Taking the pulse of the economy 24th March 2003

Zbigniew Struzik draws parallels between two complex systems: that of the heart, as observed through the rate of heartbeat, and the economy, measured by the stock index record.

Econophysics versus cardiophysics?

There are a number of reasons why one may want to have a simultaneous look at the cardiac and economic systems. Both exhibit complex behaviour classified as scale invariant and multifractal. Both can be seen as systems of a number of coupled oscillators, subject to stochastic regulation. Both consist of antagonistic actors; in the neuro-regulatory cardiac system, in addition to the pacemaker, the periodic pulse generating node, heart rhythm is driven by sympathetic and parasympathetic nodes, in constant imbalance. The sympathetic system slows down the heart rate, while the parasympathetic one accelerates it. One might think that during activity, only the parasympathetic system would be active, while during rest or sleep only the sympathetic one would be in action. Contrary to this, both systems are at work at any moment in time. In fact, it is a constant, simultaneous 'struggle' of both systems which gives rise to the actual heartbeat ratio, characterised by continuously changing dynamics.

This scenario resembles a view of the market, with *pessimists* who sell and *optimists* who buy stock options at any given instant. Obviously, passive observers do not play role in shaping the instantaneous index value - similarly the pacemaker node is irrelevant to the shape of the heart rate record - pacemaker rhythm only would yield a constant line of fixed heart rate. But the number of *active* actors changes as the market changes. Some pessimists join the optimist group and vice versa. Additionally, there is leakage of information, or simply similar action, clustering the behaviour of agents in both the pessimistic and the optimistic groups. This leads to a continuously varying number of *independent* active agents in the system, which may be related to the number of effectively active degrees of freedom of the abstract market system under consideration.

In particular, in the market scenario, polarisation of opinions leads to clusters of agents, effectively reducing the number of *independent* degrees of freedom in the system. The resultant behaviour of the stock becomes (anti-)correlated and bends towards anti-persistent fractional Brownian motion characterised by $H < 0.5$. The number of effectively active degrees of freedom for a fully developed economic system is likely to be somewhere in the range of $10 - 100$.

The behaviour of the healthy heartbeat is dominated by the coherent, concerted action of the inputs within the nodes of the two antagonistic systems (sympathetic and

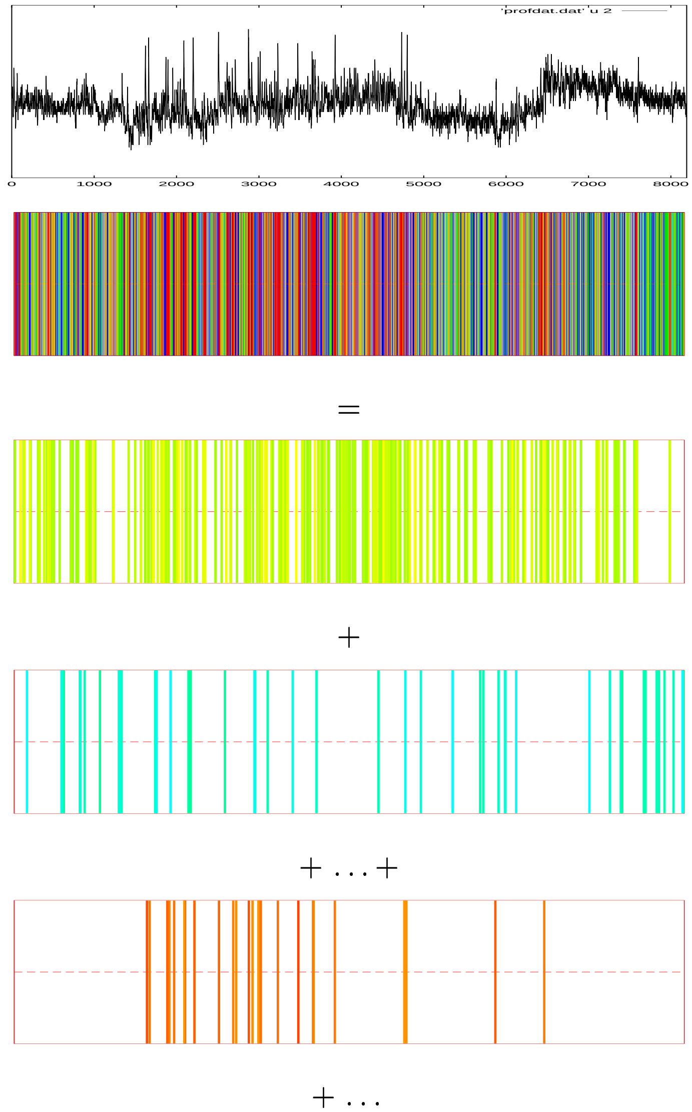

Figure 1. Local contributions to the multifractal spectrum of a healthy adult heartbeat record with all its monochromatic, *i.e. mono-fractal*, components separable.

parasympathetic), leading to an extremely high level of anti-persistence at the range of $H = 0.0$ to $H = 0.1$ (for the normal healthy heartbeat at rest). In the case of pathology, exercise or stress, this behaviour becomes less anti-persistent and less (anti-)correlated, potentially indicating less coherence in the functioning of the two antagonistic systems and thus possibly the increase of the effective number of the independent degrees of freedom in the system. It can be speculated that the range of the degrees of freedom in the healthy cardiac system is extremely low (less than 10, probably closer to 2) while for pathology it grows above this range.

The similarities drawn between both systems of market self-regulation and heart neuroregulation can be expanded even further. In the condition of extreme stress during labour, the heartbeat of the child being born can be shown to undergo a dynamical evolution which can resemble, and be compared to, severe stock market crashes.

Traditional objective - Predicting the crash

It is the unstated holy grail when in dealing with the economy to discover a law or design machinery capable of predicting stock market crashes. Similarly econophysics has not escaped this trend - efforts have been made to identify crash precursors, and predictive models based on critical behaviour have been proposed (to various degrees of testability). Most data is, of course, analysed a posteriori, leaving the actual assessment of the capability of crash prediction open to new studies on new data.

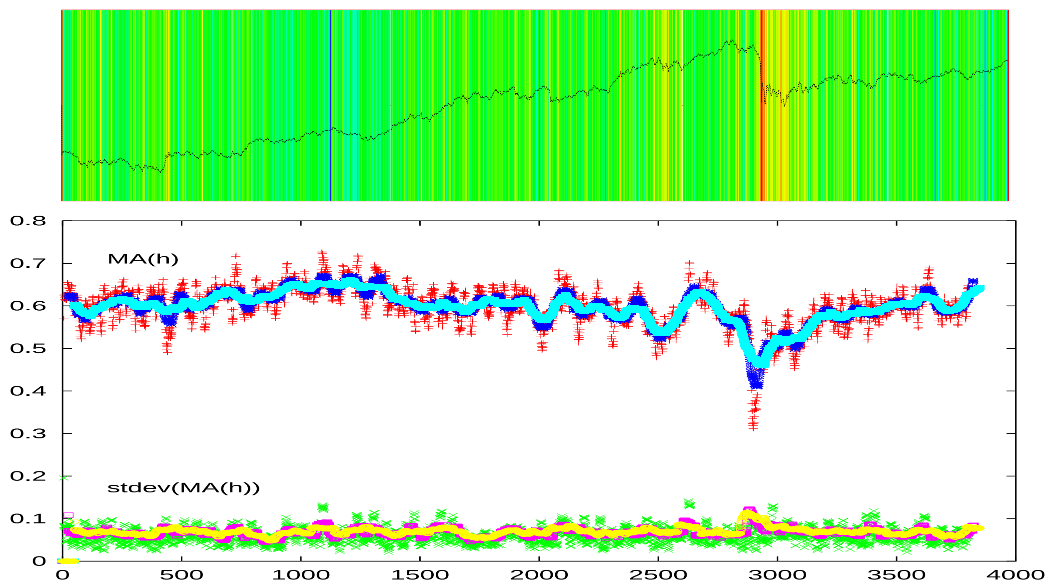

Figure 2. Local roughness expressed in colour range in the upper plot. In the lower plot, the corresponding numerical values smoothed with moving average filters of three lengths. Standard deviation in each MA filter length plotted below.

Such analyses take into account only the largest crashes. This is both due to the fact that such are the most spectacular and damaging, and because there is only a very limited number of crashes that can be analysed by hand in the same way as in the approaches mentioned. In view of the scale invariance, this approach does not, however, seem correct - large crashes should not be different from small crashes at a smaller scale (higher resolution). The crashes, large or small, should, in view of the scale invariant paradigm, be equally well predictable and should follow the same mechanism. This issue, the abundance of small scale crashes for prognosis testing, has not been explored to anywhere near a satisfactory degree. One of the possible explanations for this fact is that the large scale crashes may actually follow a different mechanism than the bulk of smaller crashes constituting the scale invariant index structure. This is the subject of an ongoing debate, as to whether the large crashes are 'outliers' or belong to the scale invariant distribution.

In the context of this debate, it may be interesting to compare to the index crashes the dramatic events of heartbeat 'crashes' in a fetus at the time of birth. These crashes are generally considered to fall outside the 'normal' statistical range of heartbeat dynamics. They are the result of temporary hypoxia as a result of an 'external' cause, like e.g. obstruction of the umbilical cord, or as a result of the 'internal' response of the system to prolonged deficiency of oxygen.

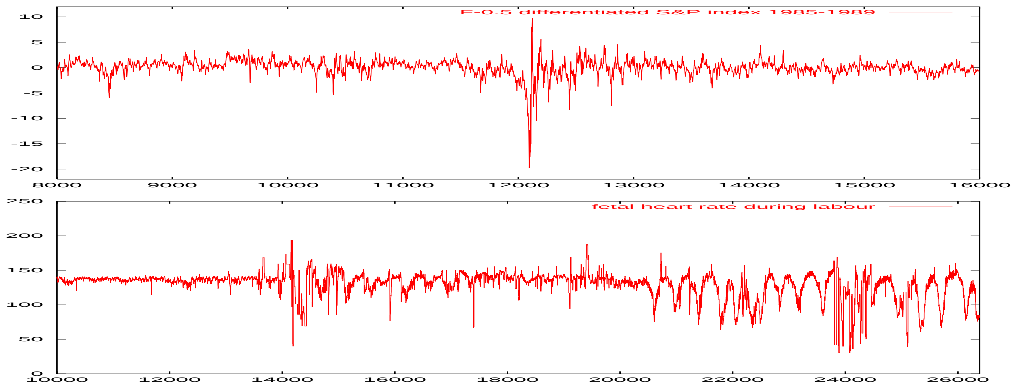

Figure 3. Top: a fractionally differentiated S&P500 record F(-0.5) about four years long centred at the '87 crash. Bottom: a fetal heartbeat rate record during labour, about ten hours long.

A different perspective - What happens after the crash?

The focus of perinatology is always after-crash oriented: it is the good health of the baby delivered which is the goal and the objective of the labour and the decision making process involved. Monitoring the fetal heart rate is a critical component of the decision making process, and in fact the only means of assessing fetal well-being during delivery. There is, however, no one-to-one relationship between the level of heart rate and its characteristics and the state of the fetus.

The healthy heart of the fetus has the ability to slow down the heart rate in the condition of temporary hypoxia (insufficiency of oxygen) during labour. This effect, sometimes informally referred to as the 'whale effect', leads to so-called decelerations in the rate of heartbeat. Whales, by slowing down their heart rate, temporarily slow down their metabolism during stays underwater. Similarly the fetus adapts to the temporary deficiency in oxygen by slowing down the heart rate without damage to the neural system. The onset of deceleration does not, therefore, provide sufficient information about the condition of the fetus. Repeated decelerations over a prolonged period of time may indicate hypoxia as well as a diminished dynamic range of the variability of the heart rate. The dynamics of recovery from the decelerations is another important criterium by which fetal well-being may be estimated. At the other end of the spectrum are the sudden shots of adrenaline, 'rallies' in financial terms, when the heartbeat suddenly rises for a short period of time. This is generally a good sign and is not associated with tachycardia. It proves the dynamic capability of the fetal heart and the ability to cope with the extreme stress which the fetus undergoes.

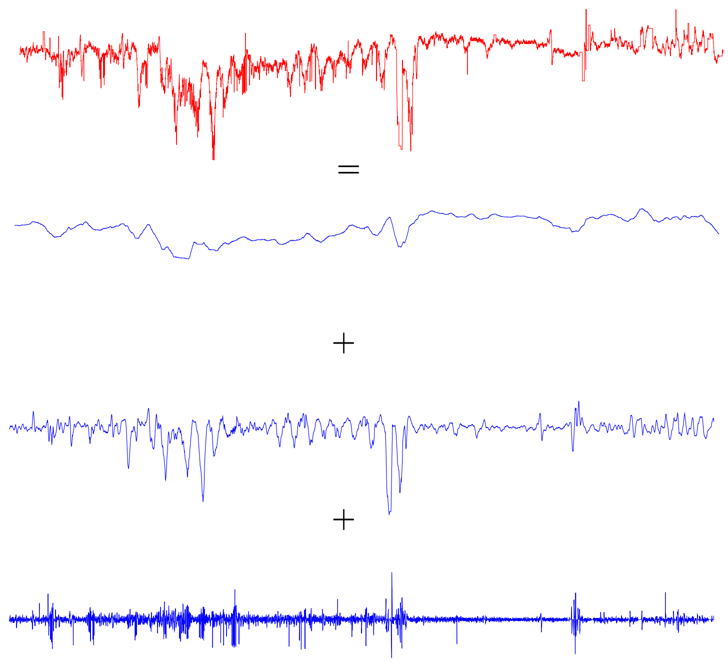

Figure 4. Fetal heart rate record decomposed into orthogonal meaningful components; the low frequency backbone: the *baseline* or trend, the 'crashes' and 'rallies' component: the *decelerations* and *accelerations*, and the high frequency 'volatility' component: the heart rate *variability*.

Thus, both decelerations and accelerations are features normally found in heartbeat records during labour resulting in the birth of a healthy child (positive fetal outcome). The decelerations are an indication of possible hypoxia but do not necessarily indicate that there is a need for intervention. The fetus may well cope with the temporary deficiency of oxygen without any damage to its neural system. The actual decision that the obstetrician takes is based on a number of observations, among them the rate of recovery from decelerations. Other observations include the baseline level of heart rate, the variability level, the recent history of heart rate, etc.

A simple transformation of the record of the fetal heart rate during labour shows a striking resemblance to financial records - the linearly integrated record of the heartbeat plunges upon encountering a series of decelerations. However, integration of the heart rate brings it into the range of a strongly correlated persistent record. In order to make a comparison with financial records, one needs to have a record centred at about $H = 0.5$, corresponding with non-correlated random walk. This can be achieved by fractionally integrating the time series. Fractional integration/differentiation is a linear operation which, just like integration or differentiation, preserves the nonlinear component - the scale invariant, multiplicative cascade-like properties of the signal, together with the associated multi-scaling and the multifractal spectrum. The result is depicted in the figure 5, together with the associated multifractal spectrum.

It has been previously recognised that post-crash behaviour may provide a relevant

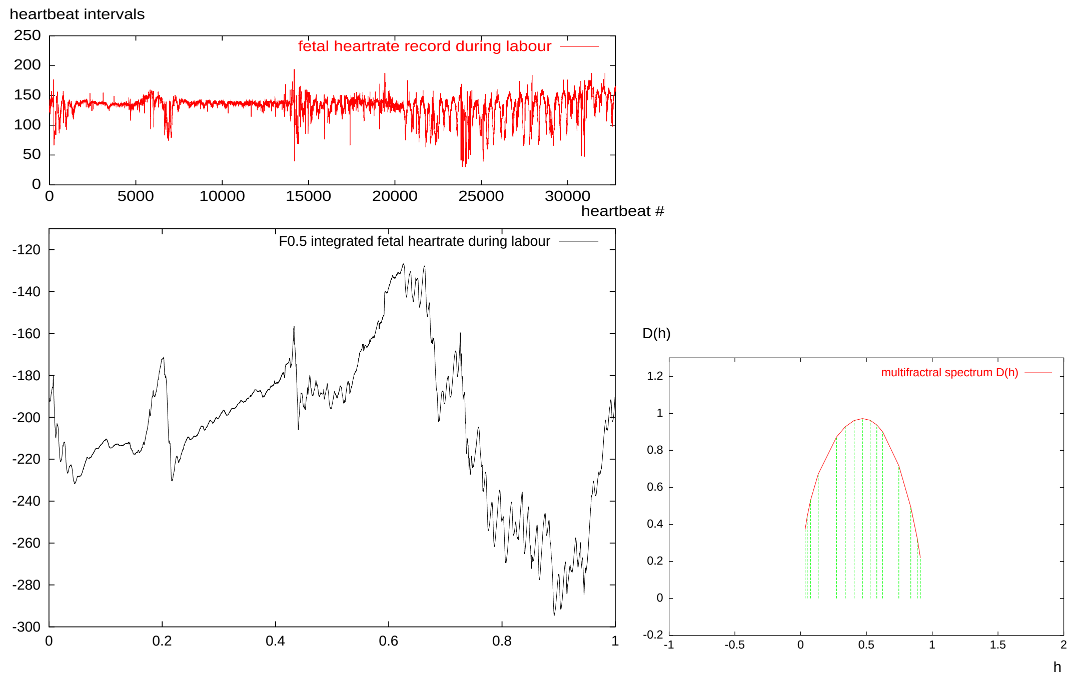

Figure 5. Top: the input time series: fetal heartbeat record. Left: fractionally integrated input heartbeat record. Right: the corresponding multifractal spectrum.

means of completing financial time series analysis. In a very recent shift of interest towards the analysis of intra- and post-crash behaviour, this counterpart of the traditional crash-precursory behaviour has been proven to constitute a meaningful 'diagnostic' tool.

Breaking with the universality picture - Reasoning from non-stationarity

As already mentioned above, the focus of perinatology is after-crash oriented: the obstetrician observes the heart rate online during the (series of) decelerations (usually using a relatively short window of $5-10$ minutes recordings; for comparison the record in figure 5 is about 6 hours long) and in case of encountering decelerations, evaluates the risk of continuing with the natural delivery or orders intervention. This may mean taking a fetal blood sample to determine the level of oxygen in the blood of the fetus, or immediate delivery through Caesarian section. The decision making is based on a complex evaluation which considers both the history of the heart rate and its current instantaneous values and the deviating, *non-stationary* values of its characteristics like baseline, accelerations and decelerations and the variability level. Additionally, external data about the history of the pregnancy and the history of the mother enters the reasoning and decision making process.

A true multifractal process suggested for both financial time series and heartbeat rate would share the same parameters (like MF spectrum) for any sub-part of the record. In particular, for an ideal multifractal system, any new data recorded would not affect the spectrum already estimated. However, in real life systems which are not isolated, the interaction with the environment is reflected in the systems' characteristics at any moment. The dynamical process, in constant interaction with the environment, will reflect the current mode of interaction with the environment in its characteristics. This holds for externally induced crashes - decelerations due to events like obstruction of the umbilical cord or financial crashes due to a single piece of very bad news (outbreak of war etc). But, it also holds for the systems' characteristics at any moment, and can be related to external information in an instantaneous fashion.

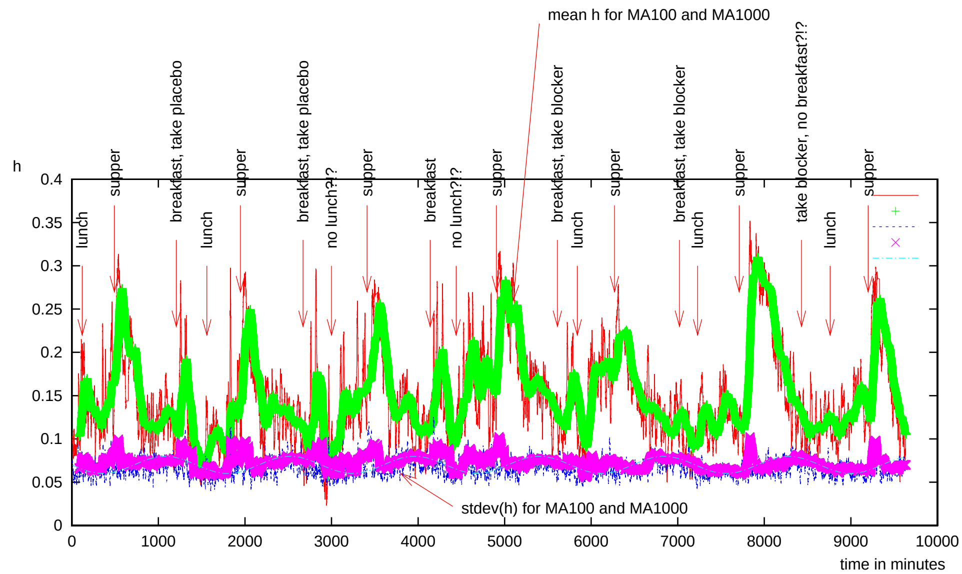

Figure 6. The variability plot from a long run of experiments on adults where the test persons were given placebo or beta-blocker. Two runs of MA filter were performed with 100 and 1000 maxima long window. An interesting pattern of response to food is evident.

In particular, short-time collective behaviour of the local scaling or roughness exponent h (Hölder exponent) contributing to the global multifractal (MF) spectrum can be studied. [4]. In the (7 days long) example record of the heartbeat of an adult, an interesting pattern of 'surprising' features can be identified. Upon verification, it confirms a pattern of response to activity, suggesting novel links to external information. Without going into much detail of the record given, there is a particularly strong response of the person in question to food. The observed shift towards higher values as the result of eating (it is almost possible to estimate the volume of the meal!) may indicate some nearly pathologic response in this individual case.

Similar analysis of financial records can be done. In fact, linking roughness exponents with external influences has been postulated previously [5]. The roughess estimated there was, however, not linked to the multifractality model of the financial time series. Indeed, universal descriptions of complex phenomena, like that using multifractal cascades, may prove to be inadequate for a complete description, even if valid on a restricted temporal range or in isolated or free-running conditions. The inadequacy of such models is often demonstrated by the non-stationarity of their characteristics. This aspect is usually neglected (through the selection of examples supporting the theory) or filtered out.

However, failure of the model to explain the phenomenon fully may well provide significant insight into the dependence of the system on external conditions. Structures emerging from the non-stationarity of the (most) sophisticated description available very likely indicate that more dimensional embedding may be needed for a proper description of the system under study. Such embedding and the right parameter choice for the more adequate model may not be easy to identify. In science, this search trajectory for better models has been routinely carried out at the cost of repeated experiments under varied input or environmental conditions (or modes of interaction with the environment).

Such a search for a better model may be shortened by automatically investigating the additional 'environmental' information often available, as has been done 'by hand' in the previous sections. Due to the modern technology of 'intelligent' data analysis and reasoning, in particular using Bayesian techniques, such 'automated research' may be possible. By incorporating the most advanced models available with explicitly identified instances where they fail to describe the model (as is e.g. manifested in non-stationarity), new theories can be generated and (dis-)proven by the system. The scientist, of course, remains at the centre of such an automated research trajectory - in fact the automated reasoning is merely meant to add to the expert's guided process, as a tool capturing, (dis-)proving and visualising the non-evident dependences in a large, high-dimensional Bayesian net. It is important to realise that in such an approach, one is looking for an unknown model, therefore not just model-fitting [6].

Can lessons be learned for economics?

Possibly, yes, but more likely both domains will profit from exploring the link between them. There is no analytic, exactly solvable model of the neuro-anatomic regulation of the heart rate in general and fetal heart rate in particular; neither is there a precise model of market behaviour. In both domains, expert knowledge and experience are used to process both the environmental information as well as the current status and the history of the observables - time series of heartbeat or index levels respectively. In this article, we have suggested that similar approaches can be used for both phenomena, not only because of the technical convenience but also due to the deeper reasons which we associate with the underlying dynamical regulation, as revealed in the extreme events of 'crashes'.

Crashes, both multi-scale and the largest, are a natural component of market behaviour. However, both may differ in the underlying mechanism. This observation is known in fetal heartbeat analysis during labour, and every obstretician evaluates the crash, or the sequence of decelerations, by assessing to the best of his knowledge the danger to the health of the baby at birth. This approach is probably not different from the assessment that traders use in evaluating the effects of a stock market crash.

However, comparing one single trader with the obstretician in charge is inadequate. More accurate seems to be the comparison with governmental ministries of economics or finance and central banks. Their responsibility lies in taking decisions ensuring the proper conduct of economic 'health'. While, similarly to the case of the obstretician, years of training and knowledge acquisition ensure high levels of expertise among decision takers, it seems that the knowledge of one person may be generalised and enlarged by combining it with the knowledge of several other experts. This is the primary concept behind the 'expert system' or 'decision support system', which would be capable of evaluating the risks of particular decisions.

It would be another objective - the health of the economy which should be evaluated using data from the stock market, just as the health of the baby - fetal well-being - is estimated from its heart rate. Looking into the dynamics of the crashes and the period of recovery from each crash may provide valuable information for determining both fetal and economic well-being in the cases under study. Some heavy crashes will be within the possibilities of the system to cope with, some will not, resulting in a degradation of the system. In case of the fetus, the feared of structural degradation is the cerebral palsy - various degrees of damage to the neural system. Economic parallels are difficult to draw, but heavy recessions, which may ultimately lead to new economies (or wars), may be suggested.

It is the task of decision makers to estimate the risk of such system degradation and order the appropriate intervention in case there is a need for this. Such a decision making process is far from obvious and is not directly related to a single known characteristic observable from the system. In the case of fetal heartbeat, such a decision is not only taken on the basis of the current instantaneous level of decelerations or variability. Neither in the case of the economy is it based on the instantaneous level of volatility etc.

Due to the complexity of the system, it may not be possible to determine exactly the consequences of a particular decision (for example changing the interest rates). This is where new possibilities open up in addition to pure expert-learned, rule-based decision systems. In the modern framework of probabilistic Bayesian net based decision support systems, there is a highly non-trivial possibility of learning from the data in addition to learning from the experts. It seems, therefore, that the complex decision making task in the economy may in fact profit from taking a similar path to that suggested in the domain of obstetrics: the development of a decision support system.

Bibliography

[1] A. Fisher, L. Calvet, B.B. Mandelbrot, Multifractality of the Deutschmark/US Dollar Exchange Rate, Cowles Foundation Discussion Paper, (1997).

[2] P.Ch. Ivanov, M.G. Rosenblum, L.A. Nunes Amaral, Z.R. Struzik, S. Havlin, A.L. Goldberger and H.E. Stanley, Multifractality in Human Heartbeat Dynamics, *Nature* **399**, 461-465, (1999).

[3] D. Sornette, Y. Malevergne, J.F. Muzy, Volatility Fingerprints of Large Shocks: Endogeneous Versus Exogeneous, arXiv:cond-mat/0204626, (2002).

[4] Z. R. Struzik. Revealing Local Variablity Properties of Human Heartbeat Intervals with the Local Effective Hölder Exponent. Fractals **9**, No 1, 77-93 (2001).

[5] N. Vandewalle, M. Ausloos, Coherent and Random Sequences in Financial Fluctuations, Physica A, **246**, 454-459, (1997).

[6] Z. R. Struzik, W. J. van Wijngaarden, R. Castelo. Reasoning from Non-stationarity. Physica A: Statistical Mechanics and its Applications, 314(1-4):245-254, 2002.