Corrigendum


The author is grateful to Christoph Pflaum for pointing out a mistake in the statement and proof of theorem 2.2. The correct version is:

Theorem 2.2
If we consider an expansion of a $C^3(\Omega)$-function, $u$, in piecewise constant functions on the grid $\Omega_n$, for an arbitrary $n \in \mathbb{Z}^3$, $n > 0$, and if we write

$$ R_n u = v_0 + \sum_{0 \leq j \leq n} u_j, \quad (1) $$

with $v_0 \in V_0$ and $u_j \in W_j$, $0 \leq j \leq n$, then

$$ \|u_j\| \leq 2^{\frac{j}{2}|u|}, \quad (2) $$

and we get an estimate for the approximation error

$$ \|u - R_n u\| \leq \frac{1}{3} \sqrt{2} \frac{1}{3} (h_1 + h_2 + h_3)|u|. \quad (3) $$

Proof

We take the normalised $\{\tilde{\psi}_j^k\} = \{2^{(j-e)/2}\psi_j^k\}$ as a basis in $W_j$, $0 \leq j \leq n$, $j \neq 0$. Clearly, all these functions are orthogonal to all functions $v_0 \in V_0$ and mutually they form an orthonormal set in $W_j \subset L^2(\Omega)$. We see further $\psi_k^l \in W_j$ and support($\psi_k^l$) = $\Omega_{j-e,k}$, or, in other words, $\psi_k^l \in V_j$, but $\psi_k^l$ scales like a basis function in $V_{j-e}$. Hence

$$ \int 2^{(j-e)/2}\psi_j^k 2^{(j-e)/2}\psi_m^j d\Omega = 0 \quad \text{for} \quad k \neq m, $$

and

$$ \int 2^{(j-e)/2}\psi_j^k 2^{(j-e)/2}\psi_k^j d\Omega = 2^{(j-e)} \int_{\Omega_{j-e,k}} d\Omega = 1. $$

Thus, we find (1) with

$$ u_j = \sum_k a_k \tilde{\psi}_k^j = \sum_k (u, \tilde{\psi}_k^j) \psi_k^j. $$

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Now
\[ a_{jk} = (u, \tilde{\psi}_k^j) = \int_{\Omega} u \tilde{\psi}_k^j \ d\Omega = \int_{\Omega_{j-\epsilon,k}} u \tilde{\psi}_k^j \ d\Omega. \]

By Taylor expansion around \( z_k^{i-\epsilon} \), we have
\[ \left| \int_{\Omega_{j-\epsilon,k}} u \tilde{\psi}_k^j \ d\Omega \right| \leq 2^{-2j} \| 2^{j-\epsilon} u \|_2. \quad (4) \]

For \( j \geq \epsilon \) the point \( z_k^{i-\epsilon} \) lies in the interior of \( \Omega \) and the estimate holds with
\[ |u| = \max \left| \frac{\partial^3 u(x)}{\partial x_1 \ldots \partial x_3} \right|. \]

For \( \psi_k^j \) with a \( j \)-component equal to zero, the point \( z_k^{i-\epsilon} \) lies on the boundary and the function \( \psi_k^j \) is constant in one direction over the whole domain \( \Omega \), and it is of Haar-wavelet type for the non-zero indices (or index). In this situation the same estimate (4) holds with, e.g. if \( j_1 = 0 \),
\[ |u| = \max \left| \frac{\partial^2 u(x)}{\partial x_2 \ldots \partial x_3} \right|. \]

For \( j = 0 \) the relation (4) is trivially satisfied. Hence, the estimate (4) holds for \( j \geq 0 \) if we use the seminorm (21), and we find
\[ |a_{j,k}| \leq 2^{-3/2} 2^{-3/2 |j|} |u|, \]
\[ |u_j|^2 = \sum_k |a_{jk}|^2 \leq \sum_k 2^{-3 |j| - 3} |u|^2 = 2^{-2 |j| - 3} |u|^2, \]
so that
\[ |u_j| \leq 2^{1/2 |j| - 3/2} |u|, \]
which leads to (2), and
\[ |u - R_n u|^2 = \sum_{j_1 > n_1} \sum_{j_2 > n_2} \sum_{j_3 > n_3} |u_j|^2 \leq \sum_{j_1 > n_1} \sum_{j_2 > n_2} \sum_{j_3 > n_3} 2^{-2 |j| - 3} |u|^2 \]
\[ \leq 3^{-3} 2 (2^{-2n_1} + 2^{-2n_2} + 2^{-2n_3}) |u|^2. \]

and it follows that
\[ |u - R_n u| \leq \left( \frac{2}{3} \right)^{1/2} (2^{-n_1} + 2^{-n_2} + 2^{-n_3}) |u| = \frac{1}{3} \sqrt[3]{2} (h_1 + h_2 + h_3) |u|. \]

\[ \square \]