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MATHEMATICAL CENTRE TRACTS 130

ILP

INTERMEDIATE LANGUAGE FOR PICTURES

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CONTENTS

1. INTRODUCTION

1.1. Final version (1980)

This second edition contains the definition of ILP [14] and as such replaces and invalidates the definition given in the preliminary version. As foreseen, implementing ILP and playing with the implementation revealed deficiencies in the original design, leading to changes in the definition. Just as we preferred not to justify the chosen constructs in the original report, we now prefer not to explain in great detail our reasons to change some of them. A few words can be said here though.

Changes fall into three categories:

1. The original construct turned out to be insufficient.

2. The original construct proved to be impractical.

3. The original definition of the construct turned out to be inconsistent with other language constructs.

Changes have been made to the following constructs, for the reasons denoted between parentheses: **LINE** (2), *text_quality* (1 and 2), pen position (3), *projection* (2) and the effect of the prefix **ABS** and of inhibiting *attribute_matches* within *subspaces* (2 and 3).

Besides changing syntax and/or semantics of language constructs, we sometimes changed our way of explaining them. Improved insight due to experience gained during the implementation and effect of reviewing a text after a considerable amount of time revealed the clumsiness or inadequacy of some of our original semantic descriptions. Especially the part on attribute mixing (formerly attribute concatenation) was thoroughly revised and reduced to a fifth of its original length.

We will conclude this introduction with some more general remarks about the state of our graphics work. The embedding of ILP into a high level language (ALGOL68) was completed in 1978 [5]. Ideas about I/O symmetry based on ILP as a representation language were expressed in [6]. An input module based on the detection mechanism is being developed; some underlying concepts are described in [7]. We abandoned our original, too optimistic, plan of developing a full-fledged, well documented, user oriented graphics system. ILP itself, ALGOL68G-0, and the input module do not yet make up such a system. Two groups of people are essential to complete such an effort:

-- A group of people who can afford to spend the bulk of their time writing programs (e.g. to connect other drawing machines) and documentation.

-- A bunch of critical, non-specialist users who are willing to work with an experimental system and to keep complaining about anything they find hard to understand or awkward to use.

The nature of the Mathematical Centre (an institute for more or less fundamental research without regular students) is such that none of these groups is available. We feel however that the work done sofar has given new insights in the design methodology of graphics systems.

1.2. The kernel of an Interactive Graphics System

The language defined in this report is a special purpose data description language. The restrictions implied by the term "special purpose" are twofold. First of all, the language is only intended for the description of pictures*). Every construction in the language is justified by the requirement that it should cover a part of this descriptive function. Useful constructs that might have been added because of its function as a programming language have been omitted. The second restriction is derived from the fact that the language is an 'intermediate' language. This means that its second function is to fill in the gap that exists between a picture description in the form of instructions for a physical drawing machine on the one hand and a picture description as part of a more sophisticated language or data structure for an application area on the other hand. The intermediate language may be a low level language in the sense that for each feature required the most simple constructions can be chosen. All these aspects are emphasized by the name Intermediate Language for Pictures or ILP for short.

The definition and implementation of the language constitute the design and implementation of the kernel of an interactive graphics system. The design goal of this system has been published in [4]. Although practical limitations have restricted the scope and goals of this research, we still believe that the basic philosophy is sound and that it may lead to the design of better structured graphics systems. In this philosophy, ILP plays a key role in all graphics system facilities:

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*) A picture is defined as a description of some object such that a visible image of that object can be obtained from this description in a uniform way. The description may include both geometrical (shape, size) and non-geometrical (colour, weight) properties of the object.

-- A high level graphical language is obtained by embedding ILP in an existing high level general purpose programming language.

-- The control of every drawing machine in the graphics system is defined by a conversion between ILP code and device code. This is true for input as well as output. In principle, full symmetry between input and output can be obtained.

-- A picture file system is defined and organized as a library for ILP programs in which the latter can be stored, retrieved and classified.

All other graphics facilities can be defined as transformations of ILP programs.

All these modules (and others that might be added) greatly profit from the conceptual uniformity provided by ILP.

1.3. The design of ILP

A further function of ILP, which as such is only implicitly present in a graphics system, is that it provides a means of communicating about the graphics system during the design phase. To support this communication, a symbolic notation for ILP programs has been introduced, which makes ILP look like an ordinary programming language. The success of this symbolic code was so convincing that it was decided to use the same code for the definition of ILP in this report. Moreover, each module of the system is implemented in such a way that it is able to accept and produce symbolic ILP code. in which it communicates through symbolic ILP code. This constitutes a very useful testing facility and also proves that conceptual uniformity has been preserved.

In the period when the designers decided to work according to the scheme explained above, they assumed overly optimistic that either an existing language or a collection of features taken from existing languages could be used for this purpose. Neither turned out to be the case. Most existing languages suffered from the fact that they had been forced into the frame of a so-called FORTRAN interface. Since the only two means of expression here are subroutine identifier and simple parameters, designers of these packages always argue that the number of identifiers and parameters should be kept small, and above all that the interrelation between function calls must be exceedingly simple, because each structuring function (like opening and closing brackets) requires subroutine calls scattered throughout the application program.

The effect of this type of limitations is that everybody chooses a subset of desired features. No two subsets have the same representation in terms of identifiers and parameters and moreover all subsets differ from each other, and all are declared to be the best of all possible

choices.

Given this state of affairs, the designers decided to adhere to the principle that if a feature would be included it would be included completely. One of the consequences is that a FORTRAN subroutine library is most unlikely to be a suitable representation of ILP.

More interesting material was provided by graphic languages that support data structures. In these cases efficiency of problem representation plays a major role. Complicated data structures for graphics are justified by the fact that the application program can use the same data structure. In this way the problem of representing graphical data structures is generalized towards structuring associative data or towards hierarchies of cyclical data. This type of languages cancels itself out for the bulk of the moderate applications of computer graphics. From this observation the designers drew the conclusion that it made sense to try to characterize the complexity of purely graphical information. The best way to do this seemed to define a complete graphical language and to find the simplest representation for it.

The language ILP deals with four major facets of graphical information:

-- The elementary drawing actions.

-- Modifications of such drawing actions under control of state information.

-- Structuring (and combining) states and actions.

-- Specification of entry points for external references on which interaction and association of non graphical data can be based.

As such ILP constitutes a so-called general purpose modelling system.

The elementary drawing actions must be understood as a means to visualize elementary geometrical objects. Typical actions are to draw a point, line, contour (closed polygon) and curve. Less typical but useful is text. In fact the exclusive (exceptional) function of text has caused a number of unsolved problems with respect to the orthogonality of the design. Typical state information consists of transformations, coordinate mode (absolute or incremental) and style functions (line style, typographic style for text etc.). All non-geometrical aspects have been isolated from the actions and are controlled by independent state information. For instance, invisible moves that are used for positioning are not considered as drawing actions. This type of information is entirely included in the state. In so far as invisible moves can be found among the drawing actions, they represent part of a geometrical object (e.g. invisible line or invisible curve). Here the prefix "invisible" is state information.

A second important consequence of the distinction between geometrical and non-geometrical information is, that an exact specification is possible of the effect on the pen position of both actions and the state.

The state information follows two important principles. A complete state vector can be split in a number of subvectors which are all manipulated independently, i.e. a change of one subvector never has consequences for the effect of the other subvectors. The state manipulations are chosen in such a way that a new state can be obtained from an existing one by respecifying or adjusting a minimal number of values. The second important principle is that all independent subvectors in the state have the same basic structure. Moreover, the same basic manipulations can be applied to all of them. In other words a uniform scheme has been found that allows a large variety of properties to be associated with geometrical information. The basic manipulations can produce the right values as well as the right structure.

The modifying effect of state information on actions can be specified for each subvector separately, provided that priority rules are obeyed which define (as far as necessary) the order in which the state subvectors must be applied.

The simplicity of the semantic primitives and the limited ways in which they can be combined have turned out to impose surprisingly few restrictions on the expressive power of ILP. In order to make this clear we have, throughout the report, put a strong emphasis on such restrictions. Especially the criterion that only complete features should be included was (almost) never violated.

The structuring of ILP data is obtained by grouping and combining. Grouping means that a number of similar constructions is put together as a unit on a higher level in the hierarchy. Combining means that two different constructs are put together in a unit. At the level of elementary actions, the grouping of sequences of similar actions is implicit. At the level of complete pictures, one or more of them can be put together for the purpose of multiple referencing (subpictures) or as a conceptual unit (embracing). Both forms of grouping can be found in the representation. Moreover, grouping itself can be specified without having elementary actions. In this way, the structure skeleton of a picture can be specified. Combining always involves state information on the one hand and actions on the other. Combining is used for setting up the right state.

The facilities for structuring are not allowed to produce cyclic structures. This would introduce the need for conditions in ILP, that break the cycle. This is an example of excluding features that are convenient for programming but not fundamental for picture representation.

All references in ILP have the same form and are represented by a symbolic name. All entry points for external references are represented

in the same way.

1.4. The description of ILP

ILP is described in chapters 2 and 3. Chapter 2 gives an introduction to the basic concepts of ILP by means of simple examples. The function of chapter 2 is to provide an overview of ILP before plunging into all the syntactic and semantic details presented in chapter 3. Obviously chapter 3 is the most important one. Here, indeed everything is brought together that concerns the basic function of ILP: the representation of pictures.

Two interesting subjects concerning ILP have been left out of chapter 3. First of all the justification of most constructions of ILP is omitted. In most cases the justification can be deduced from the fact that it contributes to the representation of a particular construct. Moreover, it was felt important to concentrate on a precise definition leaving aside all matters that make the definition more complicated (like for instance, defining alternative constructions). Secondly the role of ILP in the various modules of the graphics system is not further explained.

The reader of chapter 3 will notice that some constructions of ILP have been specified in great detail and an attempt has been made to be very precise about them. Other constructions are presented in a more or less vague way. The detailed descriptions concern new or for ILP important constructions. The less precise definitions have been used to avoid lengthy descriptions of what is intuitively clear (e.g. the conversion of ILP primitives to, say, a plot file). The reason for being less precise (or incomplete) is in most cases given in the form of remarks, which as such are not part of the definition.

ILP can be extended in two directions. New primitive actions and state information can be added to cover the representation of other classes of pictures (e.g. grey scales). New constructions for structuring and building hierarchies of states (vectors of vectors) can be introduced to allow all kinds of manipulation (e.g. movies). For both type of extensions ILP must preferably constitute the kernel. Because in that case ILP can close the gap between "classical" and "modern" computer graphics.

To facilitate reading of the remainder we now give an overview of notational conventions throughout the report. Most basic concepts of ILP are at the same time non-terminals of the syntax. They are denoted in a special font, e.g.: *non_terminal*. Basic concepts that are not a syntactical category are underlined at first (and defining) occurrence. Syntactic terminals are denoted in capital letters, e.g. **TERMINAL**. There is an index that references the occurrences of most concepts. Footnotes and REMARKs are used to add comments to the text in places where it is

important to separate the essential from the explanatory.

On the primitive level of ILP the designers have hardly attempted to introduce new concepts in picture description. It is in the field of structuring graphical information that a new, more uniform framework is introduced. This framework, it is hoped, unites the large variety of elementary constructions needed for picture description.

1.5. ILP and the graphics standards

By the end of 1976 we became acquainted with various groups working on graphics standards. Ever since 1977 members of the ILP-team have participated in the attempts of defining an international graphics standard. Although ILP is not a CORE-like system and although it aims at future graphics facilities rather than adhering to current common practice there has been some mutual influence.

First of all the ILP designers have always supported including input functions in the standard which are more sophisticated than the five logical devices, for instance, as presented in the GSPC'77 proposal. Since then a lot of discussion about input has taken place. Among other things, IFIP WG5.2's graphics subcommittee has organised a workshop on "Methodology of Interaction" [8], the so-called SeillacII workshop. On the one hand, in current standards proposals [9] and [10] richer input functions are included. On the other hand, we believe, it became clear at the SeillacII workshop, that putting too high demands on a standard with respect to interaction is still premature.

The revised version of ILP has benefitted from current standards proposals especially with respect to the treatment of **TEXT**. The preliminary version of ILP was quite vague here. The definition of ILP **TEXT** in this report is greatly influenced by GSPC's 1979 CORE.

Recently it has become clear that as part of the standardisation effort a so-called Graphics METAFILE will have to be defined. We believe in retrospect that ILP can be characterised as a "very high level METAFILE".

2. AN OVERVIEW OF ILP

2.1. Introduction

In this chapter, ILP will be presented in an informal way. The chapter has a tutorial character and will heavily rely on short examples. No attempt has been made to cover the subject exhaustively. Only aspects that are characteristic of ILP and distinguish it from other graphics languages get attention. In particular, standard concepts from computer graphics (linear transformations, styles etc.) will not be discussed in their own right. For these topics, we refer to the introductory texts [1] and [2].

In all cases where the examples leave some doubt about what is precisely possible in ILP and what the exact semantics of ILP constructions are, chapter 3, which contains the formal definitions, should provide the answers.

All drawings in this chapter where produced by the ILP system on a HRD-1 laser display/plotter.

2.2. Picture elements

Picture elements are language primitives, used to describe basic drawing actions. They represent lines, points and the like, drawn in some user-selectable Euclidean space of an arbitrary dimension, called user space. How this space can be selected, will be described in 2.6.. Until then, in all examples a two-dimensional space, with orthogonal coordinate axes, is assumed.

Example 1

```
PICT ( 2 ) ex1
        WITH { VISIBLE ; FIXED }
        DRAW {
                LINE ( [ 0 , 0 ] , [ 0.25 , 0.5 ] ,
                       [ 0.5, 0] , [ 0 , 0 ] ) ;
                POINT ( [ 0.25 , 0.25 ] )
                } .
```

A two dimensional picture (a picture that must be drawn in a plane) is defined, having name ex1. The dimension is specified by the number "2" surrounded by parenthesis, immediately following keyword **PICT**. In general, any positive integer may be used in this place.

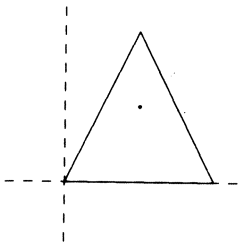
The picture consists of three line segments and a point. All elements of this picture are explicitly declared visible (by **VISIBLE**). The use of **FIXED** causes all coordinate pairs [a , b] to be interpreted as absolute positions in user space. (The other possibility will be dealt with in 2.3.).

The essential elements in this example are the picture elements **LINE** and **POINT**.

The picture element **LINE** states, that lines have to be drawn from the first coordinate specified to the last one. If necessary, an invisible move is generated to the first coordinate.

The picture element **POINT** says, that a point must be drawn at [-0.5,-1]. In general, **POINT** too, can have a number of coordinates as its arguments.

The drawing defined by the program above looks like:

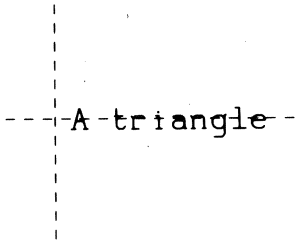


Here, as well as in the following examples, the coordinate axes are only added for illustrative purposes. They are not normally part of ILP output.

Another ILP primitive is the picture element **TEXT**. Its use is shown in example 2.

Example 2

```
PICT ( 2 ) ex2
        WITH VISIBLE
        DRAW
                TEXT ("A" , " triangle" ) .
```



Here, the string "A" and "triangle" are drawn, starting again at the untransformed pen position.

2.3. Attribute classes

We have already encountered pieces of ILP programs, enclosed between the "brackets" **WITH** and **DRAW**. These program parts consisted of sequences of attribute class elements, separated by semicolons. Attributes are instruments to influence the way in which a picture element is drawn, or to associate non graphical information with it.

All attributes together, that are relevant for a particular picture element, determine the so-called state of that element. This state determines, what will actually happen, when that element is drawn. Attributes are divided in attribute classes, each corresponding to a particular type of operation on picture elements. In the following, some

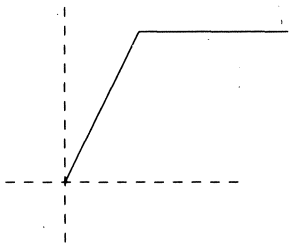
examples will be given on the attribute classes coordinate mode, transformation and style.

The class coordinate mode can have either the value **FIXED** or **FREE**. It operates on the coordinates of picture elements. In the case of **FIXED**, coordinates denote absolute positions in user space, as illustrated in example 1. When **FREE** is used, the coordinates denote increments relative to the untransformed pen position. In that case also, the first move is invisible.

Example 3

Replace in example 1 **FIXED** by **FREE**.

The resulting drawing is:

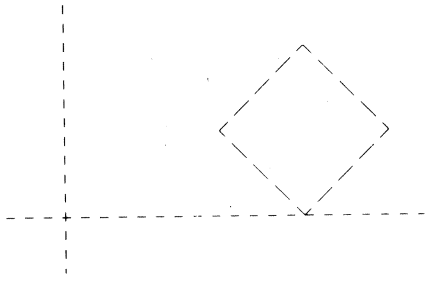


Note in particular, that only two visible lines are shown in this drawing. This is so, because an increment [0 , 0] represents a line of zero length, which coincides with the end point of the second line.

Example 4

```
PICT ( 2 ) ex4
        WITH {
                FIXED ;
                CALE [ 0.1 , 0.1 ];
                TRANSLATE [ 1,0 ] ;
                ROTATE -45 AROUND (  [ 1,0 ] ) ;
                PERIOD ( 50 , 25 , 25 ) ;
                MAP ( 0.04 CONTINUE )
                }
        DRAW
                LINE ( [ 0,0 ], [ 1,0 ], [ 1,1 ],
                        [ 0,1 ], [ 0,0 ] ) .
```

The corresponding drawing is:



A square is drawn, rotated clockwise through 45 degrees, around its lower right-hand corner. This square is translated 1 unit in the x-direction and 0 units in the y-direction, and finally scaled to one tenth of the original size in both directions. **TRANSLATE**, **ROTATE**, and **SCALE** denote transformations. (Note that the rightmost transformation is applied first!). **PERIOD** and **MAP** denote elements of the attribute class style, they determine line style. In this case, the style pattern, defined by **PERIOD** consists of a dash, a gap and again a dash, with respective length of 50, 25 and 25 units. **MAP** specifies that the actual length of this pattern in user space is 0.04, and that the pattern continues from one line (element) to the next. The scale factor serves the additional purpose of keeping all coordinates in user space within the prescribed bounds ([-1,-1] to [+1,+1]). In section 2.6 a more convenient mechanism wil be presented.

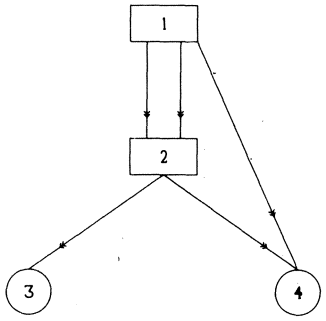
2.4. Data structure

ILP can be viewed as a language to describe data structures, which in turn correspond to drawings.

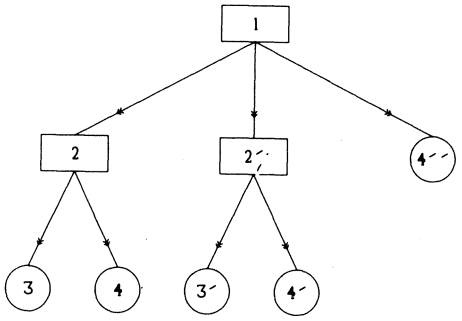
An ILP data structure has the form of a directed acyclic graph. The pictures and attributes correspond to nodes of the graph, the references to pictures and attributes correspond to arcs. The acyclicity results from the semantic rule, that ILP programs may not be recursive. The graph can be converted into a tree, by making copies of all multiply referenced nodes, and creating appropriate references to these new nodes.

Example 5

Suppose, an ILP graph has the form:



The corresponding tree looks like:



In this tree, 2´ is a copy of node 2, 3´ of 3, and 4´´ and 4´ of node 4.

The drawing represented by the tree, can be produced by a process called elaboration. During this process, the tree is traversed in preorder [3], which is recursively defined by:

-- Visit the root of the tree.

-- Traverse its descendant sub-trees in preorder. Descendants are traversed one by one, starting with the leftmost subtree, then proceeding to its rightmost neighbour and so on, until the rightmost subtree has been traversed.

The nodes of the tree in example 5 are visited in the following order:

1 2 3 4 2´ 3´ 4´ 4´´.

At every node where, according to the ILP program some action must take place (drawing, evaluating of attributes and updating the state, subspace selection), this action is initiated by the elaboration process when this node is encountered. Only at the picture leaves (representing picture elements), drawing actions are performed.

In sections 2.4.1. till 2.4.3. these data structure aspects of ILP will be elucidated with the help of some examples.

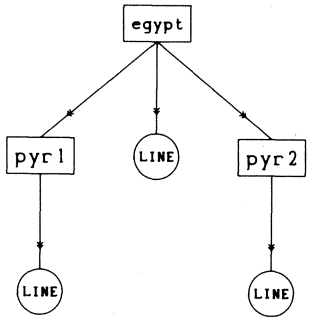
2.4.1. Pure pictures

Pure pictures correspond to subtrees (graphs) of the full ILP tree (graph). They are characterized by the property that they do not contain attributes. A pure picture can constitute a correct and complete ILP program in which all attributes have default values. We will ignore attributes for the moment and introduce some ILP concepts using pure pictures as examples.

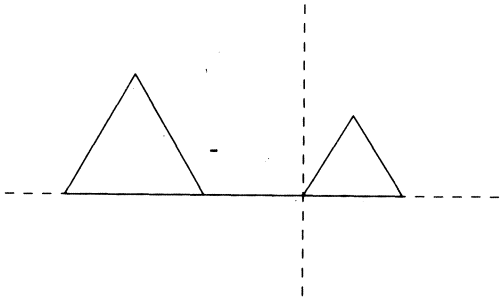
Example 6

```
SUBPICT ( 2 ) pyr1
        LINE ( [ 0,0 ],[ 0.5,0.8 ],[ 1,0 ],[ 0,0 ] ) .

SUBPICT ( 2 ) pyr2
        LINE ( [ -1,0 ],[ -1.6,1.2 ],[ -2.4,0 ],[ -1,0 ] ) .

PICT ( 2 ) egypt
        { pyr1 ; LINE ( [ 0,0 ],[ -1,0 ] ) ; pyr2 } .

PICT ( 2 ) ex6
        WITH { FIXED ; SCALE [ 0.1,0.1 ] }
        DRAW egypt .
```

The tree, defined by ex6 contains a pure picture tree, corresponding to



egypt. The drawing looks like:



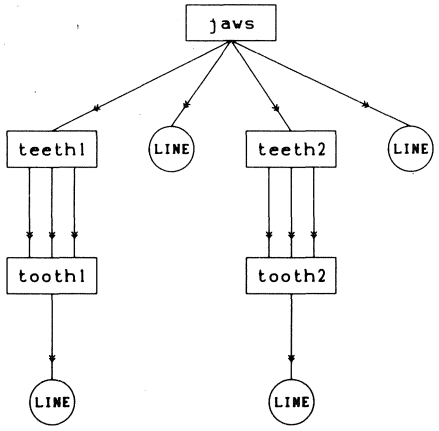
This last example illustrates that there are two kinds of named pictures (pictures having a name); root pictures (designated by **PICT**) and sub pictures (**SUBPICT**). Root pictures are the only ones that may be referred to from outside the ILP program in which they are defined. The root of an ILP graph must correspond to a root picture or in other words, elaboration can only start in a root picture.

Another example of a named picture, defining a pure picture graph, is the following:

Example 7

```
SUBPICT ( 2 ) tooth1
        LINE ( [ 0 , 0 ] , [ 1 , 4 ] , [ 1 , -4 ] ) .

SUBPICT ( 2 ) tooth2
        LINE ( [ 0 , 0 ] , [ -1 , -4 ] , [ -1 , 4 ] ) .

SUBPICT ( 2 ) teeth1
        { tooth1 ; tooth1 ; tooth1 } .

SUBPICT ( 2 ) teeth2
        { tooth2 ; tooth2 ; tooth2 } .

PICT ( 2 ) jaws
        { teeth1 ; LINE ( [ 0 , 0 ] , [ 0 , 10 ] ) ;
          teeth2 ; LINE ( [ 0 , 0 ] , [ 0 , -10 ] ) } .
```

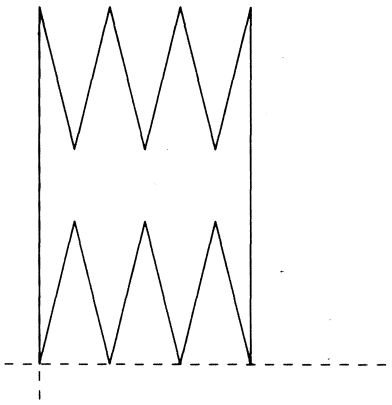
The picture graph defined by jaws is:



The ILP statement:

```
PICT ( 2 ) ex7 WITH { FREE ; SCALE [ 0.1 , 0.1 ] }
               DRAW jaws .
```

defines the drawing:



All elements of a pure picture are elaborated in the same state. The structure of example 7 can therefore be reduced (but not compactified) to a linear list of LINE's. However in that case the logical distinction between tooth and teeth is lost.

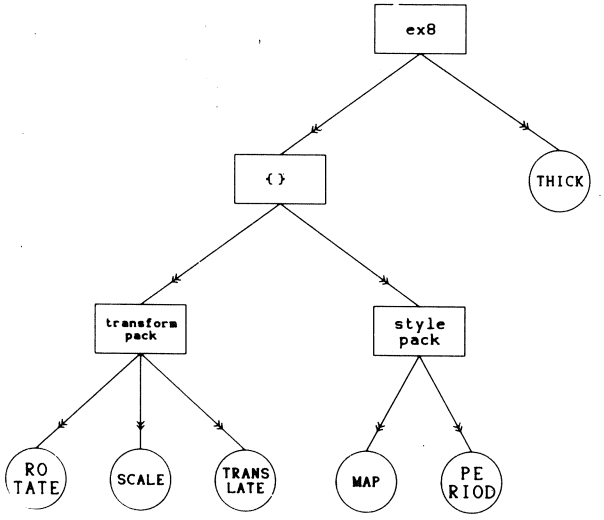
2.4.2. Pure attribute graphs and picture nodes

As with named pictures, attributes can be grouped in named units too, called attribute packs.

Example 8

```
ATTR ( 2 ) transformpack {
        ROTATE 90 AROUND ( [ 1 , 1 ] ) ;
        SCALE [ 2 , 3 ] ;
        TRANSLATE [ -1 , 0.5 ]
        } .

ATTR DIMLESS stylepack {
        MAP ( 10 CONTINUE ) ;
        PERIOD ( 10 , 3 , 11 )
        } .

ATTR ( 2 ) ex8 {
        { transformpack ; stylepack } ; THICK ( 10 ) } .
```

Attribute pack ex8 defines a pure attribute graph of the form:



Just as named pictures, attribute packs have a dimension, which is specified in the same way. This dimension is obviously meaningful when the pack contains for instance transformations. In other cases (for instance for a style pack), the pack could be used in combination with pictures of arbitrary dimension. Arbitrary dimension is specified by **DIMLESS**

This example illustrates another property of ILP programs: by means

of brackets, structure can be enforced, without using explicit references to (using names of) objects. In example 8 the attribute node labelled "{}" is added because of the construction:

{ transformpack ; stylepack }

In the same way, picture nodes can be created.

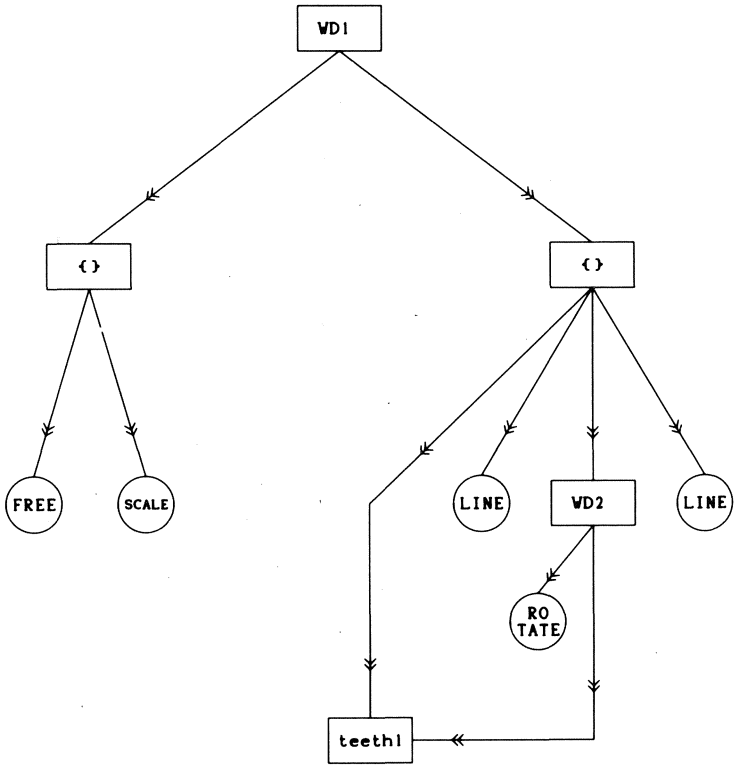
As can be seen from the examples already given, the **WITH...DRAW** construction links attributes to pictures. In the data structure, a **WITH...DRAW** node is itself a picture node, i.e. at any place in the data structure where a reference to a pure picture graph is permissible, a reference to a **WITH...DRAW** node is allowed as well. A picture graph has a structure similar to that of a pure picture graph, but with the extra property, that certain picture nodes (**WITH...DRAW** nodes) have pure attribute graphs also as descendants. In other words, a **WITH...DRAW** node of a picture graph, has a number of pure attribute graphs, as well as a number of picture graphs as its descendants.

The data structure defined by an ILP program, is a picture graph.

Example 9

```
PICT ( 2 ) ex9
        WITH { FREE ; SCALE [ 0.1 , 0.1 ] } DRAW {
                teethl ;
                LINE ( [ 0 , 0 ] , [ 0 , 10 ] ) ;
                WITH ROTATE 180 AROUND ( [ 6 , 10 ] )
                DRAW {
                        teethl ;
                        LINE ( [ 0 , 0 ] ,  [ 0 , 10 ] ) }
        } .
```

The data structure has the form:



This data structure contains two **WITH...DRAW** nodes, labelled "WD1" respectively "WD2". When for teeth1 the subpicture defined in example 7 is taken, the drawing "jaws" results again. The lower jaw is only subjected to the attribute from WD1, the upper jaw comes in its anatomicatly correct position, because it is subjected to the rotation attribute from WD2 as well.

2.4.3. Combining attributes into a state

As shown in the previous examples, a variety of attributes (possibly specified in different **WITH...DRAW** constructions), can influence a picture element. Clearly it is necessary, to combine these various entities in units that can be meaningfully applied to picture elements. Only elements from one and the same attribute class will be mutually combined

(mixed), in a way that may be specific for the class to which they belong. Next, these combinations are packed into the state. The combinations are applied to picture elements in some fixed order, defined by priority rules.

Example 10

```
PICT ( 2 ) ex10
        WITH { SCALE [ 1 , 2 ] ; SCALE [ 2 , 1 ] }
        DRAW P .
```

A scaling is an attribute of the class transformation. Transformations are simply applied one after the other, starting with the rightmost one (the one, textually closest to the picture element). Hence, this program is semantically equivalent to:

```
PICT ( 2 ) ex10
        WITH SCALE [ 2 , 2 ]
        DRAW P .
```

Example 11

```
PICT ( 2 ) ex11
        WITH SCALE [ 1 , 2 ]
        DRAW
                WITH SCALE [ 2 , 1 ]
                DRAW P .
```

Again this program is semantically equivalent with the previous two.

Example 12

```
PICT ( 3 ) ex12
        WITH MAP ( 3 CONTINUE )
        DRAW {
                P1;
                WITH MAP ( 5 RESETLINE )
                DRAW P2
                } .
```

P1 is drawn under influence of the first map specification, P2 under influence of both the first and the second. Clearly it is meaningless, to apply two map specifications in succession, so they have to be

combined into one single map.

In general, this combining is done by mixing rules, which look like:

A <> B -> C

where A, B and C are elements from the same attribute class. The meaning is, that A concatenated with B, gives C.

For matrix transformations, this rule reads:

A <> B -> A * B

where * denotes matrix multiplication.

In case of map, this rule reads:

A <> B -> B

showing simply, that the second map definition replaces the first.

Example 13

```
PICT ( 1 ) ex13
        WITH { SCALE [ 3 ] ; MAP ( 2 CONTINUE ) }
        DRAW P .
```

Here, the priority rules require, that first the transformation, (**SCALE**) and then the style element (**MAP**) is applied. This has consequences, because a transformed picture element drawn with a certain style, can look quite different from a picture element with a certain style applied to it which is thereafter transformed. (The latter is impossible in ILP.)

Example 14

```
PICT ( 2 ) ex14
        WITH A1
        DRAW {
                WITH A2 DRAW P1 ;
                WITH A3 DRAW P2
                } .
```

When the corresponding data structure is traversed, first the collection of attributes contained in A_1 is encountered, then those in A_2 and finally those in A_3. P_2 is affected both by attributes A_1 and A_3, P_1

by A_1 and A_2. Attribute combination is defined in such a way, that the following efficient combination scheme can be employed:

at A_1: Combine all attributes from A_1 in a (partial) state SP_1. SP_1 is identical to state S_1.

at A_2: Combine the attributes from A_2 in a (partial) state SP_2. Combine S_1 and SP_2, this gives state S_2. S_2 is applied to P_1.

at A_3: Combine the attributes from A_3 in a (partial) state SP_3. Combine S_1 and SP_3, this gives state S_3. S_3 is applied to P_2.

Hence, attributes within one **WITH...DRAW** construction have to be combined only once during elaboration. Attributes from nested **WITH...DRAW** constructions can be combined and retrieved, using a stack.

2.5. Default attribute, matches and prefixes

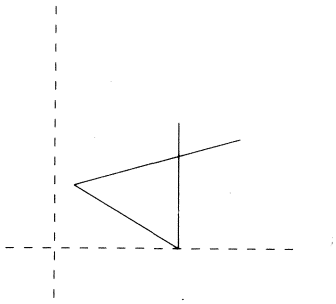
Every attribute class has a default element. If, during elaboration, a picture element is reached and the state does not contain a fully specified element for a certain attribute class, the default element is used. For instance, the default transformation is a unit matrix, the default for visibility is **VISIBLE**. Defaults release the user of the burden to specify values for all attribute classes.

With most attribute classes, an attribute match is associated. Its function is, to switch at the picture element level, between a default value for the associated class and the value specified in the program. This default is equal to the default valid at the root, as long as no **SUBSPACE** has been entered. On subspace entry, the default is reset to the then current value. See 2.6 for further information.

Example 15

```
PICT ( 2 ) ex15
        WITH { FIXED ; ROTATE -30 AROUND ( [ 0.5,0 ] ) }
        DRAW
        LINE ( [ 0,0 ],[ 0.5, 0 ],~TF [ 0.5, 0.5 ], [ 0,0 ] ) .
```

The drawing is:



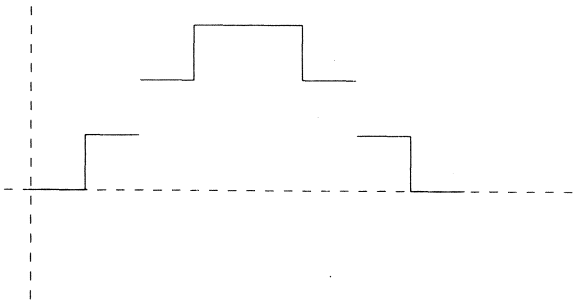
TF is the match for transformations. ~**TF** here signifies, that the second line element must not be rotated, but subjected to the default transformation (unit matrix) instead.

Matches not only can be applied to "arguments" (for instance coordinates) of a picture element, but also to the element as a whole. This leads to the possibility of two levels of matches. The one, directly preceding an "argument", locally replaces the match of a whole element.

Example 16

```
PICT ( 2 ) ex16
        WITH { INVISIBLE ; SCALE [ 0.1 , 0.1] ; FREE }
        DRAW
        LINE ~VS( [ 0,0 ] , [ 1,0 ] , [ 0,1 ] , [ 1,0 ],
        VS [ 0,1 ] , [ 1,0 ] , [ 0,1 ] , [ 2,0 ] ,
        [ 0,-1 ] , [ 1,0 ] , VS [ 0,-1 ] , [ 1,0 ] ,
        [ 0,-1 ] , [ 1,0 ] , [ 0,-1 ] ) .
```

The drawing is:



Visibility gets value **INVISIBLE**. The match **~VS** directly following "**LINE**", replaces this class value by the default value **VISIBLE**, so the line as a whole is made visible. Locally, the explicit class value **INVISIBLE** is reinstalled by the match **VS**, causing some line segments to become invisible.

Every attribute can be prefixed either by **ABS** or by **REL**. If no prefix is present (as in all our examples until here), prefix **REL** is assumed. If an attribute has prefix **REL**, it will be combined with the appropriate class value, contained in the current state. If it has prefix **ABS**, the attribute is combined with the current default (either the predefined default valid at the root, or the class value on the most recent subspace entry).

Absolute transformations are for example useful to draw some picture at a fixed position and with a fixed size and orientation in user space, regardless of the transformation class value on the program point from which the picture was called. Obvious applications are drawing legendae with maps or illustrations, and putting something in a menu during elaboration of a complicated picture on the screen.

2.6. Subspace

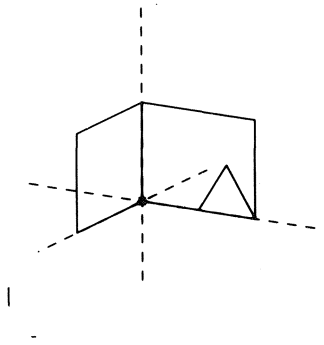
The subspace construction is the mechanism to redefine the coordinate system of user space. It can be used to change axes, without changing the dimension of user space and to specify proper subspaces (i.e. with lower dimension) of an envelopping space. Hence dimension can change in an ILP program. The dimension of subpictures, root pictures and attribute packs is explicitly specified and determines the number of components of coordinates, matrices etc. Hence it can be statically checked, whether ILP statements within the scope of a subspace selection,

use elements of the proper dimension. A second effect of subspace selection is the redefinition of all default class values to the value as accumulated on subspace entry. In this way, a subspace serves as an enclosed area: nothind defined in the outside world can be changed.

Example 17

```
PICT ( 3 ) ex17
        WITH { FIXED;
               SCALE [ 0.5 , 0.4 , 0.5 ] ;
               ROTATE  20 AROUND ( [ 0,0,0 ],[ 1,0,0 ] );
               ROTATE -30 AROUND ( [ 0,0,0 ],[ 0,1,0 ] )
                   }
        DRAW {
        LINE ( [ 0,0,0 ],[ 1,0,0 ],[ 1,1,0 ],[ 0,1,0 ],
        [ 0,0,0 ],[ 0,0,1 ],
        [ 0,1,1 ],[ 0,1,0 ] ) ;
        SUBSPACE ( 2 )
                ORIGIN ( [ 0.5,0,0 ],[ 1,0,0 ],[ 0,1,0 ] )
                WITH FREE
                DRAW LINE ( [ 0,0 ],[ 0.25,0.5 ],[ 0.25,-0.5 ] )
        } .
```

The drawing is:



First, squares are drawn in the (x,y) plane, resp (y,z) plane. Then the (x,y) plane is selected as a two dimensional subspace. The subspace origin coincides with the point [0.5 , 0 , 0]. Its x and y axes are identical to those of the envelopping space. Note also the scale factor and the two rotations, which conveniently specify a viewing transformation.

In general, the first "argument" of **ORIGIN** specifies the new origin,

the further "arguments" specify the new axes as vectors in the old coordinate system. In this subspace a triangle is drawn. The coordinates of this triangle must be specified by two numbers instead of three.

The dimension of the root picture where elaboration starts, is defined by that picture itself. The coordinate axes of the user space at the root (the untransformed user space) form by default a right handed, orthogonal coordinate system. After all transformations have been applied to coordinates in a picture element a position in untransformed user space results. This position must lie in the user unit cube, i.e. all its coordinate components must have absolute values less than or equal to one. As a consequence, there seems to be a choice between using picture elements with only small coordinate values, which is quite impractical, or applying a scale transformation at the root. The second possibility is also unpleasant, because it prohibits the use of "ABS" with lower level transformations, which would switch off the scale. The subspace mechanism provides a practical third alternative however. Suppose, elaboration starts in the following root picture:

```
PICT ( 2 ) ex18
        SUBSPACE ( 2 )
          ORIGIN ( [ 0,0 ] ,  [ 0.0001,0 ] , [ 0,0.001 ] )
                "rest of root picture" .
```

Immediately, a new coordinate system is introduced, with its origin and axes coincident with those of the two dimensional untransformed user space. The length unit in this new space is 0.001 of that of the untransformed space however. As a consequence, the coordinate values produced by "rest of root pict" may have absolute values <= 1000. This transformation (and any other defined outside the subspace) can never be switched off, as the effect of ABS and attribute matches never reaches beyond a subspace boundary. Furthermore, the subspace transformation, which is not an attribute cannot be switched of by "ABS".

Example 19

```
SUBSPACE ( 2 )
        ORIGIN ( [ 0,0 ],[ 0.001,0 ],[ 0.001,0.001 ] )
```

Now, not only coordinate values are expressed in different units, but an additional affine transformation is introduced, because the y-axes of the new space coincide with the line y=x in the envelopping space.

2.7. Miscellaneous topics

In the preceding paragraphs, we have focussed attention on the highlights of ILP and have consequently omitted other features. To make the picture given in this overview more complete, we will very briefly discuss them now.

The set of picture elements provided in ILP contains, apart from points, lines and text, also contours (closed polygons) and generators (an elaborate library facility). Only generators will be discussed here.

Whenever the elaboration process (the process that traverses the ILP data structure, see 3.2.3.) encounters a generator, a new data structure is obtained (in some way) and inserted in the place where the generator occurs. Several types of generators exist, which differ in the way they produce a new data structure:

-- symbols: correspond with a previously defined root picture, and can hence completely be specified as ILP program.

-- curves: correspond with a recipe to produce picture elements according to a certain specification (e.g., a sinus curve). These picture elements need not be given in the form of an ILP program. Curves can only produce data structures from a limited class.

-- templates: correspond with a recipe to produce any legal ILP data structure, which may be produced in any way.

Templates form the most general library facility. However, this generality must be paid for, since the data structures produced by templates have to be checked dynamically for correctness, while the correctness of the data structures produced by curves and symbols can be determined statically.

The set of attribute classes contains, apart from transformations and coordinate mode, also style, pen, detection and control.

Coordinate mode deals with absolute and incremental drawing. Examples were given in section 2.3..

Transformations have, apart from a few exceptions the meaning as normally used in computer graphics systems ([1],[2]). An exhaustive list of transformations is:

-- rotate, scale, matrix transformation, affine transformation, homogeneous matrix transformation all with standard meaning.

-- projection, a central or parallel projection which does not reduce the dimension of a picture.

-- window, viewport which resemble the usual concepts of window and viewport, apart from some additions. It is worth mentioning that windows may be arbitrarily nested and that the nested windows may be rotated relative to each other.

Style determines what kind of picture elements must be produced by a drawing machine. In the preceding paragraphs line style (i.e., a style associated with lines) was already mentioned. A style can also be associated with points (point style: determines the symbol to be used for the representation of points) and text (typographic style: determines boldness, italicity, alphabet and the like for text values).

Pen determines the reproduction method to be used for the visualization of picture elements. Examples are colour and intensity.

Detection determines which parts of the ILP data structure can be pointed at by devices such as lightpen and cursor. The result of such an operation is not simply the picture element pointed at, but may be a part of the data structure in which the picture element is contained. In this manner ambiguities can be resolved: when pointing at a door-in-a-house, is the door of the house intended?

In Appendix 3 a more elaborate example is given in which many ILP features are exposed. Comment is given along with the ILP program. Note in particular the convenient way of structuring the picture, which has the desirable effect that only few and simple coordinate values need to be specified.

3. THE SYNTAX AND SEMANTICS OF ILP

3.1. Overall Structure

The complete syntax of ILP is given in Appendices 1 and 2. In this chapter we will use extracts from it as a guide to the discussion. No attempt has been made to exclude all possible syntactical forms that have no semantic meaning. This would make the syntax extremely difficult to read. Instead we tried to keep it as simple as possible.

The syntax rules are grouped in such a way that the basic structure of the language is reflected as much as possible. The syntax is split in two parts: the set of units that will be produced by lexical scanning and the so-called main syntax. Only the main syntax will be described in this chapter, the other part is given in Appendix 2.

The semantic meaning that corresponds with each syntactical construction will be described by means of an interpretation process referred to as elaboration. In the sequel no distinction will be made between the semantic meaning associated with a certain syntactical construction and the result of the elaboration of that construction. When the elaboration of a particular language construction is carried out, the overall interpretation process is in some intermediate stage. This intermediate stage can be considered as the context in which that particular language construction is elaborated and will be referred to as environment. The elaboration process is only used as a description method and is not intended as an implementation proposal.

An ILP program (*picture_program*) consists of three distinct sets: a set of *root_pictures*, a set of *subpictures* and a set of *attribute_packs*:

```
picture_program:
                pictstruct |
                picture_program pictstruct ;

pictstruct:     named_picture |
                attribute_pack ;

named_picture:  root_picture |
                subpicture ;
```

A *root_picture* has two properties that distinguish it from a *subpicture*:

-- The only *pictures* of an ILP program, that can be referenced from another ILP program, are its *root_pictures*.

-- The elaboration of an ILP program starts in a *root_picture* and not in a *subpicture*. *Subpictures* can only be activated via a *named_picture* in the same ILP program. The elements of the three distinct sets are:

root_picture: **PICT** *dimension pname*
picture . ;

subpicture: **SUBPICT** *dimension pname*
picture . ;

attribute_pack: **ATTR** *dimension aname*
attribute . ;

The only connection between a *picture* and an *attribute_pack* is by means of the "**WITH ... DRAW**" construction, e.g.

WITH A **DRAW** P .

The resulting construction is again of type *picture*. The rules for *picture* and *attribute* are:

picture: *pname* |
picture_element |
{ *pictures* } |
subspace picture |
WITH *attribute*
DRAW *picture* ;

attribute: **ABS** *basic_attribute* |
REL *basic_attribute* |
basic_attribute ;

basic_attribute:
attribute_class |
aname |
{ *attributes* } |
NIL ;

Note that a list of *pictures* between brackets is again a *picture* and that a list of *attributes* between brackets is again an *attribute*.

The result of the elaboration of a *picture* depends on the specification of *attribute*s. Section 3.2. describes the global organization of ILP programs and the relationship between *picture*s and *attribute*s.

The environment contains two groups of values:

- One group the so called state, changes as a result of elaborating *attribute*s.
- The remainder changes as a result of two kinds of actions namely, elaboration of *picture_element*s or external actions. The initial environment contains unique values for the members of both groups.

With every *root_picture*, *subpicture* and *attribute_pack* a *dimension* is associated. It determines the number of components of which coordinates and matrices consist, that occur in these constructions. In an environment with a certain dimension, only constructions of the same *dimension* may be referenced. The dimension can be changed by a *subspace* selection. *Dimension* and *subspace* are described in detail in section 3.3..

*Attribute*s are divided into classes. It sometimes matters in which order *attribute*s from the same class are specified. *Attribute*s from different classes are mutually unrelated. A complete treatment of *attribute*s is given in section 3.4..

The language primitives for which some visual representation exists on drawing machines are called *picture_element*s. Examples are points, lines and characters. They are described in section 3.5..

3.2. Graph Structure

An ILP program has no block structure. All *named_picture*s and *attribute_pack*s are on the same level. However, each ILP program can be considered as the representation of some directed graph structure. The terminology used for graphs is taken from KNUTH [3]. Such a graph is formed by the statical nesting of *picture*s and *attribute*s. These objects are nested either as a result of referring to one object from inside another or nested textually by means of brackets. Recursive calls are explicitly forbidden, hence the graph is an oriented graph without cycles. The graph can be expanded into a tree by replacing all multiple referenced subgraphs (*named_picture*s, *attribute_pack*s) by separate copies. Inside an *attribute* only other *attribute*s may be referenced; this gives rise to attribute nests. Attribute nests only contain *attribute*s. Through the **WITH ... DRAW** construction (see section 3.1.), *picture*s may contain references to both *attribute*s (attribute nests) and other *picture*s, resulting in picture nests.

In correspondence with the syntax, the graph has two types of nodes

namely picture nodes and attribute nodes. There are corresponding types of arcs namely arcs pointing to a picture node (picture arcs) and arcs pointing to an attribute node (attribute arcs). Every *root_picture* constitutes a connected (directed) subgraph. All picture nodes not connected to this subgraph have no meaning with respect to an elaboration of this particular *root_picture*. In the following we will restrict ourselves to such connected subgraphs, which will be called picture graphs. If we remove the picture nodes and picture arcs from the complete graph, then for every "**WITH...DRAW**" node we obtain an isolated attribute graph, which contains only attribute arcs and attribute nodes.

3.2.1. Picture nodes

The alternatives in the following syntax rule are the constructions that can represent a picture node:

```
picture:    pname |
            picture_element |
            { pictures } |
            subspace picture |
            WITH attribute
                    DRAW picture ;
```

A *picture_element* (c.f. 3.5.) is an end node (leave). The other alternatives of the rule are nodes (but not leaves). Note that,

{ *picture_element* }

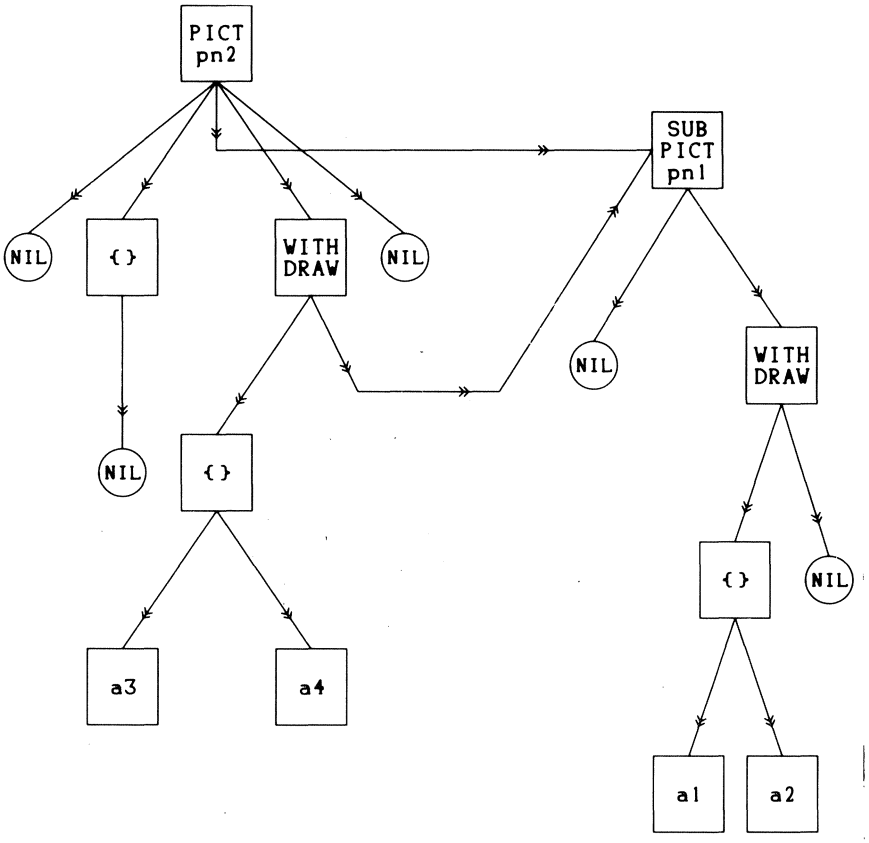
is a special case of

{ *pictures* }

which is not a leave. Because a *picture_element* may have value **NIL**, arbitrary graph-structures can be specified, even without writing down any other action than **NIL** e.g.:

```
SUBPICT (3) pn1  {
        NIL ;
        WITH { a1 ; a2 }
        DRAW NIL
}.

PICT (3) pn2 {
        NIL ; { NIL } ; pn1 ;
        WITH { a3 ; a4 } DRAW pn1 ; NIL
}.
```

The graph for this ILP program is:



3.2.2. Attribute nodes

Attribute nodes are represented by *basic_attribute*s as can be seen in the syntax rules:

attribute: **ABS** *basic_attribute* |
REL *basic_attribute* |
basic_attribute ;

basic_attribute:
attribute_class |
aname |
{ *attributes* } |
NIL ;

The terminal nodes are *attribute_class* and NIL. The other two, *aname* and { *attributes* }, are the non-terminal nodes. An *aname* represents a reference to an *attribute_pack*. The prefixes **ABS** and **REL** have no influence on the graph structure, but specify how the *attribute* has to be mixed with members of the same *attribute_class* (see 3.4.).

3.2.3. Traversing process

3.2.3.1. Basic rules

The structure explained above plays a vital role in the semantics of ILP programs. The description of ILP semantics proceeds in stages. In each stage an algorithm is used that simplifies the graph towards a canonical form. The basic semantic rules associated with the graph are the following:

-- Each (maximal) subgraph containing only *attribute*s is converted into one list of *attribute*s (algorithm ETA, see 3.4.1.). In this list all references to *attribute*s (*aname*s) are replaced by the *attribute*s themselves (algorithm RAP, see 3.4.2.1.). Hence references to *attribute*s are semantically equivalent with textual insertion of the *attribute*s referred to. After further simplification (algorithm LIN, see 3.4.2.1.), the resulting list of *attribute*s (called state component, see 3.4.3.) is applied to the picture node from which the attribute graph is a direct descendant.

-- A state (a combination of state components, see 3.4.3.) can only be applied to picture nodes, in the way described by the following application rule:

Application of a state to a picture node means one of two things:

- If the picture node is a *picture_element* then all *attribute*s in the state are applied as described in 3.2.3.2..

- If the picture node is not a *picture_element*, the state is applied to all its direct descendants as follows: Whenever a descendant is not a *picture_element*, the state is combined with the state component (if present) of that descendant into a new state, otherwise no action takes place. Next this application

rule is used recursively.

As a result of this we have to define three semantic operations on *attribute*s:

-- To combine the *attribute*s in an attribute-graph into one state component.

-- To combine states and state components.

-- To apply an *attribute* to a *picture_element*.

The combination rules for *attribute*s will be given in section 3.4.. The third operation is a special case of applying a state to a *picture*. This will be discussed in general in the next section and for each type of *picture_element* in particular in sections 3.4.4. till 3.4.10..

3.2.3.2. Pictures and picture elements

When the elaboration begins, an initial state is set up as part of the initial environment. Then the traversing process starts in the initial *root_picture*.

When during the traversing process a *picture*, which is not a *picture_element*, is encountered, the following rules apply:

-- Set up a state for that node, by combining the state component (if present) of that node and the previous (either parent or initial) state.

-- Visit all descendants of the node in left-to-right order (which corresponds to textual order in the ILP program).

-- Return to the parent node and restore the original state of that node. In terms of the semantically equivalent tree (the expanded graph), nodes are visited in preorder.

Nodes that are *picture_element*s, represent drawing operations. If these operations are executed by a drawing machine the following happens:

-- The mode of the drawing machine is updated according to the state.

-- Whenever necessary, the *picture_element* is changed into zero or more new *picture_element*s by applying the state to it. *)

-- Each resulting *picture_element* obtained is used to drive the drawing machine.

Thus in addition to the combination rules for state and state component, the semantic operations needed in order to elaborate a *picture* are:

-- Restore, save and combine state(component)s.

-- Return from and call a *picture*.

-- Elaborate *picture_element*s.

So the general scheme is that while traversing the subgraph containing all *picture*s the current state is either updated or applied to a *picture_element*.

3.3. Dimension and subspace

Pictures considered as geometrical objects are defined in an Euclidean space with coordinate axes and a certain dimension. The description of a picture can be simplified by choosing a space of minimal dimension. In many cases, for the user, the position of the picture with respect to the axes is another means to simplify the description. The ILP *subspace* mechanism makes it possible to temporarily change the dimension of the space in which a picture is being constructed. It can reduce the dimension in order to reflect the inherent dimension of that picture. It can also redefine the position and orientation of the axes. If a picture lies, for example, in a given plane then two coordinates are sufficient to specify a point of that picture. In this case the given plane can be selected by *subspace* and as a consequence all redundant coordinates in the picture specification must be omitted.

---

*) The result of the application of the state can partly be described by means of ILP primitives. When this method is used in the sequel, this does not imply, that in an actual implementation the modified *picture_element*s must be available as ILP objects.

3.3.1. Dimension

Before we go into the details of *subspace* selection, some attention must be paid to coordinate systems. The *coordinate*s in an ILP program are expressed in user coordinates. At every point during elaboration all relevant subspace and other transformations (see 3.4.4) concatenate into one current transformation matrix, which defines the mapping from the user coordinates into transformed coordinates These transformed coordinates form a right handed Cartesian coordinate system of dimension equal to the dimension of the root picture. As long as no subspace or other transformation has been specified the current transformation matrix is the unit matrix and the two coordinate systems coincide. In general, coordinates can have arbitrary real values, but there is one important restriction: *coordinate*s, subjected to all relevant transformations, can be divided into two groups: those that pass through all *window*s involved (see 3.4.4.8.) and those that lie outside at least one *window*. The transformed coordinates of the first group all must lie in the unit cube, i.e. have values in the real interval [-1.0,+1.0].

Finally, there exists for each drawing machine a fixed, device-dependent mapping from the unit cube onto points in the addressing area of that device. This mapping is established at the moment of device-selection and is parameterized outside ILP. Because the position and orientation of this addressing area relative to the unit cube can be chosen freely, devices with non-square (or non-cubic) addressing areas can be handled. In this way the mapping on the physical addressing area of an actual drawing device has to be specified for the unit cube only.

A *dimensional_value* is the ILP equivalent of what is elsewhere known as a "coordinate pair" or "coordinates". As can be seen in the syntax rules:

dimensional_value:
[*values*] ;

values: *value* |
values , *value* ;

In ILP, *coordinate*s contain *dimensional_value*s as a special case. For instance, a *coordinate* also specifies whether the *value*s of the *dimensional_value*s are absolute or incremental. When in the sequel the term *dimensional_value*s is used some meaning must be assigned to the special properties that come with *dimensional_value*s only. In all other cases the term *coordinate*s is maintained.

The dimension of a *dimensional_value* (i.e. the number of *value*s of which the *dimensional_value* consists) is not dictated by the syntax. On the other hand, *subspace* (see 3.3.2.) fixes, among other things, the dimension of the environment. Therefore the following semantic rule

(general dimension rule) is required to enforce the right dimension of *dimensional_value*s in various contexts:

In an environment of a certain dimension, the following constructions may only occur with the same dimension as that of the *subspace*:

-- *dimensional_value*;

-- reference to a *subpicture* and a *root_picture*;

-- reference to an *attribute_pack*;

-- *subspace* selection.

To enforce this rule, a *dimension* is associated with each *root_picture*, *subpicture*, *attribute_pack* or *subspace*. This dimension is either explicitly specified or assumes the default value 2. This implies, for example, that in a *subpicture* with *dimension* two, only *dimensional_value*s consisting of two *value*s may occur. *Dimension* is syntactically described by:

dimension: **DIMLESS** |
dim ;

dim: (*value*) |
empty ;

Because some *attribute*s (like colour and intensity) and *picture_element*s (e.g. **NIL**) are dimension independent the *dimension* specification **DIMLESS** exists. A **DIMLESS** *attribute_pack*, *root_picture* or *subpicture* may be referenced in any environment, regardless of its dimension.

The mechanism just described is extended further to cater for matrices of *dimensional_value*s:

matrix_value: [*dimensional_values*] ;

dimensional_values:
dimensional_value |
dimensional_values ,
dimensional_value ;

A *matrix_value* consists of a number of *dimensional_value*s equal to the dimension of the current environment.

The other constructions which must fit dimension, are *subspace*,

rotate and *homogeneous_matrix*. The restrictions on their values are discussed in 3.3.2., 3.4.4.1. and 3.4.4.7..

3.3.2. Subspaces

With the aid of this conceptual framework, the *subspace* selection mechanism can now be explained. Syntactically a *subspace* is specified as follows:

subspace: **SUBSPACE** *dim new_axes* ;

new_axes: *position* (*shift axes*) ;

shift: *dimensional_value* ;

position: **CURRENT** |
ORIGIN ;

axes: *empty* |
, *dimensional_values* ;

The *subspace* construction defines new coordinate axes with respect to the ones, still valid during its elaboration. The origin of the *subspace* follows from *position* and *shift*. In the **CURRENT** case, it is the untransformed pen position (UPP, see 3.5.) shifted by the vector corresponding to *shift*, otherwise it is the origin defined by the previous *subspace* selection, shifted by the same amount.

In a *subspace* selection, two dimensions are involved, the dimension of the environment in which the selection occurs, and the dimension of the subspace being selected, specified by *dim*. This latter dimension becomes the new dimension of the environment, during the elaboration of the *picture* which starts with the *subspace*. *axes* must contain a number of *dimensional_values*, equal to the value of *dim*. These *dimensional_values* specify the direction of the coordinate axes and the units in which coordinates are measured, in the subspace. The directions are those of the vectors defined by *dimensional_values*, the metric follows from the rule, that those vectors have unit length in the *subspace*. It should be noted that we do not require that these axes are orthogonal, only that they are defined by independent vectors. The default value for *axes* is the first *dim* axes of the environment.

The general dimension rule excludes the selection of a *subspace* with higher dimension than the environment in which the selection occurs.

The *dimensional_value*s required to specify such a selection would have been of a higher dimension than the dimension of the environment and are thus illegal.

Let the *dimensional_value*s (considered as column vectors) defining the *subspace* be extended with a zero at the bottom, and the result be denoted by the columns $D_1,\ldots,D_n$. Let the column vector from the previous origin to the new origin be extended with a one at the bottom, and be denoted by D. Then the transformation from subspace to environment is given by the matrix:

$$(D_1, \ldots, D_n, D).$$

3.4. Attributes

The syntax rules describing the various *attribute*s are:

attribute: **ABS** *basic_attribute* |
REL *basic_attribute* |
basic_attribute ;

basic_attribute:
attribute_class |
aname |
{ *attributes* } |
NIL ;

attributes: *attribute* |
attributes ; *attribute* ;

attribute_class:
transformation |
detection |
style |
control |
pen |
coordinate_mode |
visibility ;

With every *attribute_class* (except *control*), corresponds an *attribute_match*, defined by the syntax rules:

attribute_matches:
empty |
attribute_matches
deny attribute_match ;

attribute_match:
TF |
DT |
ST |
PN |
CM |
VS ;

deny: *empty* |
~ |
NOT ;

Attribute_matches are part of *picture_elements*.

An *attribute_class* is a terminal *attribute* node. The *attribute_class* values can range from simple constructs to complex structures. For each class, however, the format of the value is fixed. Here we must differentiate between a complete class value (which as such is not a syntactical category) and a contribution to such values by an individual *attribute_class* element (which is a terminal production of *attribute_class*).

For some *attribute_classes* (e.g. *style* and *pen*) the class value is described as an ordered n tuple of so called atoms. An atom has the following properties: It can have a unit value with respect to combining:

$$a * unit = unit * a = a$$

Each element of such an *attribute_class* specifies precisely one atom. Hence, for a complete class value at least n *attribute_class* elements are required. A unit class value consists of the n unit atom values. A set of $k < n$ different atom values can be expanded to a class value by adding a unit value to each missing atom. In this sense each individual atom can also be considered as a class value ($k = 1$).

Unit values cannot (and need not) be specified. They only serve to simplify the semantic description.

Apart from a unit value for *attribute_classes* there exists a default value for each *attribute_class* and also for each atom. This value is taken when an *attribute_class* must be applied to a *picture_element* and the unit value (for a class or atom) is specified. For some classes the

default value can also be selected explicitly as *attribute_class* element.

In the following, we will elucidate, how *attribute*s act upon *picture_element*s. From a semantic point of view, two major steps are needed in the process of applying *attribute*s to a *picture_element*.

In the first step, the attribute structure is simplified by applying combination rules for *attribute*s, to the effect that *attribute* nests and nested "**WITH** ... **DRAW**" constructions are removed. By this process an ILP program can be converted into a so called basic ILP program, that consists of a linear list of "**WITH** A **DRAW** P" constructions, where A denotes a linear list of *attribute_class* values and P a *picture_element*. The linear list A contains all *attribute*s that have been specified for P. The order of the *picture_element*s in the basic ILP program must be the same as in the picture tree when traversed in preorder. The important reason for this is that each *picture_element* partly sets the environment for its successors. *subspace*s and *picture_element*s can only be elaborated when the environment is known. For a given *picture_element* the major steps must be fully completed before the same steps can be taken for its successor. The first algorithm of the first step takes care of all environment specifications for the subspaces. From there all steps can be carried out independent of any environment. When finally the *picture_element* itself is elaborated, the environment is first used to complete the *picture_element*, next the *attribute*s are applied, then the element is drawn and finally the environment is updated. As already stated, there is a correspondence between ILP programs and directed acyclic graph structures. For convenience, we will split the description of the first step in two parts. The first part is described as a conversion of graphs (section 3.4.1.), the second as a conversion of programs (3.4.2.).

In the second major step, the *attribute_class* elements from each "**WITH** ... **DRAW**" construction of the basic program, are concatenated or combined, and then applied one after the other. The general features of this step are described in sections 3.4.2.2. and 3.4.2.3., while the aspects that are characteristic for individual *attribute_class*es, are described class wise in sections 3.4.4. till 3.4.10..

3.4.1. Decomposition of the picture tree

There exists a unique path in the picture tree (see 3.2.) from the root to each *picture_element*, called element path. For every element path, we will construct a new tree, called element tree, as follows:

Algorithm ET: construct an element tree

ET1 Start with a node of the form "**WITH** U **DRAW** NIL", where "U" contains the *attribute_class* unit value followed by a "subspace marker" one for each class. Traverse the element path.

ET2 Every time a *subspace* node is encountered, generate the corresponding *subspace* transformation S (see 3.3.), using the *subspace* specification and the value of the untransformed pen position (UPP see 3.3.2.), which is given in the environment. The UPP is set to the origin. Replace in the original program the *subspace* by "**WITH** { S; $SM_1;SM_2;...;SM_n$ } **DRAW** (so the *subspace* is evaluated only once and in the right environment). SM_i are subspace markers, one for each attribute class. They are considered to be special elements of that class. S is considered to be an element of the transformation class. Continue with the same node.

ET3 Every time a "**WITH...DRAW**" node is encountered, replace the last "NIL" of the element tree by "**WITH** A **DRAW** NIL. Here "A" is the *attribute* of the node at hand.

ET4 When the *picture_element* is reached it replaces the last NIL of the element tree.

A picture tree with *picture_element*s is converted by ET into a semantically equivalent picture forest T_1, ..., T_n of element trees. Tree T_i contains *picture_element* P_i, which is the i-th *picture_element* encountered, when the picture tree is traversed in preorder.

With every element tree T_i, corresponds an *attribute* A_i. A description in the form of a string of every A_i is produced by algorithm ETA , and modified by algorithms RAP and LIN. In this and the following sections manipulations on descriptions of picture and attribute graphs are used. The algorithms as presented, ignore the layout characters in such descriptions.

Algorithm ETA: compute element tree attributes

ETA1 Initialize A_i with "REL{". Traverse T_i from root to leave.

ETA2 Every time a "**WITH** X **DRAW** Y" node occurs, append "X;" to the right of A_i.

ETA3 Finally, replace the last (rightmost) ";" of A_i by "}".

The application of algorithm ETA results in an ILP program with body:

WITH A_1 **DRAW** P_1;
WITH A_2 **DRAW** P_2;
.
.
.
WITH A_n **DRAW** P_n;

3.4.2. Attribute mixing

The process of combining and simplifying *attribute*s that will be described in 3.4.2.1. and 3.4.2.2. is called attribute mixing. It can be applied to any sequence of *attribute*s, whether this sequence is derived from an element tree or not. The result of mixing is again a construction of type *attribute*.

3.4.2.1. Simplification of attributes

Every A_i in the program produced by algorithm ETA, is simplified in the following steps:

Algorithm RAP: remove *aname*s, add prefix

RAP1 Replace all references to *attribute*s in A_i by their body, e.g. for every *aname* substitute the *attribute* from the *attribute_pack* with that aname. Repeatedly perform this step, as long as references to *attribute*s are present (note that recursion is not allowed).

RAP2 Prefix every not prefixed "{" or *attribute_class* with "REL".

Finally, A_i is converted into one list without sublists of *attribute_class*es .

Algorithm LIN: linearize *attribute*

LIN1 Find a construction B of form **ABS** { *attributes* } or **REL** { *attributes* } which contains only prefixed *attribute_class* elements. When no such construction can be found, then, for every attribute class, remove all subspace markers except the last one and terminate.

LIN2 Sort the elements of B class wise, without disturbing the sub-order in each class (result: construction B´).

LIN3 Apply the following substitutions to adjacent elements x and y of B´, belonging to the same *attribute_class* until no further substitutions are possible:

"RFL x; ABS y" -> "ABS y"
"ABS x; ABS y" -> "ABS y"

whenever x is not a subspace marker.

The effect of this substitution rule is that, within an attribute class, all elements between an "ABS" and the previous *subspace* are deleted. The result of this step is construction B´´.

LIN4 Apply the following substitutions to adjacent elements of B´´, belonging to the same *attribute_class* until no further substitutions are possible:

"REL s; ABS y" -> "REL s; REL y"

where s is a subspace marker. As a consequence, in B´´ only the leftmost element belonging to a certain *attribute_class* (called leftmost class element), can have prefix "ABS".

LIN5 If the left bracket is preceded by "ABS" then replace the prefix of every leftmost class element by "ABS". Next remove (the only and outermost) "ABS {" or "REL {" and "}". The result is labelled B´´´.

LIN6 Finally, replace the original construction B in A_i by B´´´ and continue at LIN1.

As a result of algorithms RAP and LIN, the *attributes* A_i in the program produced by ETA are transformed into a simple list of prefixed *attribute_class* elements. It should be noted that, occurrences of prefix "**ABS**" have been removed.

3.4.2.2 Mixing Rule

The description of the semantics of *attributes* always consists of at least two steps:

- describe the semantics of a class value
- describe the semantics of combining a sequence of class elements into a class value. For every *attribute_class* there exists a mixing rule of the form:

A <> B -> C

where <> denotes mixing, A and C are class values and B is a class

element. A, B and C are all of the same *attribute_class*. During elaboration this rule is applied repeatedly for every *attribute_class* on the LIN list, starting with a unit class value and working from left to right. When the subspace marker is encountered, the class value is copied as the "subspace class value". The algorithm continues untill the last element of the LIN list. The final class value is the "current class value". This series of class values, one pair for each *attribute_class*, is then applied to the picture elements. The order of application is determined by the priority of the *attribute_class* (from high to low):

control
coordinate_mode
transformation
visibility
style
detect

3.4.3 States

The subspace class values for all *attribute_class*es combined define the subspace state, while the current class values define the current state. The *attribute_matches* within the *picture_elements* determine which of these states (the selected state) is to be applied.

The elaboration process maintains a record of the Drawing Machine State (DMS) which starts at the initial state and can be changed in combination with some action on the drawing machine. At appropriate points during elaboration of a *picture_element* the DMS is compared with the selected state and if necessary adjusted. This adjustment is usually accompanied by some machine action. For each of these actions an inverse action has to be defined which undoes the effect on the drawing machine. In all cases after this process the DMS corresponds to the selected state.

3.4.4. Transformations

From a semantic point of view, transformations are applied one after the other, although in an actual implementation, matrix transformations (see below), will probably be concatenated. The result of applying a transformation T to a *picture_element* P can be described as an ILP program P´ that consists of a linear list of transformed *picture_element*s. Transformations are window definitions, quality definitions or matrix transformations.

The semantics of matrix transformations have some general aspects that will be discussed first.

When a matrix transformation is applied to a *coordinate_type picture_element* (see 3.5.1.) the resulting ILP program P´ consists of one *picture_element* of the same category as the original *picture_element*.

*Picture_element*s, either contain a row of *coordinate*s (e.g. *line*) or generate a sequence of *coordinate*s (*generator*, *text*). A *coordinate* contains a *dimensional_value* which, if the dimension of the environment is n, consists of the row of *value*s $[v_1, v_2, \ldots, v_n]$. With such a *dimensional_value*, then corresponds a column vector v with n+1 components, defined as:

$$v = \begin{vmatrix} v_1 \\ v_2 \\ . \\ . \\ . \\ v_n \\ 1 \end{vmatrix}$$

In the sequel this extended form (i.e. homogeneous coordinates) will be used).

With every matrix transformation either a n,n-matrix, or a (n+1),(n+1)-matrix can be associated, where n is again the dimension of the environment. n,n-matrices will be extended to (n+1),(n+1)-matrices by first extending every row with a rightmost element with value zero, and then adding an extra (bottom) row of n+1 elements which are all zero, except for the rightmost one, which has value one.

Hence every matrix transformation is represented by a (n+1),(n+1)-matrix A. To vector v corresponds a transformed vector w, defined by:

$$w = A * v$$

where "*" denotes ordinary matrix multiplication. Because column vectors are used, the order of multiplication must be matrix times vector.

To vector w corresponds a *dimensional_value*

$$\left[\frac{w_1}{w_{n+1}}, \frac{w_2}{w_{n+1}}, \ldots, \frac{w_n}{w_{n+1}} \right]$$

which is called the transformed of *dimensional_value* $[v_1, \ldots, v_n]$. The result of applying a matrix transformation to a *picture_element* is now obtained by replacing all (generated) *dimensional_value*s by their

transformed *dimensional_value*s.

A transformation class value consists of

$$\{ M, W, Q \}$$

where
M is the transformation matrix
W is the window
Q is the text quality
The unit value is

$$\{ \text{unit matrix, empty, LOW} \}$$

The mixing rule for transformation is:

$$\{ M_a, W_a, Q_a \} \ <> \ B \ => \ \{ M_c, W_c, Q_c \}$$

when

- B is a matrix:

$M_c = M_a <> B$ (matrix concatenation)
$W_c = W_a$
$Q_c = Q_a$

- B is a window:

$M_c = M_a$
$W_c = W_a <> (M_a * B)$
$Q_c = Q_a$
<> is concatenation.
* is matrix-window multiplication.
Note that the transformed window is mixed with the class value.

- B is a text quality:

$M_c = M_a$
$W_c = W_a$
$Q_c = B$

The *transformation*s are listed in the following syntax rules:

```
transformation: rotate |
                scale |
                translate |
                matrix |
                projection |
                affine |
                homogeneous_matrix |
                port
                text_quality ;

rotate:         ROTATE value
                       AROUND invariant ;

scale:          SCALE dimensional_value ;

translate:      TRANSLATE dimensional_value ;

matrix:         MATRIX matrix_value ;

affine:         AFFINE matrix_value
                       dimensional_value ;

projection:     projection_type eye_position
                       ON projection_space ;

homogeneous_matrix:
                HOMMATRIX homogeneous_matrix_value ;

port:           window |
                window , viewport ;

text_quality:   QUALITY( quality ) ;
```

3.4.4.1. Rotation

An elementary rotation in n-dimensional Euclidean space can be specified by:

-- Selection of a plane V in the n-dimensional space.

-- Selection of a point P in this plane.

-- Definition of a rotation angle phi.
The matrix R:

$$\begin{vmatrix} \cos\varphi & \sin\varphi & 0 & \cdots & 0 \\ -\sin\varphi & \cos\varphi & & & \\ 0 & & 1 & & \\ \cdot & & & \cdot & \\ \cdot & & & \cdot & \\ \cdot & & & \cdot & \\ 0 & & & & 1 \end{vmatrix}$$

describes this elementary rotation under the condition that a new set of coordinate axis $x_1, \ldots, x_n$ is chosen with:

-- The origin coincident with P.

-- x_1 and x_2 contained in V. Let the matrix which transforms the original coordinate axis into the set $x_1, \ldots, x_n$ is given by T, then the rotation in the untransformed coordinate system is given by

$$T^{-1} * R * T$$

A rotation in n-dimensional Euclidean space can be considered as the product of a number of elementary rotations.

In ILP, an elementary rotation is syntactically specified by:

rotate: **ROTATE** *value*
AROUND *invariant* ;

invariant: (*dimensional_values*) ;

The rotation angle is determined by *value*, while the rotation plane and point are specified by *invariant*. The *invariant* contains a number of *dimensional_values* which is one less then the dimension of the environment. The first *dimensional_value* specifies the rotation point P, the following define (n-2) independent vectors orthogonal to the rotation plane. Rotation takes place clockwise (defined with respect to the normal from the origin to the plane), through a number of degrees, specified by *value*.

In the two dimensional case, the set of n-2 vectors is empty, in the three dimensional case it is the familiar axis of rotation. As a consequence, in the two or three dimensional case a general rotation can

be specified by one single *rotate*.

REMARK

It should be clear that we are confronted with a tradeoff here: if the dimension of the environment is less then four, it is economical to specify a plane by its normals, if the dimension is more then four, specifying the plane with two vectors contained in it is cheapest. We have chosen the first alternative.

3.4.4.2. Scale

By scaling, the *value*s of the *dimensional_value* of a *coordinate* are changed independently of each other. Scaling can be represented by a diagonal matrix. The syntax rule is:

scale: **SCALE** *dimensional_value* ;

Each *value* in the *dimensional_value* specifies a diagonal element of the unextended transformation matrix.

3.4.4.3. Translate

A translation maps all points in user space on points displaced by a fixed amount. Translation is syntactically described by:

translate: **TRANSLATE** *dimensional_value* ;

Each *value* in the *dimensional_value* specifies the displacement along the corresponding coordinate axis.

In an n-dimensional environment a translation, characterized by *dimensional_value* $[v_1, \ldots, v_n]$, is represented by a (n+1),(n+1)-matrix with diagonal elements of unit value, the rightmost element of the k-th row (k = 1, ..., n) with value v_k, and all other elements zero.

3.4.4.4. Matrix

A matrix transformation specifies a linear transformation of the user space. A matrix transformation is syntactically described by

matrix: **MATRIX** *matrix_value* ;

Each *dimensional_value* in the *matrix_value* (see 3.3.1.) specifies a column in the transformation matrix. As a consequence of the general

dimension rule (see 3.3.1.), a matrix contains a number of rows and columns equal to the *dimension* of the environment.

3.4.4.5. Projection

Projection is syntactically described by:

projection: *projection_type eye_position*
ON *projection_space* ;

projection_space:
dimensional_value |
ORIGIN *dimensional_value* ;

eye_position: *dimensional_value* ;

projection_type:
PROJECT |
PROJECT PERSPECTIVE |
PROJECT REVERSIBLE ;

If the keyword **REVERSIBLE** is not used the coordinate space is projected onto *projection_space*. It is a space of dimension one less than the environment, perpendicular to the the vector specified by *dimensional_value* in *projection_space*. Nevertheless, the projected image has the dimension of the environment, but there exists a linear relation between its coordinates, for instance, $x_n = 0$. If only a *dimensional_value* is present in the specification of the *projection_space*, the space contains the end point of the vector defined by *dimensional_value*. If the keyword **ORIGIN** is used it contains the origin of the current coordinate system.

A **PROJECT PERSPECTIVE** specifies a central projection with the *dimensional_value* as centre. **PROJECT** specifies a parallel projection to a direction defined by *dimensional_value*. A **PROJECT REVERSIBLE** specifies a perspective distortion equivalent to the central projection. The sequence

PROJECT [0,...,1] **ON ORIGIN** [0,...,1]; **PROJECT REVERSIBLE**

is equivalent with

PROJECT PERSPECTIVE

Let the coordinate axis of an n-dimensional Euclidean space be $x_1, \ldots, x_n$. A projection with the point $(x_1 = x_2 = \ldots = x_{n-1} = 0, x_n=c)$ as centre, on the space $x_n=0$ is given by the (n+1),(n+1)-matrix P.

$$\begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & & 1 & & \\ 0 & \ldots & 0 & a & 0 \\ 0 & \ldots & 0 & b & 1 \end{pmatrix}$$

For **PROJECT REVERSIBLE** and **PROJECT PERSPECTIVE** b equals -1/c where c equals the distance from the centre to the *projection_space*. Else b equals 0. If the keyword **REVERSIBLE** is used a is 1 else 0. Let T_1 be the transformation that translates the projection of the *eye _position* onto the projection space to the origin, T_2 the transformation that rotates the normal on this space to the direction of coordinate axis x_n. The projection is then given by the matrix:

$$T_1^{-1} * T_2^{-1} * P * T_2 * T_1$$

In the following example a three dimensional environment is assumed, with coordinate axes denoted by x, y and z.

PROJECT PERSPECTIVE [1, 1, 1] **ON** [0, 0, 1]

defines a central projection on the plane z=1. With the point x=1, y=1, z=1 as projection centre.

PROJECT [0, 0, 1] **ON ORIGIN** [0, 0, 1]

defines a projection parallel to the z-axis on the plane z=0.

Caution should be given to the sequence

{ **PROJECT** ...; **WINDOW** ... }

where the **PROJECT** is not **REVERSIBLE**. According to the mixing rule for transformation the clipping is done in the *projection_space* itself. As a consequence, under a non **REVERSIBLE** projection, some information is lost that might be necessary to correctly decide on the visibility of all picture elements.

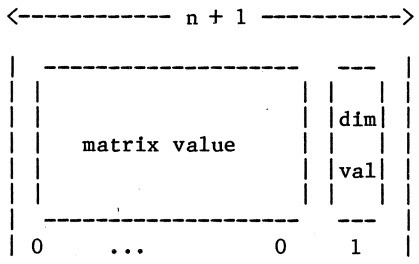
3.4.4.6. Affine

affine: **AFFINE** *matrix_value*
dimensional_value ;

An affine transformation is represented by a square matrix with:

-- A number of rows (columns), one more than the dimension of the environment.

-- A bottom row with all elements zero, except for the rightmost, which has value one. If the dimension of the environment is n, *matrix_value* specifies a n,n-matrix A, *dimensional_value* a column of n elements. The resulting affine matrix is:



3.4.4.7. Homogeneous matrix

In an n-dimensional environment a *homogeneous_matrix* transformation is represented by a (n+1),(n+1)-matrix. Every element of this matrix is explicitly specified with the help of the following syntactical construction :

homogeneous_matrix:
HOMMATRIX *homogeneous_matrix_value* ;

homogeneous_matrix_value:
[*homogeneous_dimensional_values*] ;

homogeneous_dimensional_values:
homogeneous_dimensional_value |
homogeneous_dimensional_values ,
homogeneous_dimensional_value ;

homogeneous_dimensional_value:
[*values*] ;

The *homogeneous_matrix_value* consists of (n+1) *homogeneous_dimensional_values*, which each specify a column of the matrix. Every *homogeneous_value* consists of (n+1) *values*, which each specify an element of the column.

3.4.4.8. Window and viewport

The *port* transformation is syntactically described by:

port: *window* |
window , *viewport* ;

window: **WINDOW** (*dimensional_value* ,
dimensional_value) ;

viewport: **VIEWPORT** (*dimensional_value* ,
dimensional_value) ;

Window and *viewport* select rectangular areas in user space.

The *dimensional_value* pairs in both the *window* and *viewport* definition, determine the end points of a principal diagonal of the *window* and *viewport* areas. As a consequence of the general dimension rule (see 3.3.1.), the dimension of a *window* or *viewport* is equal to the dimension of the environment in which the *window* and *viewport* are specified.

The selected areas are fully determined by the requirements that they are block shaped, and that they have their edges parallel to the coordinate axis.

When *coordinate_mode* (see 3.4.8.) has value **FREE**, the relative position of the *port*s and the free coordinates may not be known from context. The *dimensional_value*s in *port* denote absolute positions in the current coordinate system. Selection of a new origin at the untransformed pen position with the help of the **SUBSPACE** mechanism, solves this problem.

If the *port transformation* does not contain a *viewport*, only a clipping boundary is defined. Only those parts of the picture, that lie inside the window, are preserved. Without going into detail, we summarize in the table below, for every type of *picture_element* the possible elements of the result set (see 3.4.2.2.), if this set is not empty.

Picture_element result set elements

POINT	zero or one **POINT**
LINE	zero or more **LINE**´s
CONTOUR	zero or more **LINE**s or **CONTOUR**

The *picture_element generator* ultimately generates elements contained in this table, which determines its behaviour under the *port* transformation. The effect on **TEXT** depends on the value for *text_quality* (see 3.4.4.9).

If the *port* contains a *viewport* (which must be preceded by a *window*), first the matrix that maps the *window* onto the *viewport* is mixed with the class value and next the *window* itself is mixed.

The following observations can be made:

— The effect of the application of a number of *window*s (separated by matrix transformations) is identical to the effect of clipping to the intersection of the leftmost *window* and the (transformed) further *window*s.

— When two *window*, *viewport* pairs are applied, the visible part of the *viewport* area of the second pair, is always contained in the *viewport* area of the first pair.

3.4.4.9 Text quality

text_quality: **QUALITY(** *quality* **)** ;

quality: **LOW** | **MEDIUM** | **HIGH** ;

The value for *quality* determines the influence of transformations on *picture_elements* of type **TEXT**.

LOW The only influence is on the position of the first character. If the start of the first character is outside of a window the whole **TEXT** string is made invisible. Otherwise the string is not clipped.

MEDIUM The position of each character is influenced by the transformations. Clipping is done character by character. All characters are either totally visible or totally invisible.

HIGH All coordinates generated by **TEXT** are fully transformed and

clipped. Characters can be partially visible. For further details see 3.5.2..

3.4.4.10. Subspace and transformations

The major similarity between *subspace*s and *transformation*s is that the effect of any matrix *transformation* (except *homogeneous_matrix* or *projection*), can also be achieved by a *subspace* transformation.

The basic differences between *subspace* and other *transformation*s are the following:

-- A *subspace* can reduce the dimension.

-- A *subpsace* forms a blockstructure which cannot be penetrated by any internal reference:

- **ABS** and *attribute_matches* refer to the attribute class values at the moment the last enclosing *subspace* was entered.

- When a *subspace* is entered the Untransformed Pen Position and the Picture Position are set to the origin.

3.4.5. Style functions

3.4.5.1. Introduction

Style functions describe what kind of lines, points and characters (and in the future shades and greyscales) are to be produced by a drawing machine. The description is as machine independent as possible. In view of the enormous variety of drawing machines, the *style*-function package has to be extendible and is inevitably incomplete.

The given functions are all specified in such a way that the same *style* functions produce similar results on all drawing machines, that is, if they are expressible in terms of the existing hardware. With the exception of *text_quality*, which determines the effect transformations have on **TEXT**, no functions exist in ILP to express the quality required of the result of application of a *style attribute*. *) When necessary an extra software layer has to be provided to produce or approximate *style*s

---

*) It can be considered to parameterize quality outside ILP by providing a quick-and-dirty, and a high-quality mode for the representation of the same *style*.

for which no direct hardware functions are available. Since *style* has more to do with taste and clearness of expression than with accuracy, it will cause no trouble when *style* is not defined with mathematical precision (as would be the case with, say, *transformation*s).

The three classes of *style* functions that exist so far, e.g. *line_style*, *point_style* and *typographic* are mutually unrelated. The syntax for *style* is:

style: *line_style* |
point_style |
typographic ;

The class value of *style* is a 12-tuple with atoms represented by

PERIOD, MAP, THICK,
FONT, SIZE,
ITALIC, BOLDNESS,
POINTSTYLE FONT, POINTSTYLE SIZE,
POINTSTYLE ITALIC, POINTSTYLE BOLD,
POINTSTYLE *marker*.

Let C_1, ..., C_{12} and C_1', ..., C_{12}' denote *style* class values. Then the mixing rules for *style* are:

$$\{ A_1,\ldots,A_i,\ldots,A_{12}\} \ <> \ B \ \text{->} \ \{ A_1,\ldots,A_{i-1},B,A_{i+1},\ldots,A_{12} \}$$

when B is a class element corresponding to atom A_i.

3.4.5.2. Linestyle

Line_style conforms to the syntax:

line_style: **PERIOD** (*period_description*) |
MAP (*value reset*) |
THICK (*value*) ;

*Line_style*s are applied to *picture_element*s of type **LINE**. They are also applied when the **LINE** is produced indirectly, through a *contour*, or a *generator* .

The *line_style* determines what will be drawn along the straight lines that connect the successive positions of the *picture_element*s.

The *line_style* can produce a large variety of dotted and dashed lines. The definition of such a pattern goes in two steps.

3.4.5.2.1. Period definition

PERIOD describes a basic pattern which is repeatedly produced going along the *line*.

period_description:
dash |
dash , *gap* |
dash , *gap* , *dash* ;

dash: DOT |
value ;

gap: *value* ;

The period is defined on a straight line piece of 100 units in length, which is filled out by:

$$dash_1 \quad gap_1 \quad dash_2 \quad gap_2$$

Hence $dash_1 + gap_1 + dash_2 + gap_2 = 100$. Gap_1 through gap_2 may be omitted, implying that the first missing one adds up to 100. Gap_2 always is omitted. If *dash* has value DOT, a point is produced on the spot with has a length of 0 units with respect to the period. This concept **DOT** is the same, as the one used in *point_style*, see 3.4.5.4..

Examples:

PERIOD (100)	Solid line.
PERIOD (**DOT**)	One point at the beginning of each period.
PERIOD (0,100)	Blank (invisible) line.
PERIOD (50)	Dashed line with gaps equal to dashes.
PERIOD (25,50)	Dashed line with gaps equal to dashes. It starts however, with a half dash.

3.4.5.2.2. Map definition

The *value* of **MAP** specifies the actual length of the pattern described by *period_description*. This length is defined in transformed coordinates, valid at the root. A pattern of the given actual length is rolled along the line, to produce the style.

reset: **RESETCOORDINATE** |
CONTINUE |
RESETLINE ;

The three different values for *reset* tell, whether the periodic pattern has to be continued from one **LINE** to the next (value: **CONTINUE**), to be reset at the start of every new **LINE** (value: **RESETLINE**) or to be reset whenever a new *coordinate* within a **LINE** is encountered (value: **RESETCOORDINATE**).

Reset is one of the few *attributes* which influence the Drawing Machine State (DMS see 3.4.3) directly rather than just through the influence on *picture_elements*. One component of DMS records the state of the period generator. If *reset* does not have the value **CONTINUE** this component is reinitialized.

3.4.5.2.3. Thickness

The value of THICK determines the linewidth, when drawing **LINE**s. It is expressed in the same unit as used in the map definition for linestyles (see above). Thick lines are cylindrical. They are drawn with constant diameter. Thick lines are not modified by *projection transformation*s, i.e. they do not become conic.

3.4.5.3. Typographic style

The *typographic style* is in fact nothing else than a means to specify a given character set out of the available sets.

typographic: **TYPFAULT** |
font |
size |
italic |
bold ;

Characters are grouped in sets of at most 256 tokens, called a basic set.

A basic set can contain tokens of any kind, up to complete *picture*s. If *text_quality* has value **MEDIUM** or **LOW** their internal structure is inaccessible and can therefore not be manipulated. If it has the value **HIGH** they are subject to all transformations (including clipping) however.

A *font* consists of a basic set plus a description how the character data are to be interpreted, and what the effect of *size*, *italic* and *bold* is, on the individual tokens. In view of the use of *typographic* for *pointstyle*s also a default token for DOT must be given. A *font* is selected by the *font attribute*. The tokens can be modified, by

explicitly specifying *size*, *italic* and *bold*.

It is clear, that the *typographic attribute*, allows the specification of an unlimited collection of characters. **TYPFAULT** is shorthand for selection of *font*, *size*, *italic* and *bold*. Its effect is device dependent. It denotes a character set, whose elements can be drawn as efficient as possible on the device at hand, if necessary disregarding high quality demands (see 3.4.5.1.).

3.4.5.4. Point style

The syntax rules for point style are:

point_style: **DOT** |
POINTSTYLE *typographic* |
POINTSTYLE *marker* ;

Marker selects a token from the font specified by **POINTSTYLE** *font*. This token is modified by **POINTSTYLE** *size* etc. At point positions, this token is displayed, drawn in a centered fashion. It will be drawn in the x_1, x_2 plane of the current coordinate system, with its "bottom line" parallel to the x_1 axis. When the alternative **DOT** is used, a device dependent "point" will be displayed. **DOT** is shorthand for a device dependent character set (*typographic*) and for a specific token (the point) out of this set. When only **POINTSTYLE** *typographic* or **POINTSTYLE** *marker* is specified, the other atom of *point_style* has its default value (see 3.4.12.).

3.4.6. Pen functions

Pen functions determine the reproduction method to be used when a *picture_element* is drawn. As a consequence, pen functions influence only the final appearance of a drawing but do not affect the structural information contained in it. The effect of pen functions can not be described in terms of ILP primitives. *Pen* is a 3-atomic *attribute*, its mixing rule is analogous to that of *style*.

The syntax for *pen* is:

pen: **PENFAULT** |
contrast |
intens |
colour ;

Just as in the case of **TYPFAULT**, **PENFAULT** selects device dependent values for *contrast*, *intens* and *colour*.

3.4.6.1. Contrast

The syntax of *contrast* is:

contrast: **CONTRAST** (*value* , *value*) ;

It is assumed, that any physical drawing device can draw with a minimal and a maximal intensity, which are the end points of its physical intensity range. (The maximal intensity always represents "light", the minimal "dark", i.e. on a plotter, these two intensities are determined by the reflectivity of the paper, respectively the blackness of the ink.)

For every device a mapping must be defined from the interval [0,100] (the contrast range) to the physical intensity range. *Contrast* specifies a subrange of the contrast range, i.e. fixes indirectly the lowest and highest physical intensity, that can be used.

Examples:

CONTRAST (0 , 100): highest possible contrast.
CONTRAST (50 , 50): no contrast, one intensity.

3.4.6.2. Intensity

Intensity is syntactically described by:

intens: **INTENS** (*value*) ;

and determines the brightness of the registration method. *Value* may have as value a real number from the interval [0,100]. The corresponding physical intensity used by the drawing device is determined as follows. There is a linear mapping from the intensity range [0,100], to the contrast range [a,b] (0 <= a <= b <= 100), specified by *contrast*. So, a value in the intensity range determines a value in the contrast range, which determines the physical intensity, via the mapping from the contrast range to the physical intensity range.

Examples:

INTENS(100) = maximal intensity
INTENS(0) = minimal intensity

There is an important distinction between invisible lines (i.e. lines drawn with value **INVISIBLE** for *visibility*) and lines with zero intensity. In the former case the order in which the invisible lines are drawn is not defined and consecutive invisible lines may even be replaced

by one invisible line. In the latter case the drawing order is completely defined and the kind of optimizations just mentioned are not allowed.

3.4.6.3. Colour

On a mono colour (black/white) drawing device, *contrast* and *intensity* are sufficient for the specification of the different shades of "grey" in the drawing.

Examples:

```
INTENS ( 100 )   white.
INTENS ( 50  )    grey.
INTENS ( 0 )     black.
```

(These examples assume a contrast range with length not equal to zero.)

On a multi-colour device, the contributions of the three primary colours (red, yellow, blue) to the total intensity, specified by *contrast* and *intensity* are defined by *colour*. *Colour* is syntactically described by:

colour: **COLOUR** (*value* , *value* , *value*) ;

The ratio between the three *value*s is the ratio between the primary colour intensities; *value*s may denote arbitrary real numbers.

Examples:

COLOUR (100 , 0 , 0):
red, with an intensity equal to the total intensity.

COLOUR (0 , 10 , 10):
green; Yellow and blue each have half of the total intensity.

COLOUR (1000 , 1000 , 1000):
white; Red, yellow and blue each have one third of the total intensity.

3.4.7. Detection

In this section it will be shown how *attribute*s can be used to model the characteristics of a detection mechanism. *Detection* provides external references to parts of the *picture*. It divides the *picture* in units that may be subjected to further manipulations.

REMARK

The *detection attribute* provides the bridge between the interactive and not interactive parts of ILP. It is clear that this bridge should be designed carefully and that it affects both parts of the language. At this moment, only the not interactive part of ILP is defined. Major problems are involved in the design of this bridge if the interactive function of ILP in a computer graphics system is taken into account:

-- A labelling or addressing scheme must be designed to allow selection of any part of the ILP graph structure.

-- Modification operations on the ILP graph structure must be defined, which result in a compact representation of the modifications (design goal).

The important facility of picture manipulation must be designed with the help of ILP primitives. We want to apply ILP to structure this part of the graphics system just as it structures the basic graphics operations. In accordance with the overall functions of ILP, it is therefore required to solve these problems in such a way, that an ILP graph structure can be manipulated inside ILP itself. At present this is not the case. Some manipulation on these graphs can, however, be described in this report through the description method for the semantics of ILP, for which an (informal) metalanguage is used. More general manipulations, like for example edit operations, can be described neither inside ILP, nor in the metalanguage. One could invent another metalanguage for that purpose. It is far better however, to extend ILP with appropriate constructions to achieve i/o symmetry. The detection mechanism, only solves the first of the four problems: An addressing scheme for picture nodes is given.

Three entities are required to describe detection. A detector is an external process (which for example can involve lightpen, tracking cross or even some combination of these), that is used to select nodes in the ILP data structure. A detector has a name which is part of the environment when this detector is active. Nodes in the data structure must define by which named detectors they can be selected and for each of these, which identification string must be returned to the user if selection occurs. Thus detectors with different names can be used to search the data structure. The remaining entities are the detectant set and possibly a detectant.

Only *picture_element*s can be pointed at. Nevertheless, all nodes on the path from *root_picture* to this particular *picture_element* must be

potential candidates for selection. The detectant set is a subset of these nodes, and the detectant (if defined) is a preferred element of this subset. They are formed by applying combination rules to the *detection attributes* (see below). Whenever a node is detected, the string associated with it (for the currently active detector) can be returned to the user. This provides him with a facility for identification of the various detection points. During elaboration the detectant set and detectant are constructed, and preserved in the state. Their value can be returned to the user or to the application program, when, during elaboration of a *picture_element*, this element is subjected to a selection action. Initially detector and detectant are undefined and the detectant set is empty.

The *detection attribute* has the following syntax:

detection: **DETECT** *detector proper_string* |
SETDEL *detector proper_string* |
UNDETECT *detector* ;

detector: *empty* |
dname ;

The *proper_string* is the label returned to the user when the node is detected. Each *detector* is identified by a name (*dname*). There is a common *detector* which has no name. Switching from one *detector* to another is possible by external action which consists of selecting a new name or the common *detector*.

The class value of *detection* is:

$$\{ A_1, \ldots, A_i, \ldots, A_k \}$$

where A_i = ($detector_i$, set_i, $detectant_i$)
$detector_i$, set_i, $detectant_i$ can all be empty
$detector_i$ = $detector_j$ if and only if i = j.

The mixing rule for detection is:

$$A \diamond B \Rightarrow C$$

B consists of the following cases:

1 **DETECT** name string

2 **SETDEL** name string

3 **UNDETECT** name

C is given by:

-- if there is no i such that $detector_i$ = name then

- in cases 1 and 2:

$$C = \{ A_1,\ldots,A_k, D \}$$

where D = (name, string, string)

- in case 3: C = A

-- if name = $detector_i$ then

- case 1:

$$C = \{ A_1,\ldots,A_{i-1},D,A_{i+1},\ldots,A_k \}$$

where D = (name, set_{di} <> string, string)

- case 2:

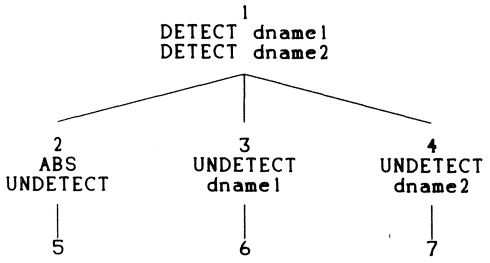
$$C = \{ A_1,\ldots,A_{i-1},D,A_{i+1},\ldots,A_k \}$$

where D = (name, set_i <> string, $detectant_i$)

- case 3:

$$C = \{ A_1,\ldots,A_{i-1},A_{i+1},\ldots,A_k \}$$

Example:
Consider the following ILP graph, in which nodes 1 through 4 are WITH...DRAW nodes and nodes 5 through 7 are *picture_element* nodes.



Node 1 can be detected by the detectors named $dname_1$ and $dname_2$. It is impossible to detect this node by selecting *picture_element* 5, selection of *picture_element* 6, respectively 7, only leads to detection of node 1,

when the detector $dname_2$, respectively $dname_1$ is active.

The nodes of this graph are visited during elaboration in the order:

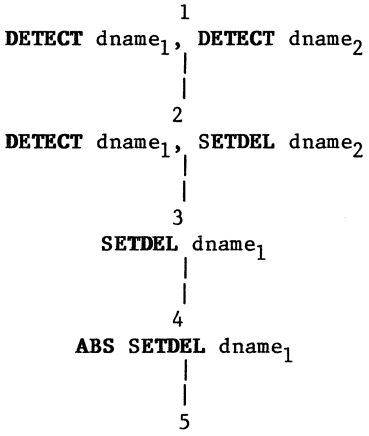
1, 2, 5, 3, 6, 4, 7.

When detector "$dname_1$" is active, the class values at these nodes are shown in the following table:

node	detectant set	detectant
1	empty	none
2	1	1
5	empty	none
3	1	1
6	empty	none
4	1	1
7	1	1

If detector $dname_2$ is active instead of $dname_1$ this table is valid after the rows for node 7 and node 6 have been interchanged.

Example:



The class value table would be:

node	dname1		dname2	
	det set	detectant	det set	detectant
1	empty	none	empty	none
2	1	1	1	1
3	1 2	2	1 2	1
4	1 2 3	2	1 2	1
5	4	none	empty	none

REMARK

So far we have not related the pointing action to visibility aspects. Apart from detectable, each primitive can also be visible or invisible. Many hardware pointing devices (e.g. lightpen) identify detectability and visibility. We have deliberately chosen for the separate concepts, because we can give a meaningful interpretation for each combination of (in)visibility and (un)detectability. For instance, in order to change an invisible move, one must first identify it.

3.4.8. Coordinate mode

The *coordinate_mode attribute_class* is specified by the syntax rule:

coordinate_mode:
FIXED |
FREE ;

When the *coordinate_mode* has value **FIXED**, positioning information represented by a *dimensional_value* is taken to mean an absolute position. When it has value **FREE**, the absolute position is found, by adding the *dimensional_value* to the untransformed pen position (see 3.5.1.). The mixing rule for *coordinate_mode* is:

A <> B => B

In other words, at any time during elaboration, the part of the state, representing *coordinate_mode*, has simply the value that has last been encountered.

3.4.9. Control

The syntax for *control* is:

control: **MACHINEDEPENDENTCONTROL** *proper_string* ;

Control is an instrument for the specification of drawing machine dependent control information, like paper feed, clear screen and so on. In general nothing can be said about the oddities of machine typical control information. Hence only a, further unspecified, *proper_string*, is transmitted to the drawing machine. The mixing rule for *control* amounts to string concatenation.

The elaboration process compares the Drawing Machine State with this resulting string. Certain strings will correspond to actions of the drawing machine.

Example:

```
ATTR new_page MACHINEDEPENDENTCONTROL "next_page";

PICT root
{       p1;
        WITH new_page DRAW
        {       p2;
                WITH new_page DRAW p3;
                p4
        };
        p5;
        WITH new_page DRAW p6
}.
```

Assume that "next_page" corresponds to an action of the drawing machine that provides a new page. Then p1 and p5 should appear on the first page, p2, p4 and p6 on the second and p3 on the third. Since p1-p5 are elaborated in order, the drawing machine has to be capable of reversing the "next_page" action, to provide the desired effect. If this "previous_page" action is not provided, the DMS is nevertheless adjusted. In that case the effect will be that the first page contains p1, the second page contains p2, the third page contains p3, p4 and p5 and the fourth page contains p6.

3.4.10. Visibility

The *attribute visibility* has the syntax:

visibility: **VISIBLE** |
INVISIBLE ;

When the state of a *picture* contains value **INVISIBLE** for the *visibility attribute_class*, this *picture* will not be drawn during elaboration. Nevertheless it will be elaborated, to update the environment properly. The current pen position (see 3.5.1.) must be updated, and the *detection attribute_class* elements must be evaluated, since invisible *pictures* may be detected.

The mixing rule for *visibility* is the same as that for *coordinate_mode*.

3.4.11. Attribute matches

How *attribute_matches* contribute to a state has formally been described in 3.4.3..

Conceptually, *attribute_matches* are a primitive form of the **WITH...DRAW** construction, operating on the *picture_element* level. They inhibit or permit the effect of all elements of their class that lie on the element path between the picture element and the smallest enclosing *subspace*. If an inhibiting match is used, these elements are replaced by the class value at the entrance of this last *subspace*. *picture_elements* may contain two levels of *attribute_matches*. The matches of the first level are written directly following the *picture_element* tag (e.g. LINE). The matches of the second level are written directly preceding *picture_element* values like *dimensional_values*, *curve_values* etc. The first level of matches apply to all *picture_element* values unless a second level match of the same class is specified. In that case only, the *picture_element* value directly following has the second level match for that class. All *attribute_matches* not specified on any of the two levels are taken to be non inhibitive, i.e. those that leave the current state unchanged. In this way, the concept of a global state with local exceptions is also realized at the *picture_element* level.

The correspondence between *attribute_matches* and *attribute_classes* is given in the following table:

match	class
TF	transformation
DT	detection
ST	style
PN	pen
CM	coordinate mode
VS	visibility

3.4.12. The default attribute

With every *attribute_class* corresponds a default element, according to the following table:

class	default value
transformation	unit matrix transformation
detection	**UNDETECT**, i.e. undetectable
control	**MACHINEDEPENDENTCONTROL** "", i.e. the empty string
pen	**PENFAULT**
coordinate_mode	**FIXED**, i.e. abs. positioning
style line_style	**PERIOD** (100),i.e. solid line **MAP**(1, **RESETCOORDINATE**) **THICK**(thickfault)
style typographic	**TYPFAULT**
style point_style	**DOT**
visibility	**VISIBLE**

Apart from style values, the defaults are self explanatory. The defaults for style are as follows. Default *linestyle* is a solid line, when however the *period* is specified explicitly, default *map* is such, that the pattern is reset for every new **LINE**. Thickfault stands for the most convenient thickness, available on the device, on which the drawing defined by the ILP program is to be drawn. Hence, thickfault is device dependent. The default value for *typographic* is **TYPFAULT** which is discussed in 3.4.5.3.. However, *typographic* has the atoms *font*, *size*, *italic* and *bold*. When certain atoms are specified, but others not, the latter again take device dependent values. The default for *point_style* is **DOT**, which denotes a device dependent spot. The default value for **POINTSTYLE** *typographic* is the same as for ordinary *typographic*. The default **POINTSTYLE** *token* depends on the selected *font*, but will be a 'point' when the font contains one. The default for *pen* is **PENFAULT** (see 3.4.6.). If only one atom of *pen* is specified, the other again assumes a device dependent value.

3.5. Picture Elements

A *picture_element* is a language primitive of ILP. Each ILP-program eventually specifies a list of *picture_element*s (end nodes of the graph represented by the ILP program). A *picture_element* is syntactically described by:

picture_element:
coordinate_type |
text |
generator |
NIL ;

We will now discuss the various *picture_element*s.

3.5.1. Coordinate type

The syntax rules for *coordinate_type picture_element*s are:

coordinate_type:
type attribute_matches
(*coordinates*) ;

Such a *picture_element* consists of a *type*, *attribute_matches* and *coordinates*, in conformity with the syntax rules:

type: **POINT** |
LINE |
CONTOUR ;

coordinates: *coordinate* |
coordinates
, *coordinate* ;

coordinate: *attribute_matches*
coordinate_value |
attribute_matches
(*coordinate_values*) ;

coordinate_values:
coordinate_value |
coordinate_values ,
coordinate_value ;

coordinate_value:
dimensional_value |
PP |
EP ;

The *attribute_matches* in the syntax rule for *coordinate_type* are the first level matches. Those in the rule for *coordinate* are the second level matches.

Whenever during elaboration of a picture element a coordinate is generated the last generated user coordinate is stored in the environment as the Untransformed Pen Position UPP. This corresponds to a penposition in the transformed coordinate system TPP. If CTM is the Current Transformation Matrix, then TPP can be found through

TPP = CTM * UPP

When, e.g. through a new transformation, the CTM has changed the UPP stays invariant but the TPP changes and an invisible move to this new TPP is generated.

With the help of the untransformed pen position, two special *coordinates* are defined: **EP** and **PP**. **EP** is mnemonic for element position, **PP** for picture position. During elaboration of a *picture_element*, **EP** denotes the value of the untransformed pen position just prior to the elaboration of this element. **PP** denotes the value of the untransformed pen position at the start of the elaboration of the smallest *named_picture* or *subspace* enclosing the *picture_element* in which **PP** is referenced. So, in the case of *subspace* it refers to its origin.

At the start of the elaboration of a *root_picture* or *subspace* the UPP is set to the origin of the user coordinate system. For a *subspace* the old UPP is stored in the environment to be restored upon exit of the *subspace*. **PP** allows among other things the specification of *subpictures* that leave the pen position where it was at the start, by adding a *picture* like

WITH INVISIBLE DRAW POINT PP

as the last element to the *subpicture*.

Upon return from *subspace picture*, the UPP and the **PP** are restored from the environment. **EP** needs not to be restored at all. It can only be used inside *picture_elements*. Hence, it will be copied from the most recent untransformed pen position at the beginning of that *picture_element*.

The primitive action embodied by a *coordinate_type picture_element* can be described as follows. First of all the row of *coordinates* specifies a series of positions. The positions are found in either of two ways, depending on the value of the *coordinate_mode* (see 3.4.8.):

-- In the **FIXED**-state, the *coordinate_value*s are absolute values with respect to the current origin.

-- In the **FREE**-state, the *coordinate_value*s are offsets from the untransformed pen position (incremental mode).

This series of positions is the same for all *type*s. The *type* is used to specify a "polygon", that contains these positions as vertices. The last vertex of the polygon however is different for different *type*s. Let the series of positions be represented by c_1, c_2, ..., c_n. Then the polygon to be drawn is:

- In case of *type* **POINT** and **LINE**: c_1-c_2- ... -c_n .

- In case of *type* **CONTOUR**: c_1-c_2- ... c_n-c_1.

The possibility UPP-c_1- ... -c_n-UPP, can be obtained by adding the special *coordinate* denoted as **EP** to the head of the row of *coordinate*s of *type* **CONTOUR**. This produces a closed polygon with the original pen position as the first (and last) value, e.g.:

WITH FREE DRAW CONTOUR CM (EP,[0, 1],[1, 0],[0, -1])

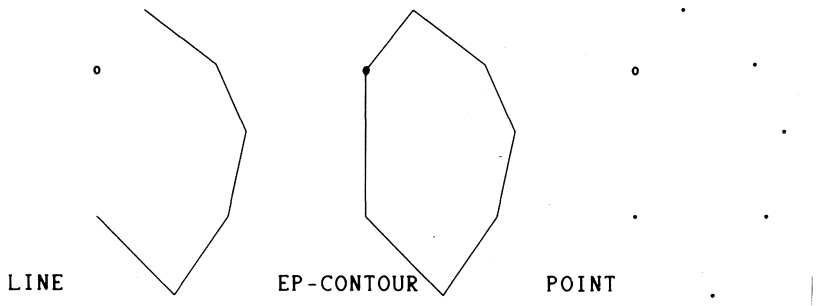
specifies a square that begins and ends in the untransformed pen position, valid at the start of the elaboration of this *picture*. If we negate **CM** in this example, we also get a closed polygon which starts and ends in the pen position. However, we cannot say what the shape will be until we know the pen position.

Next all *transformation*s of the current state are applied to the *dimensional_value*s of the *coordinate*s. This establishes which positions the pen will visit while a **POINT**, **LINE** or **CONTOUR** is elaborated. What is actually drawn, and what route is actually taken, going from one position to the next, depends on the *type* and the *attribute*s. The *attribute_class visibility*, and its match **VS** specify, whether anything will be drawn at all. In the state **INVISIBLE**, the route is followed as a sequence of invisible moves. In the state **VISIBLE**, it depends on the value of the *attribute_class*es *style* and *pen*, and their matches **ST** and **PN**, how the moves will actually be drawn.

There is a fundamental difference between **POINT**s and **LINE**s. For **LINE**s the route between successive positions defined by the *coordinate*s is always a straight line, which will be drawn according to the current *style* functions. The route between **POINT** positions is undefined. For this reason it is impossible to apply any *line_style* function to the route between these points. It is not defined in which order the positions have to be visited, with the exception of the last one. Hence the only *style* functions for **POINT**s are those which specify by which symbol (centered around the "point" position), the **POINT** will be represented. On the other hand, it is possible to specify a *line_style* for **LINE**s which

shows the positions as points. With respect to *style* functions the **CONTOUR** behaves in a **LINE**-like manner.

In the next example the same row of *coordinate*s is drawn as **LINE**, **EP-CONTOUR** and **POINT** respectively. In each case the initial pen position, marked as 0 is the same.



3.5.2. Text

Objects with *type* **TEXT** enable the production of texts as part of a *picture*. The syntax rules are:

```
text:           TEXT attribute_matches
                        ( strings ) ;

strings:        string |
                strings , string ;

string:         attribute_matches proper_string |
                attribute_matches
                        ( proper_strings ) ;

proper_strings: proper_string |
                proper_strings , proper_string ;
```

The value of *text* is a row of *string*s, which are build up from tokens. Tokens are selected from fonts. Each font contains at most 256 tokens. If the size of the character set of some device is smaller than 256, a device dependent escape mechanism is required to provide token values in

the range [0-255]. Change of font is possible by means of the *typographic style attribute*. In principle an unlimited set of fonts can be used in an ILP program.

An important aspect of *text* values is the way they are positioned, since nowhere in a *text* value, a *coordinate* can be specified, the position must be deduced from the current environment. No explicit page or layout *attribute* exists. *text* values are always positioned relative to the pen position. No limit is set to the maximal size of *text* values. Layout characters have a meaning, relative to the pen position (EP) of the current *text* value or relative to the current line of text. If *text* and other *picture_elements* are mixed, layout characters cannot have a meaning, relative to previous *text* values.

Dependent on the current values of the **FONT**, **SIZE**, **ITALIC** and **BOLD** atoms, each character of a text string generates a series of dimensional values, which defines a series of visible and invisible moves. The last of this series of moves is always an invisible one from the pen position before the drawing of the character to the final pen position. This move is called the character spacing move.

The effects of attribute classes on **TEXT** picture elements are the same as on coordinate type elements except for:

-- *line_style* and *point_style* have no effect, *typographic_style* determines the *dimensional_values* generated by the **TEXT** string.

-- *coordinate_mode* is always **FREE**.

-- *transformation*:

text_quality: **QUALITY(** *quality* **)** ;

quality: **LOW** | **MEDIUM** | **HIGH** ;

- **QUALITY(LOW)** no effect; clipping is on string level
- **QUALITY(MEDIUM)** for the character spacing move the normal effect, for all other moves no effect; clipping is on character level
- **QUALITY(HIGH)** for all moves the normal effect; clipping is on coordinate level

3.5.3. Generator

So far we have encountered primitives with explicit values. The remaining three *type*s are generators of values.

A *generator* is syntactically described by:

```
generator:      symbol |
                curve |
                template ;
```

The semantics of a *picture_element* of type *generator* are defined as follows.

Each *generator* contains a number of *gname*s. When a *generator* is encountered by the elaboration process, this process activates some external mechanism for every *gname* of the *generator*. Each mechanism generates an ILP graph, corresponding to a *picture*, whereafter these graphs are combined into a new graph of the same type. This graph replaces the picture node corresponding to the *generator*, after which the elaboration process continues with the subgraph just inserted. The *picture* generated, however, is considered as one indivisible action. This means that manipulations can only be defined for that *picture* as a whole. In particular *detection* of parts of the elaborated *picture* is impossible.

To guarantee, that the result of the replacement is again a correct graph, two demands must be met:

-- The generated ILP graph must be complete, i.e. it may not contain references to undefined nodes. To facilitate statical checking of this property, the following rule must be obeyed. The *picture* corresponding to the graph may not contain *pname*s or *aname*s of objects, defined in the *picture_program* that contains the *generator*, unless these references (*pname*s or *aname*s) are passed as *template_parameter*s (see 3.5.3.2.2.).

— In the *picture* describing the generated graph, all generated *dimensional_value*s, *matrix_value*s etc. must have dimensions in accordance with that of the environment and eventually generated *subspace*s.

*Generator*s provide a library facility. Because the nature of the library elements is not defined inside ILP, they are implementation, and application dependent. Nevertheless, the interface between the library and ILP (the *generator*) is defined inside ILP, and hence does not depend on a specific implementation.

3.5.3.1. Symbol

The syntax for *symbol* is:

symbol: **SYMBOL** *gnames* ;

gnames: *gname* |
gnames , *gname* ;

Every *gname* of a *symbol* corresponds with a *root_picture* in a previously defined ILP program. In this case, the picture graph is generated as follows.

Every *gname* represents a picture graph, as defined in 3.2.. If the *symbol* contains more then one *gname*, all picture graphs are combined into one, by creating a picture node, having all these graphs as direct descendants. The (left-right) order of the descendants corresponds to the textual order of the *gname*s. In this case it is necessary, that all *gname*s correspond to *root_picture*s of the same dimension.

3.5.3.2. Curve and template

The generation mechanism activated by a *curve* or *template* can be of arbitrary nature, as long as it produces picture graphs of the correct kind. The only demand is, that the mechanism is a program that can be invoked by the elaboration process and generates an ILP picture graph accessible to it. The distinction between *curve*s and *template*s lies in the structure of the picture graphs they produce.

3.5.3.2.1. Curve

The syntax for *curve* is:

curve: **CURVE** *type attribute_matches*
(*curve_generators*) ;

curve_generators:
curve_generator |
curve_generators , *curve_generator* ;

```
curve_generator:
                attribute_matches
                curve_determinator |
                attribute_matches
                        ( curve_determinators ) ;

curve_determinators:
                curve_determinator |
                curve_determinators
                        , curve_determinator ;

curve_determinator:
                gname |
                gname ( interval ,
                        curve_parameters ) |
                gname ( curve_parameters ) ;

interval:       UNIT |
                ( value , value ) ;

curve_parameters:
                curve_parameter |
                curve_parameters
                        , curve_parameter ;

curve_parameter:
                value |
                dimensional_value ;
```

The semantics of *curve*s will be described in terms of elements from ILP programs rather then in terms of the corresponding graphs. This will lead to a clearer description. In case of a *curve* , an object of type *dimensional_value*s corresponds to every *gname*. In other words, every *gname* represents a mechanism for the generation of *dimensional_value*s. These *dimensional_value*s, together with the *attribute_matches* of the *curve_determinator* containing the *gname* can be combined into an object of type *coordinate*. Then, using the *attribute_matches* (if present) of the *curve* a *picture_element* of type *type* can be formed out of these *coordinate*s. The order of the *coordinate*s in the *picture_element* corresponds to the textual order of the *gname*s. The *picture_element* thus constructed, is equivalent with the generated picture graph, that will replace the *generator* node.

The parameters of a curve can be (at most) one *interval*, and a number of *values* or *dimensional_values*. If there is an *interval*, we have a parameter curve. The *interval* is the domain of a parameter t. The *dimensional_values* of the generated *picture_element*, correspond to different values of t, when t steps through the interval. The stepsize can be calculated by the *curve* itself, can depend on a given device, or can be a parameter to the curve (a *value*). The other parameters (*curve_parameters*) are either *values* or *dimensional_values*. Their number and meaning is specific for each particular *gname*. *Dimensional_values* could for instance be used, to define some fixed points on-, or tangents to the curve.

3.5.3.2.2. Template

The syntax for *template* is:

template: **TEMPLATE** (*template_generators*) ;

template_generators:
template_generator |
template_generators
, *template_generator* ;

template_generator:
gname |
gname (*template_parameters*) ;

template_parameters:
template_parameter |
template_parameters
, *template_parameter* ;

template_parameter:
value |
dimensional_value |
pname |
aname |
dname ;

A *template generator* may produce an ILP picture graph of arbitrary structure. Because of the fact, that this picture graph not necessarily represents a *picture_element*, the syntax rules for *template* do not contain *attribute_matches*.

Each *gname* identifying a generation mechanism has its own specific set of parameters, described by *template_generator*. *pname*s or *aname*s used as parameters must correspond to *root_picture*s, *sub_picture*s or *attribute_pack*s defined in the ILP program containing the *template*. These parameters specify the references corresponding to *pname*s and *aname*s, allowed in the generated graph. Name conflicts must be avoided by using unique names.

The picture graphs generated, (one for every *gname*) are combined in one single picture graph, in the same way as is done for *symbol*s.

4. DESIGN GOALS AND EVALUATION

In this chapter, the design criteria of ILP are considered and an analysis is given, to show whether and if so, how, the stated goals are achieved.

4.1. Design goals

Five major design goals can be distinguished:

-- Compactness of picture representations, to reduce the enormous amounts of data which are normally required for the representation of pictures.

-- Mutual independence of attributes, to isolate the effects of individual attributes and forbid side effects caused by attributes from one class on attributes from another class.

-- Symmetry of input and output, which obviates the need for separate languages for input and output descriptions.

-- Embedding, which allows the incorporation of ILP in other (high level) programming languages.

-- Self modification of ILP programs, which allows the description of changes in a picture in ILP itself.

Compactness of picture representations can be achieved in several ways:

-- Multiple occurrences of the same subpicture are included only once in the data structure.

-- Only necessary coordinate values need to be specified, i.e. in a two dimensional space two numbers are sufficient to determine a coordinate value.

-- Coordinate values are packed, i.e. a priori knowledge of the range in which coordinate values lie is used to determine the most compact representation of coordinate values.

In ILP only the first two methods are used explicitly. The first is realized via the subpicture, root picture and attribute pack mechanisms. The second is realized with the subspace mechanism. Note that the dimension of each coordinate value can be determined statically. The third

method can be applied by an optimizing compiler.

Apart from the influence of these explicit methods, the ILP attribute mechanism has the beneficial effect of factoring out common subpictures, since the same subpicture can be drawn in contexts with completely different attribute values.

Independence of attributes restricts the ways in which attributes can influence each other. This restriction has several advantages:

-- The semantics of individual attributes can be studied in isolation, thus obviating the need to consider complex interactions between attributes.

-- The attributes are easily extensible, since new attribute can (by definition) not influence the already existing attributes.

The restriction of attribute independence seems to be justified, if the already considerable complexity of the semantics of independent attributes is taken into account. On the other hand certain useful applications of attribute interaction are forbidden by this restriction. Line style that adapts itself to transformations is an example.

Symmetry of input and output, means that the same intermediate representation is used both for drawing and reading pictures. The advantage of this method is obvious: only one intermediate representation is required. Although this scheme is simple to explain, it is difficult to implement. Especially on the input side, a completely new organization is required, since input can only be provided in the form of ILP primitives like picture elements, primitive attributes or references to pictures.

Embedding means incorporation of ILP in existing programming languages. In other words, ILP can be used as a model for a graphics system, which can be incorporated in an existing programming language. Although the embedding methods may be different, the various user interfaces and the underlying model graphics system remain the same. ALGOL 68G is an example of such an embedding, in which ALGOL 68 serves as a host language. A major consequence of this embedding strategy is that many features (variables, loop constructions) need not be included in ILP since the host language provides such facilities.

Self modification means that, with the help of a local editor for building and changing ILP constructions, elaborate edit operations can be described in ILP itself. Not only the resulting picture, but also the way it was constructed can be remembered, if necessary.

For both editing and modifying, a sophisticated reference mechanism is required. It is felt that the attribute mechanism can be used to model such a reference scheme. The detection mechanism is the first (and

sofar the only) step in this direction.

4.2. Omissions

Several features and concepts are not incorporated in ILP. Some are not yet understood well enough (time, modifications), others are omitted as a consequence of the embedding strategy. Some of the omitted features are:

-- Variables, recursion, loop constructions. The host language already provides these facilities.

-- Subpictures with parameters, which could be used to further compress the picture data.

-- Modifications of pictures. It is not yet clear how modification operations on the ILP data structure must be described in ILP itself and how selective modifications (changing one line in a subpicture) must be realized.

-- Time and moving pictures. The problems are comparable to those for picture modifications.

-- Surfaces. The present contours can be used to delimit a surface, but better tools are needed.

-- Surface style, the equivalent of line style and point style. Grey-scale forms an example.

-- Association of non graphical information with a picture.

4.3. Evaluation

Some of the lessons that can be learned from the design of ILP are:

-- The level of intermediate representation as provided by ILP seems adequate. Attention is focussed on a restricted problem area and many problems related to high level graphics languages and machine dependent issues can be (partly) ignored. However, ILP presents the designer with similar problems as other intermediate languages do. How does one decide at which level (above or below the intermediate language) a certain feature belongs? For example, should primitives for hidden line removal be part of the intermediate language or not? In the former case only very general algorithms can be used which can not use problem specific properties, while in the latter case simple and efficient algorithms, which use low level information, are ruled out.

-- A careful description of the semantics of drawing operations and attributes reveal problems which were not recognized before. Such an analysis required a considerably greater effort, than was anticipated.

-- ILP provides a uniform interface, during the design phase of a graphics system. This implies that every modification of ILP must be reflected in all interfaces between system modules. Note that only the interface is fully specified and that implementation techniques may differ from module to module.

In 1979 ILP was implemented on a PDP 11/45. The implementation consists of a compiler and an interpreter. The compiler checks for syntactical correctness and produces an efficient encoding of the programs. The interpreter executes these encoded programs and thus forms a realization of an ILP machine. Some interesting conclusions drawn from this implementation are:

-- The considerable effort spent during the design phase to produce a detailed and consistent semantic description was not waisted. This report shows that remarkably few modifications were needed to keep the language consistent and complete.

-- Implementing ILP is a software project of easily manageable complexity. The detailed description of the semantics of the language made cooperation in the team which produced the necessary software easy. Four programmers spent three months each writing the compiler and the interpreter which amounted to some 150 page code in the programming language C. The code of the interpreter occupies about 60 Kbytes on the PDP 11.

-- The interpreter drives a drawing machine with rather sophisticated and sometimes baroque hardware capabilities. The interpreter attempts to use these capabilities whenever this would lead to higher efficiency. This turned out to be possible to a great extent which indicates that the level of ILP primitives is sufficiently high to make it possible to use a wide range of drawing devices efficiently.

-- A serious drawback of a highly structured and recursively defined language like ILP could be that interpreting a program written in such a language consumes much more computing time than executing a more conventional low level display file. However, measurements of the behaviour of the interpreter show that, even for a highly structured picture like example 20 in chapter 2, the time consumed by typical ILP algorithms like state administration and attribute mixing consume less than 15 % of the total computing time, while standard graphics algortihms like transformation and clipping take more than 50 %.

REFERENCES

[1] NEWMAN, W.M. & R.F. SPROULL, *Principles of Interactive Computer Graphics*, McGraw-Hill, 1973.

[2] ENCARNAÇÃO, Jose L., *Computer Graphics*, Programmierung und Anwendung von Graphischen Systemen, R. Oldenburg Verlag, 1975.

[3] KNUTH, Donald E., *The Art of Computer Programming*, Vol. 1/Fundamental Algorithms, Addison-Wesley, 1968, 305-357.

[4] TEN HAGEN, P.J.W., P. KLINT, H. NOOT & T. HAGEN, *Design of an interactive graphics system*, Report IW 36/75, Mathematical Centre, Amsterdam, 1975.

[5] SINT, H.J., *Design of an ALGOL 68 Extension for Graphics*, Computer Graphics, Vol. 13-4, Siggraph-ACM, 1980, 332-354.

[6] KLINT, P. & H.J. SINT, *A framework for the interface between graphics and pattern recognition*, Methodology of Interaction, Proc. IFIP WG5.2 Workshop, North-Holland, 1980.

[7] TEN HAGEN, P.J.W., *A conceptual basis for graphical input and interaction*, Methodology of Interaction, Proc. IFIP WG 5.2 Workshop, North-Holland, 1980.

[8] GUEDJ, Richard A. (ed.), *Methodology of Interaction*, Proc. IFIP WG 5.2 Workshop, North-Holland, 1980.

[9] SIGGRAPH-ACM (GSPC), *Status of the Graphics Standards Planning Committee*, Computer Graphics, Vol. 13-3, Siggraph-ACM, 1979.

[10] DIN, Graphical Kernel Systems (GKS), *Functional Description*, Din 00 66 252, 1979.

[11] DEHNERT, E., G. ERNST & H. WETZEL, *GRAPHEX 68, Graphical Language Features in ALGOL 68*, Report T.U. Berlin, August 1974.

[12] HURWITZ, A., J.P. CITRON & J.B. YEATON, *GRAF: Graphic Addition to Fortran*, Proc. AFIPS' SJCC 1967, 553-557.

[13] SMITH, D.N., *GPL/I, APL/I Extension for Computer Graphics*, SJCC 1971, AFIPS Press, Montreal, New York, p. 511.

[14] HAGEN, T., P.J.W. TEN HAGEN, P. KLINT & H. NOOT, *ILP, Intermediate Language for Pictures*, Preliminary Report IW 68/77, Mathematical Centre, Amsterdam.

[15] HAGEN, T., P.J.W. TEN HAGEN, P. KLINT & H. NOOT, *ILP, Intermediate Language for Pictures*, Proc. IFIP conference 1977, North Holland 1977.

Appendix 1 Syntax

The syntax rules are given in BNF. Non-terminals are denoted in the form *non_terminal*. The syntax is context free. The non-terminal that is defined in a rule is separated by a colon (:). Alternatives are separated by a bar (|). The end of a rule is marked with the symbol ;. Terminal symbols are either special single characters from the following list:

( ) { } , . ; [ ] ~

or they are delimeters denoted in bold capitals e.g. **TERMINAL.** The non terminals not defined in this syntax are all defined in Appendix 2. They constitute the so called lexical units.

The syntax as presented is directly fed into the parser generator for ILP. For this reason usual notational conventions to make the syntax look more compact, have been omitted.

picture_program:
pictstruct |
picture_program pictstruct ;

pictstruct: *named_picture* |
attribute_pack ;

named_picture: *root_picture* |
subpicture ;

root_picture: **PICT** *dimension pname*
picture . ;

dimension: **DIMLESS** |
dim ;

dim: (*value*) |
empty ;

subpicture: **SUBPICT** *dimension pname*
picture . ;

```
attribute_pack: ATTR dimension aname
                        attribute . ;

picture:        pname |
                picture_element |
                { pictures } |
                subspace picture |
                WITH attribute
                        DRAW picture ;

pictures:       picture |
                pictures ; picture ;

picture_element:
                coordinate_type |
                text |
                generator |
                NIL ;

coordinate_type:
                type attribute_matches
                        ( coordinates ) ;

coordinates:    coordinate |
                coordinates
                        , coordinate ;

coordinate:     attribute_matches
                        coordinate_value |
                attribute_matches
                        ( coordinate_values ) ;

coordinate_values:
                coordinate_value |
                coordinate_values ,
                        coordinate_value ;
```

```
coordinate_value:
                dimensional_value |
                PP |
                EP ;

dimensional_value:
                [ values ] ;

dimensional_values:
                dimensional_value |
                dimensional_values ,
                        dimensional_value ;

matrix_value:   [ dimensional_values ] ;

values:         value |
                values , value ;

type:           POINT |
                LINE |
                CONTOUR ;

subspace:       SUBSPACE dim new_axes ;

new_axes:       position ( shift axes ) ;

shift:          dimensional_value ;

position:       CURRENT |
                ORIGIN ;

axes:           empty |
                , dimensional_values ;

generator:      symbol |
                curve |
                template ;
```

```
symbol:         SYMBOL gnames ;

gnames:         gname |
                gnames , gname ;

curve:          CURVE type attribute_matches
                        ( curve_generators ) ;

curve_generators:
                curve_generator |
                curve_generators , curve_generator ;

curve_generator:
                attribute_matches
                curve_determinator |
                attribute_matches
                        ( curve_determinators ) ;

curve_determinators:
                curve_determinator |
                curve_determinators
                        , curve_determinator ;

curve_determinator:
                gname |
                gname ( interval ,
                        curve_parameters ) |
                gname ( curve_parameters ) ;

interval:       UNIT |
                ( value , value ) ;

curve_parameters:
                curve_parameter |
                curve_parameters
                        , curve_parameter ;
```

```
curve_parameter:
                value |
                dimensional_value ;

template:       TEMPLATE ( template_generators ) ;

template_generators:
                template_generator |
                template_generators
                        , template_generator ;

template_generator:
                gname |
                gname ( template_parameters ) ;

template_parameters:
                template_parameter |
                template_parameters
                        , template_parameter ;

template_parameter:
                value |
                dimensional_value |
                pname |
                aname |
                dname ;

text:           TEXT attribute_matches
                        ( strings ) ;

strings:        string |
                strings , string ;

string:         attribute_matches proper_string |
                attribute_matches
                        ( proper_strings ) ;

proper_strings: proper_string |
                proper_strings , proper_string ;
```

```
attribute_matches:
                empty |
                attribute_matches
                        deny attribute_match ;

deny:           empty |
                ~ |
                NOT ;

attribute_match:
                TF |
                DT |
                ST |
                PN |
                CM |
                VS ;

attribute:      ABS basic_attribute |
                REL basic_attribute |
                basic_attribute ;

basic_attribute:
                attribute_class |
                aname |
                { attributes } |
                NIL ;

attributes:     attribute |
                attributes ; attribute ;

attribute_class:
                transformation |
                detection |
                style |
                control |
                pen |
                coordinate_mode |
                visibility ;
```

```
transformation: rotate |
                scale |
                translate |
                matrix |
                projection |
                affine |
                homogeneous_matrix |
                port
                text_quality ;

rotate:         ROTATE value
                        AROUND invariant ;

invariant:      ( dimensional_values ) ;

scale:          SCALE dimensional_value ;

translate:      TRANSLATE dimensional_value ;

matrix:         MATRIX matrix_value ;

affine:         AFFINE matrix_value
                       dimensional_value ;

projection:     projection_type eye_position
                        ON projection_space ;

projection_space:
                dimensional_value |
                ORIGIN dimensional_value ;

projection_type:
                PROJECT |
                PROJECT PERSPECTIVE |
                PROJECT REVERSIBLE ;

eye_position:   dimensional_value ;
```

homogeneous_matrix:
HOMMATRIX *homogeneous_matrix_value* ;

homogeneous_matrix_value:
[*homogeneous_dimensional_values*] ;

homogeneous_dimensional_values:
homogeneous_dimensional_value |
homogeneous_dimensional_values ,
homogeneous_dimensional_value ;

homogeneous_dimensional_value:
[*values*] ;

port: *window* |
window , *viewport* ;

window: **WINDOW** (*dimensional_value* ,
dimensional_value) ;

viewport: **VIEWPORT** (*dimensional_value* ,
dimensional_value) ;

style: *line_style* |
point_style |
typographic ;

line_style: **PERIOD** (*period_description*) |
MAP (*value reset*) |
THICK (*value*) ;

pen: **PENFAULT** |
contrast |
intens |
colour ;

```
period_description:
                dash |
                dash , gap |
                dash , gap , dash ;

dash:           DOT |
                value ;

gap:            value ;

reset:          RESETCOORDINATE |
                CONTINUE |
                RESETLINE ;

contrast:       CONTRAST ( value , value ) ;

intens:         INTENS ( value ) ;

colour:         COLOUR ( value , value , value ) ;

text_quality:   QUALITY( quality ) ;

quality:        LOW | MEDIUM | HIGH ;

typographic:    TYPFAULT |
                font |
                size |
                italic |
                bold ;

font:           FONT ( value ) ;

size:           SIZE ( value ) ;

italic:         ITALIC ( value ) ;
```

```
bold:           BOLD ( value ) ;

point_style:    DOT |
                POINTSTYLE typographic |
                POINTSTYLE marker ;

control:        MACHINEDEPENDENTCONTROL proper_string ;

coordinate_mode:
                FIXED |
                FREE ;

visibility:     VISIBLE |
                INVISIBLE ;

detection:      DETECT detector proper_string |
                SETDEL detector proper_string |
                UNDETECT detector ;

detector:       empty |
                dname ;

empty:          ;
```

Appendix 2 Lexical units

```
value:          unsigned_value |
                + unsigned_value |
                - unsigned_value ;

unsigned_value: unsigned_integer |
                decimal_fraction |
                unsigned_integer exponent_part |
                decimal_fraction exponent_part ;

decimal_fraction:
                unsigned_integer . unsigned_integer ;

exponent_part:  e + unsigned_integer |
                e - unsigned_integer ;

unsigned_integer:
                digit |
                unsigned_integer digit ;

aname:          name ;

pname:          name ;

gname:          name ;

dname:          name ;

name:           letter |
                name letter |
                name digit ;

proper_string:  " any_sequence_of_symbols_not_containing_" " ;
```

letter:

```
a | b | c | d | e | f | g |
h | i | j | k | l | m | n |
o | p | q | r | s | t | u |
v | w | x | y | z |
A | B | C | D | E | F | G |
H | I | J | K | L | M | N |
O | P | Q | R | S | T | U |
V | W | X | Y | Z ;
```

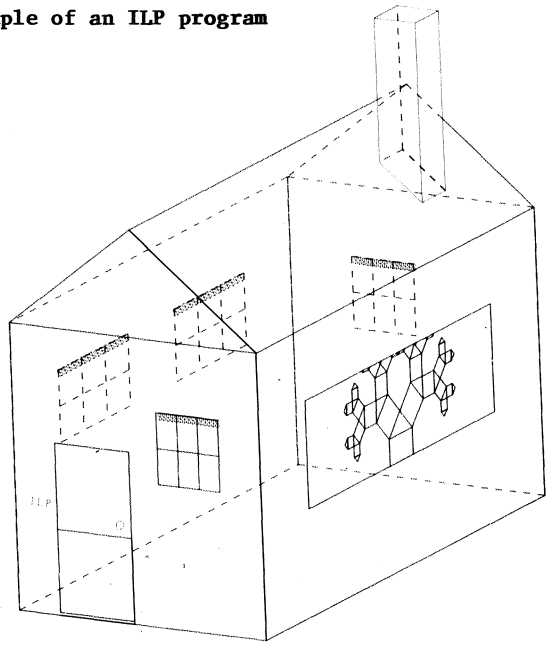
digit:

```
1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 ;
```

marker:

```
" any_symbol,_except_" " ;
```

Appendix 3 An example of an ILP program



```
# THIS IS A COMMENT #
# This ILP program tests a majority of the attributes
  and picture elements #

# THE ROOT PICTURES #
# One for direct viewing and
  one for recording on diazo film #

PICT(3) root WITH {scale; rotate; center}
     DRAW house.

PICT (3) diazo
     WITH { diazostart; diazofeed; THICK(0.004) }
      DRAW root.

# THE VIEWING ATTRIBUTE PACKS #
#Three-dimensional transformations#

ATTR(3) scale SCALE[0.1,0.08,0.1].

ATTR(3) rotate
{    ROTATE 20 AROUND ([0,0,0],[1,0,0]);
     ROTATE -30 AROUND ([0,0,0],[0,1,0])
}.
```

```
ATTR(3) center TRANSLATE [-3,-4,6].

# THE DIAZO PART # # HRD-DEPENDENT CONTROL #

ATTR DIMLESS diazostart
     MACHINEDEPENDENTCONTROL "HRD:diazo".
ATTR DIMLESS diazofeed
     MACHINEDEPENDENTCONTROL "HRD:feed".

# BUILDING THE HOUSE #
# three-dimensional subspaces #

SUBPICT (3) house
{    SUBSPACE ORIGIN([0,0,0])
          # default axes #
          frontwall;
     WITH dotted DRAW
     {    SUBSPACE ORIGIN([0,0,0],[0,0,-1],[0,1,0])
               leftwall;
          SUBSPACE ORIGIN([0,0,-12])
               backwall;
     };
     WITH TRANSLATE [0,8,0] DRAW roof;
     SUBSPACE ORIGIN([6,0,0],[0,0,-1],[0,1,0])
          rightwall;
}.

SUBPICT frontwall
{    shortwall;
     WITH  TRANSLATE[0.3,3] DRAW sign;
     WITH { TRANSLATE [1,0] ; SCALE [0.9,0.8]}
          DRAW door;
     WITH { TRANSLATE [3.5,4]; SCALE [0.5,1] }
          DRAW window
}.

SUBPICT shortwall
     CONTOUR([0,0],[6,0],[6,8],[0,8]).

SUBPICT sign                # Text and quality #
{    LINE([0,0]);
        WITH {  SIZE(0.2); QUALITY(HIGH)}
           DRAW TEXT("ILP")
}.

SUBPICT door                          # Pointstyle #
{    LINE([0,0],[2,0],[2,6],[0,6],
         [0,0.1],[2,0.1],[2,3],[0,3]);
     WITH {POINTSTYLE "o" ; POINTSTYLE SIZE(0.5) }
          DRAW POINT([1.7,3.5])
```

```
}.

SUBPICT window # two-dimensional nested transformations #
{    hline;
     WITH TRANSLATE [0,1] DRAW hline;
     WITH TRANSLATE [0,2] DRAW hline;
     WITH TRANSLATE [0,2] DRAW vline;
     WITH TRANSLATE [1,2] DRAW vline;
     WITH TRANSLATE [2,2] DRAW vline;
     WITH TRANSLATE [3,2] DRAW vline;
     WITH TRANSLATE [0.0,1.8] DRAW curtain
}.

SUBPICT hline LINE( [0,0], [3,0]).

SUBPICT vline WITH { rot90; SCALE [0.66666,1]}
      DRAW hline.

ATTR rot90 ROTATE -90 AROUND ([0,0]).

SUBPICT curtain
{    LINE([0,0]);
     WITH {SIZE(0.2); QUALITY(HIGH)} DRAW
          TEXT("$$$$$$$$$$$$$$$")
}.

SUBPICT (3) roof        # one-dimensional subspace #
{    LINE(EP,[3,3,0],[6,0,0],[3,3,0],[3,3,-10.68]);
     WITH dotted DRAW LINE(EP, [3,3,-12]);
     SUBSPACE (1) CURRENT ([0,0,0],[3,-3,0])
             WITH dotted
               DRAW LINE(EP, [0.28], ~ST [1]);
     WITH dotted DRAW LINE(EP,[0,0,-12]);
     LINE([5,1,-9]); chimney
}.

SUBPICT (3) chimney            # attribute matches #
     WITH { dotted; FREE } DRAW
        LINE([0,0,0], ~ST[0,0,-1],[-1,1,0],[0,0, 1],
          ~ST[1,-1,0],
          ~ST([0,5,0],[0,0,-1],[-1,0,0],[0,0,1]),
          ~ST([1,0,0],[0,0,-1],[0,-5,0]),[-1,1,0],
          [0,4,0], ~ST[0,0,1], ~ST[0,-4,0]).

SUBPICT backwall
{    shortwall;
     WITH {  TRANSLATE[1.5,4] ; SCALE [0.5,1] }
          DRAW window
}.
```

```
SUBPICT sidewall
     CONTOUR( [0,0],[12,0],[12,8],[0,8]).

SUBPICT leftwall
{    sidewall;
     WITH TRANSLATE[2,4] DRAW window;
     WITH TRANSLATE[7,4] DRAW window
}.

SUBPICT rightwall
{    sidewall;
     CONTOUR([2,3],[10,3],[10,6],[2,6]);
     WITH WINDOW([2,3],[10,6]) DRAW
     WITH TRANSLATE[5.5,2.5] DRAW plant
}.

# to simulate hidden lines #          # Linestyle #

ATTR DIMLESS dotted  { PERIOD(50);
           MAP( 0.03 RESETCOORDINATE) }.

SUBPICT plant                  # Coordinate mode #
{    LINE ([0,0]);
     WITH { FREE; SCALE [1,1.25]} DRAW pyth5
}.

# A pythagoras tree with recursion depth 5 #
# Nested subspaces #

SUBPICT pyth5
{    LINE([0,0],[0,1],[1,0]);
     SUBSPACE CURRENT ([-1,0],[0.5,0.5],[-0.5,0.5])
          pyth4;
     SUBSPACE CURRENT ([-0.5,0.5],[0.5,-0.5],[0.5,0.5])
          pyth4;
     LINE([0,0],[0,-1],[-1,0])
}.

SUBPICT pyth4
{    LINE([0,0],[0,1],[1,0]);
     SUBSPACE CURRENT ([-1,0],[0.5,0.5],[-0.5,0.5])
          pyth3;
     SUBSPACE CURRENT ([-0.5,0.5],[0.5,-0.5],[0.5,0.5])
          pyth3;
     LINE([0,0],[0,-1],[-1,0])
}.

SUBPICT pyth3
{    LINE([0,0],[0,1],[1,0]);
     SUBSPACE CURRENT ([-1,0],[0.5,0.5],[-0.5,0.5])
```

```
          pyth2;
     SUBSPACE CURRENT ([-0.5,0.5[0.5,-0.5],[0.5,0.5])
          pyth2;
     LINE([0,0],[0,-1],[-1,0])
}.

SUBPICT pyth2
{    LINE([0,0],[0,1],[1,0]);
     SUBSPACE CURRENT ([-1,0],[0.5,0.5],[-0.5,0.5])
          pyth1;
     SUBSPACE CURRENT ([-0.5,0.5],[0.5,-0.5],[0.5,0.5])
          pyth1;
     LINE([0,0],[0,-1],[-1,0])
}.

SUBPICT pyth1
{    LINE([0,0],[0,1],,[1,0],
          [-0.5,0.5],[-0.5,-0.5]);
     LINE([0,0],[1,0],[0,-1],[-1,0])
}.
```

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