 1879.
6. One is tempted to regret that, although Montague gave us the "method of fragments", he forgot to give us the "method of aspects" with respect to meaning.
7. Cf. NAGEL 1961.
8. Much in the manner of URSULA LeGUIN's "Earth Sea", 1979. Who would deny that contemporary semantics is more exotic, to the ordinary speakers of the languages we study, than LeGuin's witchcraft?
9. Paradoxically, the only interesting generalizations are those which do nót go smoothly- say, owing to "interference" of the various concepts being combined. More specifically, why should new analyses of semantical phenomena always be pasted on to already existing Montague fragements?
10. And even there, theories of natural deduction will usually require attention for contextual phenomena like dependence, cross-reference and definition.
11. Cf. VAN BENTHEM JUTTING 1979 for the AUTOMATH-languages, developed for the purpose of computer verification of mathematical proofs.
12. This is a simple-minded explanation of comparatives, of course- but it

will serve for the purpose of illustration. For a clever explication, cf. KLEIN 1979.

13. Cf. GOLDBLATT 1979.
14. Cf. the semantics for empirical theories given in PRZEŁECKI 1969.
15. Such a policy was investigated in MANDERS 1979, again in a context of philosophy of science.
16. Whose logical theory is investigated in GALLIN 1975.
17. Cf. the more detailed discussion in VAN BENTHEM 1980b (especially section 3).
18. This is the motivation behind intuitionistic semantics; cf. DUMMETT 1977.
19. There is a connection here with the traditional holistic semantics localizing meaning in inferential behaviour.
20. Notice, however, that Montague's PTQ-syntax- which is often claimed to provide a prime example of semantic regulation- does nót make inference very perspicuous; witness the "rodent"-example in Thomason's introduction to MONTAGUE 1974.
21. E.g., the syllogistic schemata form a clear theory of deduction, but a poor theory of denotation.
22. The answer to the first question is YES. To the second one it is NO- since predicate-logical non-provability is not a recursively axiomatizable notion.
23. In the same connection, the "incompleteness theorems" of modal logic (cf. VAN BENTHEM 1980a) show that the intended notion of provability at the level of the modal language is not captured by the obvious notion of provability at the level of the representation language, handling possible worlds with an alternative relation. In more technical terms: this theoretical extension is not *conservative* over the original theory.
24. E.g., the truth table verification of propositional axioms uses not only propositional logic itself, but even arithmetic.
25. Cf. DUMMETT 1977, p.218.
26. A reconstruction which need not always coincide with what these philosophers taught. E.g., the explication of comparatives in KLEIN 1979 goes flatly against Carnap's teachings as regards the relations between classificatory and comparative notions.
27. In the limiting case, where the semantical language is no other than the original one, you are doing lexical semantics after all.

28. Cf. CHANG & KEISLER 1973.
29. For the term "automorphism", cf. the text books. This is just an example, mind you!
30. Students of CHANG & KEISLER 1973 can easily supply a proof.
31. Cf. KEISLER 1977.
32. An exciting possibility pointed out by Kees Doets.
33. "Mere paraphrase" can be a highly creative activity!
34. The earlier uses of "models" in geometry did not even distinguish between the two senses.
35. Cf. LINDSTRÖM 1979.
36. Stating that T has a model if and only if each finite subset of T has one.
37. The answer is NO. What cán be proven is that, if the translations considered map predicates in $L \cap L'$ onto themselves- a reasonable condition-, then the mentioned "finite interpretability" of T into T' implies "total interpretability" of T in some conservative extension of T'.
38. This example may be found explained in more detail in VAN BENTHEM 1979.
39. What ís guaranteed by the assumption is the existence of an L_1-elementary *extension* D^+ of D which is an L_1+L_2-model for T. (Cf. VAN BENTHEM 1978; where also the connection is spelt out with Ramsey's ideas referred to in section 2.)
40. This is not an isolated speculation. In a quite different context, Jon Barwise has shown that there exist abundantly many so-called "resplendent models", which are so rich as to guarantee compactness in the above formulation with respect to any T. (Cf. BARWISE 1976.)

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ON THE WHY, THE HOW, AND THE WHETHER OF A COUNT/MASS DISTINCTION AMONG ADJECTIVES

by

Harry Bunt

1. INTRODUCTION

At various places in the literature on mass terms one can find the suggestion that "mass terms" should not only comprise mass nouns, but also certain adjectives, "mass adjectives" (e.g. QUINE, 1960; MORAVCSIK, 1973). This suggestion is based on two arguments: (1) the semantic argument that certain adjectives have the semantic property, considered to be characteristic for mass nouns (the property of "cumulative reference"); (2) the syntactic argument that certain adjectives ("count adjectives") may not be used to modify mass nouns. Yet, to my knowledge no serious proposals have been put forward so far for a semantic treatment of mass terms using a count/mass distinction among adjectives. This may be due to the fact that both the semantic and the syntactic argument are not clearly convincing though having some intuitive appeal, and are in need of clarification.

I will try to clarify the semantic argument, using the formal semantic theory of mass nouns that I have been developing (see BUNT 1978; 1979) as a starting-point. In this theory, the semantic characteristic of mass nouns is the property called "homogeneous reference". I will extend the concept of homogeneous reference to apply to adjectives and show that the theory predicts the application of an adjective as a restrictive modifier to a mass noun to be meaningful only if the adjective has the property of homogeneous reference. This naturally leads to the suggestion to call these adjectives "mass adjectives" and to conjecture that only mass adjectives can function as restrictive mass noun modifiers. This conjecture then takes the place of the syntactic argument mentioned above.

However, using data provided by native speakers of English and native speakers of Dutch, I will argue that this conjecture has to be qualified. I will argue that an adequate treatment of adjective-mass noun combinations calls for the distinction of a number of semantically different ways in which an adjective can modify a mass noun restrictively. On the basis of a distinction of four types of restrictive modification, the original syntactic argument will be clarified and I will try to formulate accurately what co-occurrence restrictions actually obtain for adjectives and mass nouns.

This paper is in fact a by-product of an investigation into the formal semantics of adjective-mass noun combinations. Regardless of one's position concerning the scope of the term "mass terms", it is clear that any theory of mass term semantics should be able to handle occurrences of mass nouns, modified by adjectives. Most approaches to mass term semantics in the literature do not pay much attention to such occurrences. In fact, some of the most prominent proposals for mass term semantics, such as those of Parsons, Quine, and Moravcsik encounter serious difficulties with certain adjective-mass noun combinations. Even for the purpose of developing a theory that restricts itself to nominal mass terms, it is therefore worth considering in more detail the semantic functions that adjectives can have.

This paper is organised as follows. In Section 2 the concept of mass terms is briefly discussed. The notion of cumulative reference is introduced, together with the semantic argument for distinguishing mass adjectives. In Section 3 the syntactic argument to this effect is introduced and the problems arising with adjective-mass noun combinations in some proposals for mass term semantics are discussed. Section 4 contains a brief review of the relevant features of the theory of mass noun semantics that I have proposed. The notion of homogeneous reference is introduced. In Section 5 it is argued on semantic grounds that adjectives, not having the property of homogeneous reference, cannot modify mass nouns restrictively in a meaningful way. In Section 6 "mass adjectives" are defined as those adjectives that refer homogeneously, and the conjecture is put forward that only mass adjectives can function as restrictive mass noun modifiers. This conjecture is examined in some detail, using empirical data, in Section 7. It is qualified in terms of the different types of modification that are introduced. Finally, in Section 8 the consequences of making a count/mass distinction among adjectives for the lexical component of a grammar are considered.

2. MASS TERMS

The distinction of "mass adjectives" is partly a matter of definition of the concept of mass terms. At the basis of the count/mass distinction is a conceptual distinction of a certain class of nouns. JESPERSEN (1924) introduced this distinction as follows: "There are many words which do not call up the idea of some definite thing with a certain shape or precise limits. I call these 'mass words': they may be either material, in which case they denote some substance in itself independent of form, such as 'silver', 'quicksilver', 'water', 'butter', 'gas', 'air', etc., or else immaterial, such as 'leisure', 'music', 'traffic', 'success', ..."

Nowadays one finds roughly two conceptions of mass terms: a syntax-based one and a semantics-based one.

In the syntax-based conception a certain class of *nouns* is identified as "mass nouns" and contrasted with "count nouns" on the basis of the different syntactic patterns in which they can occur. The classical examples are that a mass noun can be combined with 'much', and 'a little' whereas a count noun instead takes 'many' and 'few', that a mass noun does not permit an indefinite article or a count word and does not have both a singular and a plural form.

However, it turns out to be a tricky matter to define a class of "mass nouns", as opposed to "count nouns", on the basis of syntactic differences in a generally satisfactory way - so tricky that most writers on mass term semantics avoid the issue (see further BUNT, 1979, pp.249-251).

Besides attempts to define a notion "mass nouns" on syntactic grounds, suggestions have also been made to define a more general concept "mass terms", subsuming the class of "mass nouns", from a semantic point of view. The general idea is that there is a semantic difference between "count terms" and "mass terms" which is in the way they refer: a "count term" is always used to refer to discrete, well-delineated entities, whereas a "mass term" is used to refer in a "diffuse" sort of way, without making explicit how its referent is "individuated" or "divided" into individual objects. As QUINE (1960) puts it: "To learn 'apple' it is not sufficient to learn how much of what goes on counts as apple; we must learn how much counts as an apple, and how much as another. Such terms possess built-in modes, however arbitrary, of dividing their reference...consider 'shoe', 'pair of shoes', and 'footwear': all three range over exactly the same scattered stuff, and differ from one another solely in that two of them divide their reference differently, and the third not at all." (Word and Object, p.91).

The diffuse, non-individuating way of referring that mass terms display, is related to the phenomenon that mass terms in general seem to have the possibility of referring to each of certain objects as well as to these objects as a whole. QUINE (1960) has coined the term *cumulative reference* for this phenomenon, and suggested that it is a semantic characteristic of mass terms: "So-called *mass* terms like 'water', 'footwear', and 'red' have the semantical property of referring cumulatively: any sum of parts which are water is water." (Op. cit. p.91).

In this conception there is no restriction of "mass terms" to the syntactic category of nouns; it extends in principle to all categories of referring terms. As the examples in the quotation indicate, we could consider "mass adjectives" besides mass nouns, and we could for instance also think of "mass verbs".

Any conception of mass terms must recognise that the count/mass distinction is not really a distinction among *words,* but rather a distinction among word-senses, word-occurrences, or ways of using words.

On the one hand, almost any "mass noun" can be used as a count noun, as sentence (1) illustrates:

(1) Hungary produces many excellent wines.

The general phenomenon illustrated here is that a mass noun can be used as a count noun with the reading 'kind of ...'.

On the other hand, many "count nouns" can be used as mass nouns with the reading '... stuff'. For instance, the word 'apple' occurs as a mass noun in:

(2) Don't put so much apple in the salad.

PELLETIER (1972) has played with the idea of a machine, the "Universal Grinder", that can chop any object into a homogeneous mass: it would turn a steak into steak, apples into apple, books into book, etc. Indeed, this suggests that at least for any noun, normally used as a count noun referring to concrete objects, one can imagine a context in which it could be used as a mass noun.

3. ADJECTIVE - MASS NOUN COMBINATIONS

It is not immediately clear that a distinction of "mass adjectives", as suggested by Quine, would be a fruitful one. MORAVCSIK (1973) notes that the cumulative reference condition would for instance render 'heavy' and 'large' as "mass adjectives", whereas 'light' and 'small' would be in a different subcategory ("count adjectives"?), though intuitively one would expect these adjectives to belong to the same subcategory. Whether it is fruitful to distinguish "mass adjectives" is largely determined by the question whether such a distinction would have grammatical correlates, such as co-occurrence restrictions. There are indications that adjectives which do not refer cumulatively do not combine well with mass nouns. For instance, the combination 'small water' in

(3) *There is some small water on the floor

does not seem to be acceptable, or at least not to make sense. However, replacing 'small' by 'large', which is cumulative in reference, does not change the situation in this respect. The sentence

(4) *There is some large water on the floor

is equally unacceptable as (3). It therefore seems doubtful that cumulativity of reference would make the difference. McCAWLEY (1975) has drawn attention to the fact that the situation is not the same for all mass nouns: whereas 'large water' is unacceptable, 'large furniture' is reasonable. It seems therefore worth while examining more carefully what co-occurrence restrictions there actually are.

Whatever definition of mass terms is used, the cumulativity and non-individuating nature of their reference are generally viewed as basic semantic properties of mass nouns. Saying that a mass noun refers without individuating does not mean to say that it refers as to a single, unstructured entity. Consider, for instance, the sentence:

(5) All the water in the Rhine comes from the Alps.

Apparently, we have here a universal quantification over the referent of 'the water in the Rhine', and within the subject NP we have a selection over 'water' by means of the modifier 'in the Rhine'. In order to understand such quantifications and selections we must assume that 'water' refers to a structured entity, of which "samples" can be considered for the application of such predicates as 'in the Rhine' and 'comes from the Alps'.

What is meant by saying that a mass noun does not individuate is that it gives no indication of a logical structure of the referent as a collection of "individuals", smallest entities that the noun may refer to. This can be illustrated by comparing sentence (5) with:

(6) All the flowers in the shop come from Holland.

We have similar quantifications and selections here, but the count noun 'flowers' does individuate: in order to verify (6) we check whether the predicate 'come from Holland' is true of each of the individual flowers of which the predicate 'in the shop' is true. We treat the referent of 'flowers' as a set. But treating the referent of 'water' in (5) as a set conflicts with out intuitions: to understand the quantification and selection in this case we consider water-samples to which we apply the predicates 'in the Rhine' and 'comes from the Alps'; these water-samples do not seem to stand in membership-relation to the totality of all water (as the individual flowers stand in membership-relation to the totality of all flowers), for it is in general the case that a water-sample has indefinitely many sub-samples which are again water-samples, and which stand in the same relation to the larger sample as the larger sample to the whole. Water-samples are thus not "individuals" in the sense of minimal portions, and it seems more appropriate to view mass noun referents as having a *part*-whole structure than as having a *membership*-whole structure.

Many authors on mass term semantics have therefore proposed to invoke the use of a logical formalism, called "mereology", which may be viewed as a logic of part-whole relations. It has, besides the part-whole notion, a notion called "sum" or "fusion", by which is meant the operation of forming the "whole" or "totality" of several objects ("mereological wholes"). The sum of two mereological wholes includes all the parts of the wholes being "fused", and may be regarded as the counterpart in this formalism of the union operation on sets. There is also a notion of "overlap": the overlap of two mereological wholes includes the common parts of the two wholes.

QUINE (1960) is one of those who have suggested to use mereology. In particular he proposes to analyse adjective-mass noun combinations such as 'red wine' as denoting the overlap of the mereological wholes denoted by 'red' and 'wine'. (Remember that 'red' is a mass term in Quine's view.) This raises the question how to treat an adjective that is not a mass term, such as 'spherical'. Such adjectives would be treated like count nouns, as denoting sets. But since there is no formalism in which the overlap of a set and a mereological whole is defined, this does not provide a way of interpreting such adjective-noun combinations. Quine's escape is to conjecture that such combinations do not occur: "It is reassuring to note that adjectives, not cumulative in reference simply tend not to occur next to mass terms." (Word and Object, p.104).

I will refer to the conjecture that only cumulatively referring adjectives occur next to mass nouns as the *cumulative combination conjecture*. As it stands, the conjecture is surely not correct, as the following counterexample indicates:

(7) There is small furniture in the doll house.

(See also ALLEN, 1980, n.9.)

MORAVCSIK (1973) has suggested two other, alternative approaches to mass term semantics, using variants of mereology, both of which run into the same problem.

PARSONS (1970) has proposed a semantic theory in which mass nouns are regarded as denoting "substances". Substances occur at one of the three ontological levels of this theory, the other levels being one of physical objects and one of "bits of matter". Two primitive relations, "is constituted of" and "is a quantity of" respectively relate physical objects to bits of matter and bits of matter to substances. Unfortunately, Parsons only gives a quasi-formal characterisation of the substance notion by means of an analogy. Noting the formal similarity between the sentences

(8) Men are widespread

(9) Water is widespread

and observing that the count noun case in (8) is usually viewed as involving a predicate applied to the class of men, Parsons suggests that " ... in general, to talk about substances, we need some sort of higher-order terminology like class terminology in the case of count nouns ... I suggest then, that we introduce a "substance abstraction operatoı", on a par with the class abstraction operator. Let us use σx[...] for the substance abstraction operator. Inside the brackets go formulas which are true of quantities of a substance (pursuing the analogy suggested above). The resulting term is to refer to that substance which has as quantities all and only things which the formula inside is true of; i.e., we are to have:

(10) x Q σy[...y...] if and only if ...x..."

where 'Q' designates the "quantity of" relation (PARSONS 1970, p.375). For instance, the nominal complex "soft clay' is analysed as:

(11) $\sigma x[\text{SOFT}(x)\ \&\ x\,Q\,\text{CLAY}]$.

However, this notion of substance does not rest on safe grounds, as we can see by taking, instead of 'soft', a property which is not cumulative, such as 'spherical'. When we have two bits of matter m_1 and m_2, which are both quantities of a substance M, then the bit of matter m_3 made up by m_1 and m_2 is also a quantity of M. However, if m_1 and m_2 are spherical, m_3 does not need to be spherical. The notion of a substance $\sigma x[\text{SPHERICAL}(x)\ \&\ x\,Q\,M]$, that would have as quantities those and *only those* objects that are spherical quantities of M, is logically inconsistent.

Mass nouns, modified by non-cumulative adjectives, thus cause equally serious problems for Parsons' proposal as for Quine's and Moravcsik's proposals.

4. A THEORY OF THE FORMAL SEMANTICS OF MASS NOUNS

The present study of adjective-mass noun combinations has its roots in a theory of mass noun semantics, described in BUNT (1979) and BUNT (1978). I shall briefly mention here a few features of this theory which are relevant in the context of this paper.

1. "Mass nouns" and "count nouns" are defined as occurrences of nouns (or larger nominal complexes) in particular syntactic patterns. A noun occurrence in an expression that does not provide syntactic clues for deciding whether the noun is used as a mass noun or as a count noun can often be disambiguated in this respect with the help of contextual information, allowing one to construct a variant of the expression which does provide the necessary clues, and where the noun is used in the same way as in the original expression.
2. Semantically, the fundamental difference between a count noun and a mass noun is thought to be a difference in the way of referring. A count noun refers to an entity as a discrete class of objects, while a mass noun refers to an entity as having a part-whole structure, without singling out any particular parts. In particular, a mass noun does not designate certain parts of the referent as "minimal parts" or "individuals". Therefore, mass nouns not only refer cumulatively (Quine: "any sum of

parts which are water is water") but also "distributively": any part of something which is water is water. (At this point I disagree with Quine's "minimal parts hypothesis", see BUNT 1979, pp.254-256.) The combination of cumulative reference and distributive reference is called *homogeneous reference.*

3. The theory focusses on those semantic aspects in which mass nouns differ from count nouns. It is believed that these aspects are separable from those that require a general semantic theory to be intensional. Therefore a purely extensional framework is used.
4. In order to be able to describe in formal terms the way mass nouns refer and to investigate its consequences, I have devised a novel logical formalism for dealing with such notions as "sum" and "part".

Concerning the last point, let me briefly explain why I do not use mereology.

Mereology was devised by Lesniewski as part of an attempt to create a sound logical system for the foundations of mathematics. In particular, mereology would, together with the other components of this logical system, constitute an alternative to set theory which at that time suffered from logical antinomies. Now a fundamental issue to decide when mereology is invoked is whether the other parts of Lesniewski's logical framework are invoked as well; if not, mereology has to be "interfaced" with the more standard logical framework based on modern set theory. Lesniewski would turn around in his grave upon hearing that anyone should try to combine mereology and set theory in one framework. None of the authors on mass term semantics who invoke mereology have given any attention to this problem.

GOODMAN & LEONARD (1940) have developed a logical formalism called the *Calculus of Individuals,* which does in fact "integrate" mereology and set theory into one formalism. In standard set theory one considers no other formal objects than sets: members of sets are again sets. (Alternatively, in some conceptions of set theory sets are allowed to have "individuals" as members, entities that do not have any internal structure or, as far as they have an internal structure it is not a set structure, and set theory therefore does not deal with it.) Leonard & Goodman have devised an axiomatisation of the logic of part-whole, calling the entities satisfying these axioms "individuals". They claim that their calculus is formally equivalent to mereology, but it should be noted that their axioms are formulated in set-theoretical terms; therefore, their system is formally defined only in combination with set theory. In this sense they "integrate" set theory and mereology. Although they do not discuss the integration explicitly, the suggestion is that the axioms of the Calculus of Individuals would simply be added to an axiomatisation of set theory.

Even if this can be done in a formally correct way, there is something unsatisfactory about this approach. The part-whole structure of the individuals is formally the same as the part-whole structure of sets, defined by the subset-relation. It is transitive and reflexive, it serves as a basis for defining such notions as the union (called "fusion" or "sum") and the intersection (called "overlap") of two or more individuals, etc. Consequently, the same part-whole structure is in fact defined twice by the axioms: once indirectly, via the axioms for the membership-relation, and once directly by the part-whole axioms of the Calculus of Individuals. There is thus an awkward overlap in the axiom system. Since the subset-structure of a set has the same logical properties as the part-whole structure of an individual, one would prefer to view a set as a particular kind of individual, viz. an individual that has not only a part-whole structure but also a membership-structure.

In working out this idea, I have devised a formalism with the following properties:

(i) it defines the logic of "individuals" having a part-whole structure without necessarily having a membership-structure;

(ii) it defines sets as formal objects having the part-whole structure of "individuals" and also having a membership-structure.

Since, in this setup, sets are particular instances of "individuals", it seems no longer appropriate to use the term "individual". Instead, I use the term "ensemble", and name the formalism *Ensemble Theory*.

The following features of ensemble theory are relevant to mention here.

There is a primitive relation called *"part-of"* or *"sample-of"*, symbolised as $\subseteq$, which is transitive and reflexive. *Equality* of two ensembles 'x' and 'y' is defined as mutual inclusion: $x = y$ iff $x \subseteq y$ and $y \subseteq x$.

The *"sum"* or *"merge"* of two (or more) ensembles 'x' and 'y' is defined as the smallest ensemble 'z' having both x and y as parts; "smallest" in the sense that any other ensemble having both x and y as parts will have z as a part. If 's' is a collection of ensembles, I will write 'U(s)' to denote the merge of all these ensembles.

Similarly, the *"overlap"* of two ensembles 'x' and 'y' is defined as the smallest ensemble including all common parts of 'x' and 'y'.

The overlap of some ensembles may be empty. The *empty ensemble* '$\emptyset$' is defined by the property of having no other parts than itself. The ensemble axioms make sure that $\emptyset$ is uniquely determined by this definition, and that $\emptyset \subseteq x$ for any ensemble x.

A theorem of central importance for the application of ensemble theory is the following:

<u>THEOREM</u>. *For any ensemble* 'x' *and property* 'P' *there exists a uniquely determined ensemble which is the smallest ensemble including all parts of* x *having the property* P.

I designate this ensemble by:

(12) $$[z \subseteq x \mid P(z)].$$

From the definition of merge it follows that this ensemble is equal to the merge of all the parts of x that have the property P.

Two particularly interesting kinds of ensembles are those called "continuous" and those called "discrete". To introduce these, I first introduce the relation called *"genuine part-of"*, symbolised as '$\bar{\subset}$', defined as non—empty proper part:

(13) $$x \mathrel{\bar{\subset}} y \underset{D}{=} x \subseteq y \mathbin{\&} \neg(x=y) \mathbin{\&} \neg(x=\emptyset).$$

An ensemble 'x' is continuous iff it has a genuine part and each of its genuine parts has again a genuine part, i.e.:

(14) $$(\exists z)(z \mathrel{\bar{\subset}} x) \mathbin{\&} (\forall z)(z \mathrel{\bar{\subset}} x \rightarrow (\exists y)(y \mathrel{\bar{\subset}} z)).$$

In a continuous ensemble one can continue ad infinitum to take ever smaller parts.

To introduce the notion of a discrete ensemble, I first mention the notion of an *"atomic ensemble"*: an atomic ensemble is a non-empty ensemble having no other parts than itself and $\emptyset$.

An ensemble 'x' is now called *discrete* iff it is equal to the merge of its atomic parts.

A continuous ensemble is, clearly, not discrete since it has no atomic parts.

The $\subseteq$-relation of ensemble theory can hardly fail to remind us of the $\subseteq$-relation of set theory. In fact, it can be shown that the subset-relation is a particular case of the more general part-whole relation of ensemble theory. If we add to the axiom system of ensemble theory an axiom saying that all ensembles are discrete, i.e. we restrict the ensemble concept to the discrete case, the resulting axiom system can be proved to be equivalent to the Zermelo-Fraenkel axiom system for set theory. In other words, *discrete ensembles are sets*. Ensemble theory is thus an extension of standard set theory; the notion of an ensemble is a more general notion than that of a set.

Before leaving ensemble theory, I want to mention one of its axioms, relating ensembles of whatever type to discrete ensembles. This is the *power axiom*, which states that for every ensemble 'x' there exists an ensemble 'y' which is the smallest ensemble having all parts of 'x' as elements. This ensemble 'y' can be proved to be uniquely determined and to be discrete; it is thus an extension of the concept of a power set. I will designate the power set of x by $\mathcal{P}(x)$.

5. SEMANTIC CONSTRAINTS ON MASS NOUN MODIFICATION

We have seen above that the treatment of adjective-mass noun combinations as denoting the intersection of the mereological whole and the set, denoted by the noun and the adjective, leads to problems. Use of ensemble theory represents an advance in this respect, since the interplay of sets and non-discrete wholes is well-defined in this formalism. For example, I will interpret a mass noun-adjective combination like 'soft clay' as:

(15) $[x \subseteq \text{CLAY} \mid \text{SOFT}(x)]$,

which denotes the merge of all clay-samples satisfying the predicate 'SOFT'.

Of course, such an analysis is only correct for those adjectives that are *intersective,* i.e. adjectives having the property that the extension of an adjective - noun combination depends only on the extensions of adjective and noun (cf. MONTAGUE (1970); BENNETT (1974).

On the basis of our analysis of adjective-mass noun combinations, illustrated by (15), we can predict that the application of certain adjectives to mass nouns leads to logically strange situations. As a first example I consider the adjective 'heavy'.

Imagine a situation in which I am carrying a heavy bag of sand. Notice, first of all, that I can use the adjective 'heavy' in combination with the mass noun 'sand' in saying:

(16) The sand in my bag is heavy.

Let us focus on that interpretation of (16) where 'heavy' means that a certain weight is exceeded. (On this interpretation, 'heavy' is an intersective adjective.) Suppose I now ask you:

(17) Put the heavy sand from my bag in this container, please.

This is a rather strange sentence, I think, which can be explained on semantic grounds. For what would you do to fulfill this request? You might compare the situation with the case where I have a bag of stones, and you are being asked:

(18) Put the heavy stones from my bag in this container.

A plausible way to proceed in this case would be to consider each of the stones from the bag, decide whether it is heavy or not, put it in the container if it is, and put it back into the bag if it is not. Now let us try to apply a similar procedure in the case of (18): we take some sand from the bag, decide whether it is heavy or not; if it is we put it in the container and else we put it back into the bag. Clearly, such a procedure runs into problems. For instance, when you have done this I can take a small sample from the container, which is not heavy, and accuse you of having put not only heavy sand but also light sand in the container. This is due to the fact that the predicate 'heavy' has the property that, when 'x' is a part of a mass noun extension that the predicate is true of, x will in general have a part 'y' that it is not true of. It is therefore logically impossible to select heavy sand-samples *only*. If, instead of 'heavy', we take a predicate that does not have such a property, like 'dry', there is no problem with sentences of this form. A sentence like:

(19) Put the dry sand from my bag in this container, please,

is perfectly all right. The problem with (18) is thus indeed caused by the adjective 'heavy'.

Generalising from this example we can say that from a logical point of view, a predicate which is to function as a restrictive mass noun modifier should have the property of being *distributive*; formally, a predicate P is distributive iff:

(20) $(\forall x)(P(x) \rightarrow ((\forall y \subseteq x)P(y)))$.

Formally, the problems with 'heavy' can be captured nicely in ensemble-theoretical terms. If we treat 'the heavy sand from the bag' in the way illustrated above (see (15)) for 'soft clay', i.e. as:

(21) $[x \subseteq [y \subseteq SAND \mid frombag(y)] \mid heavy(x)]$,

where 'frombag' abbreviates "from the bag", etc., we can observe two things. Firstly, the fact that any part 'x' of a mass noun extension 'M' which the predicate 'heavy' is true of has a part 'y' which it is not true of, is expressed by:

(22) $(\forall x \subseteq M)(heavy(x) \rightarrow (\exists y \subseteq x) \neg heavy(y))$.

Substituting (21) for 'M' in this formula, we see that the sand we end up with in the container will include parts that are not heavy. Secondly, we can see that 'heavy' cannot be a meaningful restrictive modifier, for there are two situations possible: either (a) there was no heavy sand in the bag at all, in which case (21) is equal to the empty ensemble, or (b) there was heavy sand in the bag, in which case (21) is equal to the ensemble formed by all the sand in the bag. Generally, for any ensemble 'E' we have:

(23) $[x \subseteq E \mid heavy(x)] = \emptyset$ or $= E$.

In other words, 'heavy' cannot possibly restrict any mass noun extension to a nontrivial part.

A second requirement on restrictive mass noun modifiers is examplified by the fact that very similar problems as those arising with 'heavy sand' would arise with 'light sand', as in:

(24) Put the light sand from my bag in this container, please.

When trying to imagine how to fulfill this request, we readily see that we get into the same kind of troubles as in the case of 'heavy sand'. In particular, even if we select only light samples from the bag, we will end up having heavy sand in the container. The source of the trouble is here that the property 'light' is not conserved when two or more samples with this property are merged. Conservation of a property 'P' under merging is expressed in ensemble-theoretical terms by:

(25) $(\forall x)((\forall y \in x)P(y)) \rightarrow P(\cup(x))$,

i.e. the predicate 'P' is true of the merge of any set of objects of which P is true. This is what Quine called "cumulativity". It thus seems that, from a semantical point of view, a restrictive mass noun modifier should be required to be *cumulative*.

As in the case of 'heavy sand', it is illuminating to consider the ensemble-theoretical analysis of 'light sand'. It is easy to see that, just as for 'heavy sand', we have for any ensemble 'E':

(26) $[x \subseteq E \mid \text{light}(x)] = \emptyset \quad \text{or} \quad = E.$

So 'light' cannot restrict a mass noun extension in a nontrivial way.

What has been said about 'light' also applies to 'small', 'low', 'narrow', 'short', 'cheap', etc., and what has been said about 'heavy' also applies to 'large', 'high', 'long', 'wide', 'expensive', 'spacious', etc. In sum, nontrivial restrictive mass noun modification requires the modifier to have the semantic properties of distributivity and cumulativity.

6. 'MASS ADJECTIVES' REDEFINED

Earlier, I have introduced the concept of *homogeneous reference* for nouns as the conjunction of cumulative and distributive reference. I now extend this concept to adjectives, calling an adjective homogeneously referring iff it corresponds with a predicate that is both cumulative and distributive (a 'homogeneous predicate'). It follows that homogeneous predicates meet the two requirements we have formulated for restrictive mass noun modification.

It has been argued that homogeneous reference is semantically characteristic for mass nouns; by defining *mass adjectives* as those adjectives that refer homogeneously, we now obtain a more general notion of "mass terms", comprising nouns and adjectives, which is characterised uniformly by the semantic property of homogeneous reference. All adjectives that do not refer homogeneously are then "count adjectives".

In the light of the above mentioned requirements on restrictive mass noun modifiers, this leads us to the conjecture that *only mass adjectives can function as restrictive mass noun modifiers*. I call this conjecture the "homogeneous combination principle", and list it for convenience of reference as follows:

(27) Only homogeneously referring adjectives can function as restrictive mass noun modifiers (*Homogeneous combination principle*).

If this principle is valid, it gives us good reasons why it would be interesting to make a count/mass distinction among adjectives.

7. SYNTACTIC CONSTRAINTS ON MASS NOUN MODIFICATION

I will now examine to what extent the homogeneous combination principle holds.

Comparing the principle to the cumulative combination conjecture, two differences may be noted: 1. homogeneous reference (cumulative and distributive reference) is taken here as the decisive property, rather than cumulative reference alone; 2. a claim is made here only about adjectives functioning as restrictive modifiers, rather than about adjectives that "occur next to mass terms". The necessity of this qualification is illustrated by the fact that a sentence such as:

(28) I'm tired of carrying the heavy sand in my bag

is quite all right.

We have already come across several counterexamples to the cumulative combination conjecture, and the question arises whether these are also counterexamples to our conjecture. For instance, as a counterexample to the cumulative combination conjecture we have seen sentence (7):

(7) There is small furniture in the doll house.

Another counterexample, suggested by BURGE (1972), is:

(29) There is some cylindrically shaped marble on the artist's workbench.

Actually, I doubt whether this is a very convincing counterexample; isn't (29) a somewhat strange sentence? We have seen similar doubts concerning the sentence

(24) Put the light sand from my bag in this container, please,

which would also contradict the cumulative combination conjecture, and there may also be doubts about the acceptability of (7).

It turns out to be generally the case that sentences, involving mass nouns modified by a count adjective (in our sense of the term) are more or less tortuous and give rise to diverging opinions on their syntactic or semantic well-formedness. We have therefore consulted a number of informants about a veriety of such sentences. 15 Native speakers of English were consulted about 50 English sentences, a number of which involved count adjective-mass noun combinations, and 30 native speakers of Dutch were consulted about similar Dutch sentences. The informants were asked to choose one of the following judgements: "fine", "somewhat strange", "quite strange", or "wrong". To facilitate comparison of judgements, the numerical values 0, 1, 2, and 3, are assigned to these judgements. The number, corresponding to the average judgement of a sentence, is then a certain measure of its "degree of deviance".

It turns out that the sentences (29) and (24) are found unacceptable by most of my informants (degrees of deviance 1.9 and 2.5, resp.); sentence (7) is considerably better but not beyond controversy, having degree of deviance 0.9.

However, there are also quite uncontroversial sentences with a mass noun, modified by a count adjective. Examples are:

(30a) You have heavy luggage.

(30b) The ship was loaded with 300 tons of flexible copper.

Both have degree of deviance 0.1. Also, sentence (7), though criticised by some informants, is found quite normal by a large enough number of informants that we have to take it seriously, and the same can be said about the sentence:

(31) I have heavy sand in my bag.

(Degree of deviance 1.4.)

In order to obtain a good understanding of what these sentences, which at least at first sight are counterexamples to the homogeneous combination principle, mean for this conjecture, we should take a closer look at the semantic relation between the adjective and the noun. I will argue that a good understanding of the situation requires the distinction of four different types of (restrictive) modification, and that the homogeneous combination principle should be qualified in terms of these modification types.

7.1. Collective modification

Among the sentences I consulted informants about are the following two:

(31) I have heavy sand in my bag,

(32) Please remove only the heavy sand from my bag.

Both sentences contain the count adjective-mass noun combination 'heavy sand', but their average appreciation is quite different: (31) has degree of deviance 1.4; (32) degree of deviance 2.5.

How is this difference explained? The difference in the numerical values of the degrees of deviance comes about largely because a number of informants found (31) quite fine, while *all* informants, without exception found (32) more or less deviant. Several informants who judged (31) "fine" motivated their judgement by saying that it is fine only if interpreted as:

(33) I have a heavy bag of sand.

In this reading, 'heavy' is considered as applying to the (bag of) sand *as a whole*, a reading which is not present for (32).

The difference between (31) and (32) is parallelled by the difference in quantification type in the sentences:

(34) The sand in my bag is heavy

(35) Most of the sand in my bag is heavy.

The first of these sentences has a collective reading, the second one does not. In view of the analogy, I call the type of modification occurring in (31) on the reading (33) *collective modification*.

Mass nouns may be modified collectively by count adjectives. We thus have to qualify the homogeneous combination principle; more accurately, it says:

(36) Only homogeneously referring adjectives can function as non-collective restrictive mass noun modifiers (Homogeneous combination principle, second formulation).

7.2. Generic modification

Sentence (30):

(30) The ship was loaded with 300 tons of flexible copper

presents another case of a mass noun modified by an adjective that may be said to be non-homogeneous, since some flexible pieces of copper together may form a piece, too thick to be flexible.

Could we have a case of collective modification in (30)? Surely we don't, assuming it is not the totality of the 300 tons of copper which is meant to be flexible. Could we have a case of non-restrictive modification? In principle we could, but then consider the sentence:

(37) The ship was loaded with 300 tons of flexible copper and 500 tons of hard copper.

'Flexible' is clearly a restrictive modifier here. This sentence has degree of deviance 0.2, which means that it is generally accepted as correct.

As in the case of collective modification, it is instructive to consider a parallel case of quantification. Take the sentences:

(38a) Copper is more flexible than steel.

(38b) Bolivian copper is more flexible than Antarctic copper.

(39) This bracelet is made of flexible copper.

The quantification in the sentences (38) is usually called "generic", since something is said in these sentences about kinds ("genera") of copper and steel. Interpreting (39) as:

(40) This bracelet is made of a flexible *kind of copper*,

we can say the same about (39), and I therefore call this type of modification *generic modification*. It is in this way that sentence (39) is accepted by my informants, viz. interpreted as:

(41) The ship was loaded with 300 tons of a flexible kind of copper and 500 tons of a hard kind of copper.

This type of modification is also present in such expressions as 'heavy sirup' and 'light oil', where 'heavy' and 'light' refer to the *specific* *weight* of these liquids.

It is worth noting that adjectives like 'heavy' and 'flexible', when functioning as generic mass noun modifiers, are used in a sense in which they are in fact homogeneous, since any sample of the noun denotation has the specific properties of the genus. Therefore, sentences like (30) do not really present counterexamples to the homogeneous combination principle.

Instead of saying that adjectives which are not homogeneous in their "normal" use also have a "specific" sense ('heavy' = having a high specific weight, etc.), one might prefer to say that these adjectives can be used in combination with a mass noun in such a way that they acutally apply to "genera" of the noun denotation. In that case the homogeneous combination should be qualified accordingly. Since it seems harmless to do this anyway (when formulating the principle in its original form we did not mean to say anything about generic modification), we will do so:

(42) Only homogeneously referring adjectives can function as non-collective, non-generic restrictive mass noun modifiers (Homogeneous combination principle, third formulation.)

7.3. Homogeneous and discrete modification

There are still other cases of acceptable count adjective-mass noun combinations which cannot be explained in terms of collective or generic modification. We already came across the following examples:

(7) There is small furniture in the doll house.

(30a) You have heavy luggage.

One might think that 'small furniture' and 'heavy luggage' are acceptable due to the fact that the mass nouns 'furniture' and 'luggage' are naturally associated with certain discrete objects: chairs, tables, ... and suitcases, bags, etc. However, the following sentences show that this explanation does not work:

(43) *There is small apple in the salad.

(44) *You have heavy sausage on your plate.

Though 'apple' and 'sausage' are naturally associated with discrete apples and sausages, the combinations 'small apple' and 'heavy sausage' in these sentences are deviant. Apparently, the existence of a strong association of a mass noun with a discrete class of objects is not sufficient to allow a count adjective to modify the noun restrictively in a non-collective, non-generic way.

Let us consider a mass noun of which the extension does not naturally come in discrete parts, such as 'sugar'. Imagine the following situation. We are in a sugar refinery and are given an explanation of the process. We are being told that

(45) The wet sugar, resulting from the second stage of the process, is dried here.

(46) This container contains only wet sugar.

The sugar leaves the refinery in various forms, among which are lumps of two different sizes: small cubical lumps and larger rectangular lumps. When we get to the packing department, we are being told that

(47) The blue boxes are filled with cubical sugar, the red boxes with rectangular sugar, and the white boxes with a mixture of cubical and rectangular sugar.

(48) The blue box contains only cubical sugar.

Now let us consider the sentences (46) and (48) more closely. They have implications that can be expressed by sentences, presenting cases of quantification parallelling the cases of modification we have with 'wet sugar' and 'cubical sugar':

(49) All the sugar in this container is wet.

(50) All the sugar in the blue box is cubical.

There is a difference between these sentences in that (49) can very well be understood as asserting that all sugar-samples in the container are wet, while (50) cannot sensibly be understood as asserting that all sugar-samples in the box are cubical - only the *lumps* of sugar are meant to be cubical. Whereas the quantification in (49) ranges over all the sugar in the container, in (50) it ranges only over the set of sugar-lumps in the box.

Similarly, the domain of application of the modifier 'wet' in (46) consists of all sugar-samples, while in (48) the domain of application of the modifier 'cubical' is restricted to a particular subset of sugar-samples (the lumps). The former type of modification I call *homogeneous,* the latter type *discrete*. Homogeneous modification occurs when a mass noun modifier has the (power set of the) extension of the mass noun as its domain of application. Discrete modification occurs when a mass noun modifier has a contextually determined subset of the power set of the noun extension as its domain of application.

When we have a mass noun 'm' with extension 'M', a contextually determined subset $\mathcal{P}_c(M)$ of the power set of M, and an adjective 'p' denoting a one-place predicate 'P', I treat homogeneous modification of 'm' by 'p' as denoting the noun extension M restricted to the part, made up by all the parts of M for which P is true, i.e. as:

(51) $[x \subseteq M \mid P(x)]$

and discrete modification of 'm' by 'p' as denoting the subset of $\mathcal{P}_c(M)$ consisting of those contextually determined M-samples that P is true of, i.e. as:

(52) $\{x \in \mathcal{P}_c(M) \mid P(X)\}$

Since count adjectives are typically adjectives of size, shape, or other aspects of outward appearance, which can be applied sensibly only to well-delineated objects, it is not surprising that *homogeneous* modification of a mass noun by a count adjective leads to non-sensical situations, as we have seen in Section 5, but that a count adjective under certain conditions can apply to a mass noun as a discrete modifier. In the latter case we have counterexamples to the conjectured homogeneous combination principle (42). I will now turn to these conditions and their consequences for the conjecture.

7.4. The homogeneous combination principle revisited

According to the views on the fundamental semantic difference between count nouns and mass nouns, set forth in Section 4, the use of a mass noun in general entails that no reference is made to well-delineated physical objects. This is presumably the reason why count adjective-mass noun combinations are almost invariably tortuous. For instance, the sentence

(7) There is small furniture in the doll house,

which was used to illustrate the phenomenon of "discrete modification", is judged "fine" by only 50% of my informants (it has degree of deviance 0.9); the usual comment is that (7) is not wrong, but that it would be "better" to say:

(53) There are small pieces of furniture in the doll house.

A necessary condition for discrete modification of a mass noun is that it is contextually clear which individuation of the reference is intended. For some mass nouns there are more or less standard ways of individuating the reference (as for 'furniture', 'luggage', 'footwear', 'fruit'). For other mass nouns certain individuations are common in specific contexts only; for instance, restaurants and bars provide a context in which many mass nouns referring to food or beverage of some kind are commonly individuating. For almost any mass noun it seems possible to construct a context where the noun could be used with a particular individuation in mind - which means that it becomes semantically like a count noun. (Such mass nouns also tend to be used syntactically as count nouns in those situations; think of 'one coffee', 'two beers', etc.)

Does this undermine the homogeneous combination principle completely? I think not.

First of all, the possibility of non-generic, non-collective modification of a mass noun by a count adjective is restricted to discrete modification, which is only possible in case the use of the mass noun is to be interpreted as elliptic for a non-mass expression: 'sugar' for 'lumps of sugar', 'beer' for 'glasses of beer', etc.

Secondly, there is a class of mass nouns for which discrete modification does not seem possible, except perhaps in such peculiar circumstances that it gives rise to language use, mostly considered as deviant. This is the class of mass nouns that are also commonly used as count nouns, and where the count noun denotes a certain set of parts of the mass use denotation. Another way of characterising this class is in terms of Pelletier's Universal Grinder: the count use refers to the discrete objects considered as inputs to the grinder, the mass use to the homogeneous mass produced by the grinder. We might therefore call these mass nouns "*ground nouns*". Examples of "ground nouns" are: 'apple', 'cake', 'stone', 'onion', 'hair', 'rope', 'diamond', 'rock', 'ice cream', etc.

Sentences like:

(54) *This box is filled with heavy stone
(55) *There is round pancake on the plate
(56) *Don't put so small onion in the salad

are mostly considered deviant by my informants, with the explanation that one ought to say, instead:

(57) This box is filled with heavy stones
(58) There is a round pancake on the plate
(59) Don't put so small onions in the salad.

Generally, we can say that discrete modification of a "ground noun" is deviant, because there are two possibilities for the individuation that the modifier presupposes: either (1) it is the individuation of the count use of the noun, in which case one is supposed to take the count use instead of the mass use, or (2) it is a different individuation; because it is an individuation differing from the standard one, lexicalised as the count noun, one would be expected in this case to indicate explicitly which individuation is intended.

Summarising, what is left of the homogeneous combination principle are the following constraints on restrictive mass noun modification:

(60) a. Restrictive, non-generic modification of a mass noun by a count adjective is only possible in the form of collective or discrete modification. Homogeneous modification of a mass noun can only be achieved by a mass adjective (homogeneously referring adjective).

b. "Ground nouns", mass uses of nouns also commonly used as count nouns and where the two uses are semantically related by the "Universal Grinder" idea, cannot be modified discretely. Non-generic, non-collective modification of such nouns can only be achieved by mass adjectives.

Compared with the conjecture (27) that we started out with, the principle in its final form (60) is more modest and more detailed. The original conjecture is in fact maintained for "ground nouns", while for other mass nouns qualifications have been added in terms of the various types of modification that have been distinguished.

8. CONCLUSIONS

I think we can conclude that the homogeneous combination principle that we end up with, though perhaps less spectacular than one might have hoped at the outset, still captures a class of linguistic phenomena interesting enough to warrant a count/mass distinction among adjectives.

The principle enables us to filter out unintended readings of sentences, involving count adjective - mass noun combinations. In particular, if we meet a count adjective - "ground noun" combination, only a collective or a generic interpretation is possible; the homogeneous interpretation, which would be semantically deviant (cf. Section 5), and the discrete reading, which would be syntactically deviant (Examples (54) - (56)), are ruled out by clause b of the principle.

I conclude that the question *why* a count/mass distinction among adjectives would be useful can be answered by pointing at the homogeneous combination principle.

In order to make use of the principle, one should not only distinguish between count adjectives and mass adjectives but also between "ground nouns" and other mass nouns. Would it seem feasible to make this distinction in a grammar? I think it would be very sensible to make this distinction anyway: as the idea of the Universal Grinder makes clear, virtually every concrete count noun has a potential mass noun use, of which the meaning can be derived from the count noun meaning; it would be very uneconomical to include both the mass use and the count use in the lexicon. It would surely be preferable to include as lexical items only those mass nouns which are not "ground nouns", such as 'water', 'furniture', 'sand', etc., and to derive the "ground nouns" from lexical count nouns by the rule associated with the Universal Grinder. This rule could mark the mass nouns in question as "ground nouns", thus giving us the desired distinction.

The question *how* a count/mass distinction among adjectives could be made, has been answered here by taking homogeneity of reference as the defining characteristic of mass adjectives. Does it seem feasible to identify mass adjectives on this basis? It seems that there are very few adjectives of which we can say with certainty that they name a homogeneous property (examples are 'ripe' and 'well done'). For many adjectives this is either not clear, or we would have to distinguish various senses, some of which refer homogeneously and some of which don't. This does not seem very attractive. Fortunately, in order to make use of the homogeneous combination principle it is required in the first place that we identify count adjectives, adjectives that do not refer homogeneously. Now there are a number of adjectives of which we can safely say that they do not refer homogeneously, such as adjectives of size or amount ('large', 'small', 'great', 'tiny', 'huge',... 'heavy', 'light', 'expensive',...), or of shape or other aspects of outward appearance ('spherical', 'oval', 'round', 'cubical',... 'dusty', 'shiny', 'scratched', etc.). The extent to which it is worth while making a count/mass distinction among adjectives depends the comprehensiveness of the class of adjectives for which we can decide that they do not refer homogeneously.

Altogether, I conclude that making a count/mass distinction among adjectives could be rewarding in view of the homogeneous combination principle. The distinction would have to be made on the basis of the property of homogeneous reference. It certainly seems feasible to make the distinction in a grammar, since it is only required to identify adjectives that do not refer homogeneously (count adjectives); the more of these we can identify, the more valuable the distinction will be. It only makes sense to make the distinction if we also make a distinction between "ground nouns" and other mass nouns, but this is a distinction that would be desirable to make anyway.

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THE CONTRACT GAME

by

Östen Dahl

In DAHL 1977, I discussed various 'language games' that could be used as partial simulations of linguistic interaction. One of these games was called 'the Generalized Contract Game'. The main idea behind this game was that one could analyze a fairly large number of speech act types as parts of a contract concluding process. When I started working on the present paper, my intention was to provide a more systematic and complete treatment, preferably formalized, in which this idea would be further developed. The result is not quite what I intended: what has come out is rather a collection of more or less loosely organized thoughts on the subject.

Before getting in medias res, a few remarks on the term 'language game' may be in order. In the comprehensive account I was envisaging this term is of course motivated since one would 'simulate' linguistic interaction in terms of a set rules for how the rights and obligations of the participants change as the interaction moves on. In the slightly 'softer' approach chosen here, the term may be used more loosely to stress above all the fact that speech acts should not be seen in isolation but as integral parts of sequences of linguistic and non-linguistic acts where several people take part.

The main idea referred to above, then, could be formulated more explicitly in the following way:

Every 'round' in a language game, i.e. a minimal independent piece of linguistic interaction, will be assumed to have the following normal form: It consists of a proposal made by one or more participants and the ensuing acceptance or rejection of the proposal by the group that the proposal was directed to. The content of the proposal is that the participants jointly accept a proposition or a set of propositions as true. In addition, it may involve assigning to one or more participants the responsibility for seeing to it that the proposition(s) come(s) true. In other words, if the proposal is accepted, the result is that the participants involved have entered a kind of contract.

Let us see what this would mean for some common types of speech acts. Consider first assertions.

When describing what happens in an act of assertion, one often thinks of one person who tries to make another person believe something. In the analysis we are proposing, an act of assertion would instead be a proposal to a group of people which includes the speaker to accept one or more propositions as true. What us the difference?

The contract analysis may be said to stress or highlight some aspects of what happens which may be neglected under other analyses. Thus, the contract analysis stresses the social rather than the individual level. One aspect of this is that we clearly see the similarity between what the speaker does and what the listener(s) do. Consider the following trivial conversation:

(1) A: Carter will win the election
B: Yes.

By (1), both A and B have committed themselves to the truth of the proposition that Carter will win the election. One consequence of this is that the proposition can be treated as a 'pragmatic presupposition' later on in the discourse and denying it once it has been accepted as true is inconsistent behaviour irrespective of whether it is the speaker or the listener who does it.

What does 'accept as true' mean in this context? One way of explaining it would be to say that 'accept as true' is the same as 'agree to behave as if it were true'. For example, consider the following piece of conversation:

(2) Doctor A: The patient has cancer
Doctor B: Yes.

By (2), the two doctors have committed themselves to undertaking the proper actions that are entailed by the diagnosis they have made.

Notice the difference between 'accepting as true' in this sense and 'believing that something is true'. Cf.

(3) Teacher: Why are you late today, John?
John: My mother forgot to wake me up
Teacher: Okay.

The teacher may not believe that John told the truth. This, however, is irrelevant on the social level - by saying *OK* she accepted it as true, i.e. she has accepted to refrain from any sanctions.

The dyadic case of linguistic interaction, viz. where there is one speaker and one listener, which is often treated as paradigmatic, comes out as a special case in this approach. The first part of (3) could well be pronounced by a member of a committee of doctors discussing the condition of some important patient. This would then be a proposal to the committee to accept the proposition that the patient has cancer as true. In the dyadic case, a statement is simply accepted or rejected by the listener. But in the committee case, there may be a more or less formalized procedure for accepting or rejecting a proposal (by voting for instance).

Consider now a claim made in a scientific paper. This may be seen as a proposal to the entire scientific community to accept a certain proposition as true. As long as the community has not done so, the claim is 'pending'.

Let us now consider propositions which concern the future, e.g.

(4) Dinner will be ready at five.

If I propose to a group of people to accept (4) as true, I also exhort them to behave accordingly, that is, to come to the dinner table at five. On their part, they have the right to *expect* that dinner will be ready at that time, and if it is not, they will at least have the right to get angry. In other words, as speaker, that is, as the person making the proposal, I assume a certain responsibility for the truth of the proposition. This is even clearer if it is obvious that the proposition concerns something which is under the control of the speaker, e.g. if I say

(5) I shall see to it that dinner is ready at five.

In (5), we are clearly dealing with a promise. But it is not always the case that the speaker alone takes the responsibility. Consider the following exchange:

(6) A: Let's meet at the station at five
B: Okay.

Here, A makes a proposal for *joint action*. This is perhaps the 'prototypical' contract, which involves on the part of both participants a responsibility - to show up at the station at five - and a right - to expect that the other shows up. In other words, there is a complementary relation between responsibilities and expectations. To illustrate this even clearer, consider again (4). It involves the following rights and obligations:

	Husband	Wife
Expects	Dinner is ready at five	Husband is home at five
Undertakes to	Be home at five	Make dinner at five

Let us try to systematize what has been said so far and try to classify different kinds of situations that can be obtained.

In all the situations that interest us here there is a Proposer (=the first speaker). Furthermore, there is a set of participants who make the decision as to whether the proposal should be accepted or not. Let us call these the Decision Makers. Finally, we may or may not have one or more persons who assume responsibility for part of or all of the proposal to come true. Let us call them the Responsible Agents.

We may first distinguish situations with and without Responsible Agents. We may refer to the contracts concluded in situations with Responsible Agents as *imperative contracts* and those without as *declarative contracts*. Further, we note that if the Proposer is also a Responsible Agent, he is making a *promise*[1], and that if the Proposer is not a Responsible Agent and the Responsible Agents are also the Decision Makers (that is, the Responsible Agents decide themselves whether to accept or not), we have a *request*. If the Proposer is not a Responsible Agent but the Decision Maker, we have a *command* (that is, the person who makes the proposal also decides that it should be accepted).

Where do questions come into this taxonomy? To find the answer to this problem, we need to refine the analysis. In both questions and assertions, the speaker puts forward a proposition which is to be accepted or rejected by the listener(s). So it seems that there is no difference between questions and assertions, which is clearly an undesirable consequence. However, we can note that in a question, as opposed to an assertion, the speaker does not himself take a stand on the acceptance or rejection of the proposition. Thus we must analyze the concept of a proposal in two logically independent parts, viz. *raising an issue* and *endorsing* a certain possible decision. The difference then between assertions and questions is that in an assertion the speaker both raises the question and endorses the affirmative answer to it, whereas in a question, he does only the first thing[2].

We shall discuss a few types of utterances that the contract approach may throw some light on. The first type is explicit performatives. Consider the following example:

(7) I assume the command of this ship.

Suppose you take (7) to be an ordinary assertion. If uttered by someone in the presence of the crew of the ship in question, it would then be a proposal to the crew to accept the proposition contained in (7) as true. But this proposition expresses what Searle calls an 'institutional fact', or, if we use the language game metaphor, it refers to the rights and obligations of the participants of the game. In other words, if the participants of the game accept this proposition as true, it *becomes* true, at least 'as far as the game goes'. So when the crew accept that the speaker assumes the command of the ship, he has in fact done so. This is at least part of an explanation of how you can do something by saying it. Notice that the performative force of an utterance such as (7) would in this approach be treated as a secondary effect of its primary function (as an assertion), as opposed to theories which treat performatives as primary to everything else.

Normally, it is only 'institutional' or 'socially defined' actions that can be performed by saying that you do them. You could not, to take a trivial example, blow your nose by uttering (8).

(8) I blow my nose.

However, there are cases when even 'brute facts' can be created by words - at least in a sense. Consider the following pair of utterances:

(9) (a) The Revolutionary Court declares you an enemy of the people
(b) You are an enemy of the people (uttered by the chairman of the Revolutionary Court).

(9a) is an explicit performative, like (7). (9b), which has the same function, is what we could call a *fiat*. Fiats are utterances which have the form of assertions but which become true when uttered because of the authority of the speaker(s). In this particular case, (9b) becomes true when uttered in virtue of the fact that the Revolutionary Court has the right to decide who is an enemy of the people and who is not. The borderline between fiats and ordinary statements is not entirely clear. Consider:

(10) Smoking is hazardous to your health (said by the Surgeon-General)

Of course, whether smoking is dangerous or not is an objective fact that does not depend on human actions. But in the context of the particular society in which (10) is uttered, the Surgeon-General is the final authority on the subject and 'as far as the game goes', that is, in what concerns e.g. official policies on smoking, (10) must be treated as if it were true. This is what I meant when saying that you can sometimes create even 'brute facts'.

The third class of utterances I want to discuss has variously been called *pseudo-conditionals* and *imperative conditionals*. What is referred to are conjoined sentences which are felt to have a conditional force, e.g. (11)-(12), which may be regarded as more or less equivalent to (11'-12').

(11) Give me a dollar and I'll save your life.

(12) Give me a dollar or I'll shoot you.

(11') If you give me a dollar, I'll save your life.

(12') If you don't give me a dollar, I'll shoot you.

And and *or* are usually supposed to have roughly the same semantics as the connectives 'conjunction' and 'disjunction' in propositional calculus, i.e. a conjoined sentence with *and* is true if and only if both constituent sentences are true and a sentence with *or* is true if and only if at least one of the constituent sentences is true. But such an analysis presupposes that the constituent sentences have a truth-value, and in the case of imperative sentences, it is not clear how they should be assigned truth-values, if at all. So the question is, how are we to assign an interpretation to utterances like (11)-(12)? In particular, how do we explain that they are felt to be equivalent to conditional sentences?

Supposes now that we look at (11) as a proposal to conclude a contract. We can then propose an analysis of what this contract would mean which looks very much like what we had above:

	Speaker (A)	Listener (B)
Undertaking	A saves B's life	B gives A a dollar
Expectation	B gives A a dollar	A saves B's life

Notice that this is a 'package deal': either you accept all of it or the proposal is nullified. This suggests an analysis of what *and* means here. Suppose you say that if A is a proposal and B is a proposal, then A *and* B is a proposal to accept both the content of A and the content of B. If A and B are statements, i.e. proposals to accept the truth of some proposition, A *and* B will be the proposal to accept both the proposition expressed by A and the proposition expressed by B. If A and B are requests, i.e. proposals to accept the responsibility for some action, then A *and* B is the request to accept the responsibility for both the action expressed by A and the action expressed by B. Thus the simple cases of *and*-conjunction come out right, and the extension to combinations of declarative and imperative sentences is straightforward: if A is a request and B an assertion, then A *and* B is a proposal to accept the responsibility for the action expressed by A and the truth of the proposition expressed by B. A similar analysis is possible for *or*. But we have still not explained the conditional force of (11).

To do so, let us notice that there is a strange restriction on the order of the clauses in (11). (13) does not seem grammatical:

(13) *I'll save your life and give me a dollar.

But if you put in an overt subject in the second clause, it sounds much better:

(14) I'll save your life and you give me a dollar.

The restriction thus seems to be a rather superficial one.

But notice now that (11) and (14) differ as to their most natural interpretation: the action in the first clause is the one which is naturally assumed to be carried out first. There are other examples where this is not the case:

(15) Come back tomorrow and I'll have fixed your car.

Here, the fixing must precede the coming back, due to the choice of the perfect tense in the second clause. What is interesting here, though, is that (15) is felt to be much less 'conditional' than (11) - the reason appears to be that the speaker cannot wait to do his part until the addressee has fulfilled his. So maybe the conditional force largely depends on the temporal ordering of the events. Let us rephrase this in a perhaps clearer way: All contracts are conditional in the sense that they depend on being accepted by the contract partners. Thus, in (15), the repairman need not fix the car unless the customer agrees to leave it and come back tomorrow. But in a case like (11), the order of the events makes it possible for the person A to make his obligation conditional not only on B's acceptance but also on his execution of his part.

FOOTNOTES

[*] I have benefitted very much from discussions with participants at the conferences in Tel Aviv and Amsterdam at which earlier versions of this paper were presented.

[1] What is said here is unsatisfactory as a characterisation of a promise in that what we call promises can appear in different contexts. Cf. the following two pieces of conversation:

(i) A: I'll drive you to the airport tomorrow morning, if you like
B: Yes, that would be nice.

(ii) A: Could you drive me to the airport tomorrow morning?
B: Ok, I'll do that.

These conversations are alike in that A undertakes to do the same action in both, but they differ in who makes the proposal. In (i), where A is the

Proposer, we would perhaps characterize his speech act most naturally as an offer or a suggestion, at least as long as it has not been accepted, but after the conversation has been completed I think it can be truthfully said in both (i) and (ii) that A has promised to drive B to the airport. The essential part of a promise is thus that one undertakes to do something. This is in accordance with the analysis of promising in e.g. SEARLE 1969, but notice that if one, as Searle does, analyses promising as an isolated speech act rather than as part of a language game you cannot e.g. describe the difference between (i) and (ii) and how the 'coming into force' of the undertaking depends on the discourse context.

[2] An assertion may also just be an endorsement, if the question has already been raised, or a rejection, if a contrary claim has been made. Such assertions may have special prosodic and other features. Cf. *It is RAINING* and *It IS raining*. It should be pointed out in this connection that the concepts we have introduced make it possible to distinguish a number of ways in which a proposition may be 'presupposed' in a discourse text:

(i) the question of the truth of the proposition has been raised

(ii) the claim that the proposition is true is pending

(iii) the proposition has been accepted as true by the participants.

[3] I owe this point to Arnim V. Stechow.

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THE CONWAY PARADOX:
ITS SOLUTION IN AN EPISTEMIC FRAMEWORK

by

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1. INTRODUCTION

The aim of this paper is to describe a new application of a formalism, designed originally by the last two authors as part of a theory in which various pragmatic phenomena concerning the information of language users can be handled. Using this framework, we analyse a paradox brought to the attention of the first author by CONWAY et al. (1977). In fact, the paradox involved is much older. A description of the paradox and its history was presented in GARDNER (1977); two variants can be found in LITTLEWOOD (1953). Conway's contribution consists of an impressive generalization of the situations in which the paradox can be shown to arise. We will discuss this generalization in Section 2 of the paper, but for reasons of simplicity our analysis deals only with the original simple case, indicating the explosion of the combinatorial complexity which will arise if our analysis is extended to more complicated cases.

The paradox involves hypothetical incomplete information games, to be played by perfect logicians. In the most simple situation there are two players. Each player has a non-negative number written on his forehead, which his opponent can see but which he cannot see himself. There are no mirrors available and asking the other player for information is not permitted. However, it is known to both players that according to the rules of the game the two numbers are adjacent; so a situation like (3,4) is legal, whereas (6,9) is not. Moreover, each of the players knows that the other player is informed about the rules of the game; this again is mutual knowledge up to every level. The goal of the game is to find out which number is written on one's own forehead. Both players inform each other in alternating turns about whether they know their own number, taking into account what they see and the development of the game so far. As soon as one of the players affirms that he can decide what his number is, the game terminates.

As an example, consider the game (0,1); as soon as the player with number 1 has to answer he can affirm that he knows his number, for seeing a 0, he can conclude that his own number has to be +1 or -1. Since -1 is excluded by the rules of the game, he has complete information as to what number is written on his forehead: he knows that it is +1. Similarly, in the game (1,2) the player with number 2 can answer affirmatively, as soon as the player with number 1 has given a negative answer, for the failure of the player with number 1 to terminate the game at his first turn proves to the other player that he himself does not have a zero, so he must have a 2.

The paradox arises as soon as we start analysing the games (k,k+1) for larger values of k. On the one hand a plausible argumentation can be given which shows that the game will terminate for every value of k, whereas on the other hand a straightforward analysis of a single round during a game such as (3,4) shows that such a round does not produce any useful information at all, implying that the game will never terminate. We present both argumentations in full detail in Section 2.

Our application of the epistemic framework, developed in GROENENDIJK & STOKHOF (1980), will provide for a mathematical model within which the termination proof can be shown to be correct by explicit calculation. The model also supports the possibility of obtaining a non terminating game by restricting the structures used, where these restrictions should correspond to psychological barriers in the human mind. However, it is not our intention to claim that this model explains human behaviour: our main concern is to sharpen the mathematical tools, in order to build formalisms applicable to the more interesting hard problems involving information.

The paper is organized as follows. Section 2 presents the details of the paradox together with its generalization as described by Conway. It is argued that the termination proof is in fact based upon some a priori analysis of the game. Section 3 introduces the general epistemic models introduced by Groenendijk and Stokhof, and indicates the additional restrictions which have to be satisfied by epistemic models in order for them to be useful in the analysis of the paradox. In the models there occurs a modal, possible world component which is used for expressing information about logical and factual relations between states of affairs, information about them, etc. This component is applied in Section 4 where a model for the initial state of the game is obtained. Still the resulting model is not sufficiently general, since it does not allow for the representation of the extra information conveyed by a "no" answer from one of the players. This missing feature is added by defining an update operator, which transforms the entire structure into a new one. This operator is introduced in Section 5, and it is shown by means of an explicit example that, starting from the initial state as defined in Section 4, after finitely many updates a new state is obtained where the game will terminate. Section 6 contains some concluding remarks.

2. THE PARADOX

2.1. Termination and non-termination proof

Consider a particular instance of the two person game with numbers which are not too small, such as the game (3,4). Given the fact that the two numbers are adjacent, each player can find out that the parity of his number is opposed to the parity of the number which he sees on the forehead of his opponent, so he knows whether his own number is even or odd. Let us name the players with the even and the odd number 'Eve' and 'Ott', respectively. Each player will know at the start of the game which role he or she is playing. Now we can easily indicate why the two person game (3,4) will never terminate. It can be argued that the first two answers in the game will be "no", and moreover that both players can predict this. This shows that virtually no information is exchanged during the first round, so why play this round at all?

The argument is based upon the possibilities which both players can discern at the outset. First consider Ott. He sees his opponent carrying a 4, so he knows he must have a 3 or a 5. He does not have complete information and, if asked whether he knows his number, he can only answer "no". Moreover, in both cases the information about Ott's number which is visible to Eve, will be of no help for *her* to solve *her* problem: if Eve sees a 3 she will hesitate between 2 and 4, whereas seeing a 5 will make her hesitate between 4 and 6. Since Ott knows that these are the only two possibilities, Ott is sure that the first answer given by Eve has to be "no" as well. Note moreover that the fact that Eve actually says "no" does not convey any new information to Ott, since he knew at the outset that this was the only possible answer in the given circumstances. Ott can also figure out that a "no" answer given by himself, before Eve has had to answer, won't help Eve in solving her problem, since he knows that Eve is clever enough to infer that

Ott must say "no", regardless of whether he in fact has a 3 or a 5. Adding all this up, we conclude that, regardless of who begins, Ott is sure that the first two answers in the game will be "no".

Now consider the situation for Eve. By the same argumentation as above (where the values of all numbers have to be decreased by 1), we may infer that Eve knows as well that the first two answers in the game will be "no", regardless who begins. This indicates that there does not happen anything interesting during the first round: there is no exchange of new information, and during the next and all subsequent rounds the situation will be the same - the game does not terminate.

Next we show that the game always terminates by proving the following

<u>THEOREM</u>. *The game* (x,x+1) *is terminated at move* x+1 *by the player having the highest number in case the player with the odd number starts, and at move* x+2 *otherwise.*

<u>PROOF</u>. Note that from the fact that it is given that the two numbers are adjacent, each player is able to infer whether his number is even or odd. We prove the result by induction, keeping track of the parity of x. (Again we denote the player with the even number by 'Eve' and the other player by 'Ott'.)

<u>Base induction proof, x=0</u>.

In this situation Ott has complete information (since he sees a 0), whereas the information of Eve is incomplete (she may have a 0 or a 2). So Ott will terminate the game as soon as his turn is up; this is at move 1 in case Ott starts and at move 2 otherwise. This proves the result for x=0.

<u>Induction step, x=2k</u>.

In this situation Ott has the highest number. He knows that his number equals 2k-1 or 2k+1. If the first holds, by induction Eve will terminate the game at move 2k, in case Ott starts, and at move 2k+1 otherwise. As soon as Ott finds out that this has not happened (which situation arises at move 2k+1 or 2k+2, respectively) he can terminate the game, which proves the result for x=2k.

<u>Induction step, x=2k+1</u>.

In this situation Eve has the highest number. The possibility that her number equals 2k is ruled out by the behaviour of Ott at turn 2k+1 or 2k+2, respectively, depending on whether Ott or Eve starts. So at the next move, which is move 2k+2 or 2k+3, respectively, Eve can terminate the game. This proves the result for x=2k+1.

The structure of this proof is illutrated by the diagram below. It shows a graph whose vertices are legal configurations in the game. Two configurations are connected by an edge labeled X when player X cannot discriminate the two positions on the basis of his/her visible information. For example, the games (5,6) and (6,7) are connected by an edge labeled 'Ott', since Ott, seeing a 6, cannot decide whether he has a 5 or a 7.

```
(0,1) ---- (1,2) ---- (2,3) ---- (3,4) ---- (4,5) ---- (5,6) ---- (6,7) --
       Eve        Ott        Eve        Ott        Eve        Ott
```

<u>Diagram 1</u>

A graph describing the two person game

Note that each game in the graph as two incoming edges, labeled 'Eve' and 'Ott', respectively; the only exception being the game (0,1). This game has no edge with label 'Ott' which connects it to another game, illustrating the fact that Ott can terminate this game at his first move.

In the induction proof presented above the players are supposed to perform the following "edge-cutting" game: whenever some player says "no", both players take their copy of the above graph and remove from it all nodes which have no incoming edge labeled by the player saying "no", together with all their incoming edges. After this reduction of the graph, both players consider the collection of games which remains and investigate whether the collection of games which are compatible with the real state of the world is reduced to a singleton, in which case the player can terminate the game at his next move.

The main weakness of the termination proof now can be explained as follows: it seems that, in order to terminate the game at all, the players are supposed to stop playing the real game, and start playing the edge-cutting game instead. So an *a priori analysis* of the game is added to the set of rules of the game supposed to be known to both players. Moreover, this knowledge is known to the other player as well, up to each level of epistemic analysis. Presumably the assertion that the players are "logically perfect" has to be interpreted in such a way that they independently have arrived at the solution just given, before the game starts. We consider this an unreasonable assumption.

At the same time it is easy to accuse the non-termination proof of being a prime example of a *proof by intimidation*: the rhetoric question: "Why play this round at all?" obscures the fact that we did not analyze all possible information of the "A knows that B knows that C knows that ..." type which may play a role.

The present paper aims at developing an epistemic model in which information at all epistemic levels can be represented and which, moreover, obeys the rules of the game. We do not want to build into the model an a priori analysis of the game which tells in advance *which* conceivable position is removed at *which* move in the game. Instead, we want an update operator which removes from the structure those positions which are incompatible with a "no" answer given by a player, but which does this independently of at *which* move in the game this "no" answer is given.

2.2. Generalizations

Littlewood, in LITTLEWOOD (1953), presents two variants of the paradoxical situation described above. He considers cards which are showing two adjacent, non-negative integers on the two sides. The two players are seated opposite to each other. A third player (the umpire) draws a card and puts it between the two players in such a way that each player can only see one of the faces. The player having the highest number wins the round. However, each player has the right to cancel the round, so the first thing the umpire has to do is ask the two players whether they will play or whether they want to go to the next round by asking for a new card. Littlewood claims that by an induction proof it can be shown that all rounds are vetoed by some player.

In the other version the cards are drawn from an urn, containing a single copy of the card (0,1), 10 copies of the card (1,2), 100 copies of the card (2,3), etc. One can prove that under these circumstances each player has a change of one against ten of losing. This latter version brings us back to older paradoxes involving probability notions, which can be solved by basing probability theory on measure theory.

Conway has generalized the paradox in CONWAY et al. (1977) by considering games with more than two players. In this generalization there are $k \geq 2$ players, each carrying a number on his forehead. The players are seated in such a way that each player can read all numbers except his own. Moreover, there is an umpire, who has written a list of m consecutive numbers on a blackboard, one of which is the sum of the numbers on the foreheads of the players. We denote an instance of such a game by $(n_1,n_2,\ldots,n_k \mid p_1,p_2,\ldots,p_m)$, where $n_1,n_2,\ldots,n_k$ are the numbers written on the foreheads of the players A,B,..., respectively, and the numbers $p_1,p_2,\ldots,p_m$ the numbers written on the blackboard. The umpire asks the players in cyclic order whether they know their number or not, and the game ends on the first "yes" answer.

By analysing a game such as (1,1,1|3,4,5) it can be made clear that in this game the first three answers will be "no", regardless of who begins, so again non-termination is proved by asking what possible use such a round could have. On the other hand, Conway has shown by a nice induction proof, that the edge-cutting variant of this game will terminate for an arbitrary initial position, as long as the number of values on the blackboard m does not exceed the number of players k.

We illustrate the termination by illustrating the edge-cutting variant of the above game (1,1,1|3,4,5) in the diagrams 2 and 3 below. Vertices in the game are all positions sharing the public information, i.e. the values of the numbers written on the blackboard $p_1,\ldots,p_m$. In our example these are the numbers 3,4,5. A node is therefore completely determined by a triple n_1,n_2,n_3 with sum equal to 3, 4, or 5. Two positions only differing with respect to the value of n_1,n_2,n_3, respectively, are connected by an adge labeled A,B,C, respectively, indicating that these two positions can not be discriminated by player A,B,C, respectively. It is possible to embed the resulting graph in three-dimensional space in such a way that the three orthogonal directions correspond to the three edge labels - the diagram gives a plane projection of this embedding: hence the label of an edge is determined by its direction in the diagram, as indicated in the "tripod" shown above the graph.

As before each player removes at his turn those vertices not having (or no longer having) incoming edges labeled by his color; these positions correspond to configurations where he has complete information - a "no" answer denies existence of such a configuration, and the configuration is therefore removed from the graph. Diagram 3 shows for each node the number of the move at which it will be removed in the edge-cutting game. Termination of the game is equivalent with the fact that each node sooner or later gets numbered.

The reader may convince himself that it is necessary for the proof to work that the players' behaviour is competent. During move 9 player C will remove node 211 together with the A-edge connecting it to the considered actual game 111. If A fails to terminate the game at move 10 by answering "yes", the four nodes numbered 10 will be removed and the graph will become empty, representing the situation where the game gets blocked.

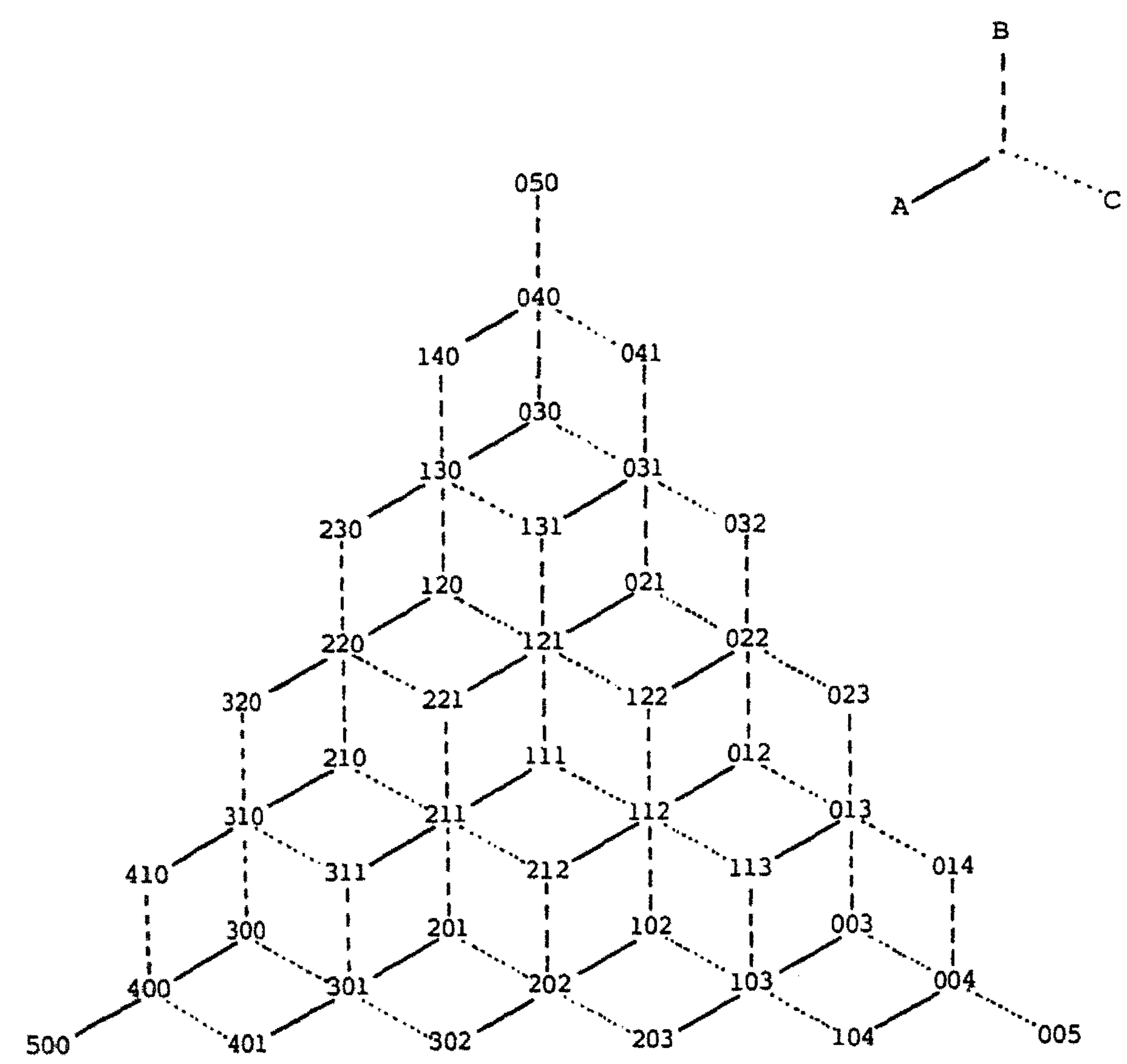


Diagram 2

Graph of possible games (a,b,c| 3,4,5). The label of an edge is determined by its direction.

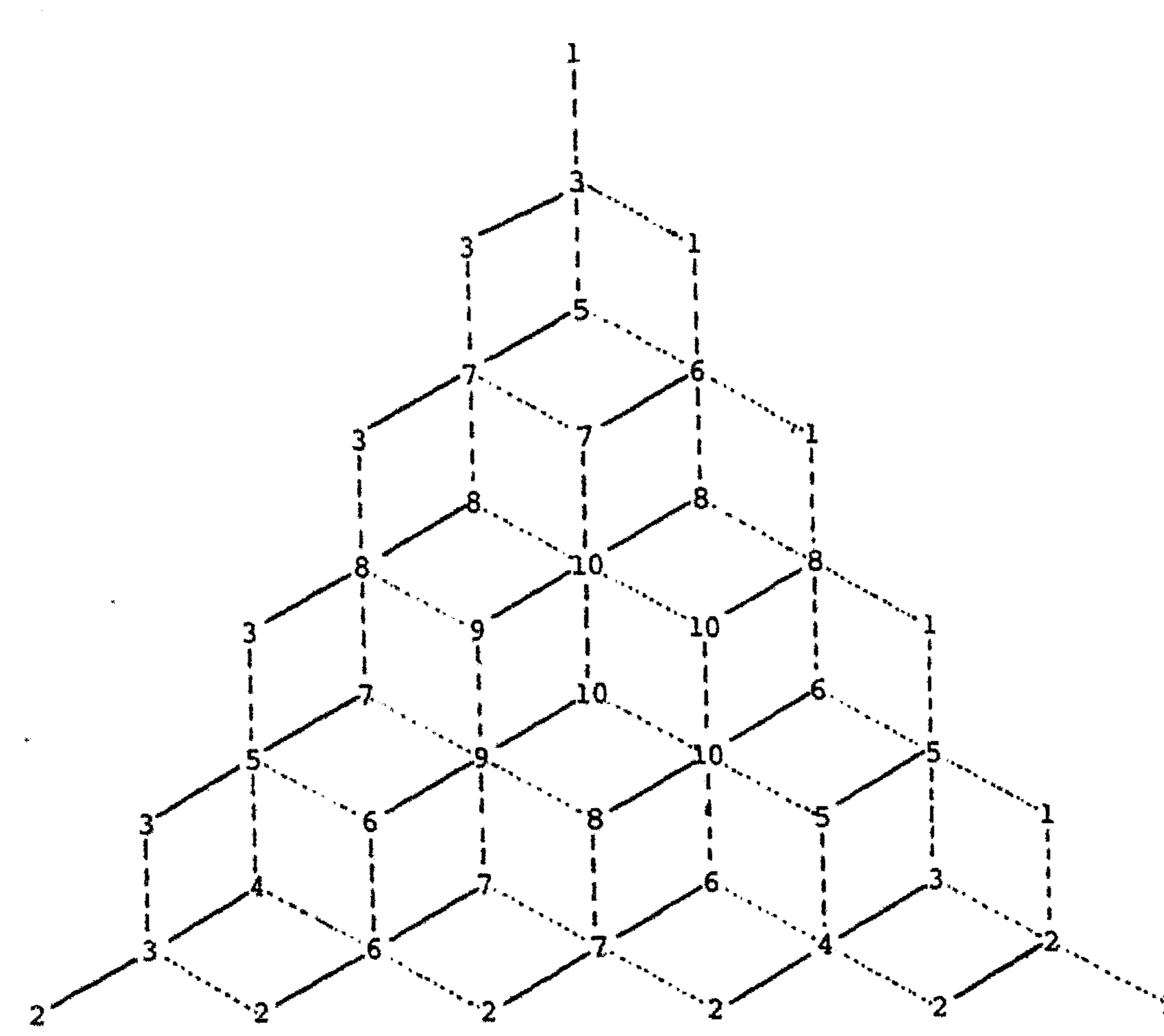


Diagram 3

Edge-cutting game for the graph of Diagram 2. A is the first to move. The numbers indicate at which move the position becomes incompatible with a "no" answer.

3. EPISTEMIC FRAMEWORK

3.1. Preliminaries

An epistemic model should not only encode the state of the actual world, but also the information that individuals in this world have about that state of the world and about the information of other individuals about the world or information of other individuals, etc. Disregarding psychological limits inherent to the human mind, this formulation leads to rather complex, infinite structures. Groenendijk and Stokhof have introduced a set theoretic framework for representing this kind of information, which we will describe briefly. But first we define some mathematical tools.

For A, a (finite or infinite) set, we define inductively the sequence of sets A^i by:

$$A^0 := A, \quad A^{k+1} := P_f(A^k)\setminus\{\emptyset\},$$

where P_f is the finite powerset operation.

A^+ denotes the disjoint union of all sets A^i, $i = 0,1,\ldots$. This union is called a *graded set*.

If $f\colon A \to B$ is a mapping then we can define a mapping $f^+\colon A^+ \to B^+$ by defining a sequence of functions $f^k\colon A^k \to B^k$ inductively by

$$f^0 := f, \quad f^{k+1}(w) := \{f^k(x) \mid x \in w\},$$

and letting f^+ be the disjoint union of the sequence f^i. We call f^+ a *graded mapping*.

Consider the following example. Let $A = \{0,1\}$ be the set of truth values. We can take for f the operation $\neg$ (negation). Then the operation $\neg^+$ is defined by

$$\neg^+(x) = \neg(x) \qquad , \text{ if } x \in A^0,$$

$$\neg^+(w) = \{\neg^+(x) \mid x \in w\}, \quad \text{otherwise.}$$

So, since $A = \{0,1\}$, $\{\{0,1\},\{0\}\} \in A^2$, and $\neg^2(\{\{0,1\},\{0\}\}) = \{\neg^1(\{0,1\}),\neg^1(\{0\})\} = \{\{\neg(0),\neg(1)\},\{\neg(0)\}\} = \{\{1,0\},\{1\}\}$.

3.2. Some sideremarks

Before going on, we will make some brief remarks on the various ways in which graded mappings in more arguments can be defined. What follows is not essential to the paper and may safely be skipped.

The operator + introduced above actually yields a functor from the category of sets and mappings to the category of graded sets and graded mappings. If this functor behaved in a certain way with respect to Cartesian products, this would lead to a simple theory for functions with more than one argument. This turns out not to be the case. There are two ways to extend functions in more arguments. First of all, one can simply apply the functor + to the mapping $f\colon A{\times}B \to C$, yielding a graded mapping

$$f^{+}\colon (A{\times}B)^{+} \to C^{+}.$$

Note however that $(A{\times}B)^{+} \neq A^{+} \times B^{+}$, the latter object being the Cartesian product of A^{+} and B^{+} in the category of graded sets. Let us call $A^{+}{\times}B^{+} := (A{\times}B)^{\%}$. We can define the graded mapping $f^{\%}$ to be the union of the mappings $f^{\%i}\colon A^{i}{\times}B^{i} \to C^{i}$, where $f^{\%i}$ is defined inductively by:

$$f^{\%0} = f, \quad f^{\%i+1}(U,V) := \{f^{\%i}(u,v) \mid u \in U,\ v \in V\}.$$

It is clear from the contents of GROENENDIJK & STOKHOF (1980) that these authors intended to use the construction % for products rather than the functor +. It can also be seen by considering small examples that the functor + does not preserve products (taking the union of products of the component sets as a definition of product in the category of graded sets, as suggested by the definition of %). The connection between the operations + and % for products is illustrated by Diagram 4.

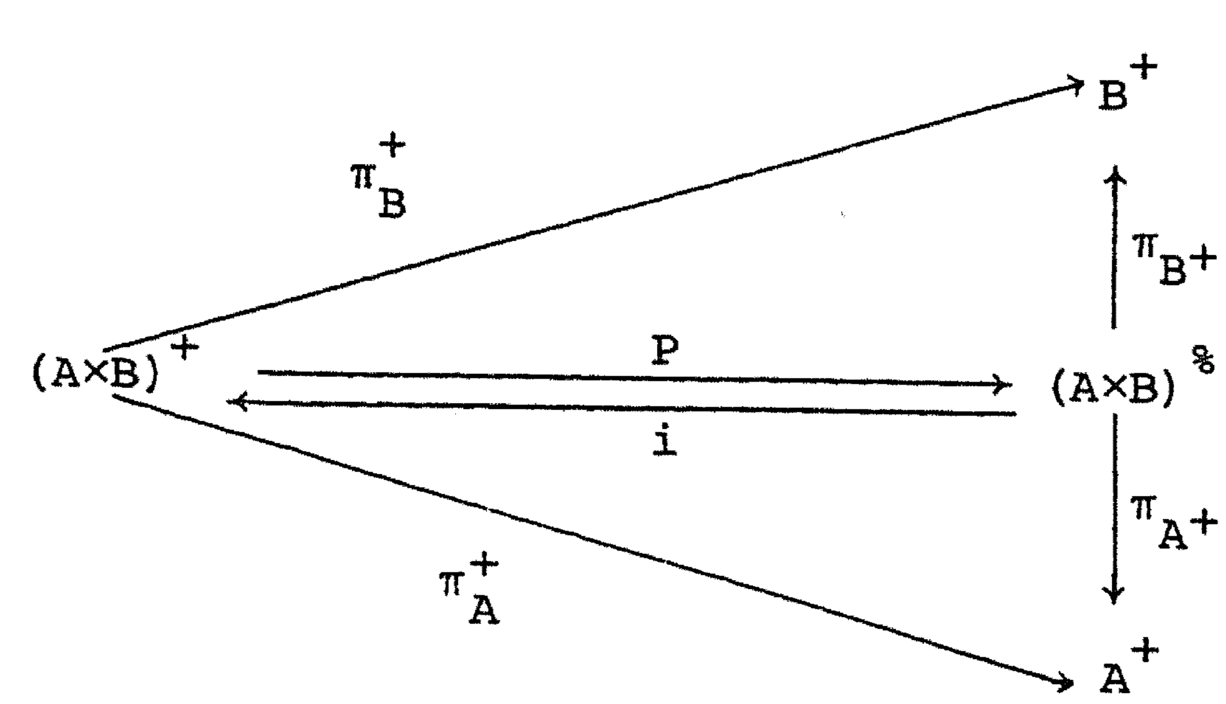


Diagram 4

The embedding i is obtained as $id^{\%}_{A\times B}$, whereas the projection p is obtained from the pair of mappings π^{+}_{A} and π^{+}_{B}, using the fact that $(A\times B)^{\%}$ is the product of A^{+} and B^{+} in the category of graded sets and graded mappings. A straightforward induction proof shows that $p(i) = \underline{id}_{(A\times B)^{\%}}$.

This non-preservation of products by the functors described above, has, as Groenendijk and Stokhof have observed, as one of its consequences that some logical laws concerning the usual logical connectives are no longer valid at higher levels. This matter will not be pursued further in this paper.

3.3. Epistemic models

Returning to the topic of this paper, we are now in a position to define the notion of an *epistemic model,* using the tools defined in 3.1.

If Σ is a finite alphabet, then we let Σ^{*} denote the set of finite strings over Σ, letting ε denote the empty string. The length of a string s is denoted by $|s|$.

DEFINITION. A *general epistemic model* is a quintuple $\langle L,\Sigma,W,A,V\rangle$, where
L represents a *language,* the elements of which are to be interpreted,
Σ is a finite alphabet, the elements of which are called *conscious entities* or *persons,*
W is a set of *possible worlds* (its role will become clear in the next section),
A is some *domain* of interpretation for the elements of L,
V is the *interpretation function*; it is a mapping $V\colon L\times\Sigma^{*}\times W \to A^{+}$, such that $V(f,s,w) \in A^{|s|}$.

The intended meaning of the valuation function V is expressed as follows:

$V(f,\varepsilon,w) = a$	means: in world w the interpretation of f equals *a*
$V(f,Zs,w) = \{q_1,\ldots,q_r\}$	means: in world w person Z has the information that one out of the r possibilities expressed by $V(f,s,w) = q_i$ is the case, but Z does not have the information which one of these possibilities is in fact the case.

As usual in semantic frameworks, the mapping V is required to obey the so-called Fregean principle of compositionality, which expresses that the meaning of a compound expression is a function of the meanings of its constituent parts (see for example VAN EMDE BOAS & JANSSEN (1979) for a discussion of this principle). The framework as it was originally proposed in GROENENDIJK & STOKHOF (1980), obeys this principle. In the present paper compositionality is not under discussion, since the language considered consists of just two atomic expressions.

For the remainder of this paper we stipulate the following:
$L = \{\underline{X},\underline{Y}\}$ (representing the numbers of players X and Y, respectively),
$\Sigma = \{X,Y\}$ (representing the players X and Y, respectively),
$A = \mathbb{N}$ (the set of natural numbers including 0).

As an example consider the assertion expressed by the formula $V(\underline{X},YX,w) = \{\{1,3\},\{3,5\}\}$. This assertion states that in world w, player Y has the information that X is hesitating about his own number; according to Y, X is either doubting between 1 and 3 or doubting between 3 and 5, but Y does not know which of these two possibilities is in fact the case. This assertion describes a situation which arises in the two person game when X actually has the number 3 on his head. In this situation Y will hesitate whether his number is 2 or 4 and accordingly he will attribute to X corresponding hesitations about his own number: hesitation between 1 and 3 in case Y has a 2, and hesitation between 3 and 5 in case Y has a 4.

3.4. Restricting epistemic models

The kind of epistemic models covered by the definition given above are still much too general. E.g., it is not required at all that the information of various persons is connected in a reasonable way. Nor is it required that the information reflects knowledge of the rules of the two person game. These requirements can be enforced by adding further conditions which the valuation function V has to satisfy. The first condition expresses that if a person X has certain information, he also has the information that he has this information. Moreover, it is known at each level in the epistemic framework that all persons fulfill this requirement. In order to express this so-called *optimal information principle*, we need a further operator defined on the set A^+.

Let $\uparrow_i$ be the operation $A^i \to A^{i+1}$ defined by $\uparrow(U) := \{U\}$. This operation may be extended to a mapping from $\bigcup_{j=i}^{\infty} A^j \to A^+$ in the usual way. Note that the operation obtained in this way, which we denote by $\uparrow_i^+$, does not preserve the grading of the set A^+; in fact, it increases its grade by one. Note also that for $i < j \leq k$ both $\uparrow_i^+$ and $\uparrow_j^+$ are definedon A^k, but that their effect is different. For example, $\uparrow_1(\{0,1\}) = \{\{0,1\}\}$ and $\uparrow_0(\{0,1\}) = \{\{0\},\{1\}\}$.

The *optimal information principle* now can be expressed as follows: for all $w \in W$, $f \in L$, $Z \in \Sigma$ and s_1 and $s_2 \in \Sigma^*$ it holds that

$$V(f,s_1ZZs_2,w) = \uparrow_i^+(V(f,s_1Zs_2,w)),$$

where $i = |s_2| + 1$. So from $V(\underline{X},Y,w) = \{0,2\}$ we may infer that $V(\underline{X},YY,w) = \{\{0,2\}\}$. Similarly, from $V(\underline{X},XY,w) = \{\{1,3\},\{3,5\}\}$ we obtain $V(\underline{X},XYY,w) = \{\{\{1,3\}\},\{\{3,5\}\}\}$ and $V(\underline{X},XXY,w) = \{\{\{1,3\},\{3,5\}\}\}$. As a result, by assuming the *optimal information principle,* we can specify V completely restricting ourselves to values of V with respect to strings s without iterated symbols. For our two-element alphabet Σ this implies that we only have to look at alternating strings. We denote this set of strings by Σ_a^*; it may be defined by

$$\Sigma_a^* := (\varepsilon+Y)(XY)^*(\varepsilon+X),$$

where we have used the terminology of regular expressions. In the sequel we shall only consider strings from Σ_a^*.

Our next condition represents the rule of the two person game, that the two numbers $\underline{X}$ and $\underline{Y}$ are adjacent and the fact that this is known to both players at all epistemic levels. This leads to what will be called the *adjacency conditions*. The definition requires another operator S^+. Let $S\colon \mathbb{N} \to \mathbb{N}^1$ be defined by $S(0) := \{1\}$, $S(k+1) := \{k,k+2\}$. So S maps each positive number on the pair consisting of its neighbours and maps the number 0 on the singleton consisting of its only positive neighbour 1. We extend S to a mapping $S^+\colon \mathbb{N}^+ \to \mathbb{N}^+$ in the usual way. Again the mapping S^+ increases the grading by one.

The *adjacency conditions* are going to express the following facts about our two person game:

(0) The actual state is a legal position of the game.

(i) Each player sees (and consequently knows) the number of the other player.

(ii) Each player knows his number to be a neighbour of the number of his opponent.

(iii) These facts (i) and (ii) are known by each player at all epistemic levels.

The mathematical formulation of the *adjacency conditions* reads as follows: for each world $w \in W$, and each alternating string s not ending on Y, and t not ending on X, the following relations hold:

(0) $V(\underline{X},\varepsilon,w) = k$, $V(\underline{Y},\varepsilon,w) = \ell$ for adjacent, non-negative k and ℓ.

(1) $V(\underline{X},sY,w) = \uparrow_0^+(V(\underline{X},s,w))$ "Y knows $\underline{X}$"

(2) $V(\underline{Y},sY,w) = S^+(V(\underline{X},s,w))$ "Y knows $\underline{Y}$ to be a neighbour of $\underline{X}$"

(3) $V(\underline{Y},tX,w) = \uparrow_0^+(V(\underline{Y},t,w))$ "X knows $\underline{Y}$"

(4) $V(\underline{X},tX,w) = S^+(V(\underline{Y},t,w))$ "X knows $\underline{X}$ to be a neighbour of $\underline{Y}$".

Together with the *optimal information principle* the five equations above allow us to compute for every actual configuration in the game the values of the valuation function for $\underline{X}$ and $\underline{Y}$ with respect to every string s. In Diagram 5 we illustrate this for the configuration where $\underline{Y} = 3$ and $\underline{X} = 2$. In this diagram we let F(f,s) denote V(f,s,w), since w is fixed.

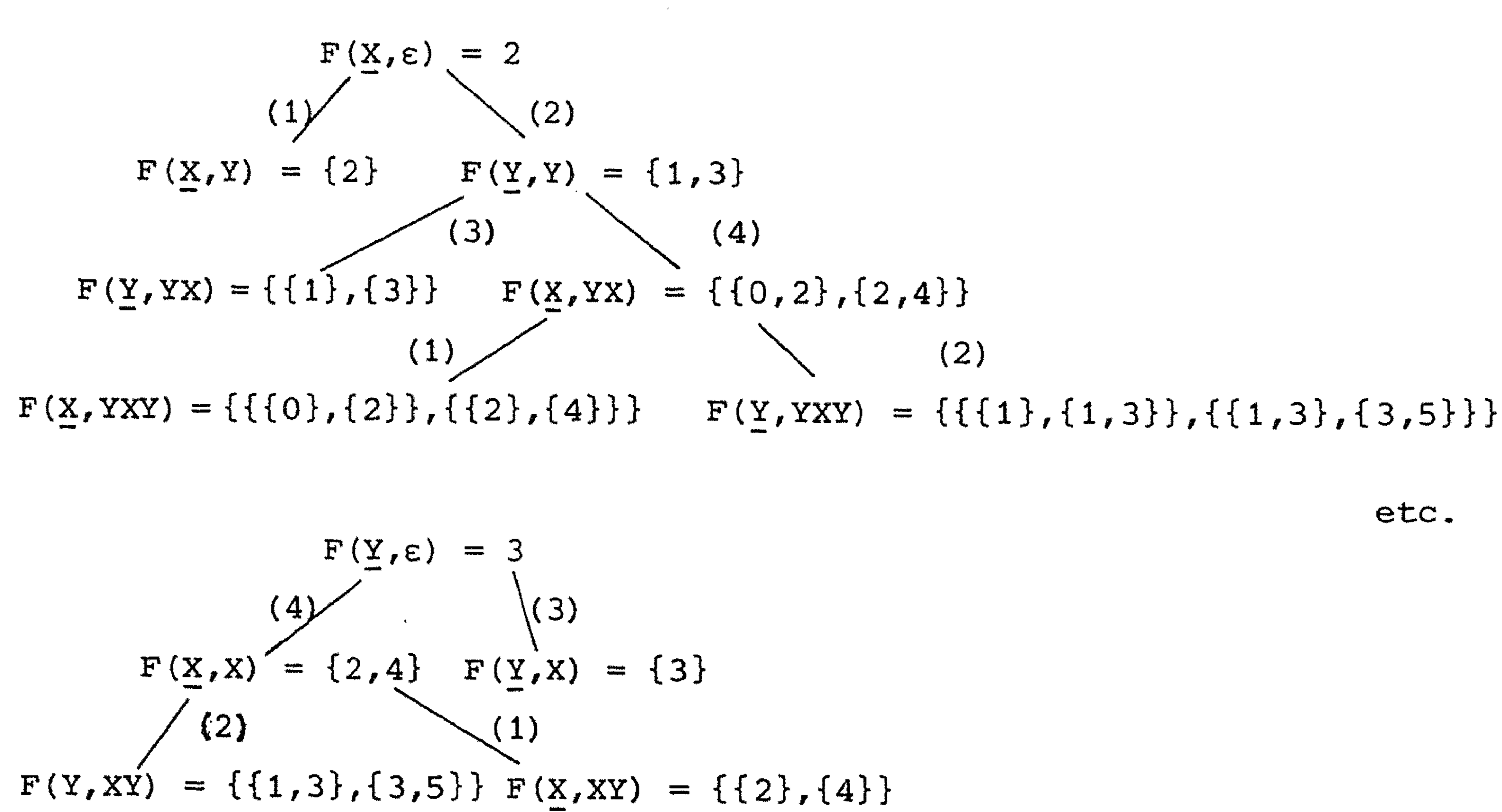


Diagram 5

Information computed in accordance with the adjacency conditions

Note that for an index s which ends on Y the values at the deepest level in the set $F(\underline{Y},s)$ are almost never singletons. This relates to the fact that Y is uncertain about the value of his own number. Note, however, the exception in the example given above: in the computation of $F(\underline{Y}, YXY)$ there occurs a single singleton {1} at the deepest level. This singleton indicates a configuration of complete information which is going to be denied by a "no" answer from Y.

Another remark we can make at this point is that the epistemic model, although it properly encodes the information the participants may have in some configuration of the game, does not account for connections between the possibilities arising at different levels. For example, the pair {2,4} in the expansion of $F(\underline{X}, YX)$ comes from the number 3 in $F(\underline{Y}, Y)$ and not from the number 1 in the latter set. Our model so far does not yet represent this part of the information which we shall need in order to complete our analysis of the paradox.

4. MODAL EPISTEMIC FRAMEWORK

4.1. The role of the possible world component

The model as described in Section 3 represents a large part of the information of the participants in the game. However, certain connections between pieces of information are not accounted for. Generally speaking, such connections represent information about logical and factual relations between states of affairs, information about them, etc. A representation of this kind of information is an essential part of a theory in which pragmatic phenomena concerning the information of language-users are to be handled. An example of information about a factual relation between possible situations in our two person game is the following.

In the situation where X is hesitating whether $\underline{X}$ equals 2 or 4, he infers that at the same time Y must be hesitating either whether $\underline{Y}$ equals 1 or 3, or whether $\underline{Y}$ equals 3 or 5. The first possibility is connected to the value 2 for $\underline{X}$, whereas the second corresponds to the possibility that $\underline{X} = 4$. The information which the participants have about these connections has not been accounted for in our model so far.

In order to represent this kind of information we will use the possible world component in our general epistemic model. Given some state in the game, which will be called the *actual world* hereafter, each player can create new possible worlds for himself where he has made *hypothetical choices* between the possibilities available to him. Moreover, he can imagine within such a world the other player making similar choices, etc. up to every level of our analysis. Since for our particular game there exist at each level at most two possibilities between which a player can choose, this leads to a structure which has the form of a (pair of) binary tree(s). The nodes of such a tree are labeled by possible worlds, described by their V-values, and its edges are labeled by strings which indicate which player created this hypothetical situation. The logic of the game is represented by the fact that, up to some particular level, the V-values are obtained by hypothesis formation (i.e. explicitly assumed by the involved conscious entity), whereas below this level the V-values are again computed using the optimal information principle and the adjacency rules. Implicitly we enlarge the collection of legally possible worlds through addition of these so-called s-extensions, to be described in more detail shortly. First, we define some more mathematical tools.

4.2. Formal implementation

Let A be some set and let q be a member of A^+. We say that q is a 1-singleton iff q is a singleton, and we say that q is a k+1 singleton for $k > 0$ iff q is a singleton whose only member is a k-singleton. We denote this property by k-sgl(q). If q is a k-singleton then its only element at level k is denoted $[q]^k$. So if $q = \{r\}$ then $[q]^k = [r]^{k-1}$.

Now let w_0 be a possible world in an epistemic model satisfying the adjacency conditions such that $V(\underline{X},\varepsilon,w_0)$ and $V(\underline{Y},\varepsilon,w_0)$ are two adjacent non-negative numbers y and y+1. With respect to string X we have $V(\underline{X},X,w_0) = \{y,y+2\}$ and $V(\underline{Y},X,w_0) = \{y+1\}$. The values of $V(f,s,w_0)$ for s starting with X are computed from these values in accordance with the adjacency conditions.

We can introduce two possible words w_1 and w_2 such that

(i) $V(\underline{X},\varepsilon,w_1) = V(\underline{X},\varepsilon,w_2) = V(\underline{X},\varepsilon,w_0)$ and similarly for $\underline{Y}$;

(ii) $V(\underline{Y},X,w_1) = V(\underline{Y},X,w_2) = V(\underline{Y},X,w_0)$;

(iii) $V(\underline{X},X,w_1) = \{y\}$, $V(\underline{X},X,w_2) = \{y+2\}$;

(iv) for other strings starting with X the values of V are computed in accordance with the adjacency conditions starting from (ii) and (iii).

The worlds w_1 and w_2 are called the elementary X-extensions of w_0. Note that we do not require anything about the values of V in the extensions with respect to strings starting with Y, but for definiteness we preserve the values at w_0. The worlds w_1 and w_2 are hypothetical situations in the mind of X and information available to Y is completely unrelated to these worlds, so it makes no difference at all what is postulated concerning the values of V with respect to strings starting with Y.

Assume that we have already defined the elementary s extensions for s starting with X of length ≤ j. Let s' = XYXY... be string of length j+1, s the string of length j resulting by removing the last element of s', and let w be one of the extensions of w_0 with respect to s. By induction hypothesis the following conditions are fulfilled:

(a) for s" starting with X and length j" ≤ j it is the case that V(f,s",w) is a j"-singleton q such that $[q]^{j''}$, its only element at level j", occurs $\underbrace{\text{as an element in an element in an element} \ldots}_{j''\text{-times}}$ in V(f,s",w_0).

(b) for f = either X̲ or Y̲ (depending on the parity of j) it is the case that V(f,s',w) is a j+1-singleton, whereas for the other it is a j-singleton q with $[q]^j$ possibly being a pair.

(c) For strings t longer than j+1 the values of V(f,t,w) are computed in accordance with the adjacency conditions starting from the values mentioned sub (a) and (b).

The s'-extensions of w are constructed as follows:

(i) for strings up to length j the values are equal to those in w; the same holds for s' and the expression X̲ or Y̲, whichever yields a j+1-singleton as mentioned sub (b).

(ii) for string s' and the remaining expression X̲ or Y̲ the value is a j+1-singleton q' with $[q']^{j+1}$ obtained by making a choice among the members of the pair mentioned sub (b).

(iii) For longer strings the values are obtained by computation in accordance with the adjacency conditions starting from the values obtained sub (i) and (ii).

The collection of s'-extensions of w_0 is obtained by performing the above construction for each s-extension of w_0. Since each s-extension yields at most two s'-extensions the system of s-extensions for strings starting with X results in a binary tree structure. The binary tree, called the *X-tree,* represents the information available to X at the initial state of the game, together with all possible hypothetical situations which X can conceive and which might have led to the situation as it is observed. The structure of the tree makes explicit the connection between hypotheses at various levels.

A similar construction can be performed for indices starting with Y.

In Diagram 6 below we give an example of a part of the (infinite) Y-tree labelling all nodes with partial information about the values of V at these nodes; only the most relevant part of the information is presented, from which the other values can be computed easily. A pair of such trees, an X-tree and Y-tree, models the initial state of the game.

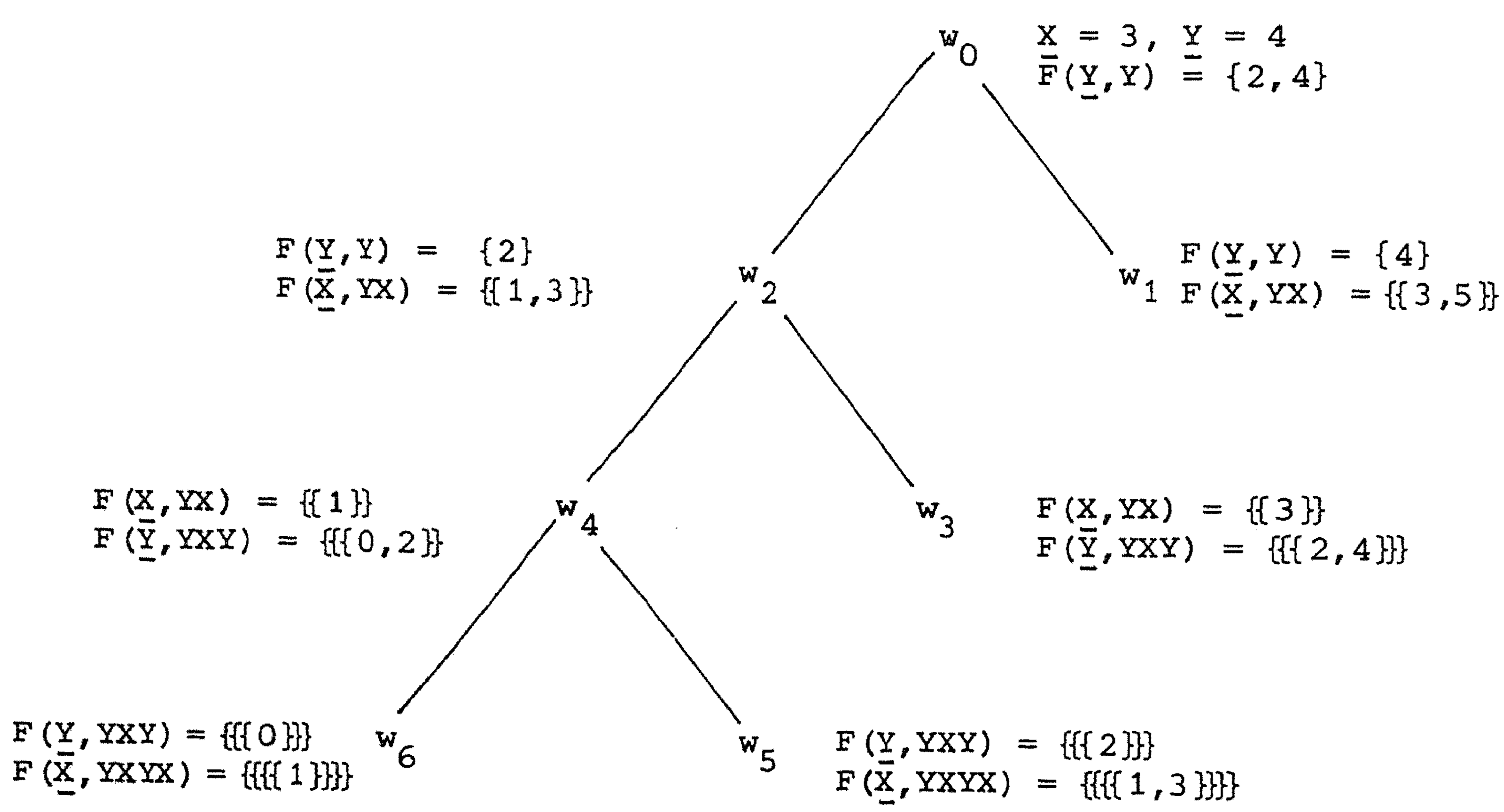


Diagram 6

Fragment of the Y-tree for the initial state of the game (3,4)

5. UPDATING THE STATE

5.1. What updating comes to

Consider the representation of the initial state of the game where $\underline{X} = 2$ and $\underline{Y} = 3$. It follows that $V(\underline{X},X,w_0) = \{2,4\}$, i.e. X is uncertain about his number. Similarly it follows that $V(\underline{Y},Y,w_0) = \{1,3\}$, i.e. Y is uncertain as well. So both players will answer "no" when asked whether they know what their number is. Further, it holds that $V(\underline{Y},XY,w_0) = \{\{1,3\},\{3,5\}\}$. This means that X knows that Y is hesitating between two possible values, although X, at his turn, is hesitating about which pair. So X knows in advance that Y will answer "no". The same holds for Y.

In order to have any progress in the game it is necessary that the players use the information conveyed by a "no"-answer being given for updating their information about the state of the game. If the players don't use this information nothing will change and the game will last forever. But how is the information conveyed by a "no"-answer to be used? Once X or Y has answered "no", it may be assumed that both players know that this answer has been given, and that they know that the other will know so as well, etc. The information must be used for ruling out hypothetical extensions of the actual world in which the player who has given the "no"-answer has the kind of complete information which he just denied to have. Note that the s-extensions of the actual world constructed in the preceding section are hypothetical situations in which the players have more information than they have in the actual world - they were constructed in that way. In some of these a player has complete information. Often this fact is the direct outcome of a choice between alternatives. But there are some worlds in which this is not the case. In these worlds the fact of complete information is not simply chosen from the alternatives, or to put it differently, it is not enforced by extending the choice that created the world upto the corresponding level.

Consider world w_6 in Diagram 6 in the preceding section. In this world choices have been made upto level 3. In this situation Y knows that X knows that Y knows the following remarkable fact: "X knows that $\underline{X} = 1$", and this instance of complete information was not created by choice-expanding upto level 4. It is the existence of such a world which is denied by the fact that, after X says "no", Y knows that X knows that Y knows that X has said "no". So w_6 no longer should be considered to be a possible world. Moreover, the possibilities higher up in the tree which led to its creation in the tree of extensions should be removed as well. This task has to be performed by an update operator which we shall now define.

5.2. The update operator

The actual world w is called a *world with complete information for Y* iff $V(\underline{Y},Y,w)$ is a singleton. Similarly for X. Let s be a string of length k ending with X, and let w' be some s-extension of world w. We say that w' is a *world with complete information for Y* iff $V(\underline{Y},sY,w')$ is a k+1-singleton.

Similarly, if s ends with Y and $V(\underline{X},sX,w')$ is a k+1-singleton then w' is a *world with complete information for X.*

In the game the answer given by a player will be "yes" if the actual world is a world of complete information for that player, and "no" otherwise. Consider the binary tree representing the information of Y, consisting of some world labelling the root (called the actual world) together with all s-extensions for strings s starting with Y. In order to represent the configuration which occurs after X says "no", we introduce the update operator $X, which modifies the tree in the following way:

(i) all words in the tree which are worlds with complete information for X are removed, together with all their descendants;

(ii) if some world w" at level k (the level of the root being 0) is removed from the tree, the information present in this world is k-extracted from the information in all worlds on the path from the actual world to w";

(iii) the resulting tree with updated information forms a new tree consisting of an actual world at the root together with its s-extensions for indices s starting with Y.

The operation of *k-extraction* used in clause (ii) above is defined as follows: let w" be a world which is removed at level k and let w' be some ancestor at level $k_1 < k$. Then w' is replaced by a new world w^* such that

$$V(f,s,w^*) = V(f,s,w') \qquad \text{if s is of length} < k,$$
$$V(f,s,w^*) = V(f,s,w') \setminus_k V(f,s,w") \qquad \text{otherwise,}$$

where the operator $\setminus_k$ is defined by:

$$A\setminus_1 B := A\setminus B, \quad A\setminus_{j+1}B := \{a\setminus_j b \mid a \in A,\ b \in B\} \quad \text{for } j \geq 1.$$

A similar definition can be given for updating the Y-tree after Y has said "no", yielding an operator $Y. Analogous definitions are required in order to explain how the operators $X and $Y modify the X-tree. Note that the actual world occurs in both trees: in order to have it updated properly the values of $V(f,s,w_0)$ are modified according to the definition for the X-tree for indices starting with X and according to the definition for the Y-tree for indices starting with Y.

We now have developed all tools needed for calculating the termination of our game. The calculation consists of two stages:

stage 1: By computing the values in accordance with the adjacency conditions a world describing the initial state of the game is defined. This world w_0 becomes the root of both an X-tree and a Y-tree which are constructed according to the methods described in Section 4.

stage 2: If it is X's turn to answer, we inspect whether the actual world is a world with complete information for X. If so, the game terminates; otherwise the operator $X is performed on both the X- and the Y-tree. Similarly, if it is Y's turn to answer. Next stage 2 is repeated.

We illustrate by an example that the calculation, starting from the situation described by Diagram 6 shown at the end of the preceding section, terminates after three answers, assuming that it is X who begins.

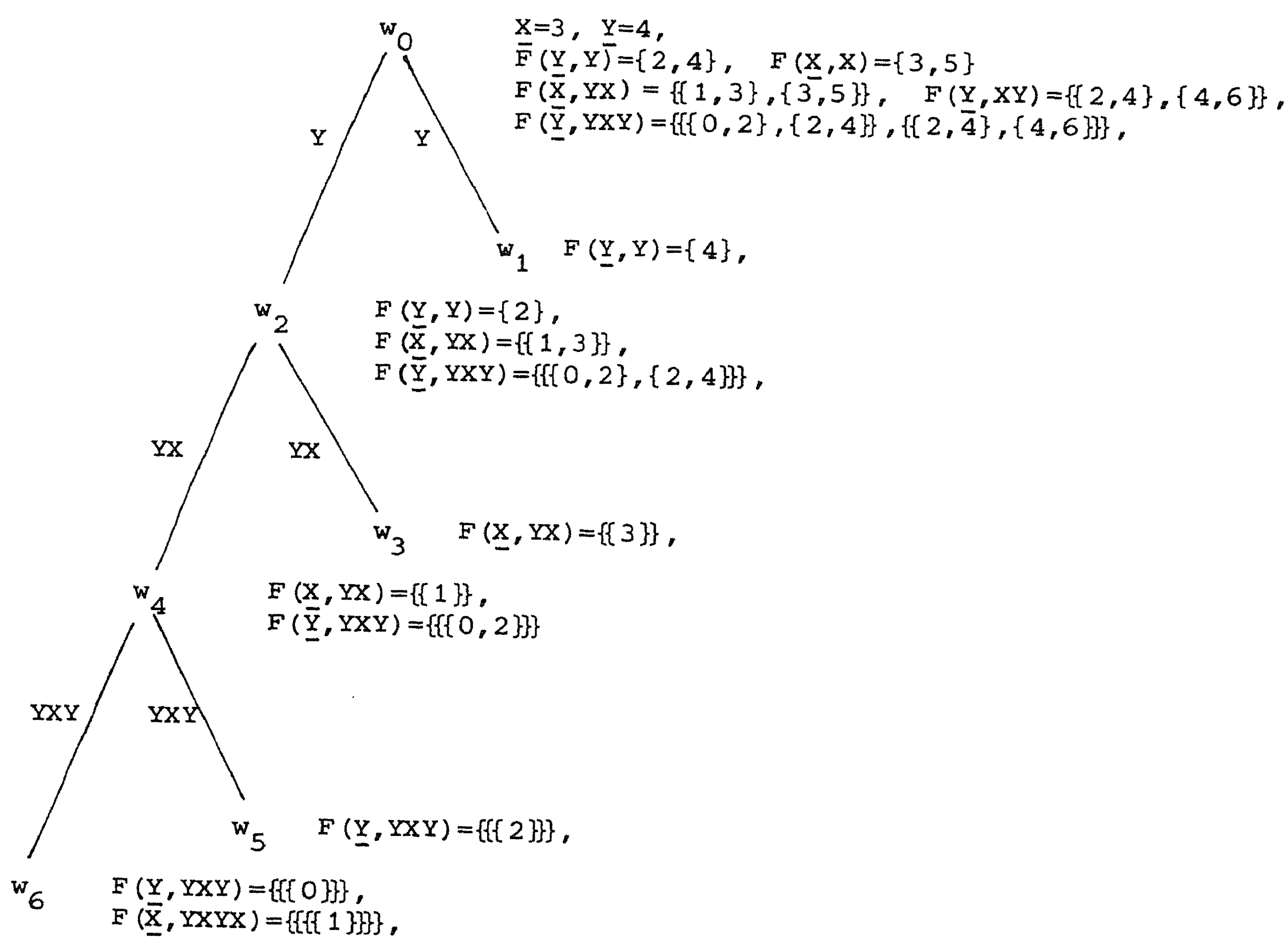


Diagram 7

Initial state: X says "no"; w_6 is a world with complete information for X; the information presented in w_6 is 3-extracted from the tree.

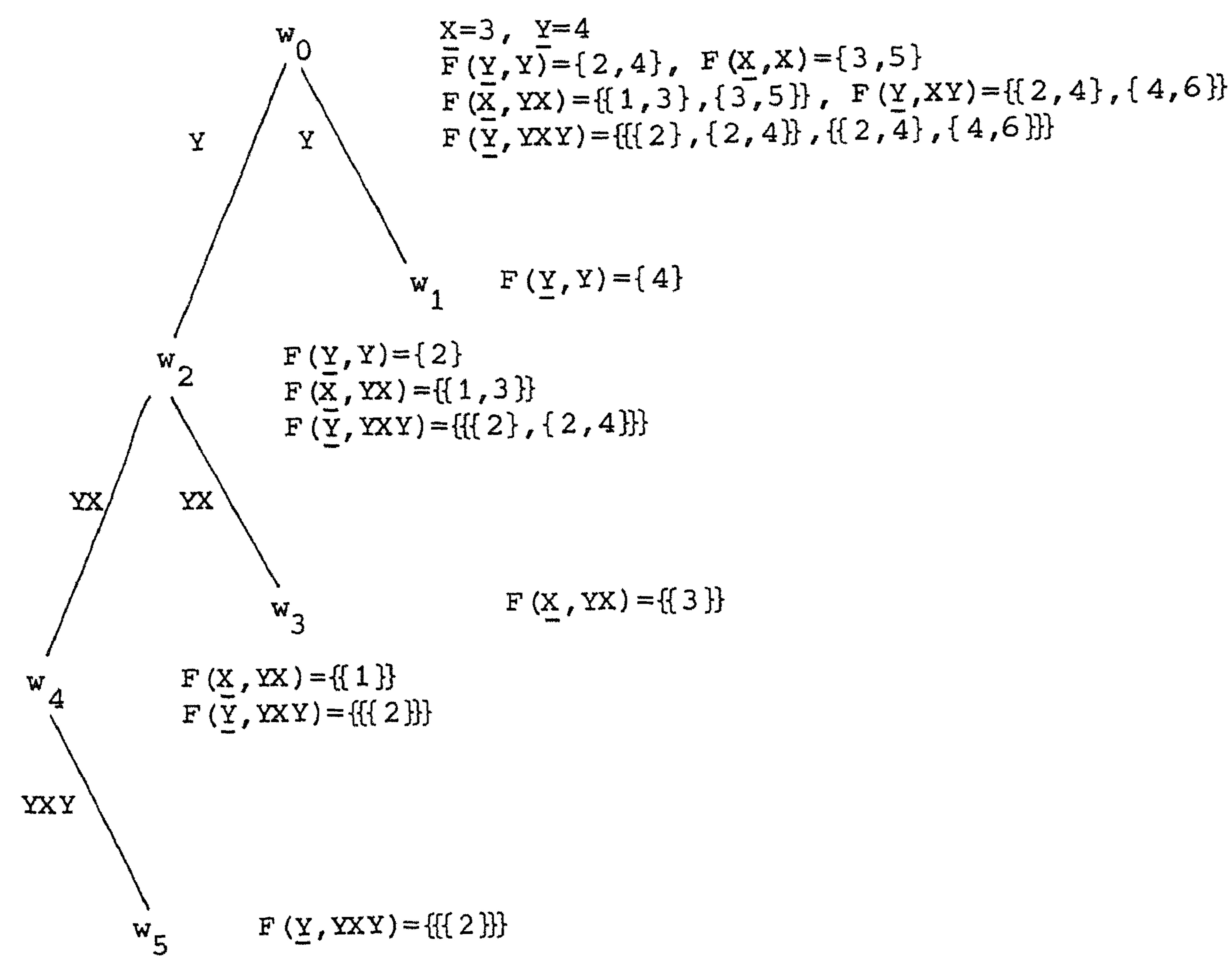


Diagram 8

Stage after X's "no" answer; Y says "no"; w_4 is a world with complete information for Y; its information is 2-extracted from the tree.

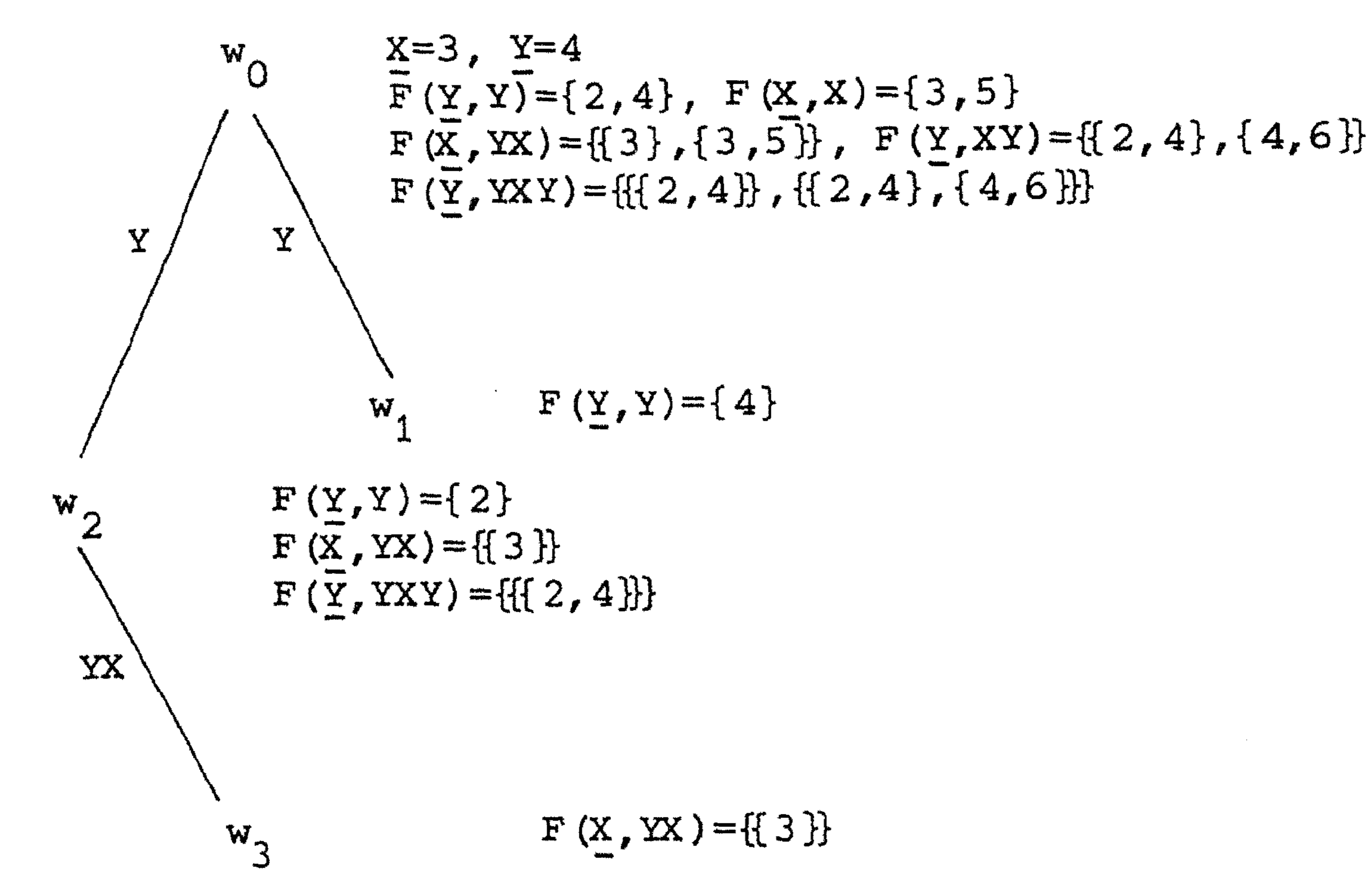
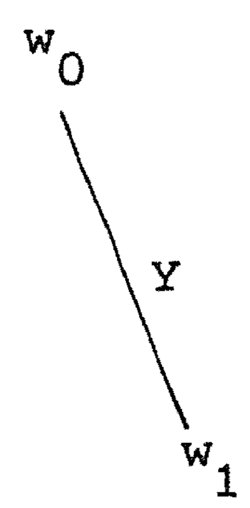


Diagram 9

Stage after Y's "no" answer; X says "no"; w_2 is a world with complete information for X; its information is 1-extracted from the tree.



$\underline{X}=3,\ \underline{Y}=4$
$F(\underline{Y},Y)=\{4\},\ F(\underline{X},X)=\{3,5\}$
$F(\underline{X},YX)=\{\{3\},\{3,5\}\},\ F(\underline{Y},XY)=\{\{2,4\},\{4,6\}\}$
$F(\underline{Y},YXY)=\{\{\{4\},\{4,6\}\}\}$

$F(\underline{Y},Y)=\{4\}$

Diagram 10

Stage after X's second "no" answer; w_0 is a world with complete information for Y, so Y says "yes" and the game terminates.

Note that in Diagram 10 the update on $F(\underline{Y},YXY)$ in the actual world is the combined result of a 1-extraction of the information at world w_2 in Diagram 9, together with a 3-extraction of a world with complete information two levels below w_3 (which is not shown in the diagram). This illustrates that indeed the entire tree has to be updated at infinitely many places at once, in order for the computation to work out correctly. If we restrict ourselves to V-values with respect to strings of bounded length, the "active" part of the tree, which we have to keep track of, will be finite.

6. CONCLUSION

As shown in the preceding two sections, the mathematical model developed in this paper has the required property: the termination of the game in the simple situation can be derived by an explicit calculation which does not involve an a priori analysis of the entire game. On the other hand the machinery involved is rather cumbersome: a complete formal definition of the tree structures involved would probably require several pages densely filled with formulas, and a formal proof that the computation works as it should, will take many more pages without presenting any new insight. A possible way of proving such a claim might be to show that after k moves, after the first answer of the player with the highest number, all numbers less than k have disappeared from the trees, yielding a new situation which is isomorfic with the initial situation under the mapping $m \to m-k$. This claim can be proved by induction by showing that it is correct for a single move (disregarding the first move in the game in case this is a move by the player with the lowest number). The proof of this induction step will require a nice recursive description of the trees. Note that in each tree there are infinitely many worlds with complete information since each node is ancestor of infinitely many worlds of this type at arbitrary distances. Therefore, the computation stages described in the previous sections actually require infinitely many steps, and at first glance, it is not at all clear that the resulting stage is always well-defined. It is conceivable that techniques for proving correctness of programs working on recursive data structures can be applied here.

If we consider the generalization of the formalism required for modelling the three-person game described by Conway, the combinatorial complexity increases strikingly: whereas the analysis given above only involves the linearly ordered chain of alternating strings, X, XY, XYX, ... , and Y, YX, YXY, ... , relevant strings in the three-person game itself form a tree, since there are two relevant ways of extending a string. For each path in this tree of strings a ternary tree of hypothetical extensions of the actual world has to be constructed. There will be some generalization of the adjacency conditions which have to be used for computing the initial structures. The update operator for processing a "no" answer probably will be more or less the same as the one presented in Section 5.

Our analysis disregards the question whether the termination of the game obtained corresponds to real human behaviour. One might argue that the model is "non-human". Consider again the tree as presented in Diagram 4 and consider world w_6. In this world, Y knows that $\underline{Y} = 2$, but on the other hand Y knows also that X is certain that Y knows that $\underline{Y} = 0$, but in fact $\underline{Y} = 4$! In this world the players not only use false hypotheses, but also hypotheses which they know by observation to be inconsistent with the real situation. In fact, they are required to disregard the real situation completely, i.e. they are required to act "as if" and to forget that they act "as if". After all it may therefore be the case that, from a psychological point of view, the non-termination argument corresponds to the real human situation, in particular for games (y,y+1), where y is sufficiently large (larger than 4 might already suffice). A similar conclusion might be obtained based upon complexity arguments. In order to terminate the game our analysis for the game (y,y+1) requires the players to develop the possible world trees up to level y at least. If one assumes that the human mind is incapable of dealing with information about information about information ... , at a level higher than three or four, these parts of the tree become inaccessible for human analysis and, consequently, the removal of worlds with complete information, which is necessary for the termination of the game, will never occur -

these worlds are too complex to be considered at all.

Clearly, the above remarks concerning human behaviour are highly speculative. However, the limit 3 or 4 is said to be reasonable by various colleagues during discussions held after talks given about the analysis presented. The reader is invited to amuse (or abuse?) his visitors at some future party be experimenting with the game, using his guests as victims. Such a test would at best affirm the existence of a limit value for y beyond which the game becomes non-terminating, without providing us with a precise explanation why this limit exists. Further psychological investigations will be needed in order to determine whether our model explains real behaviour or not.

From the above observations it now becomes clear how the paradox should be resolved; the conscious entities considered in the non-termination and termination proofs, respectively, are of different nature: humans versus robots.

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EXPRESSING LOGICAL FORMULAS IN NATURAL LANGUAGE

by

Joyce Friedman

1. INTRODUCTION

It would be of considerable practical as well as theoretical interest to be able to go from logical formulas to English sentences. We consider a highly constrained version of the problem, going from formulas of the intensional logic of Montague's PTQ [MONTAGUE 1973] to the corresponding sentences of the PTQ-fragment. PTQ is an appropriate framework in which to study the problem of obtaining sentences from logical formulas, because the relationship between syntax trees for sentences and corresponding formulas is given precisely. By working with the formulas in intensional logic that are obtainable from the syntax trees for English sentences we obtain a well-defined set of input formulas and a clear notion of the target sentences. We refer to the intensional logic as IL and the subset of IL reached by the English fragment of PTQ as IL-PTQ.

The main problem considered in this paper is finding an inverse for the function that yields a direct translation of a syntax tree. We first review the aspects of PTQ that make a solution possible. We then prove that each formula obtained as a direct translation of a PTQ sentence can be reversed to a unique source, and describe a LISP program that accepts a direct translation and finds the corresponding syntax tree.

Continuations of this work are then described briefly. These include a LISP program that finds one sentence for each lambda-reduced translation, and a proof that each lambda-reduced formula can correspond to only finitely many sentences. This shows that a program to find all sentences is possible. Finally we discuss possible extensions and applications of this work.

2. ENGLISH TO LOGIC IN PTQ

The process of going from an English sentence of the fragment to the corresponding formulas of intensional logic can be analyzed into a small set of distinct steps. We have previously described a parser that produces a set of analysis trees that show the derivation of the sentence [FRIEDMAN & WARREN 1978]. This set is constrained to be finite by omitting trees that are not interestingly different from those in the set. The rules of PTQ are then applied to the syntactic derivation to yield what we shall call a *direct translation*. The direct translation is a formula of the intensional lambda-calculus IL.

The next steps apply reductions. The processes of lambda-reduction and cancellation of the extension operator with an immediately following intension operator yield a *lambda-normal form*. [FRIEDMAN and WARREN forthcoming]. A final reduction step in PTQ replaces intensional constants by extensional ones, where this is justified by the meaning postulates [FRIEDMAN & WARREN 1979; see also JANSSEN 1976].

Global transformations such as equality reduction might then be applied, though in fact neither our programs nor Janssen's do so. For example, *Bill is a man* yields the formula $\exists x\ [\mathrm{man}'(x) \wedge [b = {}^{\vee}x]]$ which is equivalent to $\mathrm{man}'({}^{\wedge}b)$. This transformation is not applied by the programs for going from sentence to formula; it is not reversed by the inverse program. This transformation differs from those that are applied because it requires an analysis of the full structure of the formula to see the extent of the possible replacement.

3. LOGIC TO ENGLISH: IL-PTQ TO PTQ

The inverse problem for PTQ consists in reversing this process. As a starting point for reversal one might choose the direct translations, the lambda-reduced forms, the extensionalized forms, or, ideally, the full set of formulas of IL-PTQ. The first two of these constrain the problems sufficiently so that we have been able to solve it. There is reason to believe that solution is possible also for the extensionalized forms. For the full IL-PTQ it appears not to be solvable because of the undecidability of IL-PTQ [WARREN 1979].

4. PROJECTING A SYNTAX TREE TO A SENTENCE

4.1. In an approach based on PTQ the most natural way to obtain an English sentence from a logical formula is to go from the formula to a syntax tree, and then from the tree to the sentence. The process of going from syntax tree to sentence is a straight-forward application of Montague's S-rules [FRIEDMAN 1978, JANSSEN 1977], and yields a unique result. In the remainder of this paper we assume such a program is available and discuss the more interesting and difficult part of the problem - obtaining the syntax tree form the formula.

We have found it convenient in our programs to use syntax trees that differ somewhat from the analysis trees given in PTQ. The nodes are labelled with the names of syntactic rules S_n, rather than syntactic operations F_i. The tree also contains the syncategorematic items added by the rule (*the, a, every, such-that*), indicators of the subpart of rule S17 used (*not, future, past*), also the indicator he_n for the variable x_n used in rules S3, S14, S15, and S16. It is easy to map between the two sets of trees, because (1) the information in the syntax tree determines uniquely a syntactic operation F_i, (2) no two S-rules have the same operation F_i and the same input categories, and (3) no phrase is a basic phrase of two distinct categories.

Two examples of syntax trees in the LISP representation that we use are:

```
(S4 BILL (S10 TALK (S6 ABOUT (S2 A UNICORN))))
```

and

```
(S14 HEO (S2 A UNICORN) (S4 BILL (S10 TALK (S6 ABOUT HEO))))
```

An alternative display form is also used. In it the second tree above would appear as:

```
S14     HEO
        S2      A
                UNICORN
        S4      BILL
                S10     TALKS
                        S6      ABOUT
                                HEO
```

These trees are both parses of the sentence *Bill talks about a unicorn.*

4.2. Compound basis phrases

Our version of PTQ differs in a few very minor ways from the original. The primary change is that we use hyphenated constructs where a basic expression would otherwise consists of two words. Thus we have $B_{IV/t}$ = {*believe-that, assert-that*} and $B_{IV//IV}$ = {*try-to, wish-to*}. We also introduce *such-that* rather than *such that* in rule S3. Later when we refer to the *length of a phrase* these compound words will be the units. For example, the length of *every man wishes-to find a unicorn such-that it loves him* is 10. Since *that* and *to* never occur in the fragment except in these constructs, the arguments below could easily be transformed into arguments about the original version of PTQ.

5. DIRECT TRANSLATIONS

In PTQ the notion of compositionality is carried strictly through both the syntax and the translation to intensional logic. There are seventeen syntactic rules, or S-rules, S1-S17. Their basic form is:

> If a is a phrase of category A, and b is a phrase of category B, then F(a,b) is a phrase of category C.

A syntactic function F is specified for each rule; the same F may be used in more than one rule.

Corresponding to each of the S-rules, there is a translation rule, or T-rule, T1-T17, of the form:

> If a is a phrase of category A, and b is a phrase of category B, and a and b translate to a' and b' respectively, then F(a,b) translates into G(a',b').

A function G is specified for each rule; the same function may be used in more than one rule.

The exceptions to this pattern are rules S1 and T1 which form the basis step of the inductive definition. S1 specifies that the basic phrases of category A (which are listed explicitly) are to be included in the phrases of category A. T1 gives the translations of the basic phrases. In general, a translates to g(a) where g is an unspecified fixed biunique function that assigns to a basic phrase of category A, a constant of the type corresponding to the category A. The text of this paper follows Montague in using a primed version of the English word as the value of g; in the programs we take g as the identity function.

By a *direct translation* we mean the result of applying the T-rules to a syntax tree. The T-rules yield a direct translation function: a syntax tree has a unique direct translation. Thus we may speak of *the* direct translation of a syntax tree. But we cannot extend this to speak of *the* direct translation of an English phrase; a phrase may have (infinitely) many syntax trees, and hence may have different direct translations.

The problem of reversing the direct translation is thus to recognize which T-rules have been applied and to reverse their application. If, working from a translation $\underline{c}'$, we can uniquely select the function G and formulas $\underline{a}'$ and $\underline{b}'$ for which $\underline{c}' = G(\underline{a}',\underline{b}')$ then the method can be applied recursively to yield a derivation specifying G, but ambiguous with respect to which of the T-rules using that G is needed. If, further, we can uniquely select the particular T-rule, the full derivation can be identified.

6. UNIQUENESS OF ANALYSIS TREE FOR A DIRECT TRANSLATION

6.1. In this section we show that given a direct translation it is possible to find the unique phrase to which it corresponds. This proof will justify the algorithm used in the computer program. The arguments that follow rely on Table I. There we give for each syntactic category all the sources for phrases of that category and the form of their translations.

NOTATION. We use underlined letters for phrases and the same letters primed for the corresponding direct translations.

LEMMA 1. *The type of an expression* $\underline{c}'$ *of* IL *is unique and is effectively computable from* $\underline{c}'$.

PROOF. This follows from the definition of meaningful expression of IL.

LEMMA 2. *No basic expression is of two categories:*

$$A \neq C \Rightarrow B_A \cap B_C = \emptyset.$$

PROOF. By inspection of the sets of basic expressions.

LEMMA 3. *No meaningful expression translates both a phrase of category* IV *and a phrase of category* CN. *Given a direct translation of type* $\langle\langle s,e\rangle,t\rangle$ *the category of the source phrase can be effectively determined.*

PROOF. From the table we see that there are three sources for category CN and 7 sources for category IV. The category of a phrase $\underline{c}$ can be determined from the form of the direct translation $\underline{c}'$ as follows:

If $\underline{c}'$ is a constant, then $\underline{c}$ is a CN or IV according as it is in B_{CN} or B_{IV}.

TABLE I: SYNTAX AND TRANSLATION RULES OF *PTQ*

index	*name*	*source*	*translation*
e	entity	none	
IV (=t/e)	intransitive verb	B_{IV}	$\underline{a}'$
		S5 from ($\underline{a}$:IV/T,$\underline{b}$:T)	$\underline{a}'(\hat{}\,\underline{b}')$
		S7 from ($\underline{a}$:IV/t,$\underline{b}$:t)	$\underline{a}'(\hat{}\,\underline{b}')$
		S8 from ($\underline{a}$:IV//IV,$\underline{b}$:IV)	$\underline{a}'(\hat{}\,\underline{b}')$
		S10 from ($\underline{a}$:IV/IV,$\underline{b}$:IV)	$\underline{a}'(\hat{}\,\underline{b}')$
		S12 from ($\underline{a}$:IV,$\underline{d}$:IV)	$\lambda x[\underline{a}'(x) \vee \underline{d}'(x)]$
			$\lambda\ [\underline{a}'(x) \wedge \underline{d}'(x)]$
		S16 from ($\underline{a}$:T,$\underline{d}$:IV)	$\lambda y\underline{a}'(\hat{}\lambda x_n[\underline{d}'(y)])$
T (=t/IV)	term	B_T	$\lambda P[[{}^{\vee}P](\hat{}j)]$
			$\lambda P[[{}^{\vee}P](x_n)]$
		S2 from ($\underline{z}$:CN)	$\lambda P\forall x[\underline{z}(x) \rightarrow [{}^{\vee}P](x)]$
			$\lambda P\exists y[\forall x[\underline{z}'(x) \leftrightarrow x=y] \wedge [{}^{\vee}P](y)]$
			$\lambda P\exists x[\underline{z}'(x) \wedge [{}^{\vee}P](x)]$
		S13 from ($\underline{a}$:T,$\underline{b}$:T)	$\lambda P[\underline{a}'(P) \vee b'(P)]$
IAV (=IV/IV)	IV-modifying adverb	B_{IAV}	$\underline{a}'$
		S6 from ($\underline{d}$:IAV/T,$\underline{b}$:T)	$\underline{d}'(\hat{}\,\underline{b}')$
CN (=t//e)	common noun	B_{CN}	$\underline{a}'$
		S3 from ($\underline{z}$:CN,$\underline{p}$:t)	$\lambda x_n[\underline{z}'(x_n) \wedge \underline{p}']$
		S15 from ($\underline{a}$:T,$\underline{z}$:CN)	$\lambda y\underline{a}'(\hat{}\lambda x_n[\underline{z}'(y)])$
t	sentence	S4 from ($\underline{a}$:T,$\underline{b}$:IV)	$\underline{a}'(\hat{}\,\underline{b}')$
		S9 from ($\overline{\square}$:t/t,$\underline{p}$:t)	$\overline{\square}(\hat{}\,\underline{p}')$
		S11 from ($\underline{p}$:t,$\underline{q}$:t)	$\underline{p}' \vee \underline{q}'$
			$\underline{p}' \wedge \underline{q}'$
		S14 from ($\underline{a}$:T,$\underline{p}$:t)	$\underline{a}'(\hat{}\lambda x_n\underline{p}')$
		S17 from ($\underline{a}$:T,$\underline{d}$:IV)	$\neg a'(\hat{}\,\underline{d}')$
			$W\underline{a}'(\hat{}\,\underline{d}')$
			$\neg W\underline{a}'(\hat{}\,\underline{d}')$
			$H\underline{a}'(\hat{}\,\underline{d}')$
			$\neg H\underline{a}'(\hat{}\,\underline{d}')$
TV (=IV/T)	transitive verb	B_{TV}	$\underline{a}'$
IAV/T	preposition	$B_{IAV/T}$	$\underline{a}'$
t/t	sentence adverb	$B_{t/t}$	$\underline{a}'$
IV/t	sentence-taking verb phrase	$B_{IV//t}$	$\underline{a}'$
IV//IV	IV-taking verb phrase	$B_{IV//IV}$	$\underline{a}'$

If $\underline{c}'$ is $\lambda x[\underline{z}'(x) \wedge \underline{p}']$ or the same form with a disjunction, then $\underline{c}$ is a CN or IV according as $\underline{z}$ is.

If $\underline{c}'$ is of the form $\lambda y\underline{a}'(^\wedge\lambda x[\underline{z}'(y)])$, then $\underline{c}$ is a CN or IV according as $\underline{z}'$ is.

If $\underline{c}'$ is of the form $\underline{a}'(^\wedge\underline{b}')$ then $\underline{c}$ is of category IV.

Since no expression $\underline{c}'$ is of more than one of these four forms, it follows that the category of $\underline{c}$ is unique and effectively computable. The proof can be formalized as an induction on the length of the formula.

LEMMA 4. *Given a direct translation* $\underline{c}'$ *the corresponding category* C *of* $\underline{c}$ *is unique and effectively computable from* $\underline{c}'$.

PROOF. The type of $\underline{c}'$ is computable by Lemma 1. The categories with the same corresponding type are the pair IV/IV and IAV (=IV//IV) and the pair IV (=t/e) and CN (=t//e). The category IV/IV contains only the basic phrases *try-to* and *wish-to*; these are distinct from the phrases of the category IAV which are the adverbs in B_{IAV} and the prepositional phrases, beginning with *in* and *about*. The pair IV and CN follows by the preceding lemma.

THEOREM 1. *If* $\underline{c}'$ *is a direct translation, then from* $\underline{c}'$ *we can effectively determine a unique category* C *such that any phrase* $\underline{c}$ *with direct translation* $\underline{c}'$ *must be of category* C.

PROOF. By Lemma 1 we can determine the logical type for $\underline{c}'$. By lemma 4 the category of $\underline{c}$ is then unique and effectively computable.

THEOREM 2. *Given a direct translation* $\underline{c}'$ *there is a unique phrase* $\underline{c}$ *for which* $\underline{c}'$ *is the direct translation. The phrase* $\underline{c}$ *is effectively computable from* $\underline{c}'$.

PROOF. The proof will show that exactly one syntax tree is obtained from $\underline{c}'$. Since each tree has a unique projection to a sentence, the phrase $\underline{c}$ is also unique.

By Theorem 1 we can determine the category C to which any corresponding phrase $\underline{c}$ must belong. We show that, given $\underline{c}'$ and the category C of $\underline{c}$, the syntax tree for $\underline{c}$ is recursively computable.

Basic expressions. By Lemma 2 and the required biuniqueness of the translation function g from basic expressions of category A to constants of type f(A) translations of basic expressions all have unique corresponding phrases. Categories TV, IAV/T, t/t, and IV/t have only basic expressions, so they are fully covered by this case. For derived expressions we proceed by cases on the productive categories C:

Category IV. The translations are all of distinct forms except for the translations of S5, S7, S8, and S10, which are all of the form $\underline{a}'({}^\wedge\underline{b}')$. These are distinguishable by the category corresponding to $\underline{a}'$.

Category T. The four translation forms are distinct.

Category CN. The translations of the derived CN are distinct from one another and from the basic CN phrases.

Category IAV. The translations of derived adverbs are immediately distinguishable from the basic ones.

Category t. The translations for S9, S11, and S17 are clearly distinct from each other and from S4 and S14. The only possible difficulty is in distinguishing $\underline{a}'({}^\wedge\underline{b}')$ of S4 from $\underline{a}'({}^\wedge\lambda x\underline{p}')$ of S14 if $\underline{b}'$ is itself a lambda-expression. We solve this case by Lemma 5 below.

6.2. <u>Rules S4 and S14</u>

The possibility that remains to be excluded is that two t-phrases could have the same direct translation, one phrase derived by rule S4 as $\underline{a}'({}^\wedge\underline{b}')$ and the other by S14 as $\underline{a}'({}^\wedge\lambda x\underline{p}')$. This would be possible only if the IV-translation $\underline{b}'$ is identical to the lambda-expression $\lambda x\underline{p}'$, where $\underline{p}$ is a t-phrase. An IV-phrase can be a lambda-expression in two cases: S12 and S16. We must show that neither of these is the same as $\lambda x\underline{p}'$ for any sentence $\underline{p}$.

<u>EXAMPLE 1</u>. The sentence *John runs and John talks* derived by rules S14 and S11 with the tree (S14 HE0 JOHN (S11 (S4 HE0 RUN) AND (S4 HE0 TALK))) has the direct translation $j^*({}^\wedge\lambda x_0[x_0^*({}^\wedge run') \wedge x_0^*({}^\wedge talk')])$, where j^* is $\lambda P[[{}^\vee P]({}^\wedge j)]$ and x_0^* is $\lambda P[[{}^\vee P](x_0)]$. The sentence *John runs and talks,* derived by S4 and S12 with the tree (S4 JOHN (S12 RUN AND TALK)) has the translations $j^*({}^\wedge\lambda x[run'(x) \wedge talk'(x)])$. These formulas are of the same external form, $j^*({}^\wedge\lambda x[\underline{a}' \wedge \underline{b}'])$, and will match the same templates. We must find some other test to distinguish between them and between other similarly confusible S4, S14 pairs.

<u>LEMMA 5</u>. *No intransitive verb phrase has the direct translation* $\lambda x\underline{p}'$ *where* $\underline{p}'$ *is the direct translation of a sentence.*

<u>PROOF</u>. The proof is by induction on the construction of the intransitive

verb phrase b.

If b is a basic expression of category IV, b' is a constant.

If b is derived by S5, S7, S8, or S10, then b' is not a lambda-expression.

If b is derived by S12, its translation is $\lambda x[a'(x) \wedge d'(x)]$, where a and d are intransitive verb phrases, or similarly by disjunction. To see that $[a'(x) \wedge b'(x)]$ cannot be p' for a sentence p, note that a conjunction can be a sentence translation only if both of its conjuncts translate sentences. But from the Table we see that a'(x) is not the direct translation of any sentence. Similarly for the disjunction case.

If b is derived by S16, the translation is $\lambda y a'(^{\wedge}\lambda x_n[d'(y)])$, where a' translates an intransitive verb phrase. The body of this lambda-expression cannot be a sentence translation because only S4 and S9 produce the form $a'(^{\wedge}b')$ for sentences, and for S4 a is a term phrase and for S9 it is a t/t-phrase. By Lemma 4, no a' translates phrases of two distinct categories.

One solution that trivializes the above problem would be to distinguish the S14 formula from any of the S4 formulas on the basis that the lambda-abstraction is over a subscripted variable x_n. We did not take this course, since it is tied to a particular detail of the translation rules. The test on whether b' translates an IV-phrase also treats correctly formulas to which a change of bound variable has applied.

6.3. Extensions to PTQ

In Karttunen's extension of PTQ to questions KARTTUNEN [1977] it is no longer the case that the direct translation reverses to a unique sentence. His yes/no question rule produces *whether* p, *whether or not* p, and *whether* p *or not* with the same direct translation. The ambiguity in reversal here could most easily be handled by reversing the formula to a single tree, and extending the projection function for syntax trees to be a relation.

This raises the question of how far the result above will extend. The details of the proof can be expected to suggest approaches to this problem in each particular extension to PTQ.

7. THE LISP PROGRAM FOR REVERSING A DIRECT TRANSLATION

7.1. A formula of IL is represented in prefix notation. For example, the direct translation of the first parse of *The man has run* is represented as (PAST ((LAMBDA P (THERE-IS X2 (AND (FOR-ALL X1 (IFF (MAN X1) (EQ X1 X2))) ((EXT P) X2)))) (INT RUN))).

The reversal algorithm is a recursive application of functions which accept a logical formula c' that is the direct translation of a phrase of category A, and determine the translation rule and the formulas a' and b' from which c' was obtained. It follows the above proof closely.

The program consists of four sets of functions: *Inmean* functions for each of the five productive categories, *RS*$_i$ functions to reverse each of S2 through S17, a *Rulesinit* function that establishes a pattern or template for each rule, and a *Rulematch* function that compares formulas and templates. The *Rulesinit* function initializes each rule template to the translation pattern of Table I. To test if a rule S_i applies, the function *Rulematch* compares the rule template to the formula and it it matches identifies the relevant subformulas. These are then passed to the corresponding tree-building function *RS*$_i$. For each of the rules S_i of PTQ there is function *RS*$_i$. It builds a tree with S_i at the root and the trees for its subphrases as leaves.

The *Inmean* functions reverse formulas corresponding to a particular category. There is one for each productive category: *Inmeants*, *Inmeante*, *Inmeancn*, *Inmeaniv*, and *Inmeaniav*. (Because of a LISP restriction to a single case, our mnemonics use 'ts' instead of 't' for sentence, and 'te' instead of 'T' for term phrase.)

In an earlier version of this code, there was only one function *Inmean* that did the work of the five functions in the current code. Since no formula can be the direct translation of more than one phrase, the functions could clearly be combined, but at each stage information would be thrown away and then recomputed. For this reason the present version seems preferable.

The *RS*$_i$ functions take as arguments the subformulas of rule S_i and recursively call the *Inmean* functions to find their trees. For example, *RS3* is called after the pattern (LAMBDA X (AND (ZETA X) PHI)) has been matched by a CN-translation. The subformulas matching X, ZETA, and PHI are passed to *RS3*. *RS3* calls *Inmeancn* with the formula matching ZETA and *Inmeants* with the formula matching PHI. If both succeed it returns a subtree with root S3 and the CN- and t-subtrees just obtained, in the form: (S3 x CN-subtree SUCH-THAT t-subtree), where x is the subformula that matched X.

Execution for a formula a' that is the direct translation of a sentence a begins by calling *Inmeants* and proceeds recursively. The decisions about decomposition can all be made at the top level except for the possible confusion between S4 and S14 discussed above. Consequently there is one additional function *RS4/14* that carries out the analysis requisite to decide between the two rules.

EXAMPLE 2. Given the direct translation $[\lambda P[^{\vee}P](^{\wedge}j)](^{\wedge}run')$, or ((LAMBDA P ((EXT P)(INT J)))(INT RUN)), the top-level application of *Inmeants* uses *Rulematch* to determine that this is of the form $\underline{a}'(^{\wedge}\underline{b}')$ that translates rules S4 through S10 and also rule S14. Of these S4, S9, and S14 produce sentences. $\underline{a}'$ is $[\lambda P[^{\vee}P](^{\wedge}j)]$ and $\underline{b}'$ is *run'*. Since $\underline{a}'$ does not match the translation of *necessarily,* rule S9 is not possible, so *RS4/14* is called to select either S4 or S14. It applies *Inmeaniv* to $\underline{b}'$ and thus establishes that $\underline{b}$, that is, *run,* is in B_{IV}. The syntax tree must then be (S4 $\underline{a}$ $\underline{b}$) where $\underline{a}$ is a term phrase and $\underline{b}$ is *run*. *Inmeante* applied to $\underline{a}'$ finds that $\underline{a}'$ matches the pattern for translating basic terms that are proper names, and j is returned as the entity. The term *John* is found as a property of the entity j. The resulting tree is thus (S4 JOHN RUN). The corresponding sentence is of course *John runs*.

EXAMPLE 3. Consider the formula $[\lambda P\exists x_4[\forall x_3[man'(x_3) \leftrightarrow [x_3=x_4]] \wedge [^{\vee}P](x_4)]](^{\wedge}run')$, represented as ((LAMBDA P (THERE-IS X4 (AND (FOR-ALL X3 (IFF (MAN X3)(EQ X3 X4))) (EXT P) X4)))) (INT RUN))). *Inmeants* first uses *Rulematch* to establish that the formula matches the pattern of rules S4 through S10, and to find the subformulas corresponding to $\underline{a}'$ and $\underline{b}'$. As above, the pattern is recognized as S4, with the intransitive verb phrase *run*. *Inmeante* is called to reverse $\underline{a}'$. *Inmeante* calls *Rulematch* for the pattern for a term of each of the two basic types (j^* and x_n^*) and this fails. It then tries to match the templates for rule S2; this succeeds on the pattern for *the*. *RS2* is then called and returns the tree (S2 THE MAN). *Inmeants* then calls *Inmeaniv* with the argument RUN and it returns RUN. The result then returned by *Inmeants* is (S4(S2 THE MAN) RUN).

EXAMPLE 4. The sentence *John runs and John talks* of Example 1 above illustrates the choice of S14. Recall that the direct translation of this sentence is $j^*(^{\wedge}\lambda x_0[x_0{}^*(^{\wedge}run') \wedge x_0{}^*(^{\wedge}talk')])$, where j^* is $\lambda P[[^{\vee}P](^{\wedge}j)]$ and $x_0{}^*$ is $\lambda P[[^{\vee}P](x_0)]$. *Inmeants* finds that the pattern of T4/10 matches, and so calls *RS4/14*; *Inmeante* is called and reverses j^* to *John*. *Inmeaniv* is then called with the argument of j^* and finds that it is not the translation of an intransitive verb phrase. *Inmeants* is then called with this same argument, finds that the conjunction rule S11 matches, and then finds the two subphrases *John runs* and *John talks*. The correct result is then returned.

7.2. Error-checking

The program assumes that the input formula is a direct translation of some phrase of the designated category. If not, the error is detected and no result is returned. A reason for rejecting bad inputs is that the intensional logic contains many formulas that are not in IL-PTQ even though they are very close in structure to the acceptable ones. For example, the formula $\neg[\text{walk}'(^\wedge j) \wedge \text{talk}'(^\wedge b)]$ has no corresponding sentence in the PTQ-fragment; it can be expressed awkwardly in English as *It is not the case that John walks and Bill talks.*

The only errors not detected are those having to do with the types of variables; correct variable types are not enforced. The program does not now distinguish between the correct term phrase translation $[\lambda P[^\vee P](^\wedge j)]$ and the incorrectly typed formula $[\lambda x[^\vee x](^\wedge j)]$. To do this we would first have to adopt some conventions on variables types. Suitable conventions have been worked out and the type-checking functions exist [FRIEDMAN, MORAN, & WARREN 1978b], but they have not been incorporated here.

8. THE LAMBDA-REDUCED FORM

In going from sentences to formulas, the direct translation is immediately reduced to a more readable form by logical reductions. In reversing the process a next problem would be to begin with these logically reduced formulas. The formulas displayed by Montague in the examples of PTQ can be obtained from the direct translations by reductions of three kinds: lambda-reductions (including the special case of reducing $[^\vee[^\wedge a]]$ to a, global reductions, such as reducing $\exists x[\text{man}'(x) \wedge b = {}^\vee x]$ to $\text{man}'(^\wedge b)$, and the extensionalization, that is, introducing d_* for d as justified by the meaning postulates. Montague defines *logically equivalent* so that $\underline{p}$ and $\underline{q}$ are logically equivalent if and only if the biconditional $[\underline{p} \leftrightarrow \underline{q}]$ is true in every interpretation that satisfies the meaning postulates which he gives. Lambda-reduction and the global reductions preserve logical equivalence in the stronger sense that the biconditional is true in every interpretation. Extensionalization preserves equivalence only under the meaning postulates. The first reductions applied to a direct translation are lambda-contraction, that is, replacing $[\lambda x\underline{a}](\underline{b})$ by the result of substituting $\underline{b}$ for all free occurrences of x in $\underline{a}$, and $^{\vee\wedge}$-contraction, that is, replacing $[^\vee[^\wedge \underline{a}]]$ by $\underline{a}$. For example, $[\lambda P[^\vee P(^\wedge j)]](^\wedge \text{run}')$ reduces by lambda-contraction to

$[{}^{\vee}[{}^{\wedge}run']](^{\wedge}j)$, which in turn reduces to $run'(^{\wedge}j)$ by ${}^{\vee\wedge}$-contraction. For the intensional logic of PTQ lambda-contraction can occur only under certain restrictions on the functional argument or the positions into which it is substituted. For this logic, an appropriate well-defined reduction class exists. We call this the lambda-normal form, and take it as the starting point in the next step of going from formulas to sentences.

We first present the relevant definitions and results from FRIEDMAN & WARREN [forthcoming] in which we show the existence of a unique lambda-normal form for the formulas of PTQ.

A formula $[\lambda x\underline{a}](\underline{b})$ is *contractible* if $\underline{b}$ is modally closed or x does not occur in an intensional context in $\underline{a}$. Here modal closure is a syntactic condition on the formula which holds if the syntactic form guarantees that $\underline{a}^{@,i,j,g}$, that is, the extension of $\underline{a}$ with respect to a model @, a point of reference <i,j> and a variable assignment g, is independent of the point of reference. See GALLIN [1975] for a definition. The *contraction of a* contractible part $[\lambda x\underline{a}](\underline{b})$ is the result of replacing each free occurrence of x in $\underline{a}$ by $\underline{b}$, with suitable change of bound variables to avoid variable collisions. A formula $[{}^{\vee}[{}^{\wedge}\underline{a}]]$ is always *contractible*; its *contraction* is $\underline{a}$.

A formula $\underline{a}$ is *in reduced form* if it contains no contractible parts. It is *fully reduced* if it is in reduced form and contains no lambda applications $[\lambda x\underline{a}](\underline{b})$.

For PTQ, we define a *translation* of a phrase to be its direct translation $\underline{a}$ (by rules T1–T17), or any well-formed formula $\underline{d}$ to which $\underline{a}$ can be reduced by contraction of contractible parts. In the reference we prove that in translations all functional arguments are modally closed. We also show that if all functional arguments of $\underline{a}$ are modally closed, there is a unique fully-reduced $\underline{b}$ to which $\underline{a}$ reduces. The proof is by extending ANDREWS [1971] and PIETRZYKOWSKI [1973] from the typed lambda-calculus to Montague's intensional logic.

It follows that in PTQ translations of English phrases have unique fully-reduced lambda-normal forms. This lambda-normal form, then, seems a natural starting place for going from formulas to English sentences. In many cases it corresponds most closely with the usual expression of English as logic. For example, *Every man runs* has the lambda-normal form $\forall x[man'(x) \rightarrow run'(x)]$.

The same lambda-reduced form may correspond to several derivation trees, and indeed, even to several English sentences. For example, *John walks and runs, John walks and John runs,* and *John walks and he runs* all have the formula [walk'($^\wedge$j) ∧ run'($^\wedge$j)].

9. NEXT STEPS IN THE REVERSAL PROBLEM

In sequels to this paper we plan to describe in detail the next steps in this reversal problem. Here we sketch some directions to be taken, and mention results already obtained.

9.1. Reversing each formula to a single sentence

There are two ways one might attack the problem of reversing a lambda-normal form formula to a sentence. One would be to use a lambda-abstraction process to obtain direct translations from each lambda-reduced form and to then use the previous program. We had originally planned to work along this line. However, at least for the grammar at hand, this step is not a necessary one in obtaining a tree from a formula. The LISP program which we have written goes directly from the lambda-reduced form to a syntax tree, without an intermediate logical stage. It assumes that the input is the lambda-reduced form of some PTQ sentence, and may give a wrong result if it is not. We view this program as primarily a learning step in the development of a program to obtain all the syntax tree.

9.2. Obtaining all sentences

A next development of the program would be to make it produce all of the finitely many different sentences for each lambda-reduced formula. This is at least theoretically possible because there are only finitely many sentences that have any given lambda-reduced form and a bound on their length can be computed. The proof [FRIEDMAN forthcoming] uses the observation that each English word of the sentence leaves a trace of length at least one in the formula, and that these traces are in general preserved by lambda-reduction. When the traces are lost by lambda-reduction, the corresponding words will fail to appear in the sentence, because of a vacuous substitution[1].

Thus, for a formula with n traces the length of a sentence from which it could have been derived is not greater than n words. [p∧p] to p were allowed.

9.3. Reversing extensionalized formulas

In PTQ Montague introduces extensional forms of the basic expression of types IV (=t/e) corresponding to intransitive verb phrases or common noun phrases, and type TV (=IV/t) corresponding to transitive verb phrases. These are introduced by the definition:

If $\underline{d} \in ME_{f(IV)}$ then $\underline{d}_*$ is to be $\lambda u \underline{d}(^\wedge u)$; and if $\underline{d} \in ME_{f(TV)}$ then $\underline{d}_*$ is to be $\lambda v \lambda u \underline{d}(^\wedge u, ^\wedge v^*)$, where v^* is $\lambda P[^\vee P](^\wedge v)$.

In our reduction programs the extensionalized forms $\underline{d}_*$ are introduced whenever we can justify so doing by an application of the meaning postulates. Could we reverse this process by reinserting $\underline{d}$ for $\underline{d}_*$ by the definitions above and then applying lambda-reduction to get a lambda-normal form, and then apply a lambda-normal form reversal program? For example, if we begin with run'$_*$(j), the substitution will give [λu run'($^\wedge$u)](j). If the conditions for lambda-reduction were met, this would lambda-reduce to run'($^\wedge$j), which would in turn reverse to the tree (S4 JOHN RUN) and thence to *John runs*. However, there is a problem in the lambda-reduction step, which fails in IL because j is not modally closed and the context ($^\wedge$u) is an intensional context. It is likely that the meaning postulates could be used to justify the reduction in all cases arising from extensionalization, since they apply in just those cases. However, this remains to be done.

9.4. Describing the model in English

The logical formulas in the system of PTQ are really only an intermediate step in going from natural language to an interpretation in a model. Another interesting problem might be to begin with a model and attempt to describe it in PTQ-English. We have so far been unable to formulate this problem so that trivial solutions are excluded. If the problem is not properly constrained, a solution would be to enumerate the sentences of PTQ-English, translate them to IL by Montague's rules, evaluate each formula in the model, and print out only the true sentences. With the programs currently available, this could be easily done, but it seems inadequate as a solution to the problem of describing a model in English.

The same trivial solution could be applied to the problem solved in the paper. However, in this case it is clear that our solution is nontrivial: the sentence is produced by a transformation of the formula with no consideration of other irrelevant sentences. This solution is clearly more efficient that the trivial one; a complexity argument could be given.

9.5. A proposed application

A possible application of the inverse programs has been suggested by Kurt Godden of the University of Kansas [GODDEN 1980a]. In his thesis [forthcoming] he investigates using a PTQ like system for machine translation from Thai to English. As the first part of the program he proposes to use a version of our PTQ-parser, which he has modified to accept a comparable fragment of Thai. The translation and reduction programs can then be used to obtain formulas of the intensional logic. Then, using the inverse programs of this paper, the formulas will be re-expressed in English.

Interesting problems arise in this process. As Godden has pointed out, the lexical constants of IL-Thai have to be related to those of IL-English. For example, *allegedly* has a different syntactic category and different logical type in Thai. He uses meaning postulates to relate the constants of the two languages, and a transfer function to replace the Thai constants when possible.

FOOTNOTES

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1 The converse is not true. Traces may appear in the formula even though the words do not appear in the sentence. PAXTON [1973] observes that the tree (S14 HE1 (S2 A UNICORN) (S4 BILL RUN)) has the direct translation $[\lambda P \exists x_1 [unicorn'(x_1) \& [{}^{\vee}P](x_1)]] ({}^{\wedge}\lambda x_2[\lambda P[[{}^{\vee}P]({}^{\wedge}b)]]({}^{\wedge}run')])$ with the reduced form $\exists x[unicorn'(x) \& run'({}^{\wedge}b)]$. This would appear to be an error in PTQ; nonetheless it is not a difficulty here since the excess length is in the formula, not in the sentence.

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PASSIVE AND REFLEXIVES IN PHRASE STRUCTURE GRAMMAR

by

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0. INTRODUCTION

The structure of this paper is as follows. Section 1 outlines the approach to subcategorization in phrase structure grammar that is developed in GAZDAR (forthcoming). This section constitutes a necessary preliminary to the sections which follow. Section 2 shows how a single metarule applied to the VP rules listed in Section 1 suffices to handle the data involving subject-controlled reflexives. Section 3 employs the phantom category TVP in a metarule analysis of passive that is essentially a notational variant of the categorial theory of passive defended in BACH (1980). Section 4 then shows how the inclusion of the category TVP in the grammar allows the theory of reflexives advanced in Section 2 to be generalized to cover the facts of object-controlled reflexivization. Finally, in Section 5, we briefly consider some data from Breton and Modern Irish which might be used to motivate the existence of phantom VPs in VSO languages.

We turn immediately to some preliminary remarks concerning notation, semantics, and rule format. Since we will be interpreting phrase structure rules as node admissibility conditions rather than as string-to-string mapping rules, we will not use the familiar rewrite arrow notation for PS rules, but instead use a notation which reflects more directly the relation such rules bear to the (sub)trees that they admit. Instead of (1)

(1) $S \rightarrow NP\ VP$

then, we will write

(2) $[_S\ NP\ VP]$

and analogously for all other rules.

We assume that each syntactic rule in the grammar should be associated with a semantic rule which gives the meaning of the constituent created by the syntactic rule as a function of the meaning of the latter's parts. We further assume that the semantic rules should take the form of rules of translation into intensional logic. These two assumptions commit us to what BACH (1976: 2) has called the *rule-to-rule* hypothesis concerning the semantic translation relation. We take a rule of grammar to be a triple of which the first member is an arbitrary integer - the number of the rule (the role of which will become apparent shortly), the second member is a phrase structure rule, and the third is a semantic rule showing how the intensional logic representation of the expression created by the phrase structure rule is built up from the intensional logic representations of its immediate constituents. We will use a Montague-like prime convention in the semantic rules: NP' stands for the (complex) expression of intensional logic which is the translation of the subtree dominated by NP, *run'* is the constant of intensional logic which translates the word *run* in English, etc. Within this framework the first rule of a grammar of English will be this:

(3) $\langle 1, [_S \text{ NP VP}], \text{ VP}'({}^\wedge\text{NP}')\rangle$.

We will use "rule" to refer both to the triple and to its second and third members (sometimes qualifying the latter with "syntactic" and "semantic", respectively) but this should not cause confusion.

Notice that the semantic rule in (3) is not the one adopted by Montague in PTQ in which the NP' is a function taking the VP' as argument. Instead we are taking VPs to denote functions from NP intensions to truth values, following Montague's earlier treatment in his "Universal Grammar". This latter treatment has recently been strongly motivated on phonological, syntactic, and semantic grounds by THOMASON (1976), KEENAN & FALTZ (1978), KEENAN (1979a), and BACH (1979). For example, this way of doing things makes it easy to ensure that (4a) does not entail (4b):

(4) a. A unicorn seems to be approaching.
b. There exists an entity such that that entity seems to be approaching.

But making sure that (4a) does not wind up entailing (4b) is difficult to do if one retains the PTQ rule.

We assume that there are general, putatively universal, conventions of feature distribution. The most important of these is what we shall call the Head Feature Convention (HFC, hereafter):[1]

(5) HFC: In a rule of the form $\Delta \rightarrow \ldots\delta\ldots$ where δ is the head of Δ, δ carries all the features associated with Δ.

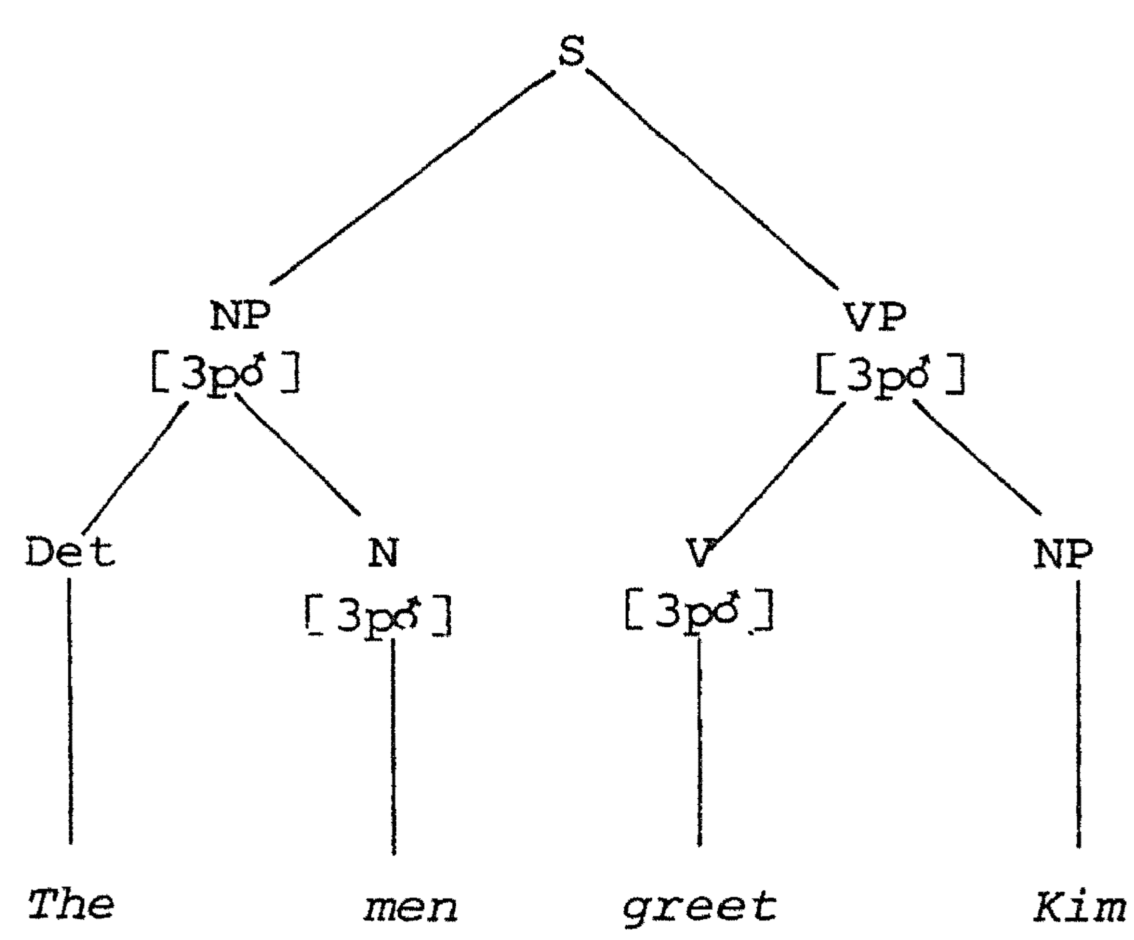
In the light of this convention, we can revise the rule given in (3) so as to capture subject-verb agreement in English:

(6) $\langle 1,\ [_S \underset{[\alpha]}{NP}\ \underset{[\alpha]}{VP}\ ],\ VP'(^\wedge NP')\rangle,$

where α ranges over permissible combinations of agreement features.

This rule, taken together with the HFC, will lead to trees such as that shown in (7):

(7)



We do not have to locate the subject noun, look up what agreement features it is carrying, then locate the first tensed verb in the VP and copy the noun's agreement features across. Instead (6) ensures that both NP and VP carry the same agreement features, and the HFC ensures that these find their way onto the relevant head noun and head verb.

Let us turn now to the rule for PP:

(8) $\langle 2,\ [_{PP}\ P\ NP],\ P'(NP')\rangle.$

A language like Latin marks NPs which stand as indirect objects, passive agents, etc., with morphological case-marking as dative, ablative, etc. In English one finds a class of PPs where in Latin one would have NPs, and these PPs are distinguished, not by case-marking but by choice of preposition. Suppose then that such PPs carry the name of the particular preposition as a feature so that the grammar employs complex symbols of the form PP[*to*], PP[*of*], PP[*for*] and PP[*by*]. These PPs can nevertheless still be expanded by means of the regular PP rule as given in (8). The HFC will carry the feature onto the prepositions that are the heads of such PPs, and then the following rule can realize the feature as the relevant preposition:

(9) $$\langle 3,\ [_{\underset{[F]}{P}} F],\ \lambda\tau[\tau] \rangle$$

where $F \in \{to, for, by, of, \ldots\}$
and τ is a variable of type $\langle\langle e,t\rangle,t\rangle$.

The semantic rule here is an identity function mapping NP extensions into themselves. Thus this kind of PP will end up having exactly the same meaning as the NP it dominates. This analysis makes the claim then that in such PPs the preposition does not carry any independent meaning but serves merely to indicate the argument place occupied by the NP.

1. SUBCATEGORIZATION

The format for rules that we are employing enables us to capture the unruly and idiosyncratic syntactic facts of subcategorization in a fairly elegant way. Suppose we have a rule of grammar *n* that introduces a lexical category X and that only a proper subset of lexical items of category X can appear under this node in the environment created by the syntactic component of rule *n*. Then we can allow *n* to be a feature on X, and interpret rule *n* as shown in (10):

(10) $$\langle n,\ [\ldots\ X\ \ldots],\ \ldots \rangle$$

to be an abbreviation-by-convention of (11):

(11) $$\langle n,\ [\ldots\ \underset{[n]}{X}\ \ldots],\ \ldots \rangle .$$

This use of rule numbers as subcategorization features eliminates the need for context-sensitive rules of lexical insertion. A context-free PS rule can allow X[*n*] to dominate only those lexical items permitted in the context defined by rule *n*. A direct consequence of this approach is that subcategorization is only allowed to be sensitive to (i) the category of the grandmother, and (ii) the category and position of the aunts, of the lexical item. An example may help elucidate this:

(12) $\langle i, [_{VP}\ V\ NP\ \underset{[to]}{PP}\ ], \ldots \rangle$

where V[i] → {*hand, sing, throw, give,* ...}.

Rule *i* says that a VP can consist of a V[i] followed by an NP followed by a dative PP. And among the lexical items that can be dominated by V[i] are *hand, sing, throw, give,* etc. Note that the use of complex symbols enables us to avoid the charge usually levelled against such context-free phrase structure proposals for lexical insertion, namely that by distinguishing V_i from V_j, say, we lose generalizations about verbs (e.g. that they all take tense). We do not lose the generalizations since V[i] and V[j] have at least two features in common (namely [+V, -N]) and it is this fact which accounts for the generalizations that can be made. We assume that tense, aspect, passive, etc. are marked as morphosyntactic features on nodes. Thus a tree may, for example, contain a node labelled V[i, +PASSIVE] immediately dominating *handed*.

The rules to be given below combine the approach to subcategorization developed above with (i) Bresnan-style claims (e.g. in her 1978) about syntactic categories and constituent structure, and (ii) a Montague-based approach to semantics. A similar Bresnan-Montague marriage has already been exploited productively by KLEIN (1978), LADUSAW (1979), and by McCLOSKEY (1979) in his grammar of Modern Irish, and the present proposals are indebted to those works. In this kind of approach, all the semantic work done in a classical transformational grammar by lexically governed syntactic rules like Equi and Raising is done by a combination of lambda abstraction and meaning postulates.

In the examples that follow, the *a*-line defines a rule of number *n*, the *b*-line lists example lexical items of category V[*n*], and the *c*-line gives an example of a constituent admitted by the rule.[2]

(13) a. $\langle 4, [_{VP}\ \underset{[\alpha]}{V}\ NP], V'(^{\wedge}NP')\rangle$

b. eat, sing, love, give, upset, ...

c. eats Fido.

(14) a. $\langle 5,\ [_{\substack{\mathrm{VP}\\ [\alpha]}}\ \mathrm{V\ NP}\ \underset{[\mathit{to}]}{\mathrm{PP}}\ ],\ \mathrm{V'}({}^{\wedge}\mathrm{PP'})\ ({}^{\wedge}\mathrm{NP'})\rangle$

b. hand, give, sing, throw, ...

c. hands Fido to Kim.

(15) a. $\langle 6,\ [_{\substack{\mathrm{VP}\\ [\alpha]}}\ \mathrm{V\ NP}\ \underset{[\mathit{for}]}{\mathrm{PP}}\ ],\ \mathrm{V'}({}^{\wedge}\mathrm{PP'})\ ({}^{\wedge}\mathrm{NP'})\rangle$

b. buy, cook reserve, ...

c. buys Fido for Kim.

(16) a. $\langle 7,\ [_{\substack{\mathrm{VP}\\ [\alpha]}}\ \mathrm{V\ NP\ NP}],\ \mathrm{V'}\ ({}^{\wedge}\mathrm{NP'})\ ({}^{\wedge}\mathrm{NP'})\rangle$

b. spare, hand, give, buy, ...

c. spares Fido a bath.

(17) a. $\langle 8,\ [_{\substack{\mathrm{VP}\\ [\alpha]}}\ \mathrm{V\ NP}\ \bar{\mathrm{S}}],\ \mathrm{V'}({}^{\wedge}\mathrm{NP'})\ ({}^{\wedge}\bar{\mathrm{S}}')\rangle$

b. promise, ...

c. promises Kim that Fido runs.

(18) a. $\langle 9,\ [_{\substack{\mathrm{VP}\\ [\alpha]}}\ \mathrm{V\ NP}\ \bar{\mathrm{S}}],\ \mathrm{V'}({}^{\wedge}\bar{\mathrm{S}}')\ ({}^{\wedge}\mathrm{NP'})\rangle$

b. persuade, tell, ...

c. persuades Kim that Fido runs.

(19) a. $\langle 10,\ [_{\substack{\mathrm{VP}\\ [\alpha]}}\ \mathrm{V}\ \underset{[\beta]}{\mathrm{NP}}\ \underset{[\beta]}{\overline{\mathrm{VP}}}\ ],\ \mathrm{V'}({}^{\wedge}\mathrm{NP'})\ ({}^{\wedge}\overline{\mathrm{VP}}')\rangle$

b. want, prefer, ...

c. wants Fido to run.

(20) a. $\langle 11,\ [_{\substack{\mathrm{VP}\\ [\alpha]}}\ \mathrm{V}\ \underset{[\beta]}{\mathrm{NP}}\ \underset{[\beta]}{\overline{\mathrm{VP}}}\ ],\ \mathrm{V'}({}^{\wedge}\overline{\mathrm{VP}}'({}^{\wedge}\mathrm{NP'}))\rangle$

b. expect, believe, ...

c. expects Fido to run.

(21) a. $\langle 12,\ [_{\underset{[\alpha]}{VP}}\ V\ \underset{[\beta]}{NP}\ \underset{[\beta]}{\overline{VP}}],\ V'({}^{\wedge}\overline{VP}')\ ({}^{\wedge}NP')\rangle$

b. persuade, ask, force, ...

c. persuades Fido to run.

(22) a. $\langle 13,\ [_{\underset{[\alpha]}{VP}}\ V\ \underset{[\beta]}{NP}\ \underset{[\beta]}{VP}],\ V'({}^{\wedge}VP')\ ({}^{\wedge}NP')\rangle$

b. make, ...

c. makes Fido to run.

2. REFLEXIVIZATION I: SUBJECT CONTROL

It does not seem to have occurred to linguists until recently that it might be possible to give an inductive definition of the set of rules in the grammar. Such an inductive definition can be seen as a grammar for the grammar. In this paper we make crucial use of what we refer to as meta-rules. These can be seen as clauses in the inductive definition of the grammar. Consider, by way of analogy, how the syntax of propositional calculus is standardly given. One begins by listing or enumerating a set of (atomic) sentences, and then one says "if A is a sentence, then $\neg$A is a sentence" and "if A and B are sentences then $A \wedge B$ is also a sentence" and so on. We can formulate a grammar for the grammar of a natural language in much the same way. We begin by listing a set of (atomic) rules, and then we say things of the form "if *r* is a rule of format *R*, then F(*r*) is also a rule, where F(*r*) is some function of *r*". We will refer to such statements as meta-rules.

Our grammar for VPs, as indicated in Section 2 above, contains a large number of rules having the general format shown in (23):

(23) $\langle n,\ [_{\underset{[\alpha]}{VP}}\ \ldots\ XP\ \ldots],\quad \rangle$, where X is N or P.

The set of rules characterized by (23) constitutes the domain of subject-controlled reflexivization. Accordingly, we will formulate subject-controlled reflexivization as an operation mapping VP rules into VP rules, namely the metarule shown in (34):[3]

(24) $$\langle n, [_{\substack{VP\\ [\alpha]}} \ldots XP \ldots], F\rangle \Longrightarrow$$

$$\langle n, [_{\substack{VP\\ [\alpha]}} \ldots \underset{\begin{bmatrix}\alpha\\ \mathrm{SELF}\end{bmatrix}}{XP} \ldots], \lambda PP\{\lambda r F(^\wedge\lambda PP(r))\}\rangle,$$ where X is N or P.

This says that for every rule in the grammar that fits the description given in (23), there is to be another rule identical in form except that the feature SELF is added to XP, and the translation of the reflexive VP involves lambda binding of a designated variable *r* in the manner indicated. We assume that independently needed feature conventions allow the agreement features and SELF to trickle down XP (e.g. in case XP = PP) to end up eventually on an NP. The SELF feature on such an NP then forces the appearance of a reflexive pronoun in virtue of such rules as that shown in (25):

(25) NP[SELF, +SING, 2PERS] → *yourself* (*yourself'* = $\lambda PP(r)$).

As can be seen from (25), the translation of a reflexive pronoun involves the designated variable *r* that gets bound by lambda abstraction in the translation of the reflexive VP rules introduced by (24). Consequently the translation of a VP of the form *upset x-self* will be $\lambda PP\{\lambda r\, [upset'\ (^\wedge\lambda PP(r))(^\wedge\lambda PP(r))]\}$, as one would want. Rules output by (24) include, for example, those shown in (26) - (29);

(26) $$\langle 4, [_{\substack{VP\\ [\alpha]}}\ V\ \underset{\begin{bmatrix}\alpha\\ \mathrm{SELF}\end{bmatrix}}{NP}\ ], \lambda PP\{\lambda r V'(^\wedge NP')(^\wedge\lambda PP(r))\}\rangle$$

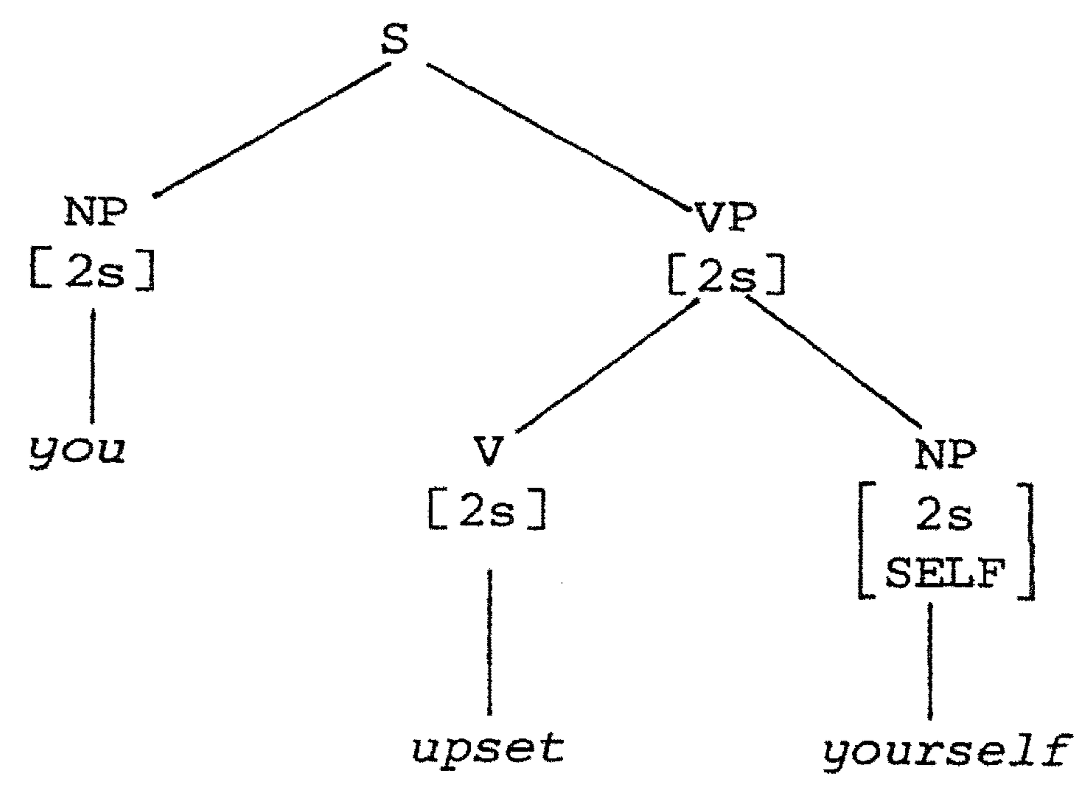
(27) $$\langle 5, [_{\substack{VP\\ [\alpha]}}\ V\ \underset{\begin{bmatrix}\alpha\\ \mathrm{SELF}\end{bmatrix}}{NP}\ \underset{[to]}{PP}\ ], \lambda PP\{\lambda r V'(^\wedge PP')(^\wedge NP')(^\wedge\lambda PP(r))\}\rangle$$

(28) $$\langle 5, [_{\substack{VP\\ [\alpha]}}\ V\ NP\ \underset{\begin{bmatrix}to\\ \alpha\\ \mathrm{SELF}\end{bmatrix}}{NP}\ ]\ \lambda PP\{\lambda r\ V'(^\wedge PP')(^\wedge NP')(^\wedge\lambda PP(r))\}\rangle$$

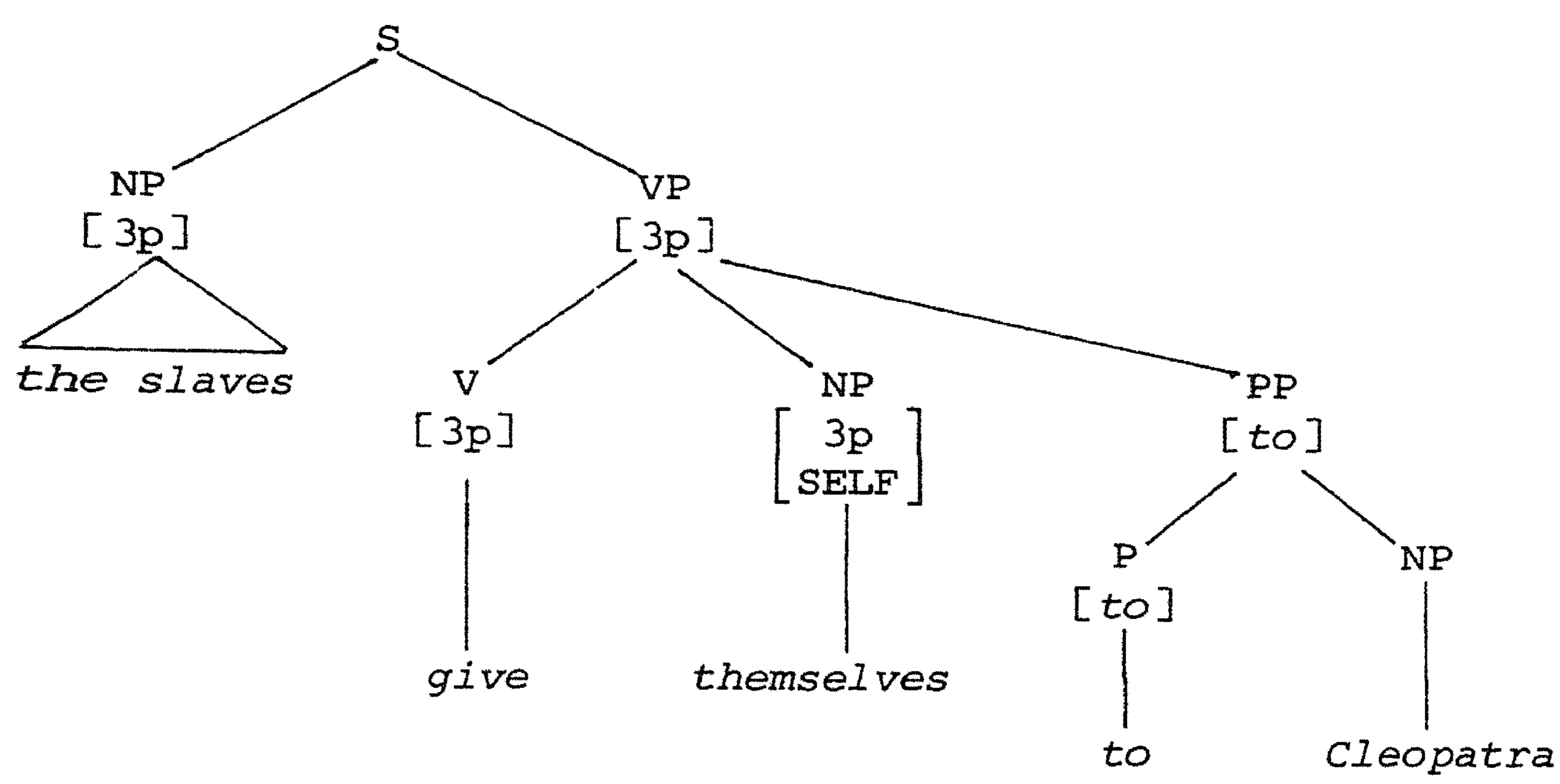
(29) $$\langle 13, [_{\substack{VP\\ [\alpha]}}\ V\ \underset{\begin{bmatrix}\alpha\\ \mathrm{SELF}\end{bmatrix}}{NP}\ \underset{[\alpha]}{VP}\ ], \lambda PP\{\lambda r V'(^\wedge VP')(^\wedge NP')(^\wedge\lambda PP(r))\}\rangle$$

And these, in turn, will induce the trees such as those shown in (30) - (34):

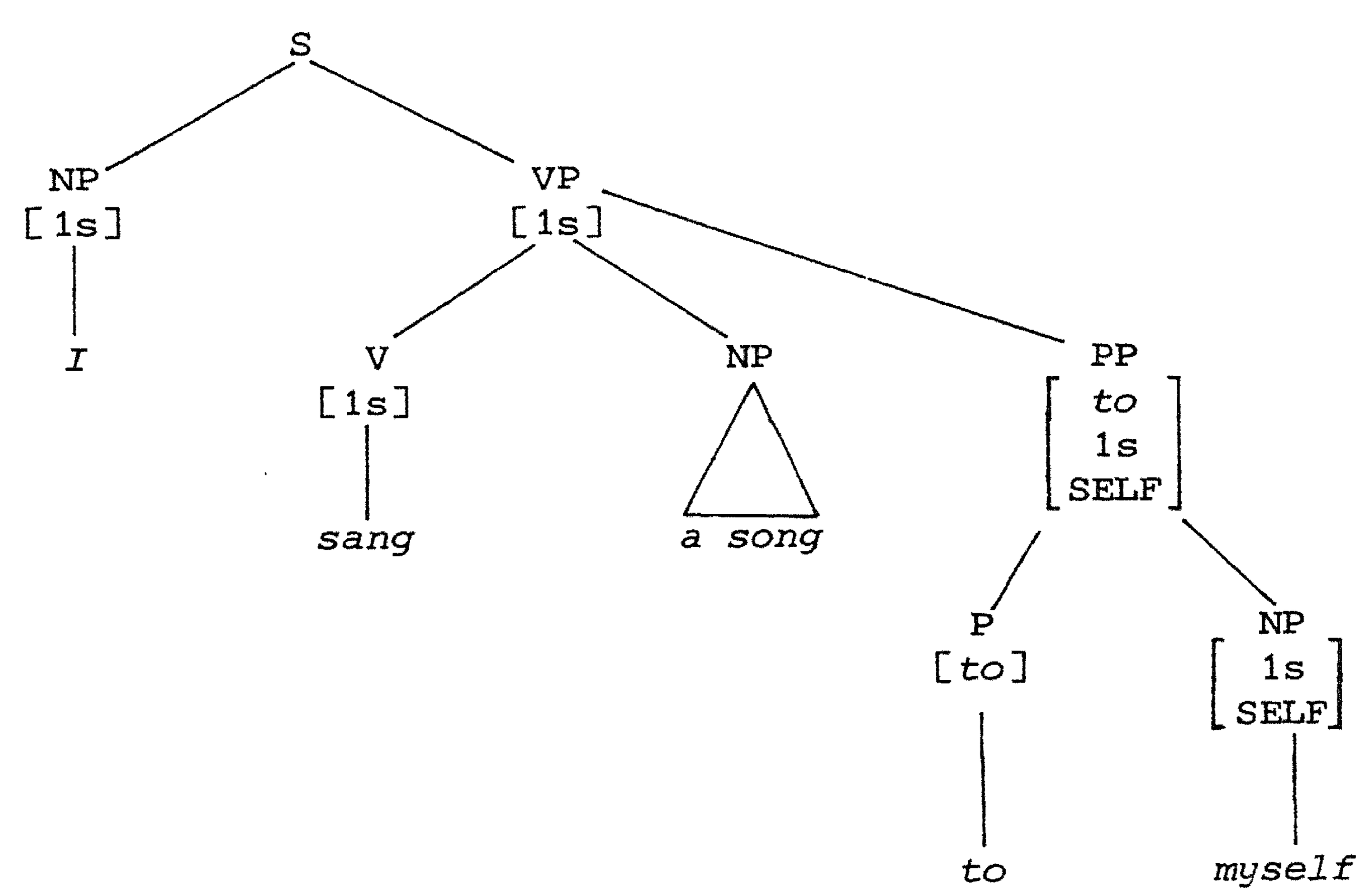
(30)


S
NP
[2s]
you
VP
[2s]
V
[2s]
upset
NP
2s
SELF
yourself

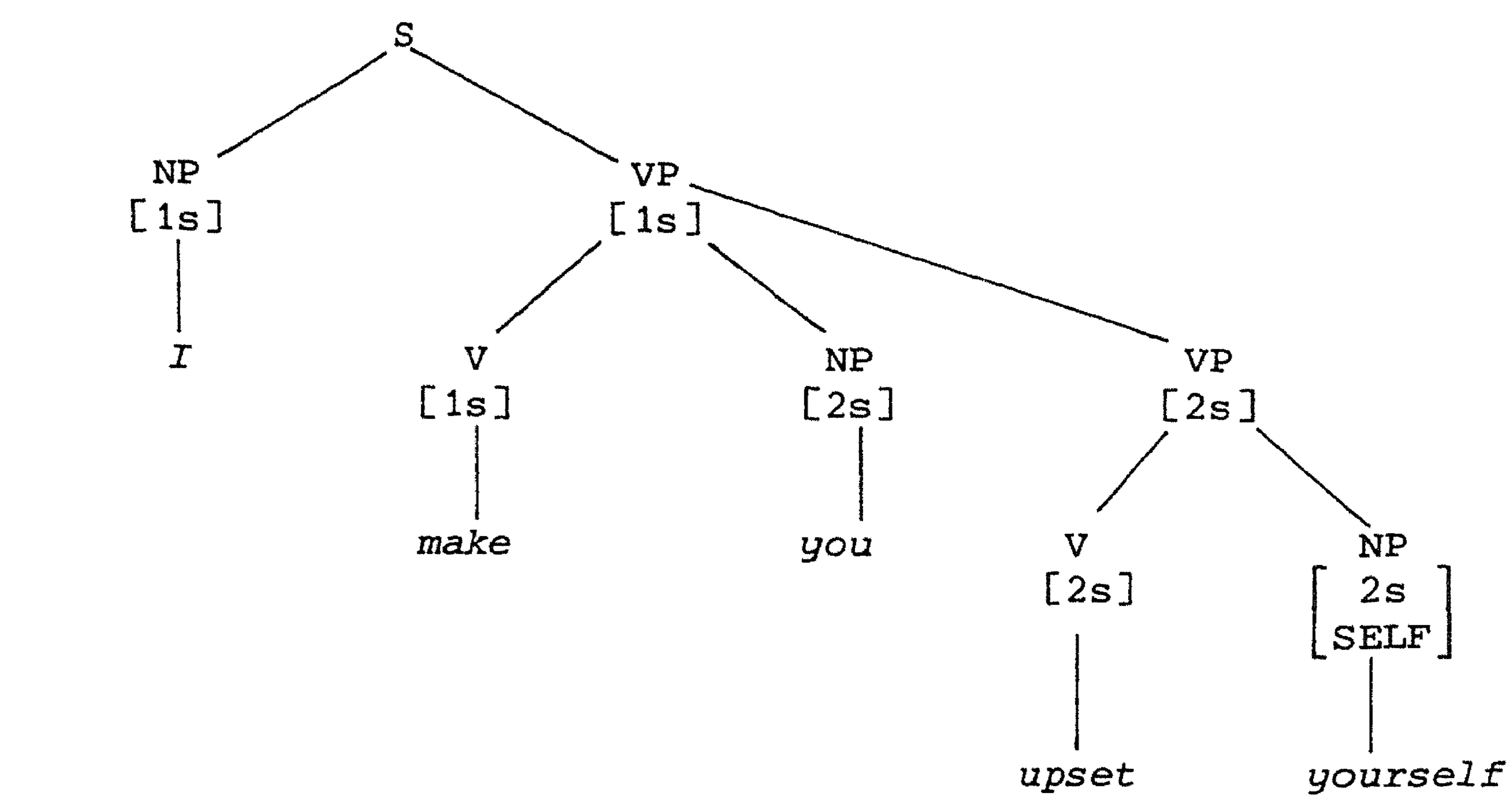
(31)


S
NP
[3p]
the slaves
VP
[3p]
V
[3p]
give
NP
3p
SELF
themselves
PP
[to]
P
[to]
to
NP
Cleopatra

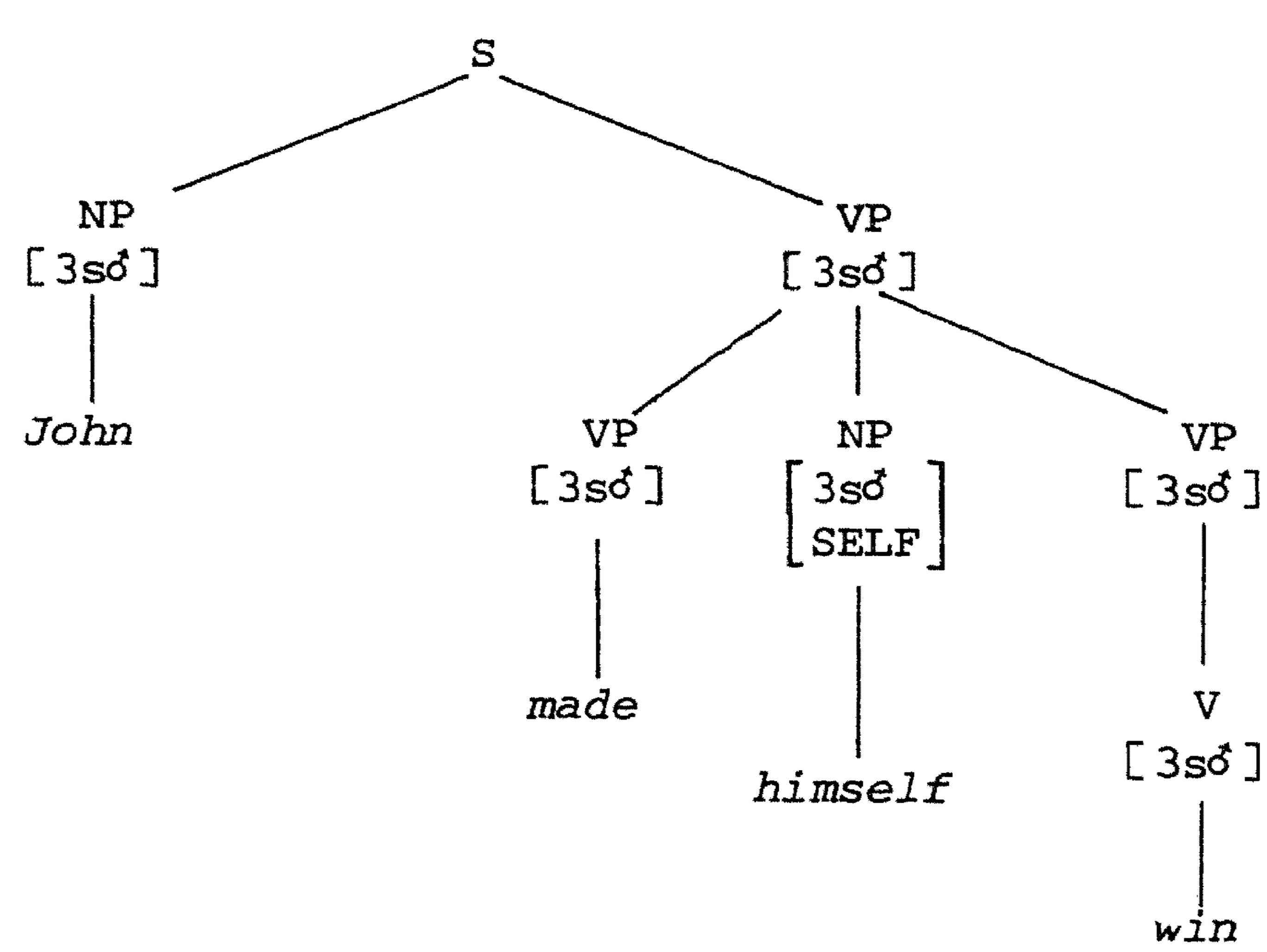
(32)


S
NP
[1s]
I
VP
[1s]
V
[1s]
sang
NP
a song
PP
to
1s
SELF
P
[to]
to
NP
1s
SELF
myself

(33)



(34)



However, metarule (24) will not output rules that admit any of the ungrammatical strings in (35):[4]

(35)
a. *I upset yourself.
b. *Myself upset you.
c. *Myself upset me.
d. *I sang a song to yourself.
e. *I made him upset yourself.
f. *I made him upset myself.
g. *I made yourself win.
h. *I made myself upset yourself.

3. PASSIVIZATION

(36) a. Kim permitted Fido to chase Felix.

b. Kim promised Fido to chase Felix.

(37) a. Fido was permitted to chase Felix by Kim.

b. *Fido was promised to chase Felix by Kim.

(38) a. Kim persuaded Lee to go.

b. Kim promised Lee to go.

(39) a. Lee was persuaded to go.

b. *Lee was promised to go.

Facts like those represented in (36) - (39) have motivated some grammarians (PARTEE (1976), THOMASON (1976), KEENAN (1979b,1980), BACH (1980), DOWTY (MS)) to employ a syntactic category of transitive verb phrase (TVP, hereafter): *permit to chase Felix* is a TVP and hence passivizes, whereas *promise to chase Felix* is not a TVP and consequently does not passivize. Thus (38a) and (38b) will be assigned the analysis trees shown in (40a) and (40b), respectively, and (39a) will be assigned that shown in (41):

(40) a. $[_S$ *Kim persuaded Lee to go*$]$

$[_{NP}$ *Kim*$]$ $[_{VP}$ *persuaded Lee to go*$]$

$[_{TVP}$ *persuaded to go*$]$ $[_{NP}$ *Lee*$]$

$[_V$ *persuaded*$]$ $[_{\overline{VP}}$ *to go*$]$

b. $[_S$ *Kim promised Lee to go*$]$

$[_{NP}$ *Kim*$]$ $[_{VP}$ *promised Lee to go*$]$

$[$*promised Lee*$]$ $[_{\overline{VP}}$ *to go*$]$

$[_V$ *promised*$]$ $[_{NP}$ *Lee*$]$

(41) $[_S$ *Lee was persuaded to go*$]$

$[_{NP}$ *Lee*$]$ $[_{VP}$ *was persuaded to go*$]$

$[_V$ *was*$]$ $[_{PVP}$ *persuaded to go*$]$

$[_{TVP}$ *persuade to go*$]$

$[_V$ *persuade*$]$ $[_{\overline{VP}}$ *to go*$]$.

However, there will be no analysis for (39b) since *promise to go* is not a TVP and cannot therefore undergo passivization. As Bach points out, the category TVP 'is not easily accommodated in a phrase-structure grammar of the sort presupposed in current transformational grammar' (BACH 1980: 320). The reason for this is that TVPs, in Bach's analysis, are discontinuous constituents - their object NP is sandwiched between the transitive verb and its complement in surface structure.

Surprisingly, it turns out that the category TVP can be very readily accommodated in a phrase-structure grammar of the sort presupposed here, namely a grammar whose set of rules is partly specified by metarules. In fact, Bach's whole analysis of passive, which is defined on the basis of a categorial grammar allowing syntactic operations like "right wrap" in addition to concatenation, can be simply mapped into the present framework. We replace all the relevant VP rules given in Section 1 with TVP rules. For example:

(42) a. $\langle 4, [_{\substack{TVP\\ [\beta]}} V], V'\rangle$

b. eat, sing, love, upset, ...

(43) a. $\langle 9, [_{\substack{TVP\\ [\beta]}} V\ \bar{S}], V'(^\wedge\bar{S}')\rangle$

b. persuade, tell, ...

(44) a. $\langle 12, [_{\substack{TVP\\ [\beta]}} V\ \underset{[\beta]}{\overline{VP}}], V'(^\wedge\overline{VP}')\rangle$

b. persuade, ask, force, ...

And then we define the following pair of metarules to provide us with active and passive VP rules:[5]

(45) $\langle n, [_{\substack{TVP\\ [\beta]}} V\ X], F\rangle$

ACTIVE $\Longrightarrow$ $\langle n, [_{\substack{VP\\ [\alpha]}} V\ \underset{[\beta]}{NP}\ X], F(^\wedge NP')\rangle$

PASSIVE $\Longrightarrow$ $\langle n, [_{\substack{VP\\ \left[\begin{smallmatrix}\beta\\ PAS\end{smallmatrix}\right]}} V\ X\ \underset{[by]}{(PP)}], \lambda P[F(P)(^\wedge PP')]\rangle$

To see how the TVP-based passive metarule works out in practice, we exhibit its output with respect to the three TVP rules given in (42) - (44) above:

(46) $\langle 4,\ [_{\begin{bmatrix}\text{VP}\\ \beta \\ \text{PAS}\end{bmatrix}}\ \text{V}\ \ (\underset{[by]}{\text{PP}})\,],\ \lambda P[\text{V}'(P)(^{\wedge}\text{PP}')]\rangle$

(47) $\langle 9,\ [_{\begin{bmatrix}\text{VP}\\ \beta \\ \text{PAS}\end{bmatrix}}\ \text{V}\ \bar{\text{S}}\ (\underset{[by]}{\text{PP}})\,],\ \lambda P[\text{V}'(^{\wedge}\bar{\text{S}}')(P)(^{\wedge}\text{PP}')\,]\rangle$

(48) $\langle 12,\ [_{\begin{bmatrix}\text{VP}\\ \beta \\ \text{PAS}\end{bmatrix}}\ \text{V}\ \ \underset{[\beta]}{\overline{\text{VP}}}\ \ (\underset{[by]}{\text{PP}})\ ],\ \lambda P[\text{V}'(^{\wedge}\overline{\text{VP}}')(P)(^{\wedge}\text{PP}')\,]\rangle$

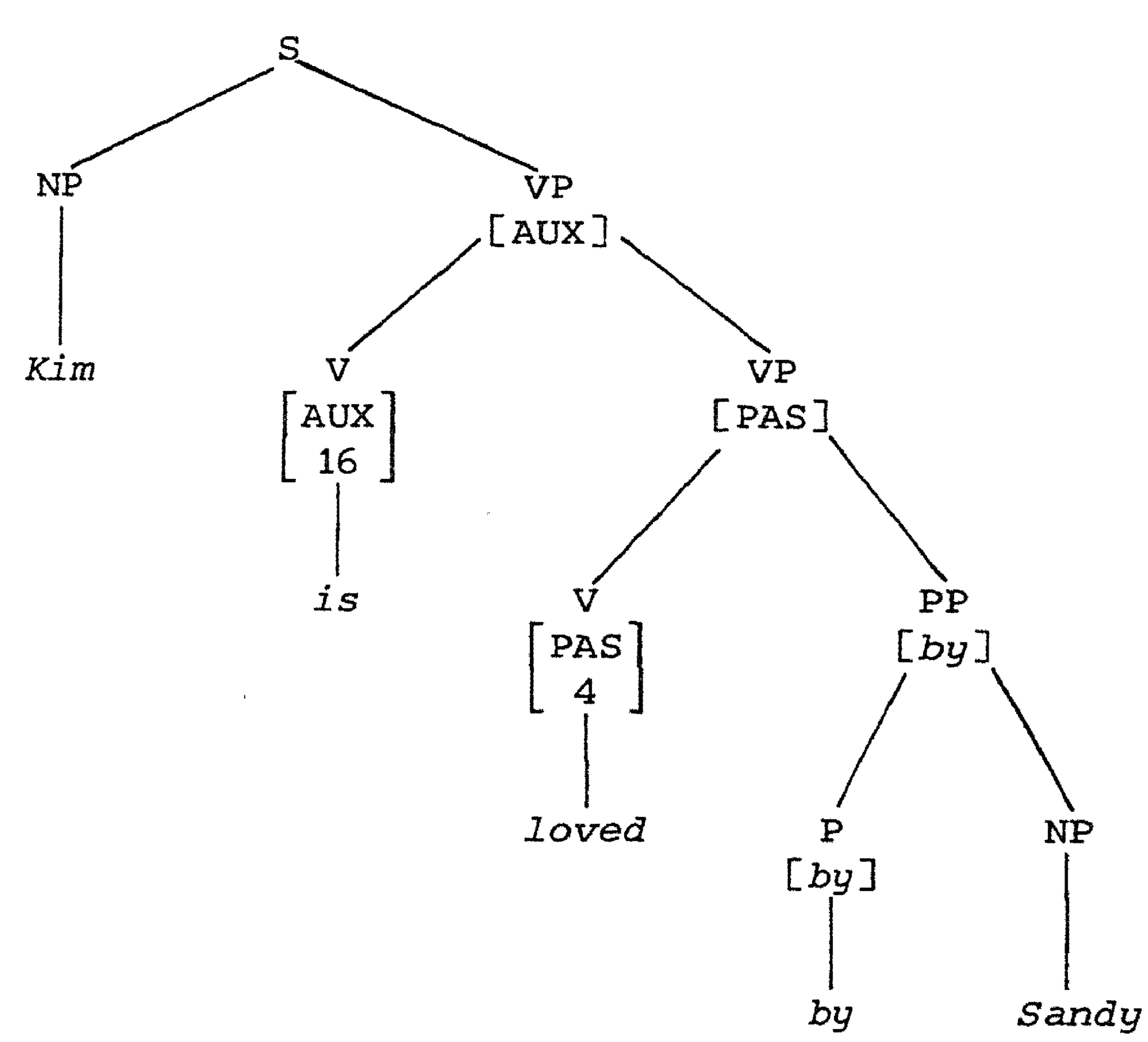
Passive VPs are introduced by the following phrase structure rule:

(49) $\langle 16,\ [_{\begin{bmatrix}\text{VP}\\ \alpha \\ \text{AUX}\end{bmatrix}}\ \text{V}\ \ \underset{\begin{bmatrix}\alpha \\ \text{PAS}\end{bmatrix}}{\text{VP}}],\ \ \text{V}'(\text{VP}')\rangle$

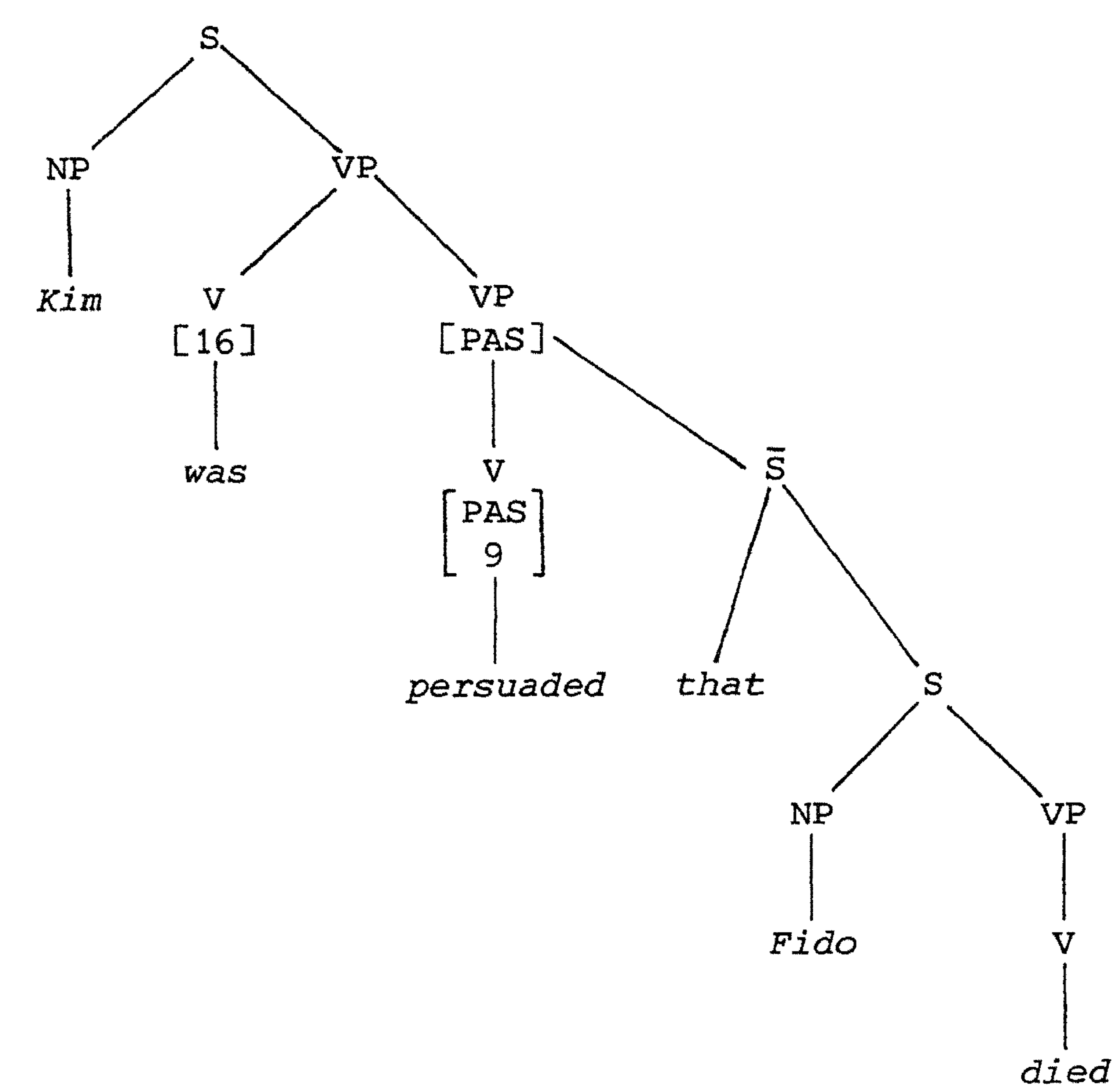
where V[16] can only be *be* (*be'* = $\lambda F[F]$).

These rules then admit such trees as the following:

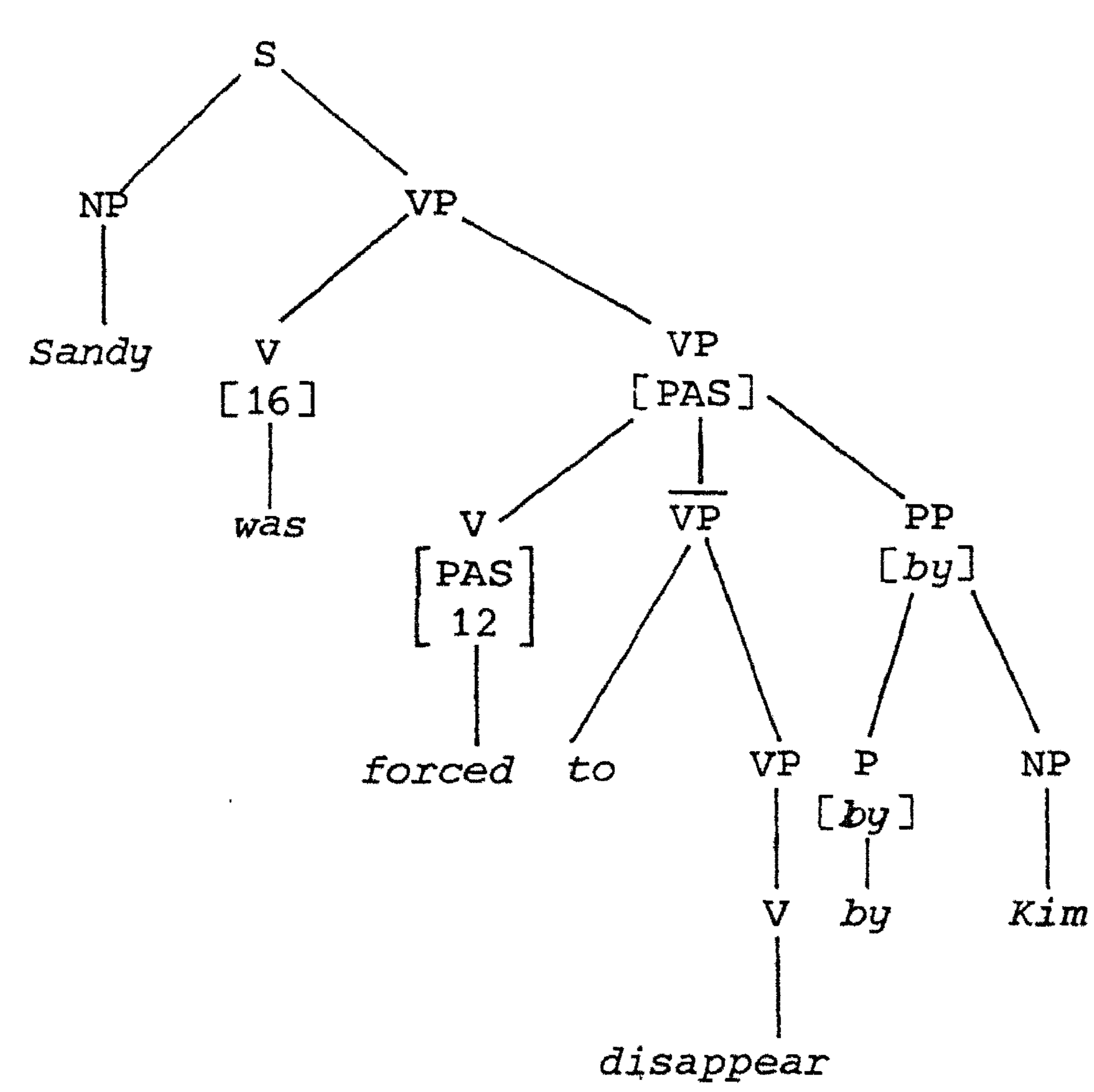
(50)



(51)



(52)



Since this treatment of passivization makes identical predictions to those made by Bach's analysis and since these are discussed by him at length in BACH (1980), we will not pursue this matter further here. The crucial point to note is that the "right-wrap" operation can be reconstructed in a meta-rule, and thus a phrase structure grammar whose rule set is partly defined via metarules can employ a category such as TVP. Assuming that the rest of the grammar is left unchanged, then the category TVP will never itself show up in the structural descriptions of English sentences for the very simple reason that the grammar contains no rules that allow a constituent to contain a TVP. We may refer to categories having this property (with respect to a given grammar) as "phantom categories".

4. REFLEXIVIZATION II: OBJECT CONTROL

The treatment of reflexivization developed in Section 3 above provides no rules which would allow us to generate examples like those in (53):

(53) a. You told me about myself.
b. We explained her to herself.

And yet such examples are perfectly grammatical in English (though not in many other languages).

Our grammar for TVPs, as indicated in Section 4 above, contains a large number of rules having the general format shown in (54):

(54) $\langle n, [_{\substack{TVP\\ [\beta]}} \ldots XP \ldots], F\rangle$, where X is N or P.

The set of rules characterized by (54) constitutes the domain of object-controlled reflexivization. Accordingly, we will formulate object-controlled reflexivization as an operation mapping TVP rules into TVP rules, namely the metarule shown in (55) (cf. (24), above):

(55) $\langle n, [_{\substack{TVP\\ [\beta]}} \ldots XP \ldots], F\rangle \Longrightarrow$

$\langle n, [_{\substack{TVP\\ [\beta]}} \ldots XP_{\begin{bmatrix}\beta\\ SELF\end{bmatrix}} \ldots], \lambda P_2 \lambda P_1 P_2\{\lambda r[F(^\wedge \lambda P P(r))(P_1)]\}\rangle$

where X is N or P.

We will illustrate the application of this metarule by reference to the example in (56):

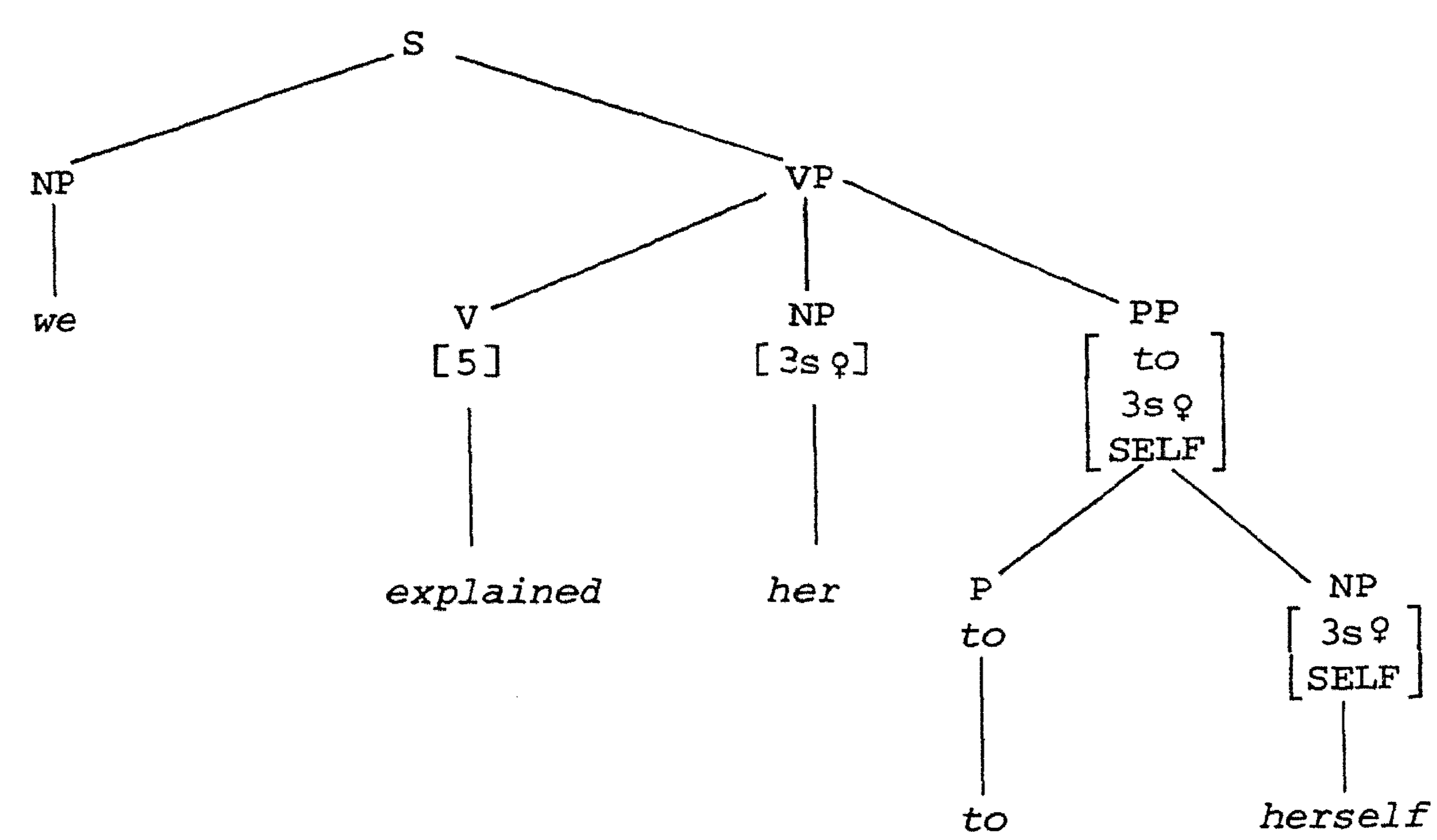
(56) a. $\langle 5, [_{\substack{TVP\\ [\beta]}} \; V \; PP_{[to]} \;], V'(^\wedge PP')\rangle$

b. $<5,\ [_{\substack{TVP \\ [\beta]}}\ V\ \ \underset{\left[\begin{smallmatrix} to \\ \beta \\ SELF \end{smallmatrix}\right]}{PP}\ ],\ \lambda P_2 \lambda P_1 P_2\{\lambda r[V'(^\wedge PP')(^\wedge \lambda PP(r))(P_1)]\}>$

c. $<5,\ [_{\substack{VP \\ [\alpha]}}\ V\ \ \underset{[\beta]}{NP}\ \ \underset{\left[\begin{smallmatrix} to \\ \beta \\ SELF \end{smallmatrix}\right]}{PP}\ ],[\lambda P_2 \lambda P_1 P_2\{\lambda r[V'(^\wedge PP')(^\wedge \lambda PP(r))(P_1)]\}](^\wedge NP')>$

Here the original TVP rule in (56a) is mapped by the metarule in (55) into the reflexivized TVP rule given in (56b) and this, in turn, is mapped by the active metarule in (45) into the active VP rule given in (56c). This rule then allows the grammar to admit trees such as that shown in (57):

(57)



However, metarule (55) will not output any rules that will lead to the grammar admitting any of the ungrammatical strings in (58):

(58) a. *We explained you to herself.
b. *We explained herself to you.
c. *We explained herself to her.

As with the analysis of subject-controlled reflexives, this analysis embodies no commitment to a particular linear order of controller and reflexive in the string. Thus the reflexive can precede the controller in sentences like (59):

(59) Mary protected from themselves all those students who came to her for aid.

As will be apparent to one who has read GAZDAR (1980, forthcoming), the binding mechanism for reflexives proposed here and the binding mechanism we assume for unbounded dependencies are distinct. It happens to be the case in English that subjects precede objects and oblique NPs, and that unbounded dependency controllers (e.g. *Wh* expressions) normally precede their traces. But according to the present proposals this is simply a contingent fact about English: nothing in our analyses of reflexivization and, say, topicalization would lead us to expect that the directions of binding will be either uniform across languages, or even parallel within any given language. This is in marked contrast to the theory of binding proposed within recent transformational work which claims that binding of reflexives and binding of unbounded dependency traces constitute a unitary phenomenon. This claim would be interesting and surprising if true. However, investigation of the binding facts in languages with other word orders than SVO shows clearly that the claim is false. The table in (60) summarises the findings of PULLUM (forthcoming) with respect to this issue:

(60)

Language	Word order	Reflexive controller	*Wh*-Contruction controller
English	SVO	Left	Left
Hixkaryana	OVS	Right	Left
Bzhedukh	SOV	Left	Right
Malagasy	VOS	Right	Left

As can be seen, the position of the reflexive controller varies with the relative order of subject and object, and is quite independent of the position of *wh*-construction controllers. This situation is exactly what our analysis would lead one to expect.

5. VPs IN VSO LANGUAGES

The treatment of object-controlled reflexives which we have given crucially depends on our use of the phantom category TVP for the analysis of English, an SVOX language that arguably never exhibits TVPs in surface structure.[6] Suppose that we were to find, as seems highly likely, that subjects control reflexives in direct objects, but not conversely, in VSO languages. Then the logic of our analysis would commit us to postulating a category VP in the grammars of such languages, even though such a category obviously cannot show up in the surface structures of simple active declarative sentences. In order to arrive at rules for generating such sentences our grammar would need to make use of a metarule mapping VP rules into S rules. Is is thus of some interest to consider whether there is any evidence for the existence of VP rules in the grammars of VSO languages. One obvious type of evidence would be the existence of VP-type constituents in the surface structures of some sentences of such languages. DOWTY (1978) has drawn attention to some curious examples of apparent VP-topicalization that can be found in a paper on Breton by ANDERSON & CHUNG (1977):

(61) *Deskiñ Brezhoneg a reomp*
to-learn Breton prt do-1pl
'We are learning Breton'

(62) *Lenn eul levr brezhoneg a ran bemdez*
to-read a book Breton prt do-1sg every-day
'I read a Breton book every day'

'The construction ... is in no way marginal but rather is perfectly productive in the language' (ANDERSON & CHUNG 1977: 22). The only solutions to the problem caused by such data in a transformational framework is to treat the VSO order as derived from an underlying SOV or SVO order with a VP constituent in the underlying representation. But, as DOWTY (1978: 112) points out, these unmotivated solutions can be avoided in a grammar which allows right-wrap of a VP around an NP. This operation would take the form of a metarule along the lines of (63) in the present framework:

(63) $[_{VP}\ V\ X\ ] \Longrightarrow [_{S}\ V\ NP\ X]$

In Modern Irish, another VSO language, McCLOSKEY (1980) has pointed out the existence of an OV constituent that can be in the scope of *only*, and which can be clefted, pseudo-clefted or topicalized. Among the relevant data that he gives are the following examples:

(64) *é féin a ghortú a rinne sé*
himself hurt[-FIN] COMP did he
'He hurt himself' (lit: 'It was hurt himself that he did')

(65) *Deir siad gur é a bhailiú a chreideann sí is fearr*
say they COMP it collect[-FIN] COMP believe she COMP best
'They say that its to collect it that she thinks is best'

(66) *Titim de chrann a rinne sé*
fall of tree COMP did he
'He fell from a tree' (lit: 'It was fall from a tree that he did')

These suggest that Modern Irish differs from Breton in having VP rules such as the following:

(67) a. $[_{VP}$ NP V$]$
b. $[_{VP}$ NP V PP$]$
c. $[_{VP}$ V PP$]$

An analysis along the lines we have been discussing would posit the following metarule, which would produce S-rules like those in (69):

(68) $[_{VP}$ X V Y$] \Longrightarrow [_{S}$ V NP X Y$]$

(69) a. $[_{S}$ V NP NP$]$
b. $[_{S}$ V NP NP PP$]$
c. $[_{S}$ V NP PP$]$

A virtue of such a proposal is that it allows one to maintain a strictly local theory of subcategorizational dependencies. This avoids the difficulties (cf. McCLOSKEY 1979: 184) that VSO languages quite generally pose for this highly desirable theory.

6. CONCLUSION

English reflexivization and passivization were two cornerstones in the early attempts to motivate transformational grammars over metatheoretically more restrictive alternatives. Since FREIDIN (1975), and more significantly BRESNAN (1978), linguists have been aware of the failure of these attempts with respect to passivization. What we hope to have shown in this paper is that the generalizations surrounding the distribution of reflexive elements in English are also naturally expressed within a much more restrictive metatheory.

The particular analysis we have proposed here allows us to express (in a phrase structure grammar defined in part by metarules) certain generalizations which are unexpressible in transformational grammars of a familiar sort. This analysis which treats certain seemingly basic rules as the output of metarules, suggests new directions for the analysis of syntactic phenomena in VSO languages which are themselves problematic for standard transformational accounts.

FOOTNOTES

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1. See Gazdar (forthcoming) for a formal definition of the "head of" relation.
2. In these rules α and β are variables ranging over permissible combinations of person, number and gender features.
3. Note that this device might also lend itself to the analysis of reciprocals, a matter that space prevents us from discussing here.
4. Since "reflexivization" is an optional process in our analysis, we provide no explanation here for the deviance of familiar examples such as those in (i):

   (i) a. *I upset me.

   b. *You upset you.

   c. *Erasmus upset Erasmus.

   Following LASNIK (1976), we assume that the deviance of these examples follows from a more general principle.
5. For the rules output by the passive metarule we assume the following convention for the semantics of optional arguments.

   Optional argument convention:
   If $\beta' = \ldots(^\wedge\alpha') \ldots$, where β immediately dominates an optional constituent α and α' is of type <<e,t>,t>, then when α is omitted, $\beta' = \ldots(^\wedge\lambda P[\exists x P(x)]) \ldots$

This has the effect of ensuring existential quantification into missing argument positions.

6. The use of the familiar term "surface structure" here should not be taken to imply that we believe that there is any such thing as "deep structure".

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SEMANTICS OF WH-COMPLEMENTS

by

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0. INTRODUCTION

This paper presents the outlines of an analysis of *wh*-complements in Montague grammar. We will be concerned primarily with semantics. Questions and *wh*-complements in Montague grammar have been studied in HAMBLIN (1976), BENNETT (1979), KARTTUNEN (1977) and HAUSSER (1978) among others. These proposals will not be discussed explicitly, but some differences with Karttunen's analysis wil be pointed out along the way.

Apart from being interesting in its own right, it may be hoped that a semantic analysis of *wh*-complements will shed some light on what a proper analysis of direct questions will look like. One reason for such an indirect approach to direct questions is the general lack of intuitions about the kind of semantic object that is to be associated with them. A survey of the literature reveals that direct questions have been analyzed in terms of propositions, sets of propositions, sets of possible answers, sets of true answers, the true answer, properties, and many other things besides. As far as *wh*-complements as such are concerned, we do not seem to fare much better, but there is this clear advantage: we do have some intuitions about the semantics of declarative sentences in which they occur embedded under such verbs as *know, tell, wonder*. What kind of semantic object we may choose to associate with *wh*-complements is restrained by various facts about the semantics of these sentences.

This paper is organized as follows. In Section 1 we discuss a number of semantic facts concerning declarative sentences containing *wh*-complements, leading to certain conclusions regarding the kind of semantic object that is to be associated with *wh*-complements. In Section 2 we show that Ty2, the language of two-sorted type theory, gives suitable means to represent the semantics of *wh*-complements, and that Ty2 can take the place of IL in PTQ as a translation medium. In Section 3 we indicate how the analysis proposed can be implemented in a Montague grammar and how the semantic facts discussed in Section 1 are accounted for. We end in Section 4 with some remarks.

1. SEMANTIC PROPERTIES OF *wh*-COMPLEMENTS

In this section a number of semantic properties of *wh*-complements will be traced by considering the validity of arguments in which sentences containing them occur. The conclusion of our considerations will be that there are good reasons to assume *wh*-complements to denote the same kind of semantic object as *that*-complements: propositions. The differences between the two kinds of complements will be explained in terms of differences in sense.

1.1. *Whether*-complements and *that*-complements

Consider the following valid argument, of which one of the premisses contains a *whether*-complement and the conclusion a *that*-complement.

(I) John knows whether Mary walks
Mary walks
———————————
John knows that Mary walks

The validity of this type of argument reflects an important fact of sentences containing *whether*-complements and, by implication, of *whether*-complements themselves. As (I) indicates, there is a relation between the semantic object denoted by *whether Mary walks* and the proposition denoted by *that Mary walks*. Similarly, the validity of (II) is based on a relation between the semantic object denoted by *whether Mary walks* and the proposition denoted by *that Mary doesn't walk*.

(II) John knows whether Mary walks
Mary doesn't walk
———————————
John knows that Mary doesn't walk

Together, (I) and (II) indicate that the actual truth value of *Mary walks* determines whether the relation holds between *whether Mary walks* and *that Mary walks*, or between *whether Mary walks* and *that Mary doesn't walk*.

The following examples show that the validity of (I) and (II) does not depend on the factivity of the verb *know*:

(III) John tells whether Mary walks
Mary walks
John tells that Mary walks

(IV) John tells whether Mary walks
Mary doesn't walk
John tells that Mary doesn't walk

Since x *tells that* ϕ does not imply that ϕ is true, the validity of (III) and (IV) cannot be accounted for in terms of factivity, and neither should the validity of (I) and (II) if, as we do, one assumes that it has to be explained in a similar way.

The overall suggestion made by (I) - (IV) is that there is a relationship between sentences in which a *whether*-complement occurs embedded under verbs as *know* or *tell* and similar sentences containing a *that*-complement. The most simple account of this relationship would be to claim that *whether* ϕ and *that (not)* ϕ denote the same kind of semantic object. Taking *that (not)* ϕ to denote a proposition, this amounts to claiming that *whether* ϕ denotes a proposition too.

1.2. Index dependency

Although on this account both *that*- and *whether*-complements denote propositions, they do this in different ways. The contrast between (I) and (III) on the one hand, and (II) and (IV) on the other hand, shows that *which* proposition *whether* ϕ denotes depends on the actual truth value of ϕ. This marks an important difference in meaning between *that*- and *whether*-complements. The denotation of *that*-complements is *index independent*: at every index *that* ϕ denotes the same proposition. The denotation of a *whether*-complement may vary from index to index, it is *index dependent*. At an index at which ϕ is true it denotes the proposition that ϕ; at an index at which ϕ is false it denotes the proposition that not ϕ. In other words, whereas the propositional concept which is the sense of a *that*-complement is a constant function from indices to propositions, the propositional concept which is the sense of a *whether*-complement (in general) is not. So, although, at a given index, a *whether*-complement and a *that*-complement may have the same denotation, their sense will in general be different.

1.3. Extensional and intensional complement embedding verbs

The difference in sense between *that*-complements and *whether*-complements plays an important role in the explanation of the semantic properties of sentences in which they are embedded. Embedding a complement under a verb semantically corresponds to applying the interpretation of the verb to the *sense* of the complement. This is the usual procedure for functional application, motivated by the assumption that no context can, *a priori,* be trusted to be extensional.

As a matter of fact, such verbs as *know* and *tell* are extensional, and moreover, the validity of the arguments (I) - (IV) is based upon this fact. In accordance with usual procedures, the extensionality of these verbs will be accounted for by a meaning postulate which reduces intensional relations between individual concepts and propositional concepts to corresponding extensional relations between individuals and propositions.

However, there are also complement embedding verbs which do create truly *intensional* contexts. The verb *wonder* is such an intensional verb. The assumption that no extensional relation corresponds to the intensional one denoted by *wonder* explains why arguments such as (I) - (IV) do not hold for this verb. In fact, sentences of the form x *wonders that* ϕ, which would have to be the conclusion of such arguments, are not even well-formed, a fact that needs to be accounted for as well.

1.4. Constituent complements

Consider the following arguments, of which one of the premisses contains a *wh*-complement with one or more occurrences of *wh*-terms such as *who, what, which girl*.

(V) John knows who walks
Bill walks
―――――――――――――
John knows that Bill walks

(VI) John knows which man walks
Bill walks
―――――――――――――
John knows that Bill walks

Given the usual semantics, these arguments are valid.[1] Again, this can be explained in a very direct way if we take the constituent complements to denote propositions. The validity of (V) - (VI) does no more depend on the factivity of *know* than does the validity of (I) and (II). This will be clear if one substitutes the non-factive *tell* for *know* in (V) - (VI). The validity of all these arguments does depend on the extensionality of *know* and *tell*. As was the case with *whether*-complements, which proposition a constituent complement denotes depends on what is in fact the case. For example, which proposition is denoted by *who walks* depends on the actual denotation of *walk*. If Bill walks, the proposition denoted by *who walks* should entail that Bill walks; if Peter walks, it should entail that Peter walks. This index dependent character can more generally be described as follows. At an index i, *who walks* denotes that proposition p, which holds true at an index k iff the denotation of *walk* at k is the same as its denotation at i.

1.5. Exhaustiveness

This more general description of the proposition denoted by *who walks* not only implies, as is supported by argument (V), that for John to know who walks he should know - *de re* - of everyone who walks that he does, but also implies that of someone who doesn't walk, he should not erroneously believe that she does. That this is right appears from the validity of the following argument:

(VII) John believes that Bill and Suzy walk
Only Bill walks
———
John doesn't know who walks

If only Bill walks and John is to know who walks, he should know that only Bill walks and he should not believe that someone else walks as well. We will call this property of propositions denoted by constituent complements their *exhaustiveness*.

Another way to make the same point is as follows. For a sentence *John knows* ψ, where ψ is a *wh*-complement, to be true, it should hold that if one asks John the direct question corresponding to ψ, one gets exactly the correct answer. So, if only Bill walks and *John knows who walks* is to be true, John should answer: 'Bill' when asked the question: 'Who walks?', and not for example: 'Bill and Suzy do'. A similar kind of exhaustiveness is exhibited by *whether*-complements of the form *whether* ϕ *or* ψ.[2] Consider the following argument:

(VIII) John knows whether Mary walks or Bill sleeps
Mary doesn't walk and Bill sleeps
———
John knows that Mary doesn't walk and that Bill sleeps

The validity of this argument illustrates that the proposition denoted by an alternative *whether*-complement is exhaustive too. At an index i, *whether* ϕ *or* ψ denotes that proposition p that holds at an index k iff the truth values of both ϕ and ψ at k are the same as at i.

Karttunen does not incorporate the property of exhaustiveness in his analysis. As a consequence, he cannot account for the validity of (VII) and (VIII). Whether he does consider them valid is unclear to us. His analysis forces him to neglect exhaustiveness for a reason not related to this, which will be discussed in the next section.

1.6. A *de dicto / de re* ambiguity of constituent complements

Sentences in which constituent complements containing *wh*-terms of the form *which* δ occur exhibit a certain kind of ambiguity, which resembles the familiar *de dicto / de re* ambiguity, and which will henceforth be referred to as such. For example, whether the following argument is valid or not depends on how the conclusion is read.

(IX) John knows who walks
 ____
 John knows which girl walks

That (IX) is *valid* could be argued for as follows. Since the set of girls is a subset of the set of individuals, and since if one knows of a set which of its elements have a certain property, one also knows this of every subset of that set, it cannot fail to hold that John knows which girl walks if he knows who walks. Here the conclusion is taken *de re*.

On the other hand, one might point out that (IX) is *not valid* by presenting the following situation. Suppose that just one individual walks. Suppose further that it is a girl. If John knows of this individual that she is the one that walks, but fails to believe that she is a girl, then the premiss of (IX) is true, but its conclusion is false. In this line of reasoning the conclusion is taken *de dicto*. It takes for granted that the conclusion should be read in such a way that if John is to know which girl walks, he should believe of every individual which is in fact a girl and walks, not only that she walks, but also that she is a girl. Within the first line of reasoning, this assumption is not made. So, whether (IX) is valid or not depends on how the conclusion is read. If we assign it a *de re* reading (IX) is valid, under a *de dicto* reading it is not. The *de re* reading of the conclusion of (IX) can be paraphrased as *of each girl, John knows whether she walks*.

This *de dicto* / *de re* ambiguity also plays a role in:

(X) John knows which man walks
____
John knows which man doesn't walk

This argument is valid iff both the premiss and the conclusion are read *de re*, its inverse is then valid as well. Under all other possible combinations of readings (X) is not valid. Consider e.g. the *de dicto/de dicto* combination. Suppose the premiss is true. This is compatible with there being an individual of which John erroneously believes that it is a man, but rightly believes that it does not walk. However, in such a situation, if the conclusion is read *de dicto*, it is false. Similar examples can be constructed to show that (IX) is also not valid on the other two combinations of readings. This shows, by the way, that the *de dicto* and *de re* readings involved are logically independent.

The possibility to distinguish *de dicto* and *de re* readings of constituent complements marks an important difference between Karttunen's analysis and ours. Karttunen can only account for the *de re* readings. Nevertheless, in his analysis (X) is not valid (although (IX) is). This is caused by the fact that Karttunen neglects exhaustiveness. Karttunen explicitly defends leaving exhaustiveness out because in his analysis it would make (X) valid, which he rightly does not think to be the case. We believe that an analysis which can both account for exhaustiveness and for the fact that the validity or invalidity of (X) (and (IX) for that matter) depends on how the conclusion is read, is to be preferred.

1.7. Implicatures versus presuppositions

From the previous discussion, in particular from Sections 1.4 and 1.5, it will be clear that we consider the following arguments to be valid ones:

(XI) John knows who walks
Nobody walks
____
John knows that nobody walks

(XII) John knows who walks
Peter and Mary walk
____
John knows that Peter and Mary walk

(XIII) John knows whether Peter walks or Mary walks
Neither Peter nor Mary walks
____
John knows that neither Peter nor Mary walks

(XIV) John knows whether Peter walks or Mary walks
Both Peter and Mary walk
____
John knows that both Peter and Mary walk

One might object to the validity of these arguments by pointing out that *John knows who walks* presupposes that at least / exactly one individual walks, and that *John knows whether Peter walks or Mary walks* presupposes that at least / exactly one of the alternatives is the case. Therefore, one might continue, the first premiss of these arguments is semantically deviant in some sense, say lacks a truth value, if the second premiss happens to be true.

We adhere to the view advocated in KARTTUNEN & PETERS (1976), that it is better to regard these phenomena as *conventional implicatures* and not as presuppositions in the strict semantic sense. More generally, we believe that many of the arguments put forward in KEMPSON (1975), WILSON (1975) and GAZDAR (1979) showing that presupposition is a pragmatic notion hold for presuppositions of *wh*-complements as well.

In Karttunen's analysis, (XI) - (XIV) are valid as well. The validity of (XI) and (XIII), however, has to be secured by a special clause in a meaning postulate relating *know* + *wh* to *know that*. The need for this special clause explains itself by the fact that the validity of (XI) and (XIII) is at odds with not incorporating exhaustiveness. One would expect that in an analysis in which (VII) and (VIII) of Section 1.5 are not valid, (XI) and (XIII) would not be valid either.

1.8. Towards a uniform treatment of complements

A distinctive feature of our analysis is that *wh*-complements are taken to be proposition denoting expressions. This is an important difference between our approach and that of others. To mention only two, in Karttunen's they denote sets of propositions, and in Hausser's they are of all sorts of different categories. From this difference other differences follow, e.g. the possibility of a uniform treatment of complements. For, besides the fact that it provides a simple and direct account of the validity of the various arguments discussed above, the hypothesis that *that*- and *wh*-complements denote the same kind of semantic objects makes it possible to assign them to the same syntactic category.[3] This seems especially attractive in view of the fact that it is possible to conjoin *wh*- and *that*-complements.

Further, if both kinds of complements can belong to the same syntactic category, we are no longer forced to assume there to be two complement taking verbs *know*, of different syntactic categories, and of different semantic types: one which takes *that*- and one which takes *wh*-complements. We need not acknowledge two different relations of knowing which are only linked indirectly, i.e. by a meaning postulate. This holds for all verbs which can take both kinds of complements.[4]

Of course, there are also verbs such as *wonder* which take only *wh*-complements and verbs such as *believe,* which take only *that*-complements. The relevant facts can easily be accounted for by means of syntactic subcategorization, or preferably, in lexical semantics, by means of meaning postulates.

2. Ty2 AND THE SEMANTIC ANALYSIS OF *wh*-COMPLEMENTS

In Section 1 we have sketched informally the outlines of a semantics for *wh*-complements. In particular, we argued that *wh*-complements denote propositions and do this in an index dependent way. The description of this index dependent character involves comparison of what is the case at different indices. This leads to the choice of a logical language in which reference can be made to indices and in which relations between indices can be expressed directly. The language of two-sorted type theory, Gallin's Ty2, is such a language. In this section we will show that it serves our purpose to express the semantics of *wh*-complements quite well.

Ty2 is a simple language. Rather than by stating the explicit definitions, we will discuss its syntax and semantics by comparing it with IL, the language of intensional logic of PTQ, thereby indicating how Ty2 can be put to the same use as IL in the PTQ system. We will also make some methodological remarks on the use of Ty2. For a formal exposition and extensive discussion of Ty2, the reader is referred to GALLIN (1975).

2.1. Ty2, the language of two-sorted type theory

The basic difference between IL and Ty2 is that s is not introduced only in constructing more complex, intensional types, but that it is a basic type, just like e and t. Complex types can be constructed with s in exactly the same way as with e and t. As is to be expected, the set of possible denotations of type s is the set of indices. Since it is a type like any other now, we will also employ constants and variables of type s. This means that it is possible to quantify and abstract over indices, making the necessity operator $\Box$ and the cap operator $^\wedge$ superfluous.

A *model* for Ty2 is a triple <A,I,F>, A and I are disjoint non-empty sets, A is to be the set of individuals, I the set of indices. F is an interpretation function which assigns to every constant a member of the set of possible denotations of its type. Notice the difference with the interpretation function F of IL-models, which assigns senses and not denotations to constants. The interpretation of a meaningful expression α of Ty2, written as $[\![\alpha]\!]_{M,g}$, is determined with respect to a model M and an assignment g only. (As usual, g assigns to every variable a member of the set of possible denotations of its type.)

The important difference with interpretations in IL is that the latter also need an index to determine the interpretation of an expression. This role of indices as a parameter in the interpretation is taken over in Ty2 by the assignment functions. The effect of interpreting in IL an expression with respect to an index i is obtained in Ty2 by interpreting expressions with respect to an assignment which assigns to a free index variable occurring in the expression the index i. To an index dependent expression of IL (an expression of which the denotation varies from index to index) there corresponds an expression in Ty2 which contains a free index variable. The result is an expression of which the interpretation varies from assignment to assignment. A formula ϕ is *true* with respect to M and g iff $[\![\phi]\!]_{M,g} = 1$; ϕ is *valid in* M iff for all g, ϕ is true with respect to M and g; ϕ is *valid* iff for all M, ϕ is valid in M.

2.2. Translating into Ty2

To illustrate the difference between IL and Ty2, consider first how the English verb *walk* translates into Ty2. Instead of simply translating it into a constant of type f(IV), it is translated into the expression walk'($v_{0,s}$), in which walk' is a constant of type <s,f(IV)>, and $v_{0,s}$ is a variable of type s, so the full translation of the verb is an expression of type f(IV).

All translations of basic expressions will contain the same free index variable. For this purpose we use $v_{0,s}$, the first variable of type s, which from now on we will write as a. Therefore, the translation of a complex expression will be interpreted with respect to the index assigned to a by the assignment function.

The rules for translating PTQ English into Ty2 can be obtained by using the fact that $\lambda a\alpha$ expresses the same function in Ty2 as $^{\wedge}\alpha$ in IL, $^{\vee}\alpha$ is the same as $\alpha(a)$; and $\Box$ corresponds to Λa. Consider the following examples of Ty2 analogues of (parts of) some PTQ translation rules, in which ~ abbreviates 'translates into'.

(T:1) (a) If α is in the domain of g, then $\alpha \sim g(\alpha)(a)$.

With the usual exceptions, g associates a basic expression α of category A with a Ty2 constant α' of type $<s,f(A)>$, giving its sense. The full translation of α, $\alpha'(a)$, gives as usual its denotation.

(T:1) (b) *be* $\sim \lambda \mathcal{P}\lambda x\mathcal{P}(a)(\lambda a\lambda y\,[x(a) = y(a)])$

(c) *necessarily* $\sim \lambda p\Lambda a(p(a))$

(d) *John* $\sim \lambda P\,[P(a)(\lambda aj)]$

(e) *he*$_n \sim \lambda P\,[P(a)(x_n)]$

(T:2) If $\delta \in P_{CN}$, and $\delta \sim \delta'$, then *every* $\delta \sim \lambda P\Lambda x[\delta'(x) \rightarrow P(a)(x)]$

(T:4) If $\alpha \in P_T$, $\delta \in P_{IV}$, $\alpha \sim \alpha'$ and $\delta \sim \delta'$, then $F_4(\alpha,\delta) \sim \alpha'(\lambda a\delta')$.

Of course, the meaning postulates of PTQ can be translated into Ty2 as well. (Notice that the rigid designator view of proper names like *John* is already implemented in the translation.) The translation of a sentence is illustrated in (1):

(1)

man
$man'(a)$

every man
$\lambda P\Lambda x[man'(a)(x) \rightarrow P(a)(x)]$

walk
$walk'(a)$

every man walks
$\lambda P\Lambda x[man'(a)(x) \rightarrow P(a)(x)](\lambda a[walk'(a)])$

$\Leftrightarrow$

$\Lambda x[man'(a)(x) \rightarrow walk'(a)(x)]$

$\Leftrightarrow$

$\Lambda u[man'_*(a)(u) \rightarrow walk'_*(a)(u)]$

2.3. *That*-complements and *whether*-complements in Ty2

The proposition denoting expression which is to be the translation of a *that*-complement *that* ϕ can be constructed from the translation of ϕ by using abstraction over indices. For example, the sentence *Mary walks* translates into the formula $walk'_*(a)(m)$; from this formula we can form the expression $\lambda a[walk'_*(a)(m)]$. Its interpretation $[\![\lambda a\ walk'_*(a)(m)]\!]_{M,g}$ is that proposition $p \in \{0,1\}^I$ such that for every index i: $p(i) = 1$ iff $[\![walk'_*(a)(m)]\!]_{M,g} = 1$. So, $\lambda a[walk'_*(a)(m)]$ denotes the characteristic function of the subset of the set of indices at which it is true that Mary walks. Notice that $\lambda a\ walk'_*(a)(m)$ does not contain a free index variable. This makes it the index independent expression it was argued to be in 1.1 and 1.2. Its sense denoted by the expression $\lambda a \lambda a[walk'_*(a)(m)]$, is a constant function from indices to propositions.

In Section 1.1 we circumscribed the denotation of *whether Mary walks* as follows: at an index at which it is true that Mary walks it denotes the proposition that Mary walks, and at an index at which it is false that Mary walks it denotes the proposition that Mary doesn't walk. Another way of saying this is that at an index i *whether Mary walks* denotes that proposition p such that for every index k, p holds true at k iff the truth value of *Mary walks* at k is the same as at i. In Ty2 this can be expressed by the index dependent proposition denoting expression (2), the interpretation of which is given in (2'). By $g[x/y]$ we will understand that assignment g' which is like g except for the possible difference that $g(y) = x$.

(2) $\lambda i[walk'_*(a)(m) = walk'_*(i)(m)]$

(2') $[\![\lambda i[walk'_*(a)(m) = walk'_*(i)(m)]]\!]_{M,g}$ is that proposition $p \in \{0,1\}^I$ such that for every index $k \in I$: $p(k) = 1$ iff
$[\![walk'_*(a)(m) = walk'_*(i)(m)]\!]_{M,g[k/i]} = 1$ iff
$[\![walk'_*(a)(m)]\!]_{M,g[k/i]} = [\![walk'_*(i)(m)]\!]_{M,g[k/i]}$ iff
$[\![walk'_*(a)(m)]\!]_{M,g} = [\![walk'_*(i)(m)]\!]_{M,g[k/i]}$.

So, at the index $g(a)$, the expression (2) denotes the characteristic function of the set of indices at which the truth value of *Mary walks* is the same as at the index $g(a)$. The index dependent character of *whether*-complements discussed in 1.1 and 1.2 is reflected by the fact that a free index variable occurs in their translation. The expression $\lambda a \lambda i[walk'_*(a)(m) =$ $= walk'_*(i)(m)]$, denoting the propositional concept which is the sense of *whether Mary walks*, does not denote a constant function. For different indices its value may be a different proposition.

2.4. Constituent complements in Ty2

The kind of expressions which denote propositions in the required index dependent way can be constructed not only from formulas, such as $\text{walk}'_*(a)(m)$ in (2), but from expressions of arbitrary type. Let $\alpha/a/$ and $\alpha/i/$ be two expressions such that where the first has free occurrences of a, the second has free occurrences of i, and vice versa. Then the expression (3) denotes a proposition in an index dependent way, as its interpretation given in (3') shows.[5]

(3) $\lambda i[\alpha/a/ = \alpha/i/]$

(3') $[\![\lambda i[\alpha/a/ = \alpha/i/]]\!]_{M,g}$ is that proposition $p \in \{0,1\}^I$ such that for every index $k \in I$, $p(k) = 1$ iff $[\![\alpha/a/]\!]_{M,g} = [\![\alpha/i/]\!]_{M,g[k/i]}$

Expressions serving as translations of *wh*-complements will always be of this form. The translation of a *whether*-complement has been given in (2). There $\alpha/a/$ is the formula $\text{walk}'_*(a)(m)$. An example of an expression which will serve as the translation of a constituent complement is:

(4) $\lambda i[\lambda u[\text{walk}'_*(a)(u)] = \lambda u[\text{walk}'_*(i)(u)]]$.

In this case, $\alpha/a/$ is $\lambda u[\text{walk}'_*(a)(u)]$, an expression of type <e,t>. At an index g(a), (4) denotes that proposition which holds at an index k iff $[\![\lambda u[\text{walk}'_*(a)(u)]]\!]_{M,g}$ is the same set as $[\![\lambda u[\text{walk}'_*(i)(u)]]\!]_{M,g[k/i]}$. I.e. at an index g(a), (4) denotes that proposition which holds true at an index k iff the denotation of walk'_* at that index k is the same as at the index g(a). And this is precisely the index dependent proposition which, in Section 1.4, we required to be the denotation of the constituent complement *who walks*.

2.5. Methodological remarks on the use of Ty2

In this section we will defend our use of Ty2 against some objections that are likely to be raised.

A first objection might be that translations in Ty2 are (even) less 'natural' than those in IL. In view of the fact that within a compositional semantic theory the level of translation, be it in Ty2 or in IL, is in principle dispensable, we do not see that there is empirical motivation for this kind of objection.

A second objection that is often raised against the use of a logical language which allows for reference to and quantification over indices, is that it involves stronger *ontological commitments* than a language in which the relevant phenomena are dealt with by means of intensional operators. We do not think that this objection holds ground. It is not the object language in isolation, but the object language together with the meta-language in which its semantics is described that determines ontological commitments. Since the statement of the semantics of intensional operators involves reference to and quantification over indices as well, the commitments are the same. The dispensability of the translation level even strengthens this point.

A more serious reason for preferring an operator approach to a quantificational approach might be that for some purposes one does not need the full *expressive power* of a quantificational language and therefore prefers a language with operators which has exactly the, restricted, expressive power one needs. In fact, in Section 4, we will point out that by the introduction of a new intensional operator to IL, one can get a long way in the semantic analysis of *wh*-complements. However, phenomena remain that escape treatment in this intensional language.

Taking the semantic analysis of tense into consideration as well, we think a lot can be said in favour of a logical language in which reference to and quantification over indices is possible. It appears that analyses set up in the Priorean fashion tend to become stronger and stronger, up to a point where if there still is a difference in expressive power with quantificational logic at all, this advantage is annihilated by the unintuitiveness and complexity of the language used. For an illuminating discussion of these points, see VAN BENTHEM (1978). In fact, we think that Ty2 provides a suitable framework for the incorporation of a semantic analysis of tense in the vein of NEEDHAM (1975) into a Montague grammar as well.

3. *Wh*-COMPLEMENTS IN A MONTAGUE GRAMMAR

In this section we will outline how the semantic representations of complements in Ty2, given in 2, can systematically be incorporated in the framework of a Montague grammar. We will not present the syntactic part of our proposal in detail. In particular, the definitions of the various syntactic functions occurring in the syntactic rules will not be stated in this paper. We will concentrate on the explanation of the semantic facts discussed in Section 1.

3.1. *Whether*-complements and *that*-complements

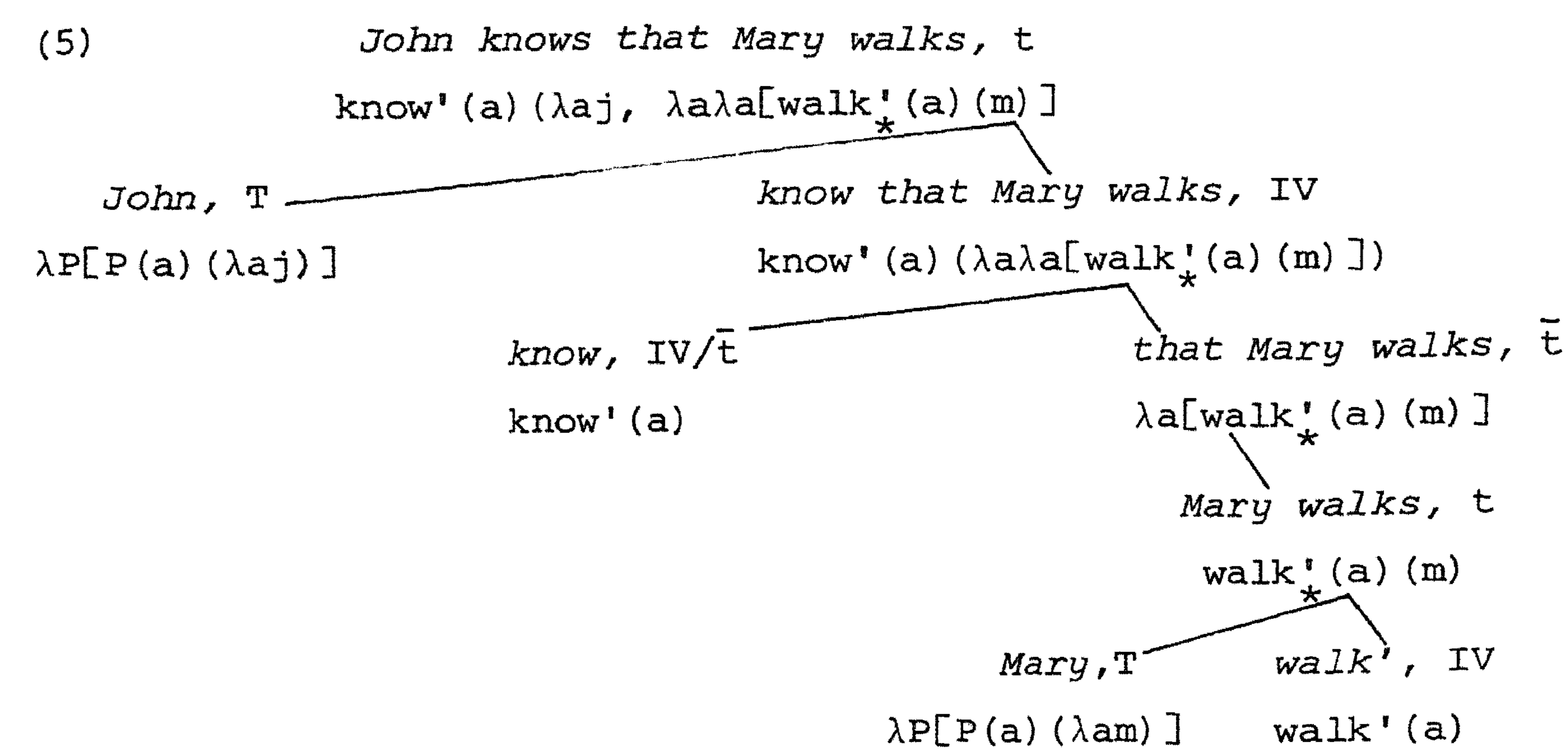
Complements are expressions which denote propositions. Therefore, they should translate into expressions of type <s,t>. In PTQ there is no syntactic category which is mapped onto this type[6], therefore we add the following clauses to the definitions of the set of categories and the function f mapping categories into types:

$$\text{If } A \in \text{CAT, then } \bar{A} \in \text{CAT}; \ f(\bar{A}) = \langle s, f(A) \rangle.$$

So, $\bar{t}$ will be the category of complements. Complement embedding verbs, such as *know, tell, wonder* and *believe* will be of category $IV/\bar{t}$. As we remarked in Section 1.8, the categories $\bar{t}$ and $IV/\bar{t}$ will have to be subcategorized, since not all of these verbs take all kinds of complements. This can be done in an obvious way, with which we will not be concerned here.

In (5) an analysis tree of a sentence containing a *that*-complement is given together with its translation. Here and elsewhere, notation conventions and meaning postulates familiar from PTQ are applied whenever possible.

(5)



The syntactic rule deriving a *that*-complement and the corresponding translation rule are:

(S:THC) If $\phi \in P_t$, then *that* $\phi \in P_{\bar{t}}$.

(T:THC) If $\phi \sim \phi'$, then *that* $\phi \sim \lambda a \phi'$.

The rule which embeds the complement under a verb is a simple rule of functional application. The corresponding rule of translation follows the usual pattern:

(S:IV/$\bar{t}$) If $\phi \in P_{IV/\bar{t}}$ and $\psi \in P_{\bar{t}}$, then $F_{IV/\bar{t}}(\delta,\psi) \in P_{IV}$.

(T:IV/$\bar{t}$) If $\delta \sim \delta'$ and $\psi \sim \psi'$, then $F_{IV/\bar{t}}(\delta,\psi) \sim \delta'(\lambda a \psi')$.

Sentence (5) expresses that an intensional relation of knowing exists between the individual concept denoted by λaj and the propositional concept denoted by $\lambda a \lambda a[walk'_*(a)(m)]$. By means of a meaning postulate, to be given below, this intensional relation will be reduced to an extensional one.

In (6) an analysis tree and its translation of a sentence containing a *whether*-complement is given:

(6) *John knows whether Mary walks,* t

$know'(a)(\lambda aj, \lambda a\lambda i[walk'_*(a)(m) = walk'_*(i)(m)])$

John, T
$\lambda P[P(a)(\lambda aj)]$

know whether Mary walks, IV
$know'(a)(\lambda a\lambda i[walk'_*(a)(m) = walk'_*(i)(m)])$

know, IV/$\bar{t}$
$know'(a)$

whether Mary walks, $\bar{t}$
$\lambda i[walk'_*(a)(m) = walk'_*(i)(m)]$

Mary walks, t
$walk'_*(a)(m)$

The rule which forms a *whether*-complement from a sentence, and the corresponding translation rule are as follows. (An asterisk indicates that a rule will later be revised.)

(S:WHC*) If $\phi \in P_t$, then *whether* $\phi \in P_{\bar{t}}$

(T:WHC*) If $\phi \sim \phi'$, then *whether* $\phi \sim \lambda i[\phi' = [\lambda a\phi'](i)]$.

Whether-complements can be generated by a more general rule:[7]

(S:WHC) If $\phi_1,\ldots,\phi_n \in P_t$, then *whether* ϕ_1 *or* ... *or* $\phi_n \in P_{\bar{t}}$

(T:WHC) If $\phi_1 \sim \phi'_1,\ldots,\phi_n \sim \phi'_n$, then *whether* ϕ_1 *or* ... *or* $\phi_n \sim$
$\sim \lambda i[\phi'_1 = [\lambda a\phi'_1](i) \wedge \ldots \wedge \phi'_n = [\lambda a\phi'_n](i)]$.

Obviously, (S:WHC*) and (T:WHC*) are special cases of (S:WHC) and (T:WHC).

In general, *whether*-complements of the form *whether* ϕ_1 *or* ... *or* ϕ_n are ambiguous between an *alternative* and a *yes/no* reading. The following two trees and their translations illustrate this ambiguity.

(7) *whether John walks or Mary walks,* $\bar{t}$

$\lambda i[(walk'_*(a)(j) = walk'_*(i)(j)) \wedge (walk'_*(a)(m) = walk'_*(i)(m))]$

John walks, t
$walk'_*(a)(j)$

Mary walks, t
$walk'_*(a)(m)$

(8) *whether John walks or Mary walks,* $\bar{t}$

$\lambda i[(walk'_*(a)(j) \vee walk'_*(a)(m)) = (walk'_*(i)(j) \vee walk'(i)(m))]$

John walks or Mary walks, t

$walk'_*(a)(j) \vee walk'_*(a)(m)$

3.2. Extensional and intensional complement embedding verbs

In Section 1.3 we stated that verbs such as *know* and *tell* are extensional. The meaning postulate guaranteeing this reads as follows:

(MP:IV/$\bar{t}$) $\vee M \, \Lambda x \, \Lambda r \, \Lambda i[\delta(i)(x,r) = M(i)(x(i),r(i))]$

M is a variable of type <s,<<s,t>,<e,t>>>; x of type <s,e>; r of type <s,<s,t>>; i of type s; and $\delta = g(\alpha)$, $\alpha \in B_{IV/\bar{t}}$, but $\alpha \neq$ *wonder, investigate, ask.*

Requiring this formula to hold in all models guarantees that to certain intensional relations between individual concepts and propositional concepts, extensional relations between individuals and propositions correspond. We extend the substar notation convention of PTQ as follows:

(SNC) $\delta_* = \lambda a \lambda p \lambda u(\delta(a)(\lambda ap)(\lambda au))$

p is a variable of type <s,t>, u of type e.

Combining (MP:IV/$\bar{t}$) with (SNC) one can prove that (9) is valid:

(9) $\Lambda i[\delta(i)(x,r) = \delta_*(i)(x(i),r(i))]$.

If we apply (9) to the translations of (5) *John knows that Mary walks* and (6) *John knows whether Mary walks,* we get the following results:

(5') $know'_*(j, \lambda a[walk'_*(a)(m)])$

(6') $know'_*(j, \lambda i[walk'_*(a)(m) = walk'_*(i)(m)])$

Formula (5') expresses that the individual John knows the proposition that Mary walks. In (6') it is expressed that John knows the proposition denoted by $\lambda i[walk'_*(a)(m) = walk'_*(i)(m)]$. As has been indicated in Section 2.2, which proposition is denoted by this expression at g(a) depends on the truth value of $walk'_*(a)(m)$ at g(a). More generally, we can prove that the following holds:

(10)

$$[\![\lambda i[\phi/a/ = \phi/i/]]\!]_{M,g} \begin{cases} = [\![\lambda i[\phi/i/]]\!]_{M,g} & \text{if } [\![\phi/a/]\!]_{M,g} = 1 \\ = [\![\lambda i[\neg\phi/i/]]\!]_{M,g} & \text{if } [\![\phi/a/]\!]_{M,g} = 0. \end{cases}$$

Given (10), it is obvious that the arguments (I) and (II) of Section 1.1 are valid. Their translations are:

(I') $$\frac{\begin{array}{l} \text{know}'_*(a)(j,\ \lambda i[\text{walk}'_*(a)(m) = \text{walk}'_*(i)(m)]) \\ \text{walk}'_*(a)(m) \end{array}}{\text{know}'_*(a)(j,\ \lambda a[\text{walk}'_*(a)(m)])}$$

(II') $$\frac{\begin{array}{l} \text{know}'_*(a)(j,\ \lambda i[\text{walk}'_*(a)(m) = \text{walk}'_*(i)(m)]) \\ \neg\text{walk}'_*(a)(m) \end{array}}{\text{know}'_*(a)(j,\ \lambda a[\neg\text{walk}'_*(a)(m)])}$$

Since (MP:IV/$\bar{t}$) also holds for *tell*, the arguments (III) and (IV) are rendered valid in exactly the same way. And precisely because (MP:IV/$\bar{t}$) does not hold for verbs like *wonder*, arguments like (I) - (IV) cannot be constructed for this and similar verbs. The relation expressed by *wonder* is not extensional in object position, its second argument is irreducibly a propositional concept. Argument (VIII), concerning the exhaustiveness of alternative *whether*-complements is discussed in 3.4. The arguments (XIII) and (XIV) of Section 1.7 are left to the reader.

3.3. Single constituent complements with *who*

First we consider constituent complements which contain just one occurrence of the *wh*-term *who*. An example of an analysis tree of a sentence containing such a complement, together with its translation, is:

(11) *John knows who walks*, t

$know'_*(a)(j, \lambda i[\lambda u[walk'_*(a)(u)] = \lambda u[walk'_*(i)(u)]])$

John, T — $\lambda P[P(a)(\lambda aj)]$

know who walks, IV — $know'(a)(\lambda a \lambda i[\lambda u[walk'_*(a)(u)] = \lambda u[walk'_*(i)(u)]])$

know, IV/$\bar{t}$ — $know'(a)$

who walks, $\bar{t}$ — $\lambda i[\lambda u[walk'_*(a)(u)] = \lambda u[walk'_*(i)(u)]]$

who walks, t///e — $\lambda x_0[walk'(a)(x_0)]$

he$_0$ *walks*, t — $walk'(a)(x_0)$

Constituent complements are formed from sentences containing a syntactic variable, but in an indirect way. First a so-called *abstract* is formed, an expression of category t///e. The *wh*-term *who*(*m*) is placed at the front of the sentence, certain occurrences of the variable are deleted, others are replaced by suitable pro-forms. In fact, our use of the phrase '*wh*-term' is rather misleading. Unlike in Karttunen's analysis for example, they do not belong to a fixed syntactic category. In this they are like their logical language counterpart, the λ-abstraction sign. This rule of abstract formation and its translation are:

(S:AB1) If $\phi \in P_t$, then $F_{AB1,n}(\phi) \in P_{t///e}$

(T:AB1) If $\phi \sim \phi'$, then $F_{AB1,n}(\phi) \sim \lambda x_n(\phi')$.

The translation of an abstract is a predicate denoting expression. From these abstracts constituent complements are formed. The syntactic rule that does this is a category changing rule. The corresponding translation rule turns predicate denoting expressions into proposition denoting expressions in the way indicated in (3) in Section 2.4.

(S:CCF*) If $\chi \in P_{t///e}$, then $F_{CCF}(\chi) \in P_{\bar{t}}$

(T:CCF*) If $\chi \sim \chi'$, then $F_{CCF}(\chi) \sim \lambda i[\chi' = [\lambda a \chi'](i)]$.

The intermediate level of abstracts is not strictly needed for single constituent complements, but it is essential for a correct analysis of constituent complements that contain more than one *wh*-term (see Section 4). Moreover, an attractive feature of our analysis is that another kind of *wh*-construction, *relative clauses*, can both syntactically and semantically be treated as abstracts as well.

We are now able to show that argument (V) of Section 1.4 is valid. Its translation is:

(V')

$$\frac{\begin{array}{l}\text{know}'_*(a)(j,\ \lambda i[\lambda u[\text{walk}'_*(a)(u)] = \lambda u[\text{walk}'_*(i)(u)]])\\ \text{walk}'_*(a)(b)\end{array}}{\text{know}'_*(a)(j,\ \lambda a[\text{walk}'_*(a)(b)])}$$

From $[\![\text{walk}'_*(a)(b)]\!]_{M,g} = 1$, it follows that $[\![\lambda u[\text{walk}'_*(a)(u)]]\!]_{M,g}([\![b]\!]_{M,g}) = 1$. So, at every index k such that $[\![\lambda i[\lambda u\ \text{walk}'_*(a)(u)] = \lambda u[\text{walk}'_*(i)(u)]]]\!]_{M,g}(k) = 1$, it also holds that $[\![\lambda u[\text{walk}'_*(i)(u)]]\!]_{M,g[k/i]}([\![b]\!]_{M,g[k/i]}) = 1$. I.e. that at every such index k: $[\![\lambda a[\text{walk}'_*(a)(b)]]\!]_{M,g}(k) = 1$. Under the not unproblematic, but at the same time quite usual assumption that to know a proposition is to know its entailments, this means that (V') is valid. The assumption in question can be laid down in a meaning postulate in a straightforward way.

3.4. Exhaustiveness

It is easy to see that argument (VII) of Section 1.5, illustrating the exhaustiveness of the proposition denoted by a constituent complement is valid too. Its translation is:[9]

(VII')

$$\frac{\begin{array}{l}\text{believe}'_*(a)(j,\ \lambda a[\text{walk}'_*(a)(b) \wedge \text{walk}'_*(a)(s)])\\ \Lambda u[b{=}u \leftrightarrow \text{walk}'_*(a)(u)]\end{array}}{\neg\text{know}'_*(a)(j,\ \lambda i[\lambda u[\text{walk}'_*(a)(u)] = \lambda u[\text{walk}'_*(i)(u)]])}$$

Suppose the conclusion is false and the second premiss is true. Then $[\![\lambda u\ \text{walk}'_*(a)(u)]\!]_{M,g}$ is (the characteristic function of) the unit set consisting of $[\![b]\!]_{M,g}$. From this it follows that $[\![\text{know}'_*(a)(j,\ \lambda a[\Lambda u[b{=}u \leftrightarrow \text{walk}'_*(a)(u)]])]\!]_{M,g} = 1$. Under the assumption that knowing implies believing, also to be laid down in a meaning postulate, it follows that the first premiss is false. So, (VII') is valid. We leave it to the reader to verify that the similar arguments (XI) and (XII) of Section 1.7 are valid too.

Argument (VIII), showing the exhaustiveness of *whether* complements translates as follows:

(VIII')

$\text{know}'_*(a)(j,\ \lambda i[(\text{walk}'_*(a)(m) = \text{walk}'_*(i)(m)) \wedge (\text{sleep}'_*(a)(b) = \text{sleep}'_*(i)(b))])$

$\neg\text{walk}'_*(a)(m)\ \wedge\ \text{sleep}'_*(a)(b)$

$\text{know}'_*(a)(j,\ \lambda a[\neg\text{walk}'_*(a)(m)\ \wedge\ \text{sleep}'_*(a)(b)])$

From the truth of the second premiss it follows that for every index k such that $[\![\lambda i[(\text{walk}'_*(a)(m) = \text{walk}'_*(i)(m)) \wedge (\text{sleep}'_*(a)(b) = \text{sleep}'_*(i)(b))]]\!]_{M,g}(k) = 1$ it holds that $[\![\neg\text{walk}'_*(a)(m) \wedge \text{sleep}'_*(a)(b)]\!]_{M,g[k/a]} = 1$ and thus that for every such index k it holds that $[\![\lambda a[\neg\text{walk}'_*(a)(m) \wedge \text{sleep}'_*(a)(b)]]\!]_{M,g}(k) = 1$.

3.5. Single constituent complements with *which*

The analysis of constituent complements in which one occurrence of a *wh*-term of the form *which* δ occurs is illustrated in the following example:

(12) *John knows which man walks*, t
$\text{know}'_*(a)(j, \lambda i[\lambda u[\text{man}'_*(a)(u) \wedge \text{walk}'_*(a)(u)] = \lambda u[\text{man}'_*(i)(u) \wedge \text{walk}'_*(i)(u)]])$

- *John*, T
  $\lambda P[P(a)(\lambda aj)]$
- *know which man walks*, IV
  $\text{know}'(a)(\lambda a\lambda i[\lambda u[\text{man}'_*(a)(u) \wedge \text{walk}'_*(a)(u)] = \lambda u[\text{man}'_*(i)(u) \wedge \text{walk}'_*(i)(u)]])$
  - *know*, IV/$\bar{t}$
    $\text{know}'(a)$
  - *which man walks*, $\bar{t}$
    $\lambda i[\lambda u[\text{man}'_*(a)(u) \wedge \text{walk}'_*(a)(u)] = \lambda u[\text{man}'_*(i)(u) \wedge \text{walk}'_*(i)(u)]]$
    - *which man walks*, t///e
      $\lambda x_0[\text{man}'(a)(x_0)\ \wedge\ \text{walk}'(a)(x_0)]$
      - *man*, CN
        $\text{man}'(a)$
      - *he*$_0$ *walks*, t
        $\text{walk}'(a)(x_0)$

Again, the complement is formed in two steps. First, from a sentence containing a syntactic variable, and a common noun phrase an abstract is formed. The syntactic function which does this is quite similar to the one forming abstracts with *who*. The syntactic rule and the translation rule are:

(S:AB2) If $\phi \in P_t$ and $\delta \in P_{CN}$, then $F_{AB2,n}(\delta,\phi) \in P_{t/\!/\!/e}$

(T:AB2) If $\phi \sim \phi'$ and $\delta \sim \delta'$, then $F_{AB2,n}(\delta,\psi) \sim \lambda x_n(\delta'(x_n) \wedge \phi')$.

The translation is a complex predicate denoting expression. It denotes the conjunction of the predicate denoted by the common noun phrase and the predicate that can be formed from the sentence.

The second step is to apply the category changing rule (S:CCF*) which turns abstracts into complements. This way of constructing complements like *which man walks* gives rise to the *de dicto* reading discussed in 1.6. The proposition $[\![\lambda i[\lambda u[man'_*(a)(u) \wedge walk'_*(a)(u)] = \lambda u[man'_*(i)(u) \wedge walk'_*(i)(u)]]]\!]_{M,g}$ holds at an index k iff the intersection of the set of men and the set of walkers at k is the same as at g(a). If John knows this proposition, it is implied that if a certain individual is a walking man, John knows both that it is a man and that it walks. In view of this, (X'), the translation of (X) with both the premiss and the conclusion in the *de dicto* reading is not valid:

$$\text{(X')}\quad \frac{know'_*(a)(j,\lambda i[\lambda u[man'_*(a)(u) \wedge walk'_*(a)(u)] = \lambda u[man'_*(i)(u) \wedge walk'_*(i)(u)]])}{know'_*(a)(j,\lambda i[\lambda u[man'_*(a)(u) \wedge \neg walk'_*(a)(u)] = \lambda u[man'_*(i)(u) \wedge \neg walk'_*(i)(u)]])}$$

A counterexample can be constructed as follows. Suppose that for some assignment g and for some individual d it holds that: $[\![walk'_*(a)]\!]_{M,g}(d) =$ $= [\![man'_*(i)]\!]_{M,g}(d) = [\![walk'_*(i)]\!]_{M,g}(d) = 0$, and $[\![man'_*(a)]\!]_{M,g}(d) = 1$. Then the proposition which is the argument in the premiss holds at g(i), whereas the proposition which is the argument in the conclusion does not. So, the proposition in the premiss does not entail the proposition in the conclusion, which, given the usual semantics of *know* would be the only way in which the premiss could imply the conclusion. By a similar argument it can be shown that the inverse of (X') is not valid either.

3.6. *De re* readings of constituent complements

In 1.6 we argued that (X) is valid iff both its premiss and its conclusion are read *de re*. This means that a second way to derive sentences containing constituent complements should be added to the syntax. In this derivation process common noun phrases are quantified into sentences containing a common noun variable in a constituent complement. For this purpose, we add common noun variables one_0, one_1,... to the set of basic expressions of category CN. They translate into variables o_0, o_1,... of type $\langle\langle s,e\rangle,t\rangle$. The rule of common noun quantification and the corresponding translation rule are as follows:

(S:CNQ) If $\phi \in P_t$ and $\delta \in P_{CN}$, then $F_{CNQ,n}(\delta,\phi) \in P_t$

(T:CNQ) If $\phi \sim \phi'$ and $\delta \sim \delta'$, then $F_{CNQ,n}(\delta,\phi) \sim \lambda o_n \phi'(\delta')$.

The sentence *John knows which man walks* can now also be derived as follows:

(13) *John knows which man walks*

$know'_*(a)(j,\lambda i[\lambda u[man'_*(a)(u) \wedge walk'_*(a)(u)] = \lambda u[man'_*(a)(u) \wedge walk'_*(i)(u)]])$

man
$man'(a)$

John knows which one$_2$ *walks*
$know'_*(a)(j,\lambda i[\lambda x[o_2(x) \wedge walk'(a)(x)] = \lambda x[o_2(x) \wedge walk'(i)(x)]])$

John
$\lambda P[P(a)(\lambda aj)]$

know which one$_2$ *walks*
$know'(a)(\lambda a\lambda i[\lambda x[o_2(x) \wedge walk'(a)(x)] = \lambda x[o_2(x) \wedge walk'(i)(x)]])$

know, $IV/\bar{t}$
$know'(a)$

which one$_2$ *walks*, $\bar{t}$
$\lambda i[\lambda x[o_2(x) \wedge walk'(a)(x)] = \lambda x[o_2(x) \wedge walk'(i)(x)]]$

which one$_2$ *walks*, $t///e$
$\lambda x_5[o_2(x_5) \wedge walk'(a)(x_5)]$

one$_2$, CN
o_2

he$_5$ *walks*, t
$walk'(a)(x_5)$

The translation of (X) with both premiss and conclusion read *de re* is now:

(X'')

$$\frac{know'_*(a)(j,\lambda i[\lambda u\ man'_*(a)(u) \wedge walk'_*(a)(u)] = \lambda u[man'_*(a)(u) \wedge walk'_*(i)(u)]])}{know'_*(a)(j,\lambda i[\lambda u[man'_*(i)(u) \wedge \neg walk'_*(a)(u)] = \lambda u[man'_*(a)(u) \wedge \neg walk'_*(i)(u)]])}$$

The proposition denoted by the complement in the premiss at g(a) is the same as the one denoted by the complement of the conclusion at g(a). The first proposition holds true at an index *k* iff the intersection of the set of men at g(a) and the set of walkers at g(a) is the same as the intersection of the set of men at g(a) and the set of walkers at k. Clearly, this is the case iff the intersection of the set of men at g(a) and the set of non-walkers at g(a) is the same as the intersection of the set of men at g(a) and the set of non-walkers at k, i.e. iff the second proposition holds true at k. So, both (X'') and its inverse are valid arguments.

We leave it to the reader to satisfy her/himself that (IX) with its conclusion read *de dicto* is not valid, whereas with the conclusion read *de re* it is. It should be noted that the following *is* a valid argument:

(XV) *John knows who walks*

$\text{know}'_*(a)(j,\lambda i[\lambda u[\text{walk}'_*(a)(u)] = \lambda u[\text{walk}'_*(i)(u)]])$

John knows who doesn't walk

$\text{know}'_*(a)(j,\lambda i[\lambda u[\neg\text{walk}'_*(a)(u)] = \lambda u[\neg\text{walk}'_*(i)(u)]])$

Perhaps this is not as it should be. If not, there is the possibility to complicate (T:AB1) in such a way that the range of *who* is restricted to some contextually defined set. Then counterexamples to (XV) can be given.

4. CONCLUDING REMARKS

This last section contains some remarks on matters which cannot be dealt with in this paper. They will be discussed in detail in GROENENDIJK & STOKHOF (in preparation).

The rules given sofar deal only with single constituent complements. Some new rules have to be added to account for multiple constituent complements as well. The semantic results are completely analogous to those obtained in the previous sections. Consider the following examples:

(14) *who loves whom*, $\bar{\bar{t}}$

$\lambda i[\lambda u\lambda v[\text{love}'_*(a)(u,v)] = \lambda u\lambda v[\text{love}'_*(i)(u,v)]]$

(15) *which man which girl loves*, $\bar{\bar{t}}$

$\lambda i[\lambda u\lambda v\ \text{girl}'_*(a)(u) \wedge \text{man}'_*(a)(v) \wedge \text{love}'_*(a)(u,v)] =$
$[\lambda u\lambda v\ \text{girl}'_*(i)(u) \wedge \text{man}'_*(i)(v) \wedge \text{love}'_*(i)(u,v)]]$.

It is in the analysis of multiple constituent complements that an essential use is made of the level of abstracts.

The analysis presented above does not yet account for the conjunction of complements. In order to do so properly, we ultimately analyze complements as a kind of terms, as expressions denoting not propositions, but sets of properties of propositional concepts. This also enables us to account for that reading of (16) which is equivalent to (17) (cf. KARTTUNEN & PETERS (1980)):

(16) John wonders which professor recommended each candidate

(17) John wonders which professor recommended which candidate

In Section 2.5 we said that one can get a long way in the analysis of complements by adding a new intensional operator to IL. As a matter of fact, one could come quite as far as the end of this paper, since the phenomena that resist adequate treatment in such an intensional language are not treated here. The operator in question, called Δ, can be introduced in IL as follows:

(18) If $\alpha \in ME_a$, then $\Delta\alpha \in ME_{<s,t>}$;

$[\![\Delta\alpha]\!]_{M,k,g}$ is that $p \in \{0,1\}^I$ such that for every $i \in I$: $p(i) = 1$ iff $[\![\alpha]\!]_{M,k,g} = [\![\alpha]\!]_{M,i,g}$.

Complements could then be formed from sentences and abstracts simply by putting Δ in front of them. The phenomena that cause this approach to fail have in common that their treatment requires the quantification of terms into complements.

At the beginning of this paper we said that an adequate semantics for *wh*-complements might give a clue to the semantics of direct questions as well. At first sight, little or nothing seems to speak against simply associating direct questions with the same semantic object we associated *wh*-complements with. An objection that might come to mind is this. Suppose ϕ is true. Then the direct questions *Does John know whether* ϕ? and *Does John know that* ϕ? denote the same proposition. Wouldn't this mean that asking the first question comes to the same thing as asking the second one? No, no more than that asserting a declarative sentence ϕ comes to the same thing as asserting a declarative sentence ψ in case ϕ and ψ happen to have the same truth value. Although the *denotations* of the two questions are the same, their *senses* are still different. Other interesting issues are e.g. to what extent we could consider the proposition denoted by a question to be the proposition expressed by an answer to it. A discussion of these matters is, however, beyond the scope and limitations of the present paper.

FOOTNOTES

* We would like to thank Roland Hausser, Alice ter Meulen and Zeno Swijtink, and in particular Johan van Benthem, Theo Janssen and Lauri Karttunen for their remarks on an earlier version, which have led to many improvements.

1. If their conclusions are read *de re*, these arguments are valid. If their conclusions are read *de dicto,* however, they are not. However, it turns out that the combination of treating proper names as rigid designators and verbs such as *know* as relations between individuals and propositions does not make it possible to distinguish *de dicto* readings of the conclusions of these arguments. This is not correct, it should be possible to distinguish a *de dicto* reading of these sentences, while maintaining a rigid designator view of proper names at the same time.
2. Complements of this form are ambiguous between an *alternative* and a *yes/no* reading. The latter might be indicated as *whether* (ϕ *or* ψ). In Section 3.1 we show how this ambiguity is accounted for. In (VIII) the alternative reading is meant.
3. For a proposal which makes it possible to consider infinitival complements to be proposition denoting expressions as well, see GROENENDIJK & STOKHOF (1979).
4. There still remains the verb *know* which takes NP's, as in *John knows Mary*. An argument in favour of regarding this verb to be different from the one taking complements might be that in such languages as German and Dutch the difference is lexicalized. On the other hand, in a sentence like *John knows Mary's phone number,* the verb *know* seems to be like the complement taking *know* in many respects. (See also footnote 5.)
5. The possibility to construct these proposition denoting expressions from expressions α of arbitrary type is quite interesting also in view of sentences like *John knows Mary's phone number,* mentioned in footnote 4. If we simply apply procedure (3) with the translation of the term *Mary's phone number* substituted for α/a/ we seem to obtain exactly the proposition John needs to know if he is to know Mary's phone number. This point was brought to our attention by Barbara Partee.
6. Notice that in PTQ complements are in fact taken to be of category t. When embedded under complement taking verbs, we semantically apply the interpretation of the verb to the sense of the complement. This makes that proposition denoting expressions do occur in PTQ translations. Because of this, one might think that the new category $\bar{t}$ is superfluous. But it is

not, since we want complements to *denote* propositions and to have propositional concepts as their sense.

7. For those who find it unbearable, c.q. unnatural, that the translation of *whether* ϕ *or* ψ does not contain a disjunction, we present the following equivalent alternative:

$$\text{(T:WHC')} \quad \lambda i[\lambda p[p(a) \wedge [p = \lambda a\phi_1' \vee \ldots \vee p = \lambda a\phi_n']] = \lambda p[p(i) \wedge [p = \lambda a\phi_1' \vee \ldots \vee p = \lambda a\phi_n']]].$$

8. As (10) shows, *whether*-complements resemble *if then else* statements of certain programming languages. In JANSSEN (1980) the latter are used as counterexamples to the validity of cap-cup elimination in IL. It seems that *wh*-complements are natural language counterexamples. If ψ translates a *wh*-complement, then $\lambda a(\psi(a)) \neq \psi$, i.e. ${}^{\wedge\vee}\psi \neq \psi$.
9. For the treatment of *only* used here, see GROENENDIJK & STOKHOF (1976).

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THE PLACE OF PRAGMATICS IN MODEL THEORY

by

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0. INTRODUCTORY REMARKS

How much semantics can be handled in the syntax? How much pragmatics can be handled in the semantics? And conversely, how much syntax can be handled in the semantics? How much semantics can be handled in the pragmatics? The answer to all these questions is the same: of the components of grammar actually proposed in the literature, each has been expanded to handle a lot more phenomena than advisable for its own good. For example, the treatment of semantic generalizations in the syntax is amply illustrated in the various stages of transformational grammar, while the treatment of pragmatic generalizations in the semantics is exemplified by the various performative analyses of non-declarative sentence moods (FN.1). It is the goal of the present paper to outline a theory of discourse which

(a) provides clear standards for the borderlines between the components of a general framework consisting of a *Syntax, Semantics, Lexicon, Context,* and *Pragmatics*;
(b) indicates how the different components interact in the course of interpreting the use of an expression by a speaker relative to a context.

The interaction of components will be summarized in terms of a "Speaker Simulation Device" (SID).

Our point of departure will be Montague Grammar, since this type of grammar is the only framework presently known which relates syntax and semantics of natural language in a systematic and coherent manner. Montague Grammar and the standard model theoretic approach on which it is based fail, however, to account for the distinction as well as the interaction of semantics and pragmatics. We may therefore interpret the following pages as an attempt to find a place for pragmatics in model theory.

Our method of creating such a place is both radical and simple. Based on a reinterpretation of model-theoretic semantics from the 'verifying mode' to the 'synthesizing mode', we arrive at *two* types of model, the token-model and the context-model, both of which are assumed to be part of a speaker simulation device, neither containing any real objects. It is proposed to treat the literal meaning of expressions in terms of the token-model, the subjective reality of the speaker in terms of the context-model, and the use of expressions relative to the context (i.e. pragmatics) in terms of matching the token-model and the context-model.

1. MEANING AND USE

What is a meaning? Of the many answers that have been given to this question, we will be concerned here only with two, namely

1) speech-act theory, as presented in various forms by AUSTIN (1962), GRICE (1957), SEARLE (1969), WUNDERLICH (1976), and others,
2) model-theoretic semantics, as developed by TARSKI (1936), CARNAP (1947), KRIPKE (1963), MONTAGUE (1974), and others.

Speech-act theory defines meaning as what the speaker intends, as what a speaker really meant when (s)he said something. This intentional approach to meaning is closely related to aspects of *language use*. In the following let us refer to meaning defined in terms of speech acts, rules of conversation, felicity conditions, or use conditions as $meaning^2$.

Model-theoretic semantics, on the other hand, defines meaning as a relation between expressions and the objects, or sets of objects, to which the expressions refer, or which the expressions are said to denote. The paradigmatic case of this approach to meaning is the logical concept of a *proper name*. For example, the meaning, or denotation, or referent of the proper name *John* is the actual person so named. A *predicate* like *walk,* furthermore, is said to denote a set of individuals, containing elements which have the property of walking. FREGE (1892) completed the assignment of kinds of objects to the major parts of speech by proposing that *declarative* sentences should be defined to denote truth-values. This proposal developed into the view (DAVIDSON 1967) that the meaning of a sentence may be equated with its truth-conditions. Recent developments in model-theoretic semantics, finally have led to quite detailed analyses of meaning in natural language by formally specifying the model-theoretic objects which serve as referents, either in terms of complex translations or in terms of meaning postulates (MONTAGUE 1974).

Each of the two approaches to meaning mentioned in (1) and (2) above captures a legitimate and important aspect of meaning in natural language. But unfortunately, in their present form the two approaches are pursued in a way that renders them incompatible. Speech-act theory has no account of how the literal meaning of an expression depends on its surface structure. The speaker meaning is furthermore claimed to represent the primary notion of meaning, so that all other accounts are derivative (GRICE 1957). Model-theoretic semantics, on the other hand, while providing a highly developed technique to analyze the literal meaning of expressions, is in its present form unable to provide natural accommodations for the *use-aspect* of natural language.

Before we turn to the question of how to reinterpret the speech-act approach (meaning^2) and the model-theoretic approach (meaning^1) in such a way as to make them compatible, let us consider how meaning^1 and meaning^2 should in general be related. Since meaning^1 is defined as the literal meaning of expressions and meaning^2 is defined as what the speaker/hearer has in mind in a certain utterance situation it is reasonable to relate them in the following way:

3) use of meaning^1 = meaning^2.

In other words, by using a certain expression with a certain literal meaning (meaning^1) relative to a context we may achieve a communicative effect (meaning^2) which goes far beyond the literal meaning encoded in the token surface. In ironic use, for example, meaning^2 may be even directly contrary to meaning^1.

The necessity to distinguish between meaning^1 and meaning^2 may also be illustrated in connection with the somewhat hackneyed example (4):

4) Can you pass the salt?

Uttered at the dinner table, (4) is used as a request (normally) and the intended response is passing the salt. Uttered to someone disabled by disease or accident, on the other hand, (4) may be used as a *bona fide* question, and the intended response would be 'yes' or 'no'.

So does (4) have two meanings depending on the context? The answer is yes if 'meanings' in the preceding question is read as meaning^2. The answer is no, however, if 'meanings' is read as meaning^1. (4) has only one literal meaning, but this meaning may be *used* in many different ways in many different contexts, creating a whole spectrum of meanings^2.

The content of formula (3) may be found implicitly in the previous literature, especially-speech act literature. But nobody seems to have drawn the stringent consequences which follow from it, both for speech-act theory and model-theoretic semantics. The consequence for speech-act theory is that we cannot study the use of a meaning by a speaker in a context unless we have an independent description of the literal meaning that is being used, much as we cannot study the use of a tool relative to a certain object before we know the tool's, exact shape, size, and material. Which brings us back to the analysis of meaning^1 in general and model-theoretic semantics in particular.

As shown by Montague, we may formally describe the literal meaning of expressions in a fragment of English in terms of translation into a model-theoretically interpretable language (intensional logic). Thus, given any linguistic expression in the surface fragment, we may characterize its literal meaning (meaning^1) in terms of the denotation conditions associated with its formal translation. But how can we get in this system from a formal characterization of meaning^1 to a formal characterization of meaning^2?

It is curious that the standard model-theoretic approach, as represented by Carnap, Kripke, and Montague (a) completely abstracts from the speaker/hearer and (b) provides no analysis of lexical meaning. Rather, the formal model is seen as a *representation of reality,* and the denotation conditions (truth-conditions) are read as if it were the purpose of a formal interpretation to find out whether a formula is 1 or 0 relative to a model at an index. *In praxi,* however, the model structure is not independently given, but must be specified by the logician before (s)he can start with a formal interpretation of a formula. In as much as we may imagine different states of affairs, we may define the formal model structure as we see fit. Thus, the explicit specifications of a formal model is logically and empirically unrewarding on the standard approach. The sole purpose for actually defining a formal model structure would be to *illustrate* how a model-theoretic interpretation works (on the compositional or non-lexical level).

2. PROBLEMS OF THE STANDARD MODEL THEORETIC APPROACH

While we may define the formal model to represent any state of affairs we like, there are systematic restrictions on the definition of the model structure imposed by the meaning of the words of the language under interpretation (assuming the model structure is used to interpret a natural language). Compare for example (5) and (6):

5) The red circle rises.

6) The square circle rises.

Whereas we may define a model structure such that (5) is 1 (true) relative to one index and 0 (false) relative to another, intuition requires that there should be no index in the model structure relative to which (6) would be 1.

One way to treat the restrictions induced by the intuitive word meanings on the definition of the model structure is to exclude certain model structures from consideration. This is the *meaning postulate approach,* as used by Montague in PTQ (MONTAGUE 1974, chapter 8). Meaning postulates are external restrictions on model structures which delimit the class of what Montague calls 'logically possible' models. This terminology is somewhat misleading, however. What is at issue is not logical possibility but rather the speakers intuitions regarding the semantic interdependence in the denotation of different words. For example, a model where the denotation of *man* does not overlap with the denotation of *human* would be no more linguistically reasonable than a model where the denotations of *square* and *round* are not disjoint.

While the method of meaning postulates permits to maintain that assumption of the standard approach according to which the model structure is viewed as a representation of reality and the denotation conditions are viewed as instructions to find out whether a sentence is 1 or 0 relative to an index, meaning postulates are an extremely cumbersome method for formally implementing lexical interdependencies. This leads to the question: how could we separate the *lexical aspect* of word meaning from what we might call the *referential aspect*? This question is quite parallel to our earlier question of how to separate the description of literal meaning from the speaker meaning in model theoretic semantics.

The traditional model-theoretic approach, according to which meaning (denotation, reference) is stipulated to be a *direct relation* between expressions and model-theoretic objects not only eliminates the possibility for a well-defined lexical semantics in model-theoretic terms, but also raises serious *ontological problems.* If meaning is the relation between an expression and the object it refers to, must the object be real? If yes (and philosophers in the traditions of nominalism and realism decidedly think so), we are faced with the question of what to do with language expressions for which there simply are no real objects as possible referents. Take for example *the smallest prime number greater than 11, John's last hope,* but also expressions other than noun phrases such as *in, and, to,* etc. There are no real objects to which these expressions may be said to refer. Thus one either has to expand ones notion of what is real in order to give these expressions meanings, or one has to deny meanings to incomplete expressions, postulating that only complete sentences have a meaning by themselves (FN.2). The latter view (which originated with RUSSELL (1906) later lent implicit support to the performative analysis of non-declarative sentences.

Another problem with the traditional model-theoretic approach concerns the treatment of *context-dependent expressions* or indexicals. Compare for example (7) and (8):

7) Bill saw Mary at the station.

8) I saw you here.

In (7) the truth-value depends on the denotation of the constants *Bill, see, Mary,* and *at the station,* as specified by the model. In particular, *Bill, Mary,* and *at the station* are to be defined as denoting particular individuals and a particular place, respectively. In (8), however, the situation is quite different insofar as it would be intuitively wrong to assign fixed denotations to the indexicals *I, you,* and *here.*

One way to treat indexicals within the standard model-theoretic approach is the so-called *coordinates approach* (MONTAGUE 1974, chapter 3, LEWIS 1972): in addition to the coordinates specifying a possible world and a moment of time, additional coordinates are defined for each context-dendency aspect to be treated. LEWIS (1972), for example, uses a different coordinate for possible speakers (pronoun *I*), possible hearers (pronoun *you*), possible places (pronoun *here*), possible indicated objects (pronoun *this*), and even for possible previous discourse, respectively. In short, the coordinates approach permits to retain the assumption according to which meaning is a direct relation between expressions and referents by defining a context of use as an extended point of reference.

The intuitive interpretation of the model structure as a representation of reality, however, suffers under the coordinate approach. Since the model structure is assumed to specify a state of affairs at an index, one would expect that this state of affairs *is* the context. Instead, the coordinates approach introduces a second kind of reference mechanism: while the denotation of regular constants is specified over the denotation function, the denotation of indexicals is specified over numerous additional context-coordinates. Furthermore, to define the context as an arbitrary n-tuple of external coordinates fails to capture the highly specific interaction between context-dependent expressions and a coherent context (i.e. situation).

Additional problems raised by the standard approach concern non-literal reference such as vague reference and metaphoric reference. Since the standard approach characterizes the meaning relation as a direct mapping between the expression and the state of affairs provided by the model (denotation), the only way to handle non-literal meaning assignments is to postulate ambiguities.

While Montague's model-theoretic analysis is oriented towards the analysis of literal meaning of surface structures and essentially limited to sentence semantics, there is another approach, called *discourse semantics*, which is oriented towards the utterance situation and intersentential inferences. There is no question that model-theory may also be interpreted in the sense of discourse semantics. Is the discourse-semantic version of model-theory subject to the same difficulties as the sentence-semantic version?

In discourse semantics, a context is usually defined as a *set of propositions*. One aim of the analysis is to study the inferences of a context, or how the inferences of an expression vary in conjunction with different contexts. This approach, represented in various forms by HINTIKKA (1976), STALNAKER (1970, 1978), BARTSCH (1979), GROENENDIJK & STOKHOF (1975), KARTTUNEN & PETERS (1978) and others, is of special interest because (a) it is based on alternative notion of *context* (different from the coordinates approach), and (b) it may be viewed as a study in modeling contexts and speech-act situations.

However, modeling situations and the literal meaning of surface expressions in the *same* model, with a *direct* relation between expressions and referents, inevitably leads either to extremely 'standard' contexts or extremely 'non-standard' meaning assignments to surface structures. It also leads to violations of the Fregean Principle (FN.3).

In summary, both the sentence-semantic and the discourse-semantic version of the model-theoretic approach suffer from the same old problem of the standard approach, though in different from. This problem is what we have described as the fusion of the lexical and the referential aspect, which follows from the assumption that 'meaning' should be defined as a direct relation between expressions and referents. The source of the problem must be sought in the failure to distinguish between literal meaning and the use aspect of meaning in natural language. After all, which 'meaning' is in the standard approach supposed to be constituted as a direct relation between the expression and the model-theoretic object, meaning^1 or meaning^2? It is the problem of the standard approach that it cannot provide a clear answer to this question. Since sentence-semantics has no room for pragmatics defined as a coherent theory of use and discourse-semantics has no convincing account of literal meaning of expressions, neither version of the standard model theoretic approach can provide for a clear distinction between semantics and pragmatics.

Such a distinction is indispensable, however. Every time we study the meaning of a word or sentence we must decide what to treat as part of the literal meaning and what in terms of use. If we rob the field of pragmatics of its legitimate regularities, we gravely obstruct our ability to develop a viable theory of pragmatics. As the same time we obstruct our ability to arrive at a viable theory of semantics (overloading).

3. REINTERPRETING THE FORMAL MODEL-STRUCTURE

We have seen that the difficulties of the standard model-theoretic approach all stem from problems arising with the semantic treatment of natural language. For example, the need for providing interpretations to context-dependent expressions (indexicals) and the problems constituted by vague and metaphoric reference come from natural language. And the need for a model-theoretic account of the lexical intuitions of the speaker/hearer comes likewise from natural language. This has led the representatives of the standard approach to occasionally scoff at natural language as illogical or even as beyond any consistent logical analysis. The source of the problem, however, must be sought in the failure to distinguish between the literal meaning and the use-aspect of meaning in natural language.

Let us turn now to an alternative approach which preserves the formal and descriptive merits of model-theoretic semantics while accommodating formula (3):

3) use of meaning^1 = meaning^2.

This new grammatical framework, first presented in HAUSSER (1979a), separates the lexical aspect of meaning from the referential aspect (cf. Section 2) by treating

9i) *literal meaning* in terms of model-theoretic *synthesis* in a lexical space representing the speaker/hearer's lexical intuition;

9ii) *context* in terms of a model-theoretic representation of what the speaker/hearer perceives and remembers in a given utterance situation;

9iii) *reference* in terms of matching the synthesized literal meaning with the context.

Thus our alternative approach is based on the construction of *two* models, one representing the literal meaning of the token, the other representing the context. The former model is called the *token-model,* the latter is called the *context-model.* The speaker's *use* of a literal meaning (meaning^1) relative to a context is treated in our system as the *matching* of the two model theoretic structures. Thus pragmatics is sandwiched between the token-model and the context-model, inside the head of the speaker/hearer. The process of *reference* is regarded as part of pragmatics, while the construction of the token- and the context-model shares to a degree the goals of sentence- and discourse-semantics, respectively.

We arrive at the token-model by reinterpreting the intuitive role of the formal model-structure. Rather than treating the model-structure as a representation of reality and the denotation conditions as instructions to determine the truth value of formulas relative to an index, let us view the model structure as a representation of the lexical intuition of the speaker/hearer and the denotation conditions of a sentence token as instructions to synthesize or construct a model (or set of models) relative to which the sentence would be true. Thus the purpose of semantically interpreting an expression is not to determine its denotation relative to a model (in a model structure at an index) given in advance and regarded as a representation of reality (at that index), but rather to construct a denotation (or model) that would satisfy the expression and that is regarded as a *formal representation of its literal meaning* (meaning^1).

We assume that the synthesis of a token meaning is executed in a partially defined model structure, called *lexical space,* which is assumed to be part of a speaker simulation device (SID). What is required for the synthesis of a token meaning? While the logical operators like $\neg$, $\wedge$, λ, etc.

in the translation of a token receive their meaning in terms of the denotation conditions associated with these operators (where the denotation conditions are specified in a metalanguage or in terms of certain operations), unanalyzed logical constants like *man'* or *walk'* are assigned their denotations by the model-structure.

The structuring principles of a partially defined model structure regarded as a lexical space are

10i) the category/type/denotation correspondence inherent in Montague Grammar, and

10ii) the speaker's intuition concerning the semantic interrelations between constants of equal type, such as inclusion, overlap, etc. of the sets denoted.

Take for example the expressions *cat, dog,* and *mammal*, which are of equal category, namely t//e. They translate into the unanalyzed constants *cat', dog',* and *mammal'*, which are of type <s,<<s,e>,t>>. This type uniquely determines the *domain/range structure* of the functions which serve as the denotation of these expressions:

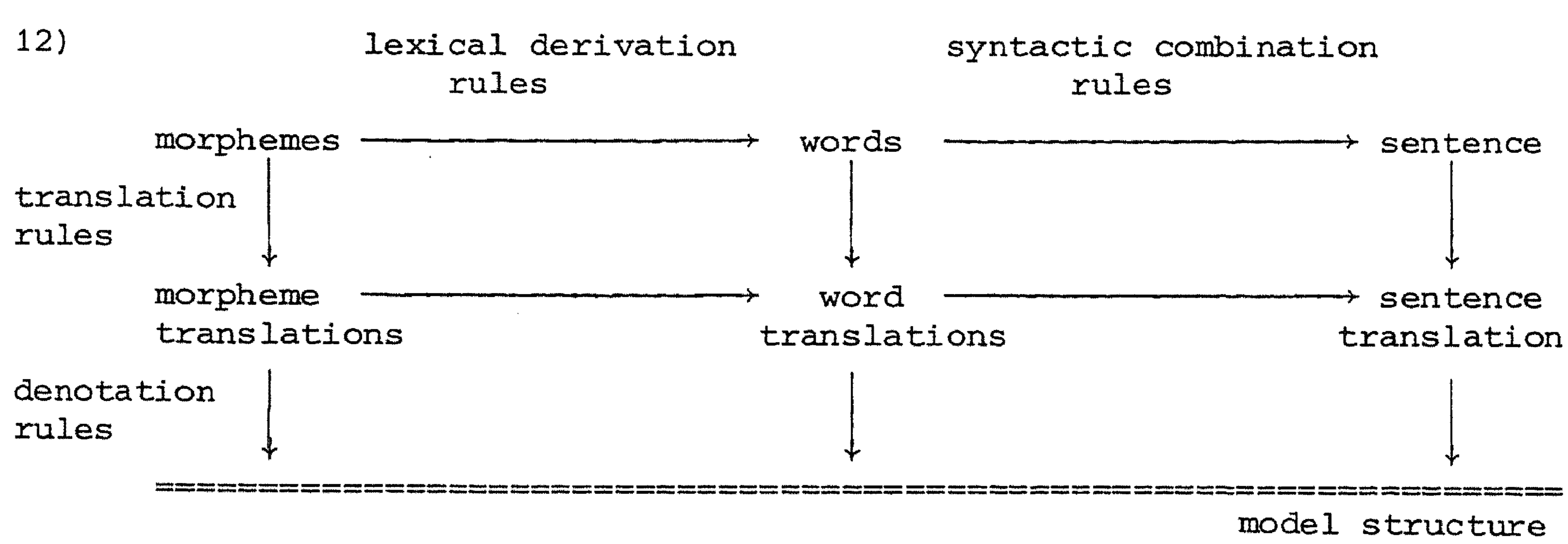
11) $(I \times J \rightarrow ((I \times J \rightarrow A) \rightarrow \{0,1\}))$.

In order to implement the lexical intuition of an English speaker/hearer we define the denotation of *cat'* and *dog'* in the lexical model as disjunct sets (extensionally speaking). Furthermore, we define the denotation of *cat'* and *dog'* as subsets of the denotation of *mammal'*. In this way, we arrive at a definition of lexical meaning which avoids the use of paraphrase (which would be circular) and which employs the model theoretic technique without identifying the model structure with reality. Our new form of model-theoretic *lexical semantics* is clearly compatible with Montague's *sentence semantics* (e.g. PTQ, EFL, UG). All that is changed by our reinterpretation is the process of assigning denotations to the unanalyzed constants in the translation formulas.

To synthesize a token in a lexical space of an SID means to set the denotations of the constants in the translation formula into certain interrelationships specified by the logical operators in the formula. For example, to synthesize the meaning of *John walks.* we have to set the denotation of *j* as an element of the denotation of walk'. Note that the partially defined model structure of our lexical space differs from the partial models proposed in FRIEDMAN et al. (1978). In Friedman et al. the model is conceived as a partially defined representation of reality, which means that as new expressions come up in a text, new denotations are defined in the model. Thus, in order to interpret *John walks.* at an index a denotation is assigned to, e.g. *walk'*, if it has not been specified already. The evaluation of expressions relative to indices in the Friedman model structure is still intended to determine truth values. Our lexical space, on the other hand, is a partially defined model structure not because certain aspects of reality have not been filled in yet, but because the model structure specifies only the semantic interrelations of constants according to the speaker's lexical intuition. A completely specified model (or denotation) comes about only once the synthesis instructions associated with the logical operators present in the translation of a token have been executed.

Since the lexical space serves solely for the interpretation of unanalyzed logical constants (on the basis of which the token-model is synthesized), some remarks on the structure of the surface lexicon are in order. In line with philological tradition, we distinguish three kinds of surface entities: morphemes (or lexemes), words, and sentences (of various moods and degrees of elipsis). We assume that words are *derived* from a limited number of morphemes (or 'roots', cf. VENNEMANN 1974, p.348) via lexical derivation rules. Sentences are derived from words via the usual syntactic rules. Lexical derivation rules differ from syntactic rules not only with regards to domain and range of the respective rule types, but also in that syncategorematic operations are strictly prohibited in the definition of syntactic rules, while they are permitted (and quite frequent) in the definition of lexical derivation rules. Schematically, the syntactic and semantic derivation of a sentence in our grammar may be characterized as follows:

12)



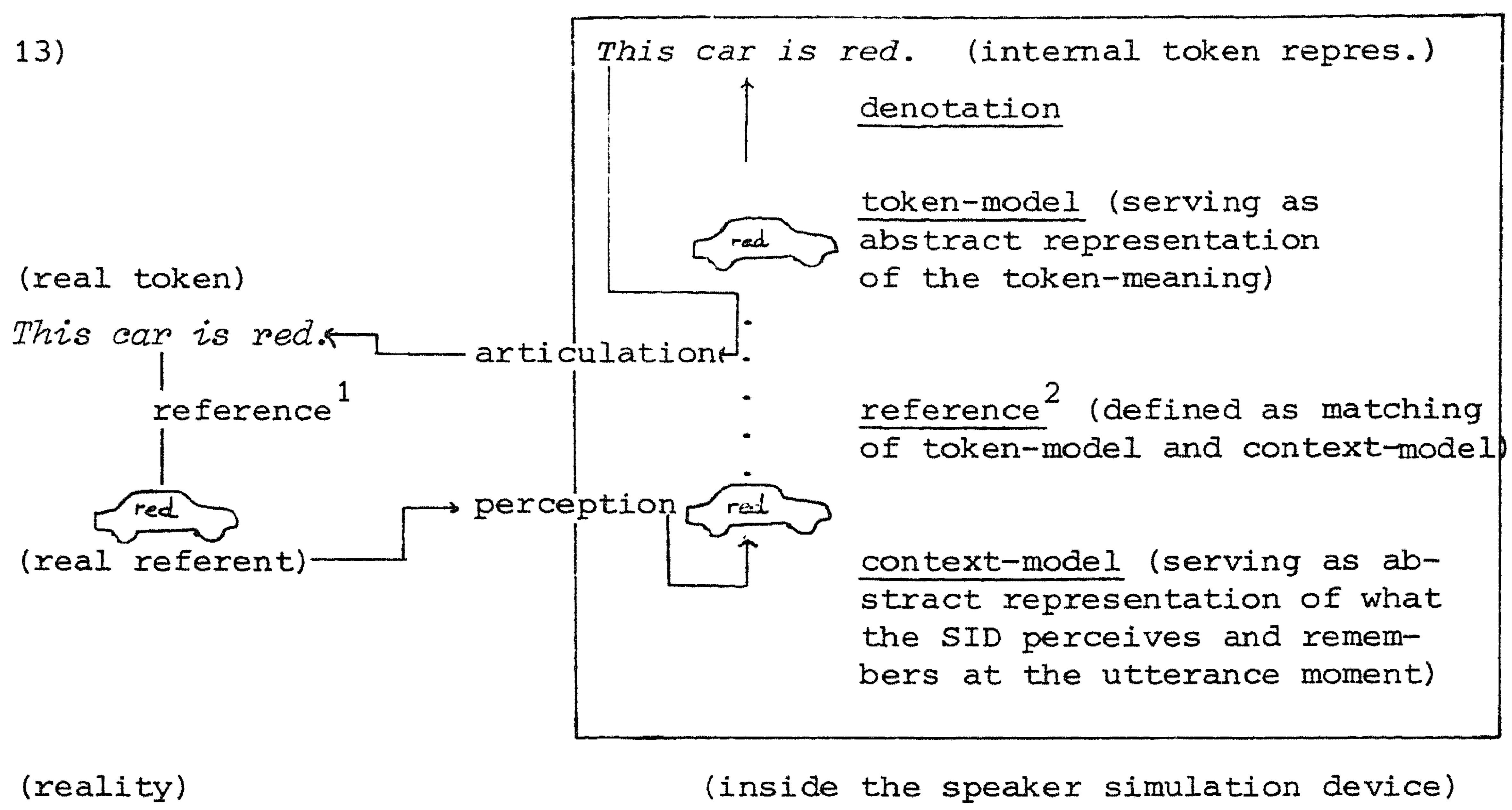
It is assumed that unanalyzed logical constants are introduced only via *morpheme translations* (and possibly lexical derivation rules on the translation level). As illustrated in (12), the model-theoretic synthesis of a token meaning starts with the morphemes (or rather the unanalyzed logical constants in their translation), whereby the model-theoretic construction of the complex sentence meaning is simultaneous to (or parallel with) the surface syntactic derivation of the sentence. Our lexical analysis differs from Montague as well as Dowty in that these authors take *words* as the basic entity of their lexical analysis, rather than morphemes. Thus, in Dowty's analysis lexical derivation rules map words into derived words, whereby the lexical rules are regarded as a variant of the syntactic rules. The model-structure, furthermore, is interpreted in the traditional way as a representation of reality, whereby some lexical intuitions are implemented in terms of meaning postulates while others are implemented in terms of complex translations. At the center of our lexical theory, on the other hand, is the idea to treat the model structure as a lexical space. Complex lexical meanings on the word level are characterized solely in terms of complex logical translations, and not in terms of meaning postulates. (For a discussion and examples of surface lexical analysis see HAUSSER 1979b.)

4. THE SPEAKER SIMULATION DEVICE (SID)

While the switch from the "verifying mode" to the "synthesizing mode" in the interpretation of model-theory provides for an analysis of the lexicon and removes the indicated ontological problems of the standard approach, it cannot by itself suffice as a complete analysis of meaning, in particular that aspect of meaning which is constituted by the *use* of a literal meaning by a speaker relative to a context. Furthermore, in order to satisfy the needs and purpose of traditional language philosopy, we must somehow reestablish the connection to *reality* which was severed when we reinterpreted the formal model-structure as a lexical space. The question then is: how do synthesized models relate to reality?

As already indicated, in order to handle the use-aspect of natural language and as a bridge pier between the token-meaning and reality, we complement the synthesized token meaning in our system with a formal *context*. This formal context is regarded as a model-theoretic representation of what the speaker/hearer perceives and remembers at the moment of a token interpretation. Schematically, the interaction between the token-model, the context-model, and reality may be indicated as follows:

13)



(13) pictures an SID (speaker simulation device) in that kind of speech-act situation which is taken as the *paradigmatic case* by the standard approach. That is a situation with an expression (i.e. "This car is red.") and a state of affairs containing a 'real'referent (car) and property (red) such that there is a *correspondence* between the expression and the 'model' (which is identified with a real situation). The basic goal of the standard approach is to capture the Aristotelian notion of truth, which is defined as a correspondence between what is said and what is (cf. TARSKI 1944).

While the standard approach limits attention to the relation between the 'real token' and the 'real referent' in (13), thus defining meaning as a direct relation between expressions and model-structures, our alternative approach takes this relation apart into several sub-mappings by routing the relation between the 'real token' and the 'real referent' through a speaker (SID). This has numerous consequences:

14i) Since the literal meaning of the 'token representation' in the SID is characterized in terms of a *synthesized* model, where the basic sets A, I, and J of the model-structure (cf. MONTAGUE 1974, chapter 8) cannot possibly contain any real objects, but must be interpreted as consisting of purely abstract memory spaces in the SID, *the ontological objections* justly raised against the standard approach *do not apply* (FN.4).

14ii) Since we distinguish between denotation (i.e. the relation between the token-representation and its synthesized meaning) and reference (i.e. the relation between the token-model and the context-model), *semantics and pragmatics are effectively separated* and distinguished.

14iii) By reinterpreting the model-structure as a lexical space, which assigns partially defined denotations to the unanalyzed logical constants in the translation language according to the speaker/hearer's lexical intuitions, we arrive at a *viable theory of the lexicon.*

14iv) At the same time we create the need, and the room, for a *coherent notion of context,* defined as a model theoretic representation of what the speaker/hearer perceives and remembers at an utterance moment under consideration.

14v) By distinguishing between the formal context and reality, we are able to describe *cases of perception or memory error*. (Such a case is discussed in DONNELLAN 1966.) (FN.5).

14vi) By distinguishing between the real token and the token representation in the SID, we are able to describe cases of *acoustic misunderstanding* as well as cases of *high-level speech errors*.

The standard approach describes the speech-act situation from the view point of an outside observer, who looks at the expression and the state of affairs, but has no access to the inside of the speaker/hearer. Our alternative approach, on the other hand, describes the speech-act situation from the view point of the speaker/hearer. While the standard approach is interested solely in modeling valid inferences of expressions in a literal or standard interpretation, our alternative approach is interested in the general phenomenon of communication. In order to analyse different types or *uses* of expressions, our alternative approach models not only literal meaning, but also the interpretation of this literal meaning relative to a context inside the speaker/hearer. After all, the utterance or interpretation of an expression presupposes in principle a speaker and/or hearer, and thus tokens in principle have a use aspect relative to the utterance- and/or interpretation-context. Thus, the goal of our formal analysis is similar to that of artificial intelligence, whereas our methods in the analysis of literal meaning employ, preserve, and extend the formal techniques of model-theoretic semantics (FN.6).

5. THE STRUCTURE OF THE INTERNAL CONTEXT

On the whole, the alternative approach is more complicated than the standard approach. But then, the alternative approach can handle phenomena (e.g. metaphoric reference, propositional attitudes, cf. HAUSSER 1979b), which the standard approach, in virtue of its basic set up, cannot treat. Also, the alternative approach provides the framework for a natural treatment of phenomena which have been analyzed within the standard approach in rather unsatisfactory ways (e.g. context-dependency (FN.7), non-declarative sentence moods (FN.8) and the lexicon (FN.9). And conversely, the alternative approach can account for those cases which the standard approach has been specifically designed to handle. Consider once more example (13). Assuming that

15i) articulation is proper,

15ii) reference is an instance of *literal reference* (defined as a complete match between the token-model and the context-model), and

15iii) perception is accurate,

our alternative approach comes to the same result as the standard approach. That is, the expression in (13) is evaluated as true relative to the indicated situation. Thus, our alternative approach captures as a *special case* both, the Aristotelian notion of truth and the prototype of utterance situation analyzed by the standard approach.

At this point, the following two questions need to be raised:

16i) How much of the new framework is worked out in detail and how much is presently only intended?

16ii) How much of the new system needs to be complete in order to be viable as a framework for ongoing linguistic analysis in syntax, semantics, and pragmatics?

Of the subsegments of the token/referent loop there are two the formal nature of which need not be of concern to the linguist. These two subsegments are (a) articulation and (b) perception. For the linguistic analysis it is sufficient to limit attention to the relation between the token-representation and the context-representation inside the SID, whereby the assumption of properly working articulation and perception in the SID is a presupposition for the study of normal discourse. While ultimately the difficult problem of simulating articulation and perception has to be solved in order to arrive at the distant goal of building a SID that can actually communicate in a natural language, this particular subject matter has no direct influence on the formal analysis of the syntax, semantics, and pragmatics of natural language. In those cases discussed in the literature which crucially depend on misperceptions (DONELLAN 1966) (or mispronouncements, though no actual example comes to mind) it is sufficient to describe the discrepancy between different speaker contexts (FN.10).

Let us turn now to the remaining segments of the token/referent loop. The by far best developed sub-segment is the mapping from the token-representation to the representation of its literal meaning, that is, the logical translation and the associated synthesized model. The reason is that this segment has been analyzed in detail within Montague Grammar, and we have shown that only a relatively minor reinterpretation of formal model-theory permits to utilize the results of Montague Grammar within our alternative framework.

The next sub-segment indicated in (13) is the mapping from the token-model to the context model, called *reference*^{2}. Intuitively, we view reference^2 as a matching of the two formal models. in HAUSSER (1979b, section 4) three different types of reference^2 (i.e. literal, vague and metaphoric reference) are informally described in terms of three different kinds of matching. But the question is now: what are the formal rules of reference^2 (and pragmatics in general)?

In order to formally analyse the matching of the two models we need to know their formal nature. In the case of the token-model, the formal structure is determined by the surface structure of the token representation under interpretation. In the case of the context-model, on the other hand, we have made no assumptions besides that it should be a model-theoretic representation of what the SID perceives and remembers. This assumption, however, naturally induces a number of structural properties on the context which go far beyond the structural features induced by either the coordinates approach or the proposition approach (cf. Section 2 above).

One important distinction in the definition of context is that between a *speaker-context* (or utterance-context) and a *hearer-context*. The speaker-context and the hearer-context may be quite distinct, which is one reason why attempts to base the analysis of meaning on the notion of a "standard context" are not appropriate. Take for example a letter. The author of the letter (in short, the speaker) synthesizes the token meaning in relation to his speaker-context, and then articulates the real token on paper. The real token in this case has an extended existence and may travel to far away places. The recipient of the letter (in short, the hearer) synthesizes the same token meaning as the author of the letter (provided the two speak the same language) and interprets the token relative to his hearer-context, which will differ from the speaker context in time, place, personal history, etc. Indeed, the only occasion where the speaker- and the hearer-context are identical is when a person talks to him- or herself.

Whether a token is interpreted relative to the speaker- or the hearer-context has consequences on the interpretation of indexicals. Take for example the sentence (17).

17) I see you.

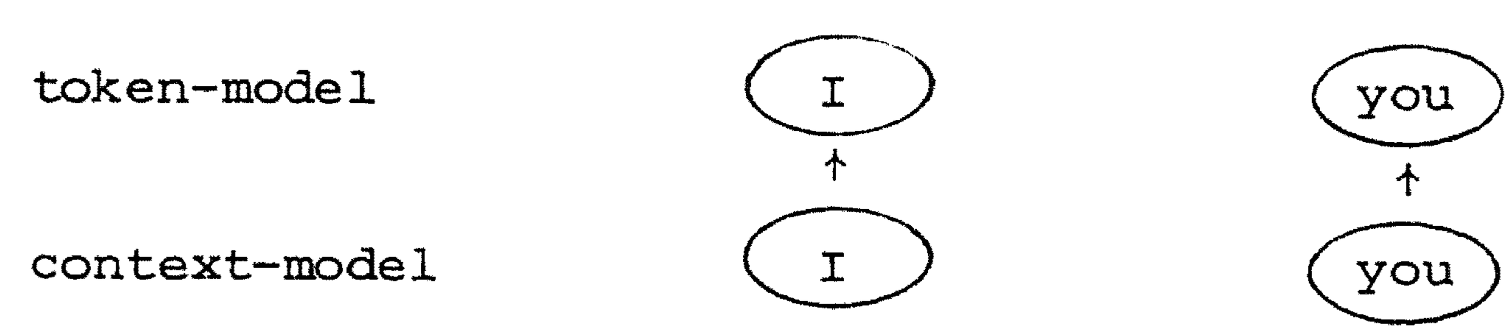
According to our analysis of context-dependency in HAUSSER 1979b,c, (17) translates into (17'):

17') $\Lambda x \varepsilon[\Gamma_1(x)]^1 \text{ see}'(x, \lambda P \Lambda y \varepsilon[\Gamma_2(x)]^1 P(x))$,

where the context-dependency aspects introduced by *I* and *you* are formally treated in the translation in terms of the context variables Γ_1 and Γ_2, respectively.

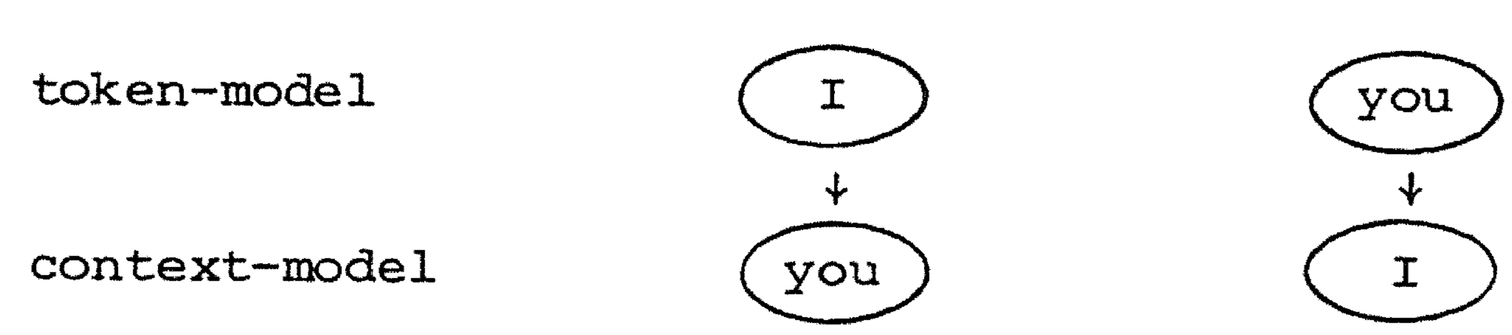
If (17) is interpreted relative to a speaker-context, the direction of the reference mapping is bottom up and the interpretation of *I* and *you* is as indicated below:

18)



If (17) is interpreted relative to a hearer-context, on the other hand, the direction of the reference mapping is top down and the interpretation of *I* and *you* is as follows:

19)



On the level of the context, *I* and *you* link up with the SID in question and the addressee of this SID, respectively. Note that we regard the reconstruction conditions of context-variables as the definition of the meaning of these context-dependent expressions. Thus the literal meaning of an expression like (17) is characterized independent of any particular context.

Next consider the interpretation of tense and modal operators in the token-representation. While on the standard approach expressions may be interpreted relative to different indices in the model-structure, we assume that in our framework the token-representation is synthesized always relative to the same abstract index, called the *zero-index of the token*. This zero-index is then equated with the 'present' moment and place of the speaker- or hearer-context. If the token-translation contains tense or modal operators, the interpretation of these operators is relative to the zero-index of the token.

While the nature of the rules for the interpretation of context-dependent expressions relative to a context in our system is fairly straightforward and has been discussed in more detail at other places, the formal nature of the pragmatic strategies that lead to non-literal interpretations is still mysterious. Generally speaking, in the interpretation of non-literal uses we assume that the system proceeds from the literal use to the derived use via a sequence of pragmatic inferences. Consider our earlier example "Can you pass the salt?" in its use at the dinner table:

20)

token-model	Can you pass the salt?
context-model	You ask me whether I can pass the salt. I may assume that you know that I can pass the salt. ⇒ You want me to pass the salt to you.

Similar analyses can be given for other instances of non-literal uses, as described in HAUSSER (1979b). It is a matter of further research to systematize such informal description in order to arrive at a theory of pragmatic inferences suited to describe metaphoric, ironic, etc. uses, which are so common in natural language.

Let us turn now to the internal structure of the context-model. One problem with the traditional treatment of context and the model-structure in general is that the external reality presents an infinity of facts, some known, some unknown, some present, some past, and some still in the future. Thus it is practically impossible to incorporate all these details in a formal representation (though this is what has to be done in an approach that regards the formal model-structure as a representation of reality). In our system, on the other hand, we need only account for what the speaker/ hearer knows or believes at the utterance or interpretation moment, whereby we treat the difference between knowledge and believe simply in terms of different degrees of subjective certainty.

In terms of which parameters should the context of the SID be organized? Let us take the *present* moment and place of the SID as the *zero-index of the context*. The subjective past of the SID is organized along the internal time axis, backward from the zero-index, while the spatial orientation of the context may be organized according to the subjective notions of front, back, left, right, up and down of the SID at the zero-index. Besides these primary notions of time and space, we may incorporate derivative time and space structures in the memory of the SID, such as knowledge of history, cities, or countries.

Further parameters organizing the context at each successive zero-index are the so-called external input parameters. Assuming that the SID is modeled after a person, the input parameters would be something like

I see:
I hear:
I feel:
I taste:
I smell:

In addition we must assume so-called internal input parameters representing desires, fears, instincts, etc.

Whereas the actual content of these parameters will be in a form characteristic of the particular medium (optical, acoustical, etc.), we may assume (for reasons of linguistic analysis) an intermediate *context-representation,* where the content of the input parameters is stated in the form of *propositions* of a suitable context-language. These propositions are then synthesized as the context-model, on the basis of the same lexical space as the token model.

We postulate the above parameters not merely to 'psychologize' our notion of context. Rather, they are necessary for the interpretation of context-dependent expressions (context-variables). With regards to the interpretation of non-literal use, furthermore, we cannot expect that a theory will render linguistically satisfactory formalizations if the framework operates on a smaller basis of contextual information than the speaker/hearer does in daily life.

In as much as we state the content of the context-parameters in terms of propositions, our approach is similar to the propositional approach to context. In as much as we treat different aspects of context-dependency in terms of different context-variables and distinguish between different parameters, on the other hand, our approach shares intuitive similarities with the coordinates approach. The basic difference between our notion of context and the other two notions, however, is that we regard the context as a *speaker internal* representation of structures which may be real as well as fictional, whereas the coordinates approach and the propositions approach treat the context as a speaker external addition to the representation of reality constituted by the traditional model structure.

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FOOTNOTES

1. This paper is based in part on HAUSSER (1978a,b,1979a,b,c), where specific encroachments onto the territory of neighbouring components have been described and analyzed.
2. For a discussion see HAUSSER (1979b), section 2.
3. A grammar where the Fregean Principle is applied to the natural surface (taking words as the basic elements) is called a *surface compositional grammar*. The Surface Compositionality Constraint, formalized in HAUSSER (1978b), provides the principled standard for drawing the line between semantic and pragmatic aspects of meaning in natural language.
4. Cf. the discussion of a strictly intensional logic in HAUSSER (1979b).
5. For a reanalysis of the Donnellan example (concerning "The man with the Martini...") see HAUSSER (1979b), section 4.
6. An example is the analysis of propositional attitudes in HAUSSER (1979b), section 6.
7. See HAUSSER (1979a,1979c) for a criticism of traditional model-theoretic notions of context, as well as an alternative proposal (i.e. to treat context as a model-theoretic representation of what the speaker perceives

and remembers).

8. See HAUSSER (1978a), (1980b) for criticism of model-theoretic treatments of non-declarative sentence moods in terms of mood operators or underlying performative clauses, as well as an alternative proposal (i.e. to treat mood as a particular mode of syntactic composition which results in characteristic types of possible denotation).
9. See HAUSSER (1979b) for an account of the structure of the lexicon within the framework of our alternative model theoretic approach (i.e. within the SID).
10. This was shown in HAUSSER (1979b, section 4).

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ON QUESTIONS

by

Jaap Hoepelman

1. INTRODUCTION

Since M. and A. Prior's paper on 'Erotetic Logic' (PRIOR & PRIOR, 1955), the logico-linguistic literature on questions has proliferated so enormously, that one feels slightly embarrassed to add yet another proposal to the existing ones. Nevertheless it seems to me, that there are certain phenomena which a theory of questions should be able to handle, but which are not dealt with jointly by the existing theories. These phenomena can be summped up under the following headings:

1) Questions and argumentation
2) The role of negative questions
3) The function of the particles "yes" and "no" and their counterparts in other languages (and in older stages of English)
4) The scope of questions
5) The relation between interrogative pronouns and relative pronouns
6) The problem of asking equivalent questions
7) Questions, tautologies and contradictions.

Let us briefly consider each of these points.

2.

Ad 1. Questions can play a role in argumentation, just like imperatives can (RESCHER, 1966, p.5). There are logical relationships between questions. To cite ÅQVIST (1965): "Some questions seem to be logical consequences of others, two questions may be logically equivalent, a set of questions may be consistent, or inconsistent, and so on" (p.3). Connections like these also seem to hold between questions and assertions. Moreover, expressions which one can reasonably assume to be of sentence type can be built up of "assertive" parts and of "question" parts. To give a few examples: From the two questions

(1) Have you seen a picture of Picasso?

and

(2) Have you seen a picture of Dali?

the following seems to be a consequence (still in an intuitive sense of course):

(3) Have you seen a picture of Picasso and a picture of Dali?

But also the following seems to be a consequence:

(4) Have you seen a picture of Picasso or a picture of Dali?

The following assertion and question seem to be incompatible (when asserted and asked by the same person of course):

(5) No one has ever seen the abominable snowman.

(6) Has John seen the abominable snowman?

We must be a little bit on guard in this case. One can very well imagine a fragment like the following, spoken at a meeting of the Royal Geographic Society:

(8) No one has ever seen the abominable snowman!
Has Sir Edmund Hillary seen him? Has his sherpa seen him?
Has Reinhold Messner seen him? No! No one has seen - etc.

In this case the questions are used rhetorically or, as Åqvist would say, in a secondary way, and one would suppose that it is *because of* this incompatibility that they cannot be accepted as real questions. The speaker could as well have said:

(9) No one has seen the abominable snowman!
Sir Edmund Hillary hasn't seen him! His sherpa hasn't seen him!
Reinhold Messner hasn't seen him!, etc.

To give a few examples of combined assertion-question sentences:

(10) Hübner is a great chess-player all right, but can he stand the stress of the tournament?

(11) This pie doesn't taste good, you have forgotten the spinach or the tomatoes, or have you forgotten the garlic?

(12) Is the red button turned on? Then push the green button and run away fast!

(13) Did they pay him? He went right to the tavern and spent his wages down to the last dime (BOLINGER, 1978, p.101).

Ad. 2. Negative questions are an interesting and quite confusing phenomenon, that seems to have received too little attention in the literature. Compare the following questions and their answers:

(14) -Is two an even number?
-Yes./-Yes, two is an even number.
-No./-No, two is not an even number.

(15) -Isn't two an even number?
-Yes./-Yes, two is an even number.
-No./-No, two is not an even number.

Notice that one can give the same answers to the negative and to the positive question, and that these answers have the same force. One couldn't answer "Isn't two an even number?" in the following ways:

(16) *Yes, two is not an even number.

(17) *-No, two is an even number.

Nevertheless, one cannot say that

(14') Is two an even number?

and

(15') Isn't two an even number?

are the same question. A moment's reflection on the situations in which you would ask (14') or (15') makes this clear. You are likely to ask (14') when you hear somebody talk about the difference between even and odd numbers, a difference you have never heard of before. You didn't know that two is an even number. On the other hand, you will ask (15') in a situation like this: You have always thought that two is an even number. Now you hear a talk of a famous mathematician on the properties of numbers and something in what he says make you think that two might not be an even number after all. So you ask: "Isn't two an even number?" If you do not think that two is an even number, then it is inappropriate for you to ask "Isn't two an even number", because it would make other people think that you hold "Two is an even number" to be true. And if you think that two is an even number, then asking "Is two an even number" is inappropriate, because it would make people think that it is not the case that you hold "Two is an even number" to be true.

Ad. 3. The observations made above mean that we cannot consider "yes" and "no" as abbreviations for the assertion or the negation of the questioned sentence. We will see that some of the theories to be discussed lead to precisely the wrong results in this respect. Modern English has only "yes" and "no", but in French one has "oui", "si" and "non", in German "ja", "doch" and "nein", in Dutch "ja", "toch wel" and "nee". In sixteenth century English we have "yes", "no", "yea" and "nay" (cf. BÄUERLE, 1979). The use of these particles is sketched in the following scheme:

(18)

Question	Answer				
	modern English	old English	French	German	Dutch
Is two an even number?	yes/no	yea/nay	oui/non	ja/nein	ja/nee
Isn't two an even number?	yes/no	yes/no	si/non	doch/nein	{ja / toch wel}/nee

A theory of questions should explain the use of these particles.

Ad. 4. Mostly by means of stress one can indicate which element of the sentence it is that is actually questioned. E.g. (19) and (20) are two different questions:

(19) Did John meet Mary in the párk?

(20) Did John meet Máry in the park?

One can extend this set of examples simply by shifting the stress. A sentence like (19) can be paraphrazed like

(19') (I know that) John met Mary somewhere, but did he meet her in the park?

whereas (20) would give

(20') (I know that) John met someone in the park, but was it Mary whom he met?

A theory of questions should be able to deal with this phenomenon.

Ad. 5. In most of the languages I know, (some) relative pronouns and (some) interrogative pronouns have the same form. We may suppose that this is no coincidence. It may not be our first concern, and a theory of questions may be a very good one without explaining this sameness of form, but of two equally good theories I would prefer the one which, as an additional feature, does explain it.

Ad. 6.

(21) two plus two equals four

and

(22) three plus one equals four

are two logically equivalent sentences. Nevertheless, asking whether two plus two equals four is not asking whether three plus one equals four, i.e. (23) and (24) are not equivalent questions:

(23) Does two plus two equal four?

(24) Does three plus one equal four?

Any theory which treats them as being equivalent is in need of revision and/or amendment, I think.

Ad. 7. It makes no sense to ask a contradiction or a tautology. A question like

(25) *Does John work and not work?

is anomalous, and so is a question like

(26) *Does John work if he works?

A theory of questions should be able to sort out such deviant ones as (25) and (26). Of course (27) is perfectly o.k. as a question:

(27) Does John work or not?

This only means that (27) should not be analyzed as an inquiry after the truth of the tautology "John works or John doesn't work". On the other hand, one *can* ask a logical truth (which is not a tautology) or falsehood, as is witnessed by (28):

(28) Is two plus two five?

But again a question like (29)

(29) Is two plus two four or not?

should not be constructed as an inquiry after the truth of the tautology "two plus two is four or two plus two is not four".

3.

We will now discuss some representatives of the main tendencies in question theories, and see whether they can deal with the points made above. Our division of theories on questions is taken from BÄUERLE (1979). As is well known questions can be divided in three main categories, so called *wh-questions:*

(30) Who is the prime minister of the Netherlands?

yes-no questions:

(31) Is Amsterdam the capital of the Netherlands?

and *alternative questions:*

(32) Is Amsterdam or the Hague the capital of the Netherlands?

It is of course attractive to try to reduce these categories and there exist several proposals to do so, depending on the (formalized) conceptions of questions. (I closely follow Bäuerle hereafter.)

3.1.

The *propositional approach* is represented by such authors who claim that any question is to be identified with a list of sentences, possible answers, which are offered as choices, loosely speaking. E.g. HAMBLIN (1973) takes a question to denote an at least two membered set of propositions. Statements on the other hand denote one membered sets of propositions. A wh-question like

(33) Who walks?

represents the set containing the propositions denoted by "Mary walks", "John walks", ... (a possibly infinite list).

A yes-no question like

(34) Does Peter walk?

represents the set containing the pair of propositions denoted by "Peter walks" and its negation.

Hamblin doesn't pay attention to alternative questions, but we may, with Bäuerle, assume that an alternative question can be thought to represent a set containing the propositions which correspond to the alternatives. KARTTUNEN (1978) slightly modifies this approach in that he assumes a question to represent a set containing only the true answers to it. Karttunen's analysis is carried out within the framework of Montague grammar. We will take his paper as a representive of the propositional approach, and see how it copes with out problems. Karttunen does not treat direct questions, because he assumes that direct questions can be reduced to indirect ones by means of paraphrases like

(35) I ask you (to tell me) whether it is raining

for

(35') Is it raining?

and

(36) I ask you (to tell me) which book Mary read

for

(36') Which book did Mary read? (p.165).

Therefore, Karttunen concentrates on the semantics of indirect questions. It is perhaps not fair to make critical remarks in connection with these examples already, because they are not pursued in the paper and because one can safely assume that Karttunen would have noticed these problems if he had worked them out. Let us say, then, that the following remarks are preventive criticism. If "Is it raining?" is analyzed as "I ask you (to tell me) whether it is raining", then presumably

(37) Isn't it raining?

will be analyzed as:

(37') I ask you (to tell me) whether it is not raining.

One can satisfy this request in the case of (35) (I ask you (to tell me) whether it is raining) by the statement

(38) It is raining

and in the case of (37') by the statement

(39) It is not raining.

You couldn't answer (37') by the statement

(40) Yes, it is raining.

But the question-answer pair

(41) -Isn't it raining?
-Yes, it is raining

is perfectly normal.

Another problem, it seems to me, resides in the use of the phrase "I ask you...". It is to be expected that the semantics of the verb "ask" will be connected intimately with the semantics of direct questions, so analyzing direct questions as being built up from indirect questions plus "I ask you ..." takes for granted that which is to be explained. It will perhaps be better to drop the "I ask you ..." part and to take the imperative of "tell" instead:

(42) Tell me! whether it is raining.

We will return to the imperative later on.

The core of Karttunen's proposal is his PROTO-QUESTION RULE (p.174):

(43) PQ: If $\phi \in P_t$, then $\ulcorner ?\phi \urcorner \in P_Q$.
If ϕ translates to ϕ', then $\ulcorner ?\phi \urcorner$ translates to $\lambda p[{}^{\vee}p \wedge p = {}^{\wedge}\phi]$.

Here Q is the category of indirect questions, defined as t//t. The proto-question "?Mary cooks" is translated to $\lambda p[{}^{\vee}p \wedge p = {}^{\wedge}cook'_*(m)]$. This expression denotes a function from propositions to truth values, i.e. a set of propositions. If Mary cooks, then "?Mary cooks" denotes a set whose only member is the proposition that Mary cooks; if Mary doesn't cook, the empty set is denoted. Proto-questions are the building blocks out of which expressions of English are constructed, like "whether Mary cooks or John eats out", "whether Mary cooks or not" and "which girl cooks" by means of rules, of which we will cite the YES/NO-QUESTION RULE as an example:

(44) YNQ: If $\ulcorner ?\phi \urcorner \in P_Q$, then $\ulcorner$whether $\phi\urcorner$, $\ulcorner$whether or not $\phi\urcorner$ and $\ulcorner$whether ϕ or not$\urcorner \in P_Q$.

If $\ulcorner ?\phi \urcorner$ translates to ψ', then $\ulcorner$whether $\phi\urcorner$, $\ulcorner$whether or not $\phi\urcorner$ and $\ulcorner$whether ϕ or not$\urcorner$ translate to $\lambda p[\psi'(p) \vee [\neg\exists q[\psi'(q)] \wedge p = {}^\wedge\neg\exists q[\psi'(q)]]]$.

"Whether Mary cooks", "whether or not Mary cooks" and "whether Mary cooks or not" are all translated to a formula that turns out to be equivalent to

(45) $\lambda p[{}^\vee p \wedge [p = {}^\wedge cook'_*(m) \vee p = {}^\wedge\neg cook'_*(m)]]$.

These fragments suffice to bring forward my main criticisms against Karttunen's proposal. In the first place, there is nothing which prevents a tautology or a contradiction to be asked in a yes/no-question. Secondly, (this has also been pointed out by HIGGINBOTHAM & MAY, 1978 p.21), if the sentence ϕ occurs in the scope of "?" in a proto-question, then the set denoted by the proto-question will contain all propositions which are logically equivalent to ϕ. In the case of a question like (23) (Does two plus two equal four?) this would lead to the undesirable result that one could answer with "Yes. Five plus five equals ten". Thirdly, and we mentioned this point before, negative questions do not come out right. It is easily seen (by substituting a negated sentence for ϕ in the yes/no-question rule) that negative sentences will be treated in exactly the same way as positive ones, so that we will be led to an analysis of (37) (Isn't it raining?) as (37') (I ask you to tell me whether it is not raining), which, as I hope to have made plausible, is wrong.

In the fourth place - but this is an objection that can also be directed against other theories which try to explain direct questions via indirect ones - in certain cases sentences which combine an assertive and a question part lead to ugly results if one replaces the question by its proposed paraphrase. E.g. (10) (Hübner is a great chess-player all right, but can he stand the stress of the tournament?) will lead to

(46) Hübner is a great chess-player all right, but I ask you to tell me whether he can stand the stress of the tournament

and it is easy to imagine a situation in which you utter (10) without it being the case that you require someone to tell you such a thing. Whether Hübner can stand the stress of the tournament is something "the future will learn" and it may very well be the case that you utter (10) knowing that there is no one around to whom you could sensibly direct a request like the one contained in (46). The point is even stronger in the case of (11) (This pie doesn't taste good, you have forgotten the spinach or the tomatoes, or have you forgotten the garlic?) which would lead to

(47) This pie doesn't taste good, you have forgotten the spinach or the tomatoes, or I ask you to tell me whether you have forgotten the garlic.

In all likeness the addressee will not be able to tell you what he or she has forgotten.

The points 3) (the function of "yes" and "no") and 4) (the scope of questions) are not dealt with by Karttunen, but in the case of 3) we fear that an eventual theory might lead to problems because of the difficulties with negative questions mentioned above. The scope of questions is quite another problem. It is not dealt with in any of the theories I have seen. We shall try to sketch a solution for it at the end of this paper.

3.2.

The *categorial approach* supposes that questions are to be considered as functions from categorial answers to propositions. Representatives of this line of thought are e.g. COHEN (1929) and EGLI (1976). A question like

(48) Who comes?

can be analyzed as

(49) $\lambda x_{NP}[x_{NP}$ comes$]$

and a question like

(50) When does he come?

like

(51) $\lambda x_{ADV_t}(x_{ADV_t}$ (he comes))

(BÄUERLE, 1979 p.64). Notice that in this approach it will be difficult to account for sentences which are built up from an assertive and a question part, like (11). EGLI (1976) proposes the following reduction of yes/no-questions to categorial questions: Any assertion is assumed to consist of two parts, *modus* and *dictum*. The *modi* are "yes" and "no", and the structure of

(52) Will he come?

is supposed to be

(53) $\lambda x_M[x_M$ (he will come)]

(BÃUERLE, 1979 p.65). Thus yes/no-questions are reduced to categorial ones.

Again, there seems to be nothing which prevents asking tautologies or contradictions in this approach (or at least to mark these questions in some way). And again, negative questions are not treated in the right way. If, as Bäuerle says, Egli considers "no" to be equivalent to "it is not the case that", then answering the negative question

(54) Isn't John ill?

with "no" will amount to the assertion that John is ill. But, as we have pointed out before, a dialogue like

(55) -Isn't John ill?
-No, John is ill

is deviant.

The same objection can be directed against the analysis in HAUSSER (1977) of yes/no-questions (as rendered by BÃUERLE, 1979 p.65). Hausser's analysis of the question

(56) Did John leave?

is carried out in Montague grammar and has the following form:

(57) $^\wedge\lambda y_n[y_n(^\wedge leave'(^\wedge j)] \wedge [(y_n = \lambda p[^\vee p]] \wedge [y_n = \lambda p[^\vee\neg p]]]$,

where y_n represents the modus variable "it is the case that" or "it is not the case that". Taking the negative

(58) Didn't John leave?

instead of (56), then substituting "yes" or "no" for y_n, one easily sees that according to this theory the answers to (58) will be

(59) Yes, John didn't leave

and

(60) No, John left

which is wrong.

3.3.

As a third approach I would like to mention the epistemic-imperative theory of ÅQVIST (1965) and HINTIKKA (1974). In Åqvist's theory a question like

(61) Is Linguistic Philosophy still alive?

is analyzed as

(62) Bring it about that either I know that Linguistic Philosophy is still alive or I know that Linguistic Philosophy is not alive any longer

and

(63) Which is the smallest prime greater than 500?

is analyzed as

(64) Bring it about that there is an object of which I know that it is the smallest prime greater than 500

(ÅQVIST, 1965 p.4).

We will not go into the semantics for the imperative and for "know" presented by Åqvist, but just give a few of his formulae which are easily understood intuitively. This will be enough for our purposes. Let

(65) Brutus killed Caesar

be represented by p, and

(66) Cassius killed Caesar

by q. Then the question

(67) Did Brutus kill Caesar or did Cassius?

will be analyzed as

(68) $!(Kp \vee Kq)$

i.e. "Bring it about that I know that p or that I know that q". This leads to the definition of an operator $?_2(p,q)$ as follows:

(69) $?_2(p,q) =_{def.} !(Kp \vee Kq)$

and in general

(70) $?_n(p_1,p_2,\ldots,p_n) =_{def.} !(Kp_1 \vee Kp_2 \vee \ldots \vee Kp_n)$

for any number n. Single yes/no-questions are made by means of $?_1$:

(71) $?_1p = ?_2(p, \neg p)$.

But now we see immediately that in Åqvist's theory too negative questions will be treated in the wrong way - no difference can be made between (65) and (72).

(72) Didn't Brutus kill Caesar?

And a question like

(73) Doesn't two plus two equal four?

certainly does not mean

(74) Bring it about that I know that two plus two is not four.

As may be seen in (70), Åqvist introduces a multiplicity of question operators already when dealing with questions on the propositional level. This trait is reinforced when question operators are combined with predicate logic (containing imperative and epistemic operators). Here an enormous proliferation of quantifiers is needed to deal with possible readings of interrogatives (see e.g. the tableau in ÅQVIST, 1965, p.128). I think that Åqvist is right in the way he distinguishes these readings. But if a theory can be found which deals with them in a formally less complicated way, it is to be favoured.) There are other problems connected with the epistemic-imperative theory of questions. Some of these have been pointed out and partially solved by HINTIKKA (1974, p.4). Yet his solution is not convincing everywhere and leads to problems of its own. Limitations of space prevent us from going into this here.

Arguing on a more general level, one may ask whether it is not desirable to try to find a logic of questions which does not consist in a reduction to imperative and epistemic notions. There surely are connections between imperatives and questions. E.g. a request can be formulated as an order or as a question:

(75) Get me a glass of water!

(76) Will you get me a glass of water?

(77) Get me a glass of water, please. (Question?, Order?)

(78) Will you get me a glass of water, please?

With some effort one could try to reduce imperatives to questions as well as the other way around - although I doubt whether a complete reduction will be possible. So one could analyze (75) as

(79) You know what is going to happen to you if you don't get met a glass of water, don't you? It won't happen if you get me a glass of water.

Perhaps a better understanding of the connection between imperatives and questions can be achieved if we first try to develop a logic of imperatives and a logic of questions independently from one another. I will sketch a proposal for the latter in the next sections.

4.

As may already be guessed from the foregoing I favour a treatment of all questions as expressions of sentence type. With Åqvist I think that it may be profitable to assign truth values to questions, which would e.g. simplify the treatment of consequences of questions and the valuation of combined assertive-question sentences. Under certain circumstances a question can be considered to be a "true question", in others as "false" or, if you don't like these expressions, as "correct" and "incorrect" (ÅQVIST, 1965 p.26). This can be the case (for a certain individual) without there being anyone around to whom the order "relieve me from my ignorance concerning ϕ" can be directed. Sticking to the level of propositional logic, I want to give a logic for the propositional operator "?" which can be read as "It is the question whether" (German: "Es ist die Frage ob", Dutch: "Het is de vraag of"). For a sentence p, when is it the question whether p? Certainly not when p is true or false. One would rather say that it is the question whether p, when the truth value of p itself is undetermined. Thus we might be led to an interpretation of "?" in some system of many valued logic, having intermediate truth values, representing indeterminacy. I have tried this for Łukasiewicz system of three-valued logic (as proposed in ŁUKASIEWICZ 1920, see RESCHER 1969) as well as for its four-valued extension, but both systems do not lead to wholly satisfactory results. My interest for a four-valued system arose from the consideration of negative questions. It is true that you are uncertain about "John is ill" if you ask either

(80) Is John ill?

or

(81) Isn't John ill?

But if you ask (81) you don't desire an answer in the negative any more than if you ask (80). On the contrary: someone who asks (81) expects a positive answer. We might say that there are two kinds of uncertainty here, which are "directed" differently. Let us therefore consider a four-valued logic which might do justice to this intuition.

If we have two many-valued logics, C_n and C_m, respectively, the *product system* $C_n \times C_m$ can be given in the following way (I follow RESCHER 1969, p.97 on these pages):

(82)

a. The truth values of system $C_n \times C_m$ are to be ordered pairs (v_1, v_2) of truth values, the first of which, v_1, is a truth value of C_n and the second of which, v_2, is one of C_m.

b. The truth value of a proposition is to be (v_1, v_2) in $C_n \times C_m$ iff its truth value is v_1 in C_n and v_2 in C_m.

c. Correspondingly, negation ("$\neg$") and arbitrary binary logical connectives ("$\circ$") are to be so specified for $C_n \times C_m$ that their truth tables are governed by the rules:

$$\neg(v_1, v_2) = (\neg v_1, \neg v_2) \quad \text{and} \quad (v_1, v_2) \circ (v_3, v_4) = (v_1 \circ v_3, v_2 \circ v_4).$$

We will now turn our attention to the particular four-valued product system $C_2 \times C_2$, which is the product of the classical two-valued propositional calculus with itself. The tables for the connectives in $C_2 \times C_2$ are as follows:

p	$\neg$p		$p \wedge q$				$p \vee q$				$p \rightarrow q$				$p \leftrightarrow q$			
		p \ q	11	10	01	00	11	10	01	00	11	10	01	00	11	10	01	00
11	00	11	11	10	01	00	11	11	11	11	11	10	01	00	11	10	01	00
10	01	10	10	10	00	00	11	10	11	10	11	11	01	01	10	11	00	01
01	10	01	01	00	01	00	11	11	01	01	11	10	11	10	01	00	11	10
00	11	00	00	00	00	00	11	10	01	00	11	11	11	11	00	01	10	11

To this we add the following table for "?" (the intuitive reason for doing it this way will become apparent later):

(84)

p	?p	(and so: $? \neg p$
11	00	00
10	00	10
01	10	00
00	00	00)

We will now give a few examples of interesting valid and non-valid formulae containing "?" in this system. Valid are e.g.:

(85)
$\neg ?(p \vee \neg p)$
$\neg ?(p \wedge \neg p)$
$\neg ? \ldots ?p$ (for any number of ?'s)
$? \neg p \rightarrow \neg ?p$
$?p \rightarrow ? \neg ? \neg \ldots \neg ?p$ (any number of $? \neg ?$'s)
$?(p \rightarrow q) \rightarrow (?p \rightarrow ?q)$
$?(p \vee q) \rightarrow (?p \vee ?q)$
$(?p \wedge ?q) \rightarrow ?(p \wedge q)$
$(?(p \rightarrow q) \wedge p) \rightarrow ?q$

The reader can easily verify this for himself.

A few interesting non-valid formulae are:

86)
$?p \rightarrow ? \neg p$
$? \neg p \rightarrow ?p$
$(p \leftrightarrow q) \rightarrow (?p \leftrightarrow ?q)$

That $p \leftrightarrow q$ does not imply that $?p \leftrightarrow ?q$ is of course particularly satisfying in the light of our discussion of equivalent questions. We see that the interpretation of "?" in $C_2 \times C_2$ has certain desirable properties: It is e.g. valid that it is not the question whether $p \vee \neg p$, and likewise for $p \wedge \neg p$. $[?(p \rightarrow q) \wedge p] \rightarrow ?q$ is a formula which we will need when discussing the scope of questions.

We will now say that a positive yes/no-question, like "Is John ill?" is of the form ?p, a negative yes/no-question like "Isn't John ill?" is of the form $? \neg p$ and a yes/no-question like "Is John ill or not?" is of the form $?p \vee ? \neg p$. In this last case we should be careful. It has been maintained e.g. by BÄUERLE (1979, p.62) that alternative questions like

(87) Did you meet my brother or my sister?

cannot be answered by "yes" or "no". I think that this claim is only partially right. A sentence like

(88) Have you ever seen a film of Keaton or Chaplin?

can perfectly well be answered by "yes" or "no". Also the following dialogue seems to be acceptable:

(89) -Here is John. Look at the red spots on his face!
Is he ill or not?
-Yes. He probably got the measles./-No. That's marmelade.

Not in all cases answering questions like (87) with a "yes" or a "no" is acceptable. There seems to be an ambiguity here. If you ask (87), may be all you want to know is whether the addressee saw either one of your brother or sister (for then it might e.g. be the case that you know that your father is safe). Here a single "yes" is justified as an answer. Another possibility, however, is that you want to know which one of your brother and sister was seen by the addressee. Now a "yes" alone will not do. We may explain this ambiguity by assuming that (87) can formally be expressed in two ways:

(90) ?(you saw my brother-or-sister)

(91) ?(you saw my brother) or ?(you saw my sister).

The case of sentences like "Is John ill or not" is a little bit different. Although, as we have seen, a "yes" can be acceptable as an answer to a question of the form "ϕ or not", most likely, a father who asks his son

(92) -Have you passed this exam or not?

will not be content with a "yes" alone. This, I feel, is even more the case when instead of (92) the following had been asked:

(93) -Have you passed the exam or haven't you?

At the end of this paper a solution for this problem will be suggested which will depend on the respective scopes of "not" and "?".

There is another way to look at the product logic $C_2 \times C_2$ (again I follow RESCHER (1969, p.113)). Let w_1 and w_2 be two alternatives (possible worlds). To any proposition we will assign the truth value 1,2,3,4 according as it is

(94) a. true in w_1 and in w_2
b. true in w_1 but not in w_2
c. false in w_1 but true in w_2
d. false in both w_1 and w_2.

For negation and conjunction we will get the following truth tables:

(95)

p	¬p
1	4
2	3
3	2
4	1

p ∧ q

p \ q	1	2	3	4
1	1	2	3	4
2	2	2	4	4
3	3	4	3	4
4	4	4	4	4

If you compare these tables with those given for $C_2 \times C_2$, replacing 11 by 1, 10 by 2, 01 by 3 and 00 by 4, you will find out that they are exactly the same, and this holds for the other connectives as well. We may now intuitively reinterpret our calculus of questions, by saying that a person's certainty rests on the comparison of two alternatives or worlds, of which the second has greater "authority" (in an epistemological sense) than the first. The first world may be thought of as the world as it might be for all the questioner knows, the second one as the world of the authority he is addressing himself to (a person, perhaps "Nature", God or what have you) *as the questioner conceives of it*. This is the reason why I have assigned 10 to "?p" in case 01 is assigned to "p". If a proposition p might not be true in "your" world, but seems to you to be true in the world of your "epistemological authority" then *for you* it is appropriate to ask "?p", but you will not think that for your "epistemological authority" it is the question whether p.

5.

We have seen that we cannot consider "yes" and "no" as abbreviations for "it is the case that -" and "it is not the case that-" followed by the sentence that has been asked. I agree with BÄUERLE (1979, p.68-69) that "yes" and "no" are not to be taken as answers, but as "discourse elements that relate the answer to the question in some way or other". Let us see how we can make this more precise in the theory of questions proposed above.

If the question

(96) Is four an even number?

is addressed to someone, then, assuming that the person who asks it is honest, the addressee will know that the truth value of

(97) Four is an even number

for the person who asks (96) is 01. Let us say that the person who asks (96) suffers from an *uncertainty of the second kind*. The addressee can try to remedy this uncertainty by answering

(98) Yes./Yes, four is an even number

or

(99) No./No, four is not an even number.

By (98) he indicates that the person who asks, should entertain a *certainty of the first kind* about "Four is an even number", i.e. that he can hold its truth value to be 11. By (99) the addressee indicates that the person who asked (96) should entertain a *certainty of the second kind* about "Four is an even number", i.e. he should hold its truth value to be 00. To put it briefly, "yes" indicates the following change: 01 ⇒ 11, and "no": 01 ⇒ 00.

Now consider negative questions.

(100) Isn't four an even number?

can be answered by

(101) Yes./Yes, four is an even number.

(102) No./No, four is not an even number.

Hearing (100) the addressee knows that the person who asks suffers from an *uncertainty of the first kind*, i.e. that for him the truth value of "Four isn't an even number" is 01, and therefore the truth value of "Four is an even number" 10. The addressee's answers "yes" and "no" are the same and indicate the same as in the case of the positive question: They try to change the uncertainty of the first kind into a certainty of the first kind (truth value 11) or a certainty of the second kind (truth value 00). Briefly: "yes" 10 ⇒ 11 and "no" 10 ⇒ 00. Thus we see that "yes" and "no" in modern English are ambiguous in a way. Sixteenth century English, having "yes", "no", "yea" and "nay" would give the following picture:

(103) "yea" 01 ⇒ 11
"nay" 01 ⇒ 00
"yes" 10 ⇒ 11
"no" 10 ⇒ 00

We can give the following scheme for English, French, German and Dutch:

(104)

	English (mod.)				(16th C.)			
	yes	no	yes	no	yea	nay	yes	no
01	11	00			11	00		
10			11	00			11	00

	French				German				Dutch			
	oui	non	si	non	ja	nein	doch	nein	ja	nee	ja toch wel	nee
01	11	00			11	00			11	00		
10			11	00			11	00			11	00

Now we can also explain why the following answers to (96) and (100), respectively, are incorrect:

(105) *Yes, four is not an even number

*No, four is an even number.

"Yes" would indicate that the truth value to be entertained by the person who asks either (96) or (100) is to be 11, whereas asserting "four is not an even number" indicates that it has to be 00. Conversely, "no" indicates that the truth value of the questioned sentences should be 00, whereas asserting "two is an even number" indicates the truth value 11.

6.

Armed with our new machinery, let us now turn to wh-questions and see what we can do with them. It is clear that in addition we will need the language of predicate logic. Let us assume that putting "?" in front of any expression of sentence type, results in a new expression of sentence type. Let us also assume that we have two models M_1 and M_2,

(106) $M_1 = (D_1, F_1)$

$M_2 = (D_2, F_2)$,

where D_1 and D_2 are domains of individuals, $D_1 \subseteq D_2$, and F_1, F_2 are assignments to the constants, $F_1 \subseteq F_2$. In M_1 values are assigned to the individual variables by the functions $g_1, g_1', \ldots$, in M_2 by the functions $g_2, g_2', \ldots$.

We make the product system $PC_2 \times PC_2$ according to the following rules:

(107) a. The truth values of $PC_2 \times PC_2$ are to be ordered pairs of truth values (v_1, v_2), the first of which, v_1, is a truth value of PC_2 in M_1 according to the assignment g_1, the second of which, v_2, is a truth value of PC_2 in M_2 according to the assignment g_2.

b. The truth value of an expression of sentence type ϕ under the assignment $g_{1,2}$ is to be (v_1, v_2) in $PC_2 \times PC_2$ if its truth value in M_1 is v_1 under g_1, and v_2 in M_2 under g_2.

c. For expressions of sentence type the connectives are specified according to the following rules:

$$\neg(\phi_{M_1,g_1}, \phi_{M_2,g_2}) = (\neg\phi_{M_1,g_1}, \neg\phi_{M_2,g_2})$$

$$(\phi_{M_1,g_1}, \phi_{M_2,g_2}) \circ (\psi_{M_1,g_1}, \psi_{M_2,g_2}) =$$

$$= ((\phi_{M_1,g_1} \circ \psi_{M_1,g_1}), (\phi_{M_2,g_2} \circ \psi_{M_2,g_2})),$$

where ϕ_{M_1,g_1} is the value assigned to ϕ in M_1 under g_1.

For "?" we have the following rule:

d. (i) Let ϕ be a closed formula without individual constants. Then $?\phi_{g_{1,2}} = 10$ iff $\phi_{g_{1,2}} = 01$. Otherwise $?\phi_{g_{1,2}} = 00$.

(ii) Let $\phi(x_1, \ldots, x_n)(a_1, \ldots, a_m)$ be a formula containing the n free variables $x_1, \ldots, x_n$ and the m individual constants $a_1, \ldots, a_m$ and no others (m or n may equal 0). Then
$?\phi(x_1, \ldots, x_n)(a_1, \ldots, a_m)_{g_{1,2}} = 10$ if
$\phi(x_1, \ldots, x_n)(a_1, \ldots, a_m) = 01$ <u>and</u>
$g_2(x_1), \ldots, g_2(x_n), F_2(a_1), \ldots, F_2(a_m) \in D_1$.
Otherwise $?\phi(x_1, \ldots, x_n)(a_1, \ldots, a_m)_{g_{1,2}} = 00$.

We order the truth values as follows: $00 < 01 < 10 < 11$, and define $\forall x$ and $\exists x$.

e. $\forall x\ \phi_{g_{1,2}} = \min[g'_{1,2}(\phi_{g'_{1,2}})]$, $\exists x\ \phi_{g_{1,2}} = \max[g'_{1,2}(\phi_{g'_{1,2}})]$, where $\min[g'_{1,2}(\phi_{g'_{1,2}})]$ is the minimum truth value of ϕ for all $g'_{1,2}$ which are like $g_{1,2}$ apart possibly from the assignment to x, and likewise for $\max[g'_{1,2}(\phi_{g'_{1,2}})]$.

If we now assume that $D_1 = D_2$, then formulae of the following form will be valid:

(108) $\forall x?\phi \rightarrow ?\forall x\phi$

$\forall x?\phi \rightarrow ?\exists x\phi$

$?\exists x\phi \rightarrow \exists x?\phi$

The last of these formulae isn't very intuitive, but it doesn't hold when we assume that $D_1 \subset D_2$. The converse formulae of the above do not hold if $D_1 = D_2$, nor if $D_1 \subset D_2$.

If we add "=" to our set of predicate constants then (among others) the following will hold:

(109) $\forall x\forall y\forall z[x=y \wedge ?(y=z) \rightarrow ?(x=z)]$

which is an instance of

(110) $\forall x\forall y[x=y \wedge ?P(y) \rightarrow ?P(x)]$.

The following do <u>not</u> hold (which is as it intuitively should be):

(111) $\forall x\forall y[?(x=y) \wedge P(x) \rightarrow ?P(y)]$

(112) $\forall x\forall y[?(x=y) \wedge ?P(x) \rightarrow ?P(y)]$.

7.

Now I propose to give a wh-question like

(113) Which men work?

the following intuitive reading:

(114) For every man x, is x a man which works?

In Montague grammar we could achieve a formal rendering of (113) in the following way:

first we form *man which works,* which is translated to $\lambda x[man'_*(^{\vee}x) \wedge work'_*(^{\vee}x)]$, then we form *is he$_n$ a man which works?,* which is translated to $?\exists y[man'_*(^{\vee}y) \wedge work'_*(^{\vee}y) \wedge {}^{\vee}y = {}^{\vee}x_n]$; then, from the expressions *every man* and *is he$_n$ a man which works?* we form, by a rule of quantification, the expression *Which men work?*, which is translated to

(115) $$\forall x[\mathrm{man}'_*({}^\vee x) \rightarrow ?\exists y\ \mathrm{man}'_*({}^\vee y) \wedge \mathrm{work}'_*({}^\vee y) \wedge {}^\vee y = {}^\vee x]].$$

Likewise,

(116) Which man works?

will be read as

(117) There is a man x: is x a man which works?

And by the same steps as those sketched above, we would arrive at the translation

(118) $$\exists x[\mathrm{man}'_*({}^\vee x) \wedge ?\exists y[\mathrm{man}'_*({}^\vee y) \wedge \mathrm{work}'_*({}^\vee y) \wedge {}^\vee y = {}^\vee x]].$$

The sentence

(119) Who works?

seems to be ambiguous, and can be treated as

(119') Which persons work?

and

(119") Which person works?

respectively, leading to the translations

(120) $$\forall x[\mathrm{person}'_*({}^\vee x) \rightarrow ?\exists y(\mathrm{person}'_*({}^\vee y) \wedge \mathrm{work}'({}^\vee y) \wedge {}^\vee y = {}^\vee x)]$$

and

(121) $$\exists x[\mathrm{person}'_*({}^\vee x) \wedge ?\exists y(\mathrm{person}'_*({}^\vee y) \wedge \mathrm{work}'_*({}^\vee y) \wedge {}^\vee y = {}^\vee x)].$$

We see that a question of the form

(122) Where ϕ?

could also be rendered in two ways:

(123) $$\forall x[\mathrm{place}'_*({}^\vee x) \rightarrow ?\exists y(\mathrm{place}'_*({}^\vee y) \wedge \phi({}^\vee y) \wedge {}^\vee y = {}^\vee x)]$$

and

(124) $$\exists x[\mathrm{place}'_*({}^\vee x) \wedge ?\exists y[\mathrm{place}'_*({}^\vee y) \wedge \phi({}^\vee y) \wedge {}^\vee y = {}^\vee x]].$$

I think that this corresponds to a real ambiguity, the difference between what Åqvist calls "monadic complete list what-questions" and "monadic at-least-what questions" (ÅQVIST, 1965 pp.85-91).

Multiple questions seem to pose no particular problems in this framework (see e.g. KARTTUNEN & PETERS, 1979). E.g.

(125) Which student does each professor recommend?

(KARTTUNEN & PETERS, 1979 p.1) can be arrived at in the following way: first we form *he*$_1$ *recommends he*$_2$, then we form *student which he*$_1$ *recommends*, then *a student which he*$_1$ *recommends,* followed by *is he*$_3$ *a student which he*$_2$ *recommends*?, the translation of which will be

(126) $$?\exists y[\text{student}'_*({}^{\vee}y) \wedge \text{recommend}'_*({}^{\vee}x_1, {}^{\vee}y) \wedge {}^{\vee}y = {}^{\vee}x_3].$$

Out of (126) and *a student* we then make *which student does he*$_1$ *recommend?* by a rule of quantification, and translate it to

(127) $$\exists x[\text{student}'_*({}^{\vee}x) \wedge ?\exists y[\text{student}'_*({}^{\vee}y) \wedge \text{recommend}'_*({}^{\vee}x_1, {}^{\vee}y) \wedge {}^{\vee}y = {}^{\vee}x]].$$

Finally, again by a rule of quantification, from *each professor* and *which student does he*$_1$ *recommend?* we form

(128) Which student does each professor recommend?

which has the following translation:

(129) $$\forall z[\text{professor}'_*({}^{\vee}z) \rightarrow [\exists x[\text{student}'_*({}^{\vee}x) \wedge ?\exists y[\text{student}'_*({}^{\vee}y) \wedge \text{recommend}'_*({}^{\vee}z, {}^{\vee}y) \wedge {}^{\vee}y = {}^{\vee}x]]]].$$

It is easy to see how one can arrive at the other possible reading of (125), namely by first pushing *each professor* into *student which he*$_1$ *recommends*. As a last example we directly give the translation of

(130) Which professor recommends which student for the job?

(131) $$\exists z[\text{professor}'_*({}^{\vee}z) \wedge ?\exists v[\text{professor}'_*({}^{\vee}v) \wedge \exists x[\text{student}'_*({}^{\vee}x) \wedge ?\exists y\ \text{student}'_*({}^{\vee}y) \wedge \text{recommend}'_*({}^{\vee}v, {}^{\vee}y) \wedge {}^{\vee}y = {}^{\vee}x]] \wedge {}^{\vee}z = {}^{\vee}v]].$$

I will conclude this chapter by putting forward a few suggestions on the structure of sentences containing "know wh-". In the first place we notice that the English verb "know" can be used in (at least) two different ways, as examplified in

(132) John knows Peter

and

(133) John knows that Peter comes.

Dutch and German have two different verbs for these uses, namely "kennen" and "weten (D.)/ wissen (G.)". I think that

(134) John knows who comes

in which *know* is used in the first sense, can be read as:

(135) There is someone who comes and John knows him.

It is easy to see how (134) could be dealt with in Montague grammar: We form *a person who comes* and *John knows* him_n and by a rule of quantification we get *John knows who comes* which is translated to

(136) $\exists x[\text{person}'_*({}^{\vee}x) \wedge \text{come}'_*({}^{\vee}x) \wedge \text{know}'_*(j, {}^{\vee}x)]$.

Sentence (134) in its plural reading will be formed analogously and translates to

(137) $\forall x[\text{person}'_*({}^{\vee}x) \wedge \text{come}'_*({}^{\vee}x) \rightarrow \text{know}'_*(j,x)]$.

If *know* is used in the second sense, then I think that (134) has several readings. (134) may be read as

(138) There is a person who comes and John knows that he comes

or as

(139) For a person x, John knows whether x comes or not.

Again, it is easy to see how we can form (138) in Montague grammar: We make *a person who comes* and *John knows that* he_n *comes* and by a rule of quantification we get *John knows who comes,* translated to

(140) $\exists x[\text{person}'_*({}^{\vee}x) \wedge \text{come}'_*({}^{\vee}x) \wedge \text{know}'_*(j, {}^{\wedge}\text{come}'_*({}^{\vee}x))]$

and analoguously for the plural reading.

I suppose that (139) can be dealt with along the following lines: From *John knows that* he_n *comes* and *John knows that* he_n *doesn't come* we form *John knows whether* he_n *comes or not*. From this and *a person* we form *John knows who comes,* which is translated to

(141) $\exists x[\text{person}'_*({}^{\vee}x) \wedge \text{know}'_*(j, {}^{\wedge}\text{come}'_*({}^{\vee}x)) \vee \text{know}'_*(j, {}^{\wedge}\neg\text{come}'_*({}^{\vee}x))]$.

Finally, I would like to point out that there might still be another reading for (134) namely

(142) For a certain person of whom it is the question whether he comes, John knows whether he comes or not.

It is clear how we could deal with the part of this sentence following the comma. The part preceding it can be handled in the following way: we make *he_m comes?* and out of this and *person* we make *person who comes?* Then we make *a person who comes?* and out of this and *John knows whether he_n comes or not* we make *John knows who comes*, which is translated as

(143) $$\exists x[\text{person}'_*({}^{\vee}x) \wedge ?(\text{come}'_*({}^{\vee}x)) \wedge \text{know}'_*(j, {}^{\wedge}\text{come}'({}^{\vee}x)) \vee \text{know}'_*(j, {}^{\wedge}\neg\text{come}'({}^{\vee}x))].$$

The plural readings for (134) are treated accordingly.

8.

Finally I will make a few remarks on the scope of questions. We have seen that questions like

(144) Does Jóhn kiss Mary in the park?

and

(145) Does John kíss Mary in the park?

are two different questions. Intuitively it is clear what this difference consists in: in (144) the speaker is uncertain about the person who kisses Mary, in (145) the speaker is uncertain about what John does to Mary in the park.

In an earlier paper I have proposed a treatment for denial in Montague grammar (HOEPELMAN, 1980), which I will extend to questions. Compare the following sentences:

(146) Jóhn doesn't kiss Mary in the park.

(147) John doesn't kíss Mary in the park.

Here, too, the difference is intuitively clear. In (146) the speaker accepts that someone kisses Mary, but he denies that it is John who does the kissing. In (147) the speaker accepts that John does something to Mary in the park, but he denies that it is kissing. In HOEPELMAN (1980) I introduce a dummy element for each basic category of the Montague grammar, which in the case of terms could be read as "something" (or "someone" if terms are split up in *human* and *non-human*.) Let us denote the dummy of category a by O_a. Then (146) is analyzed as

(148) $\neg\exists x\forall y[park'_*(^\vee y) \rightarrow {}^\vee x = {}^\vee y \wedge in'(^\vee x)(^\wedge\lambda z[date'_*(^\vee z,m)])(^\wedge j) \wedge$
$\wedge\ O'_T(in'(^\wedge\lambda P\exists x\forall y[park'_*(^\vee y) \rightarrow {}^\vee x = {}^\vee y \wedge P\{x\}]))(^\wedge\lambda z[date'_*(^\vee z,m)],$

where O'_T is the translation of the dummy of category T.

Generally speaking, if $\phi(O'_a,d_a)'$ is obtained from ϕ' by replacing the element d_a of type a by the translation O'_a of a dummy element with the same type, then the proposed translation for a sentence ϕ in which d is the denied element, will be

(149) $(\phi(O_a,d_a))' \wedge \neg(\phi')$.

As this translation gives certain undesirable results when we try to handle the scopes of adverbs like "necessarily" (in e.g. "necessarily John kisses Mary in the park"), Gabbay (personal communication) proposed to replace (149) by (150):

(150) $[\phi(O_a,d_a)]' \wedge \S([\phi(O_a,d_a)]' \rightarrow \phi'),$

where § is the adverb in question. It is easily seen that (150) is equivalent to (149) in case $\S = \neg$.

If we now apply the same technique to question sentences, then (144) can be given the form:

(151) $[\phi(O_T,John)]' \wedge ?([\phi(O_T,John)]' \rightarrow \phi'),$

which, by the formula $p \wedge ?(p\rightarrow q) \rightarrow ?q$ is seen to imply

(152) $[\phi(O_T,John)]' \wedge ?\phi.$

Let us now return to "-or not?" questions, as announced at the end of Chapter 5. Suppose that you want to prove to someone that four is an even number. After long and complicated calculations you look him in the face triumphantly and ask

(153) Is four an even number or not?

"Yes", he has to admit. But if you had asked

(154) Is four an even or an odd number?

the answer could have been neither "yes" nor "no". As we have seen, an "-or not?" question sometimes (perhaps most of the time) behaves like (154) in this respect. My hypothesis is, that in such cases the *not* is the denial of a specific element of the sentence.

To take a simple example, I suppose that in such cases

(155) Is John ill or not?

has the following form:

(156) $?[ill(John)] \vee ?[[\phi(O_{adj},ill)] \wedge \neg[ill(John)]]$,

which clearly is not of the form $?p \vee ?\neg p$.

FOOTNOTES

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COMPOSITIONAL SEMANTICS AND RELATIVE CLAUSE FORMATION IN MONTAGUE GRAMMAR

by

Theo M.V. Janssen

0. INTRODUCTION

The principle of compositionality (or the Fregean principle) reads as follows:

The meaning of a compound expression is built up from the meanings of its constituent parts.

This principle is a fundamental principle for Montague grammar. It implies that for each construction step in the syntax, there has to be a corresponding semantic step. Formulated in the algebraic terminology of 'Universal Grammar' (MONTAGUE 1970), the principle says that the syntax and semantics are algebras, and that the meaning assignment is a homomorphism relating these two algebras. We now may ask the question to what extend this organization of the grammar restricts the options we have in the syntax to describe a particular phenomenon.

PARTEE (1973) raised this question with respect to relative clause constructions, and her answer was that we should use the CN-S analysis. She concluded that the framework puts very strong constraints on the syntax, with the consequence that 'it is a serious open question whether natural languages can be so described' (PARTEE 1973, p.55). Her argumentation is used in CHOMSKY (1975) to support his ideas of an autonomous syntax in transformational grammars. Partee's conclusion that a CN-S analysis is required, has been disputed by BACH & COOPER (1978), who give a T-S analysis of English relative clause constructions. In the present article I will investigate the thematic question: *does the framework of Montague grammar compel us to a specific choice for the syntactic analysis for restrictive relative clauses?* The arguments from the literature are considered, and new arguments are put forward. In the course of the discussion positive and negative answers to the thematic question will alternate. An answer to the general version of the question is obtained as well. It will turn out that syntactic variables (like he_n) play an important role in relative clause constructions. This role is investigated, and this gives rise to the introduction of a new principle for Montague grammar: the variable principle.

1. THE CN-S ANALYSIS

1.1. The discussion in Partee 1973

PARTEE (1973) considers three kinds of analyses of relative clause constructions which were proposed in the literature in the framework of transformational grammar. She investigates which of them constitutes a good basis for a compositional semantics. Below I will summarize her argumentation. Of the three kinds of construction (CN-S, T-S, Det-S) the second was the most popular one among transformational grammarians. The analyses will be presented in the categorial terminology of Montague grammars instead of the terminology of transformational grammars. An exception to this is that for the category of sentences S will be used instead of t. The trees in the figures below have the same status as the trees in MONTAGUE (1973) (henceforth PTQ): they are a representation of the derivational history of a phrase. The only difference is that the nodes are labelled by the category of the expression produced in that stage of the construction process, rather than with the expressions themselves. The three kinds of analysis are called after the configuration in which the relative clauses are introduced. The analyses are:

1. CN-S : the Common Noun-Sentence analysis (Figure 1)
2. T-S : the Term-Sentence analysis (Figure 2)
3. Det-S: the Determiner-Sentence analysis (figure 3).

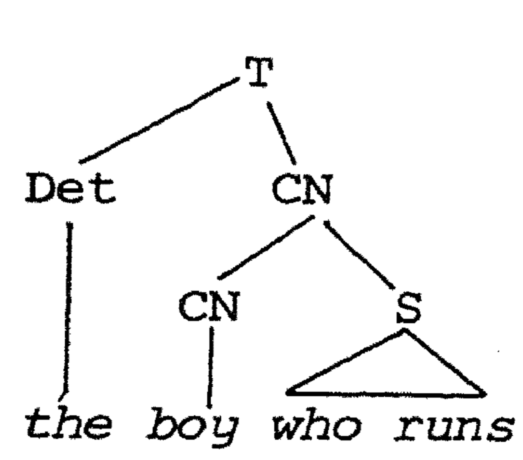

Fig. 1

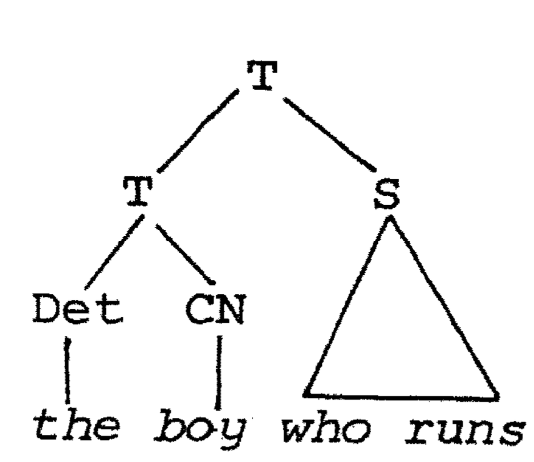

Fig. 2

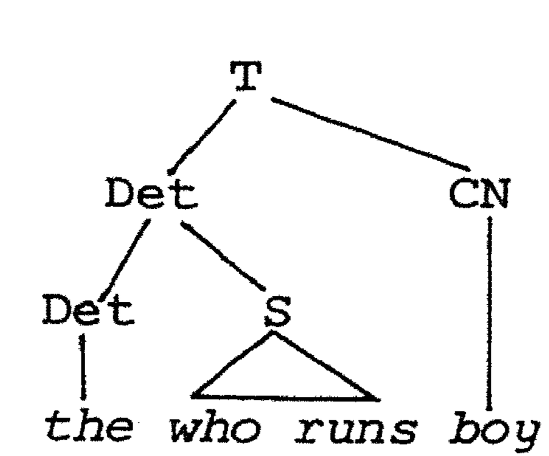

Fig. 3

In the analysis presented in Figure 1, the common noun *boy* can be interpreted as expressing the property of being a boy, and the phrase *who runs* as expressing the property of running. The conjunction of these properties is expressed by the noun phrase *boy who runs*. The determiner *the* expresses that there is one and only one individual which has these two properties. So the CN-S analysis provides a good basis for obtaining the desired meaning in a compositional way.

In the T-S analysis as presented in Figure 2, the term *the boy* is interpreted as expressing that there is one and only one individual with the property of being a boy. Then the information that the individual is running can only be additional. So in a compositional approach to semantics *who runs* has to be a non-restrictive relative clause. Therefore Partee's conclusion is that the T-S analysis does not provide a good basis for a compositional semantics of restrictive relative clauses.

The Det-S analysis from Figure 3 does not provide a good basis either. The phrase dominated by the uppermost Det-node (i.e. *the who runs*), expresses that there is one and only one individual with the property of running, and the information that this individual is a boy, can only be additional.

Of course, these arguments do not constitute a proof that it is impossible to obtain the desired meanings from the T-S and Det-S analyses. It is, in general, very difficult to prove that a given approach is not possible, because it is unlikely that one can be sure that all variants of a certain approach have been considered. This is noted by Partee when she says: 'I realize that negative arguments such as given against analyses 2. and 3. can never be fully conclusive. [..]' (PARTEE 1973, p.74 - numbers and category names adapted T.J.). She proceeds: 'The argument against 3. is weaker than that against 2., since only in 2. the intermediate constituent is called a T.' (ibid.). Her carefully formulated conclusion is 'that a structure like 1. can provide a direct basis for the semantic interpretation in a way that 2. and 3. cannot' (ibid. p.54).

1.2. The PTQ-rules

Accepting the argumentation given in Section 1.1, is not sufficient to accept the claim that one should use the CN-S analysis. It remains to be shown that such an analysis is indeed possible, and this means providing explicit syntactic and semantic rules. Partee does not need to do so because in her discussion she assumes the rules for relative clause formation which are given in PTQ. Although these rules do not produce litterary the same strings as she discusses, the same argumentation applies to them.

The production of the relative clause corresponding to Figure 1 according to the PTQ rules, roughly proceeds as follows. The relative clause is not formed from the sentence *who runs* but from one of the form he_n *runs*. Here n is some natural number. The indexed pronoun he_n is called a 'syntactic variable'. I will use 'variable', when confusion between syntactic and logical variables is unlikely, or when a single indication is required for both kinds. The rule scheme for relative clause formation says that from a sentence (e.g. he_2 *runs*) and a noun (e.g. *boy*), a compound noun phrase can be formed (*boy such that he runs*). This rule scheme, which for each choice of n constitutes a rule, reads as follows.

S3,n: If $\alpha \in P_{CN}$ and $\beta \in P_S$ then $F3n(\alpha,\beta) \in P_{CN}$
where $F3,n(\alpha,\beta) = \alpha$ *such that* $\tilde{\beta}$
and $\tilde{\beta}$ comes from β by replacing each occurrence of he_n by *he/she/it* and of him_n by *him/her/it,* according to the gender of the first CN in α.

The corresponding translation rule reads

T3,n: If α,β translate into α',β', respectively,
then $F3,n(\alpha,\beta)$ translates into $\lambda x_n[\alpha'(x_n) \wedge \beta']$.

The formulation of these rules contains a lot of redundancy, and therefore the rules will be presented more consisely here. I adopt the convention that the syntactic function used in a rule will bear the same index as that rule. Then without loss of information, the first line of S3,n can be given by mentioning the relevant sequence of categories (category of first argument, of second argument, category of result). The corresponding operation on strings is presented by listing the steps which have to be performed successively. Here the convention is used that α always denotes the first argument of the rule and β the second. If e.g. the first argument is changed in some step, then from that moment on α denotes the thus changed argument. Rule S3,n presented in this new format reads as follows:

S3,n CN + S $\rightarrow$ CN
F3,n Replace he_n in β by *he/she/it* and him_n by *him/her/it* according to

the gender of the first CN in α;
concatenate (α, *such that*, β).

The translation rule contains a lot of redundancy too. Let us adopt the convention that by α' is understood the translation of the first and by β' of the second argument of the syntactic rule. Then T3,n can be described by just giving the relevant logical expression. Rule T3,n presented in this way reads:

T3,n: $\lambda x_n[\alpha'(x_n) \wedge \beta']$.

The sentence *He*$_3$ *runs* translates into $run(x_3)$. Application of instance T3,3 of translation scheme T3,n to this formula and to *boy* (being the translation of the common noun boy - notice the difference in type face) yields:

(1) $\lambda x_n[boy(x_n) \wedge run(x_n)]$.

This expression is interpreted as the property which holds for an individual if he both is a boy and is running. This is completely in accordance with the interpretation sketched for Figure 1.

Notice that S3,n can be applied two times in succession (or even more). Then sentences are obtained like (2) (due to Bresnan, see PARTEE, 1975, p.263) and (3) (due to PARTEE - ibid).

(2) *Every girl who attended a women's college who made a large donation to it was included in the list.*

(3) *Every man who has lost a pen who does not find it will walk slowly.*

In these sentences two relative clauses are attached to a single head noun. This construction is known under the name stacking (of relative clauses). In Dutch and German stacking is not a grammatical construction.

Rules S3,n and T3,n do not give a correct treatment of all phenomena which arise in connection with relative clauses. Some examples are:

1. The rule produces the *such-that* form of relative clauses, and this is not their standard form. A rule which produces a form with relative pronouns cannot be obtained by means of a straightforward reformulation of S3,n, since complications arise (see RODMAN 1976).
2. In certain circumstances T3,n may give rise to an, unintended, collision

of variables. This problem can be avoided by renaming, in certain cases, bound variables (THOMASON 1974, p.261), or by using each index in only one rule (JANSSEN 1980a). We will return to this point in Section 5.1.

3. Some famous problematic sentences do not get a proper treatment with this rule. Examples are the so called 'Bach-Peters sentences' and the 'Donkey sentences'. There are several proposals for dealing with them For instance HAUSSER (1979) presents a treatment for the Bach-Peters sentence (4), and COOPER (1979) for the donkey sentence (5).

(4) *The man who deserves it gets the price he wants.*

(5) *Every man who owns a donkey beats it.*

For a large class of sentences, however, the PTQ rule yields correct results, and I will restrict the discussion to this class. The class contains the relative clause constructions in the *such-that* form, the relative clause is a single (i.e. unconjoined) sentence, and stacking is allowed. Bach-Peters sentences and Donkey sentences are not included. For this class, the CN-S analysis gives a correct treatment in a compositional way, whereas for the T-S and Det-S analyses it is argued that this is not the case. So in this stage of our investigations, the answer to the thematic question has to be positive: the compositionality principle compels us to a certain analysis of relative clause constructions.

1.3. Fundamental problems

The PTQ rule for relative clause formation is essentially based on the use of variables in the syntax (he_n), and the use of unbound variables in the logic (x_n). This device gives rise to two problems which are of a more fundamental nature than the problems mentioned in Section 1.2. The latter concerned phenomena which were not described correctly by the given rule, but it is thinkable that some ingenious reformulation might deal with them. The fundamental problems I have in mind are problems which arise from the use of variables as such. It is essential for the entire approach to obtain a solution for these problems, since in case they are not solved satisfactorilly, we cannot use the tool at all. This aspect distinguishes them from the problems mentioned in Section 1.2. The problems also arise in connection with other rules dealing with variables (S14,n, .. S17,n). Note that the epithet 'fundamental' is not used to make a suggestion about the degree of difficulty of the problem, but to indicate the importance that some answer to it is given. The two fundamental problems are the following.

1) 'left-over'

The first problem is: what happens in case a variable is introduced that is never dealt with by S3,n or any other rule. On the syntactic side it means that we may end up with a sentence like *he*$_7$ *runs*. Since *he*$_7$ is not an English word, this is not a well-formed sentence, and something has to be done about it. On the semantic side it means that we may end up with an expression containing an unbound logical variable. From the discussion in Section 4 it will appear that it is not obvious how we should interpret the formulas thus obtained.

2) 'not-there'

The second problem is: what happens when a rule involving variables with a given index is applied in case such variables are not there. I give two examples of such situations. The first is obtained if one applies S3,1 to the common noun *man*, and the sentence *Mary talks*. Then the noun-phrase (6) is produced, which is ill-formed because there is no pronoun which is relativized.

(6) *man such that Mary talks.*

On the semantic side (6) gives rise to a lambda operator which does not bind a variable. The second example (GROENENDIJK & STOKHOF 1976) is obtained by an application of S3,1 to *man* and *he*$_2$ *walks*. Then the common noun phrase (7) is formed.

(7) *man such that he*$_2$ *walks.*

Out of (7) we can build the object term of (8).

(8) *He*$_2$ *loves the man such that he*$_2$ *walks.*

By an application of S14,2 we finally obtain

(9) *John loves the man such that he walks.*

This sentence has just one reading, viz. that John loves a running man. The translation rules of PTQ however, yield (10) as reduced translation for (9).

(10) $\exists u[\forall v[[man_*(v) \wedge walk_*(j)] \leftrightarrow u=v] \wedge love_*(j,u)].$

This formula expresses that the one who walks is John. THOMASON (1976) makes a related observation by counting the number of ambiguities of (11).

(11) *Bill tells his father that John resembles a man such that he shaves him.*

For the first problem it is evident that it is the use of variables which creates it, and that it are not the phenomena themselves: if there were no variables in the syntax, they could not be 'left-over', nor remain 'unbound' in their translation. For the second problem it is rather a matter of conviction that it is the use of variables that creates the problem. Even if (6) would be well-formed, I would consider its production in the way sketched above, as an undesirable side effect of the use of variables, because it does not exhibit a phenomenon for which variables are required.

In the literature there are some proposals for dealing with these two fundamental problems. One proposal (implicitly given in RODMAN 1976) is of a purely syntactic nature and simply says: the 'left-over' and 'not-there' constructions are not acceptable, and in case such a construction threatens to arise, it is filtered out. This approach is not considered here in detail, because it played no role in the discussion concerning our thematic question. In the approach of COOPER (1975) the 'left-over' constructions are accepted, an answer is given to the semantic questions, and the 'not-there' constructions are dealt with in the semantics. In the next sections his proposal will be discussed in detail. A proposal combining syntactic and semantic aspects (JANSSEN 1980b) is considered in Section 4.

2. THE T-S ANALYSIS

2.1. Cooper 1975 on Hittite

COOPER (1975) considers the construction in Hittite which corresponds to the relative clause construction in English. In Hittite the relative clause is a sentence which is adjoined to the left or the right of the main sentence. For this and other reasons, Cooper wishes to obtain such constructions by first producing two sentences and then concatenating them. A simplified example is the Hittite sentence which might be translated as (12), and has surface realization (13). The sentence is produced with the structure given in Figure 4. For ease of discussion English lexical items are used instead of Hittite ones. 'Genitive' is abbreviated as 'gen', 'plural' as 'pl', 'particle' as 'ptc', and 'which' as 'wh'. The example is taken from BACH & COOPER (1978) (here and in the sequel category names are adapted).

(12) *And every hearth which is made of stones costs 1 shekel.*

(13) *SA NA4 HI.A-ia kuies GUNNI.MES nu kuissa 1 GIN*
gen.stone-pl.-and which hearth-pl. ptc. each(one) 1 shekel

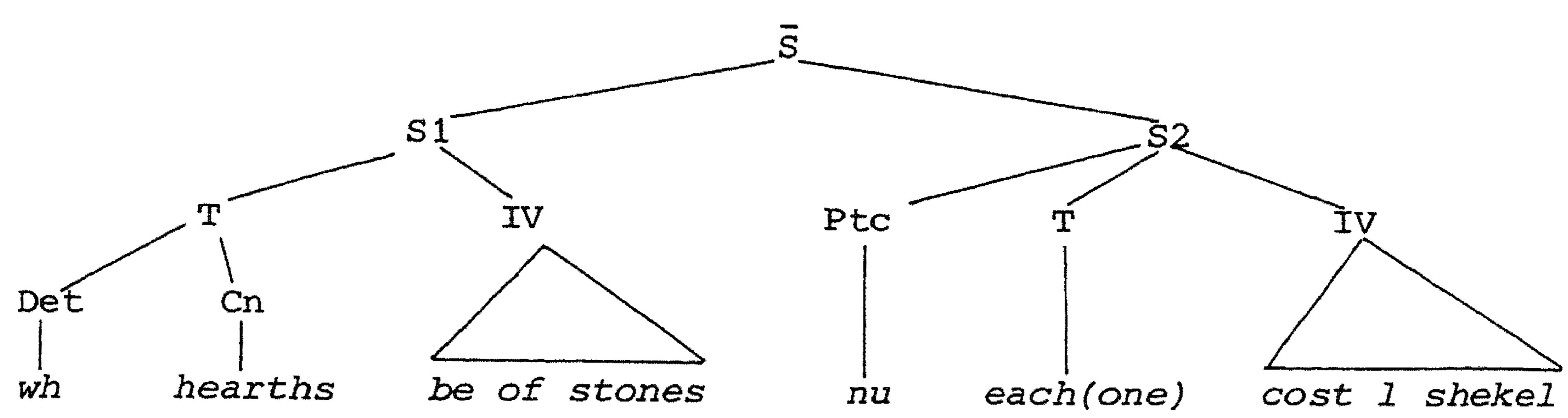

Figure 4

Sentence (13) is assumed to have the same meaning as the corresponding English sentence (12). There seems to be a conflict between the arguments in favor of a CN-S analysis is given in Section 1, and the wish to use the S-S analysis for Hittite. Cooper's solution is to allow the Term-phrase *each(one)* 'to denote the set of properties possessed by every entity having property *R*' (BACH & COOPER 1978, p.147). Which property *R* is, is specified by the relative clause S1. The translations of S1 and S2 are (14) and (15), respectively (here and in the sequel $^{\vee}$, $^{\wedge}$ and $_{*}$ symbols are added).

(14) $\forall x[{}^{\vee}R(x) \rightarrow Cost\text{-}one\text{-}shekel(x)]$

(15) $Hearth(z) \wedge Made\text{-}of\text{-}stone(z)$.

The syntactic rule which combines S1 and S2 to a phrase of the category S, has as corresponding translation rule

$$\lambda R[S2'](^{\wedge}\lambda z[S1']).$$

Here S1' and S2' are the translations of S1 and S2, respectively. When this rule is applied to (14) and (15), we obtain (16) as reduced translation.

(16) $\forall x[hearth(x) \wedge made\text{-}of\text{-}stone(x) \rightarrow cost\text{-}one\text{-}shekel(x)]$.

Since $\bar{S}$ is of another category than S1 and S2, this production process does not allow for stacking, which is claimed to be correct for Hittite.

2.2. Bach & Cooper 1978 on English

BACH & COOPER (1978) argue that the treatment of COOPER (1975) of Hittite relative clauses can be used to obtain a T-S analysis for English relative clause constructions which is consistent with the compositionality principle. Terms are treated analogously to (the Hittite version of) *each(one)*. The term *every man* is assumed to denote, in addition to the PTQ interpretation, the set of properties possessed by *every man* which has the property R. Then the term-phrase *every man who loves Mary* is obtained from the structure given in Figure 5.

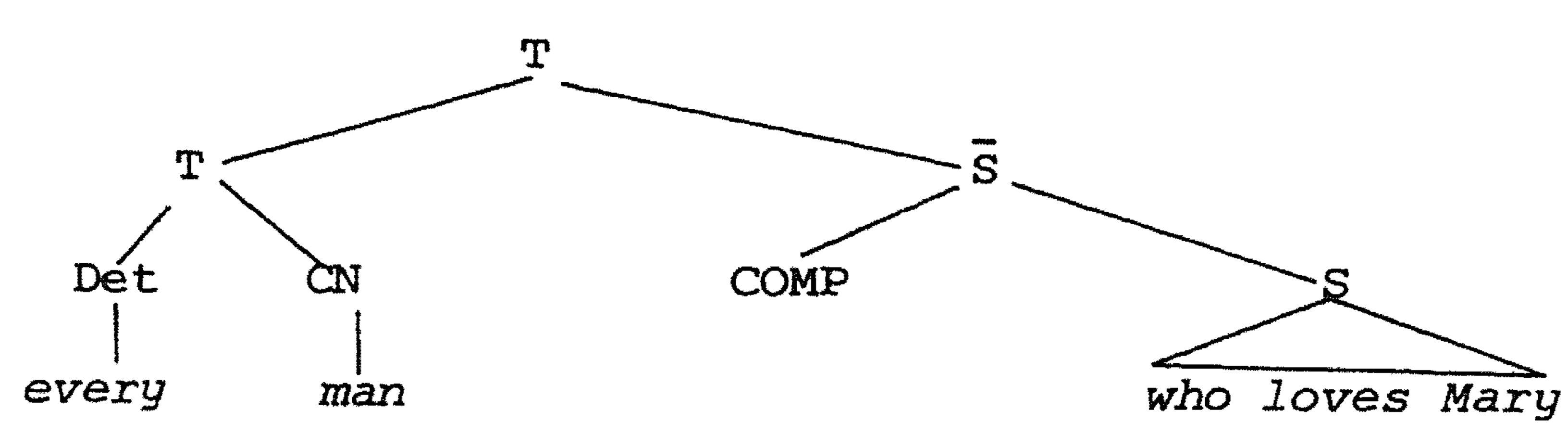

Figure 5

The rule for combining the translation of the term and the relative clause is:

$$\lambda R[T'](^{\wedge}S').$$

Here T' and S' are the translations of the term phrase and the relative clause, respectively. If we take (17) as translation of *every man*, and (18) as translation of the relative clause $\bar{S}$, then we obtain (19) as translation of the whole term (after reduction).

(17) $\lambda P[\forall x[man(x) \wedge {}^{\vee}R(x)] \rightarrow {}^{\vee}P(x)]$

(18) $\lambda z[love_{*}({}^{\vee}z,m)]$

(19) $\lambda P[\forall x[man(x) \wedge love_{*}({}^{\vee}x,m)] \rightarrow {}^{\vee}P(x)]$.

Thus a T-S analysis is obtained for relative clause constructions, of which the translation is equivalent to the translation in the case of a CN-S analysis.

As Bach and Cooper notice, if we follow this approach, a complication has to be solved, since English allows for undefinite stacking of relative clauses. The proposal sketched so far, provides for one relative clause for each T. The complication can be taken care of by allowing an alternative interpretation not only for Terms, but also for relative clauses. 'Thus, for example, the relative clause *who loves Mary* can denote not only the property of loving Mary but also the property of loving Mary and having property *R*' (BACH & COOPER 1978, p.149).

Bach and Cooper remark that their compositional treatment of the T-S analysis clearly is less elegant and simple than the alternative CN-S analysis. They conclude: 'Our results seem to indicate, however, that such an analysis cannot be ruled out in principle, since any constraint on the theory that would exclude the T-S analysis, would seem to exclude the Hittite analysis as well. [...] or the happy discovery of some as yet unknown principles will allow the one, but not other.' (ibid. p.149).

The conclusion which prompts itself in this stage of our investigations is that the answer to the thematic question is a negative one: the principle of compositionality does not compel us to a special analysis of English relative clauses.

2.3. Fundamental problems

As a matter of fact, the discussion in BACH & COOPER (1978) does not provide the evidence that a T-S analysis is indeed possible for English relative clauses. They do not present explicit rules, and neither is it immediately clear what the details would look like (e.g. what is the role of $\bar{S}$ and COMP in the system of categories, and what is the translation rule which combines the translations of S and COMP). Nevertheless, the main point of their approach has become clear from their exposition.

The kernel of the approach of Bach and Cooper is to let the translations of terms and relative clauses contain a free variable R. For this variable the translation of some relative clause will be substituted. However, this variable R gives rise to the same kind of problems as mentioned in Section 1 with respect to the variables x_n.

1. 'Left-over'
We may select for a term the translation with free variable R, whereas we do not use in the remainder of the production a rule which deals with this variable. Since R has no syntactic counterpart, the produced sentences are

not per se ill-formed, but the question concerning the interpretation of unbound variables remains to be answered.

2. 'Not-there'
There may be an occurrence of the term-phrase *every man* with the translation without R, nevertheless appearing in a structure where a relative clause is attached to it. Then an incorrect meaning is obtained.

Only when these fundamental problems are solved, we may hope that the idea of Bach and Cooper leads to rules for the T-S analysis. Notice that the proposal of RODMAN (1976) for solving the two fundamental problems by filtering them out, cannot be followed here because in the syntactic expressions there is no variable which may control the filter. A solution has to be found on the semantic side. These problems for the Bach-Cooper idea, are signalized for the case of Hittite by COOPER (1975). He has proposed some solutions which are assumed by Bach and Cooper. In order to obtain further justification for the answer to the thematic question given in Section 2.2, we have to check the details of Cooper's proposals for these problems. This will be done in the next section.

3. THE PROPOSALS OF COOPER

3.1. Not-there

A translation rule which usually binds a certain variable, may be used in a situation where no occurrences of such a variable are present. To avoid problems, Cooper proposes to give no semantic interpretation to expressions of intensional logic which contain a vacuous abstraction. According to his proposal the interpretation of $\lambda R\alpha$ is undefined in case α has no occurrences of R.

Let us first consider in which way this idea might be formalised. At first glance it seems easy to obtain the desired effect. One just has to look into the expression α in order to decide whether $\lambda R\alpha$ is defined or not. However, this is not acceptable. Such an approach would disturb the homomorphic interpretation of intensional logic (henceforth IL). IL is interpreted in accordance with the principle of compositionality: for each construction of the logical language there is a corresponding interpretation instruction. To obtain the interpretation of a compound logical expression, the interpretations of the parts of that compound are relevant, but not their actual form. An important consequence of this is that two semantically equivalent expressions are interchangeable in all contexts. If we would have a condition like 'look into α' in the definition of interpretation, this basic property of logic would no longer be valid. Two IL-expressions α and β might be semantically equivalent, whereas α satisfies the 'look into'-condition, and β not. Consequently, the interpretation of just one of $\lambda R\alpha$ and $\lambda R\beta$ would be defined. Such a violation of the fundamental law of substitution of equivalents is of course not acceptable. Therefore, a 'look into' clause has to be rejected. One has to respect the homomorphic interpretation of logic, and therefore, the situations in which $\lambda R\alpha$ should receive no interpretation have to be characterized in terms of the semantic properties of α (i.e. in terms of the interpretation of α with respect to a point of reference and a variable assignment). Cooper follows this strategy.

Cooper's first step towards a characterization consists of adding a restriction to the usual definition of the interpretation of $\lambda u\alpha$. '[..] the function denoted by the abstraction expression $\lambda u\alpha$ is only defined for entities within its domain if a different assignment to the variable u will yield a different denotation for α' (COOPER 1975, p.246). As he notes, this definition has as a consequence that $\lambda u\alpha$ is 'undefined not only if α does not contain a free occurrence of u, but also if α is a tautology. Thus for instance, according to this definition $\lambda u[u=u]$ represents a function which is undefined for any entity. However, the technique of supervaluation [...] will show these expressions to be defined but not those where α is not a tautology' (ibid.). This definition is Cooper's final one, but it is not the one we need. It implies that now $\lambda R[x=x]$ is defined. This has the following consequence for relative clause formation. One might produce some sentence expressing a tautology, while its translation does not contain an occurrence of the variable R. Syntactically there needs not, in Cooper's approach, to be anything which prevents us from using this sentence in a relative clause construction, whereas, contrary to his intention, the interpretation of the translation is defined. So Cooper's definition does not provide a solution to the 'not-there' problem.

Cooper's aim was to give a semantic characterization of the IL-syntactic property 'contains an occurrence of the variable R'. I expect that there is no semantic property coinciding with the syntactic one. This is suggested by the observation that almost always a semantic irrelevant occurrence of a certain variable can be added to a given IL-expression. (ϕ and $R=R \wedge \phi$ are semantically indiscernable). Therefore, I expect that no solution in this direction can be found. Moreover, I consider the whole idea underlying Cooper's approach to be unsound. The standard interpretation of $\lambda R\alpha$ is, in case α does not contain an occurrence of R, a function that delivers for any argument of the right type, the interpretation of α as value. So $\lambda R\alpha$ denotes a constant function. Following Cooper's idea, one would loose this part of the expressive power of IL, a consequence I consider to be undesirable.

3.2. Left-over, Proposal 1

The translation of a completed syntactic production of a sentence may contain an occurrence of a free variable. The second fundamental problem was what to do with variables that are 'left over'. Cooper proposes to assign no interpretation to such an expression, and to follow this approach for special variables only. Let z be such a variable (of the type of individuals). As was the case with the first problem, discussed in Section 3.1, one has to respect the homomorphic interpretation of IL. The desired effect should not be obtained by looking into the formula, but by changing the definition of interpretation. Cooper claims that the desired effect is obtained 'by restricting the assignments to variables so that z is always assigned some particular non-entity for which no predicate is defined' (COOPER 1975, p.257). This proposal gives rise to a considerable deviation from the model for IL as it is defined in PTQ. In that model, there are for every entity predicates which hold for it, e.g. the predicate of being equal to itself (viz. $\lambda u[u=u]$). This property is lost in Cooper's approach. He does not define a model which has the desired properties, nor does he give other details. For the discussion concerning the thematic question, this point is not that relevant, because BACH & COOPER (1978) do not propose to follow this proposal in the case of English relative clause constructions, but another one, which will be discussed in Section 3.3.

3.3. Left-over, Proposal 2

A second proposal of COOPER (1975) for the treatment of unbound variables which occur in the translation of a completed production of a sentence is to let the unbound variables be interpreted by the variable assignment function, and to give some linguistic explanation of how to understand the results thus obtained. This approach assumes that in complete sentences indices of variables can be neglected, or that there is some final 'cleaning-up' rule which deletes the indices. For our discussion of relative clause formation the syntactic details of this proposal are irrelevant because the variable R leaves no trace in the syntax.

The unbound relative clause variable R only occurs in subexpressions of the form $R(x)$. These subexpressions are understood by Cooper as 'a way of representing pragmatic limitations on the scope of the quantifier [binding x]. [...]. Thus assigning a value to R in this case has the same effect as adding an unexpressed relative clause to show which particular set we are quantifying over' (COOPER 1975, p.258-259). The same strategy is employed in COOPER (1979a,b) for indexed pronouns. A pronoun he_n which has not been dealt with by a relative clause formation rule or some other rule, is considered as a personal pronoun referring to some contextually determined individual. Its translation has introduced a variable x_n, which remains unbound, and is interpreted by the variable assignment.

The basic idea underlying this approach is to consider the assignment to variables as part of the context of use as was done in 'Universal Grammar' (MONTAGUE 1970). This idea is employed too in GROENENDIJK & STOKHOF (1976). In one respect the idea leads to a deviation from PTQ. There, an expression of type t is defined to be true in case it denotes 1 for every variable assignment (MONTAGUE 1973, p.259). So, $run(x)$ would mean the same as its universal closure. In the proposal under discussion this definition has to be dropped, but this should cause no difficulties.

I have several objections against this proposal of Cooper. The first one is that it yields incorrect results; the other four argue that the whole approach is unsound. My objections are explained below.

1. If the translation of a phrase contains two occurrence of R, and a relative clause is combined with that phrase, then the translation of the relative clause is, by λ-conversion, substituted for both occurrences of R. As Cooper mentions, this phenomenon arises in his grammar for Hittite for (the Hittite variant of):

(20) *That(one) adorns that(one).*

Here the translation of both occurrences of *that(one)* contain an occurrence of the variable R. If this sentence is combined with a sentence containing two occurrences of a *wh*-phrase, semantically strange things happen. Cooper notes this problem and he says: "My intuition is, however, that if there were such sentences, they would not receive the interpretation assigned in this fragment. [...] As it is not clear to me what exactly the facts of Hittite are here I shall make no suggestions for improving the strange predictions of the fragment as it is." (COOPER 1975, p.260).

Unfortunately, the proposal for English of BACH & COOPER (1978) runs into a related problem. Consider the structure for the term phrase given in Figure 6. It is an example taken from their article, and exhibits stacking of relative clauses (the structure is simplified by omitting Comp's).

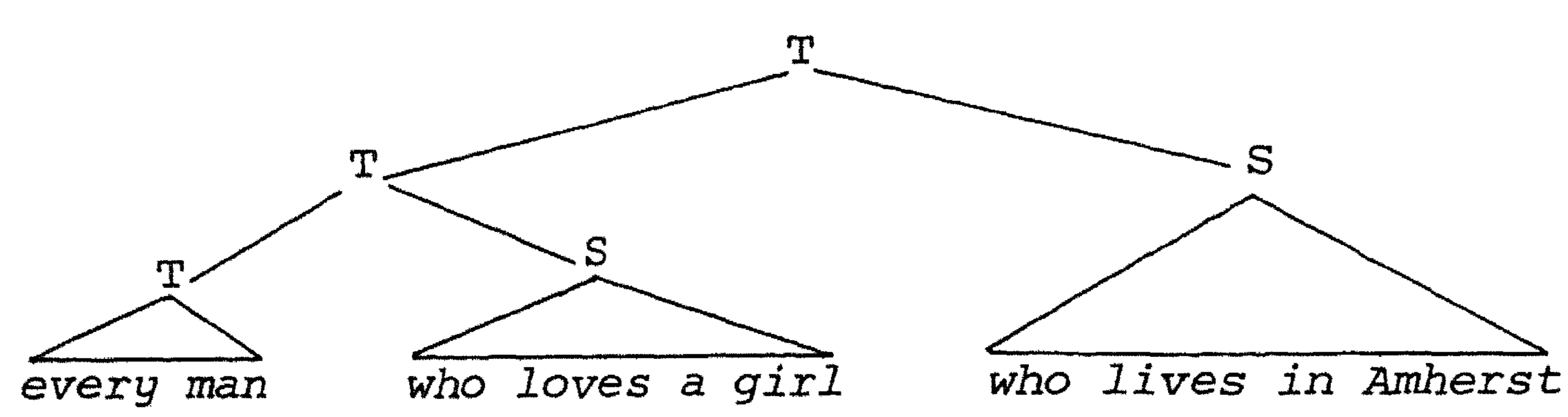

Figure 6

The translation of *every man* has to contain a variable for the relative clause. Recall that in the conception of Bach & Cooper the proposal discussed in Section 3.1 deals with the situation that we have the translation not containing R. Let us assume that we have taken the translation (21), which contains an unbound variable R.

(21) $\lambda P\,\forall x[man(x) \wedge {}^{\vee}R(x) \rightarrow {}^{\vee}P(x)]$.

Suppose now that the referent of *a girl* is to be contextually determined (this possibility is not considered by Bach and Cooper). Then the translation of a girl has to contain the variable R. Besides this variable the translation of (22) has to contain a variable R for the second relative clause. So the translation (22) has to be (23).

(22) *who loves a girl*

(23) $\lambda z\,\exists y[girl(y) \wedge {}^{\vee}R(y) \wedge love_{*}({}^{\vee}z, {}^{\vee}y) \wedge {}^{\vee}R(z)]$.

Consequently, the translation of (24) has to be (25).

(24) *every man who loves a girl*

(25) $\lambda P\,\forall x[man(x) \wedge \exists y\, girl(y) \wedge {}^{\vee}R(y) \wedge love_{*}({}^{\vee}x, {}^{\vee}y) \wedge {}^{\vee}R(x) \rightarrow {}^{\vee}P(x)]]$.

The translation of *who lives in Amherst* roughly is indicated in (26).

(26) λz[*live-in-Amherst(z)*].

The translation of the entire term-phrase in Figure 6 is described by

(27) λR[*every man who loves a girl'*] *(who lives in Amherst')*.

This yields a logical expression which says that both the man and the girl live in Amherst, which is not the intended reading of the construction with stacked relative clauses.

These incorrect predictions are not restricted to stacking. The same problems arise in case a relative clause like *who runs* is combined with a disjoined term phrase like *the man or the woman*. Then semantically both terms are restricted, whereas syntactically only the second one is. The source of all these problems is that a single variable is used for relative clauses and for contextual restrictions. These two functions should, in my opinion, be separated. But then the left-over/not-there problem for relative clause variables arises with full force again.

2. As a motivation for interpreting the R's as contextual restrictions, the argument was given that when we speak about every man, we in fact intend every man from a contextually determined set. But this argument applies with the same force in case we speak about every man who runs. It is not true that terms sometimes are contextually determined, and sometimes not. If one wishes to formalize contextual influence, then every term should be restricted. This suggests (as under 1) a system of variables for context restrictions which is independent of the system of variables for relative clauses.

3. Variables of which the interpretation is derived from the context have to receive a very special treatment. This can be shown most clearly by considering a sentence which has as translation a formula containing an occurrence of an unbound variable of the type of individuals or individual concepts: *he runs*, obtained from the sentence he_n *runs*. These sentences have as translation $run(x_n)$. For every variable assignment this translation gets an interpretation. One of the possible assignments is that x_n is the person spoken to, so *He runs* would have the same truth conditions as *You run*. Some female person might be assigned to x_n, so the sentence may have the same truth conditions as *she runs*. These are incorrect results, so there has to be some restriction on the variable assignments for x_n. There are

also semantic arguments for such a restriction. A pronoun *he* usually refers to individuals from a rather small group (e.g. the person mentioned in the last sentence, or the person pointed at by the speaker). So again some restriction has to be given. These two sources of inadequacy can be dealt with by means of a device from Universal Grammar (MONTAGUE 1970): for evaluating a complete sentence not all variable assignments are available, but only a subset thereof. In the light of the arguments given above, this subset is rather small. So the contextually determined variables are not so variable at all; they behave more like constants.

4. A rather fundamental argument against the use of variables for formalizing contextual influence is the following. In PTQ the contextual factor of the reference point under consideration (a time world pair), is formalized by means of the so called indices I and J. Several authors have proposed to incorporate other factors in the indices. LEWIS (1972), for instance, mentions as possible indices: speaker, audience, segment of surrounding discourse, and things capable of being pointed at. These indices constitute an obvious way to formalize contextual influence. In the light of this, it is very important to realize that in IL the interpretation of constants is 'index dependent', whereas variables have an 'index independent' interpretation:

$$c^{A,i,j,g} = F(c)(i,j), \qquad x^{A,i,j,g} = g(x).$$

This means that in IL it is very strange to use logical variables for the purpose of encoding contextual restrictions. The obvious method is by means of constants. This is precisely the method employed in BENNETT (1978).

3.4. Conclusion

We considered Cooper's proposals concerning the solution of the 'not-there/left-over' problems. His idea to give a semantic treatment of the 'not-there' problem was not successfully formalized. His treatment of the variables 'left-over' led to incorrect results for English sentences. We have to conclude that the technical details of the Bach & Cooper proposal are such that their approach does not work correctly. This means that at the present stage of our investigations concerning the thematic question we are back at the situation of the end of Section 1: only the CN-S analysis seems to be possible.

I have not formally proved that it is impossible to find some treatment in accordance with Cooper's aims. As I said in Section 1, such a proof is, in general, difficult to give. But I have not only showed that the proposals by Bach & Cooper do not work correctly, I have also argued that they have to be considered as unsound. They constitute a very unnatural approach, and in my opinion one should not try to correct the proposals, but rather give up the idea underlying them altogether. Since I consider such proposals as unsound, I will in the next section put forward a principle which prohibits proposals of these kinds. I have the feeling that the proposal to treat discourse pronomina as unbound variables, as put forward in COOPER (1979a,b), is unsound as well (although I do not know of an example demonstrating that that treatment does not work). For the treatment of discours pronomina one might develop a text-grammar version of Montague grammar in which quantification rules may pass the border of a sentence. But then the fundamental problems concerning variables still have to be solved for the text grammar. If one prefers a sentence grammar, then discours pronomina should, as I argued, be treated by means of constants. The aspect of Cooper's proposal to use a new complex kind of translation for certain discourse pronomina can probably be saved by using a constant in that translation instead of a variable (see PARTEE & BACH 1980).

4. THE VARIABLE PRINCIPLE

In the previous section we have considered some attempts to deal with the 'not-there/left-over' problems. These attempts do not give me the impression that the considered situations they deal with are welcome; rather they seem to be escapes from situations one would prefer not to encounter at all. In my opinion these attempts arise from a neglect of the special character of syntactic variables. Syntactic variables differ from other words in the lexicon since they are introduced for a special purpose: viz. to deal with coreferentiality and scope. In this respect they are like logical variables, and in fact they can be considered as their syntactic counterpart. One would like to encounter syntactic variables only if they are used for such purposes. This special character of syntactic variables is expressed by the variable principle, of which a first tentative version is given in (29).

(29) *Syntactic variables correspond closely to logical variables.*

The intuition behind this statement is not completely new. THOMASON (1976) draws attention to the analogy between 'that-complement' constructions in English, and the λ-abstraction operator in logic. PARTEE (1979b) proposes the constraint that any syntactic variable must be translated into an expression of the logic containing an unbound logical variable. Partee does not accept this constraint the other way around, precisely because she does not want to disallow Cooper's treatment of discourse pronouns.

The formulation of the principle given in (29) is vague, and one might be tempted to strengthen it to (30).

(30) *An expression contains a syntactic variable if and only if its unreduced translation contains a corresponding unbound logical variable.*

This is intuitively an attractive formulation. However, a major drawback is that it does not fit into the framework of Montague grammar. It would give the unreduced translation of an expression a special status which it does not have in the framework as it is. It would no longer be just one representation among others, all freely interchangeable. It would become an essential stage since the principle would have to function as a filter on it. It would no longer be allowed to reduce the intermediate steps in the translation process since then a semantically irrelevant occurrence of a logical variable might disappear, and thereby a translation that had to be rejected, might become acceptable. Therefore, I will give a formulation which turns the principle into a restriction on possible Montague grammars. The formulation below has the same consequences for the unreduced translation as (30), but it is not a filter on the unreduced translations and it leaves the framework untouched. This formulation is slightly more restrictive than (30), and than the formulation in JANSSEN (1980b).

The VARIABLE PRINCIPLE is defined as consisting of the following requirements:

1a) *A syntactic variable translates into an expression which contains a free occurrence of a logical variable, and which does not contain occurrences of constants.*

1b) *This is the only way to introduce a free occurrence of a logical variable.*

2a) *If a syntactic rule removes all occurrences of a certain syntactic variable in one of its arguments, then the corresponding translation rule binds all occurrences of the corresponding logical variable in the translation of that argument.*

2b) *If a translation rule places one of its arguments within the scope of a binder for a certain variable, then its corresponding syntactic rule removes all the occurrences of the corresponding syntactic variable from the syntactic counterpart of that argument.*

3a) *The production of a sentence is only considered as completed if each syntactic variable has been removed by some syntactic rule.*

3b) *If a syntactic rule is used which contains instructions which have the effect of removing all occurrences of a certain variable from one of its arguments, then there indeed have to be such occurrences.*

This formulation of the variable principle is not what I would like to call 'simple and elegant'. I hope that such a formulation will be possible when the algebraic theory of the organization of the syntax is further developed. Suppose that we have found which basic operations on strings are required in the syntax (following the ideas of PARTEE (1979a,b)), and that a syntactic rule can be described as a polynomial over these basic operations. Then we may hope to formulate the variable principle as a restriction on the relation between the syntactic and semantic polynomials. We might then require that these polynomials are isomorphic with respect to operations removing/binding variables.

Requirement 1a) is a restriction on the translation of lexical elements. It can easily be checked whether a given grammar satisfies the requirement. It is met by all proposals in the field of Montague grammar that I know of; e.g. the PTQ translation of he_n is $\lambda P[{}^{\vee}P(x_n)]$, and the translation of the common noun variable one_n (HAUSSER 1979) is the variable P_n.

For reasons of elegance, one might like to have formulation 1a') instead of formulation 1a).

1a') *A syntactic variable translates into a logical variable.*

In order to meet 1a') in the PTQ fragment, one could introduce a category of Proper Names containing *John*, *Mary*, he_1, he_2,... (with translations

j, m, x_1, x_2, respectively). Out of these Proper Names, Terms could be produced which obtain the standard translation ($\lambda P[{}^{\vee}P(j)]$, etc.). Since I do not know of a phenomenon, the treatment of which would be simplified using this approach, and since the variable principle then still would not have a simple formulation anyhow, I will not use it here. Requirement 1a) has as a consequence that the translation of a syntactic variable is logically equivalent to a logical variable. If constants are allowed to occur, then this would no longer be true (e.g. it is not true that for every c the formula $\forall x[x=c]$ is valid).

Requirement 1b) is a restriction both on the translation of lexical elements, and on the translation rules. This requirement is met by PTQ. It is not met by the proposals of BACH & COOPER (1978) which allow logical variables to occur which do not have a syntactic counterpart. Since they do not present explicit rules, I do now know at which stage the context variable R is introduced, as a lexical ambiguity of the noun, or by means of some syntactic rule.

Requirements 2a) and 2b) are conditions on the possible combinations of a syntactic rule with a translation rule. Whether a grammar actually meets them is easily checked by inspection (PTQ does). Requirement 2b) is not met by the Bach & Cooper proposal since their approach in some cases gives rise to the introduction and binding of logical variables without any visible syntactic effect.

Requirements 3a) and 3b) are not met by the PTQ grammar, and neither by the Bach & Cooper proposal. In a certain sense these requirements constitute the kernel of the principle. They express that certain configurations (described with respect to occurrences of variables) should not arise. When these requirements are met, the fundamental problems described in Section 1 disappear. As such, the two requirements are closely related to two instructions in JANSSEN (1980a, p.366), and to two conventions in RODMAN (1976, one mentioned on p.176, and one implicitly used on p.170). Requirements 3a) and 3b) alone, i.e. without 1) and 2), would suffice to eliminate the syntactic side of the two fundamental problems, but then the close relationship between syntactic and logical variables would not be enforced. That freedom would give us the possibility to abuse syntactic variables for other purposes than coreferentiality and scope. An extreme case is given in JANSSEN (1980b), where some rules which obey 3a) and 3b), but violate 1) and 2), are defined in such a way that the information that a rule is obligatory is encoded in the syntactic variables. I intend to prohibit this and other kinds of abuse of variables by combining the third requirement with the first and second.

Requirement 3a) says that all steps in a derivation process have to meet a certain condition. So 3a) appears to be a global filter. However, since one can tell from the final result whether the condition is met, it reduces to a final filter (this observation is not made in JANSSEN (1980b)). Requirement 3b) puts restrictions on the situations in which certain rules may be applied. It thus leads to partial rules: a rule does not apply to some of the expressions of the categories it is defined for. In case the reader has no problem in accepting the filter arising from 3a), and the partial rules from 3b), he may be satisfied with such an interpretation of the two conditions. I do have objections gainst partial rules and filters, and prefer to avoid them. Therefore, I have developed an implementation of 3a) and 3b) in which no partial rules or filters are used (JANSSEN 1980b). For the present discussion it is irrelevant how exactly 3a) and 3b) are incorporated in the system. Since we are primarily interested in the effects of the principle, it suffices to know that it can be done in some way.

Let me emphasize that the principle is intended to apply to the standard variables of intensional logic and their corresponding syntactic variables. For instance, the argument concerning the use of unbound variables for contextual influence does not apply if we do not translate into IL but into Ty2. The language Ty2 is defined in GALLIN (1975); it contains e.g. variables of type s (i.e. variables for indices). Ty2 is used in GROENENDIJK & STOKHOF (1980) for describing the semantics of questions. If Ty2 is used, the variable principle does not simply apply to all the variables of type s. Neither does the principle apply to so called 'context variables' of HAUSSER (1979), or the 'context expressions' of GROENENDIJK & STOKHOF (1979), which both are added to IL for the special purpose of dealing with contextual influence.

The principle eliminates the basic problems mentioned in Section 1 and it disallows the treatment of variables aimed at in COOPER (1975), and COOPER (1979a,b). Another example of a treatment which is disallowed is the proposal of OH (1977). For a sentence without discourse or deictic pronouns he gives a translation containing an unbound variable. A consequence of the principle is that the denotation of a sentence is determined completely by the choice of the model and the index with respect to which we determine its denotation. In other words, the denotation is completely determined by the choice of the set of basic entities, the meaning postulates, the index, and the interpretation function for constants (i.e. the interpretations of the lexical elements in the sentence). In determining the denotation the non-lingustic aspect of an assignment to logical variables plays no role. This I consider to be an attractive aspect of the principle. What the impact of the principle is for the answer on the thematic question will be investigated in the next section.

5. MANY ANALYSES

5.1. The CN-S analysis for English

Do the rules for the CN-S analysis of relative clauses obey the variable principle?

Recall the PTQ rules from Section 2.1.

S3,n CN + S → CN

F3,n Replace he_n in β by *he/she/it* and him_n by *him/her/it*, according to the gender of the first CN in α; concatenate (α, *such that*, β).

T3,n (PTQ) $\lambda x_n[\alpha'(x_n) \wedge \beta']$.

This combination of S3,n and T3,n does not obey the variable principle since possible occurrences of x_n in α' are, by λx_n, bound in the translation, whereas the occurrences of the corresponding syntactic variable he_n in α are not removed. This aspect is the source of the 'collision of variables' mentioned in Section 2.1. A reformulation of T3,n which avoids such a collision is given by THOMASON (1974, p.261).

T3,n (THOMASON)

$\lambda x_m[\alpha'(x_m) \wedge \tilde{\beta}']$

where $\tilde{\beta}'$ is the result of replacing all occurrences of x_n in β' by occurrences of x_m, where m is the least even number such that x_m has no occurrences in either α' or β'.

The syntactic rule S3,n removes the occurrences of he_n in β. Thomason's reformulation has the effect that the unbound logical variables x_n in β' do not occur free in the translation of the whole construction, whereas the same variables in α remain unbound. Nevertheless, Thomason's reformulation does not obey the variable principle since in the syntax occurrences of he_n in β are removed, whereas in the translation the occurrences of the corresponding variable (i.e. x_n) are not bound, but of a variable x_m (where $n \neq m$).

Another kind of objection against Thomason's rule is that it is not a polynomial over IL (neither ~ is an operation of IL, nor is the operation expressed by the sentence following the formula). So the framework of Universal Grammar (MONTAGUE 1970) is violated. A formulation of T3,n which is in accordance with this framework and which obeys the variable principle is as follows:

T3,n $\lambda P[\lambda x_n[{}^{\vee}P(x_n) \wedge \beta'](^{\wedge}\alpha')]$.

This formulation has as a consequence that only those occurrences of x_n are bound, of which the syntactic counterparts are removed in S3,n.

5.2. The S-S analysis for Hittite

Is an analysis of Hittite relative clause constructions possible which on the one hand satisfies the variable principle, and on the other hand produces such a construction out of two sentences?

Below I will describe an analysis which shows that the answer is affirmative. I will only deal with the example discussed in Section 2, and not with all other cases of Hittite relative clauses which are treated by COOPER (1975). My analysis is intended mainly as an illustration of the kinds of technique which are available if one obeys the variable principle.

The treatment described in Section 2 violates the variable principle because both subsentences in Figure 4 have a translation which contains an unbound variable, whereas the sentences themselves do not contain a syntactic variable. Given the principle, in both sentences there has to be an occurrence of a syntactic variable as well. The English variant of sentence S2 gives a hint on how to do this. It contains in a CN-position the word (*one*) - probably added for explanatory reasons. This word suggests the introduction in the syntax of CN variables one_1, one_2,..., which are translated into logical variables P_1,P_2,..., respectively (such CN-variables are discussed in HAUSSER (1979)). The rule which combines S1 with S2 will then give rise to a translation in which (by λ-conversion) the relevant property is substituted for P_n. In case one prefers not to introduce a new constituent one_n, a new variable of category T might be introduced alternatively: (31), translating as (32).

(31) $each_n$

(32) $\lambda Q[\forall x[{}^{\vee}P_n(x) \rightarrow {}^{\vee}Q(x)]]$.

The variable in the translation of the relative clause can be introduced by the translation of the determiner *wh*. Therefore, the category of determiners (which contains the Hittite version of *every*, etc.) is extended with a variable (33), translating as (34).

(33) wh_n

(34) $\lambda Q\lambda P[{}^{\vee}Q(z_n) \wedge {}^{\vee}P(z_n)]$.

We have to combine a relative clause containing a free variable z_n with a main sentence containing a free variable P_n. This can be done by means of a single rule binding both logical variables and performing the relevant operations on both syntactic variables, or by means of two rules, each dealing with one variable at a time. The former method would yield the tree from Figure 4, but it would implicate that a new kind of rules is introduced (rules with two indices). I will follow the two-rules approach.

First the relative clause is transformed into an expression of the new category Prop (=t//e), being a set of expressions denoting properties. We do this by means of the following rule (the numbers in the 500-series are numbers of new proposed rules).

S501,n S $\rightarrow$ Prop
F501,n Replace wh_n in α by *wh*
T501,n $\lambda z_n[\alpha']$.

The rule combining a property with a sentence is

S502,n Prop + S $\rightarrow$ S
F502,n delete all occurrences of one_n from β;
        concatenate (α,β)
T502,n $[\lambda P_n\ \beta'](^{\wedge}\alpha')$.

Using these rules, the Bach & Cooper example is obtained in the way indicated in Figure 7. Its translation is equivalent to the one given in Section 2 for Figure 4. Since we assume that it is guaranteed that the variable principle is obeyed, no problems arise with the syntactic variables.

The principle guarantees that rule S502,1 is applied only in case the main sentence contains an occurrence of one_1 and that rule S501,2 is applied only when the sentence contains an occurrence of the variable wh_2. Furthermore, it guarantees that all syntactic variables finally will have disappeared.

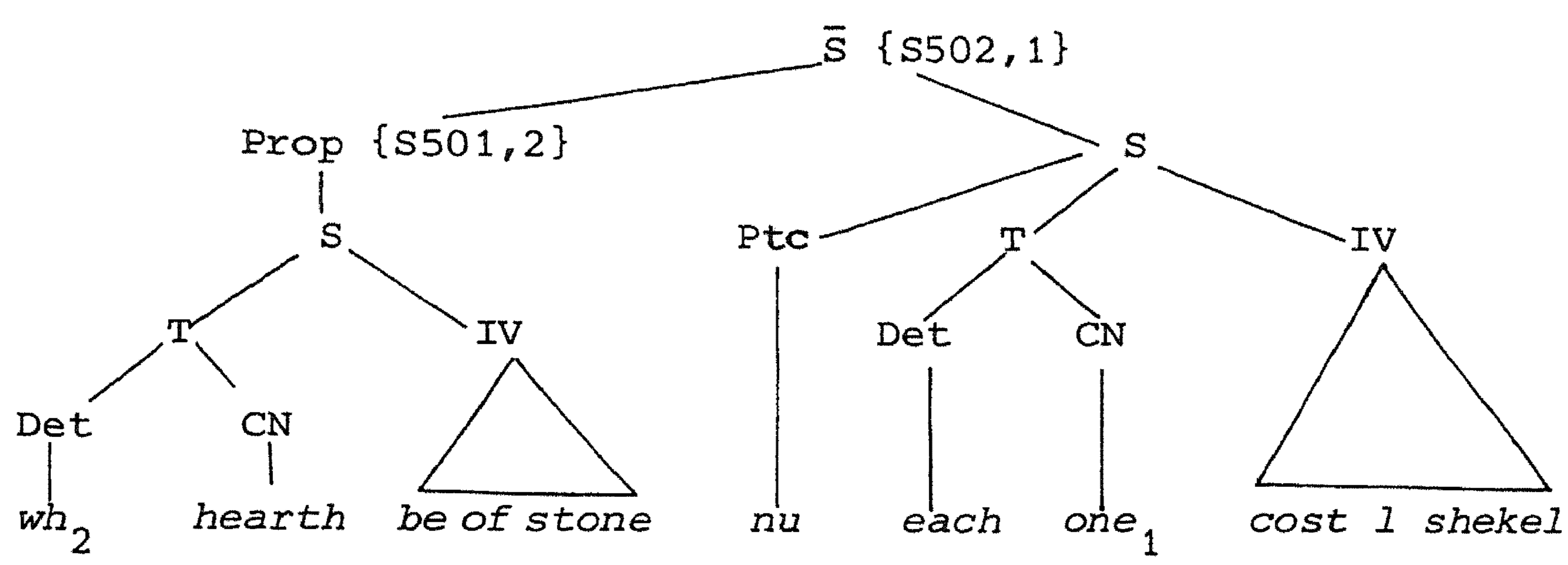

Figure 7

5.3. The T-S analysis for English

As shown in Section 5.2, an S-S analysis can be obtained simply by introducing a variable in the syntax, when such a variable is required in the translation. The same idea can be used to obtain a T-S analysis for relative clauses. In this case, we need a variable of the category Prop, written as *of* $kind_n$. It translates into the variable K_n.

A property and a common noun phrase combine to a new common noun phrase as follows:

S503	CN + Prop $\rightarrow$ CN
F503	concatenate (α,β)
T503	$\lambda y[\alpha'(y) \wedge \beta'(y)]$.

A category RC of relative clauses (RC = t///e) is introduced because RC's and Prop's will occur in different positions. The expressions of the category RC are made out of sentences as follows:

S504,n	S $\rightarrow$ RC
F504,n	delete the index n from all pronouns in α; concatenate (*such that,* α)
T504,n	$\lambda x_n[\alpha']$.

A relative clause may be 'quantified in' a term phrase by substituting the relative clause for a property variable:

S505,n T + RC → T

F505,n substitute β for $of\text{-}kind_n$ in α

T505,n $\lambda K_n[\alpha'](^{\wedge}\beta')$.

An example of a production using these rules is given in Figure 8.

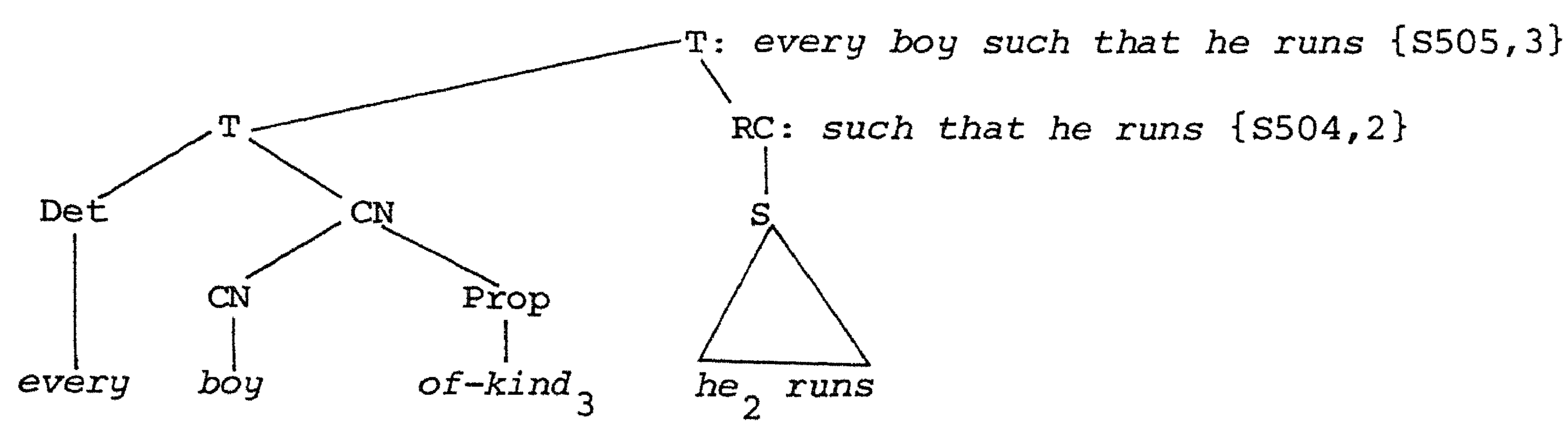

Figure 8

The translation of the lower term phrase in Figure 8 is (35), the translation of the RC phrase (36), and of the upper term phrase (after reduction) is (37).

(35) $\lambda Q \forall x[boy(x) \wedge {}^{\vee}K_3(x) \rightarrow {}^{\vee}Q(x)]$

(36) $\lambda x_2[run(x_2)]$

(37) $\lambda Q \forall x[boy(x) \wedge run(x) \rightarrow {}^{\vee}Q(x)]$.

Note that the intermediate stage of an RC is not required if S505 is a double indexed rule, dealing both with he_n and $of\text{-}kind_m$.

5.4. The Det-S analysis for English

Is a Det-S analysis possible which obeys the variable principle? Recalling the pattern underlying the S-S and T-S analyses, one might try to find such an analysis as a variant of the CN-S analysis by introducing new variables. It appeared, to my surprise, that it is possible to obtain a Det-S analysis which is not a variant of the CN-S analysis, but which is a pure Det-S analysis (recall the proviso by Partee for her argumentation concerning the Det-S analysis). I will not discuss the heuristics of this analysis, but present the rules immediately.

S506,n Det + S $\rightarrow$ Det

F506,n remove all indices n from pronouns in β;
concatenate (α, *such that*, β)

T506,n $\lambda R[\alpha'(^{\wedge}\lambda y[^{\vee}R(y) \wedge \lambda x_n[\beta'](y)])]$.

Maybe the following explanation of the translation is useful. A determiner δ is, semantically, a function which takes as argument the property η expressed by a noun and delivers a collection of properties which have a certain relation with η. S506 produces a determiner which takes a noun property η and delivers a set of properties which has that relation with the conjunction of η and the property expressed by the relative clause.

The combination of a CN with a Det-phrase, requires that the CN is placed at a suitable position in the determiner phrase. In the present fragment this position is the second position (if we had determiners like *all the*, then also other positions might under certain circumstances be suitable). The rule for this reads as follows:

S507 Det + CN $\rightarrow$ CN

F507 insert β after the first word of α

T507 $\alpha'(^{\wedge}\beta')$.

The combination of the determiner *every* with the sentence *he*$_2$ *runs* yields determiner (38), with (39) as unreduced, and (40) as reduced translation.

(38) *every such that he*$_2$ *runs*

(39) $\lambda R[\lambda Q\lambda P[\forall x[^{\vee}Q(x) \rightarrow {}^{\vee}P(x)]](\lambda y[^{\vee}R(y) \wedge \lambda x_2[run(x_2)](y)])]$

(40) $\lambda R\lambda P\forall x[^{\vee}R(x) \wedge run(x) \rightarrow {}^{\vee}P(x)]$.

The combination of (38) the common noun *man* yields the term phrase (41), which has the (usual) reduced translation (42).

(41) *every man such that he runs*

(42) $\lambda P\forall x[man(x) \wedge run(x) \rightarrow {}^{\vee}P(x)]$.

The techniques which are used to obtain a T-S analysis from a CN-S analysis can be used as well to obtain a T-S analysis which is a variant of the Det-S analysis: introduce in the Det-S analysis the variable *of-kind*$_n$, but now within the determiner. This means that at least two kinds of T-S analyses are available.

5.5. Discussion

In Section 4 a new principle was introduced: the variable principle. Obeying this principle we designed rules for relative clause constructions. It turned out that for English besides the CN-S analysis both the T-S and the Det-S analysis are possible in at least two essentially different variants. And for Hittite an S-S analysis is possible. So at the present stage of our investigations a negative answer to the thematic question has to be given: several analyses of relative clauses are possible.

Consider the CN-S analysis of 5.2 again. Is it the kind of T-S analysis meant by Partee? I do not think so. At a certain level we indeed have a T-S analysis, but on another level in the production tree there is a CN-Prop analysis which is nothing but a variant of the CN-S analysis. The opposition between the two analyses was, however, the main point in the discussion of PARTEE (1973). So one could say that her conclusion that the pure T-S analysis cannot be used, in a certain sense still holds. For the case of Hittite however, the discussion primarily aimed at obtaining an S-S analysis at some level, rather than at avoiding the CN-S analysis on all levels. In Section 1 I quoted Bach & Cooper who expressed the hope for the 'happy discovery of yet unknown principles' which exclude the TS-analysis, but allow for the S-S analysis. It seems reasonable to interpret this as the desire for a principle which prohibits the pure T-S analysis, but allows some variant of the S-S analysis. The variable principle has such an effect. But if it is interpreted as the hope for a principle which excludes all kinds of T-S analyses, or which allows a pure S-S analysis, then the variable principle is not such a principle.

We work within a framework which obeys the principle of compositionality as it is formalised in 'Universal Grammar' (MONTAGUE (1970)). This means that the syntax is organized as an algebra, the semantics is organized as an algebra, and meaning assignment is an homomorphism. So the semantic algebra is the homomorphic image of the syntactic algebra, and, therefore, each construction step in the semantic algebra corresponds to a construction step in the syntactic algebra. We have found that several kinds of analyses of English relative clauses are possible, but that they are all variants of the pure CN-S analysis, or of the pure T-S analysis. These practical results could be expected on the basis of the algebraic properties of the framework, as will be explained below.

Let us suppose that we have found a semantic operation T555 which takes two arguments, and delivers the meaning of a certain construction. So in the semantics we have the construction step T555 (α',β'). Due to the homomorphism relation, there has to be a corresponding operation F555 (α,β) in the syntax, and the two semantic arguments have to correspond with the two syntactic arguments. Instead of the semantic step T555 (α',β'), several variants are possible, each with its own consequences for the syntax. These variants amount to a construction process with two stages. We may first have T555 (α',R), where R is a variable, and introduce in a later stage a λ-operator for R taking β' as argument:

$$\lambda R[\ldots\ T(\alpha',R)\ \ldots](\beta').$$

This means that the syntactic expression β can be introduced in an arbitrary later stage of the syntactic production process. Consequently, a lot of variants of the original syntactic construction can be formed. These variants are based on the use of the construction step T555 (α',R) in the logic. Due to the variable principle, the variable R has to be introduced by the translation of some syntactic variable. Let us suppose that V is such a variable. Due to the homomorphic relation between syntax and semantics, this means that in the syntax there has to be a step F555 (α,V). So whereas we have gained the freedom to introduce β in a later stage of the syntactic construction process, step F555 is not avoided. The same argumentation applies when the first argument of T555 is replaced by a variable. It is even possible to replace both arguments by a variable, thus obtaining a large freedom in the syntax concerning the stage at which α and β are introduced. But in all these variants F555 is not avoided. Application of this argumentation to the case of relative clauses (where two basic constructions are found) means that we cannot avoid both the CN-S and the Det-S construction at the same time. So on the basis of the compositionality principle, formalized in an algebraic way, many relative clause constructions are possible. This is due to the power of λ-abstraction. This operation makes it possible that on the semantic side the effect is obtained of substituting the translation of one argument on a suitable position within the other argument, whereas in the syntax a completely different operation is performed. Referring to this power Partee once said 'Lambdas really changed my life' (Lecture for the Dutch Association for Logic, Amsterdam, 1980).

The above argumentation is not completely forcing: there is (at least) one exception to the claim that it is not possible to make a variant of a given semantic construction which avoids the corresponding syntactic construction step. An example of such an exception arose in the S-S analysis for Hittite. In the main sentence we had the Det-CN construction *each* one_n, where one_n was a variable. We obtained a variant in which there is no Det-CN construction: the logical variable introduced by one_n, could be introduced by a new variable $each_n$ (see (34)). The algebraic description of this method is as follows. Consider again T555 (α', R). The variable R might, under certain circumstances, be introduced by the translation of α, thus allowing to replace T555 by a related semantic operation which takes only one argument. That the translation of α introduces the variable R, means that in the syntax α is to be replaced by some variable, say an indexed variant of α. Its translation is then a compound expression (being a combination of the old translation α' with the variable R). This process, which avoids to have F555 in the syntax, is possible only if α is a single word with a translation which does not contain a constant (e.g. if α is a determiner). If the translation of α would contain a constant, then requirement 1a) of the variable principle would prohibit that its translation introduces a variable. If α is not a single word, then it cannot be replaced by a syntactic variable (maybe one of its parts can then be indexed). This method of creating exceptions would be prohibited when requirement 1a) of the variable principle would be replaced by the more restrictive version 1a'). In order to prove that the exception described here is the only one by which a given analysis can be avoided, the details of the relation between operations in the semantics or in the syntax have to be formalized algebraically (see also Section 3).

These algebraic considerations explain the results of our practical work. On the basis of these considerations it would be possible to explain that a Det-S analysis which is variant of the CN-S analysis, is not to be expected (in any case the described method for obtaining variants does not work). The algebraic considerations also give an answer to the general question whether the principle of compositionality restricts the options available for descriptive work. On the basis of a given construction step, a lot of variants are possible, but due to the variable principle and the homomorphic relation between syntax and semantics, this construction step cannot be avoided in these variants. So the answer to the general question is that there are indeed restrictions on the syntactic possibilities, but only in the sense that a basic step cannot, generally speaking, be avoided. The principles are not that restrictive that only a single analysis is possible. Formal proofs for these considerations require, as I said before, a further algebraisation of the syntax.

I started the present discussion by giving on the 'thematic' question the answer that we are not compelled to a certain analysis for relative clauses. On the basis of algebraic considerations this conclusion was generalized to all kinds of constructions. The answer to the thematic question was based upon an investigation of the relative clause construction as such; interaction with other phenomena was not taken into consideration. The answer to the general question was based upon arguments concerning a single operation T555. In the next section we will leave the isolation and consider the interaction of relative clause constructions with two other phenomena.

6. OTHER ARGUMENTS

6.1. Syntax: gender agreement

The relative pronoun has to agree in gender with the anteceding noun phrase. In the Det-S analysis, this poses a problem. The rule which combines a determiner with a relative clause has to specify what is to be done with the syntactic variable. The formulation I gave of rule S506,n just deletes the index, so it gives a correct result if the noun has male gender. But in the same way as we produced *every boy such that he runs*, we may produce *every girl such that he runs*. It is not possible to formulate S506 in such a way that this kind of ill-formedness is avoided, because the information which gender the noun has, is not available at the stage at which the determiner and the relative clause are combined. Not removing the index would, according to the variable principle, require a free variable in the translation of the term phrase; but I do not see how this approach might work.

The T-S analysis gives rise to a similar problem. The rule which makes the relative clause (RC) out of a sentence (S), has to specify what has to be done with he_n. The formulation I gave of S504 works correctly for male nouns only. Again, information about the gender of the noun is not yet available, and not removing the index would constitute a break with the principle. This argument does not apply to the T-S analysis in which a double indexed rule is used. In the CN-S analysis, no problems arise from gender agreement, since at the stage at which the index has to be removed, the gender of the noun is known.

One should not conclude from this discussion that it is impossible to obtain correct gender agreement in case of the Det-S or T-S analysis under discussion. I expect that it can be done by means of further subcategorization. One has to distinguish female, male, and neuter relative clauses, and female, male, and neuter determiners, and probably one needs to make similar distinctions in other categories. Then the subcategory system provides the information needed to obtain precisely the correct combinations of relative clause, determiner and noun.

There is the hidden assumption in this discussion that gender agreement has to be handled within the syntax. If we do not assume this, then a phrase as *a girl such that he runs,* is no longer considered to be syntactically ill-formed. COOPER (1975) argues in favor of dealing with gender in the semantics (at least for English). Others might prefer to handle gender in pragmatics (Karttunen, according to PARTEE (1979a)). Then the arguments given here are no longer relevant. But in languages with grammatical gender (e.g. Dutch, German), this escape is not available. Here one might adopt one of the solutions I mentioned: refined subcategorization, a T-S analysis with a double indexed rule, or simply the CN-S analysis for relative clauses.

6.2. Semantics: scope

Consider the following sentence (exhibiting stacking on the head *man*):

(43) *Every man such that he loves a girl such that he kisses her is happy.*

This sentence has a possible reading in which *every* has wider scope than *a*. In a PTQ like approach (so with the CN-S construction for relative clauses), this reading is obtained by quantification of *a girl* into the CN phrase

(44) *man such that he loves* him_n *such that he kisses* him_n.

The corresponding translation of the sentence (44) reduces to

(45) $$\forall y[\exists x[girl(x) \wedge man(y) \wedge love_*({}^{\vee}x, {}^{\vee}y) \wedge kiss_*({}^{\vee}x, {}^{\vee}y)] \rightarrow happy(y)].$$

Can this reading be obtained in other analyses of relative clauses?

In the T-S analysis this stacking of relative clauses can be obtained by means of a process indicated in Figure 9. In order to obtain coreferentiality between both occurrences of the term him_n, the term *a girl* has to be substituted at a stage in which both relative clauses are present. The earliest moment at which this is the case, is immediately after the uppermost term has been formed. Using a rule analogous to the standard quantification rules would assign the existential quantifier wider scope than the universal quantifier, thus not yielding the desired reading. So it seems to be impossible to obtain in such a T-S analysis coreferentiality and correct scope at the same time.

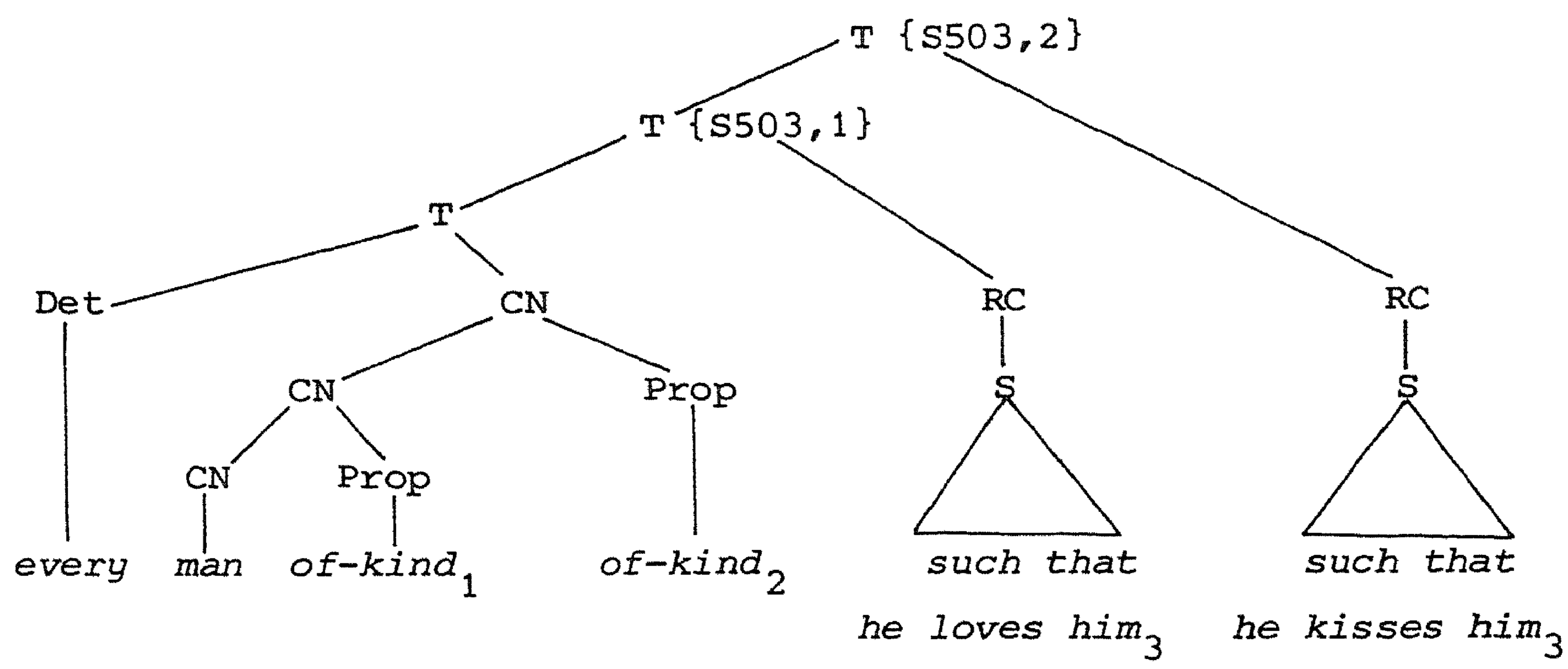

Figure 9

In the Det-S analysis the earliest stage at which the coreferentiality of *she* and *a girl* can be accounted for, is when the determiner phrase (46) has been formed.

(46) *every such that he loves* him_3 *such that he kisses* him_3.

Some later stage (e.g. the term level), might be selected as well. But in all these options, the quantification rule would give wider scope to *a* than to *every*, thus not yielding the desired reading.

Underlying this discussion is the assumption that there is something like stacking of relative clauses. If there is stacking, then the rule for quantification into a CN is essential for the PTQ fragment (FRIEDMAN & WARREN (1979)). But is stacking indeed a phenomenon of natural language? As for Hittite, BACH & COOPER (1975) inform us that no stacking occurs. As for English, no author expresses doubts, except for PARTEE (1979b). She states that the evidence for stacking is spurious. This would leave a rather small basis for our argumentation concerning an answer on the thematic question.

There is another phenomenon, however, that requires quantification into CN's. It probably is the kind of examples meant by PARTEE (1975, p.236). Example (47) assumes that there are common nouns in the fragment of the form *friend of*.

(47) *Every picture of a woman which is owned by a man who loves her is a valuable object.*

Here the intended reading is the one in which *every* has wider scope than *a*, and in which there is coreferentiality between *a woman* and *her*. This reading can easily be obtained by means of substitution of *a woman* into the CN-phrase (48).

(48) *picture of* he_1 *such that it is owned by a man such that he loves* him_1.

So even if we do not accept stacking as a phenomenon of English, a CN-S analysis appears to be required.

It is remarkable to observe that the variable principle plays no role in the discussion concerning scope. The occurrences of the Prop variables, which form a practical consequence of the principle, were not relevant. If they were omitted, which would bring us back to the original Bach & Cooper approach, then still the same problems would arise with respect to scope. So even without the variable principle a CN-S analysis appears to be required. This conclusion has to be relativized immediately. I have not given a formal proof that it is impossible to obtain a correct treatment of scope in the other analyses. I just showed that the CN-S analysis provides a direct basis for a semantic treatment of scope phenomena in a way that the considered T-S and Det-S analyses can not. This conclusion mentions another argument for relativizing. We only considered the three analyses which had our main interest. A lot more analyses are possible, and for some a correct treatment of scope may be possible. For instance, if the category of determiners contains variables for which a determiner can be substituted in a later stage, then a correct treatment of scope might be possible.

6.3. Discussion

In the previous section we observed that the framework of Montague grammar hardly restricts the possible syntactic analyses of relative clauses. In this section we investigated the possibilities for incorporating the available options in a somewhat larger fragment. It turned out that from the three main options only one was suitable. From this we learn that it is important to consider phenomena not only in isolation, but to design grammars for larger fragments. That for each isolated phenomenon there are many syntactic options available, gives us a firm basis for the hope that it is indeed possible to find a combination of syntactic constructions that fits together in a system yielding the correct semantics for the constructions involved.

Partee stated about the framework of Montague grammar, that 'it is an open question whether natural languages can so be described' (PARTEE 1973, p.55). The investigations in this article support my conviction that the framework of Montague grammar is very general and flexible, and I see no reason to have doubts about the possibility to describe natural languages within this system. In my opinion the framework of 'Universal Grammar' (MONTAGUE (1970)) is not a framework of which it can empirically be tested whether it underlies natural languges or not. It tells us how a grammar could be organized which aims to deal with both syntax and semantics. This conception of the framework is supported by the following facts:

1. The same framework (except for some technical details) has been developed independently for describing the syntax and semantics of programming languages: by a group called Adj (ADJ 1977). An application of Montague's framework, and therefore implicitly of Adj's framework, is described informally in JANSSEN & VAN EMBDE BOAS (1980).
2. Montague's framework allows to formalize rather divergent conceptions about the nature of natural languages. Examples are
   - (i) Sentences are plain strings having no internal structure. This conception is employed in PTQ.
   - (ii) Sentences have a tree structure. This conception is incorporated in Montague grammar by PARTEE (1973), and worked out in PARTEE (1979a,b), and BACH (1979).
   - (iii) Sentences have an underlying structure consisting of the frames used in functional grammar (DIK (1978,1980). In JANSSEN (1981) it is described how this conception can be incorporated in the framework.

The conception just described implies that the framework cannot be considered as a falsifiable framework. But this does not make the enterprise without challenge. Some experience learns that it is difficult enough to design a grammar for a larger fragment which produces only the correct sentences and assigns them all readings they should get. It are the predictions of such grammars which are falsifiable. Furthermore, within the framework one might formulate restrictions which have empirical content (e.g. along the lines of PARTEE (1979a,b)). But the framework as it is gives hardly any restrictions on the syntax.

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A THEORY OF TRUTH AND SEMANTIC REPRESENTATION

by

Hans Kamp

1. INTRODUCTION

Two conceptions of meaning have dominated formal semantics of natural language. The first of these sees meaning principally as that which determines conditions of truth. This notion, whose advocates are found mostly among philosophers and logicians, has inspired the disciplines of truth-theoretic and model-theoretic semantics. According to the second conception meaning is, first and foremost, that which a language user grasps when he understands the words he hears or reads. This second conception is implicit in many studies by computer scientists (especially those involved with artificial intelligence), psychologists and linguists - studies which have been concerned to articulate the structure of the representations which speakers construct in response to verbal inputs.

It appears that these two conceptions, and with them the theoretical concerns that derive from them, have remained largely separated for a considerable period of time. This separation has become an obstacle to the development of semantic theory, impeding progress on either side of the line of division it has created.

The theory presented here is an attempt to remove this obstacle. It combines a definition of truth with a systematic account of semantic representations. These two components are linked in the following manner. The representations postulated here are (like those proposed by others; cf. e.g. HENDRIX (1975) or KARTTUNEN (1976)) similar in structure to the models familiar from model-theoretic semantics. In fact, formally they are nothing other than partial models, typically with small finite domains. Such similarity should not surprise; for the representation of, say, an indicative sentence ought to embody those conditions which the world must satisfy in order that the sentence be true; and a particularly natural representation of those conditions is provided by a partial model with which the (model describing the) real world will be compatible just in case the conditions are fulfilled.

Interpreting the truth-conditional significance of representations in this way we are led to the following characterization of truth: A sentence S, or discourse D, with representation m is true in a model M if and only if M is compatible with m; and compatibility of M with m, we shall see, can be defined as the existence of a proper embedding of m into M, where a *proper embedding* is a map from the universe of m into that of M which, roughly speaking, preserves all the properties and relations which m specifies of the elements of its domain.

A theory of this form differs fundamentally from those familiar from the truth-theoretical and model-theoretical literature, and thus a substantial argument will be wanted that such a radical departure from existing frameworks is really necessary. The particular analysis carried out in the main part of this paper should be seen as a first attempt to provide such an argument. The analysis deals with only a small number of linguistic problems, but careful reflection upon just those problems already reveals, I suggest, that a major revision of semantic theory is called for.

The English fragment with which the analysis deals contains sentences built up from these constituents: common nouns, certain transitive and intransitive verbs (all in the third person singular present tense), personal and relative pronouns, proper names, and the particles *a, every,* and *if ... (then)*. These can be combined to yield the following sorts of compounds:

(i) complex singular terms such as *a man, every woman, a man who loves every woman, every woman whom a man who owns a donkey loves,* etc. (We can embed relative clauses inside others and there is no upper bound to the depth of embedding!);

(ii) singular terms - i.e. complex terms of the kind just exemplified, proper names and personal pronouns - can be combined with verbs to yield sentences;

(iii) sentences may be joined with the help of *if* to form larger sentences of conditional form; sentences serve moreover as the sources of relative clauses.

The choice of this fragment is motivated by two central concerns: (a) to study the anaphoric behaviour of personal pronouns; and (b) to formulate a plausible account of the truth conditions of the so-called 'donkey-sentences' (which owe their name to the particular examples in GEACH (1962), the work that kindled contemporary interest in sentences of this type). As these donkey-sentences will play a prominent role in the theory developed below, let me briefly review the problem that they have been taken to present. We shall concentrate on the following two instances:

(1) If Pedro owns a donkey he beats it

(2) Every farmer who owns a donkey beats it.

For what needs to be said at this point it will suffice to focus on (1). For many speakers, including the author of this paper, the truth conditions of (1) are those determined by the first order formula

(3) $(\forall x)(\text{Donkey}(x) \wedge \text{Owns}(\text{Pedro},x) \rightarrow \text{Beats}(\text{Pedro},x))$.

(As a matter of fact not all English speakers seem to agree that (3) correctly states the truth conditions of (1). Unfortunately an adequate discussion of diverging intuitions is not possible within the confines of the present contribution.)

The problem with (1) and (3) is that the indefinite description *a donkey* of (1) reemerges in (3) as a universal quantifier. How does an expression of a type which standardly (or so it always seemed) conveys existence manage to express universality in a sentence such as (1)? One way in which one might hope to explain this is by refering to the familiar equivalence between universal quantifiers with wide and existential quantifiers with narrow scope. Sentence (4), for instance, can be symbolized not only as (5) but also as (6).

(4) If Pedro owns a donkey he is rich

(5) $(\forall x)(\text{Donkey}(x) \wedge \text{Owns}(\text{Pedro},x) \rightarrow \text{Rich}(\text{Pedro}))$

(6) $(\exists x)(\text{Donkey}(x) \wedge \text{Owns}(\text{Pedro},x)) \rightarrow \text{Rich}(\text{Pedro})$.

Out of these two (6) would appear to be the 'natural' symbolization of (4) as it renders the indefinite *a donkey* as an existential quantifier. (5), we might be inclined to say, is adequate only for indirect reasons, viz. in virtue of its logical equivalence to (6). Note, however, that (1) cannot be captured by an analogue of (6). For in such a formula the scope of the existential quantifier would have to be restricted, just as it is in (6), to the antecedent alone; but then the quantifier would be incapable of binding the position corresponding to that occupied by *it* in the main clause of (1).

No one of the solutions to this problem that can be found in the existing literature strikes me as fully satisfactory. As I see the problem a proper solution should provide: (i) a general account of the conditional; (ii) a general account of the meaning of indefinite descriptions; and (iii) a general account of pronominal anaphora; which when jointly applied to (1) assign to it those truth conditions which our intuitions attribute to it. These requirements are met, I wish to claim, by the theory stated in the next two sections.

As earlier remarks implied, there are three main parts to that theory:

1. A generative syntax for the mentioned fragment of English (I have cast the syntax in a form reminiscent of the syntactic descriptions which are used by Montague; the reader may verify, however, that many other syntactic descriptions would be equally compatible with the remaining components of the theory);
2. a set of rules which from the syntactic analysis of a sentence, or sequence of sentences, derives one of a small finite set of possible non-equivalent representations; and
3. a definition of what it is for a map from the universe of a representation into that of a model to be a proper embedding, and, with that definition, a definition of truth.

The analysis thus obtained not only yields an account of the truth conditions of the donkey sentences (as well as of certain other notoriously problematic sentences which the fragment admits, such as e.g. some types of Bach-Peters sentences), it also reveals two more general insights concerning, respectively, personal pronouns and indefinite descriptions.

1) Personal pronouns, it has been pointed out, have a number of apparently distinct functions. Sometimes they seem to behave as genuinely referential terms, as e.g. the *he* in *Pedro owns a donkey. He beats it.* Sometimes, as the *him* of *Every man who loves a woman who loves him is happy,* they appear to do precisely what is done by the bound variables of formal logic. Yet another occurrence, noted in particular by EVANS (1977,1980), who coined the term 'E-type pronoun' for it, cannot be understood, or so it has been claimed, either on the model of a simple referential expression or on that of a bound variable. An example is the occurrence of *it* in *If Pedro owns a donkey he beats it.* The present theory brings out what these three different types have in common in that it offers, at the level of representation-formation a single rule which equally applies to each of them. This rule may interact

in various ways with other rules, which are associated with different syntactic constructions, and this gives rise to the seeming multiplicity of functions which the recent philosophical and linguistic literature has noted. (There are several pronoun uses, such as 'pronouns of laziness' and deictic pronouns, which have no instances within the fragment of English studied in this paper and which, therefore, cannot be discussed here. Such occurrences, however, can also be accomodated along the lines sketched in this paper.)

2) Indefinite descriptions are, on the account given here, referential terms, not existential quantifiers. When an indefinite has existential force it has that force in virtue of the particular role played by the clause containing it within the sentence or discourse of which it is part. It is true that the clausal roles which impose an existential, rather than a universal, reading upon indefinites are the more prominent; and this, I take it, has been responsible for the familiar identification of the indefinite article as a device of existential quantification. But that are not the only roles. The antecedent of a conditional, for instance, plays a role which is not of this kind; a simple clause which occurs in this role confers a universal interpretation on the indefinite descriptions it contains.

There is much that ought to be said about the conceptual implications of the present theory and about the range of its possible applications. But, as space is limited, I shall confine myself to a couple of brief remarks.

1) It should be stressed that truth as it is defined here applies not only to single sentences but also to multi-sentence discourse. This is of special importance where intersentential relations within the discourse (such as intersentential anaphoric links) contribute to its meaning. As will be seen below the links between anaphoric pronouns and their antecedents invariably have their impact on the discourse representation (irrespective of whether pronoun and antecedent occur in the same, or in different sentences) and thus on the truth conditions of the discourse, which the discourse representation embodies. Other intersentential relations, such as the relation which obtains between the sentences of past tense narratives on account of their sequential order - which is typically understood to convey the temporal relations between the events which the sentences report - can be encoded into the discourse representation with equal ease.

2) The role representations are made to play within the theory

developed in this paper places substantial constraints on their internal structure. (Careful reading of the subsequent sections will, I hope, confirm this assessment.) This is of particular significance if, as I have already more or less implied, discourse representations can be regarded as the mental representations which speakers form in response to the verbal inputs they receive. I should point out that the specific theory that is presented below does not render such identification essential. Even if the representations it posits are thought of as purely theoretical devices whose raison d'être is to be found solely in the contribution they make to an effective account of certain semantic properties of sentences and sentence complexes, the theory may merit comparison with other schemes of linguistic description which have been applied to the same phenomena. But this is not how I would like to see the proposal of this paper myself. I conjecture that the structures which speakers of a language can be non-trivially described as forming to represent verbal contents are, if not formally identical, then at least very similar to the representations here defined.

If this identification is legitimate then a theory of the sort I have tried to develop brings to bear on the nature of mental representation and the structure of thought, a large and intricate array of data relating to our (comparatively firm and consistent) intuitions about the truth-conditions of the sentences and sentence sequences we employ. I very much hope that along these lines it may prove possible to gain insights into the objects of cognitive operations, as well as into these operations themselves which are unattainable if these data are ignored, and which have thus far been inaccessible to psychology and the philosophy of mind precisely because those disciplines were in no position to exploit the wealth of linguistic evidence in any systematic fashion.

2. THE THEORY: INFORMAL PRELIMINARIES

2.1. Anaphoric Pronouns

The analysis of pronominal anaphora I shall sketch is informed by the conviction that the mechanisms which govern deictic and anaphoric occurrences of pronouns are basically the same. This is an intuition that has guided many recent theories of pronominal reference; inevitably the account given here will resemble some of these in various respects.[1]

Our point of departure will be the hypothesis that both deictic and anaphoric pronouns select their referents from certain sets of antecedently available entities. The two pronoun uses differ with regard to the nature of these sets. In the case of a deictic pronoun the set contains entities that belong to the real world, whereas the selection set for an anaphoric pronoun is made up of constituents of the representation that has been constructed in response to antecedent discourse.

About deixis I shall have no more to say in this paper. But a little more needs to be said about anaphoric pronouns before we can proceed to the detailed analysis of some particular pieces of discourse.

The strategies used in selecting the referents of anaphoric pronouns are notoriously complex; they usually employ background assumptions about the real world, "grammatical" clues, such as the requirement of number and gender agreement between the anaphor and its antecedent, and the order in which the potential referents were introduced by the preceding discourse.[2]

The integration of these various factors often involves, moreover, what seem to be quite intricate patterns of inference. Efforts to understand these strategies have claimed much thought and hard work, but, in its general form at least, the problem appears to be far too complex to permit solution with the limited analytic tools that are available at the present time.[3]

About the strategies I shall have nothing more to say. Our concern will be, rather, with the sets of referential candidates from which they select. These entities will constitute the universes of the representations of which I spoke in Section 1. I have already said that these discourse representations, or DR's as I will call them for short, are formed in response to the discourses they represent and that their formation is governed by certain rules. These rules - and this is a new, and crucial, assumption of the theory - operate on the syntactic structures of the sentences of the discourse, and it is via them that syntactic form determines what the resulting DR will be like. This determination is not complete however. The syntactic structure does not, for instance, determine the anaphoric links between pronouns and their antecedents, which the DR makes explicit.

Most of the real work that the present theory will require us to do concerns the exact formulation of the rules of DR-formation. The exact formulation of these rules will be rather compact, and will betray, I suspect, little of either motivation or empirical implications to any but the initiated. I have decided therefore to first present a number of applications of the theory. I hope that if we proceed in this manner its formal features will reveal themselves more naturally and that the subsequent reading of the exact definitions in Section 3 will thus be less disagreeable than it would be without such preparation.

Let us begin by considering the two sentence discourse:

(7) Pedro owns Chiquita. He beats her.

The DR for the first sentence of (7) will contain two elements, call them *u* and *v*, which represent, respectively, Pedro and Chiquita, and furthermore the information that the first of these, *u*, owns the second, *v*. Schematically we shall represent this information as follows:

$m_1(7)$

u	v
.	.
Pedro owns Chiquita	
u = Pedro	
v = Chiquita	
u owns v	

To incorporate the information contained in the second sentence of (7) we must extend structure $m_1(7)$. But to do that we must make two decisions, regarding the reference of, respectively, *he* and *her*. It is natural to understand *he* as referring back to *Pedro* and *her* as referring back to *Chiquita*. Let us agree to interpret the pronouns in this way and to expand $m_1(7)$ accordingly. What we get is:

m(7)

u	v
.	.
Pedro owns Chiquita	
u = Pedro	
v = Chiquita	
u owns v	
He beats her	
u beats her	
u beats v	

I said that linking *he* with *Pedro* and *her* with *Chiquita* yields what seems the most natural reading of (7). "But", you might ask, "what *other* readings could (7) have?". The answer to that question depends on the setting, or context, in which (7) is supposed to be used. If (7) were uttered by a speaker who points at some individual other than Pedro while saying *he,* or at some being distinct from Chiquita when he says *her,* the gesture would recruit this demonstrated individual as referent for the pronoun. Similarly, if (7) were part of a larger discourse *he* or *her* could conceivably refer back to some other individual introduced by an earlier part of that discourse; and this could result in a genuine referential ambiguity. However, if (7) is used by itself, i.e., without preceding verbal introduction, and also in the absence of any act of demonstration, then - and this is another important hypothesis of our theory - there are no other potential referents for *he* and *her* than the discourse referents which have been introduced in response to *Pedro* and *Chiquita.* Let us agree that henceforth (except where the contrary is indicated explicitly) all our examples of simple and multi-sentence discourses shall be understood in the last of these three ways, i.e., as used without accompanying deictic gestures and not preceded by any related discourse.

Even when we understand (7) in this third way its anaphoric links are not fully determined by what we have said. For why cannot *he* and *her* both refer to *u*, say, or *he* to *v* and *her* to *u*? The reason is of course obvious: *he* must refer to a male individual, and *her* to a female one. But, obvious as the determining principle may be, it is not quite so easy to state it in a form that is both general and accurate. For what is it that determines an antecedently introduced discourse referent as male, rather than female, or neither male nor female? (7) allows us to infer that *u* is male because we know that *Pedro,* typically, refers to male individuals. But often the antecedent term which led to the introduction of a discourse item is not quite so explicit about the gender of its referent. Consider for example such terms as: *Robin, Hilary, the surgeon, the president, an officer in the Air Force, the professor, the professor's secretary, the first inhabitant of this cave.* Often we can do no better than guess whether the referent is male or female, or human or non-human. Some of these guesses are more educated than others. And not infrequently where the anaphoric link between the antecedent and some particular pronoun is clear on independent grounds it is in fact the gender of the pronoun which resolves the uncertainty.[4]

Applying the principle of gender agreement will thus often involve drawing various inferences from the information that is given explicitly; and as in all other processes where inference can be involved, there appears to be no clear upper bound to its potential complexity.

There is a further complication that an exact statement of the principle must take into account. The gender of the pronoun that is used to refer to a certain object is not exclusively determined by the nature of that object, but, to some extent, also by the actual *form* of the anaphoric antecedent which made it available as a referent. Thus let us suppose that the name *Chiquita* in (7) actually refers to a donkey. In most situations we refer, or at any rate may refer, to a donkey by means of *it*. But in a discourse such as (7) this would be inappropriate. The name *Chiquita* highlights, one might wish to say, the fact that its referent is female, and this makes *she* the correct resumptive pronoun. But nonetheless the task of giving even an approximate formulation of the principle appears to be well beyond our present means. In what follows we shall ignore the principle of gender agreement, just as we ignore all other factors that help to disambiguate the reference of anaphoric pronouns. But where, in subsequent examples, the need for gender agreement clearly excludes certain anaphoric links I shall not bother to mention those without referring to the principle explicitly.

Clearly (7) is true, on the reading of it that is given by m(7) if and only if the real Pedro stands to the real Chiquita in a relation of ownership and also in the relation expressed by the verb *beat*. Put differently, if M is a model, representing the world - consisting of a domain U_M and an interpretation function F_M which assigns to the names *Pedro* and *Chiquita* members of U_M and to the transitive verbs *own* and *beat* sets of pairs of such members - then (7) is *true in* M iff the pair $\langle F_M$ (*Pedro*), F_M (*Chiquita*)$\rangle$ belongs both to F_M (*own*) and to F_M (*beat*). Moreover, the right hand side of this last biconditional is fulfilled if there is a map f of the universe of m(7), i.e. the set {u,v}, into U_M so that all specifications of m(7) are satisfied in M - i.e., f(u) is the individual denoted in M by *Pedro*, f(v) is the individual F_M (*Chiquita*), and it is true in M that f(u) both owns and beats f(v), in other words, that $\langle f(u),f(v)\rangle$ belongs to both F_M (*own*) and F_M (*beat*).

Let us now consider

(8) Pedro owns a donkey. He beats it.

The first sentence of (8) induces a DR that can be represented thus:

$m_1(8)$

u	v
.	.
Pedro owns a donkey	
u = Pedro	
u owns a donkey	
donkey (v)	
u owns v	

Once again there is no choice for the anaphoric antecedent of either *he* or *it* in the second sentence of (8). So the complete DR of (8) becomes:

m(8)

u	v
.	.
Pedro owns a donkey	
u = Pedro	
u owns a donkey	
donkey (v)	
u owns v	
He beats it	
u beats it	
u beats v	

(8) is true in the model M provided there is an element d of U_M such that $\langle F_M$ (*Pedro*), d$\rangle$ belongs to both F_M (*own*) and F_M (*beat*); and furthermore d is a donkey in M - formally $d \in F_M$ (*donkey*), if we assume that common nouns are interpreted in the model by their extensions. This condition is fulfilled if there is a map g from $U_{m(8)}$ (= {u,v}) into U_M which preserves all conditions specified in m(8). Note that g(v) is not required to be the bearer in M of some particular name, but only to belong to the extension of the noun *donkey*.

Before turning to the donkey sentences (1) and (2) of Section 1.2 let us take stock of some principles applied in the construction of the DR's which we have encountered so far:

(1) Certain singular terms, among them proper nouns and indefinite descriptions, provoke the introduction of items into the DR that function as the 'referents' of these terms. We shall later address the question which singular terms give rise to such introductions and whether these introductions are obligatory or optional.

(2) Other singular terms, viz. personal pronouns, do not introduce elements into the DR; instead they can only refer to items which the DR already contains.[5]

2.2. Conditionals

Our next aim is to construct a representation for the 'donkey sentence' (1), which for convenience we repeat here:

(1) If Pedro owns a donkey he beats it.

Before we can deal with (1) however, we must say something about conditionals in general.

The semantic analysis of natural language conditionals is a notoriously complicated matter, and it seems unlikely that any formally precise theory will do justice to our intuitions about all possible uses of sentences of this form. The literature on conditionals now comprises a number of sophisticated formal theories, each of which captures some of the factors that determine the meaning of conditionals in actual use.[6] Although these theories differ considerably from each other they all seem to agree on one principle, namely that a conditional

(9) If A then B

is true if and only if

(10) Every one of a number of ways in which A can be true constitutes, or carries with it, a way of B's being true.

Up to now this principle has generally been interpreted as meaning that B is true in, or is implied by, every one of a certain set of *relevant possible situations* in which A is true. (This is true in particular of each of the theories mentioned in the last footnote.) The analysis of truth in terms of DR-imbeddability, however, creates room for a slightly different implementation of (10).

Where M is a model and m a DR for the antecedent A there may be various proper embeddings of m into M, various ways, we might say, of showing that A is true in M. This suggests another interpretation of (10), viz. that each such way of verifying A carries with it a verification of B. In what sense, however, could such a way of verifying A - i.e. such a proper embedding of m - entail a verification of B? To verify B, in that sense of the term in which we have just been using it, we need a representation of B; but as a rule the content of B will not be represented in the DR m of A. To verify B in a manner consistent with some particular verification of A we must therefore extend the DR m involved in that verification to a DR m' in which B is represented as well. Thus we are led to an implementation of (10) according to which the conditional (9) is *true, given* a pair (m,m'), consisting of a DR m of A and an extension m' of m which represents B as well, iff

(11) every proper embedding of m can be extended to a proper embedding of m'.[7]

This is not yet an explicit statement of the truth conditions of (9), for it fails to tell us anything about the target structures of the verifying embeddings, and about their relation to the situation, or model, with respect to which (9) is evaluated. Here we face all the options that have confronted earlier investigators. We may elaborate (11) by stipulating that (9) is true in a model M iff every proper embedding of m into M is, or is extendable to, a proper embedding of m' on M. Or we may insist that (9) is true in the possible world w iff every proper embedding of m into any of the (models representing the) nearest A-worlds induces some proper embedding m' into that world. Indeed, any one of the existing theories could be combined with the principle conveyed by (11).

Here we shall, primarily for expository simplicity, adopt the first of the options mentioned:

Let m be a DR of A and m' an extension of m which incorporates the content of B. Let M be a model. Then *if* A *then* B is true in M, given (m,m'), iff

(12) every proper embedding of m into M can be extended to a proper embedding of m' into M.

For conditionals in which there are no anaphoric links between antecedent and consequent, (12) boils down to the truth conditions for the material conditional. But where such a link exists its implications are somewhat different. To see this let us apply the condition to (1). We have already constructed DR's of the kind needed in the application of (12) to (1), namely $m_1(8)$, and $m(8)$. According to (12), (1) is true in M given $(m_1(8), m(8))$, iff every function f from $U_{m_1(8)}$ (= {u,v}) into U_M such that

(i) $f(u) = F_M$ (*Pedro*), (ii) $f(v) \in F_M$ (*donkey*), and (iii) $\langle f(u), f(v)\rangle \in$ $\in F_M$ (*own*), can be extended to a function g from $U_{m(8)}$ into U_M such that $\langle g(u), g(v)\rangle \in F_M$ (*beat*). Of course, in the present case $U_{m(8)} = U_{m_1(8)}$ and consequently there is no question of *extending* f to g. So the above condition reduces to the stipulation that every f as described has the additional property that $\langle f(u), f(v)\rangle \in F_M$(*beat*). Clearly this condition is equivalent to the truth in M of the formula (3) which we adopted in Section 1.2 as giving the truth conditions of (1).

It is easy enough, however, to come up with examples which do involve the extension of embeddings, e.g.:

(13) If Pedro owns a donkey he lent it to a merchant.

If we extend $m_1(8)$ to a DR which incorporates the content of the consequent of (13) we get something like:

m(13)

u	v	w
.	.	.
Pedro owns a donkey		
u = Pedro		
u owns a donkey		
donkey (v)		
u owns v		
he lent it to a merchant		
u lent it to a merchant		
u lent v to a merchant		
merchant (w)		

In relation to $m_1(8)$ and m(13), (12) requires that every mapping f of the kind described in the preceding analysis of (1) can be extended to a function g from {u,v,w} into U_M such that – if we assume for simplicity that *lent to* is interpreted in M as a set of ordered triples of members of U_M – (i) $g(w) \in F_M$ (*merchant*); and (ii) $\langle g(u), g(v), g(w)\rangle \in F_M$(*lent to*).

2.3. Universals

One of the important insights that went into Frege's discovery of the predicate calculus was that the restricted quantification typical of natural language is expressible in terms of unrestricted quantifiers and truth functions. Our handling of indefinite descriptions, which formal logic treats as expressions of existential quantification, harmonizes with this insight. For, as can be seen for instance from $m_1(8)$, the introduction of a discourse referent u for an indefinite term is accompanied by two conditions, one to the effect that u has the property expressed by the common noun phrase of the term, and the other resulting from substituting u for the term in the sentence in which it occurs.

I wish to propose a treatment of terms of the form *every α* that is in similar accord with Frege's analysis of restricted universal quantification. Again it will be easier to illustrate the proposal before I state it. Consider:

(14) Every widow admires Pedro.

A representation for (14), like those for conditional sentences, involves a pair of DR's. The first of these states that some 'arbitrary' item x satisfies the common noun *widow*; the second extends this DR by incorporating the content of the condition *x admires Pedro*. Thus we obtain:

$m_1(14)$

x
.
widow (x)

$m_2(14)$

x
.
widow (x)
x admires Pedro
u = Pedro
x admires u

The truth value of (14) in M is to be determined by $(m_1(14), m_2(14))$ in precisely the same way as that of (1) is determined by $(m_1(8), m(8))$. Thus (14) is true iff every correlation of x with an element a of U_M such that $a \in F_M(\textit{widow})$ can be extended to a proper embedding of $m_2(14)$, i.e., to a function g such that $g(u) = F_M(\textit{Pedro})$ and $\langle g(x), g(u)\rangle = \langle a, g(u)\rangle \in$ $\in F_M(\textit{admires})$. Clearly this confers upon (14) the intuitively correct truth conditions.

In the same way

(15) Every widow admires a farmer

licenses the construction of the following pair of DR's:

$m_1(15)$

x
.
widow (x)

$m_2(15)$

x	u
.	.
widow (x)	
x admires a farmer	
farmer (u)	
x admires u	

Again the condition that every association of x with an object a that is a widow in the sense of M can be extended to a proper embedding of $m_2(15)$ gives the correct truth conditions of (15); or, to be precise, the truth conditions it has on what is generally considered its most natural reading.

Consider now the second donkey sentence of Section 1.2:

(2) Every farmer who owns a donkey beats it.

Sentence (2) gives rise to the following pair of DR's:

$m_1(2)$

x
.
farmer (x)
x owns a donkey
donkey (v)
x owns v

$m_2(2)$

x	v
.	.
farmer (x)	
x owns a donkey	
donkey (v)	
x owns v	
x beats it	
x beats v	

So (2) is true in M iff every f such that $f(x) \in F_M(\textit{farmer})$, $f(v) \in$ $\in F_M(\textit{donkey})$, and $\langle f(x), f(v)\rangle \in F_M(\textit{own})$ has the additional property that $\langle f(x), f(v)\rangle \in F_M(\textit{beat})$. This is exactly as it should be.

Our treatment of conditionals and universal sentences gives - for the cases, at any rate, that we have thus far considered - intuitively correct conditions of truth. But it seems at odds with the *general* definition of truth which I put forward earlier, according to which a discourse is true in M, given some representation m of it, iff *there is some* proper embedding of m into M. The semantic analyses of the sentences we have considered in this section refer to pairs of DR's rather than single DR's and involve conditions on *all* proper embeddings of a certain kind, instead of demanding the existence of at least one proper embedding.

To resolve this apparent conflict I must say a little more about the intuitive ideas behind the DR constructions of which we have now seen a few instances. Essential to the analysis of the majority of our examples was the way in which we have treated indefinite descriptions. It would be quite unsatisfactory if there were no other justification for that treatment than the observation that, combined with additional principles for DR-construction they give the truth conditions that speakers in fact associate with the sentences we have sampled. There is, however, a reason why we should *expect* a construction principle for indefinites such as we have applied, but no direct analogue of it for phrases of the form *every* α. Let us go back to the first sentence of (8). What justifies us in adding to the partial DR of (8) the element v as a 'referent' for *a donkey* is this: as I already argued, the DR of a sentence functions as a partial description of how the world ought to be if the sentence is true. To fulfill that role the DR must represent whatever information has been encoded into it in such a way that the significance of that representation is unaffected when one extends it to incorporate further information - or, what comes in this connection to much the same, when the DR is identified as a certain substructure of a larger 'real world' model via some proper embedding. The conditions u = *Pedro*, *donkey*(v) and u *owns* v which make up $m_1(8)$ clearly satisfy this requirement. They convey precisely the same information in any extension of $m_1(8)$ as they do in $m_1(8)$ itself.[8] The content of an existential sentence has been exhausted once an individual has been established which satisfies the conditions expressed by the indefinite description's common noun phrase and by the remainder of the sentence.

But a universal sentence cannot be dealt with in such a once-and-for-all manner. It acts, rather, as a standing instruction: of each individual check whether it satisfies the conditions expressed by the common noun phrase of the universal term; if it does, you may infer that the individual also satisfies the conditions expressed by the remainder of the sentence. This is a message that simply *cannot* be expressed in a form more primitive than the universal sentence itself. The universal is thus, at the level of the DR to which it belongs, *irreducible*. The same is true of conditionals. *If* A *then* B functions as an instruction to check, and keep checking, whether the antecedent A has been satisfied, and to infer, when this is found to be so, that the consequent B must also hold. This too is a piece of information that cannot be represented in any more elementary form.

This means that when we form the DR of a universal sentence, such as (14), or of a conditional, such as (1), we cannot decompose the sentence in some such fashion as we were able to decompose, say, the first sentence of (8) when constructing $m_1(8)$. So the DR for (14) cannot itself be elaborated beyond the trivial initial stage:

$m_0(14)$

Every widow admires Pedro

in which the sentence (14) occurs as a condition, but nothing else does.

There is however, another way in which we can represent the internal structure of (14), namely by constructing separate DR's for its components, and by integrating these DR's into a structure in which their connection reflects the syntactic construction by means of which these different components are amalgamated into the complex sentence. This is, in fact, essentially what I did when constructing the DR-pairs I earlier presented for (1), (14), (15), and (2).

But these pairs do not provide, by themselves, the structural representations to which we can apply our general definition of truth. To obtain such a representation for, say, (14) we must combine the pair $(m_1(14), m_2(14))$ with the DR $m_0(14)$. This gives us the following structure:

K(14)

$m_0(14)$

Every widow admires Pedro

$m_1(14)$

x
.
widow (x)

$m_2(14)$

x	u
.	.
widow (x)	
x admires Pedro	
x admires u	

Similarly the complete representation for (1) will now look thus:

K(1)

$m_0(1)$

If Pedro owns a donkey, he beats it

$m_1(1)$	$m_2(1)$
u v	u v
. .	. .
Pedro owns a donkey	Pedro owns a donkey
u = Pedro	u = Pedro
donkey (v)	donkey (v)
u owns v	u owns v
	He beats it
	u beats it
	u beats v

It may appear as if something is still missing from these structures. For what tells us that the subordinate DR's $m_1(1)$ and $m_2(1)$ represent the antecedent and consequent of a conditional, while $m_1(14)$ and $m_2(14)$ represent the components of a universal? The answer to this is simple: the necessary information is provided by the sentences in $m_0(1)$ and $m_0(14)$ whose components are represented by the subordinate DR's $m_1(1)$, $m_2(1)$, and $m_1(14)$, $m_2(14)$. In fact we shall assume that with each syntactically well-formed sentence is given a particular syntactic analysis of it, which specifies unambiguously its immediate components and the construction which forms the sentence out of these. (For the fragments we shall study in Section 3, this condition will be automatically fulfilled as each of its well-formed expressions has a unique syntactic analysis.) The role which, say, $m_1(1)$ and $m_2(1)$ play in the representation of (1) can thus be recognized by comparing their relevant entries, viz., *Pedro owns a donkey* and *he beats it,* with the syntactic analysis of the sentence (1) to be found in $m_0(1)$. All this will be discussed in detail in Section 3.

A representation of the sort just displayed, which involves structured families of DR's, will be called a *Discourse Representation Structure* or, for short, DRS. Each sentence or discourse induces the construction of such a DRS, and only where the sentence or discourse is comparatively simple will the DRS consist of a single DR only. Among the DR's that constitute a DRS there will always be one which represents the discourse as a whole. (In the two DRS's we displayed these are, respectively, $m_0(14)$ and $m_0(1)$.) This DR will be called *the principal* DR of the DRS.

Once we assign to (1) the DRS K(1) the earlier conflict between the general definition of truth and our particular account of the truth value of a conditional can be resolved. We slightly modify the truth definition to read:

(16) D *is true in* M, *given* the DRS K iff there is a proper embedding into M of the principal DR of K.

Let us try to apply (16) to (1) and its DRS K(1). (1) is true given K(1) iff there is a proper embedding of $m_0(1)$ into M. Since the universe of $m_0(1)$ is the empty set, there is only one embedding from $m_0(1)$ into M, viz. the empty function, Λ. What is it for Λ to be proper? Λ is proper iff the conditions of $m_0(1)$ are true in M of the corresponding elements of U_M. In the present case however there are no elements in $U_{m_0(1)}$, thus no corresponding elements of U_M; and there is only one condition in $m_0(1)$, namely (1) itself. Thus Λ is proper iff (1) is true in M.

It might seem at this point that we are trapped in a circle. But in fact we are not. To see that we are not it is necessary to appreciate the difference between (i) asking for the truth value in M of (1), given K(1); and (ii) asking for the truth value in M of some condition that belongs to some member of K(1). This second question has, as we saw earlier, a straightforward answer when the condition has the form of an atomic sentence. For in that case it is directly decided by the embedding and the function F_M. But when the condition is a complex sentence, e.g., a conditional or a universal, which permits no further analysis *within the very DR to which it belongs,* the answer involves an appeal to certain members of the DRS that are *subordinate* to that DR. Thus the condition (1) of $m_1(1)$ is to be taken as true in M iff it is true, in the sense defined earlier, *given* the pair $(m_1(1), m_2(1))$ of DR's subordinate to $m_0(1)$; and in *that* sense (1) is true in M, we saw already, iff M verifies the first order formula (3).

To see more clearly how the various components of our theory are to be fitted together, we should look at a few more examples.

The next example shows why it is that certain anaphoric connections are impossible. In

(17) If Pedro owns every donkey then he beats it.

it cannot have *every donkey* for its antecedent. The reason for this becomes transparent when we try to construct a DRS which gives such a reading to (17):

K(17)

m(17)

If Pedro owns every donkey he beats it.

$m_1(17)$	$m_2(17)$
u . Pedro owns every donkey u = Pedro u owns every donkey	u . Pedro owns every donkey u = Pedro u owns every donkey he beats it u beats it

$m_3(17)$	$m_4(17)$
x . donkey (x)	x . donkey (x) u owns x

We cannot complete this DRS as intended, for the discourse referent x, which we want to assign to the pronoun *it* of $m_2(17)$, is not available, as it occurs only at the level of $m_3(17)$, which is below that of $m_2(17)$. A similar explanation shows why *it* cannot be anaphorically linked to *every donkey* in

(18) Every farmer who owns every donkey beats it

and also why in

(19) If Pedro likes every woman who owns a donkey he feeds it

it cannot be co-referential with *a donkey*, whereas such a link does seem possible in

(20) If Pedro likes a woman who owns a donkey he feeds it.[9]

These last examples give, I hope, an inkling of the predictive powers of what in particular linguists might think constitutes the most unusual feature of the theory I have so far sketched: the fact that it handles singular terms of the forms *a* β and *every* β in entirely different ways. I hope that these and subsequent illustrations will help to persuade them that the conception of a perfect rule-by-rule parallelism between syntax and semantics is one that must be proved rather than taken for granted.[10] In fact, the data here presented point towards the conclusion that this conception is ultimately untenable.

Another feature that distinguishes the present account from many, albeit not all, existing theories of reference and quantification is its entirely uniform treatment of third person personal pronouns. This has already been apparent from the examples at which we have looked. It is further illustrated by such sentences as:

(21) Every farmer courts a widow who admires him.

Occurrences such as that of *him* in (21) have been put forward as paradigms of the use of pronouns as bound variables - an identification that is natural, and in fact well-nigh inescapable, when one believes that the logical forms of natural language sentences are expressions of the predicate calculus. Indeed several earlier theorists have perceived a real chasm separating these pronoun uses from those which we find exemplified by, say, *her* in (7) and *he* in (7) and (8); and, looking at pronouns from this perspective, they have often felt helpless vis-a-vis the pronoun occurrences that have been of particular concern to us in this section, viz. those exemplified by (1) and (2). Forcing these either into the mold that had been designed for uses such as that in (7), or into that measured to fit occurrences such as that of *him* in (21) turned out to be hopeless enterprises. EVANS (1977), (1980) gives conclusive evidence against the latter of these two; but his own suggestions, which go some way towards assimilating the problematic pronouns to definite descriptions, do not appear to be fully satisfactory either.[11]

Note that the more unified treatment of these pronoun uses given here is possible partly because the same construction rule for pronouns operates both at the level of the principal DR's and at subordinate levels. Thus the DRS for (21) is constructed as follows (the numbers in parentheses which precede discourse referents and conditions indicate the order in which the operations are carried out; we shall often use this notational device):

$m_0(21)$

(0) Every farmer courts a widow who admires him.

$m_1(21)$

(1)	x
	.
(1)	farmer (x)

$m_2(21)$

(1)	x	(2)	y
	.		.
(1)	farmer (x)		
(1)	x courts a widow who admires him		
(2)	widow (v)		
(2)	v admires him		
(2)	x courts v		
(3)	v admires x		

The rule for pronouns applies here in just the same way to the *him* of *v admires him* in $m_2(21)$ as it does for example to the *he* and *it* in the DRS construction of (8) or the *it* of (1) in the construction of the DR of (1).

3. THE FORMAL THEORY

3.1. Syntax

The time has come for a more formal and systematic presentation. We shall consider a fragment of English for which I shall give an explicit syntax and explicit formal rules for DRS construction. Our fragment will be exceedingly simple to start with, much simpler even than that of MONTAGUE (1973).[12] The syntax adopted resembles Montague's, but the resemblance is rather superficial; for the syntactic analysis of a sentence will play a much more modest role in the determination of its interpretation than it does in Montague grammar. In presenting the syntax I shall presume some familiarity with Montague grammar, specifically with MONTAGUE (1970a) and (1973). Our fragment, to which I shall refer as L_0, contains expressions of the following categories with the following basic members:

1) T	(Term)	: *Pedro, Chiquita, John, Mary, Bill,... he, she, it*
2) CN	(Common Noun phrase)	: *farmer, donkey, widow, man, woman,...*
3) IV	(Intransitive Verb phrase)	: *thrives...*
4) TV	(Transitive Verb)	: *owns, beats, loves, admires, courts, likes, feeds, loathes,...*
5) S	(Sentence)	: --
6) RC	(Relative Clause)	: --

<u>Formation Rules</u>

FR1. If $\alpha \in$ TV and $\beta \in$ T then $\alpha\beta' \in$ IV where β' = *him* if β = *he*, β' = *her* if β = *she* and $\beta' = \beta$ otherwise.

FR2. If $\alpha \in$ IV and $\beta \in$ T then $\beta\alpha \in$ S.

FR3. If $\alpha \in$ CN then (i) *a(n)* α, and (ii) *every* α are in T.

FR4.k If $\phi \in$ S and the k-th word of ϕ is a pronoun then $\beta\phi' \in$ RC, where ϕ' is the result of eliminating the k-th word from ϕ and β is *who, whom, which,* according as the pronoun is *he* or *she, him* or *her,* or *it,* respectively.

FR5. If α is a basic CN and $\beta \in$ RC then $\alpha\beta \in$ CN.

FR6. If $\phi,\psi \in$ S then <u>if</u> ϕ,ψ and <u>if</u> ϕ <u>then</u> $\psi \in$ S.

<u>Some comments</u>

1) The rule schema FR4.k is defective inasmuch as it allows for wh-movement out of forbidden positions. Within the present fragment there are only two sorts of noun phrase positions to which wh-movement may not apply, those inside relative clauses and those inside the antecedents of conditionals. It is not difficult to modify the syntax in such a way that these restrictions are observed. For instance we could stipulate that each time a relative clause is formed all pronouns it contains are marked, and that the same is done to those occurring in the antecedent of a conditional at the time when antecedent and consequent are joined together. The rule of relative clause formation can then be altered so that it applies to unmarked pronouns only. Such a solution is rather ad hoc, so as it would moreover complicate the syntax as a whole, I have afrained from incorporating it. I must beg the reader to keep in mind that the syntax of this section is intended as no more than a convenient basis for the definition of DRS-construction rules, and that it has no pretensions of capturing important syntactic generalizations.[13]

2) The present fragment differs from most familiar versions of Montague grammar in that it contains neither variables nor indexed pronouns.[14] Consequently the syntactic analysis of a sentence of the present fragment tells us nothing about anaphoric relations.

3) Every well-formed expression of L_0 has a unique syntactic analysis. This is a feature that is bound to be lost at some point as we extend the present fragment. It allows us, however, to omit, while uniqueness of syntactic analysis obtains, all explicit reference to syntactic analyses in discussions and, particularly, in definitions where such reference becomes essential as soon as well-formed strings do not unambiguously determine their analyses.

4) When defining the process of DRS construction we shall have to specify the order in which various parts of a given sentence are to be treated. What we need here is, in essence, a specification of scope order. I shall assume in this paper that the scope relations within a sentence are directly determined by its syntactic construction. Thus the subject term of a simple clause will always have wide scope over the object term; the *if* of a conditional sentence will always have wide scope over the terms occurring in antecedent and consequent, etc. Let us call the formation rule which is applied last in the construction of an expression γ the *outermost rule of* γ. Where γ is a sentence and the outermost rule is FR6, γ is called a *conditional (sentence)*. If the outermost rule of γ is FR1 or FR2 and this rule forms γ by combining some IV or TV with the term α, α is said to *have*, or to *be the term with, maximal scope in* γ. If the outermost rule is FR1 and α begins with *every*, γ is called a *universal IV*; similarly, if the outermost rule of γ is FR2 and α begins with *every*, then γ is called a *universal sentence*.

By eliminating Montague's rule of substitution and quantification we have dispensed with one natural way of distinguishing between alternative scope relations - such as, for instance, the two possible relations between *a widow* and *every farmer* in

(22) A widow admires every farmer.

Sentence (22) can be generated in only one way and according to that generation the subject has wide scope over the direct object as it enters the construction of the sentence at a later stage. No syntactic analysis would thus appear to convey upon (22) the reading given by

(23) $(\forall x)(\text{farmer}(x) \rightarrow (\exists y)(\text{widow}(y) \wedge \text{admires}(x,y)))$.

It might be thought that the construction of a DRS which imposes this latter reading upon (22) involves an order of application of the construction rules which contravenes the scope relations implied by the syntax. This problem too must be left for another paper.

5) We shall refer to the basic terms *Pedro, Chiquita, John, Mary,...* as the *proper names* of L_0 and to *he, she, it* as the *pronouns* of L_0. Terms of the form *every* β will be called *universal terms.*

6) I have admitted only compound common noun phrases consisting of a common noun and *one* relative clause. It would of course be possible to relax FR6 so that it can attach several relative clauses to the same head noun. Many of the resulting expressions, however, seem marginal at best. I have decided to cut the knot and keep such complex common nouns out of the fragment altogether.

3.2. Models and Discourse Representation

By a *model for* L_0 we shall understand a structure of the form $\langle U,F\rangle$ where (i) U is a non-empty set and (ii) F is an interpretation function which assigns an element of U to each of the proper names of L, a subset of U to each of its basic CN's and basic IV's, and a set of pairs of elements of U to each of the basic TV's.

We must now address ourselves to the main tasks of this section, the formulation of the rules of DRS-construction and of the definition of truth for L_0. To state the rules we shall have to decide on a format for DR's and DRS's. In choosing such a format I have been partly guided by considerations of notational convenience. In particular it is just a matter of convenience to specify (as I have already done in the examples discussed in the preceding section) that one or more discourse referents satisfy a certain predicate by adding to the relevant DR a sentence which is obtained by combining that predicate with, in the appropriate positions, these referents themselves; using them, that is, autonymously (a policy against which there can be no objection, given the symbolic nature which must be attributed to the discourse referents in any case). Almost all other features, however, of the DR-format I have chosen are determined by empirically significant aspects of the rules of DRS-construction.

Let V be a denumerable set of entities none of which is a basic expression of L_0 or a string of such expressions. V is the set from which the elements are drawn that make up the universes of the DR's. We shall often refer to the members of V as *discourse referents.* For any subset X of V let $L_0(X)$ be the result of adding the members of X to the set of basic terms of L_0. Where M is model for L_0 and $X \subseteq V$ there is a canonical way of expanding M to a model M' of $L_0(X)$ viz. by adding to the interpretation function F_m the pairs $\langle u,u \rangle$ for all $u \in X$. In the sequel we shall not bother to differentiate notationally between these two models and thus write "M" where strictly speaking we ought to have put "M'".[15]

As all our earlier examples showed, the introduction of a discourse referent is always accompanied either by a condition which identifies it as the referent of a proper name or else by one which stipulates that it satisfies some common noun. These conditions cannot be expressed in $L_0(X)$; so we must slightly extend the notation which that language provides. We shall allow in addition to what $L_0(X)$ contains already, sentences of the form $u = \alpha$ where α is a proper name and $u \in X$, to express the former, and sentences of the form $\beta(u)$ where, again, $u \in X$ and $\beta \in CN$, to express the latter type of condition. We shall refer to the language obtained from $L_0(X)$ through these additions as $L_0'(X)$.

We shall limit ourselves here to the simplest type of discourse, that of a discourse constituted by a finite sequence of declarative statements, made by one and the same speaker. Formally we shall idenfity - as in fact we already did implicitly in Section 1.2 - such a discourse with the sequence of the uttered sentences. So let us, where L is any language, define an L-*discourse* to be any finite string of sentences of L.

The examples we considered in the preceding section were carefully chosen so that the same singular term would never occur more than once. This made it unnecessary to distinguish between different occurrences of the same expression. In general, however, different occurrences must be kept apart. The need for this is most obvious in connection with pronouns - it is only too common a phenomenon that the very same pronoun occurs twice in a bit of discourse, but each time refers to a different individual, as e.g. might be intended by someone using the sentence

(24) If Bill courts a widow who admires hím then Pedro courts a widow who admires hím.

But in longer stretches of discourse other expressions are liable to recur as well. Although the DRS construction rules defined below only require us to keep track of the individual occurrences of certain expressions, little if anyting would be gained by introducing a mechanism for distinguishing just *those* individual occurrences. In fact probably the simplest way to distinguish the individual expression occurrences is this: Let $D = \langle \phi_1, \ldots, \phi_n \rangle$ be an L_0-discourse and let $\langle \tau_1, \ldots, \tau_n \rangle$ be the sequence of the (uniquely determined) syntactic analyses of the sentences of D. It is easy to formulate an algorithm which assigns a unique index, - say, a positive integer - to each of the nodes of these analyses, and, by proxy, also to the expressions formed at any such node. For instance we enumerate first all the nodes of τ_1, in some order fixed by its structure, then those of τ_2, etc., until we have dealt with the entire discourse. There is no point to go into greater detail here. We shall simply assume that one such algorithm has been fixed. By an *occurrence* of an expression α *in* D we shall understand a pair $\langle \alpha, n \rangle$ where n is the index of a node of the syntactic analysis of one of the sentences of D to which α is attached.

The relation which holds between two expressions α and β if α is a subexpression of β has an obvious counterpart between expression occurrences: $\langle \alpha, n \rangle$ is a 'suboccurrence' of $\langle \beta, m \rangle$ if $\langle \alpha, n \rangle$ occurs as part of the syntactic analysis of $\langle \beta, m \rangle$. I shall often speak, by a minor slight of hand, of one expression occurrence being a *subexpression* (*subformula*, etc.) *of* some other occurrence. No confusion should arise from this.

The construction of a DRS for D does not only require the separate identification of particular occurrences of expressions of L_0; we must also be able to keep track of different occurrences of the same expressions of $L'_0(X)$. However, as our examples have already indicated (and we shall soon make this fully explicit) the expressions from $L'_0(X) \setminus L_0$ which enter into DR's are always derived from corresponding expressions of L_0. To be specific, they result either (i) through one or more substitutions of members of X for singular terms in some sentence of L_0; or (ii) from placing a member of X in parentheses behind a CN of L_0; or (iii) from combining a member of X with = and a proper name of L_0. In the first case we can label the $L'_0(X)$-sentence occurrence unambiguously with the index of the occurrence of the L_0-sentence from which it is obtained through successive substitutions; in the second case we assign the index of the relevant occurrence of the common noun; and in the third the index of the relevant occurrence of the proper name. In each of the cases (i), (ii), and (iii), we shall say that the sentence of $L_0'(X)$ is a *descendant of* the relevant expression of L_0, and similarly that the occurrence of the $L_0'(X)$-sentence is a *descendant of* the the corresponding occurrence of an expression of L_0. Formally we shall represent any occurrence of such an expression also as a pair consisting of the expression together with the appropriate index.

There is one other notion which we have already defined for L_0 but which must also be extended to cover certain expressions of $L_0'(X)$ as well. This is the notion of the *outermost rule of* an expression. We shall need to refer to the outermost rule only of those sentences of $L_0'(X)\backslash L_0$ which result from making in sentences of L_0 one or more substitutions of members of V for occurrences of singular terms of $L_0(X)$. Any such substitution leaves the syntactic structure of the sentence in which it takes place essentially inviolate: it can only lead to some 'pruning' of the syntactic tree, viz. where the replaced singular term occurrence is itself complex. In that case the subtree dominated by the node to which the singular term (α) is attached is deleted and replaced by a single node to which is attached the inserted (basic) term (u). The outermost rule FRi of the resulting sentence should *not* count as outermost rule of the syntactic analysis of the substitution result. For FRi is the rule which combines u with the remainder γ of the sentence, and this is a syntactic operation which, unlike the analogous operation that combines the replaced singular term with γ, should give rise to no further step in the DRS construction (the singular term α has after all just been dealt with!). Thus we should identify as the *outermost rule of* the substitution result, rather the outermost rule of γ. Since, as we already observed, each of the $L_0'(X)$-sentences in question results from a finite sequence of such substitutions the above stipulation defines the outermost rule of each such sentence.

Having extended the concept of the outermost rule of an expression to certain sentences of $L_0(X)$ we can now also apply the notions *conditional* and *universal sentence* to those sentences. Moreover, we shall call *atomic* those sentences of $L_0'(X)$ which consist either (i) of a discourse referent followed by an IV; or (ii) a TV flanked by two discourse referents; or (iii) a CN followed by a discourse referent in parentheses; or (iv) a discourse referent followed by = and a proper name of L_0.

Here is the definition of the 'format' of Discourse Representations I have chosen, as well as of some related notions which we shall need in later definitions:

DEFINITION 1. Let D be an L_0-discourse.

1) A *possible DR (Discourse Representation) of* D is a pair <U,Con>, where
   (i) U is a subset of V; and
   (ii) Con is a set of occurrences in D of sentence of $L_0'(U)$.
2) Where m and m' are possible DR's for D we say that m' *extends* m if $U_m \subseteq U_{m'}$ and $Con_m \subseteq Con_{m'}$.
3) Let m be a possible DR for D. A sentence $\phi \in Con_m$ is called *unreduced in* m iff Con_m contains no descendant of ϕ. m is called *maximal* if each unreduced member of Con_m is either i) an atomic sentence, ii) a conditional, or iii) a universal sentence.

We have seen in Section 2 that in general we must associate with a given discourse a Discourse Representation *Structure*, i.e. a partially ordered family of DR's, rather than a single DR. As it turns out the partial orders of those DRS's which our rules enable us to construct can always be defined in terms of the internal structure of their members. This makes it possible to define a DRS simply as a set of DR's.

To show how the partial order can be defined in terms of the structure of the DR's that make up the DRS we have to make explicit the structural relationship that holds between a DR m which contains a conditional or universal sentence ϕ and the pair of DR's which must be constructed to represent the content of ϕ. But before we can do that we must first discuss, and introduce, a slight modification of the schema for representing conditionals and universals that we have used in our examples. So far we have represented a conditional *if* A (*then*) B by a DR m_1 of A together with an extension m_2 of m_1 which incorporates into it the information contained in B. There can be no objection to this schema as long as the information contained in A can be fully processed in m_1 before one extends it by processing B. It is not always possible, however, to proceed in this way, as is illustrated by (25).

(25) If a woman loves him Pedro courts her.

The order in which the construction rules must be applied to yield a DRS which links *him* with *Pedro* and *her* with *a woman*, is indicated in the following diagram:

$m_0(25)$

(0) if a woman loves him Pedro courts her

$m_1(25)$	$m_2(25)$
(2) u	(2) u (3) v
(1) a woman loves him	(1) a woman loves him
(2) woman (u)	(1) Pedro courts her
(2) u loves him	(2) woman (u)
(5) u loves v	(2) u loves him
	(3) v = Pedro
	(3) v courts her
	(4) v courts u
	(5) u loves v

Not only is there duplication here of the conditions which occur both in $m_1(25)$ and $m_2(25)$ but some of the operations have to be performed simultaneously *and in the same way*, on the identical entries of these two DR's. It would be possible to characterize DRS-construction so that such entries are treated simultaneously in all the DR's in which they occur, and give rise in each of these DR's to the same descendants. But this is awkward, particularly where the treatment produces new subordinate DR's. It is easier to introduce into the second DR of the pair representing a conditional only the information conveyed by the consequent. In the case of (25) this will lead to a DRS of the form:

$m_0(25)$

if a woman loves him Pedro courts her

$m_1(25)$	$m_2'(25$
u	u v
a woman loves him	Pedro courts her
woman (u)	v = Pedro
u loves him	v courts her
u loves v	v courts u

Similarly, we shall represent a universal sentence by a pair of DR's into the second of which we enter the information that the remainder of the sentence is true of the discourse referent which stands in for the singular term *every* β in question. For example the DRS K(15) for

(15) Every widow admires a farmer

now becomes

K(15)

$m_0(15)$

Every widow admires a farmer

$m_1(15)$	$m_2'(15)$
x	u
.	.
widow (x)	x admires a farmer
	farmer (u)
	x admires u

Evidently the second members of the representing pairs about which we have been speaking up to now can be reconstructed from these new pairs: where (m_1,m_2) is the old pair and (m_1,m_2') the pair which replaces it according to the present stipulation, m_2 is the union of m_1 and m_2', where the *union* of two DR's $\langle U_1,Con_1\rangle$, $\langle U_2,Con_2\rangle$ is the DR $\langle U_1 \cup U_2, Con_1 \cup Con_2\rangle$ – thus, in particular, $m_2(15)$ is the union of $m_1(15)$ and $m_2'(15)$, and $m_2(25)$ that of $m_1(25)$ and $m_2'(25)$. Note that the truth clause (12) for conditionals and its analogue for universal sentences are not affected by this change.

Let us now describe how we can recognize two DR's m_1 and m_2 as representing a conditional or universal sentence that occurs among the conditions of the DR m. We first assume that m contains the occurrence $\langle\phi,k\rangle$, that ϕ is a conditional and that its antecedent and consequent are, respectively, $\langle\psi,r\rangle$ and $\langle\chi,s\rangle$.[16] We say that the pair of DR's $\langle m_1,m_2\rangle$ *represents* $\langle\phi,k\rangle$ iff:

(i) $\langle\psi,r\rangle \in Con_{m_1}$ and every member of Con_{m_1} is a descendant of a subexpression of $\langle\psi,r\rangle$;

(ii) $\langle\chi,s\rangle \in Con_{m_2}$ and every member of Con_{m_2} is a descendant of a subexpression of $\langle\chi,s\rangle$.

Now suppose $<\phi,k>$ is a universal sentence. Here it is convenient to distinguish between the case where the term with maximal scope is of the form *every* β, where β is a basic CN and that where it has the form *every* $\beta\gamma$ with β a CN and γ a RC. Let us begin by considering the first of these. We say the pair $<m_1,m_2>$ *represents* $<\phi,k>$ iff for some $x \in V$ (i) $x \in U_{m_1}$; (ii) $Con_{m_1} = \{<\beta(x),i>\}$; (iii) $<\phi',k> \in Con_{m_2}$ and each member of Con_{m_2} is a descendant of a subexpression of $<\phi',k>$, where i is the index of the occurrence of β in the term (occurrence) *every* β in question and ϕ' is the result of replacing that term occurrence in ϕ by x.

Now consider the case where the term with maximal scope has the form *every* $\beta\gamma$, where β is common noun and γ a relative clause. In this case $<m_1,m_2>$ *represents* $<\phi,k>$ iff for some $x \in V$ (i) $x \in U_{m_1}$; (ii) $<\beta(x),i>$, $<\delta,r> \in Con_{m_1}$; and every member of Con_{m_1} other than $<\beta(x),i>$ is a descendant of an occurrence of a subexpression of $<\delta,r>$; and (iii) $<\phi',k> \in Con_{m_2}$ and every member of Con_{m_2} is a descendant of an occurrence of a subexpression of $<\phi',k>$ - here i and ϕ' are as above, r is the index of the occurrence of γ in the relevant occurrence of *every* $\beta\gamma$ and δ is determined as follows: let ζ be the sentence from which the relative clause has been formed through "wh-movement"; δ is obtained by substituting x in ζ for the pronoun occurrence which was eliminated in the transition from ζ to γ.

Next we must give the definition of *partial Discourse Representation Structures*.

<u>DEFINITION 2</u>. A *partial DRS (Discourse Representation Structure) for* D is a set K of possible DR's for D such that whenever m is a member of K and Con_m contains a conditional or universal sentence $<\phi,k>$ then there is at most one pair of members m_1 and m_2 of K which represents $<\phi,k>$.

We say that a member m' of K is *immediately subordinate to* m iff either (i) there is a conditional or universal sentence occurrence $<\phi,k> \in Con_m$ such that m' is the first member of a pair which represents $<\phi,k>$; or (ii) m is itself the first member of such a pair and m' is the second member of that pair. m' is *subordinate to* m iff there exists a finite chain of immediate subordinates connecting m and m'.

The rules for constructing DRS's will guarantee that they will always have a principal member. If the partial DRS K contains such a member it will be denoted as $m_0(K)$. Where K and K' are partial DRS's we say that K' *extends* K iff there is a 1-1 map f from K into K' such that for each $m \in K$

$f(m)$ extends m. For $m \in K$ we denote as $K^{\geq}(m)$ the set consisting of m and all the members of K that are superordinate to m. We shall also write "U_K" for "$\bigcup_{m \in K} U_m$" and "$U_K^{\geq}(m)$" for "$U_M \cup \bigcup\{U_{m'}: m' \in K$ and m' is superordinate to m}". We say that a partial DRS K is *complete* iff (i) every member of K is maximal; and (ii) whenever m is a member of K and Con_m contains an ocurrence of $<\phi,k>$ of a conditional or universal sentence K contains a pair which represents $<\phi,k>$.

We can now proceed to give a precise statement of the rules for DRS-construction. It is they, I must repeat here, that carry virtually all the empirical import of the theory. Their exact formulation is therefore of the greatest importance. Instead of trying to do justice to all relevant linguistic facts at once, I shall begin by stating the rules in a fairly simple manner. This will then serve as a basis for further exploration.

For the fragment L_0 there are five rules, one for proper names, one for indefinite descriptions, one for pronouns, one for conditionals and one for universal terms. The effect of applying a rule to a particular condition in some member of a DRS is always an extension of that DRS.

Only the rules for conditionals and universals lead to the introduction of new DR's. But this does not mean that the effect of each of the other rules is confined to the particular DR m which contains the condition to which the rule is applied. Thus, for instance - and this is a point we have so far neglected in our examples - the application of the rule for proper names will always result in the introduction of a new discourse referent into the principal DR of the DRS, even if the condition to which the rule is being applied belongs itself to some other member of the structure. (I shall argue below that the rule for proper names *must* operate in this fashion.) Directly connected with this is the need to refer, in the statement of the rule for pronouns, not just to the universe of the DR m that contains the relevant condition, but also to the universes of certain other members of the DRS - in fact, as it turns out, of all those members which are superordinate to m.

To state the first three rules let us assume that K is a partial DRS, that $m \in K$, that $<\phi,k> \in Con_m$ is an unreduced member of m, and that $<\alpha,i>$ is an occurrence of a term in $<\phi,k>$ which has maximal scope in $<\phi,k>$.

CR1. Suppose α is a proper name. We add to $U_{m_0(K)}$ an element u from $V \backslash U_K$. Furthermore, we add to $Con_{m_0(K)}$ the occurrence $<u=\alpha,i>$ and to Con_m the occurrence $<\phi',k>$, where ϕ' is the result of replacing the occurrence of α in $<\phi,k>$ with index i by u.

CR2. α is an indefinite singular term. (a) α is of the form *a(n)*β, where β is a common noun. We add to U_m an element u from $V \backslash U_K$ and to Con_m the occurrences $\langle \beta(u), r \rangle$ (where r is the index of the occurrence of β in $\langle \alpha, i \rangle$) and $\langle \phi', k \rangle$, where ϕ' is as under CR1. The other members of K remain unchanged. (b) α is of the form *a(n)*$\beta\gamma$, where β is a basic common noun and γ a relative clause. We add $u \in V \backslash U_K$ to U_m and expand Con_m with $\langle \beta(u), r \rangle$, $\langle \phi', k \rangle$ and the pair $\langle \delta, s \rangle$ where δ is determined as in the definition of *represents* given above, and s is the index of the occurrence of γ in $\langle \alpha, i \rangle$.

CR3. Assume α is a pronoun. Choose a 'suitable' member u from $U_K^{\geq}(m)$. Add $\langle \alpha = u, i \rangle$ and $\langle \phi', k \rangle$ to Con_m.

NB. I have given a deliberately 'fudgey' formulation of this rule by inserting the word 'suitable'. To state what, in any particular application of the rule, the set of suitable referents is, we would have to make explicit what the strategies are that speakers follow when they select the antecedents of anaphoric pronouns. In the applications we shall consider below the restriction to 'suitable' referents that I have built into CR3 will never play an overt role (although I will occasionally ignore, without comment, readings of the sampled sentences which would impose anaphoric links that are ruled out by various factors that enter into these strategies, such as e.g. the principle of gender agreement). Nonetheless, I have included 'suitable' in the formulation of CR3, as a reminder that the rule is incomplete as it stands.

To state the last two rules let us assume that K and m are as above, that $\langle \phi, k \rangle$ is an unreduced member of Con_m and that ϕ is either a universal sentence or a conditional.

CR4. Assume $\langle \phi, k \rangle$ is a conditional with antecedent $\langle \psi, r \rangle$ and consequent $\langle \chi, s \rangle$. We add to K the member $\langle \phi, \{\langle \psi, r \rangle\} \rangle$ and $\langle \phi, \{\langle \chi, s \rangle\} \rangle$.

CR5. Assume $\langle \phi, k \rangle$ is a universal sentence and the term with maximal scope is $\langle$*every* $\beta, i \rangle$, with β a basic CN. We add, for some $u \in V \backslash U_K$ $\langle \{u\}, \{\langle \beta(u), r \rangle\} \rangle$, and $\langle \phi, \{\langle \phi', k \rangle\} \rangle$, where r and ϕ' are as 2 pages above. Similarly, where the term with maximal scope is $\langle$*every* $\beta\gamma, r \rangle$ where $\beta \in CN$ and $\gamma \in RC$ the DR's that must be added are $\langle \{u\}, \{\langle \beta(u), r \rangle, \langle \delta, s \rangle\} \rangle$ and $\langle \phi, \{\langle \phi', k \rangle\} \rangle$, where, again, $u \in V \backslash V_K$ and s, δ, ϕ' are as in the statement of CR2.

Note that if K is a finite DRS, i.e. a finite set of finite DR's, then a finite number of applications of the rules CR1 - CR5 will convert it into a complete DRS. Any complete DRS obtained from K by a series of rule applications is called a *completion* of K. Clearly, if K has a principal member, then so does every completion of K.

We can at last define the notion of a *complete DRS for* a discourse D. The definition proceeds by recursion on the length of D.

DEFINITION 3. (i) Suppose D is a discourse consisting of one sentence ϕ. Let k be the index of ϕ in D. A *complete DRS (Discourse Representation Structure) for* D is any completion of the DRS $\{<\phi,\{<\phi,k>\}>\}$. (ii) Suppose that D has the form $<\phi_1,\ldots,\phi_n,\phi_{n+1}>$ and that the set of complete DRS's for the discourse $D' = <\phi_1,\ldots,\phi_n>$ has already been defined. Let k be the index of the occurrence of ϕ_{n+1} as last sentence of D. Then K is a *complete DRS for* D iff K is a completion of a DRS of the form $(K' - \{m_0(K')\}) \cup \{m\}$, where K' is some complete DRS for D' and m is the DR $<U_{m_0(K')}, Con_{m_0(K')} \cup \{<\phi,k>\}>$.

NB. It follows from this definition together with earlier remarks that every set of possible DR's which is a complete DRS for some discourse D contains a principal DR.

3.3. Truth

Our next task is to define truth. Much has already been said about this in the preceding chapters. So we can proceed with the formal definition almost at once.

There is just one feature of the definition that might be puzzling without a brief preliminary discussion. The evaluation of conditionals and universals as a rule involves only embeddings that respect certain previously assigned values to some of the discourse referents in superordinate positions. In other words we keep, in the course of such evaluations, certain functions fixed and consider only embeddings compatible with these functions. This means that the recursive definition underlying the characterization of the truth in M must be of a concept which is sensitive not only to the information encoded in the DRS but also to some partial function from the discourse referents of that DRS into U_M. If a sentence contains several nested embeddings of conditionals or universals, the maps considered in the evaluation of deeply embedded constructions may have to agree with several functions that have been stored, so to speak, along the way down to the conditional or universal concerned. However, as these stored functions must also be compatible with each other we need consider only single functions in this connection; intuitively these are the unions of the sets of different functions accumulated along the path towards the embedded construction.

Let K be a complete DRS for D and M a model of D. We shall give the definition of the *truth value of* D *in* M *given* K in two steps. The first stage will give a characterization, by simultaneous recursion, of two relations: (i) The relation which holds between a member m of K, a function f from U_m into U_M and a partial function g from U_K into U_M iff, as we shall express it, f *verifies* m *in* M *given* K, *relative to* g; and (ii) the relation which holds between m, an unreduced member $<\phi,k>$ of Con_m, a function f from U_m into U_M and a function g from U_K into U_M iff, as we shall say, $<\phi,k>$ *is true in* M *under* f, *given* K, *relative to* g. The second stage uses the first of these two relations to define truth:

<u>DEFINITION 4</u>. Let D be an L_0-discourse, K a complete DRS of D and M a model for L_0. D is *true in* M *on* K iff there is a function f from $U_{m_0(K)}$ into U_M which verifies $m_0(K)$ in M, given K, relative to Λ. (Λ is the empty function!)

The recursive part of the definition is inevitably somewhat more involved.

<u>DEFINITION 5</u>. Let D, K, M be as in Definition 4; let $m \in K$ and let g be a partial function from U_K into U_M.

(i) f *verifies* m *in* M *given* K, *relative to* g iff each unreduced member $<\phi,k>$ of Con_m is true in M under f, given K, relative to g.

(ii) Suppose $<\phi,k>$ is an occurrence of an atomic sentence in Con_m. Then ϕ has one of the following four forms:

(a) $u\alpha$, where $u \in V$ and $\alpha \in IV$;

(b) $u\alpha v$, where $u,v \in V$ and $\alpha \in TV$;

(c) $u=\alpha$, where $u \in V$ and α is a proper name;

(d) $\alpha(u)$, where $u \in V$ and α is a basic common noun.

The question whether $<\phi,k>$ *is true in* M *under* f *given* K, *relative to* g splits up into the corresponding four clauses below (we omit the qualification 'in M under f, given K relative to g'):

(a) $<\phi,k>$ *is true* iff $f(u) \in F_M(\alpha)$;

(b) $<\phi,k>$ *is true* iff $<f(u),f(v)> \in F_M(\alpha)$;

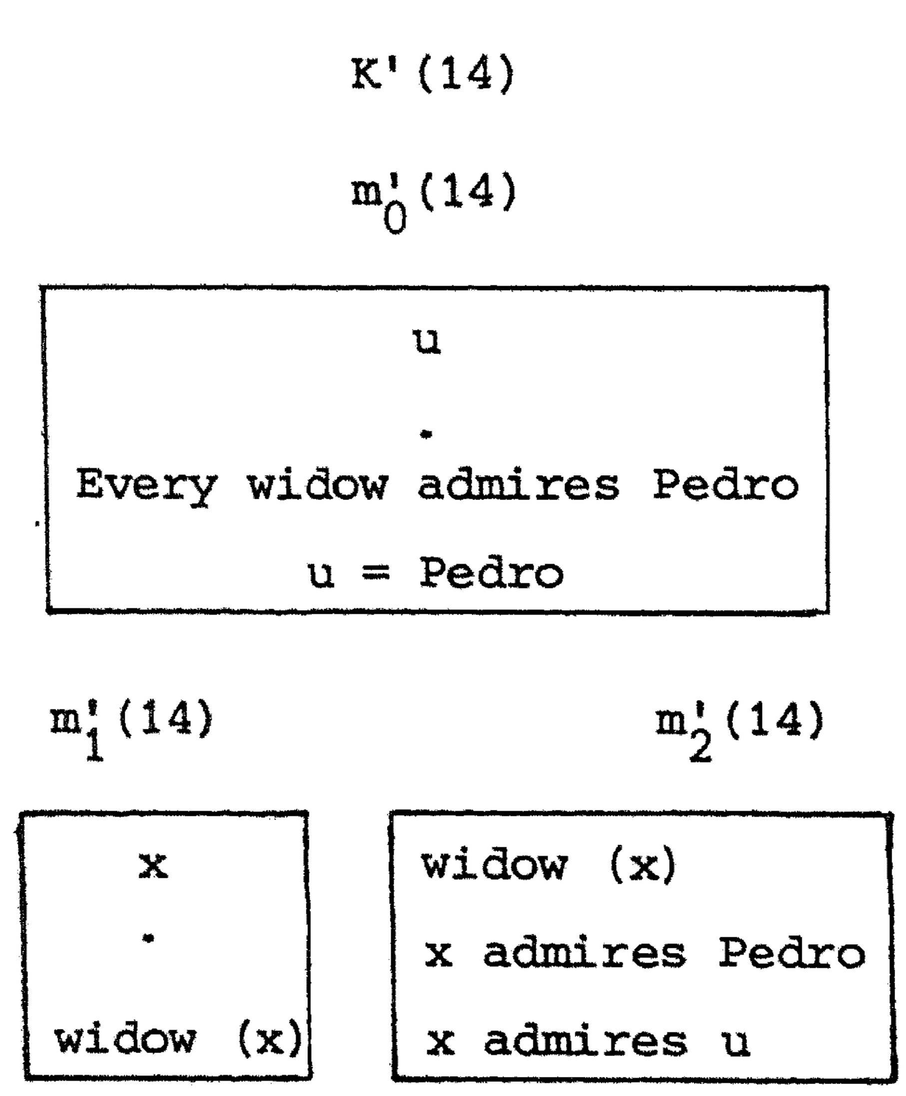
(c) $<\phi,k>$ *is true* iff $f(u) = F_M(\alpha)$;

(d) $<\phi,k>$ *is true* iff $f(u) \in F_M(\alpha)$.

(iii) Suppose $<\phi,k>$ is an occurrence of a conditional or universal sentence in Con_m. Then K will contain a unique pair $<m_1,m_2>$ which represents $<\phi,k>$. $<\phi,k>$ *is true in* M *under* f *given* K, *relative to* g iff every map h from U_{m_1} into U_M which is compatible with $g \cup f$ and which verifies m_1 in M given K relative to $g \cup f$ can be extended to a function k from U_{m_2} into U_M and verifies m_2 in M given K relative to $g \cup f$.

We shall call a function which verifies $m_0(K)$ in M, given K, relative to Λ a *verifying,* or *truthful, embedding of* K *into* M. We shall also say of such a map that it *verifies* D *in* M *on* (*the reading provided by*) K.

Many of the DRS's we have earlier displayed fail to be in complete agreement with the construction procedure as we have now formally described it. This is true, in particular, of the second representation I gave in Section 2.3 for (14). The DRS K(14) violates the rule CR1 in that the item u, which is introduced as the referent of the proper name *Pedro* should have been entered into the universe of $m_0(14)$ rather than into that of m(14). Let us give the DRS for (14) once more, this time in its proper form.



The need to place the discourse referent introduced by a proper name into the principal DR is illustrated by (25) for which I gave a DRS in Section 3.2.

This DRS is unacceptable by our rules as the referent u in m_2(25) is not accessible from the position of *him* in m_1(25), to which, at step (5) it was nonetheless assigned. This difficulty would not have arisen had CR1 been properly applied in the first place. The correct DRS for (25) looks as follows:

m'_0(25)

(3) v
(0) if a woman loves him, Pedro courts her
(3) v = Pedro

m'_1(25)	m'_2(25)
(2) u	
(1) a woman loves him	(1) Pedro courts her
(2) woman (u)	
(2) u loves him	(3) v courts her
(5) u loves v	(4) v courts u

Let us, for good measure, also give a corrected version of the DRS for (1), as the analysis of that sentence motivated so much of what I have been saying, and yet its earlier representation also contains a violation of CR1:

m'_0(1)

(2) u
(0) if Pedro owns a donkey he beats it
(2) u = Pedro

m'_1(1)	m'_2(1)
v	
(1) Pedro owns a donkey	(1) he beats it
(2) u owns a donkey	(4) u beats it
(3) donkey (v)	(5) u beats v
(3) u owns v	

We already saw in Section 2 how important it is that the discourse referents available to a given pronoun must all occur in the same, or else in some superordinate, DR. This, we saw, accounts for the fact that *it* cannot have *every donkey* as its antecedent in a sentence such as (17) or (18), or be anaphorically linked to *a donkey* in (19). The reader will inevitably ask, however, why subordination is defined in the precise way it has been. Why, for instance is, where (m_1, m_2) represents a conditional or universal, m_2 subordinate to m_1 but not m_1 subordinate to m_2; or, to put it more directly, why may the elements of m_2 not serve as referents for pronouns in sentences belonging to Con_{m_1} while the members of U_{m_1} are admitted as referents for pronouns occurring in m_2?

That the elements of m_1 must be available for the pronouns of m_2 is too central an assumption of our theory to permit tampering: our analysis of the crucial sentences (1) and (2) depended essentially on that hypothesis. But what about referents in m_2 for pronouns in m_1? Here is an example which shows that the sets of possible referents must be as we have specified them:

(26) Every farmer who admires her courts a widow.

It is my intuitive judgement that in (26) *her* can be coreferential with *a widow*, but only if *a widow* has wide scope over *every farmer*. Such 'wide scope' readings for indefinites that occupy positions which correspond to narrow scope according to our syntax are not discussed in this paper.
A reading which (26) can *not* have is, according to my intuitions, the one given by

(27) $(\forall x)(\text{farmer}(x) \rightarrow (\exists y)(\text{widow}(y) \wedge \text{admires}(x,y) \wedge \text{courts}(x,y)))$.

To block this reading we must stipulate that the element v of $m_2(26)$ is not available to the pronoun in $m_1(26)$:

$m_0(26)$

(0) every farmer who admires her, courts a widow

$m_1(26)$

(1) x
(1) farmer (x)
(1) x admires her

$m_2(26)$

(2) v
(1) x courts a widow
(2) widow (v)
(2) x courts v

Our theory seems to rule out a parallel reading for the conditional

(28) If a farmer admires her, he courts a widow.

It predicts, that is, that (28) cannot mean what is expressed by (27). Again, *her* in (28) can be understood as coreferential with *a widow* if the latter is taken to have wide scope - as it normally would in, say,

(29) If a farmer admires her he courts a certain widow I have dated and therefore know quite well.

(28) appears to have still another reading, in which *a widow* is taken as generic, a reading that is approximated by

(30) $\forall x \forall y[\text{farmer}(x) \wedge \text{widow}(y) \wedge \text{admires}(x,y) \rightarrow \text{courts}(x,y)]$.

Generics, however, are among the most recalcitrant constructions known to me. They will not be treated in this paper. Note also that

(31) If Pedro admires her he courts a widow,

though understandable, on the assumption that *her* refers to *a widow*, does not sound natural - barely, more natural in fact than do (26) and (28) on their wide scope reading, given by

(32) $\exists y[\text{widow}(y) \wedge \forall x[\text{farmer}(x) \wedge \text{admires}(x,y) \rightarrow \text{courts}(x,y)]]$.

The reason is that in order to get a reading of (31) in which *her* and *a widow* are coreferential we have to suppose - just as we must in connection with (26) and (28) - that *a widow* has wide scope over the subject Pedro. In another paper we shall have more to say about why such readings tend to be somewhat unnatural.

FOOTNOTES

* This paper was written while I held a Post-Doctoral Fellowship at the Center for Cognitive Science of the University of Texas at Austin. Anybody who has the faintest acquaintance with my personality will realize that it would not have been written had the Directors of the Center not given me this opportunity, and thus understand the depth of my indebtedness to them. I would also like to thank, among the

many who helped me during my stay in Austin, Kate Ehrlich, Alan Garnham, Lauri Karttunen and Stanley Peters for their comments and suggestions.

1. Theories that to a greater or lesser degree accord with this intuition have emerged within Artificial Intelligence and Computer Science, as well as within Linguistics. A significant contribution of this kind that comes from the first field is WEBBER (1978). Examples of such theories that have been proposed by linguists are: the theories outlined in BARTSCH (1976, 1979), COOPER (1975, 1979), HAUSSER (1974, 1979), KARTTUNEN (1976).

   By no means every recent account of pronouns is predicated on the assumption that all cases of pronominal reference can be brought under one unifying principle. Cf. for instance EVANS (1977, 1980), LASNIK (1976), PARTEE (1978).
2. There seems to be a rough preference for referents introduced by terms that appear in the discourse *before* the anaphoric pronoun over those that are introduced by subsequent terms, as well as a preference for referents that are introduced by terms that occur *near* the anaphor. (Thus the referent introduced by the last referential term preceding the anaphoric pronoun is, other factors permitting, a strong referential candidate.)
3. A large part of the research that has been done on anaphora by computer scientists and people working in Artificial Intelligence has been concerned with this problem - understandably enough, as the lack of effective routines for the detection of anaphoric antecedents has for many years been one of the main obstacles to producing satisfactory computer systems for question answering and translation. However useful some of this work may have been, I have the impression that its theoretical significance is rather limited. Indeed I much incline to the opinion expressed, for example, in PARTEE (1978, p.80) that all we can reasonably expect to achieve in this area is to articulate orders of preference among the potential referents of an anaphoric pronoun, without implying that the item that receives the highest rating is in each and every case the referent of the anaphor.
4. We are much assisted in our making of such guesses by the spectrum of our social prejudices. Sometimes, however, these may lead us astray, and embarrassingly so, as in the following riddle which advocates of Women's Lib have on occasion used to expose members of the chauvinistic

rearguard: In a head-on collision both father and son are critically wounded. They are rushed into hospital where the chief surgeon performs an emergency operation on the son. But it is too late and the boy dies on the operating table. When an assistant asks the surgeon, 'Could you have a look at the other victim?', the surgeon replies 'I could not bear it. I have already lost my son.' Someone who has the built-in conception that chief surgeons are men will find it substantially more difficult to make sense of this story than those who hold no such view.

5. As we have already observed, this is not quite correct, since a pronoun can be used deictically, in which case the referent need not belong to the DR; we shall, however, ignore the deictic use of pronouns in the course of this paper.
6. See for example LEWIS (1973), TURNER (forthcoming), VELTMAN (1976), KRATZER (1979).
7. (11) is akin in spirit to the game theoretical analysis of *if* ... *then* ... sentences proposed in HINTIKKA & CARLSON (1978), according to which a winning strategy for the defender of *if* A *then* B is a function which maps every winning strategy for the defender of A onto a winning strategy for the defender of B.
8. The fact that 'existential' quantifier phrases can be represented in this manner is closely related to the familiar model theoretic proposition that purely existential sentences are preserved under model extensions.
9. I have found at least one speaker for whom (20) is distinctly less acceptable than for instance (1).
10. See for example CARLSON (1976, Chapter I), which warns against this prejudice in similar terms.
11. Proposals similar to that of Evans can be found e.g. in COOPER (1979) and HAUSSER (1974). These suffer in my view from similar shortcomings.
12. The two fragments have roughly the same quantificational powers. But the present fragment lacks adjectives, prepositional phrases and intensional contexts.
13. One might have hoped that a theory of semantic processing such as the one attempted here could provide an explanation of why island-constraints exist and why they operate in precisely those linguistic contexts that are subject to them. I have not succeeded, however, in finding such an explanation.
14. See e.g. MONTAGUE (1970a,b; 1973), PARTEE (1975), THOMASON (1976),

COOPER & PARSONS (1976), COOPER (1979).

15. The expansion of M to M' is part of a convenient, and familiar, model-theoretic device that serves as an alternative to the notion of satisfaction: If M is a model for a first order language L and $\phi(x_1,\dots,x_n)$ is a formula of L with free occurrence of the variables $x_1,\dots,x_n$ and no others, and $u_1,\dots,u_n \in U$, then clearly $u_1,\dots,u_n$ satisfy $\phi(x_1,\dots,x_n)$ in M if M makes true the L(U)-sentence $\phi(u_1,\dots,u_n)$. Thus by passing from L to L(U) we can define *truth in* M without any need of the concept of satisfaction as such. It is in this spirit that I shall use here the sentences of L(V). Where m is a DR which contains the discourse referents $u_1,\dots,u_n$, a sentence $\phi(u_1,\dots,u_n)$ of $L(\{u_1,\dots,u_n\})$ is to express that in m $u_1,\dots,u_n$ satisfy the condition expressed by the λ-abstract $\lambda x_1,\dots,\lambda x_n\ \phi(x_1,\dots,x_n)$.
16. With the *occurrence* $\langle\phi,k\rangle$ are associated, of course, particular *occurrences* of antecedent and consequent.

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