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Spatio-temporal patterns in a DC "barrier" discharge system: numerical solutions and stability analysis

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# Spatio-temporal patterns in a DC "barrier" discharge system: numerical solutions and stability analysis

ABSTRACT

Recent numerical PDE-solutions for spatio-temporal patterns in barrier discharges are presented that confirm and transcend our earlier stability analysis results.

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*Keywords and Phrases:* pattern formation, barrier discharge, stability analysis, numerical solutions

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# SPATIO-TEMPORAL PATTERNS IN A DC "BARRIER" DISCHARGE SYSTEM: NUMERICAL SOLUTIONS AND STABILITY ANALYSIS


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Dielectric barrier discharges are known to form a wealth of structures in space and time: oscillations, filaments, dynamic filaments that blink or move, stripes, spirals etc. [1]. These structures form spontaneously and are not imposed by specific conditions on the electrodes; they are an example of spontaneous pattern formation as studied in many other branches of the natural sciences [2]. The present work deals with a particularly simple version of a barrier discharge, namely a DC driven layered structure as shown in Fig. 1 where the barrier is replaced by a high-ohmic semiconductor layer to allow current flow under DC conditions. The function of the barrier in the AC system and of the semiconductor in the DC system is comparable: the cathode surface adjacent to the gas discharge is not equipotential and can carry a surface charge. The gas discharge is in the regime between Townsend and glow; hence it is also characterized by space charge effects, now in the interior of the gas discharge. A system of this type with wide lateral aspect ratio was investigated extensively in Münster [3,4]. On varying parameters, these experiments showed stationary and oscillating states that were homogeneous in the plane orthogonal to the layers as well as spatial patterns that again can be stationary or time dependent and form stripes, spots or spirals. Can these patterns and their parameter dependence be predicted and understood?

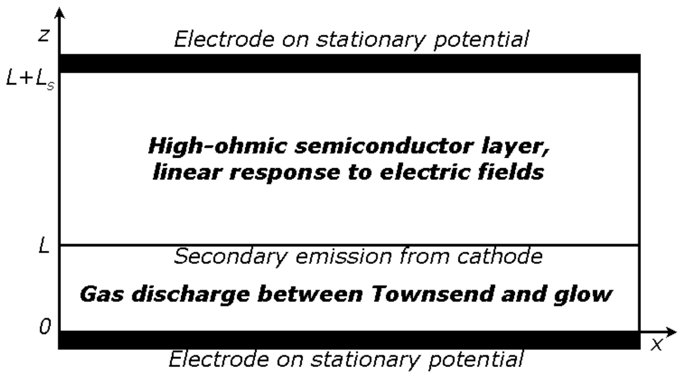


**Fig. 1**: The system of semiconductor and gas discharge

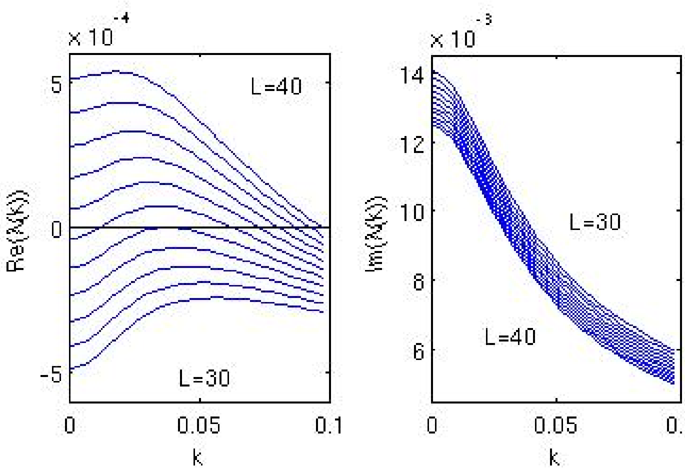


**Fig. 2**: Real and imaginary part of the dispersion relation as function of width of the gas layer *L*, where *L* varies from 30 to 40. $L_s = 54$, $R_s = 7000$, and $j_0 = 1.32 \times 10^{-5}$. The influence of conductivity and thickness of semiconductor layer etc. was investigated in a similar manner.

In previous work [5,6], we have investigated under which conditions the system stays stationary or exhibits temporal oscillations while no spatial structure developed in the transversal direction. In the present paper and a consecutive preprint [7], we investigate the onset of spatial and spatio-temporal patterns. This can be done in two ways: either the full temporal evolution is evaluated numerically or the generic linear instabilities of the system are investigated. The second approach has tremendous advantages as one basically has to calculate eigenvalues and easily can explore parameter space. On the other hand, linear stability analysis is restricted to the onset of the instability while full numerical solutions show the full development of the solution, also in the nonlinear regime. Here we present results of both approaches.

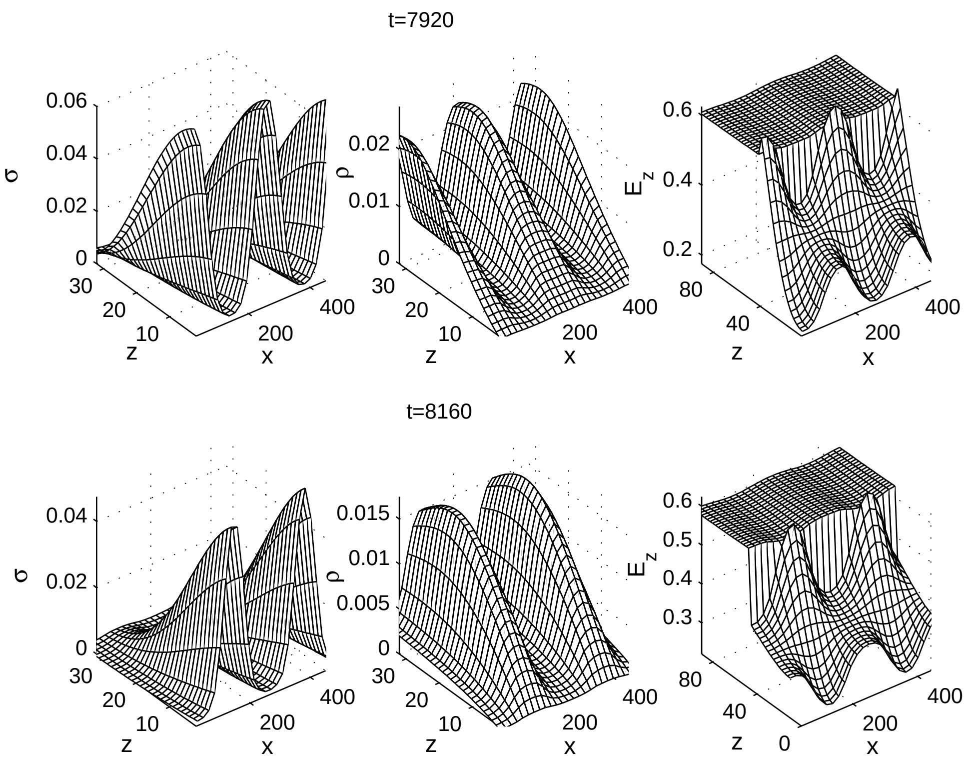


**Fig. 3**: Profiles of electron and ion densities $\sigma$ and $\rho$ and the electric field component $E_z$. Coordinates *x* and *z* are as in Fig. 1, for $\sigma$ and $\rho$ only the gas discharge part is shown, for $E_z$ the full system. $L$ = 36, $L_s = 54$, $R_s = 7000$, $U_t = 46$. Two temporal stages of the oscillation are shown: t = 7920 and 8160.

Fig. 2 shows the results of linear stability analysis for a particular set of parameters. k is the wavenumber of the perturbation, Re(λ(k)) is its growth rate. The maximal growth rate here is attained for a finite wavenumber. If the growth rate is positive which is the case if the dimensionless width L of the gas discharge layer is larger than 34, then the stationary homogeneous system is unstable and the k with the maximal growth rate will dominate the behavior. Moreover, the frequency Im(λ(k)) is finite. This means that the spatially modulated perturbation will not simply grow, but also oscillate in time.

The predicted behavior can be tested on the full numerical solutions of the initial value problem. First, we state that both results agree, which confirms the accuracy of both numerical methods. The linear stability analysis can be used to determine interesting parameter regimes, and the full numerical solutions can be used to explore the behavior beyond the validity of linearization. In Fig. 3, we show examples of the growing and oscillating behavior. These structures resemble the experimentally observed blinking filaments [3].

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