

Registration of 3D Face Scans with Average Face Models

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Registration of 3D Face Scans with Average Face Models*

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The accuracy of a 3D face recognition system depends on a correct registration that aligns the facial surfaces and makes a comparison possible. The best results obtained so far use a costly one-to-all registration approach, which requires the registration of each facial surface to all faces in the gallery. We explore the approach of registering the new facial surface to an average face model (AFM), which automatically establishes correspondence to the pre-registered gallery faces. We propose a new algorithm for constructing an AFM, and show that it works better than a recent approach. We inspect thin-plate spline and iterative closest point based registration schemes under manual or automatic landmark detection prior to registration. Extending the single-AFM approach, we consider employing category-specific alternative AFMs for registration, and evaluate the effect on subsequent classification. We perform simulations with multiple AFMs that correspond to different clusters in the face shape space and compare these with gender and morphology based groupings. We show that the automatic clustering approach separates the faces into gender and morphology groups, consistent with the other race effect reported in the psychology literature. Finally, we describe and analyse a regular re-sampling method that significantly increases the accuracy of registration.

Keywords: 3D face recognition, registration, average face model, ICP, TPS, Procrustes analysis, automatic landmark localization, other race effect

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I. INTRODUCTION

With the advances in acquisition hardware and 3D recognition algorithms, 3D face recognition has become an important biometric modality. The illumination effects that impair 2D face recognition are alleviated to a large degree when using the surface of the face. Additionally, 3D face recognition offers new possibilities of representation when compared to 2D face recognition, and the choice of representation for the surface has a bearing on the accuracy and the speed of the 3D face recognition algorithm. For instance selecting a depth map to represent a 3D face makes it possible to use almost all the techniques developed for 2D face recognition, whereas storing true 3D information potentially increases the accuracy.

The speed and accuracy bottleneck in 3D face recognition is in the registration step, which follows the preprocessing. Test faces need to be aligned to the gallery faces for comparison. Most of the algorithms start by coarsely aligning the faces, either by their centres of mass [2], nose tip [3], the eyes [4], or by fitting a plane to the face and aligning it with that of the camera [5]. Good registration of the images is important for all local similarity measures.

In 3D face classification applications, the most frequently employed registration approach is the iterative closest point (ICP) algorithm, which establishes a dense correspondence between two point clouds in a rigid manner [6]. Typically, a test face is registered to each gallery face separately [7–9], and a point set distance is adopted for classification.

In the work of İrfanoğlu et al., an alternative and fast method was proposed to register faces [10], where an average face model (AFM) was employed to determine a single point-to-point correspondence. The gallery faces, previously used in generating the AFM, are already in dense correspondence with it. Thus, a single ICP is enough to register a test face, which is much faster for a reasonably sized gallery. This overcomes what has been reported as the major drawback of ICP [8, 11].

In this paper, we explore and expand AFM-based registration. Our methodology is summarized in Section II, where we propose an approach that uses one AFM for each facial category. Section III provides a brief summary of the rigid and nonrigid registration methods that we use, along with a description of the automatic landmarking algorithm. The novel AFM construction algorithm is presented in Section IV, where we also describe two different approaches for creating category-specific AFMs. We report our results on the state-of-the-art FRGC face database in Section V, and conclude in Section VI.

II. METHODOLOGY

The goal of this paper is to investigate AFM-based registration for 3D face recognition. Our setup therefore is a full 3D face recognition system with preprocessing, landmarking, registration and recognition. This setup is shown in Figure II, which shows the stages of processing for a new query.

The query image, acquired with the 3D scanner, is first pre-processed to clear it from imaging artifacts, and a landmarking algorithm is run to detect seven landmarks (eye and mouth corners, and the nose tip) on the face. The next step is a coarse registration, which is important for ICP, followed by rigid or non-rigid registration. The AFM (or multiple AFMs) are registered to the test face, and after the dense correspondence is established, the points that do not have correspondences in the AFM are removed. This effectively crops the face area, and produces a set S of 3D points. After the cropping, the depth values on the test face are resampled from a regular $x - y$ grid. We have used a simple triangle-based nearest-neighbour interpolation for this purpose [12]. After the dense correspondence, the point vectors representing the faces are of the same size, and it becomes possible to have a PCA projection to store much smaller templates.

Figure II makes it clear that there are many design choices for the complete system, and each of these choices (manual vs. automatic landmarking, type of coarse registration, single vs. multiple AFMs, type of AFMs, rigid vs. non-rigid registration, with or without regular re-sampling, classification with or without PCA projection) has some effect on the performance. We have designed experiments to resolve each of these issues, and inspected whether a trade-off is offered by a particular choice, or whether the setting indicates the superiority of one approach over the other.

The first set of experiments deal with the stage prior to the dense registration. The coarse registration in ICP and the non-rigid thin-plate spline (TPS) based registration both require a couple of fiducial points for guidance. We evaluate the effect of errors in landmark detection by using 3D ground-truth versus automatically located landmarks (Section IIID). This permits us to analyze the algorithms under realistic assumptions, as automatic landmarking errors are not uniformly distributed. The details of the ICP and TPS methods are given in Sections IIIB and IIIC, respectively.

For coarse registration in ICP, we test four different methods. Our results indicate that simple heuristics used in the literature leave room for improvement. Then we propose a novel AFM construction method, and obtain very good registration results with a single, generic AFM.

Using a single AFM for registration, and registering separately to each gallery face are the two extreme approaches. In between these extremes, using a few category-specific AFMs can be beneficial to accuracy and still be computationally feasible. To test this hypothesis, we propose two different methods for generating category-specific AFMs. What constitutes a facial category is an open issue; we contrast an approach based on cognitive justifications (detailed in Section IVB) with one that is based on clustering on the shape space (Section IV C). The cognitive approach allows recognition scenarios to use category information (e.g. the query for a male face is searched among the males only). But if the category information is not assumed to be available, it is still possible to register the test face to all the available AFMs, and take the best registration (one with the smallest distance) as the outcome. We test both approaches in Section V.

In the last part of the paper, we demonstrate that re-sampling of depth values from a regular grid improves accuracy significantly, and we justify this improvement by a detailed analysis. We also show that a good registration ensures that a subspace projection can be

used with little accuracy decrease.

III. REGISTRATION METHODS

We consider two different registration methods in this paper. In the first method, termed *TPS-based* in the experiments section, the test face is aligned to the average face with the thin-plate spline (TPS) method, and the points not in correspondence with the AFM are cropped [10]. This method deforms the test face to fit the AFM, and the amount of deformation is proportional to the number and spread of the landmarks. At the limit of using all facial points as landmarks, the face deforms into the AFM, losing the discriminative information completely. However with a few landmarks, corresponding facial structures are aligned.

In the second method, we use the iterative closest point method to align the test face with the AFM. ICP is a rigid registration method; hence, the test face is not deformed at all. TPS-based methods are completely guided by the landmarks, whereas ICP needs a coarse initialization. Intuitively, ICP will benefit from using category-specific AFMs, as the rigid registration is not able to cope with shape differences very well. A more similar average face will ensure that the dense correspondence will be established between points that have better structural correspondence. The TPS-based method will also benefit from category-specific AFMs, albeit for another reason: A more similar average face means that the test surface will be deformed less, and discriminatory information will not be lost.

We review the ICP and TPS algorithms here, along with the Procrustes analysis, the automatic landmark localization algorithm, and the regular re-sampling that are used by both.

A. Procrustes Analysis

Procrustes analysis is a statistical tool for the analysis of geometrical shapes [13]. A shape (or equivalently a *figure*) P in \mathbb{R}^p is represented by l landmarks. Two figures $P : l \times p$ and $P' : l \times p$ are said to have the same shape, if they are related by a special similarity transformation:

$$P' = \alpha P \Gamma + \mathbf{1}_l \gamma^T, \tag{1}$$

where the parameters of the similarity transformation are a rotation matrix $\Gamma : p \times p$, $|\Gamma| = 1$, a translation vector $\gamma : p \times 1$, a positive scaling factor α , and $\mathbf{1}_l$ is a vector of ones. By using the generalized Procrustes analysis, it is possible to derive a *consensus shape* for a collection of figures [14], which is then used in registering new shapes into alignment with the collection by an affine transformation:

1. Center all shapes P_i :

$$M = \frac{1}{N} \sum_{i=1}^N P_i \quad (2)$$

$$P_i = P_i - M \quad (3)$$

2. Bring the shapes to a common scale.
3. Set the consensus shape Y equal to the first shape P_1 , as an initialization.
4. For $i = 2, 3, \dots, N$, rotate P_i to fit Y . Y is re-evaluated after each update of P_i as

$$Y = \frac{1}{i} \sum_{j=1}^i P_j \quad (4)$$

5. Repeat updating the P_i and Y , while monitoring the residual sum-of-squares:

$$S_r = N(1 - \text{tr}(Y_t Y_t^T - Y_{t-1} Y_{t-1}^T)) \quad (5)$$

where Y_t is the consensus at iteration t , and Y_{t-1} is the consensus at iteration $t - 1$. When S_r is below a threshold (e.g. 0.0001, as suggested by Gower in [14]) stop the iterations, and output the consensus shape.

B. Iterative Closest Point

In this Section we summarize the ICP procedure as defined in [6].

Define the *unit quaternion* as a four vector $\vec{q}_H = [q_0 q_1 q_2 q_3]^T$, with $q_0 \geq 0$, and $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$. The 3×3 rotation matrix H generated by this quaternion is:

$$H = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 + q_1^2 - q_2^2 - q_3^2 \end{bmatrix} \quad (6)$$

Let $\vec{q}_T = [q_4 q_5 q_6]^T$ be a translation vector. Together with q_H , they make up the complete registration state vector $\vec{q} = [q_H | \vec{q}_T]^T$. Let $P = \{\vec{p}_i\}$ be a data point set to be aligned with the model point set $Y = \{\vec{y}_i\}$. The two models will have the same number of points, and ICP will put the points with the same indices into one-to-one correspondence. Denoting the number of points in each model with N , the objective function minimized by the ICP procedure is:

$$f(\vec{q}) = \frac{1}{N} \sum_{i=1}^N \|\vec{y}_i - H(\vec{q}_H)\vec{p}_i - \vec{q}_T\|^2 \quad (7)$$

Denoting the centre of mass of the point set P with μ_p , and that of the model set with μ_y , the cross-covariance matrix Σ_{py} is given by:

$$\Sigma_{py} = \frac{1}{N} \sum_{i=1}^N [(\vec{p}_i - \mu_p)(\vec{y}_i - \mu_y)^T] \quad (8)$$

The cyclic components of the matrix $A_{ij} = (\Sigma_{py} - \Sigma_{py}^T)_{ij}$ are used to form a column vector $\Delta = [A_{23} \ A_{31} \ A_{12}]^T$, which in turn is used to form a symmetric 4×4 matrix $Q(\Sigma_{py})$:

$$Q(\Sigma_{py}) = \begin{bmatrix} tr(\Sigma_{py}) & \Delta^T \\ \Delta & \Sigma_{py} + \Sigma_{py}^T - tr(\Sigma_{py})I \end{bmatrix} \quad (9)$$

where I is the 3×3 identity matrix. The optimum rotation is given by the unit eigenvector $\vec{q}_H = [q_0 q_1 q_2 q_3]^T$ corresponding to the maximum eigenvalue of the matrix $Q(\Sigma_{py})$. The optimum translation vector is given by:

$$\vec{q}_T = \mu_y - H(\vec{q}_H)\mu_p \quad (10)$$

We use $\vec{q}(P)$ to denote the point set P after the application of the transformation represented by \vec{q} . The ICP algorithm computes and applies these transformations iteratively. A step-by-step description of the algorithm follows:

1. Initialize ICP by setting $P_0 = P$, $\vec{q}_0 = [1, 0, 0, 0, 0, 0, 0]^T$ and $k = 0$. The registration is defined relative to P_0 , which requires a coarse registration. Steps 2-5 are applied iteratively, until convergence is achieved within a tolerance τ .
2. Compute the closest points: $Y_k = \mathcal{C}(P_k, Y)$. The computational cost of this step is $O(N_p N_y)$ at the worst case, where N_p is the number of points on the registered point cloud, and N_y is the number of points on the model shape.

3. Compute the registration: $\vec{q}_k = \mathcal{Q}(P_0, Y_k)$. The computational cost is $O(N_p)$.
4. Apply the registration: $P_{k+1} = \vec{q}_k(P_0)$. The computational cost is $O(N_p)$.
5. Terminate the iteration if the change in the mean square error is below pre-set threshold τ . A heuristic value for τ is a multiple of $\sqrt{\text{tr}(\Sigma_y)}$, where Σ_x is the covariance matrix of the model shape, and the square root of its trace is a rough indicator of model shape size.

In our simulations, the test face acts as the model shape, and the cropped gallery face is aligned to it. The points of the test scan that are put into one-to-one correspondence with the model are retained, and the rest are discarded. If the registration is correct, this procedure automatically gives a good cropping, possibly including hair and clutter removal. In the rare cases, where a streak of hair extends over the face centre, the registration and the subsequent classification will be inaccurate.

Previous work on ICP show that a good initialization is necessary for fast convergence and an accurate end-result. We compare several approaches for the coarse registration. In our first approach, the point with the greatest depth value is assumed to be the tip of the nose, and a translation is found to align it to the nose tip of the AFM. This is the fastest and simplest heuristic used in the literature [15], and we expect it to perform well with near-frontal faces. In the second approach, we use the manually determined nose tip (i.e. ground truth) in the coarse alignment. In the third approach, we use the Procrustes analysis to bring seven manually determined landmark points (inner and outer eye corners, nose tip, and the mouth corners) into alignment with the average face model. Finally, we use Procrustes analysis to align automatically determined landmarks with the average face model. The automatic landmarking errors are not random, and cannot be simulated by injecting noise to the manually determined landmarks, except by modeling the specific landmarking procedure.

C. Thin-plate Splines

The TPS model expresses the bending energy of a thin metal plate fixed at certain points [16]. At the heart of the model is a special surface function:

$$z(x, y) = -U(r) = -r^2 \log(r^2) \quad (11)$$

with $r = \sqrt{x^2 + y^2}$ equal to the Euclidean distance of point (x, y) to the origin.

For a set of anchor points $P_i = (x_i, y_i), i = 1 \dots n$, the thin-plate spline interpolation is a vector-valued function $f(x, y) = [f_x(x, y), f_y(x, y)]$ that maps the anchor points to their specified homologues $P'_i = (x'_i, y'_i), i = 1 \dots n$, and specifies a surface which has the least possible bending, as measured by an *integral bending norm*. We will give a summarizing mathematical specification of the model here.

Define $r_{ij} = |P_i - P_j|$ to be the distance between the points i and j . Also define the following matrices:

$$K = \begin{bmatrix} 0 & U(r_{12}) & \dots & U(r_{1n}) \\ U(r_{21}) & 0 & \dots & U(r_{2n}) \\ \dots & \dots & \dots & \dots \\ U(r_{n1}) & U(r_{n2}) & \dots & 0 \end{bmatrix} \quad (12)$$

$$P = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \dots & \dots & \dots \\ 1 & x_n & y_n \end{bmatrix} \quad (13)$$

and

$$L = \left[\begin{array}{c|c} K & P \\ \hline P^T & O \end{array} \right] \quad (14)$$

where O is a 3×3 matrix of zeros. Let V be a matrix made up of the homologues of the anchor points:

$$V = \begin{bmatrix} x'_1 & x'_2 & \dots & x'_n \\ y'_1 & y'_2 & \dots & y'_n \end{bmatrix} \quad (15)$$

Define w_i and the coefficients a_1, a_x , and a_y as:

$$L^{-1}(V|\mathbf{0}, \mathbf{0}, \mathbf{0}) = (w_1, w_2, \dots, w_n, a_1, a_x, a_y)^T \quad (16)$$

The function $f(x, y)$ is defined as:

$$f(x, y) = a_1 + a_x x + a_y y + \sum_{i=1}^n w_i U(|P_i - (x, y)|) \quad (17)$$

$f(x, y)$ minimizes the nonnegative integral bending norm I_f over all such interpolants:

$$I_f = \iint_{\mathbb{R}^2} \left(\left(\frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \right) dx dy \quad (18)$$

The thin-plate spline function $f(x, y)$ is invariant under rotations and translations. It maps the landmarks P_i to their homologues P'_i , and defines a smooth interpolation for the rest of the points on the surface. P_i and P'_i taken together exactly specify the function $f(x, y)$, and are therefore crucial to the accuracy of the deformation. In the TPS-based registration method we use, P_i are the landmarks defined on the average face model, and P'_i are the corresponding landmarks of the test scan.

D. Automatic Landmark Localization

Registration of facial images is usually guided by a few fiducial points of established correspondence (e.g. nose tip, eye and mouth corners). These anchor points have a great influence on the resulting registration. There are very few completely automatic systems for face recognition. Most research results are reported on cropped and aligned faces, or the existence of a few manually located landmarks is assumed. This is apparently necessary, because the quality of the landmarks can be of greater consequence to the final accuracy than the classification algorithm itself.

To understand the extent of the dependence to landmarks, we have contrasted manually obtained ground truth with the results from a recent automatic landmark localization algorithm [17–19]. In this section, we briefly summarize this algorithm.

The method we employ is a coarse-to-fine approach, based on local feature information. The inner and outer eye corners, the nose tip, and the mouth corners (7 landmarks) are localized. These landmarks were selected for the reason that statistical information (rather than heuristics) can guide the search, as they correspond to distinguishable local features (whereas the tip of the chin, for instance, does not correspond to a very clear local feature). During the training phase, 7×7 patches are cropped from around each landmark in the downsampled depth image (60×80). These patches are points in a 49-dimensional space, and their distribution is modeled with a mixture of factor analysers. We employ the Incremental Mixtures of Factor Analysers (IMoFA) algorithm that automatically tunes the model complexity to the complexity of the data distribution [20].

The main assumption in factor analysis is that the d -dimensional data \mathbf{x} are generated in a p -dimensional manifold, represented with \mathbf{z} , with $d \gg p$. The vectors that span the lower dimensional manifold are called factors, and the relationship between the factor space

and the data space is given by

$$\mathbf{x} - \boldsymbol{\mu} = \boldsymbol{\Lambda}\mathbf{z} + \boldsymbol{\epsilon} \quad (19)$$

where $\boldsymbol{\mu}$ is the data mean, $\boldsymbol{\Lambda}$ is the factor loading matrix, and $\boldsymbol{\epsilon}$ models the independent variance at each data dimension. The IMoFA algorithm starts with a simple, one-factor representation of the data, and while monitoring a separate validation set, gradually increases the complexity of the model. The resulting model is a mixture with components that have different number of factors, thereby modeling clusters with different intrinsic dimensionality.

During the testing phase, this generative model is used to produce likelihoods for each point on the image. The highest-likelihood locations for each landmark are passed to GOL-LUM (Gaussian Outlier Localization with Likelihood Margins), a structural correction algorithm that can recover from localization errors [17]. The remaining errors are locations on the face with characteristics that are statistically similar to the landmark in question, and they conform to the general face pattern within a margin, as they have passed through structural correction. For instance the corners of the eyebrows may be selected as eye corners, or the cleft between the chin and the mouth may be detected as the mouth. The landmark detection was performed on the 3D depth map, which is found to be less informative than 2D images, but much more robust to illumination changes [18].

The coarse localization is complemented with a fine-level search on the original 480×640 range image. Only a relatively small (41×41) area around the coarsely located landmark is searched. The first and second depth gradients are estimated in vertical and horizontal directions, and convolved with kernels learned during the training stage. Each kernel is a map with values (± 1), and the cascade of kernels is constructed incrementally to produce the maximum output for the desired landmark shape. These kernel outputs are combined, and the highest response is taken as an indicator of the presence of the sought landmark. This approach is computationally simple, fast, and improves the quality of the coarse landmarks. See [19] for more details on the automatic landmarking procedure.

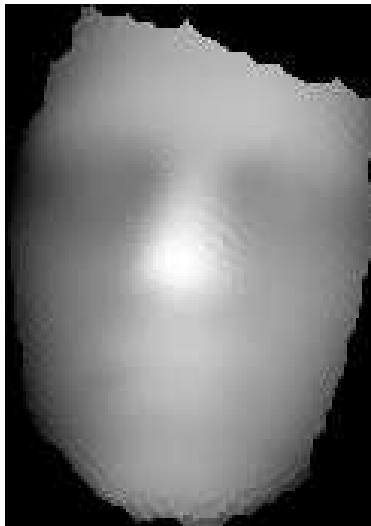
E. Regular Re-sampling

The sampling of 3D information can be a cause of error in the comparison stage. We have used a simple triangle-based nearest-neighbour interpolation for regular re-sampling of depth information from the aligned models [12]. The method relies on a Delaunay tessellation of the

original data, such that none of the original data points are contained in any circumspheres of the simplices of the tessellation. Then, for each x and y position of the regular grid, the nearest vertex in the tessellation is located and its depth value is used. The grid boundaries are derived from the generic cropped AFM, at the resolution of the range sensor. Consequently, the effect on the AFM is that the points are slightly moved to conform with a regular $x - y$ grid. We will provide a graphical intuition on why regular re-sampling is beneficial in Section V.

IV. AVERAGE FACE MODELS

In [10], a method for generating the AFM was described. In this method, the training faces are manually landmarked, and the TPS method is used to register all the training faces to a *consensus shape*. One of the faces is selected to be the AFM candidate, and from each of its vertices, distances to each other training face are computed. If a vertex has no corresponding point in every training face closer than a threshold value, that vertex is trimmed. The trimmed vertices are usually the ones at the boundaries of the face area. The remaining vertices, and their corresponding closest points for each training face are averaged to create the final AFM. This procedure creates a very smooth facial surface, as shown in Figure IV.



In this Section, we describe a novel method of generating the AFM, which is experimentally determined to produce better results. We also describe two methods of generating

category-specific AFMs, a *cognitive* method that is based on the other-race effect, and a *clustering* approach that is based on clustering in the shape space, respectively.

A. Construction of an AFM

Using a set of landmarked training faces, we generate an AFM with the following procedure:

- Using Procrustes analysis, a consensus landmark distribution is found on the training set.
- The landmarks of the consensus shape are rectified to present a fully frontal face, centered at the origin of the 3D coordinate system. This heuristic is used to facilitate the use of the transformed range image in later stages. Rectification is achieved by rotating the face so that the eye and mouth planes, as found by fitting surfaces to the appropriate landmarks, are parallel to the x -axis and the z -axis as much as possible.
- TPS deformation is computed for the training faces, which warps the landmarks of each face to the consensus shape perfectly, and interpolates the rest of the points.
- The depth values of the interpolated face are re-sampled from a regular x - y grid. This ensures that all added faces have points with overlapping x and y values, and the depth values are given for matching points. For the simple range image representation, this extra offline computation leads to much faster online model comparison.
- A cropping mask is defined to encompass the facial area, and faces are cropped before they are added to the average face model. For creating this mask, we calculate the maximum distance from the nose tip to any landmark in the consensus shape. We add a ten per cent margin to this distance, and retain all points closer than this value to the nose.
- After all the training faces are added, depth values are simply averaged.

Samples of AFMs generated with this method can be seen in Figure V F. Any irregularity in the surfaces is due to poor pre-processing of the depth data. The database we use was collected with a laser sensor that typically generates holes (especially at the eyes and the

mouth) or other artifacts. The pre-processing sometimes falls short of repairing larger errors. Since the landmark locations are retained, the facial characteristics are not smoothed out even when a large number of samples is used for AFM generation. Generating AFMs with a training set three times bigger than the original set resulted in very similar AFMs, with an average of 0.4 (± 0.3) mm. point-to-point distance to the old AFMs.

B. Cognitive Approach to Multiple AFM Generation

When humans see faces, they perceive each face individually, and not merely as specimens of a generic object category [21]. The mechanism that operates for enhanced within-category recognition is what we can call *expertise* of that category. Tong *et al.* remark that expertise-related visual judgements involve enhanced within-category discrimination, as opposed to between-category discrimination, the former requiring a magnification of differences in similar objects, and the latter calling for a neglecting of differences to group similar items together [22].

People have great difficulty recognizing faces of another race if they are not exposed to them for prolonged periods. This phenomenon is termed the *other race effect*. In the light of Tong *et al.*'s experiments, it seems reasonable that during the acquisition of face expertise, the transformations learned by the visual cortex serve to magnify the differences between individual faces, as indicated by the statistical distribution of the encountered facial features. By this reasoning, the other race effect suggests that different face morphologies exhibit different statistical distributions of distinguishing characteristics. Valentine has previously employed principal component analysis to find different subspace projections and obtained results that mimick the other race effect [23].

We stress that our aim is not to detect the *race* of a person; therefore, we use the term *morphology* to denote similar facial surface morphology characteristics. Based on the cognitive cues, we predict better recognition rates if the faces are clustered into morphological or gender groups that exhibit greater intra-group similarity and the discriminative features are learned within each group separately. This is not trivially true for all pattern recognition applications, as the grouping reduces the number of training samples and consequently runs the risk of impairing learning conditions. In this approach, the gender and morphology were determined manually, by visual inspection of the faces.

C. Clustering Approach to Multiple AFM Generation

If our hypothesis of meta-classification is correct, we expect morphology and gender to be discriminating dimensions of the face space. However, we do not wish to categorize faces into races explicitly, as this approach has ethical consequences. Can the gender and race determination during the training (and possibly, in the testing) stage be evaded? For our simulation purposes, we have roughly assigned facial images into African, Asian and Caucasian morphological face classes. The other-race effect suggests that racial-morphology based clusters *exist* in the face space, and an unsupervised clustering method can recover those clusters, among other meaningful structure. Thus, it is not necessary to recover the race and gender of a person; the clustering will hopefully provide us with a useful set of average faces to serve in meta-classification with increased discrimination within clusters.

By performing a shape space based clustering, we will also answer an interesting question: Are gender or racial morphology major factors in determining the face shape? The intuitive answer is affirmative, but the clustering provides us with an explicit way to test this hypothesis.

We propose to take a straightforward race- and gender-blind clustering approach with the k -means algorithm. The clustering is performed on the aligned coordinates of seven facial landmarks. We specify the number of clusters for the shapes, and initialize the cluster consensus shapes by random selection from the training samples. At each iteration, we align the training samples to the consensus shapes of the clusters via Procrustes analysis, and assign each sample to the cluster with minimum average distance to the cluster consensus. We then re-estimate cluster consensus shapes from the samples assigned to the cluster, and iterate until the total distance stabilizes. The number of clusters was set to six to allow comparison with the cognitive approach.

The clustering gives us a number of cluster consensus shapes, and assigns each training face to one of these clusters, without ensuring that all morphological categories are represented. We apply our AFM generation algorithm to these reduced training sets separately, and obtain one AFM for each cluster. These models can be seen in Section V G.

V. EXPERIMENTS

A. The 3D Face Database

We use the FRGC 2D-3D ver.1 face database in our experiments [24]. The database contains 943 near-frontal depth images from 275 subjects, stored in a point cloud representation. The laser scanner that was used to collect the database samples points from the surface of the face in regular intervals. Although the number of points per face can be different according to distance to the scanner at the time of acquisition, the coordinates of the points are given in a proper metric scale.

We use images from 195 subjects, with one training face in the gallery and 1-4 test faces (for a total of 659 test faces). This experimental setup is used in [33], and we chose to use the same setup to allow a direct comparison with a host of techniques. Subjects with only a single image in the dataset, and images with incorrect texture-depth correspondence are not used. We only work with 3D information for landmarking and registration; 2D is not used at all. We design a number of experiments to answer various questions. Each subsection deals with a particular question, and reports relevant simulation results. The overall system has many dimensions, ruling out a factorial experimentation protocol.

For each method, we run a recognition experiment and a verification experiment. For the recognition experiments, the rank-1 recognition rate (R1) is reported. The point set difference (PSD) is used as a distance metric for recognition. In the verification experiments, each of the 659 test faces is used for one genuine and 194 false claims. Thus, the number of false claims is two orders of magnitude higher. The equal error rate (EER), for which the false acceptance rate is equal to the false rejection rate, is reported under these conditions.

B. Preprocessing

The preprocessing of 3D information consists of several steps. Missing points and holes can be filled by local interpolation or by making use of facial symmetry [25]. However, it is a well-known fact that faces are not truly symmetrical. Gaussian smoothing and linear interpolation are used to eliminate irregularities in both texture and range images [2, 3, 26]. Background clutter and hair artifacts are usually manually removed in the literature [2, 3, 27], whereas in our approach, the cropping of the face area is fully automatic.

If there is excessive acquisition noise in a dataset, it will be difficult to assess the accuracy of algorithms, as the deterioration due to noise can be more pronounced than improvement due to a better algorithm. Datasets are frequently cleared of noisy samples [26, 28]. Mean and median filters are also employed to reduce local noise [5, 26]. The preprocessing used in this paper includes a 9×9 mean and a 9×9 median filter applied in succession, followed by linear interpolation for filling the gaps. The filter sizes are experimentally determined, and not optimized for actual shape sizes.

C. Automatic Landmarking

Figure VC shows the accuracy of the automatic landmarking procedure in terms of millimeter-distance from the manually obtained ground truth. The evaluation considers one to ten candidates for each landmark location, as indicated by the x -axis. The curves begin at the average distance for the best candidate, and decrease as better locations are found among the candidates. The manually annotated ground-truth is accurate up to several millimeters, and a lower distance to the ground truth indicates a better localization. The quality of landmarking also depends on the type of landmark; the nose tip is easier to find in 3D as expected [29–32]. The reader is referred to [19] for more detail on the performance of the algorithm.

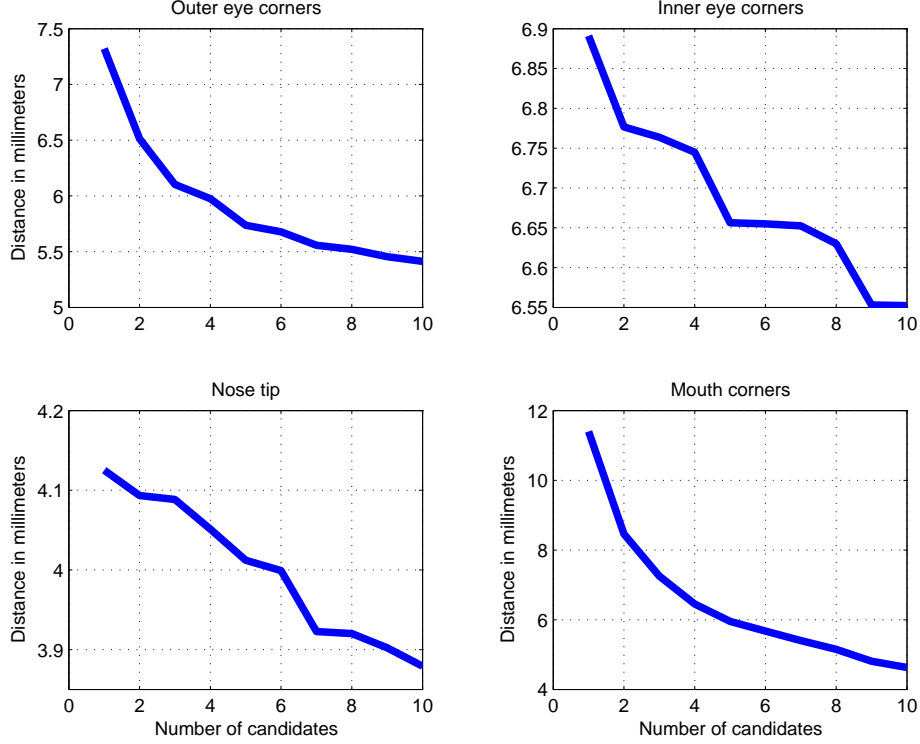
D. Classification

The most natural way of classification for point sets in dense correspondence is using the point set distance (PSD) measure. Since all points are in full correspondence, we have two 3D vectors of the same length, and PSD is simply the sum of all Euclidean distances of the points in the query and gallery faces:

$$\arg \min_j \sum_{i \in S} \|\mathbf{x}_i - \mathbf{a}_{i,j}\| \quad (20)$$

where \mathbf{x}_i is the i^{th} point of the test scan, and $\mathbf{a}_{i,j}$ is the i^{th} point of the j^{th} gallery sample. S denotes the set of points put into dense correspondence.

In the cases where we employ the regular re-sampling, the x and y coordinates of these points are the same across all samples, and therefore need not be taken into account. The



3D PSD becomes equivalent to a L1 distance computed on the depth values, and for classification, the smallest L1 distance between gallery images and the test images is sought to indicate the correct class j :

$$\arg \min_j \sum_{i \in S} |\mathbf{x}_i^z - a_{i,j}^z| \quad (21)$$

where \mathbf{x}_i^z is the depth value of the i^{th} point of the test scan, and $a_{i,j}^z$ is the depth value of the i^{th} point of the j^{th} gallery sample.

For authentication, a threshold τ is used to validate the claimed identity j :

$$\sum_{i \in S} |z_i - a_{i,j}| < \tau \quad (22)$$

and τ can be used to tune a trade-off between false acceptance rate (FAR) and false rejection rate (FRR). The equal error rate (EER) is the reported error rate for the particular τ for which FAR and FRR are equal on the test set. While it makes sense to use one-to-one ICP for the authentication setting in practice, obtaining a lower EER by just changing the AFM set is still indicative of a better registration, and therefore it is reported for some of our simulations. Furthermore, by storing only the projection coefficients for gallery faces, the storage requirement can be greatly reduced (for instance, instead of about 32.000 3D

points, a 50-dimensional vector can be stored per subject). In this case, the subject-specific 3D model is not available for registration, and an AFM will be used for authentication.

E. Coarse registration

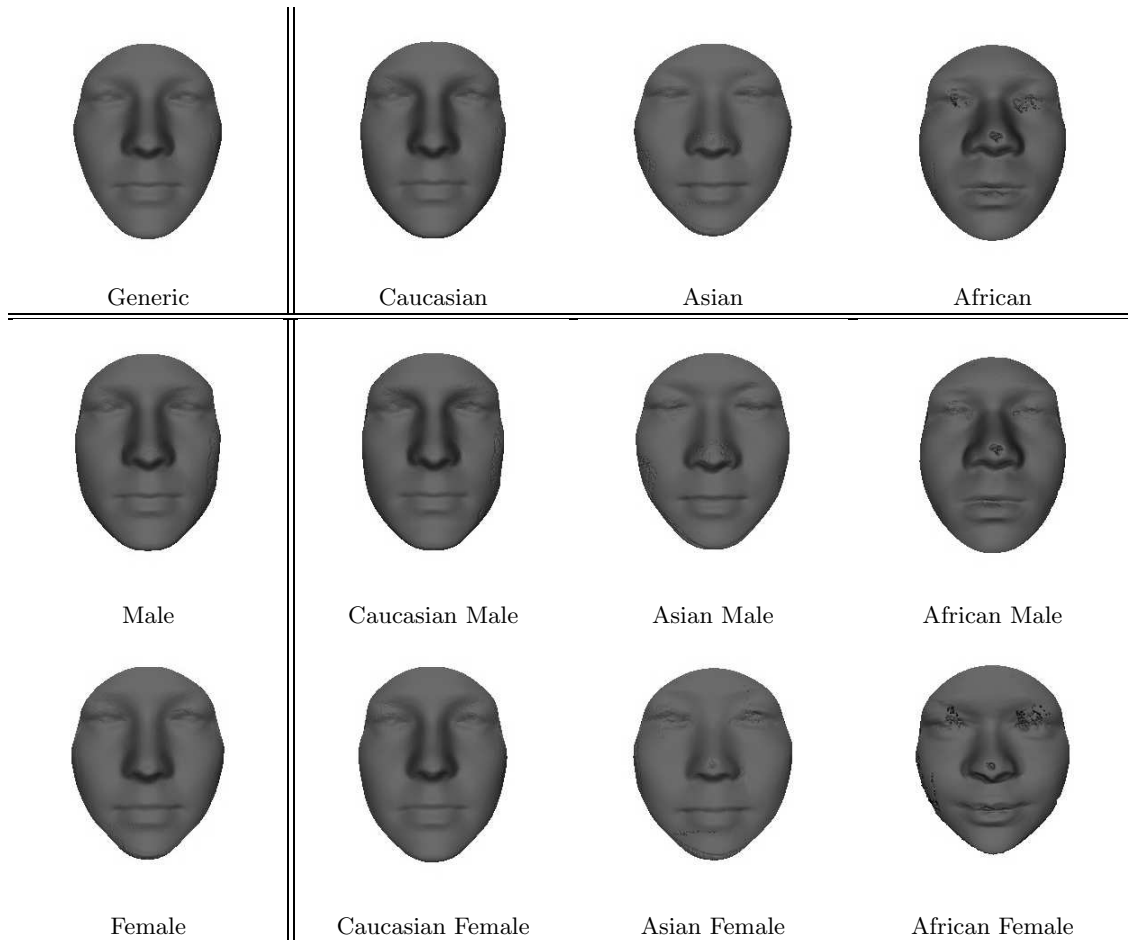
Table ?? shows the effect of coarse alignment methods on ICP-based registration. As we have suspected, the nose-tip heuristic performs the worst. Automatic localization of seven landmarks and using Procrustes alignment works better than the nose-tip heuristic. For ICP, using the nose ground truth works well in this dataset, because the faces we deal with are mostly upright and frontal. Ideally three landmarks should be used to accommodate greater pose differences in 3D. Finally, the seven manual landmarks with the Procrustes analysis give us an upper-bound on the performance of the ICP-PSD method.

We have also contrasted our AFM construction method with the method of Irfanoğlu *et al.* [10] on ICP. Manual landmarks were used, and initialization was by Procrustes alignment. With their smoother AFM, a rank-1 recognition rate of 86.34 was achieved, as opposed to our 92.11 per cent. Similarly, the EER was higher with their AFM by more than two per cent.

F. Meta-classification

Does meta-classification and more specialized individual experts increase discrimination? We have tested this hypothesis by employing the average faces that are generated from groups of training faces. We have grouped the training samples for gender and morphology, and generated average face models (AFM) for each group. Figure V F shows range images obtained for the average face, for male and female averages, for three morphological group averages (roughly Caucasian, Asian, and African), and for all combinations, respectively. The morphology does not correspond to a clear-cut distinction. The morphological group of a given face will be determined by its proximity to the average face of the group.

In Table ?? the authentication experiment results with or without specific average faces are shown for TPS-based registration. We have supplied both the generic-AFM based system and the specific-AFM based system with the categorical information, which improves the accuracy by itself. However, any improvement in the specific system with regards to the



corresponding generic system is strictly due to better registration. We have computed distances between the test face and the gallery faces with an L_1 distance metric, and trimmed the worst two per cent of the correspondences. The EER without any use of categorical information is 20.10 per cent. The results reported under these conditions show that specific AFM usage is beneficial in this case. We stress that the main purpose of the PSD is to evaluate the effect of AFM usage; the EER is generally too high for these experiments because of deformations in the registration, but the gain obtained by introducing specific models is more pronounced in comparison to ICP-based experiments.

The point set distance after aligning the face to the female and male averages can be used in gender classification. This simple method works with 80 per cent accuracy.

G. Shape Space Clustering

The shape-space method divides the training samples into clusters of similar size. The number of clusters should be small to have a maximal computational gain, and can be increased as long as the accuracy increase is considered worth the computational cost. However, the number of training samples per cluster should be sufficiently high (depending on the quality of the range data and pre-processing), else the constructed AFM will be poor. We have specified six clusters, to allow a comparison with the full morphology-gender combination case, and ran our algorithm on the training part of the FRGC ver.1 dataset. Figure V G shows the cluster means.

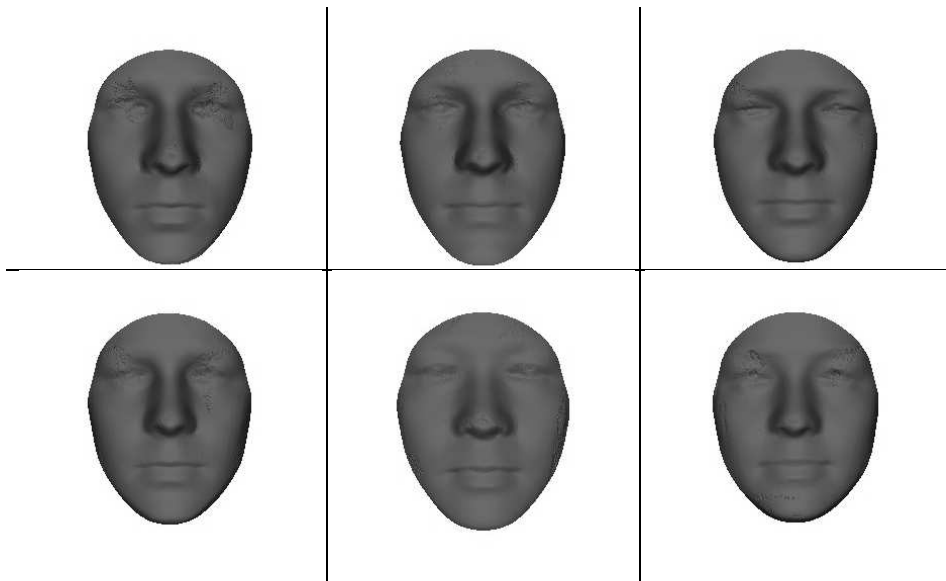
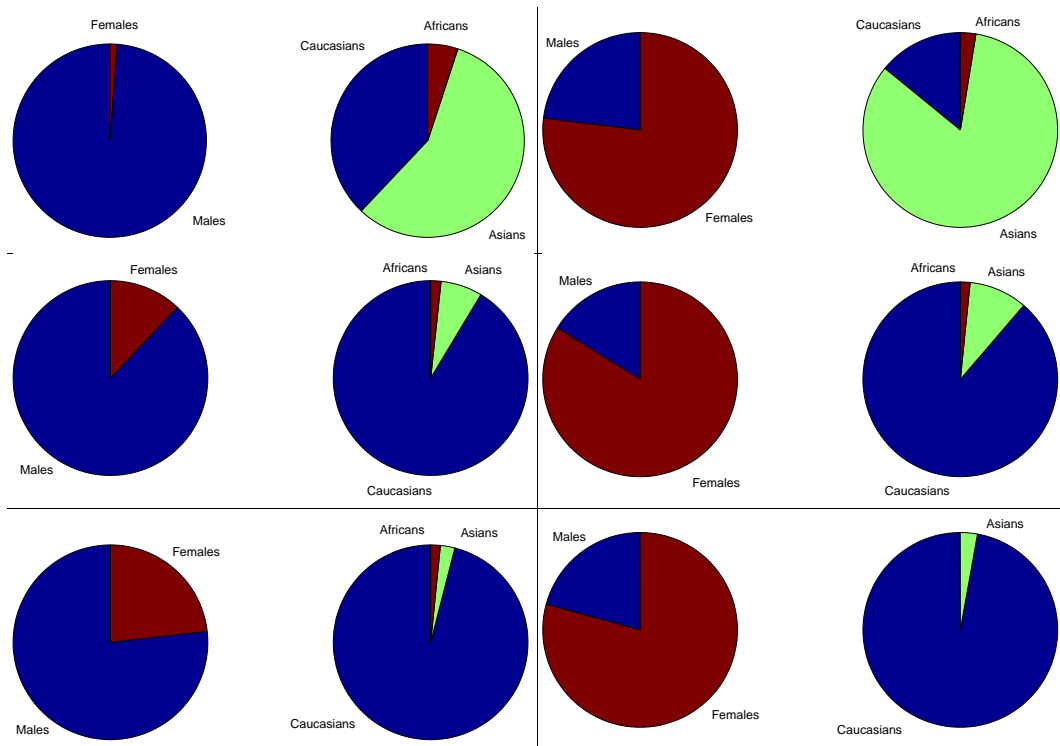


Table ?? shows recognition rates for ICP and TPS based systems with manual or automatic landmarks. The first row shows the results obtained with a single generic AFM. The next three rows show results with gender-, morphology-, and gender + morphology-based specific AFMs. The results for the last row are obtained with six shape-space derived clusters. For this last case, the registration does not benefit from the injection of categorical information, and each test sample is compared with all the training samples. The best result is obtained with shape space derived specific AFM and ICP (93.78 per cent). As a baseline experiment, we have also tested one-to-all ICP, where each test face is registered to each gallery face for distance computation. With manual landmarks, we have obtained 89.07 per cent rank 1 recognition rate, which further demonstrates the usefulness of AFM-based registration.

We have also tested adding the previously excluded single-image subjects to the gallery. This decreases the accuracy of all the recognition methods, as the 195-class problem becomes a 275-class problem. The ICP method (with manual landmarks and generic AFM) suffers 2.6 per cent accuracy loss with the enriched gallery.

Our cognitively motivated subspace hypothesis is confirmed, if the shape space clustering automatically creates clusters with a dominant gender, or puts samples from one race into a single group¹. This is what more or less happens, and we do have clusters dominant in a single gender or a single morphology. This effect is more pronounced when we increase the size of the training set by 371 images from the FRGC ver.2 dataset. Figure VG shows the distribution of morphology and gender in each group as pie charts.



¹ Currently available 3D face datasets do not have a balanced morphology-gender distribution, where the cognitive hypothesis can be tested more thoroughly.

H. Regular Re-sampling and Subspace Projection

In this section we apply the Eigenface method to 3D range images registered to different AFMs with ICP. A projection from the space of 3D points can only be computed if the number of points is the same for each query and gallery face. Our method ensures that this is indeed the case. If the accuracy loss after the projection is small, this means that the registration was successful, and the selected subspace dimensionality is sufficient to capture the discriminative dimensions of variation.

The depth values of the range images are not sampled regularly from an $x - y$ grid, and so far we have computed 3D point distances, instead of a simpler 1D comparison based on the depth. Since we deal with aligned shapes in dense correspondence, applying a regular re-sampling will make it possible to discard two dimensions from the point cloud, making the subsequent comparison and subspace projection easier. We will also show that the regular re-sampling helps classification by making the distance measurement more accurate.

The dimensionality of the subspace is determined heuristically. We set the number of eigenvectors so that at least 95 per cent of the variance is accounted for. The Eigenface method is contrasted with the PSD method, which, due to regular re-sampling, uses the sum of squared distances of depth values only.

The regular re-sampling lightens the computational burden of comparing the test sample with gallery images. Furthermore, the computed projection allows us to store much smaller gallery faces. For example in experiments with the generic AFM, roughly 32.000-dimensional face vectors are represented with 50-dimensional vectors after the subspace projection. This means that the comparison with gallery faces will be much faster. Table ?? shows that the accuracy loss due to subspace projection does not exceed one per cent, if there are sufficient training samples. For the gender & morphology combination, the training set is very limited, and consequently there are only 15 eigenvectors with non-zero eigenvalues in one of the groups (henceforth we denote the number of eigenvectors with p). This number is too small to represent the facial variation, and the accuracy decrease is about three per cent. The morphology results were obtained with $p = 33$, and the gender results with $p = 49$. For the results in this section, we have grouped African faces and Caucasian faces into a single category, as we had too few samples from the African category (i.e. two clusters for morphology, and four clusters for gender & morphology runs).

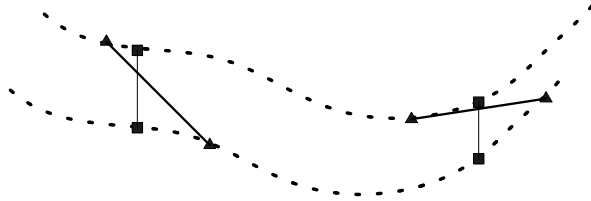
Before analysing the results, we will give some additional experimental results reported on this dataset. In [33], Gökberk *et al.* use the same experimental protocol as ours to compare a wealth of classification methods: Point set difference, non-negative matrix factorization (NMF) and independent component analysis (ICA) coefficients for point clouds, DCT, DFT, PCA, LDA, and ICA projections on depth images, shape indices, mean and principal curvatures, 3D voxel DFT coefficients and 2D Gabor wavelet coefficients. Manually annotated landmark positions were used for an AFM-based ICP registration (with an AFM generated with the method of Irfanoğlu *et al.* [10]). The best classification results are based on shape indices (90.06 per cent), principal directions (91.88 per cent) and surface normals (89.07 per cent). The best accuracy after classifier fusion is 93.63 per cent, obtained with modified plurality voting [33].

For projection-based methods, the input dimensionality needs to be fixed. Subsequently, PCA is not directly applicable in 3D prior to the cropping (which in turn uses the alignment), as we have different number of 3D points per subject. It is possible to do a PCA on the range image without 3D registration, by using the landmarks to define a bounding box on the range image. In [34], this approach is used, where manual landmarking is followed by re-scaling of the face area to a fixed bounding box. By using a cropping mask, the effect of boundaries were reduced, and a PCA transformation was used to compute Mahalanobis cosine distance for classification. With an experimental setup on the FRGC ver.1 dataset very similar to ours (198 training and 670 probe images were used, subjects with a single image are excluded), the reported accuracy is 87.04 per cent.

The ICP results reported in Table ?? are much better than the results reported in Table ??, and the results reported in the literature. With the generic AFM and the automatic landmarks (i.e. the fully automatic system) the PSD method without re-sampling has a rank-1 recognition rate of 87.86 per cent, whereas after re-sampling, it has 98.03 per cent accuracy. The reason is graphically depicted in Figure V H. The facial surface (shown symbolically as a dotted line in the figure) is irregularly sampled by the laser scanner (two points per facial surface, shown with black triangles). The ICP registration brings these surfaces into alignment by global rigid matching. Hence, the corresponding points may not be in close alignment locally, although the sum of all displacement vectors is at a local minimum.

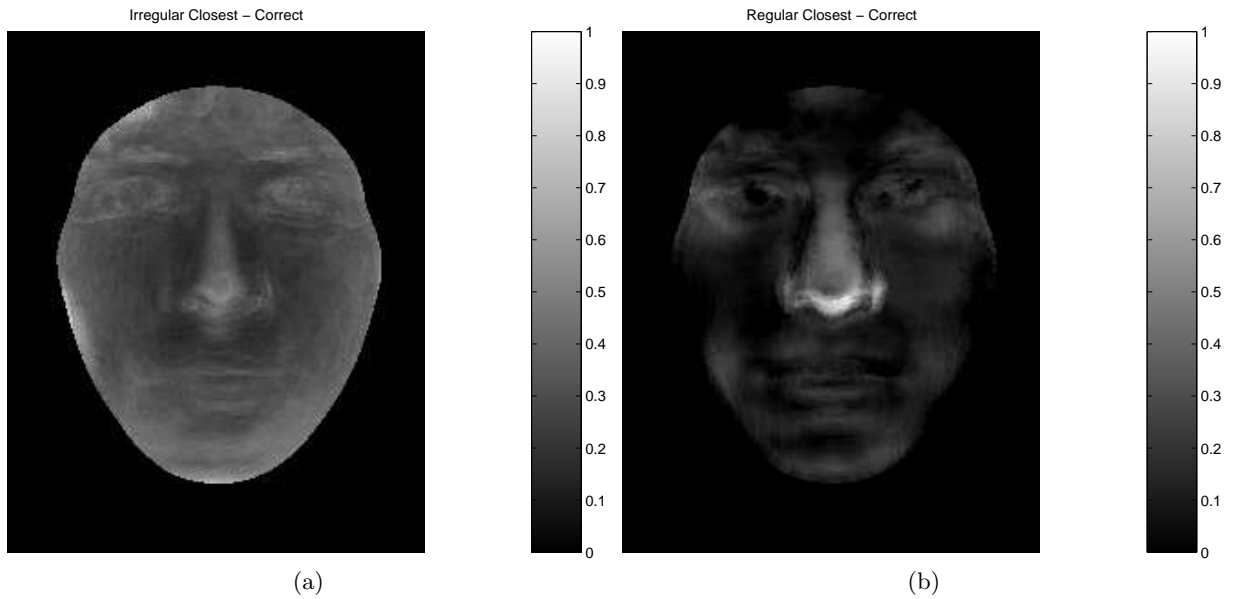
Regular re-sampling produces depth values at regular x and y intervals (shown with black square points). These points give a more realistic indication of the distance between

the two surfaces, unless the absolute depth gradient is very high. In the latter case, small displacements in the $x - y$ plane will result in big changes in depth, making an irregular, point-to-point 3D comparison the logical choice. However, the facial surface as represented by a range image has few points with sharp depth changes (i.e. the nose ridge, mouth and eye corners, and the face boundary). Our cropping procedure eliminates the face boundary, and greatly reduces the number of these points. Consequently, the regular re-sampling is indispensable for AFM-based registration.



When we inspect the samples that are classified correctly after re-sampling, but not before, we see that the error due to point-irregularities is large enough to disturb classification. Figure V H depicts the mean distance differences between the correct class and the incorrect class for these samples. For samples with irregular point distributions, the point-to-point distance terms have a large variance, and this error is distributed all over the face as shown with light colour. However, for regular re-sampled point distributions, large areas on the face have very low error, as shown with dark colour. As we predicted, error in re-sampled faces peaks for locations with greater depth gradient, and especially for the nose ridge. Since the nose ridge is a relatively small area of the face, increased error here is compensated by decreased error on the larger facial surfaces. Furthermore, the nose ridge error is increased for competing classes as well. Consequently, the nose area becomes useful in discrimination, even though sometimes it is the highest error area during registration with the correct gallery sample.

The re-sampling does not have a high computational cost, as the points are already ordered in the range image. For one-to-all ICP, it is possible to perform a similar re-sampling. However, if the gallery faces are not in alignment, the re-sampling has to be performed online for each gallery face separately. Another benefit of using the AFM is that the re-sampling is just performed once for each test face, and the computation is offline for gallery samples.



When we experiment with race and gender information, we make sure that for each comparison, the training and test samples are registered to the same AFM. For example, in the simulations where the gender is available, we register a male test face with the male AFM before comparing it to the male faces in the gallery, but we use the female AFM for comparisons with female gallery faces. Thus, we have two options when using categorical AFMs: We can either inject ground truth information (for instance by setting intra-group distances to infinity), or we can let the system decide on the face category by picking the match with the smallest distance, like we do for shape space clustering based categories. The results reported in Table ?? are obtained with the latter method. Our simulations show that injecting the ground truth increases the accuracy only by 0.5–1 per cent. This means that for this dataset, cross-gender and cross-morphology errors are relatively rare, and we obtain categorical information with great accuracy by simply selecting the best gallery face.

Another issue we have considered was the number of points in different AFMs. The AFM for the females contains roughly 20 per cent fewer points than the AFM for the males. In our first experiments, we have used a single mask to crop faces in all categories. This procedure gives faces with equal numbers of points. The results of Table ?? are obtained by allowing each category to have a different number of points after cropping, and by nor-

malizing the Euclidean distances accordingly. For the gender case, this procedure increased the recognition accuracy by a half per cent.

VI. CONCLUSIONS

For real-time 3D face recognition, the computational requirements of the algorithms must be taken into consideration. We have evaluated ICP and TPS based registration of 3D faces under automatic and manual landmarking. The much slower ICP method is only viable if the registration is speeded up through the use of average face models. We have proposed an AFM-based method for this purpose, and demonstrated its usefulness. By extending the paradigm to multiple AFMs, we have proposed two approaches for generating category-specific AFMs and contrasted these. Finally, we have proposed a regular re-sampling step that increases the accuracy and speed of classification.

Our results show that ICP is superior to the faster TPS based method in accuracy. Simulations with improved non-rigid approaches have shown that these methods depend heavily on the accuracy of landmarks, and also require a greater number of those [19]. However, the beneficial effect from specific AFM use is more evident in TPS methods that use either automatic or manual landmarks.

We propose a cognitively based approach and a clustering approach to generating specific AFMs. Clustering on shape space produces good AFMs, increases the accuracy of registration, but also reveals natural groups depending on morphology and gender in the face space. This is interesting, as it provides an indication that the *other race effect* has a physical basis even for the 3D information contained in a face.

The reasoning behind multiple AFM usage is that categorical information can act as a filter to reduce the candidates for recognition, and an average face more similar to the test face can ensure a better registration. In the case that cross-gender and cross-morphology confusions are relatively rare, the injection of categorical information does not increase the accuracy.

Another observation regarding multiple AFMs is that specific AFM models have different numbers of points. A male face usually contains 20 per cent more points than a female face, simply because female human faces are typically smaller. When we align a face to the female and the male AFMs, the distribution of distances is different in the centre of the face and at

the periphery. Using a smaller AFM (the one for the females, or Asians, for instance) will effectively remove the points close to the periphery from the distance calculation. This can be an issue for one-to-all ICP approaches as well.

Our experimental results have also confirmed that ICP is sensitive to initialization, and automatic landmarking as a pre-processing step is beneficial to ICP. Manual inspection showed that none of the test cases had gross registration deficiencies in the fully automatic method. The nose-tip heuristic may be useful in frontal faces, but the hair, clothing and sometimes the chin can be erroneously detected as the nose tip. The error due to incorrect nose localization can be gauged by looking at the results of the simulations that use the ground-truth for the nose in initialization. We should also keep in mind that the database we use is made up of near-frontal faces. The nose-tip heuristic will perform worse in other pose settings. On the other hand, our results confirm that the nose tip is more important for the registration than any other landmark we have used.

The re-sampling procedure we propose is based on the relative flatness of the facial surface and the fact that cropping eliminates the facial boundary, where the cheeks can produce a sharp depth gradient in the frontal view. With this method, the error due to irregular sampling by the laser scanner is compensated, and there is a significant increase in the accuracy. As a result of AFM based registration, the cropped models have the same number of points, and these points are matched on the depth map after re-sampling. This helps us in projecting the depth map to a much lower dimensional manifold via PCA, while retaining a high recognition accuracy. We obtain the best results with ICP and shape space clustered AFMs, and our reported results with re-sampling are significantly better than results reported in the literature without re-sampling for the same experimental protocol.

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